



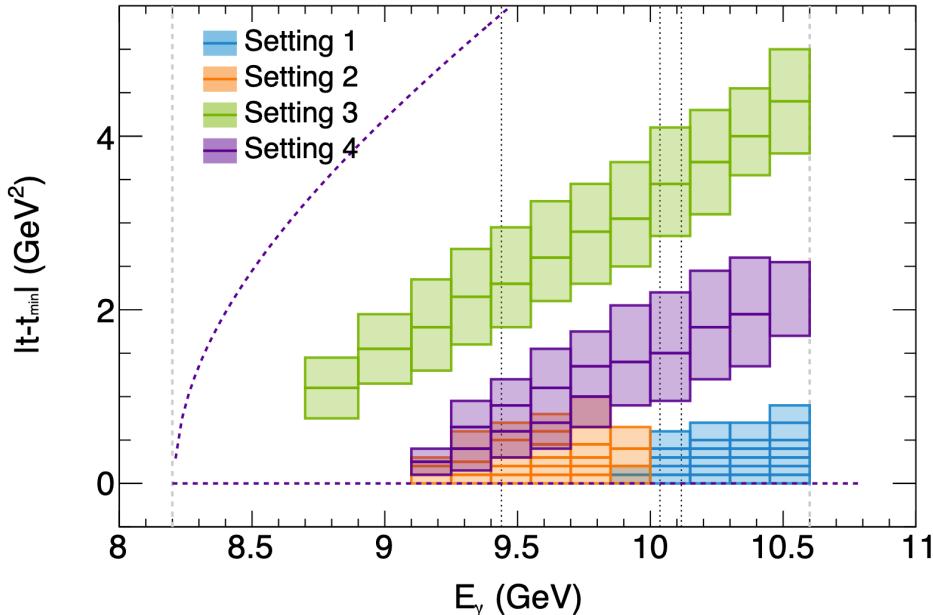
THE 007 EXPERIMENTS

HENRY KLEST
Argonne National Laboratory

On behalf of the
J/ψ-007 (E12-16-007)
& ϕ -007 (E12-25-007) Collaborations

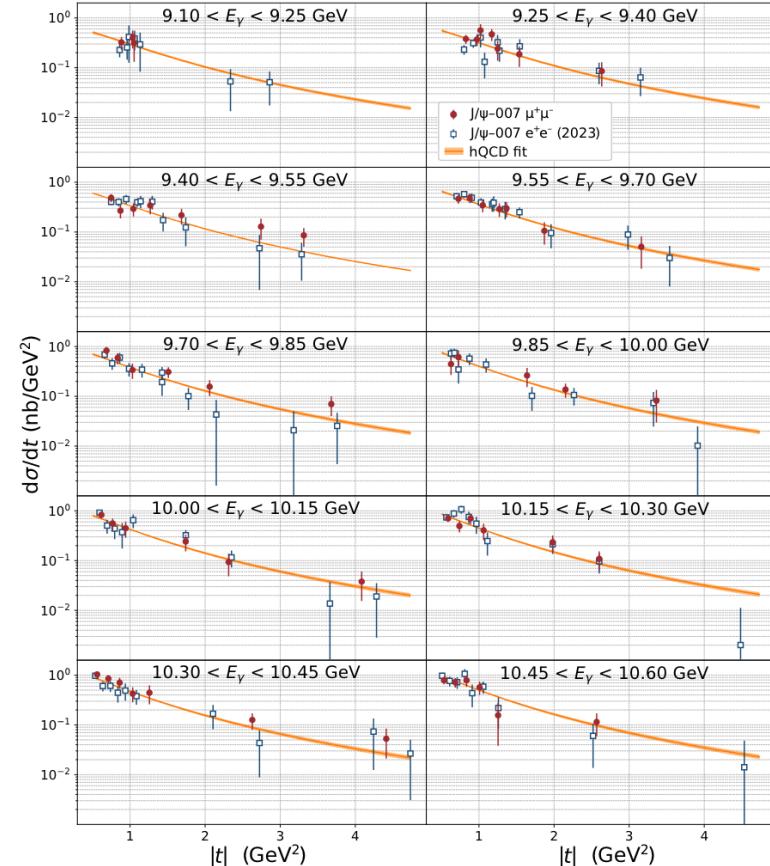
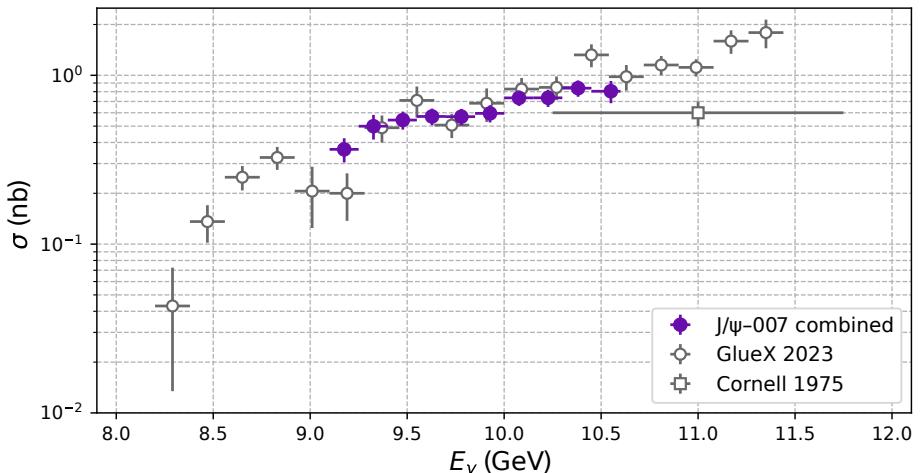
THE J/ψ -007 EXPERIMENT - STATUS

- Experiment to measure the J/ψ cross section near-threshold
 - Relevant for $uudcc\bar{c}$ pentaquark search and gluon gravitational form factors
- Ran for 8 PAC days in 2019
 - Copper radiator to enhance γ flux
 - Four spectrometer settings
 - One paper published so far in 2023
- One paper currently under review
 - $J/\psi \rightarrow \mu\mu$ cross section & GFF fits
- Ongoing analyses:
 - Pentaquark exclusion bounds - ANL
 - $H(e,e'p)X$ - NMSU



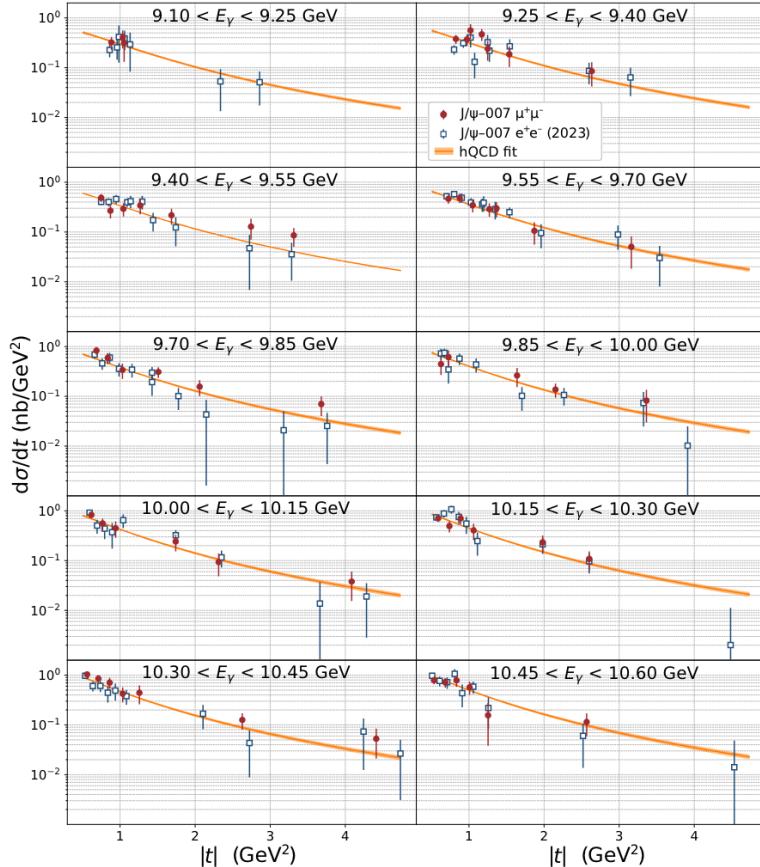
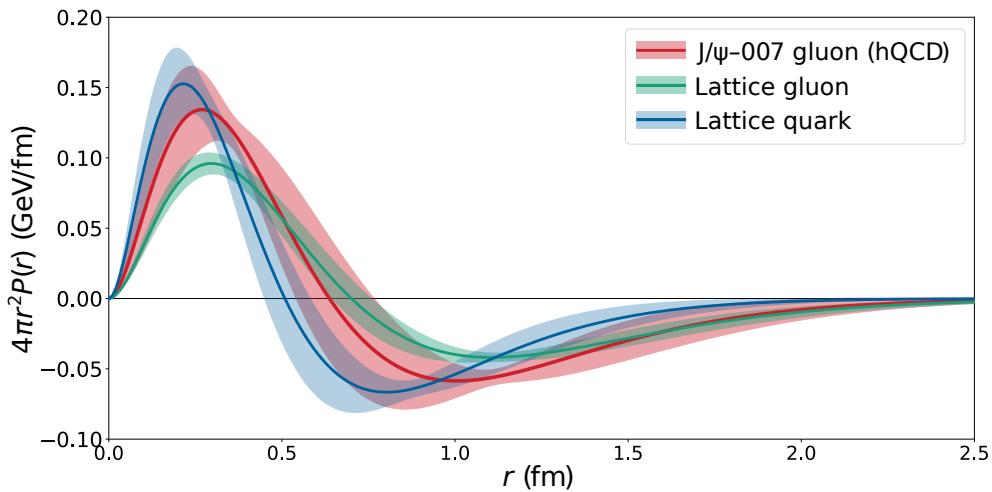
THE J/ψ -007 EXPERIMENT - $J/\psi \rightarrow \mu\mu$

- Submitted to PRL, currently in the review process
 - ~Doubles number of cross section points
 - Adds a plot of cross section vs. E_γ
 - Agrees with GlueX, but no obvious “dip” at low E_γ



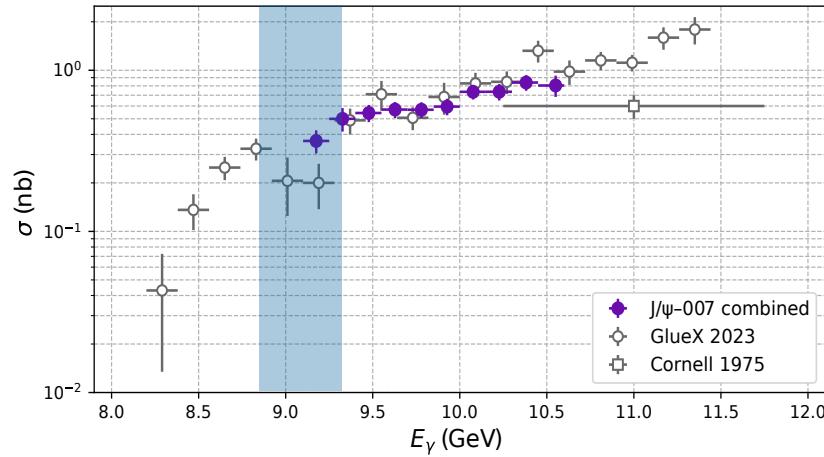
THE J/ψ -007 EXPERIMENT - $J/\psi \rightarrow \mu\mu$

- Submitted to PRL, currently in the review process
- Extraction of gluonic gravitational form factors agrees well with lattice results!



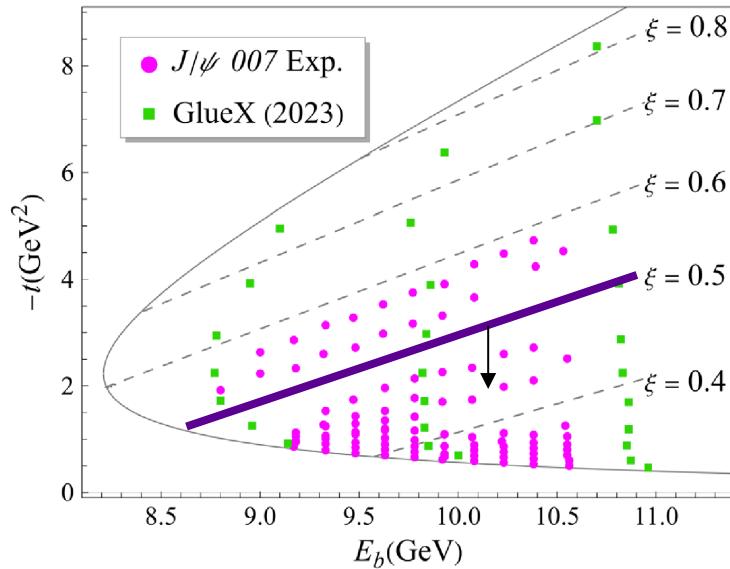
THE J/ψ -007-II EXPERIMENT – NEW PROPOSAL

- Given the excitement and questions generated by the first round of JLab J/ψ results, makes sense to run a new experiment to map this cross section in more detail
- Main Goals:
 - Study the lower E_γ region where GlueX sees bumps in the E_γ spectrum**
 - Go to larger ξ where the GFFs can be extracted more rigorously from GPD models
 - See if J/ψ exhibits a noticeable u-channel enhancement



THE J/ψ -007-II EXPERIMENT – NEW PROPOSAL

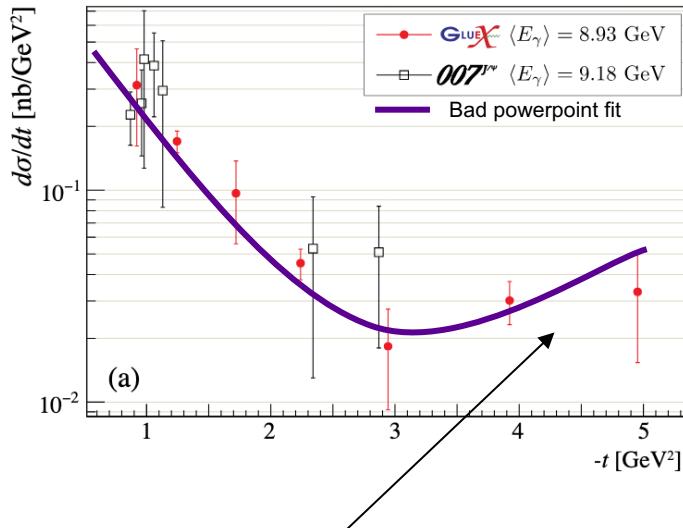
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Currently all data below $\xi = 0.5$ are **discarded!**
Only one J/ψ -007 setting is used in the fit!

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u-channel enhancement in the GlueX data? 🤔

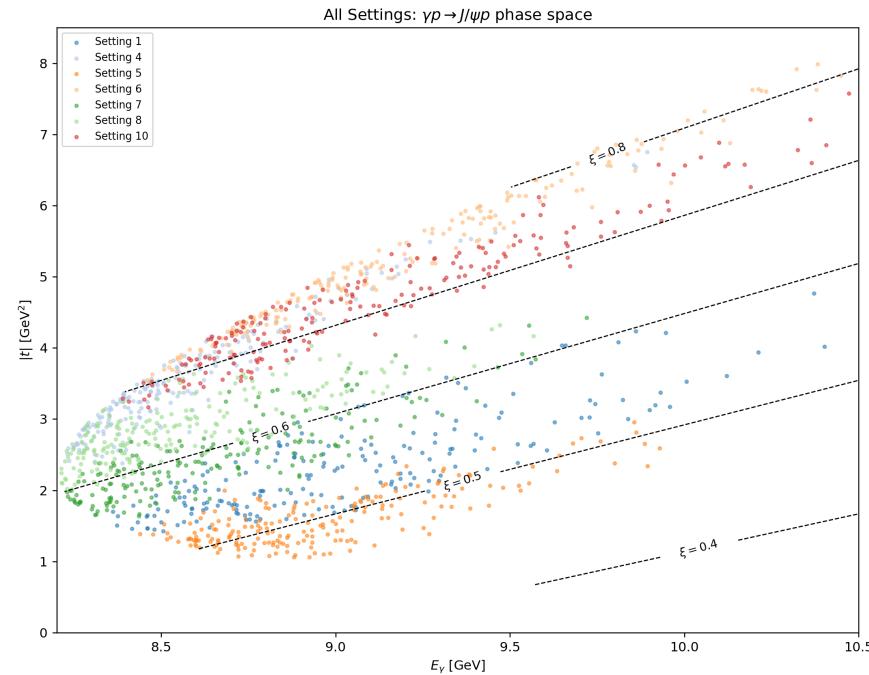
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- **Proposal effort led by ANL postdoc Fernando Flor**



J/Ψ-007-II PRELIM SETTINGS

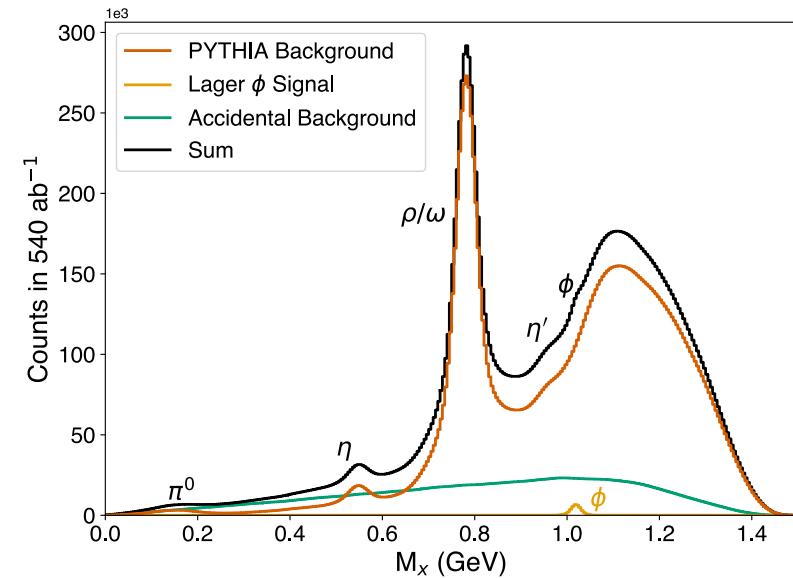
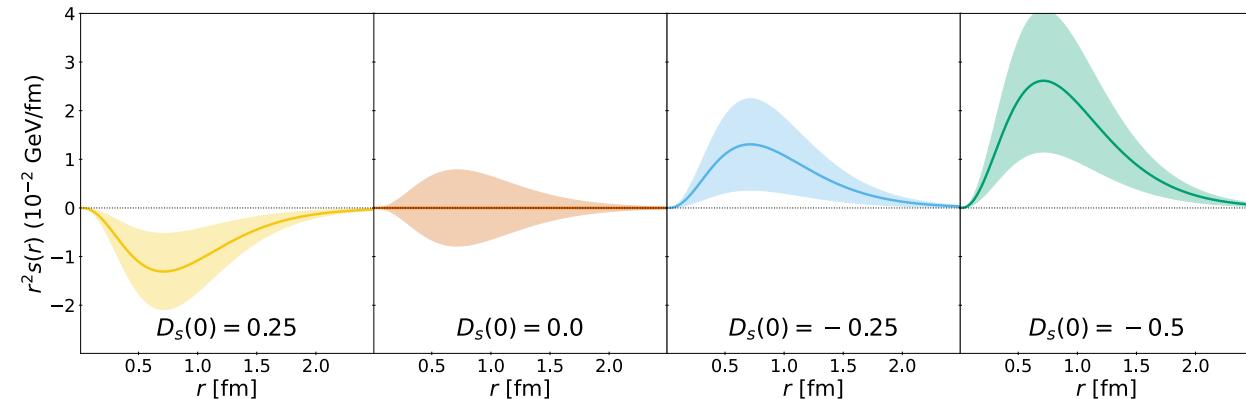
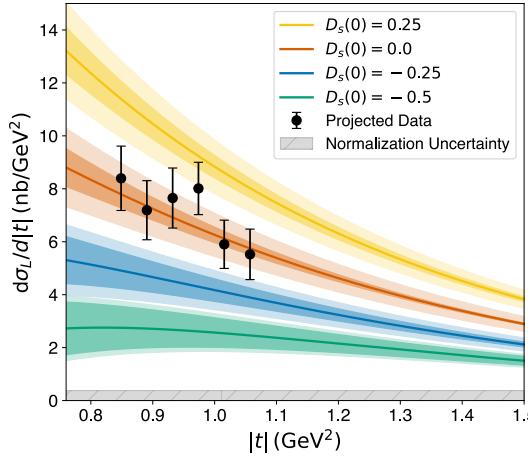
Hall C Configuration	SHMS (-)		HMS (+)		Kinematic Ranges
	$ p $ (GeV)	θ (deg.)	$ p $ (GeV)	θ (deg.)	
J/ψ -007-II Setting 1	1.96	37.5	5.83	17.1	$E_\gamma \in [8.4, 10.5] \text{ GeV}$ $ t \in [1.4, 5.0] \text{ GeV}^2$ $0.5 \leq \xi \leq 0.6$
J/ψ -007-II Setting 2	3.79	28.6	3.60	21.0	$E_\gamma \in [8.3, 10.5] \text{ GeV}$ $ t \in [1.8, 5.7] \text{ GeV}^2$ $\xi = 0.6$
J/ψ -007-II Setting 3	4.74	16.2	2.97	33.0	$E_\gamma \in [8.4, 9.3] \text{ GeV}$ $ t \in [1.0, 3.4] \text{ GeV}^2$ $0.45 \leq \xi \leq 0.6$
J/ψ -007-II Setting 4	4.11	20.2	2.51	37.8	$E_\gamma \in [8.2, 9.1] \text{ GeV}$ $ t \in [2.4, 5.0] \text{ GeV}^2$ $0.6 \leq \xi \leq 0.8$
J/ψ -007-II Setting 5	4.44	22.4	3.66	23.0	$E_\gamma \in [8.5, 10.2] \text{ GeV}$ $ t \in [1.0, 3.1] \text{ GeV}^2$ $\xi = 0.5$
J/ψ -007-II Setting 6	3.07	30.1	3.27	28.5	$E_\gamma \in [8.4, 10.0] \text{ GeV}$ $ t \in [3.3, 7.3] \text{ GeV}^2$ $\xi \geq 0.7$
J/ψ -007-II Setting 7	2.55	32.0	4.80	21.0	$E_\gamma \in [8.2, 9.8] \text{ GeV}$ $ t \in [1.6, 4.6] \text{ GeV}^2$ $\xi = 0.6$
J/ψ -007-II Setting 8	4.25	22.9	2.76	31.4	$E_\gamma \in [8.2, 9.2] \text{ GeV}$ $ t \in [2.1, 4.0] \text{ GeV}^2$ $0.6 \leq \xi \leq 0.7$
J/ψ -007-II Setting 9	3.33	31.3	3.27	25.6	$E_\gamma \in [8.3, 10.5] \text{ GeV}$ $ t \in [3.0, 7.2] \text{ GeV}^2$ $\xi = 0.7$
J/ψ -007-II Setting 10	4.20	28.8	3.00	25.7	$E_\gamma \in [8.7, 10.5] \text{ GeV}$ $ t \in [2.0, 7.0] \text{ GeV}^2$ $\xi = 0.7$
J/ψ -007-II Setting 11	3.50	28.8	3.00	25.7	$E_\gamma \in [8.3, 10.4] \text{ GeV}$ $ t \in [1.7, 7.5] \text{ GeV}^2$ $\xi = 0.7$



- **Hits $E_\gamma \approx 9 \text{ GeV}$ (GlueX bumps)** ✓
- **Hits $\xi > 0.5$** ✓
- **Hits $|t| \approx |t_{Max}|$ (u-channel)** ✓

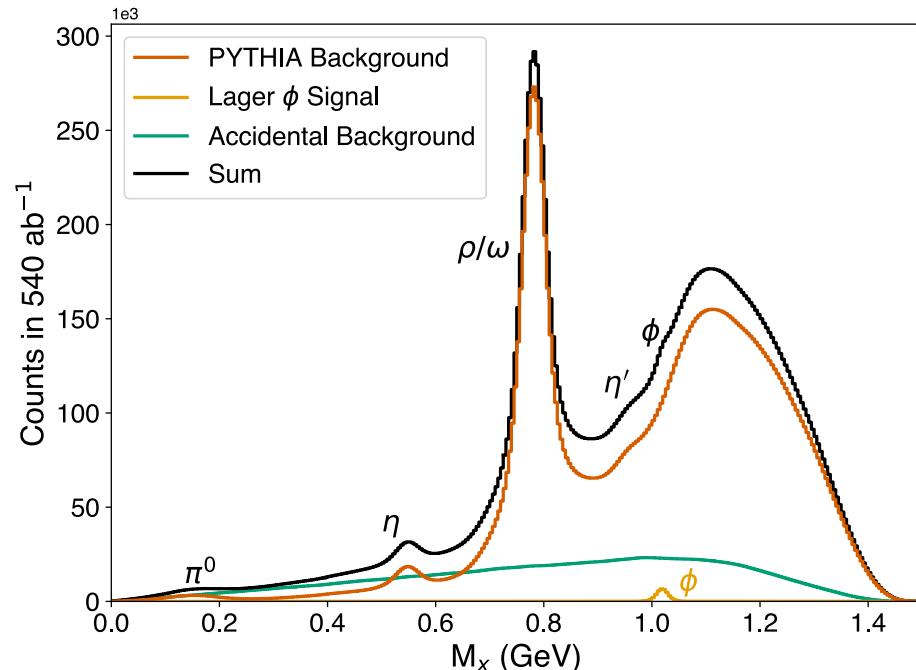
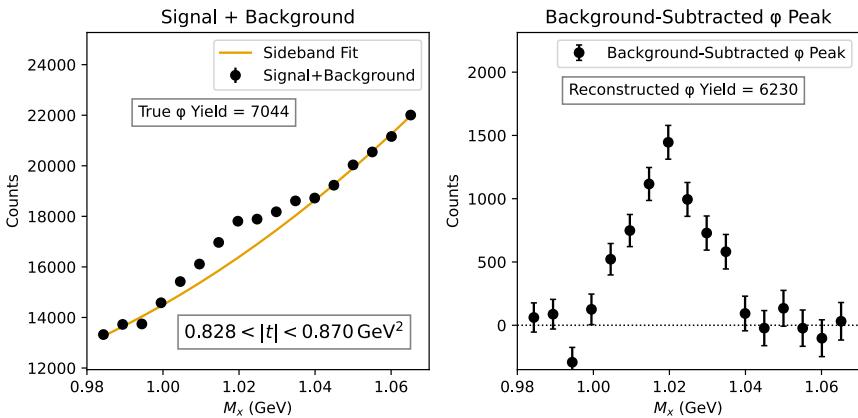
THE ϕ -007 EXPERIMENT

- Experiment to extract the strangeness gravitational form factors of the nucleon via exclusive ϕ production
 - Fit GPD model to ϕ cross section to extract D_s , contribution of strange quarks to the proton GFFs!
 - Measure ϕ in the $H(e, e' p)$ missing mass
 - One setting!**



EXPERIMENTAL MEASUREMENT

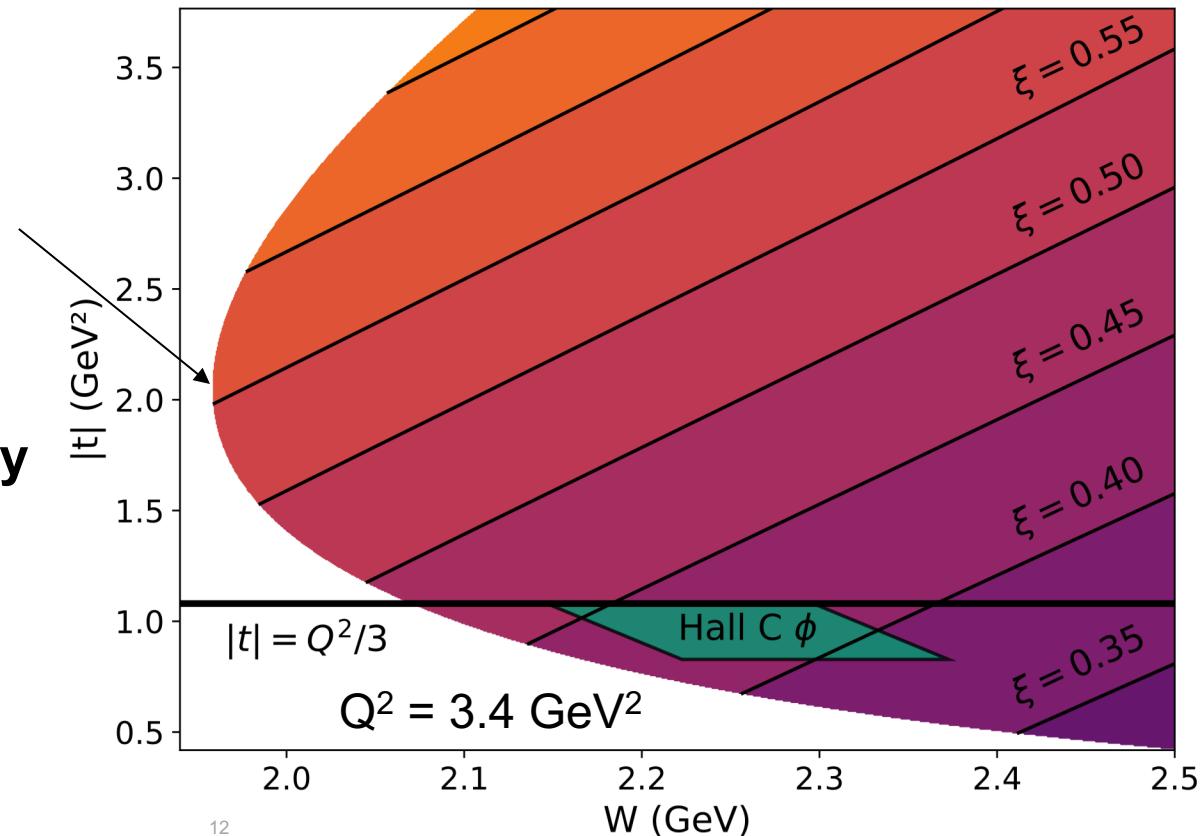
- Measure $e + p \rightarrow e' + p' + \phi$ via **missing mass technique**
- **Large and irreducible** continuum background from $e + p \rightarrow e' + p' + X$!
 - However, missing mass resolution of the Hall C spectrometers is good enough to **fit + subtract background with the data itself**



- Challenging kinematic constraints to access D_s experimentally
 - $\xi \geq 0.4$
 - $|t| \ll Q^2$
 - **Not too close** to the threshold $W = 1.96 \text{ GeV}$
- Cross section $\propto Q^{-9}!$
- Requires **high luminosity** and precise kinematic reconstruction

Use the Hall C spectrometers!

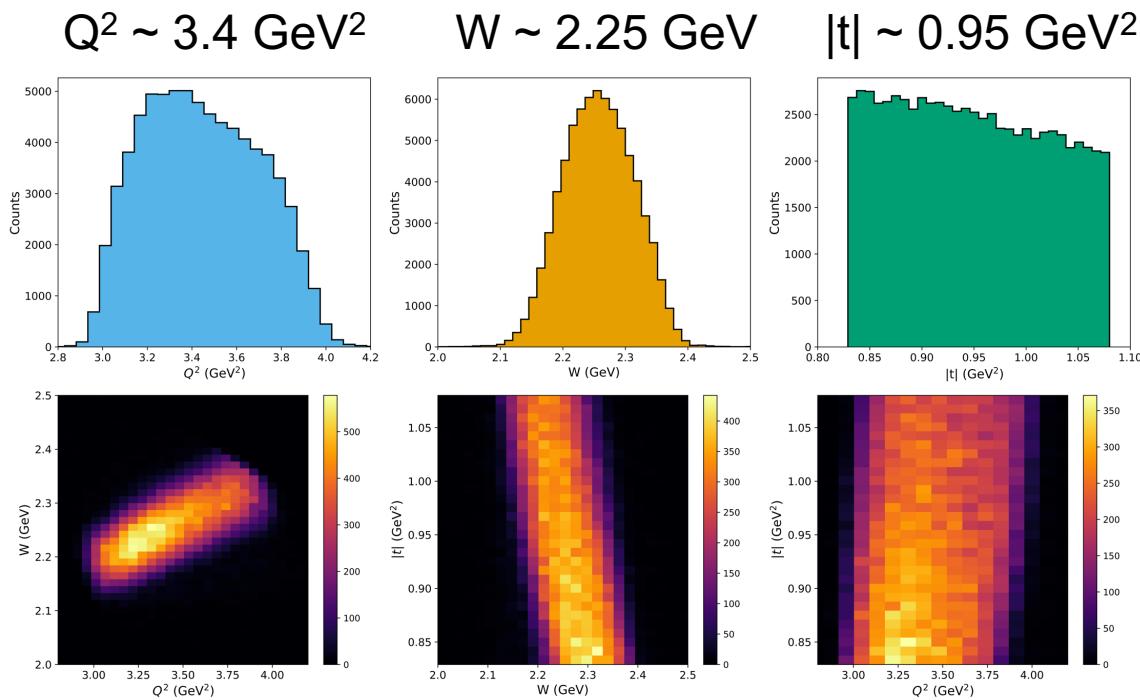
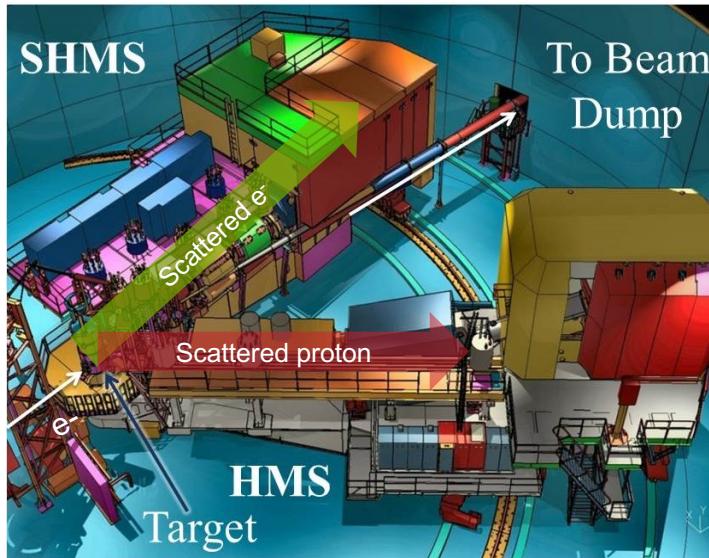
KINEMATICS



KINEMATICS

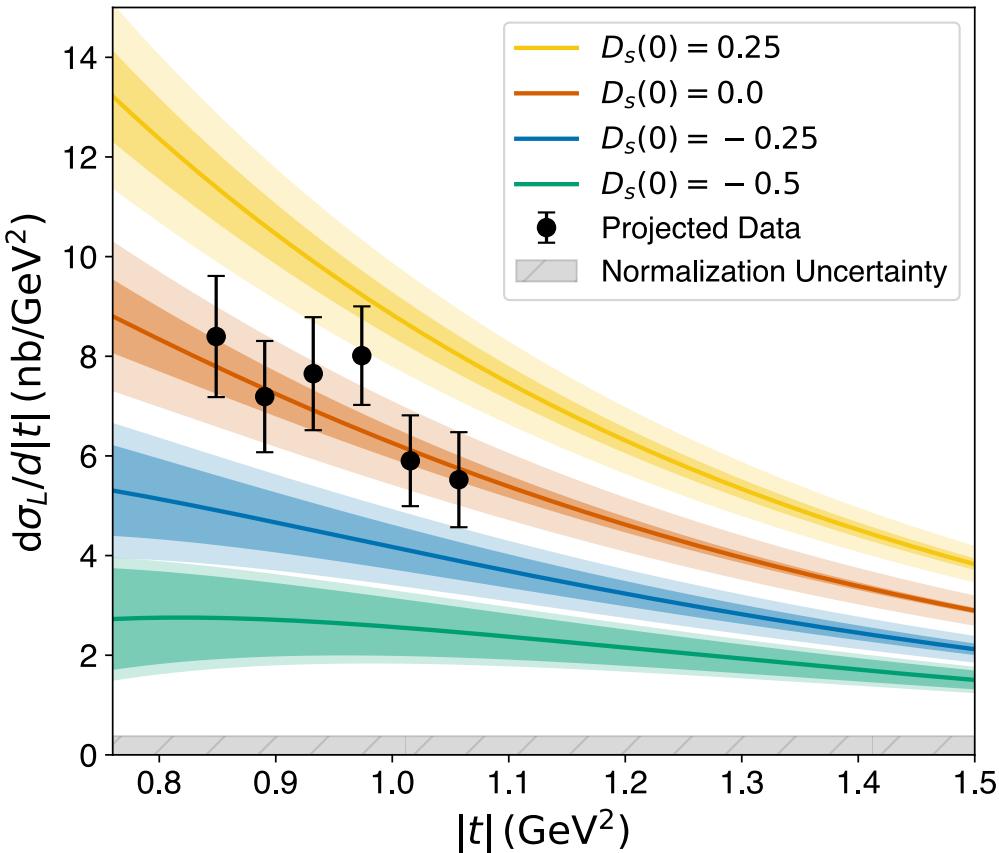
- 54 μA 10.6 GeV beam, 15 cm LH_2 target
- Measure proton in HMS, electron in SHMS
 - SHMS: $\theta_{e'} = 13^\circ$, $p_{e'} = 6.7 \text{ GeV}$
 - HMS: $\theta_{p'} = 32^\circ$, $p_{p'} = 1.1 \text{ GeV}$

Standard Hall C equipment
Ready to run!



CROSS SECTION PROJECTIONS

Linear scale



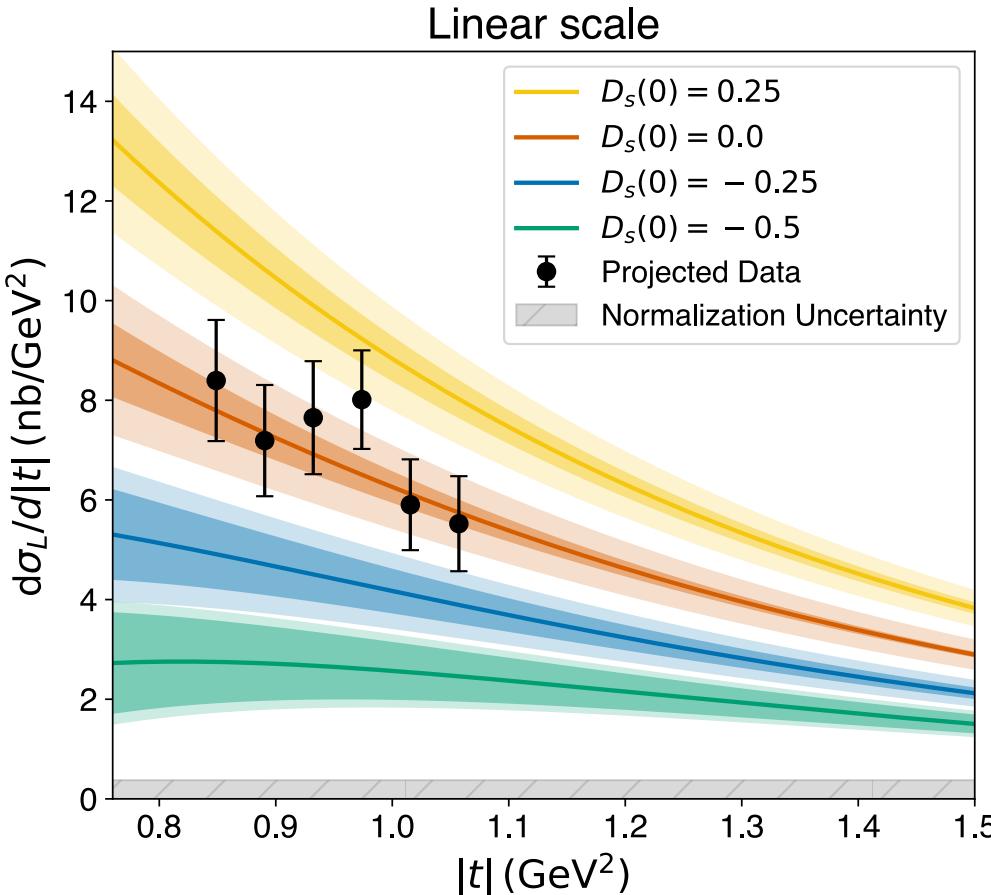
- Experimental uncertainty from these sources:

Source	Bin 1	Bin 2	Bin 3	Bin 4	Bin 5	Bin 6
Signal Extraction	14.0%	13.6%	14.9%	13.6%	13.3%	15.1%
Radiative Correction	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%
Background Modeling	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%
Tracking Efficiency	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%
Rescattering Correction	2.0%	2.0%	2.0%	2.0%	2.0%	2.0%
Other Systematics	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%
Total Point-to-point	15.6%	15.2%	16.4%	15.2%	14.9%	16.6%
Acceptance Correction	3.0%	3.0%	3.0%	3.0%	3.0%	3.0%
Value of R^{11}	3.8%	3.8%	3.8%	3.8%	3.8%	3.8%
Total Normalization	4.8%	4.8%	4.8%	4.8%	4.8%	4.8%

Dominated by signal extraction!
→ Needs high statistics, 540 ab⁻¹

- Theoretical uncertainty and experimental uncertainty are of similar size

HOW WELL CAN WE EXTRACT D_s ?



- Jitter datapoints and fit to theory predictions at different values of D_s
- Anticipate resolutions of 0.1 to 0.2 on $D_s(0)$

Precise enough to validate or invalidate the claim that $D_s = D_{u,d}$!

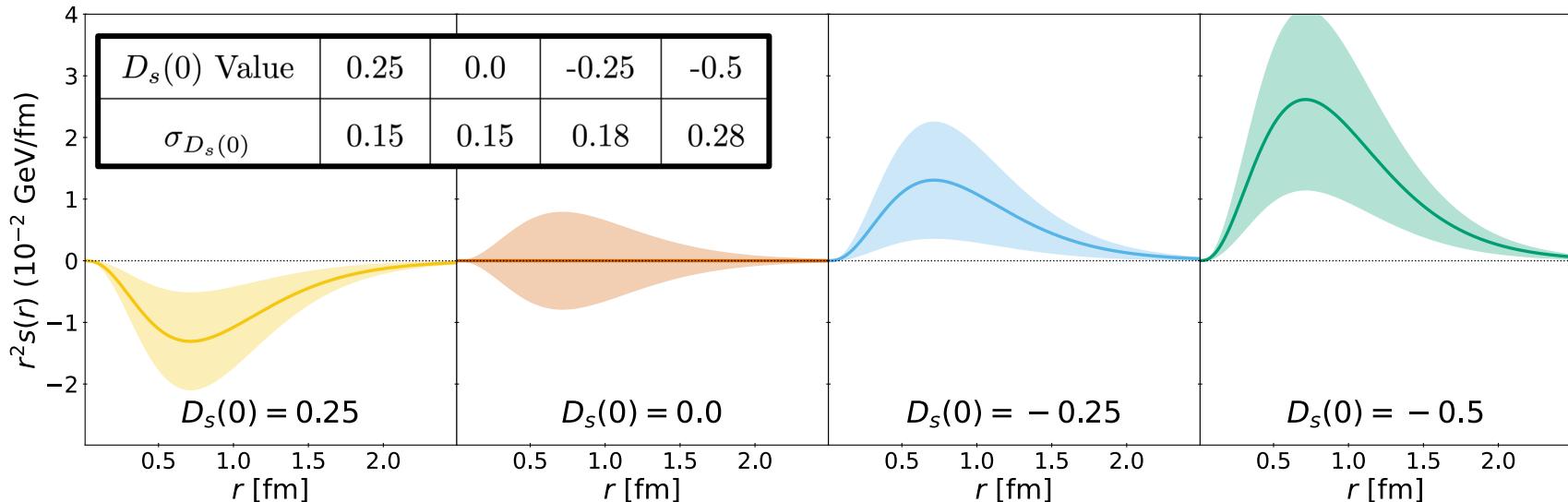
$D_s(0)$ Value	0.25	0.0	-0.25	-0.5
$\sigma_{D_s(0)}$	0.15	0.15	0.18	0.28

Extracted resolutions on $D_s(0)$ for various values of $D_s(0)$.

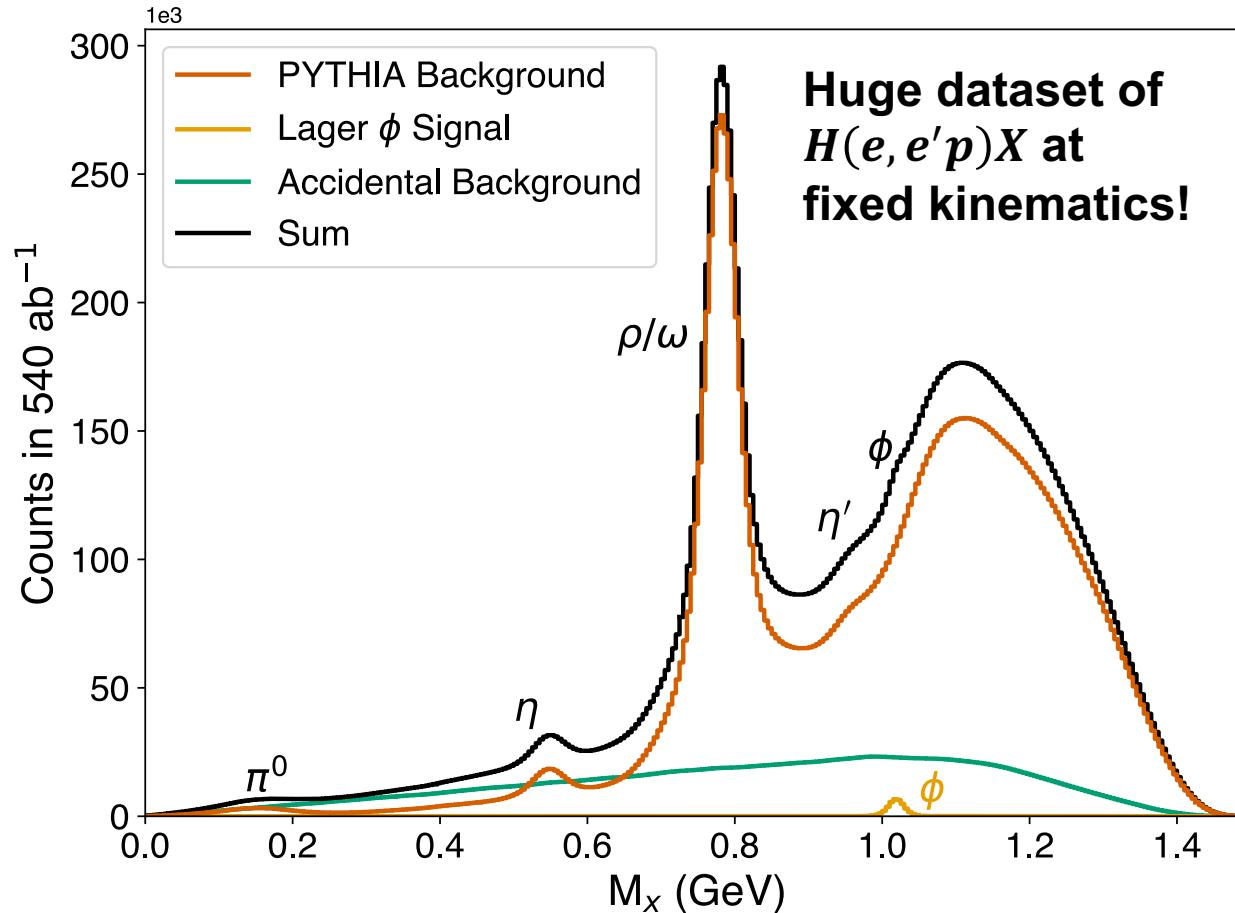
WHAT CAN WE LEARN?

Using these resolutions on $D_s(0)$ and the functional form measured on the lattice, can project uncertainties for the **strangeness shear force distribution**

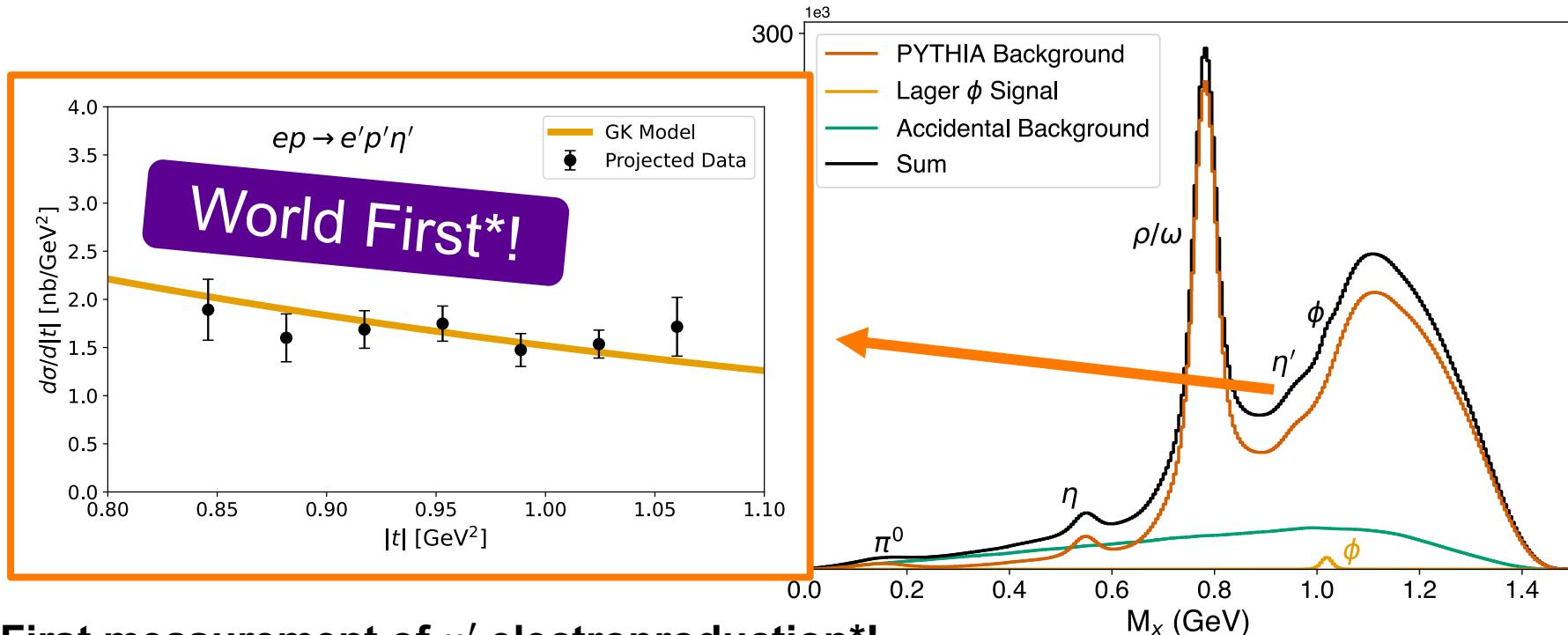
First ever extraction! Terra incognita...



WHAT ELSE CAN WE LEARN FROM THIS DATA?



WHAT ELSE CAN WE LEARN FROM THIS DATA?

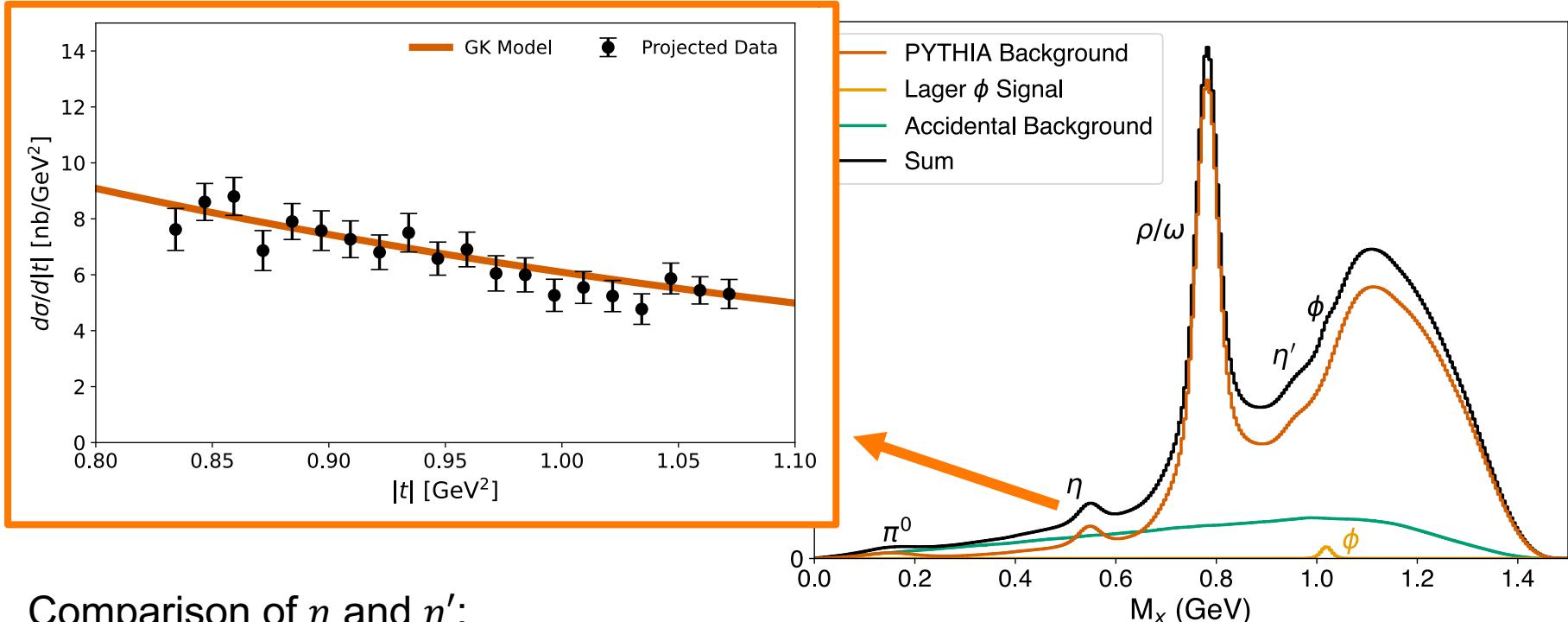


First measurement of η' electroproduction*!

Unexpectedly large η' mass is generated by the **QCD chiral anomaly**,
What can electroproduction teach us?

*Except for the recent Hall A paper at $Q^2 = 0.46$ GeV 2

WHAT ELSE CAN WE LEARN FROM THIS DATA?



Comparison of η and η' :

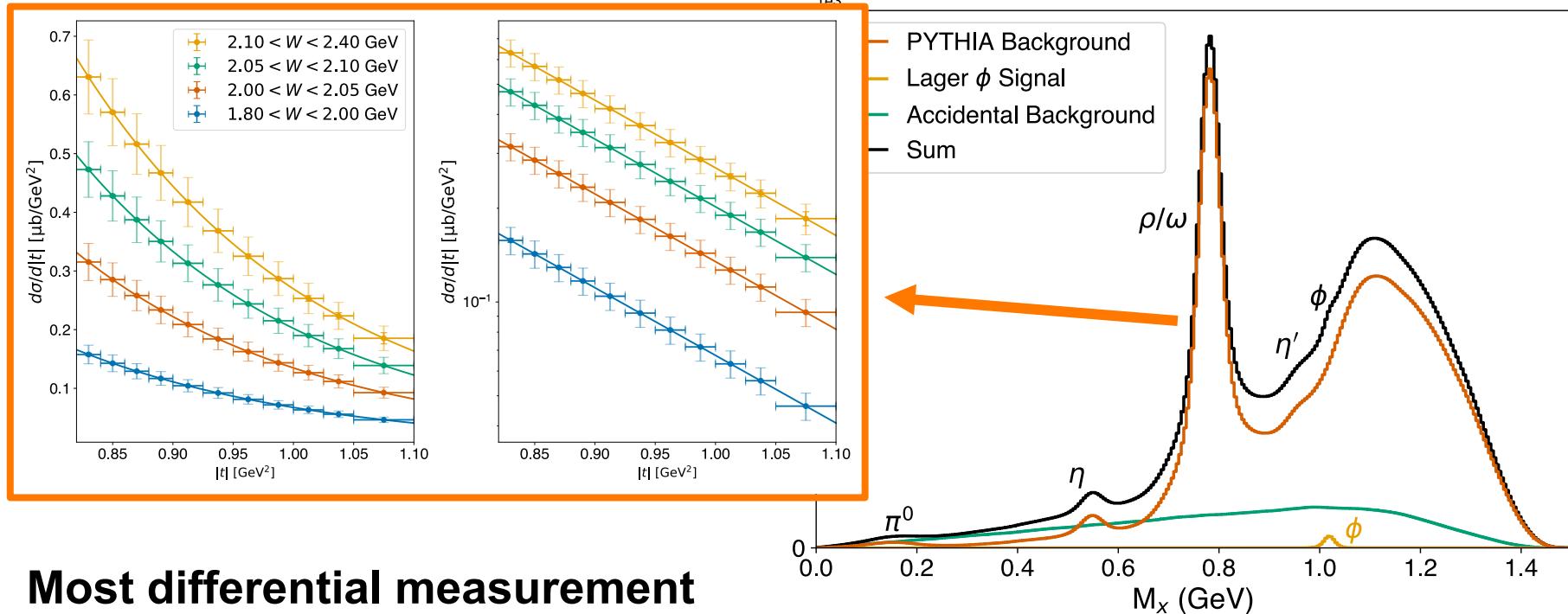
What is the role of the chiral anomaly in electroproduction^[1]?

$\eta : \eta' = 1 : 2 \rightarrow$ Naïve cross section ratios neglecting the anomaly

$\eta : \eta' = 1 : 0.87 \rightarrow$ With the anomaly included

[1] Eides, Frankfurt, Strikman - Hard Exclusive Electroproduction of Pseudoscalar Mesons and QCD axial anomaly

WHAT ELSE CAN WE LEARN FROM THIS DATA?

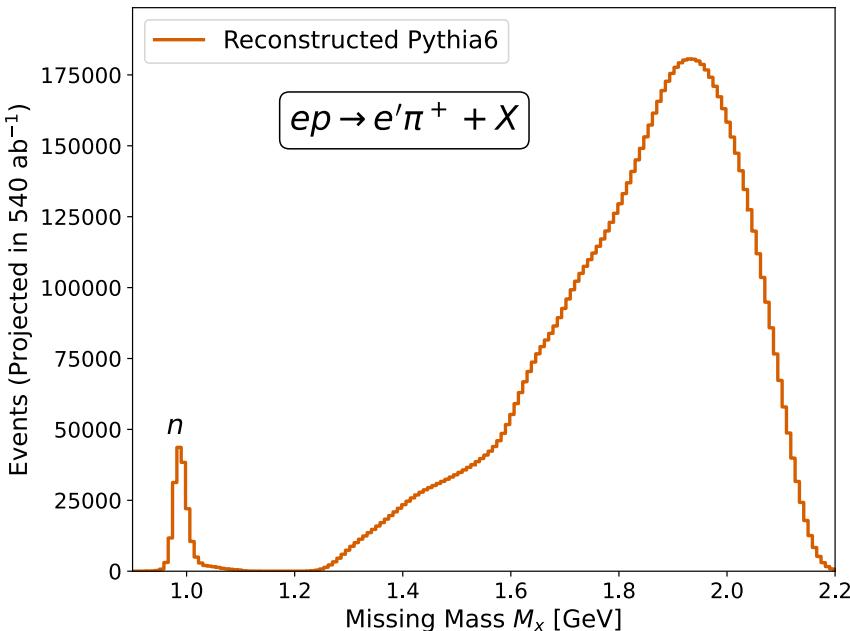


Most differential measurement
of near-threshold ω
electroproduction! + BSA!

Connection to the proton mass radius^[1]?

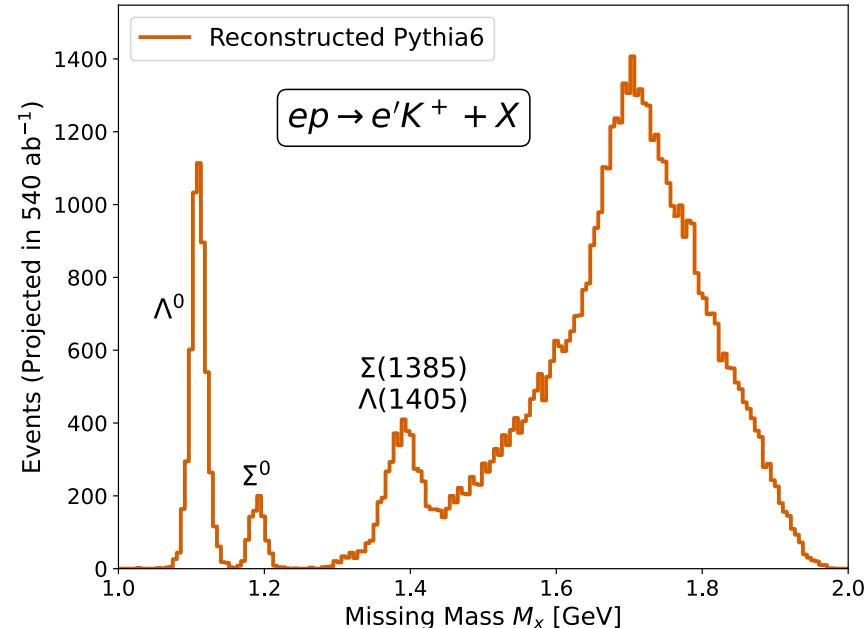
[1] - Wang et al. Extraction of the proton mass radius from
the vector meson photoproductions near thresholds

WHAT ELSE CAN WE LEARN FROM THIS DATA?



u -channel π^+n and K^+Y electroproduction

Baryon takes most of the γ^* momentum
Compare to pQCD TDA predictions

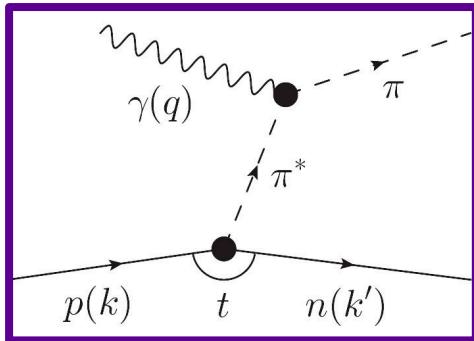


Connection to how baryon number is distributed in the nucleon^[1]?

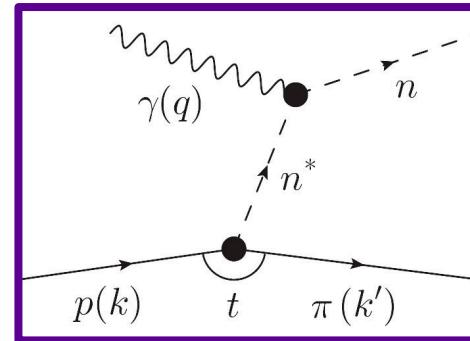
[1] Pire et al. - Toward an advanced phenomenology of πN transition distribution amplitudes

ASIDE – NEUTRON FORM FACTORS?

Can we measure the **neutron form factors** by knocking a **neutron** out of a proton in the **u-channel Sullivan process**?



Standard Sullivan Process,
sensitive to pion form factor



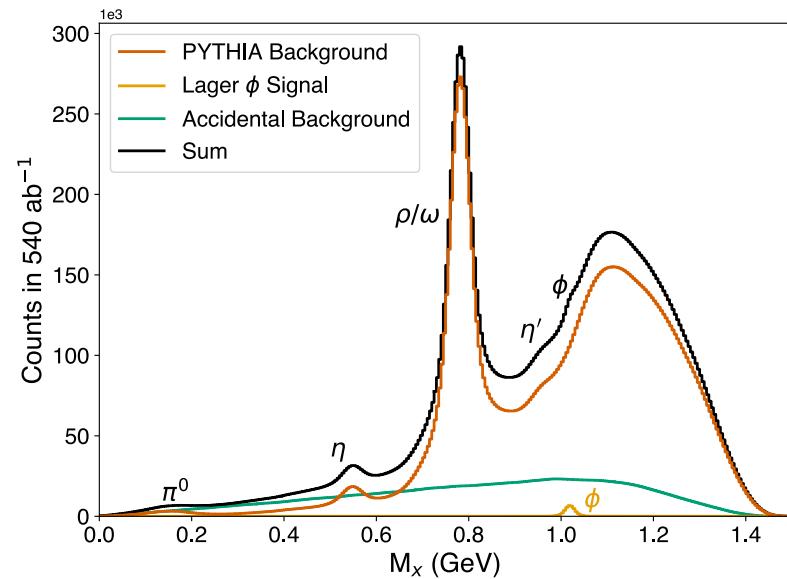
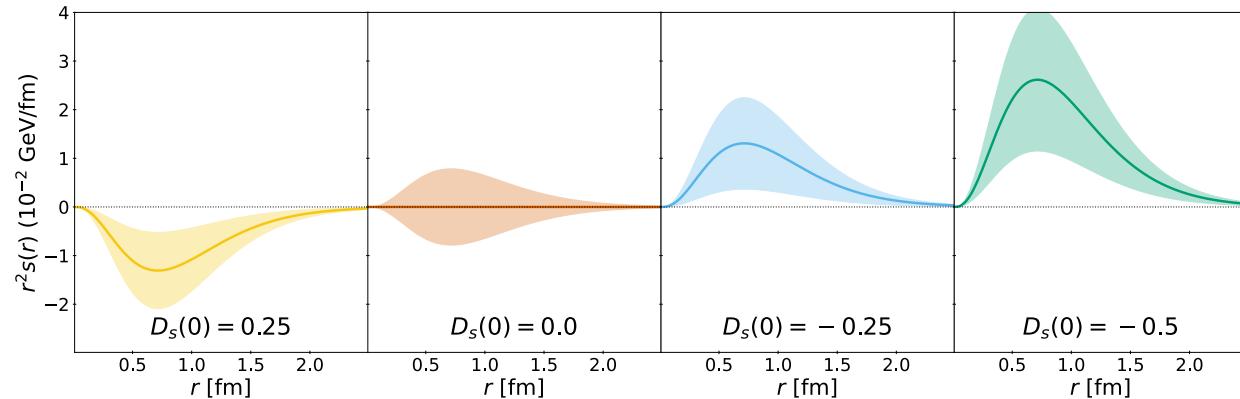
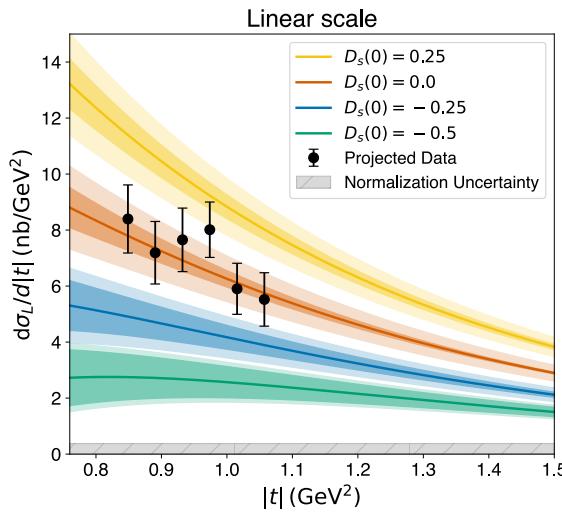
u-channel Sullivan Process,
sensitive to **neutron form factor**?

- Realistically, form factor extraction is complicated by extrapolation to pole
 - Worse at higher masses, need to extrapolate further, bad for neutrons!
- However, neutron form factors are known already!
 - Can compare what we get out of Sullivan to the known FFs!
- Can we use this to **test the procedure for extrapolation to the pole?**

Note: $\phi-007$
can't do this, no
L/T separation

THE ϕ -007 EXPERIMENT

Approved for 35 days by
PAC53! A- scientific rating



THE ϕ -007 EXPERIMENT – STATUS SINCE THE PAC

- Since the PAC, have been working towards **ERR**
 - Previously planned a vacuum coupling between scattering chamber and HMS, dropped from the plan to save floor time, simulations showed it was unnecessary anyway
 - Went from $75 \mu A$ on a 10 cm target to $54 \mu A$ on a 15 cm target
 - Easier on the accelerator and can reduce random coincidence background
 - Developed a more detailed run plan, including elastics, carbon foils, etc.
- Spokespersons submitted a document to leadership answering the standard ERR questions

ϕ – 007 **ERR Documentation**

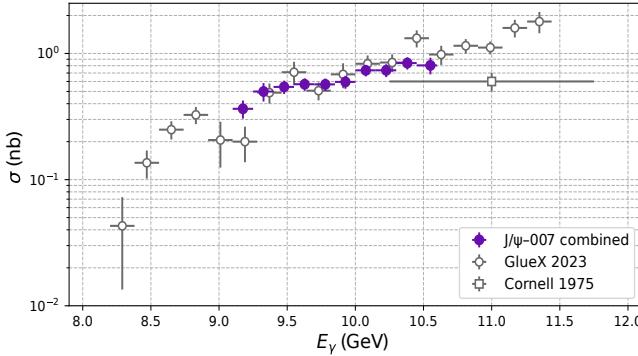
Henry Klest,^{1,*} Sylvester Joosten,¹ Holly Szumila-Vance,² and Wenliang (Bill) Li³

- ERR to be scheduled in February
- Goal is to be ready to run before MOLLER, should the scheduling situation be favorable!

CONCLUSION

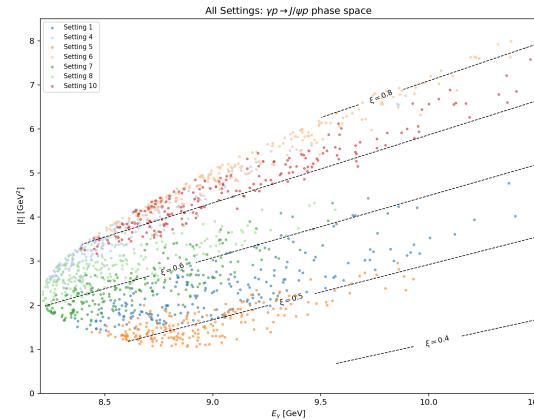
- Data analysis of J/ψ -007 is ongoing
 - New di-muon cross section PRL hopefully soon
 - Pentaquark study still underway

J/ψ -007

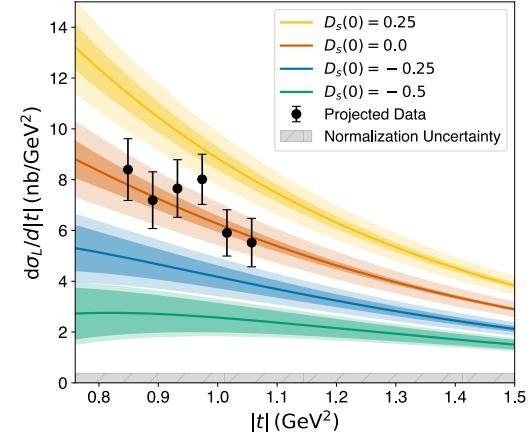


- J/ψ -007-II proposal being prepared!
 - Investigate features of the GlueX spectra, go to kinematics more amenable for GFF fits
- ϕ -007 Experiment to have an ERR soon
 - Want to be in position to run soon if we get tapped for it!
- Lots of fun physics on the way!

J/ψ -007-II



ϕ -007

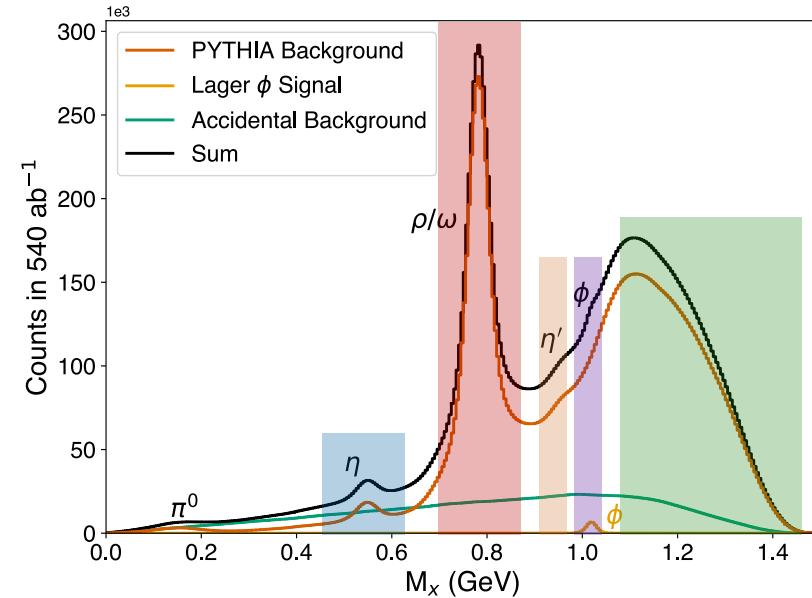


BACKUP

OTHER PHYSICS OPPORTUNITIES

Channel	Physics Goal
ϕ BSA	Compare to GPD models
η' cross section / BSA	GPDs and/or study of the chiral anomaly
ω cross section / BSA	Compare to GPD models, Mass radius
η cross section / BSA	Baseline for η' sans anomaly
u -channel $H(e, e'\pi)X$	Baryon junction / spectroscopy
u -channel $H(e, e'K)X$	Baryon junction / spectroscopy
$\pi/K/p$ SIDIS	High statistics cross check of SIDIS validity at low- M_X

TABLE II. Planned measurements in addition to the ϕ cross section and their physics goals.

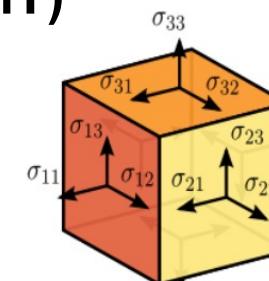


PROTON MECHANICAL STRUCTURE

Proton *mechanical* structure is defined by analogy to continuum mechanics via the **QCD energy-momentum tensor (EMT)**

$$T^{\mu\nu} = \begin{bmatrix} \text{Energy density} & \text{Momentum density} \\ \begin{matrix} T^{00} \\ T^{10} \\ T^{20} \\ T^{30} \end{matrix} & \begin{matrix} T^{01} & T^{02} & T^{03} \\ T^{11} & T^{12} & T^{13} \\ T^{21} & T^{22} & T^{23} \\ T^{31} & T^{32} & T^{33} \end{matrix} \end{bmatrix}$$

Energy flux Momentum flux



Shear stress

Normal stress (pressure)

GRAVITATIONAL FORM FACTORS

- Proton gravitational form factors (GFFs) encode information about the matrix elements of the **QCD energy-momentum tensor**

$$\langle p', s' | \hat{T}_{\mu\nu}^a(x) | p, s \rangle = \bar{u}' \left[A^a(t) \frac{\gamma_{\{\mu} P_{\nu\}}}{2} + B^a(t) \frac{i P_{\{\mu} \sigma_{\nu\}} \rho \Delta^\rho}{4m} + D^a(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4m} + m \bar{c}^a(t) g_{\mu\nu} \right] u e^{i(p' - p)x}$$

$P = \frac{p + p'}{2}$ $\Delta = p' - p = q - q'$
 $t = (p - p')^2 = \Delta^2$

EMT Matrix Elements

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & & & \\ T^{10} & T^{01} & T^{02} & T^{03} \\ T^{20} & T^{11} & T^{12} & T^{13} \\ T^{30} & T^{21} & T^{22} & T^{23} \\ & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Energy density Momentum density
Energy flux Momentum flux
Shear stress
Normal stress (pressure)

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$$P = \frac{p + p'}{2} \quad \Delta = p' - p = q - q'$$

$$t = (p - p')^2 = \Delta^2$$

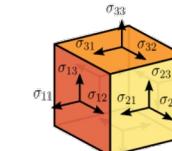
“Gravitational” Form factors

Fourier transforms of spatial distributions

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & & & \\ T^{10} & T^{01} & T^{02} & T^{03} \\ T^{20} & T^{11} & T^{12} & T^{13} \\ T^{30} & T^{21} & T^{22} & T^{23} \\ & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Diagram illustrating the components of the energy-momentum tensor $T^{\mu\nu}$ for a 3D volume element:

- Energy density**: T^{00}
- Momentum density**: T^{01}, T^{02}, T^{03}
- Energy flux**: T^{10}, T^{20}, T^{30}
- Momentum flux**: $T^{11}, T^{12}, T^{13}, T^{21}, T^{22}, T^{23}, T^{31}, T^{32}$
- Shear stress**: $T^{01}, T^{02}, T^{03}, T^{11}, T^{12}, T^{13}, T^{21}, T^{22}, T^{23}$
- Normal stress (pressure)**: T^{33}



GRAVITATIONAL FORM FACTORS

- Proton **gravitational form factors** (GFFs) encode information about the matrix elements of the **QCD energy-momentum tensor**

$$\langle p', s' | \hat{T}_{\mu\nu}^a(x) | p, s \rangle = \bar{u}' \left[A^a(t) \frac{\gamma_{\{\mu} P_{\nu\}}}{2} + B^a(t) \frac{i P_{\{\mu} \sigma_{\nu\}} \rho \Delta^\rho}{4m} + \boxed{D^a(t)} \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4m} + m \bar{c}^a(t) g_{\mu\nu} \right] u e^{i(p' - p)x}$$

$$P = \frac{p + p'}{2} \quad \Delta = p' - p = q - q'$$

$$t = (p - p')^2 = \Delta^2$$

D-term



D(0) represents a **fundamental property of the proton**

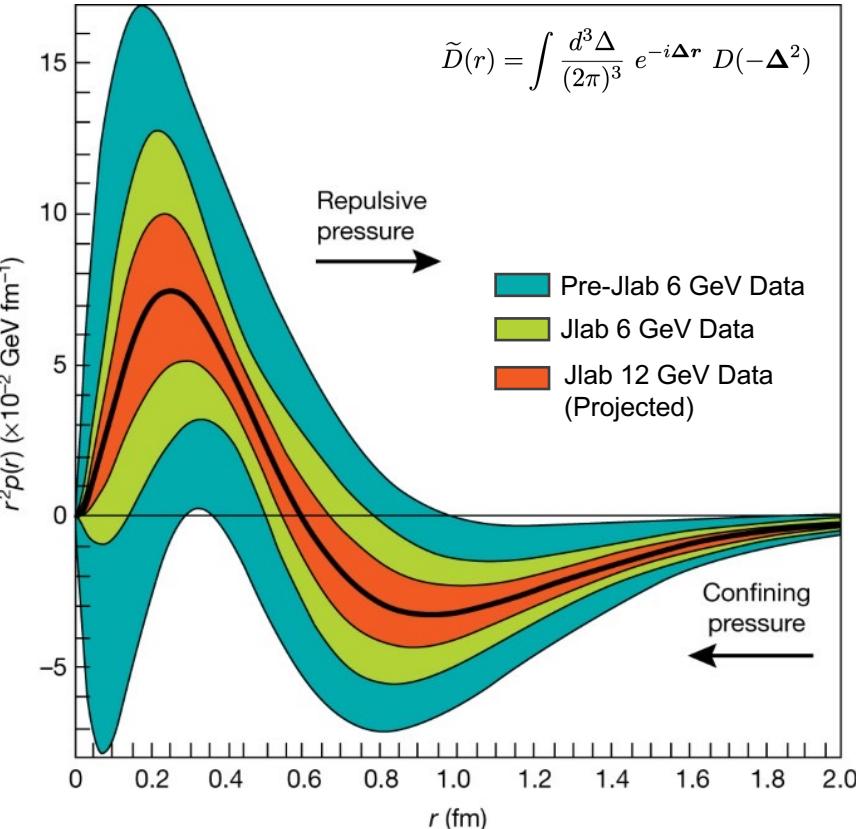
On par with spin, charge, mass!

MECHANICAL PROPERTIES

- The total D -term provides a gateway for extraction of various **mechanical** properties of the proton, including:

- Pressure distribution
- Shear force distribution
- Mechanical radius
- Tangential & normal force distributions

$$p^a(r) = \frac{1}{6m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \widetilde{D}^a(r) - m \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta r} \bar{c}^a(-\Delta^2)$$



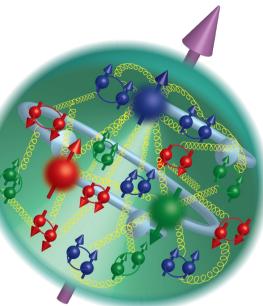
[1] - Burkert et al.: The pressure distribution inside the proton

MECHANICAL PROPERTIES

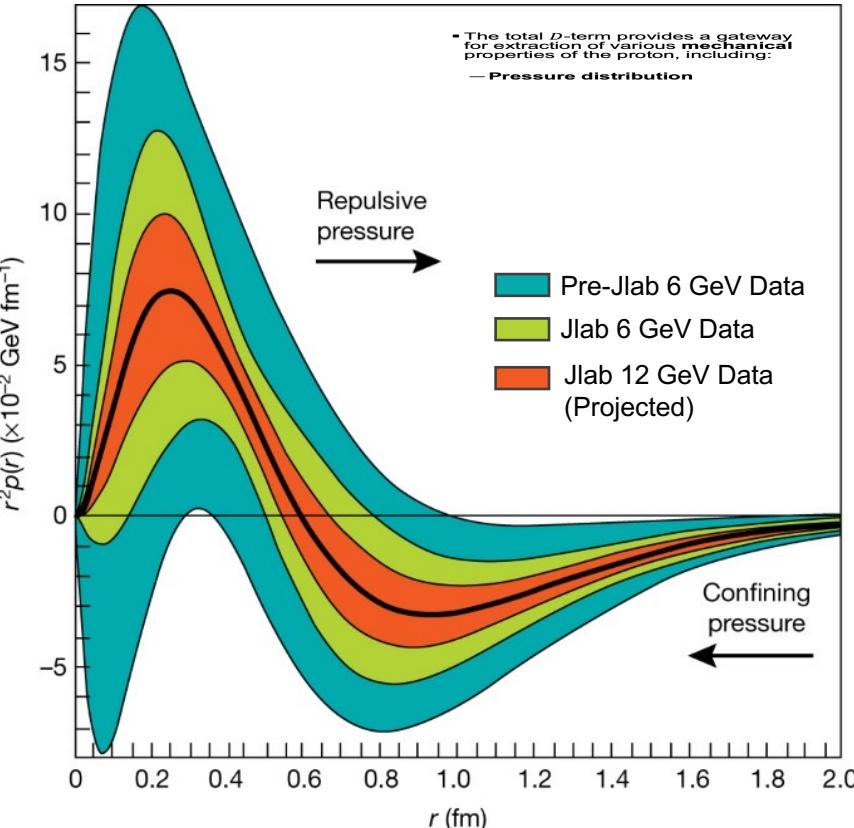
- The total D -term provides a gateway for extraction of various **mechanical** properties of the proton, including:
 - Pressure distribution

10^{29} atmospheres!?

At $r = 0.3$ fm



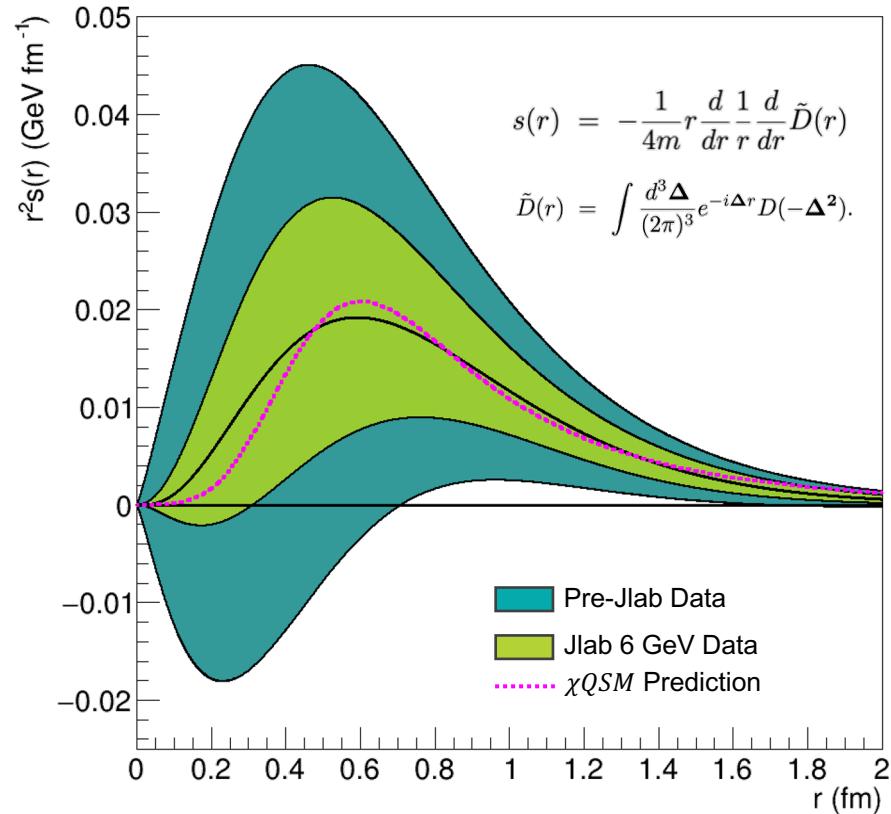
$$p^a(r) = \frac{1}{6m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \widetilde{D}^a(r) - m \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\Delta r} \bar{c}^a(-\Delta^2)$$



[1] - Burkert et al.: The pressure distribution inside the proton

MECHANICAL PROPERTIES

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 - Pressure distribution
 - **Shear force distribution**
 - Mechanical radius
 - Tangential & normal force distributions



[1] - Burkert et al.: Determination of shear forces inside the proton

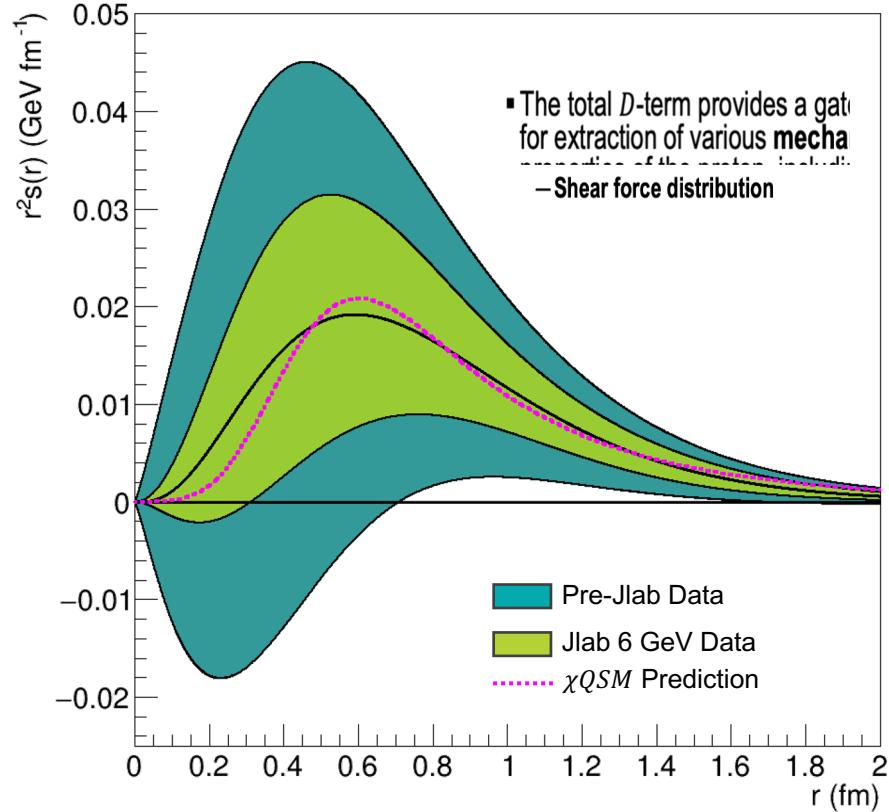
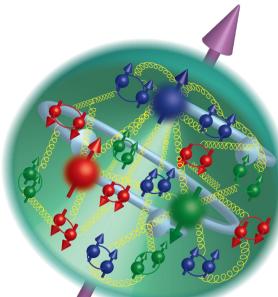
MECHANICAL PROPERTIES

- The total D -term provides a gateway for extraction of various **mechanical** properties of the proton, including:

- Shear force distribution

40000 Newtons!?

At $r = 0.6$ fm



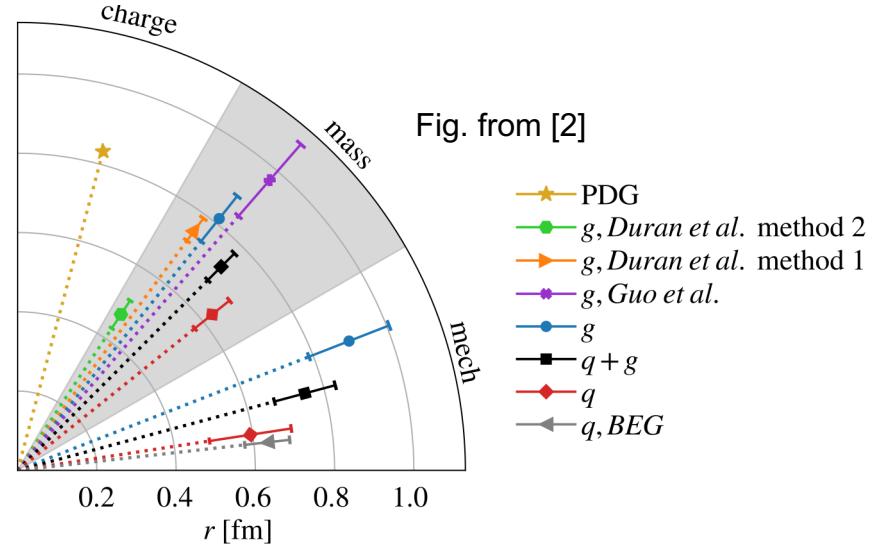
[1] - Burkert et al.: Determination of shear forces inside the proton

MECHANICAL PROPERTIES

- The total D -term provides a gateway for extraction of various **mechanical** properties of the proton, including:

- Pressure distribution
- Shear force distribution
- **Mechanical radius**
- Tangential & normal force distributions

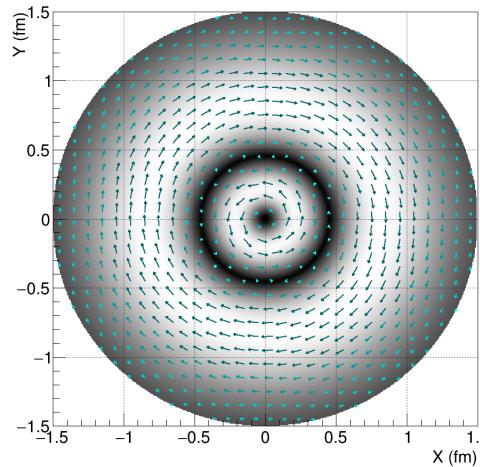
$$\langle r^2 \rangle_{\text{mech}} = \frac{\int d^3r \ r^2 \left[\frac{2}{3}s(r) + p(r) \right]}{\int d^3r \left[\frac{2}{3}s(r) + p(r) \right]} = \frac{6D}{\int_{-\infty}^0 dt \ D(t)}$$



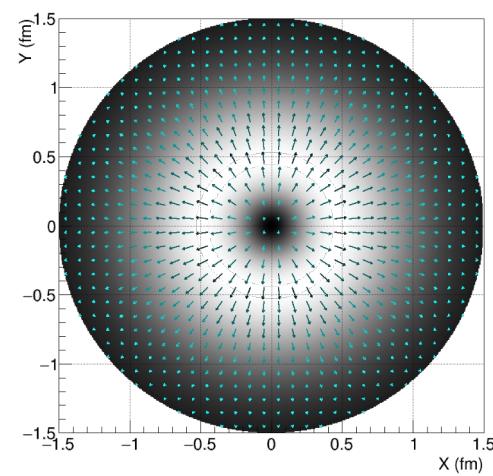
[1] – Polyakov, Schweitzer: **Forces inside hadrons: pressure, surface tension, mechanical radius, and all that**
[2] - Hackett et al.: **Gravitational form factors of the proton from lattice QCD**

MECHANICAL PROPERTIES

- The total D -term provides a gateway for extraction of various **mechanical** properties of the proton, including:
 - Pressure distribution
 - Shear force distribution
 - Mechanical radius
 - **Tangential & normal force distributions**



Tangential force^[1]



Normal force^[1]

[1] – Burkert et al.: Colloquium:
Gravitational Form Factors of the Proton

$$\frac{dF_\phi}{dS_\phi} = -\frac{1}{3}s(r) + p(r).$$

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r)$$

HOW DO WE ACCESS THE D-TERM?

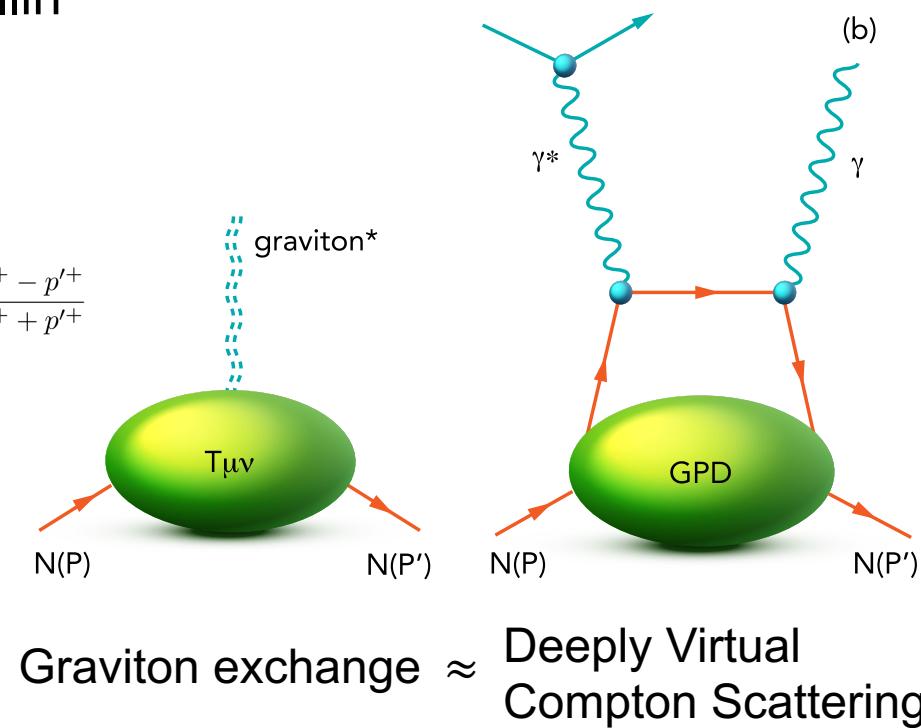
- Graviton scattering would measure directly $T^{\mu\nu}$
- Can access GFFs via the second Mellin moments of the generalized parton distributions (GPDs)

$$\int_{-1}^1 dx \ x H^a(x, \xi, t) = A^a(t) + \xi^2 D^a(t)$$

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$$

$$\int_{-1}^1 dx \ x E^a(x, \xi, t) = B^a(t) - \xi^2 D^a(t)$$

In certain regions, hard exclusive reaction cross sections reduce to **simple functions of the GFFs!**



HOW DO WE ACCESS THE D-TERM?

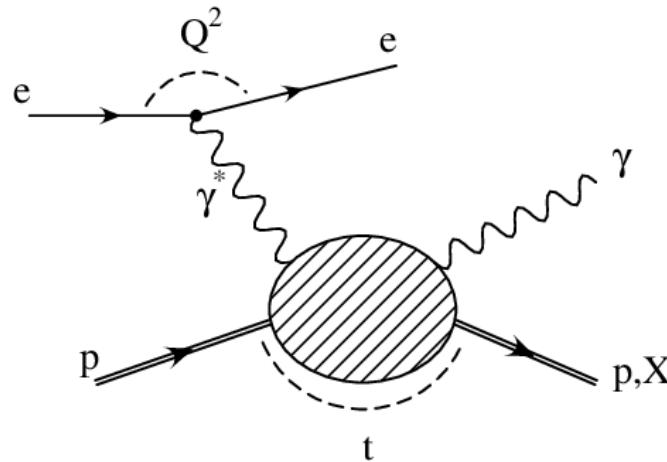
The total D -term arises from its partonic contributions via a sum rule:

$$\text{Total } D\text{-term} \quad \text{Partonic } D\text{-term contributions}$$
$$D(0) = D_g(0) + D_u(0) + D_d(0) + D_s(0) + \dots$$

Different exclusive processes access the **contributions of different parton species** to the total proton D -term!

Up & Down quarks:
Accessible via DVCS cross section &
beam-spin asymmetries

$$D(0) = D_g(0) + D_u(0) + D_d(0) + D_s(0) + \dots$$



The pressure distribution inside the proton

[V. D. Burkert](#)  , [L. Elouadrhiri](#) & [F. X. Girod](#)

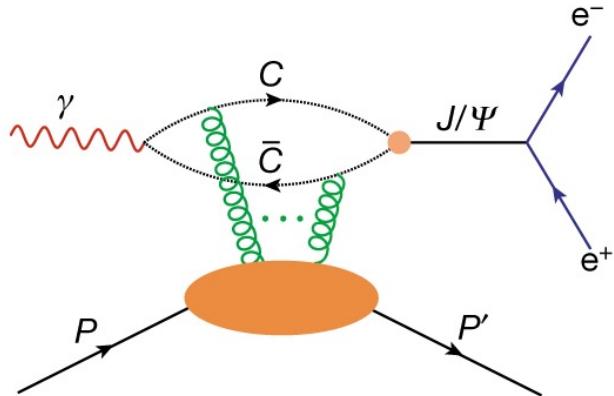
Gluons:
Accessible via near-threshold
production of J/ψ and Υ



$$D(0) = D_g(0) + D_u(0) + D_d(0) + D_s(0) + \dots$$

Determining the Proton's Gluonic Gravitational Form Factors

B. Duran^{3,1}, Z.-E. Meziani^{1,3**}, S. Joosten¹, M. K. Jones², S. Prasad¹, C. Peng¹,
W. Armstrong¹, H. Atac³, E. Chudakov², H. Bhatt⁵, D. Bhetuwal⁵, M. Boer¹¹,
A. Camsonne², J.-P. Chen², M. M. Dalton², N. Deokar³, M. Diefenthaler², J. Dunne⁵,
L. El Fassi⁵, E. Fuchey⁹, H. Gao⁴, D. Gaskell², O. Hansen², F. Hauenstein⁶,
D. Higinbotham², S. Jia³, A. Karki⁵, C. Keppel², P. King⁷, H.S. Ko¹⁰, X. Li⁴, R. Li³,
D. Mack², S. Malace², M. McCaughan², R. E. McClellan⁸, R. Michaels², D. Meekins²,
M. Paolone³, L. Pentchev², E. Pooser², A. Puckett⁹, R. Radloff⁷, M. Rehfuss³,
P. E. Reimer¹, S. Riordan¹, B. Sawatzky², A. Smith⁴, N. Sparveris³, H. Szumila-Vance²,
S. Wood², J. Xie¹, Z. Ye¹, C. Yero⁶, and Z. Zhao⁴



Only **two** existing experimental results on
gravitational form factors,
both high-profile, ~ 50 citations/year!

Interest in this subject is clear!
The field is rapidly growing!

M. A. Jackson¹, J. A. Jackson¹,
P. E. Helmer¹, S. Riordan¹, B. Sawatzky², A. Smith⁴, N. Sparveris³, H. Szumila-Vance²,
S. Wood², J. Xie¹, Z. Ye¹, C. Yero⁶, and Z. Zhao⁴

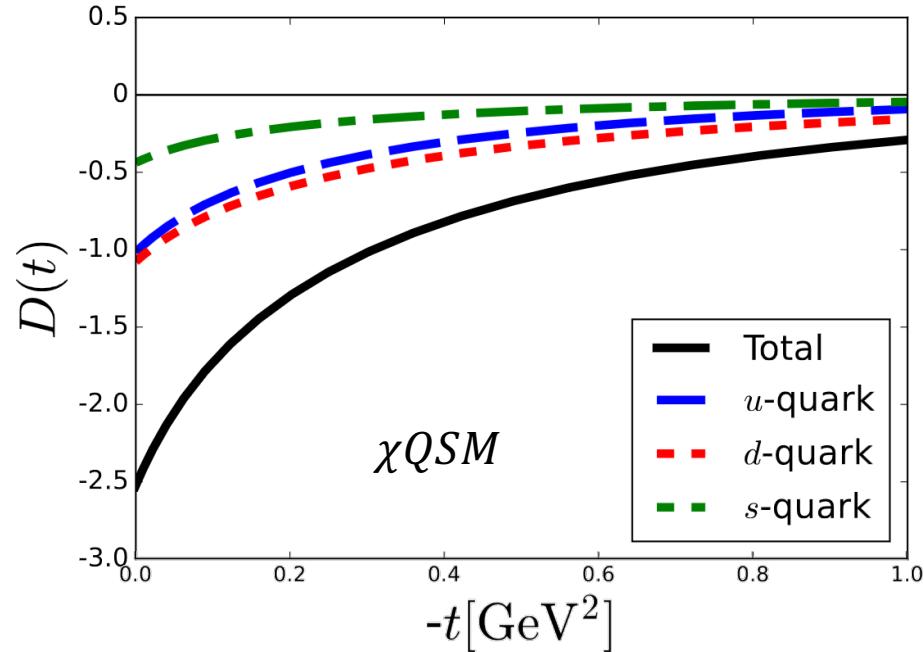


$$D(0) = D_g(0) + D_u(0) + D_d(0) + D_s(0) + \dots$$


Strange quarks:
Can we just
neglect them...?

THEORY PREDICTIONS

- Large- N_c theory predicts that the D -term is “**flavor-blind**”^[1]
 - i.e. $D_u \sim D_d$ despite their different number densities, this is supported by lattice results^[2]
- Extending this argument, could $D_u \sim D_d \sim D_s$?
- Chiral quark soliton model^[3] prediction: $D_u \sim D_d \sim 2D_s$



[1] - Goeke et al.: **Hard exclusive reactions and the structure of hadrons**

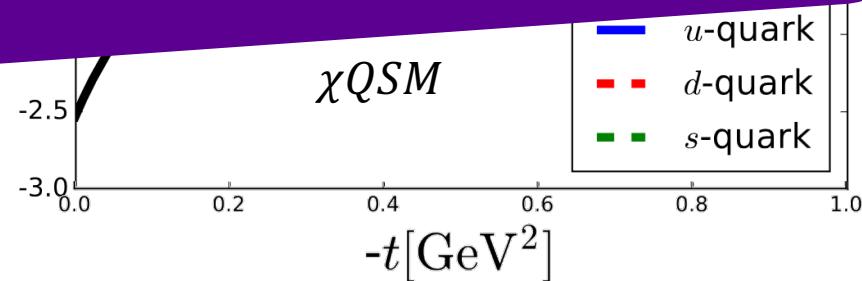
[2] - Hackett et al.: **Gravitational form factors of the proton from lattice QCD**

[3] - Won et al.: **Role of strange quarks in the D -term and cosmological constant term of the proton**

THEORY PREDICTIONS

- Large- N_c theory predicts that the D -term is “flavor blind”^[1]
– i.e. $D_u \sim D_d$ despite $m_u \neq m_d$

“The contributions of strange quarks play a particularly significant role in the D -term. Therefore, when extracting these contributions from experimental data, it is essential to take into account the influence of strange quarks.”^[3]



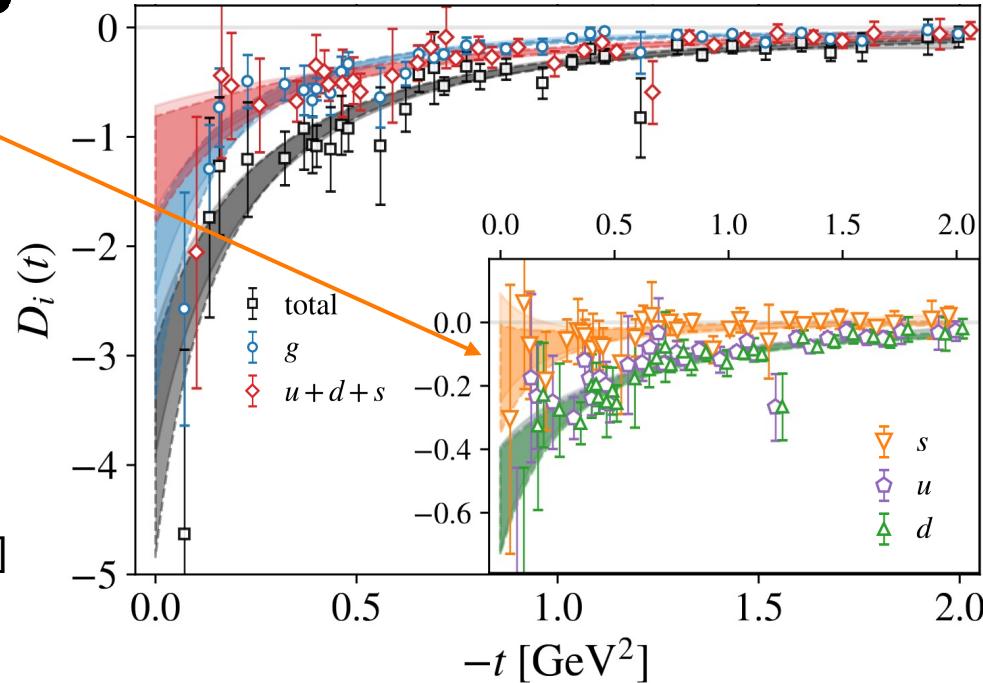
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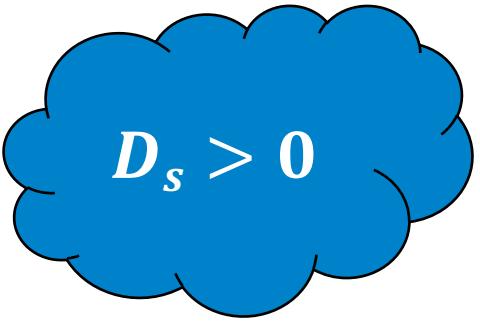
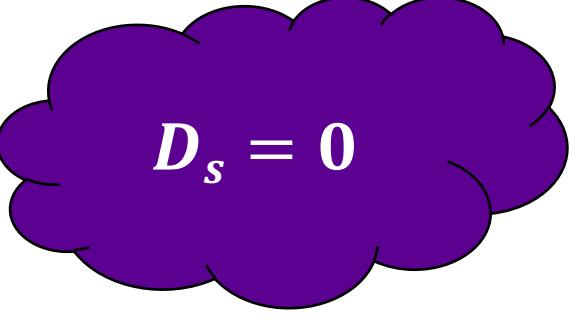
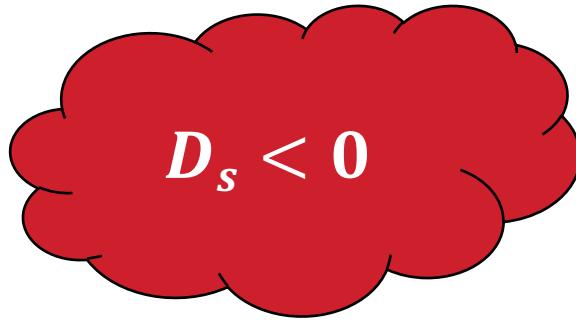
[3] - Won et al.: Role of strange quarks in the D -term and cosmological constant term of the proton

THEORY PREDICTIONS

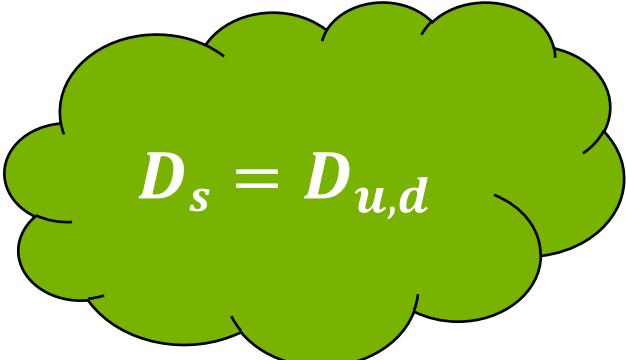
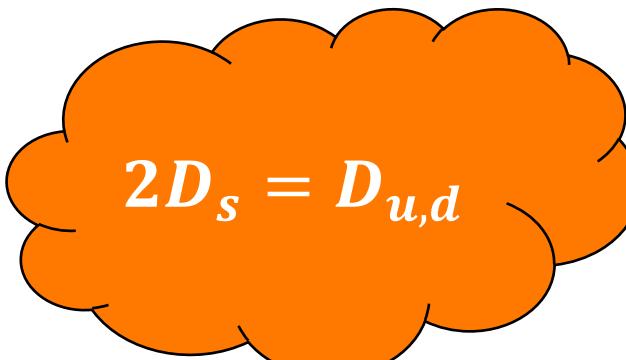
- D_s calculated on the lattice^[1]
 - Uncertainties are large!
 - Lattice does not exclude $D_u \sim D_d \sim 2D_s$ or $D_s > 0$
- Opposite signs of sea & valence quarks is a distinct possibility, predicted by χQSM ^[1]



$D_s > 0$ would mean that strange quarks feel forces of opposite direction to up & down quarks!

 $D_s > 0$  $D_s = 0$  $D_s < 0$

Variety of theory predictions giving very different values for D_s , can we extract it experimentally?

 $D_s = D_{u,d}$  $2D_s = D_{u,d}$

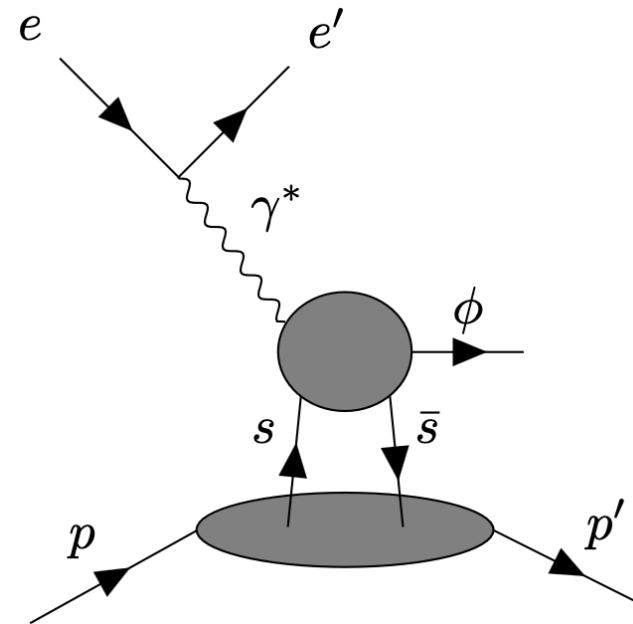
ACCESSING THE STRANGE QUARK CONTRIBUTION TO THE PROTON D-TERM

- ***Electroproduction of ϕ mesons at***

large ξ provides sensitivity to the strangeness D -term^[1,2]

- ϕ meson is very nearly a pure $s\bar{s}$ state
 - Couples strongly to strangeness in the proton
- **Only known process** to access D_s

But never measured in the required kinematic region!



[1] – Hatta, Strikman: ϕ -meson lepto-production near threshold and the strangeness D -term

[2] – Hatta, HK, et al.: Deeply virtual ϕ -meson production near threshold

THEORY PREDICTIONS

NLO GPD calculation for ϕ DVMP cross section now available^[1]!

$$\frac{d\sigma_L}{dt} = \frac{2\pi^2 \alpha_{em}}{(W^2 - M^2)W p_{cm}} \left((1 - \xi^2)|\mathcal{H}|^2 - \left(\frac{t}{4M^2} + \xi^2 \right) |\mathcal{E}|^2 - 2\xi^2 \text{Re}(\mathcal{H}\mathcal{E}^*) \right)$$

DVMP amplitudes \mathcal{H}, \mathcal{E} have **direct dependence** on partonic D -term contributions for large ξ !

$$\begin{aligned} \mathcal{H}(\xi, t) \approx & \frac{2\kappa}{\xi^2} \frac{15}{2} \left[\left\{ \alpha_s(\mu) + \frac{\alpha_s^2(\mu)}{2\pi} \left(25.7309 - 2n_f + \left(-\frac{131}{18} + \frac{n_f}{3} \right) \ln \frac{Q^2}{\mu^2} \right) \right\} (A_s(t, \mu) + \xi^2 D_s(t, \mu)) \right. \\ & \left. + \frac{3}{8} \left\{ \alpha_s + \frac{\alpha_s^2}{2\pi} \left(13.8682 - \frac{83}{18} \ln \frac{Q^2}{\mu^2} \right) \right\} (A_g + \xi^2 D_g) \right] \end{aligned}$$

Large cancellation for D_g !

$$A_g \approx 0.4, D_g \approx -2$$

While D_s contributes directly!

$$A_s \approx 0.03, D_s \approx ?$$

THEORY COMMENTS

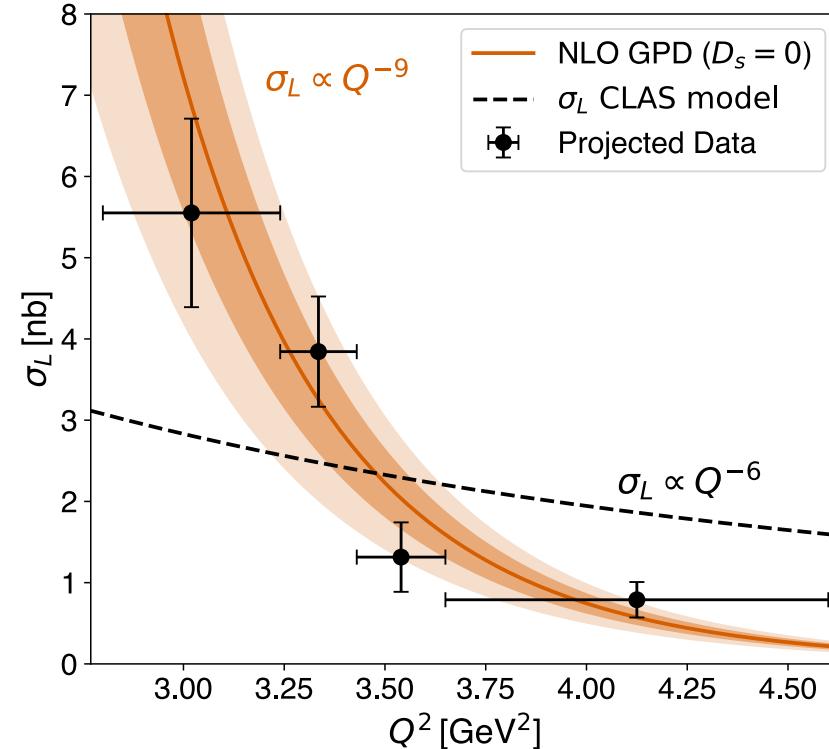
- ***Prediction of Hatta et al. based on collinear factorization needs to be tested in the near-threshold region***
 - Produced hadrons can have final-state interactions
 - Hadronic coupled-channels can contribute e.g. $ep \rightarrow K\Lambda \rightarrow \phi p$
- Theory reviewers prefer a **hadronic interpretation** of this kinematic region
 - Asymptotically near-threshold, soft hadronic interactions will dominate over hard partonic ones

These comments generated a vigorous discussion amongst the theorists!

TESTING THE THEORY

- **Q^2 scaling lets us test!**
- Hatta et al. predicts a very steep scaling with Q^2 in our range of W
 - Predicts $\sigma_L \propto Q^{-9}$ due to the **GFFs and hard coefficients**
 - Unique feature of the near-threshold framework^[1]!
 - Standard GPD predicts $\sigma_L \propto Q^{-6}$
 - VMD predicts $\sigma_L \propto Q^{-4}$

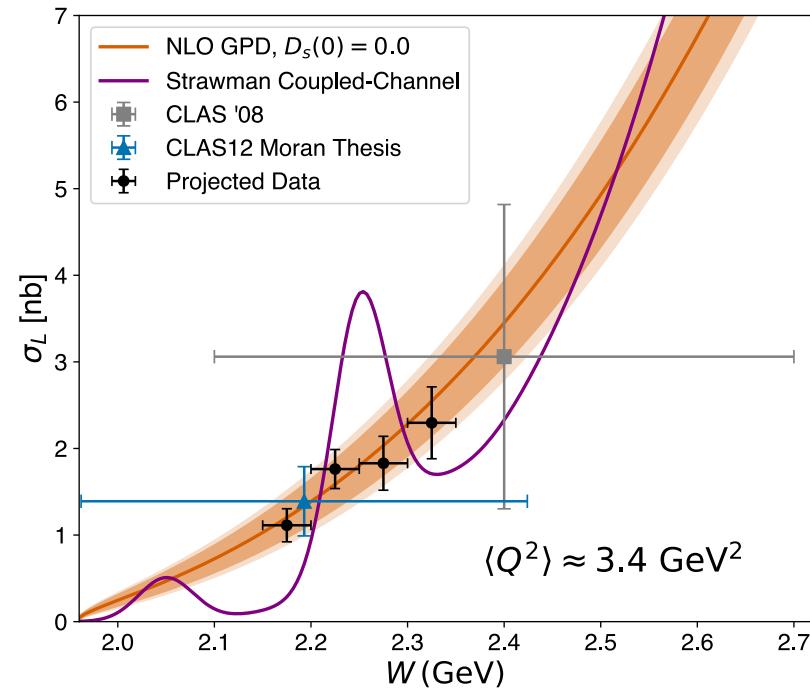
Can validate or invalidate collinear factorization at these kinematics with our data!



[1] – Hatta, HK, et al. - Deeply virtual ϕ -meson production near threshold

TESTING THE THEORY

- The prediction of Hatta et al. “**agrees**” with CLAS data and preliminary results from CLAS12
 - **Data uncertainties are very large!**
- Almost any model can describe the existing data due to poor precision & large bins
- The speculation surrounding this topic is precisely why these **data are sorely needed!**



Only way to resolve this debate is with **data!**

Data Source	$\langle W \rangle$ (GeV)	Data σ_L	GPD σ_L
CLAS '08 data	2.40	3.06 ± 1.76	3.44 ± 0.66
CLAS12 Moran thesis data	2.19	1.39 ± 0.30	1.28 ± 0.33



59 Collaborators



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59 Collaborators, international collaboration!

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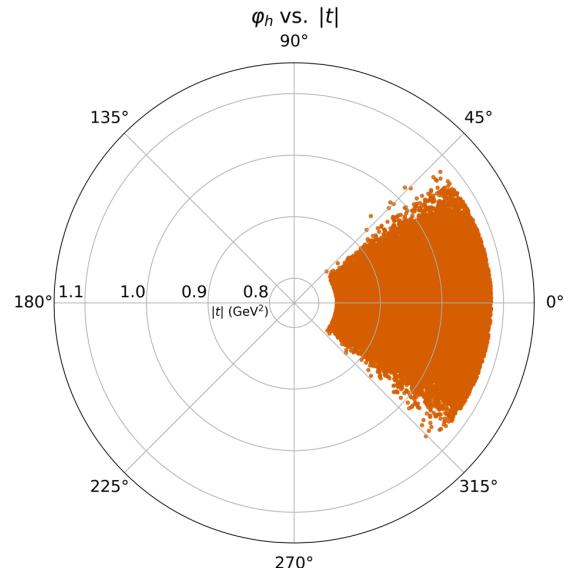
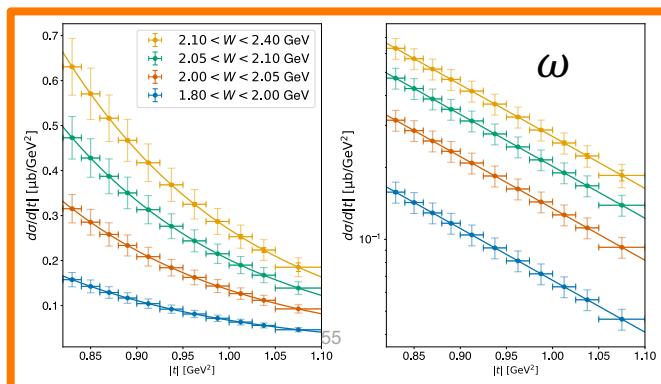
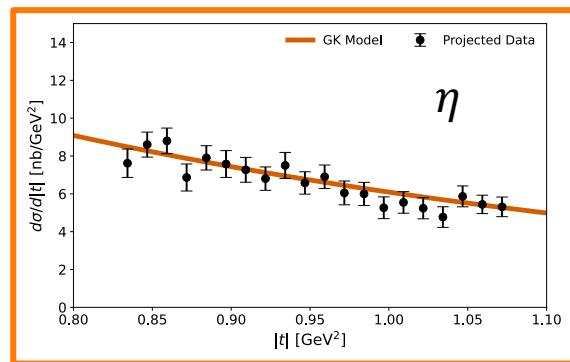
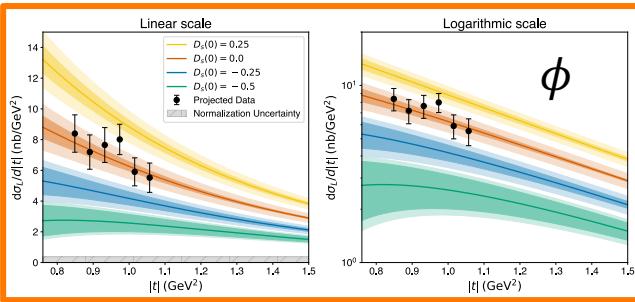
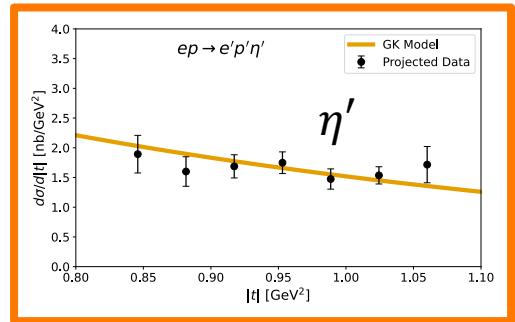
J. Datta

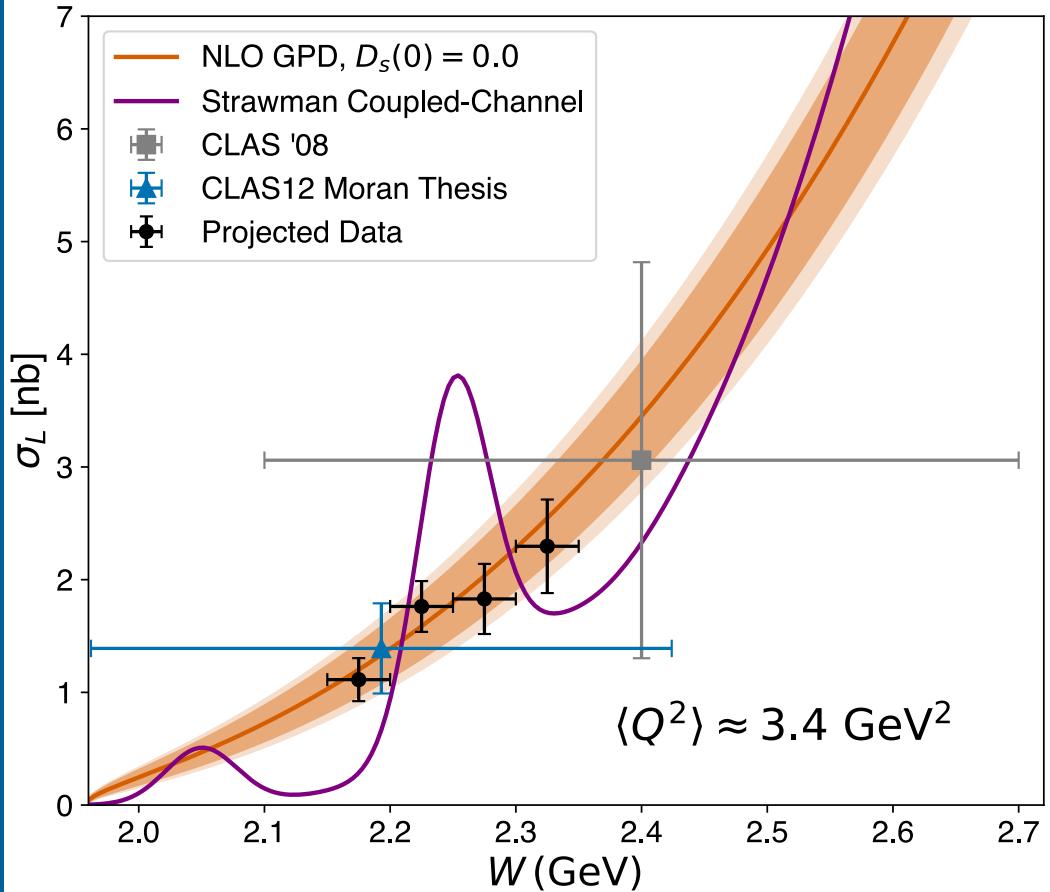
Stony Brook University, Stony Brook, NY

WHAT ELSE CAN WE LEARN FROM THIS DATA?

Beam Spin Asymmetries for all! (Partially)

$$\text{BSA} = \frac{\sqrt{2\epsilon(1-\epsilon)} \frac{\sigma_{LT'}}{\sigma_0} \sin \phi_h}{1 + \sqrt{2\epsilon(1+\epsilon)} \frac{\sigma_{LT}}{\sigma_0} \cos \phi_h + \epsilon \frac{\sigma_{TT}}{\sigma_0} \cos 2\phi_h}$$



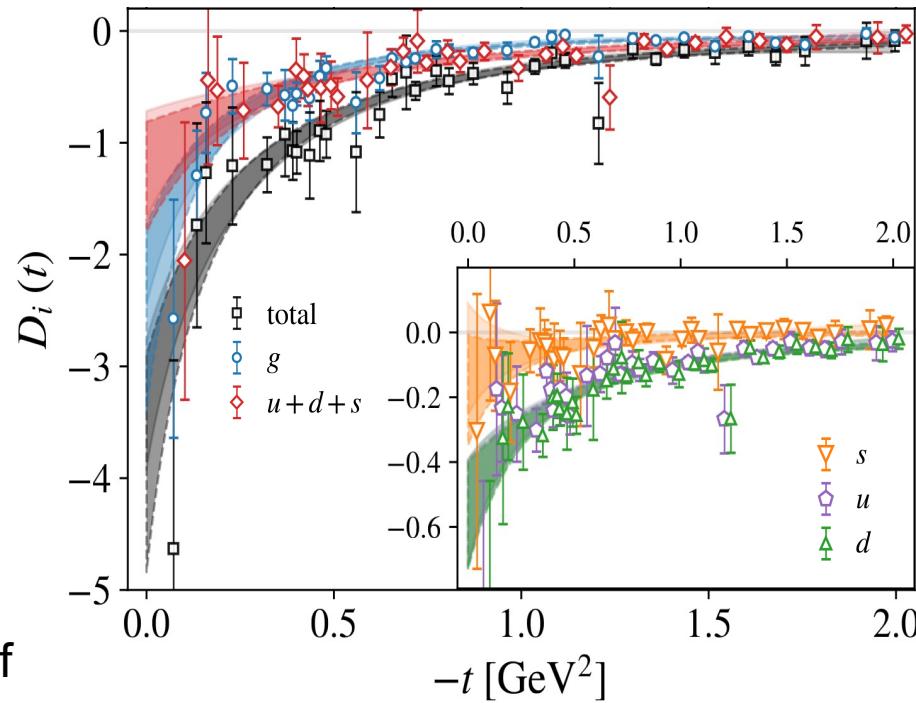


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THEORY PREDICTIONS

	Dipole	z -expansion
	D_i	D_i
u	$-0.56(17)$	$-0.56(17)$
d	$-0.57(17)$	$-0.56(17)$
s	$-0.18(17)$	$-0.08(17)$
$u + d + s$	$-1.30(49)$	$-1.20(48)$
g	$-2.57(84)$	$-2.15(32)$
Total	$-3.87(97)$	$-3.35(58)$

Quark masses tuned for a pion mass of 170 MeV, lattice spacing of 0.091 fm.
Calculations not yet in the continuum limit



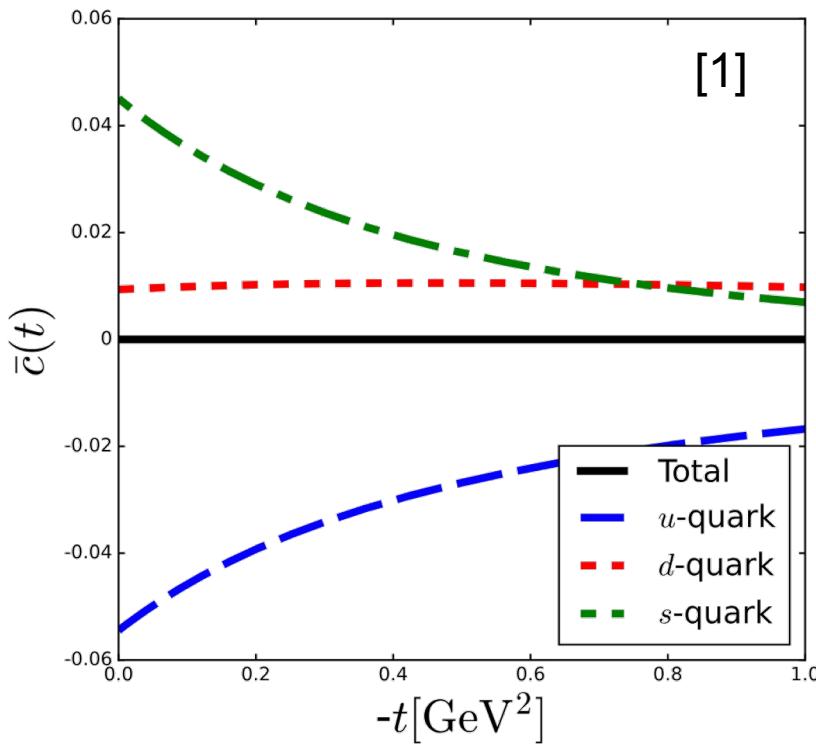
\bar{c} CAVEAT

- \bar{c} form factor contributes to many of the mechanical properties (Radial pressure, radii, etc.)
 - \bar{c} currently inaccessible to experiment

Pressure defined as:

$$p^a(r) = \frac{1}{6m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \widetilde{D}^a(r) - m \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\Delta r} \boxed{\bar{c}^a(-\Delta^2)}$$

- However, $\bar{c}_q = -\bar{c}_g$! \rightarrow Total \bar{c} **cancels** due to EMT conservation **if summing over all parton species!**

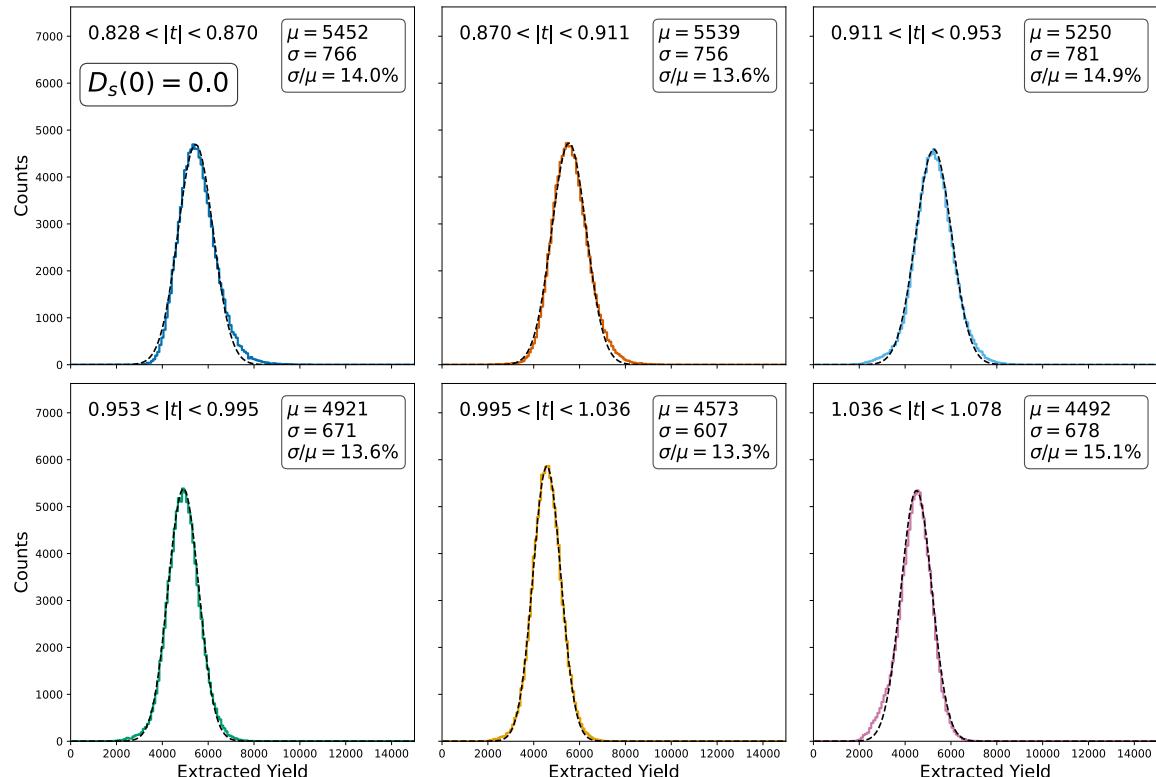


This caveat means that to extract the full set of mechanical properties, ***all partonic D-term contributions must be known!***

SIGNAL EXTRACTION

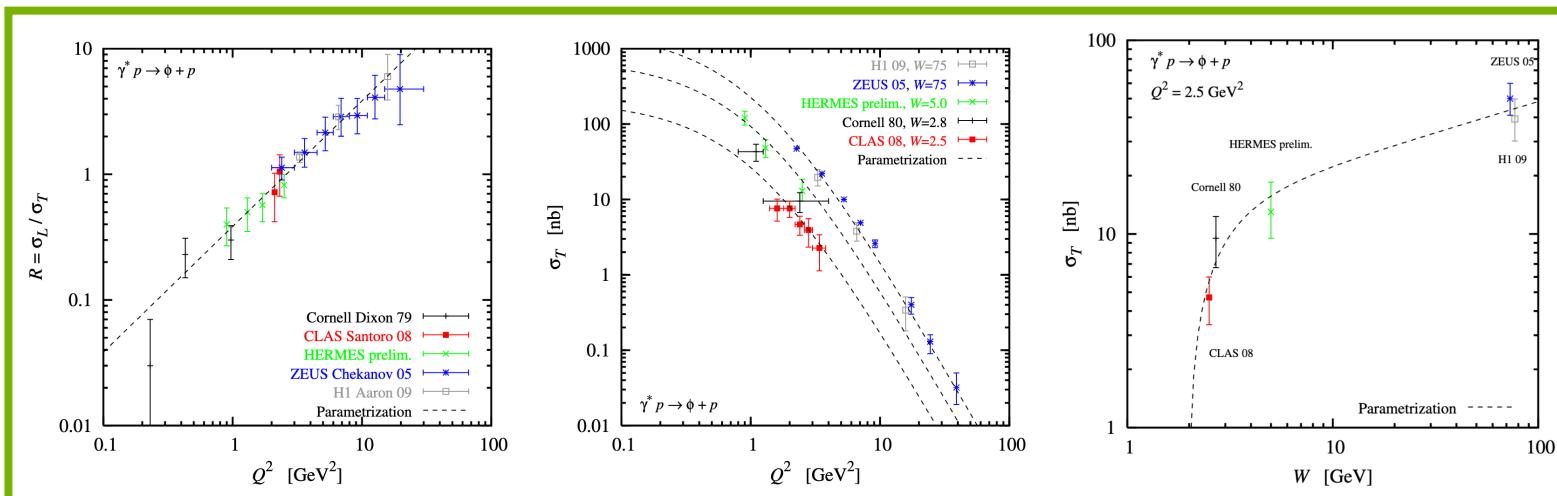
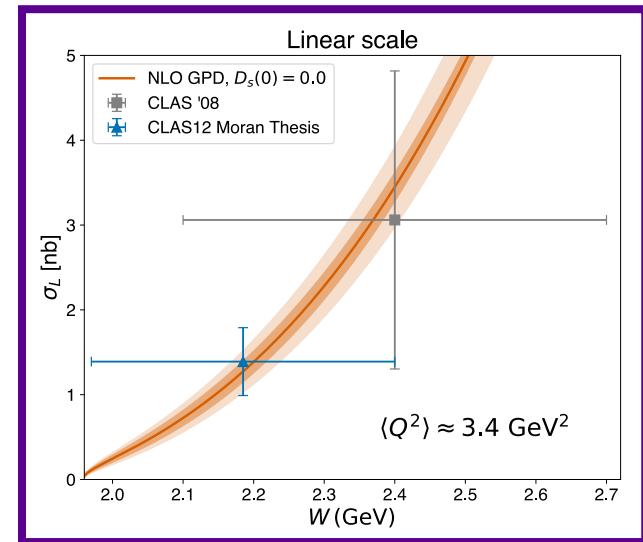
- Perform the background generation, fitting, and sideband background subtraction on pseudodata 100000 times

- Results of pseudoexperiments shown for 6 bins in $|t|$
 - Can bin less finely if cross section is smaller than predicted



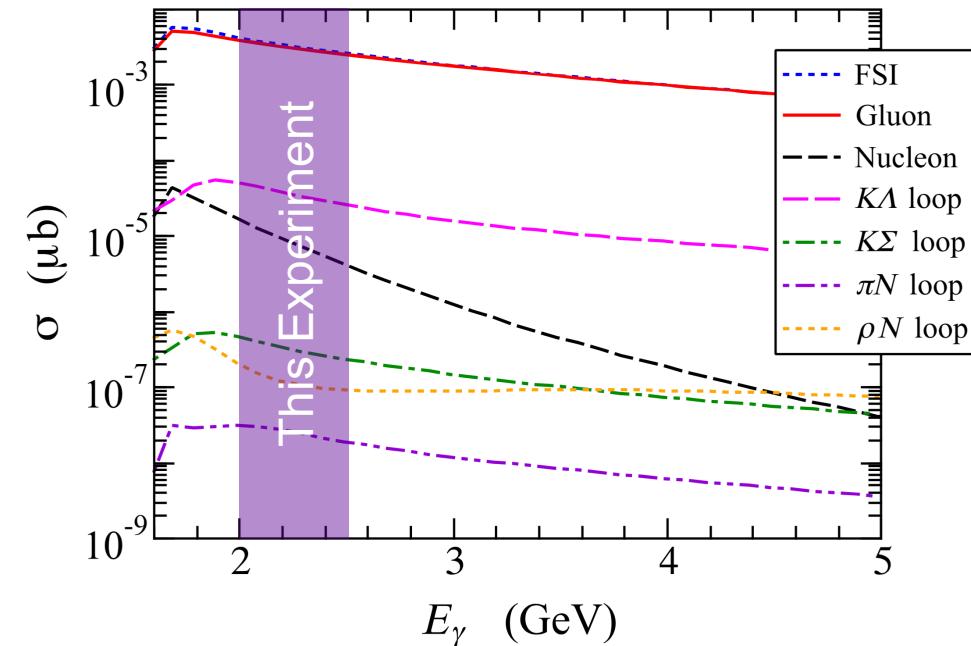
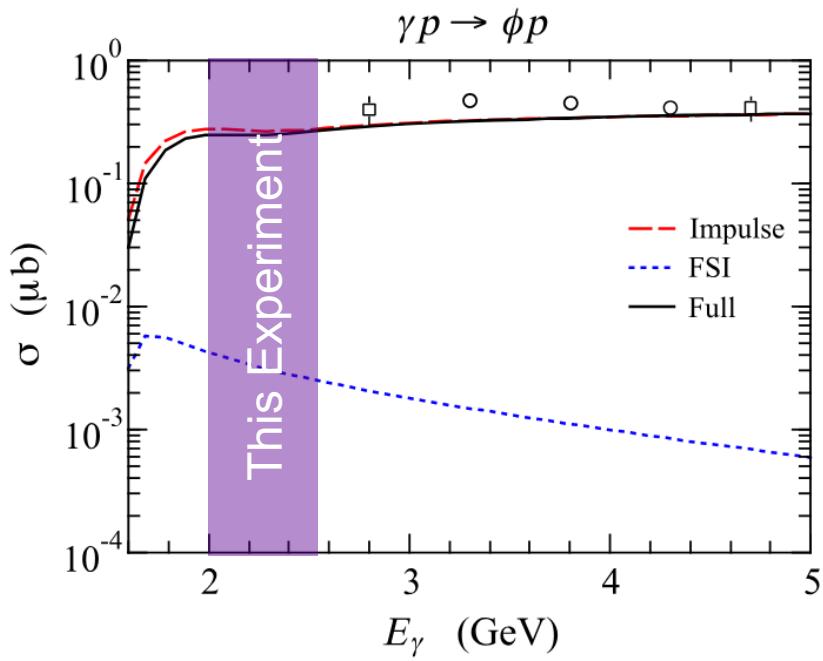
CROSS SECTION PROJECTIONS

- ϕ Cross section conservatively estimated as the smaller of two predictions in our projections
 - NLO GPD prediction —
 - Model based on existing world data developed for CLAS12



FINAL-STATE INTERACTIONS

- FSI calculated for ϕ photoproduction^[1]
 - Determined to be orders of magnitude smaller than the production cross section – **negligible**
- Even the individual FSI channels are calculated and shown to be tiny



[1] – S.H. Kim et al. **Dynamical model of ϕ meson photoproduction on the nucleon and ${}^4\text{He}$**

THEORY RESPONSES

Furthermore, **duality** tells us that partonic and hadronic descriptions are **not exclusive!**

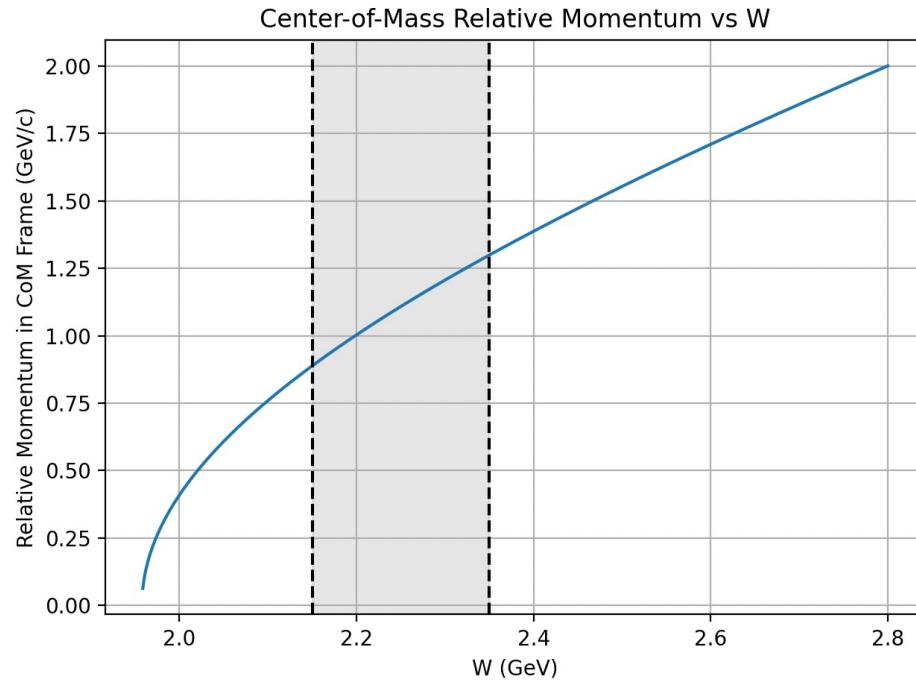
- The points raised against Hatta et al. do not apply to **holographic predictions** since holography does not rely on collinear factorization
 - Holographic predictions for ϕ already exist!^[1]
- In contact now with holographic theorists, a new calculation for our kinematics is possible

[1] - Mamo, Zahed: **Electroproduction of heavy vector mesons using holographic QCD: from near threshold to high energy regimes**

“NEAR-THRESHOLD”?

- **Asymptotically close to the threshold of $W = 1.96$ GeV, collinear factorization indeed breaks down**
- However, the **W of this experiment was chosen to be large enough that the relative momentum between the ϕ and proton is still reasonably large**
 - **“Near-threshold” is misleading!**

The only way to test whether collinear factorization holds quantitatively is with **data!**



Total momentum between the ϕ and proton in the center-of-mass frame

The forward values $A_a(\mu)$ represent the momentum fraction of the proton carried by partons $\sum_{a=q,g} A_a(\mu) = 1$. We consider their one-loop QCD evolution using

$$A_s(\mu_0) = 0.03, \quad A_g(\mu_0) = 0.42, \quad A_{u+d+c}(\mu_0) = 1 - A_s(\mu_0) - A_g(\mu_0), \quad (49)$$

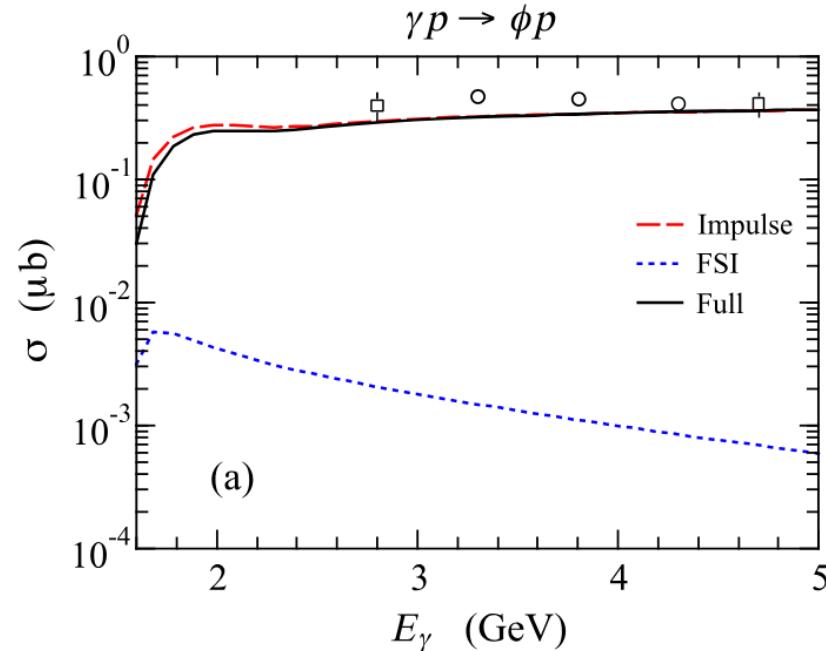
at the reference scale $\mu_0 = 2$ GeV [67]. The one-loop evolution of the D-terms $D_{q,g}(\mu)$ is the same as that for $A_{q,g}(\mu)$ and is explicitly given by

$$\begin{aligned} D_q(\mu) = & \frac{D}{4C_F + n_f} + \frac{1}{n_f (4C_F + n_f)} \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{-\frac{2}{3\beta_0}(4C_F + n_f)} (4C_F D - (4C_F + n_f) D_g(\mu_0)) \\ & + \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{-\frac{8C_F}{3\beta_0}} \left(D_q(\mu_0) - \frac{1}{n_f} \sum_{q'} D_{q'}(\mu_0) \right), \end{aligned} \quad (50)$$

$$\begin{pmatrix} \mathcal{H}(\xi, t, Q^2) \\ \mathcal{E}(\xi, t, Q^2) \end{pmatrix} = e_s \frac{C_F f_\phi}{N_c Q} \sum_{a=q,g} \begin{pmatrix} H^a(x, \xi, t, \mu^2) \\ E^a(x, \xi, t, \mu^2) \end{pmatrix} \otimes T^a \left(x, \xi, u, \frac{Q^2}{\mu^2} \right) \otimes \varphi(u, \mu^2)$$

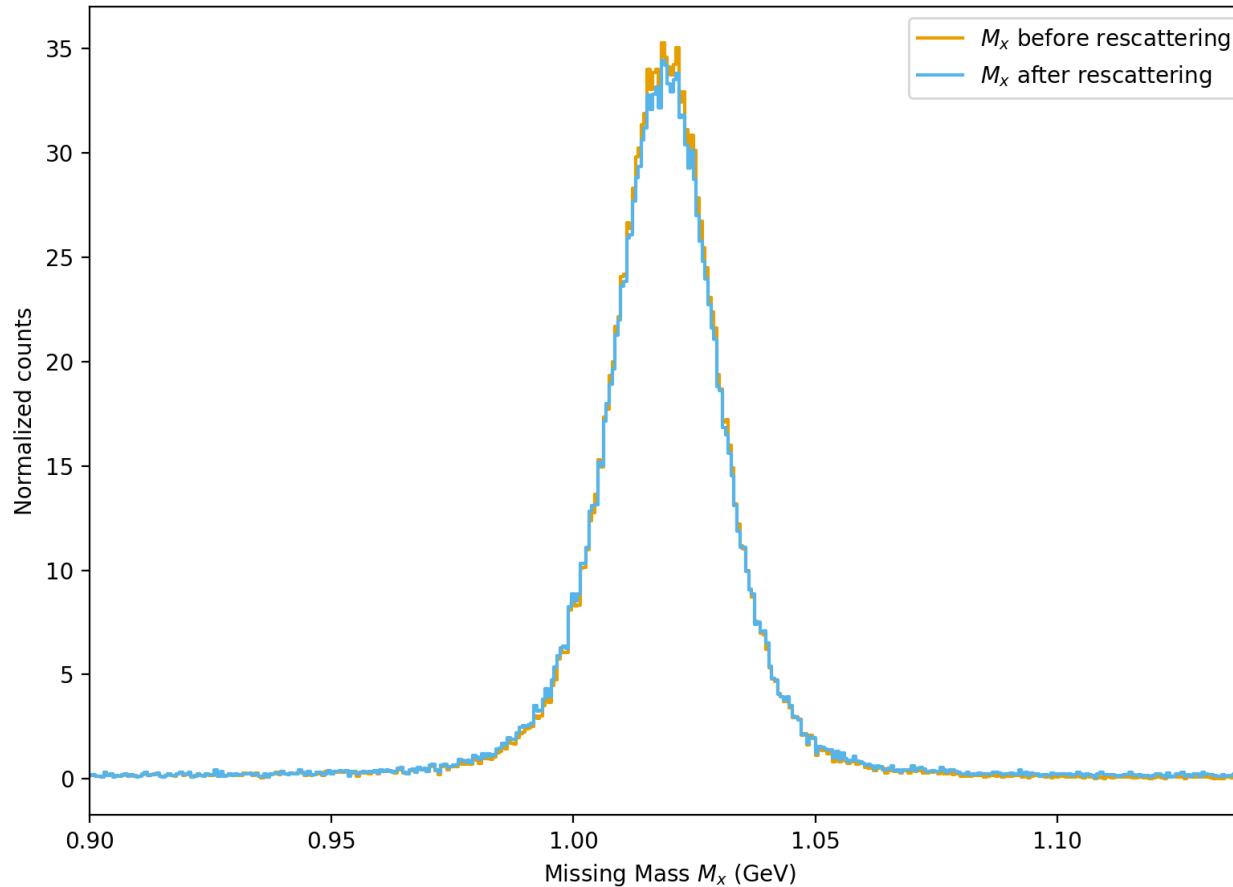
PAC52 - FINAL-STATE INTERACTIONS

- FSI calculated for ϕ photoproduction^[1]
 - Determined to be orders of magnitude smaller than the production cross section – **negligible**
- However, ϕ decay products can also rescatter on the proton!
 - $\phi \rightarrow KK$: 83% BR
 - $\phi \rightarrow \pi\pi\pi$: 15% BR
 - Estimate πp & Kp cross sections ~100 mb for our kinematics
- Using ϕ lifetime & black disk limit, **likelihood of rescattering is 2%**
 - Assume a 100% uncertainty on this for cross section measurement

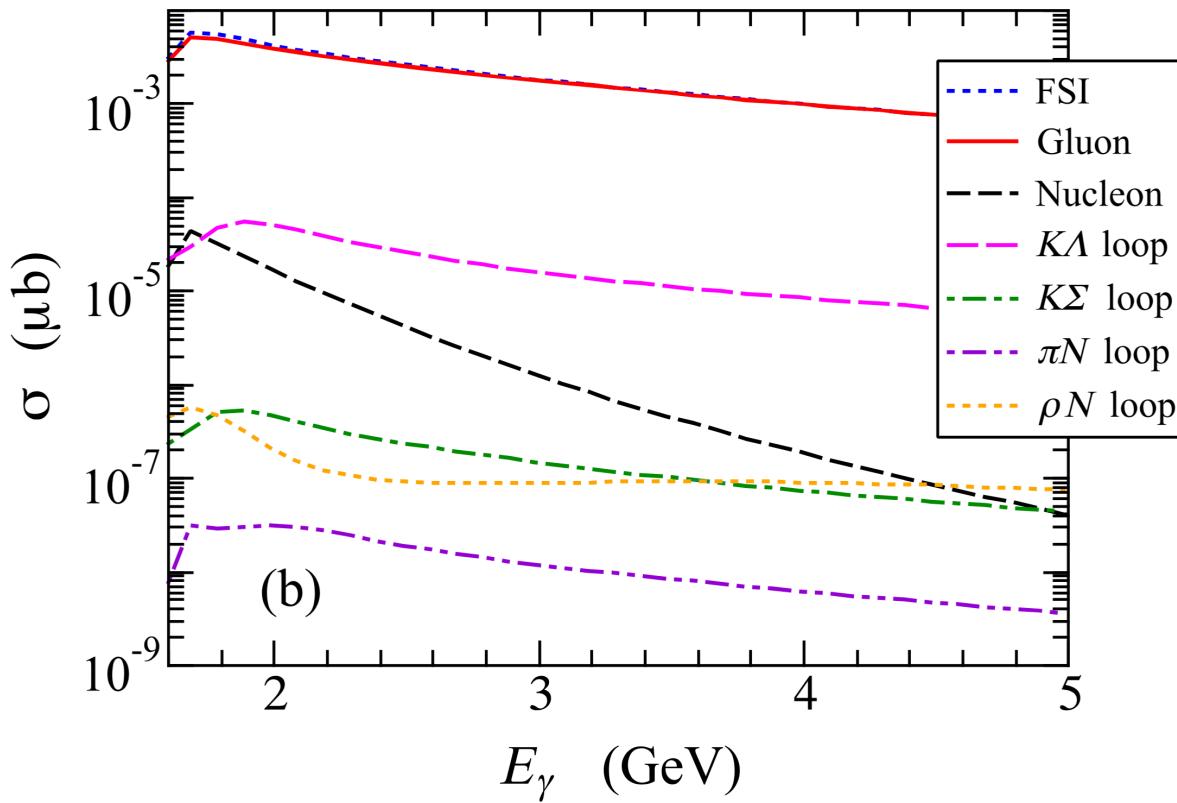


[1] – S.H. Kim et al. **Dynamical model of ϕ meson photoproduction on the nucleon and ${}^4\text{He}$**

Missing Mass Distributions



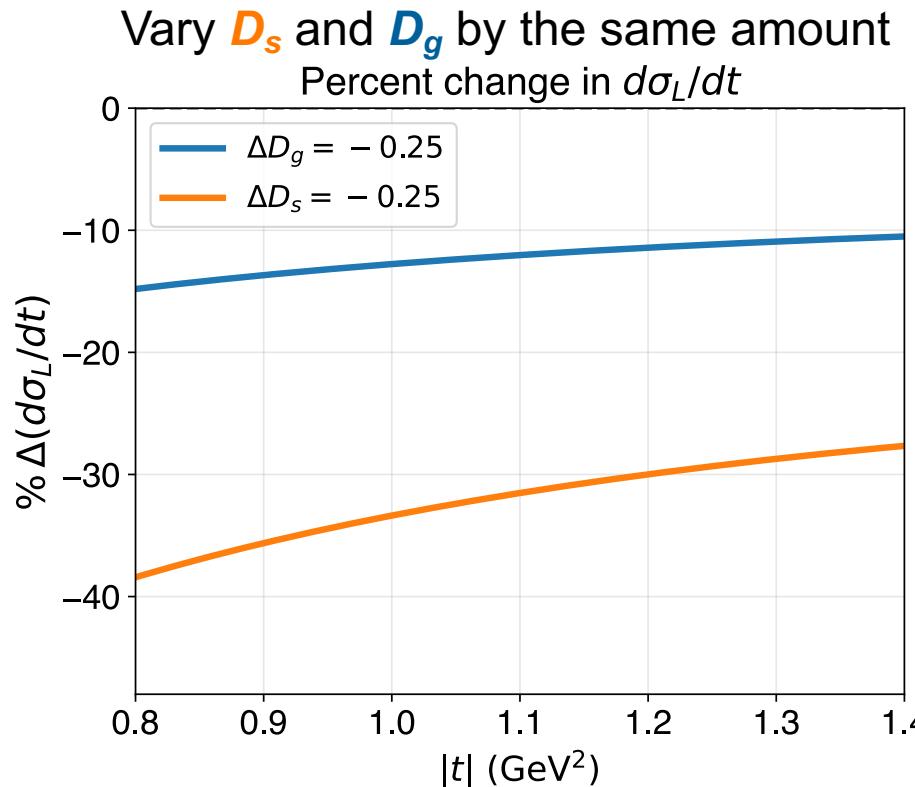
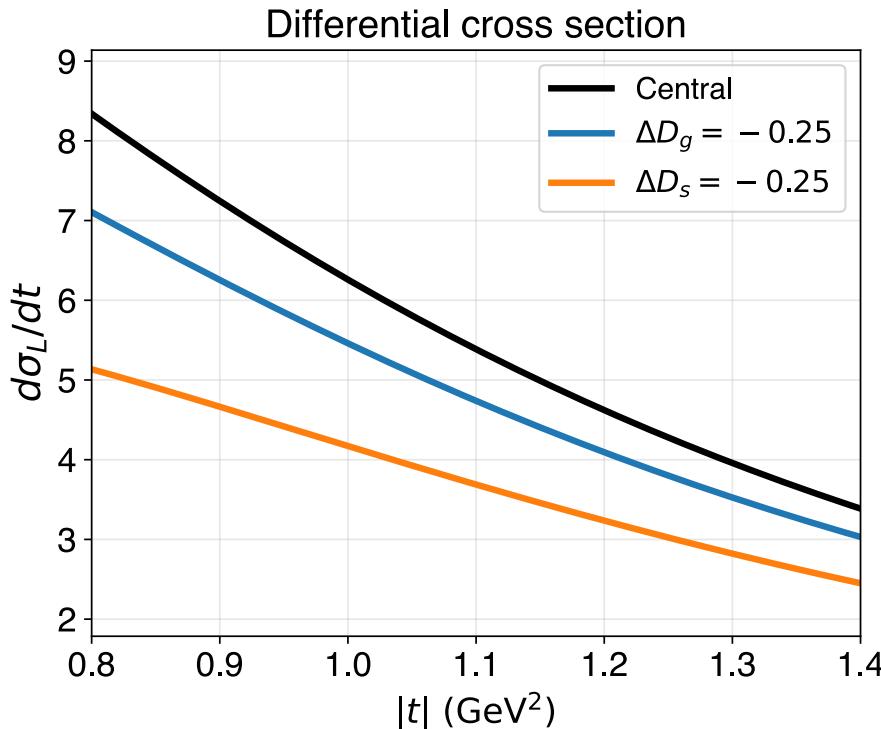
PAC52 - FINAL-STATE INTERACTIONS



- Many different coupled-channels considered in the final-state interaction calculation
- Gluon exchange dominates, but overall FSI cross section is tiny

[1] – S.H. Kim et al. **Dynamical model of ϕ meson photoproduction on the nucleon and ${}^4\text{He}$**

PAC52 - FLAVOR DEPENDENCE



Cross section significantly **more sensitive to D_s than D_g !**

PID STRATEGY

- In SHMS:
 - Electron ID'd with standard Calo+Cherenkov conditions
- In HMS:
 - Proton ID'd as slow TOF between scintillator planes, no Cherenkov signals
 - Kaon ID'd as fast TOF between scintillator planes and no Cherenkov signals
 - Timing w.r.t the RF may also provide some separation at larger momenta
 - Pion ID'd as fast TOF + Aerogel signal, but no gas Cherenkov signal
 - Positron ID'd as fast TOF, Aerogel signal, plus gas Cherenkov signal

Particle	TOF (fast)	TOF (slow)	TOF w.r.t. RF	Aerogel Cherenkov	Gas Cherenkov
Proton		✓			
Kaon	✓		✓		
Pion	✓			✓	
Positron	✓			✓	✓

The rate of single charged particles entering the spectrometer acceptances was determined using PYTHIA6 with no cuts on any kinematic variables and independently via the Wiser parameterization. We estimate based on the thickness and density of the aluminum target windows that around 10% of the rate will originate from the target windows. PYTHIA and Wiser both predict that the rate of random coincidences between a negatively charged particle in the SHMS acceptance and a positively charged particle in the HMS acceptance is around 650 Hz for a trigger coincidence time window of 70 ns. Assuming the trigger is formed by a coincidence of the HMS and SHMS hodoscopes, the rates are well within the capabilities of the data acquisition system and the livetime should be close to 100%. The rate of protons in the HMS is around 165 kHz, and the rate of electrons in the SHMS is around 25 kHz. The relevant singles rates are given in Tab. 3. The central momentum and angle of the HMS setting were chosen in part to minimize the singles rate.

	Total Rate SHMS	e^- Rate SHMS	Total Rate HMS	p^+ Rate HMS	K^+ Rate HMS
PYTHIA6	23 kHz	22.5 kHz	380 kHz	160 kHz	14 kHz
Wiser	26 kHz	25 kHz	370 kHz	170 kHz	21 kHz

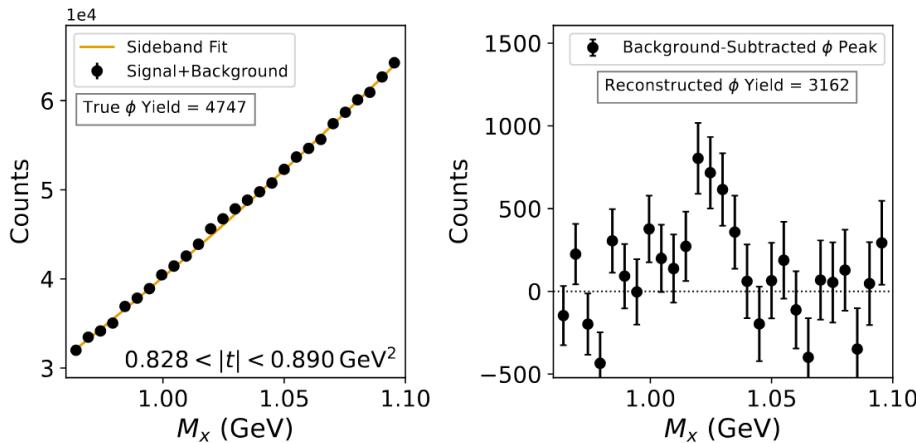
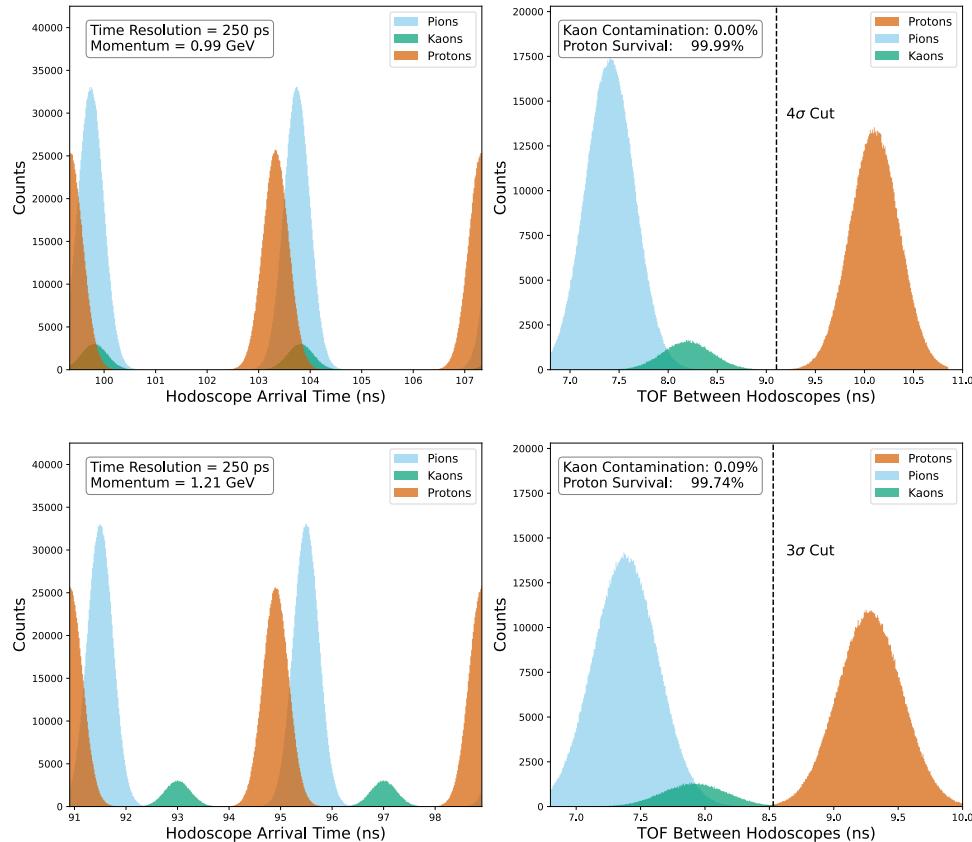
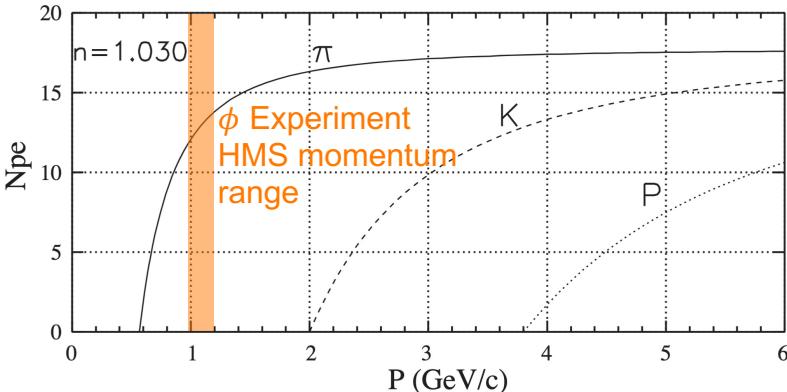


Figure 11: Example ϕ peak in the case of $D_s(0) = -0.5$. The statistical significance of the peak is around 4σ . The requirement that this peak can be resolved above background drives the requested integrated luminosity.

CAN WE DO U-CHANNEL?

- u -channel: baryon takes most of the virtual photon momentum
- Instead of $H(e, e'P)X$, can we do $H(e, e'K)X$ or $H(e, e'\pi)X$ with our dataset?
 - HMS Aerogel would likely be able to cover π/K separation
 - Kaons are below Cherenkov threshold, pions reasonably far above it



EXCLUSIVE PION PRODUCTION

[1]

- u -channel is sensitive to transition distribution amplitudes
 - Connected to **how baryon number is distributed inside of nucleons**^[1]

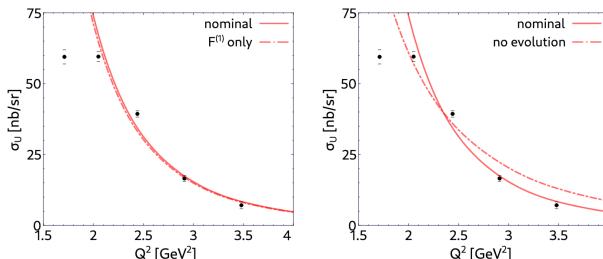
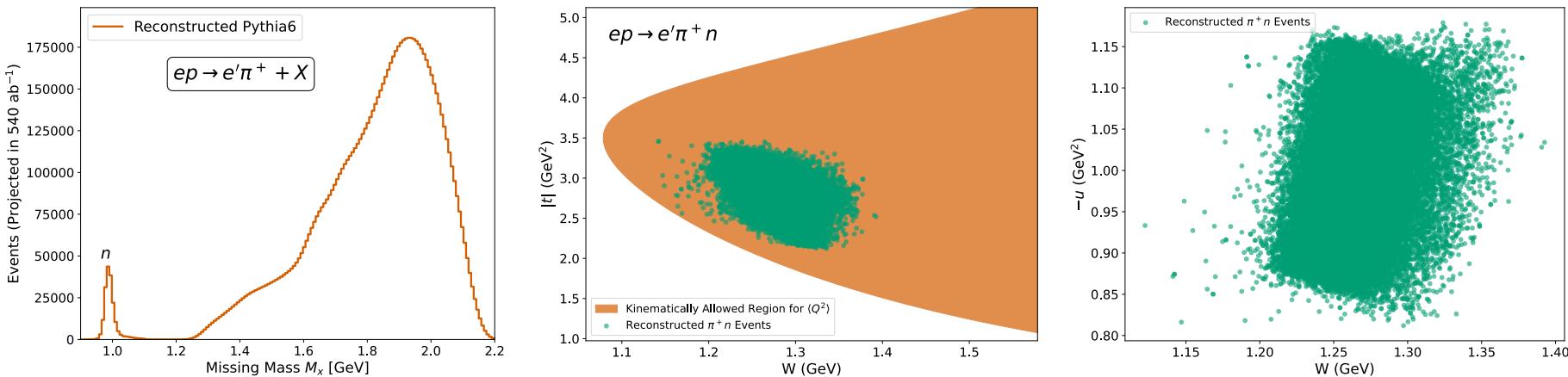
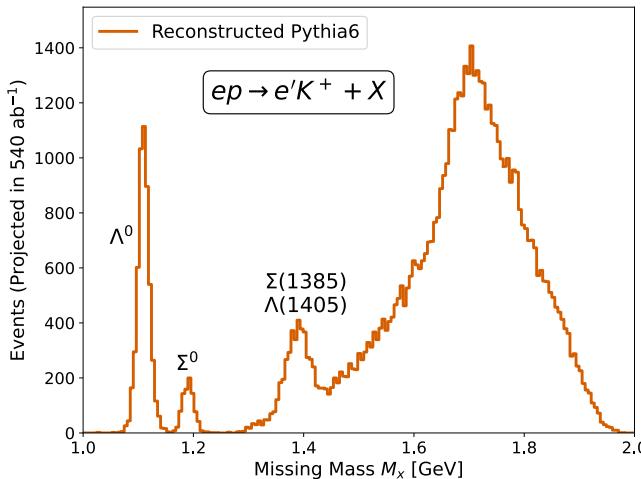


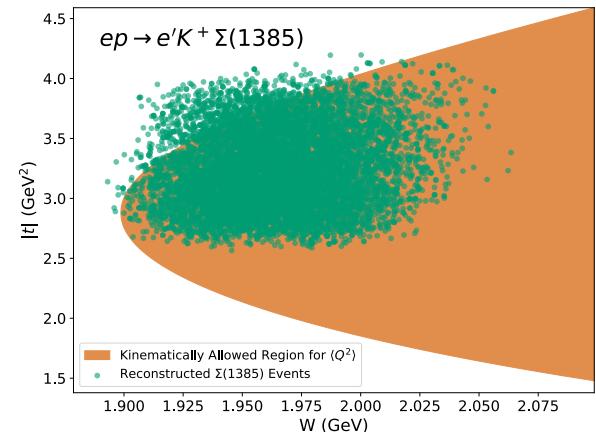
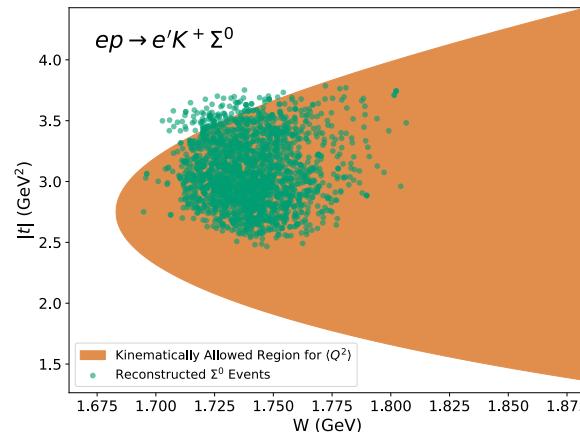
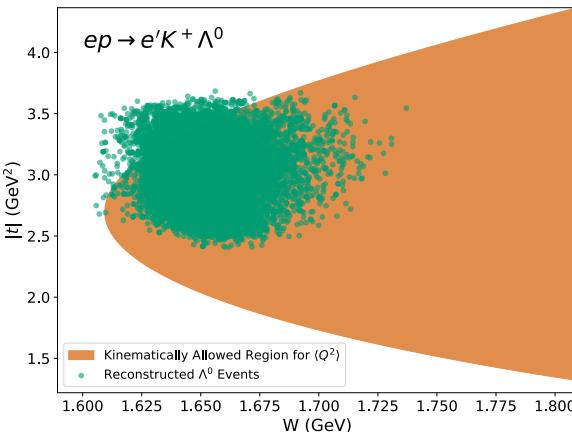
FIG. 5: CLAS data for the unseparated cross-section $\frac{d\sigma_U}{dQ^2}$ of the $\gamma^* p \rightarrow \pi^+ n$ reaction at backward angles [8], with $W = 2.2 \text{ GeV}$ and $u = -0.5 \text{ GeV}^2$. The solid curves represent the corresponding results obtained using the default version of our TDA model. The dash-dot curve in the left plot corresponds to a version of our model that includes only the second component, i.e., $F^{(1)}(\sigma, \rho, \omega, \nu, u)$, with $F^{(0)}(\kappa, \theta, \mu, \lambda, u)$ set to zero. The dotted curve shown in the right plot represents the modeling scenario when all evolution effects are neglected.



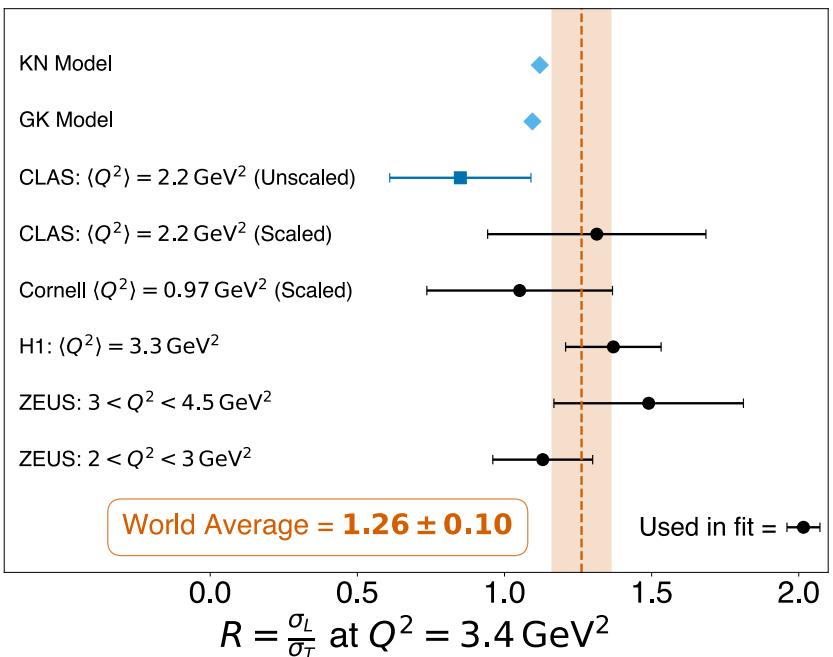
WE CAN DO U-CHANNEL!



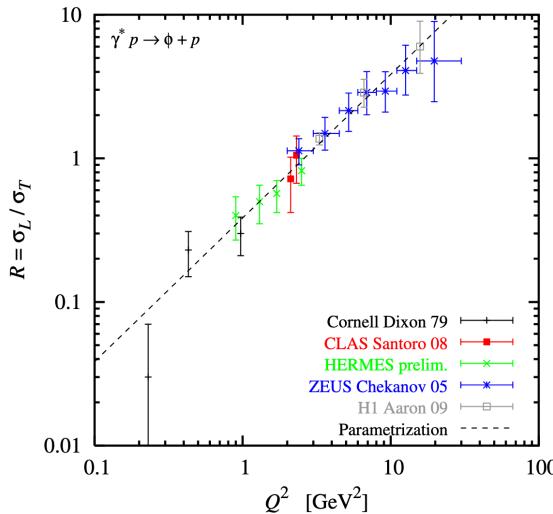
- Near-threshold, u-channel hyperon production is accessible if K^+ can be efficiently ID'd
- Likely requires refurbishment of HMS aerogel
- Note, PYTHIA6 resonance region cross sections are unreliable (especially in u-channel)
 - However, SIMC acceptance is correct, so these hyperons are well within our acceptance



GETTING $d\sigma_L/d|t|$

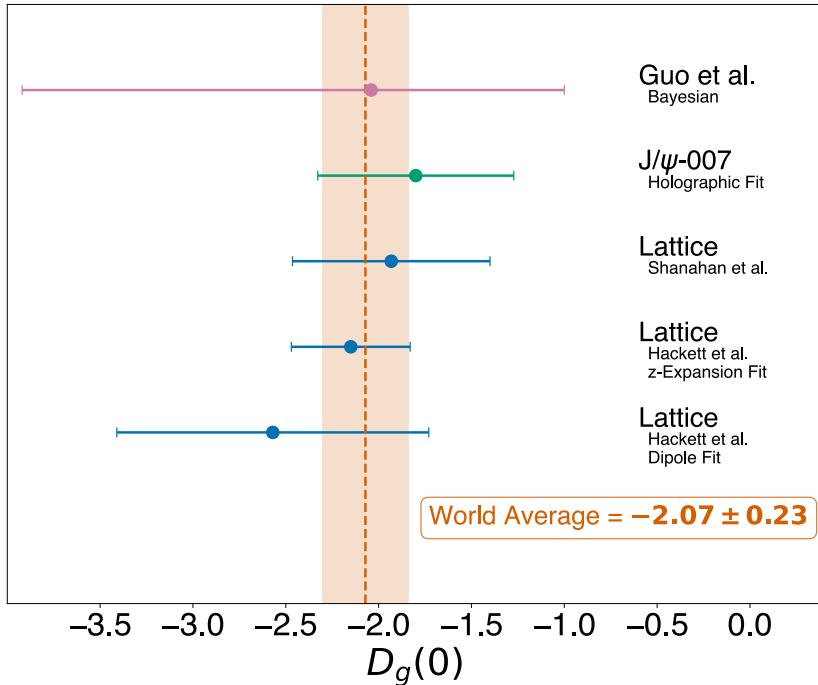


- With $d\sigma_e/d|t|$, need R to get $d\sigma_L/d|t|$
 - Fit the world data to get an idea (and uncertainty) on this quantity within our phase space ($Q^2 \sim 3.4 \text{ GeV}^2$)
- World data suggests $R(Q^2)$ **not** $R(Q^2, W, |t|)$

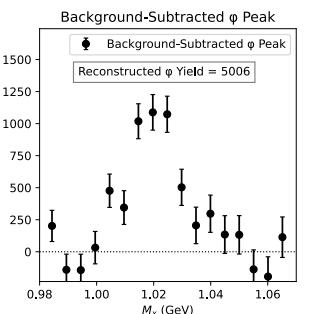
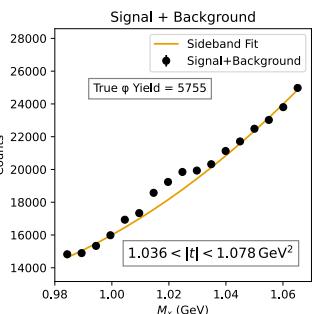
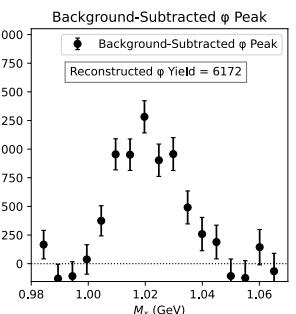
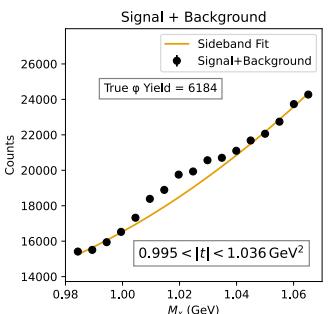
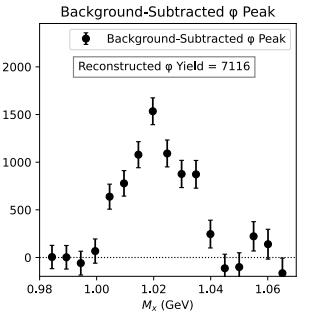
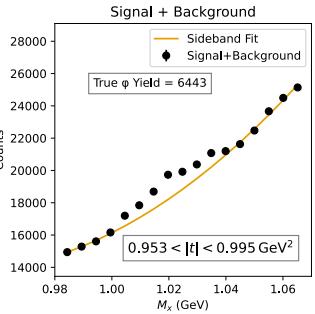
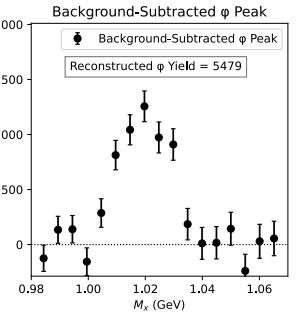
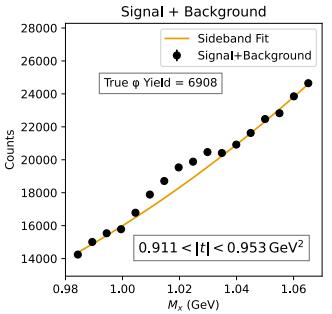
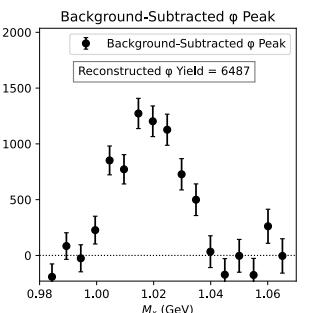
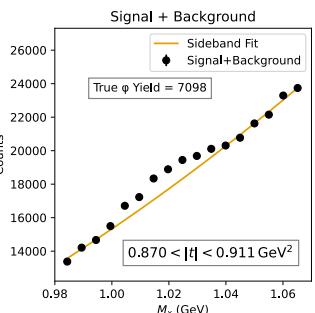
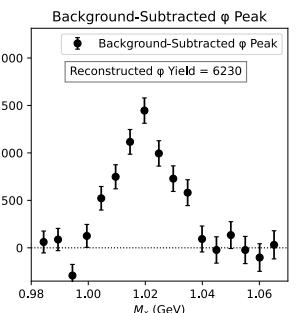
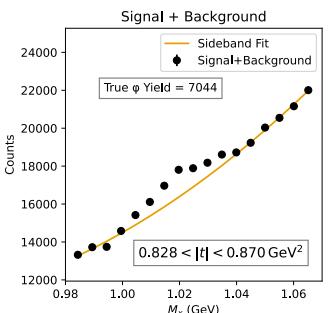


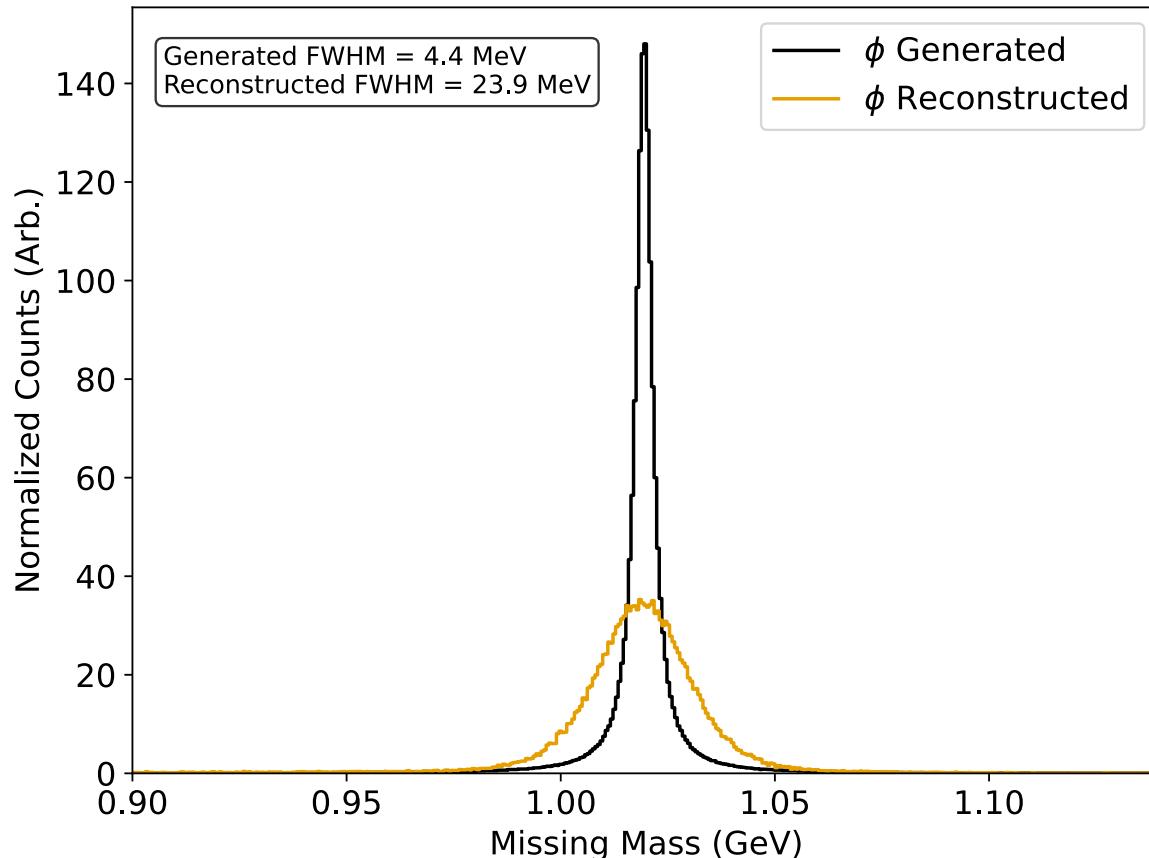
- Use CLAS12 parameterization to scale nearby world datapoints

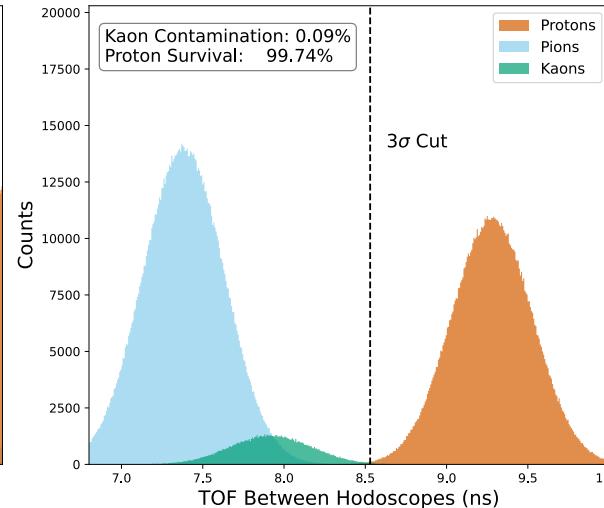
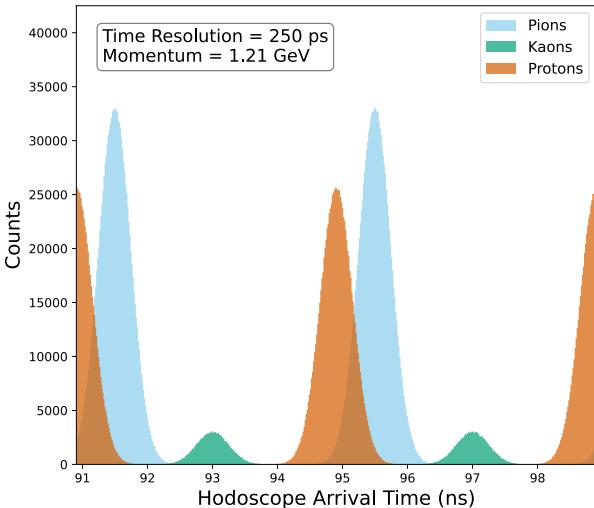
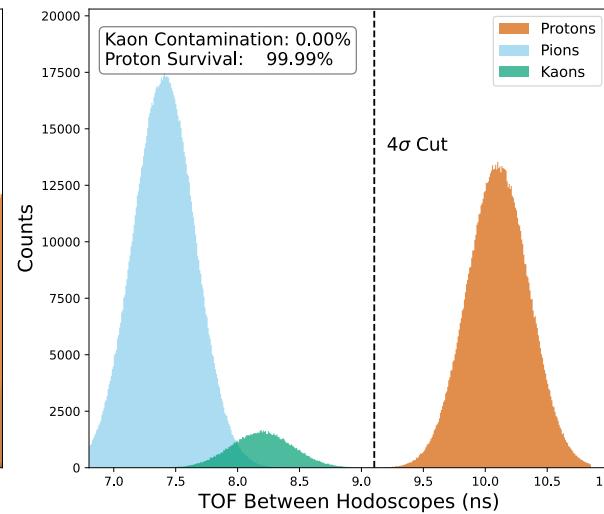
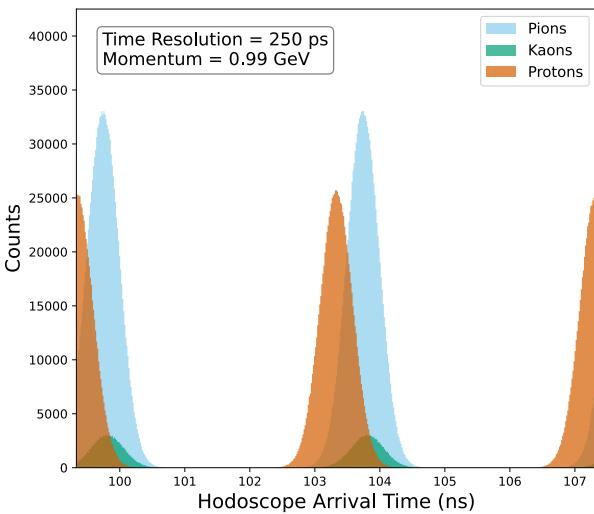
GETTING D_g



- Sensitivity of cross section to D_g isn't as large as D_s , but large uncertainties on D_g can still rain on our parade
 - Average the results of lattice + Hall C data + Guo/Yuan Bayesian analysis
 - Hopefully should more results soon (CLAS12)
 - Can also include some theoretical values in here if they seem realistic
- **In the end, it's obvious that a global fit to both D_g and D_s is the way to go!**







THEORY PREDICTIONS

Hatta, HK, Passek, Schoenleber (2501.12343)

- New predictions available from Hatta et al. using GPD framework in the near-threshold region

- Typical issue for GPDs near-threshold is final-state interactions
- FSI calculated to be 2-3 orders of magnitude smaller than production cross section for $\phi + p$ in photoproduction (S. H. Kim et al.)

- Theoretical uncertainty on cross section from this approximation is $\sim 10\%$ or less for $\xi > 0.3$!
- Focus on high ξ

$$\frac{d\sigma_L}{dt} = \frac{2\pi^2 \alpha_{em}}{(W^2 - M^2) W p_{cm}} \left((1 - \xi^2) |\mathcal{H}|^2 - \left(\frac{t}{4M^2} + \xi^2 \right) |\mathcal{E}|^2 - 2\xi^2 \text{Re}(\mathcal{H}\mathcal{E}^*) \right)$$

$$\begin{pmatrix} \mathcal{H}(\xi, t) \\ \mathcal{E}(\xi, t) \end{pmatrix} = \kappa \sum_{j=1}^{\text{odd}} \sum_{k=0}^{\text{even}} \sum_a \frac{2}{\xi^{j+1}} \begin{pmatrix} H_j^a(\xi, t) \\ E_j^a(\xi, t) \end{pmatrix} T_{jk}^a(\xi) \varphi_k, \quad \kappa \equiv e_s \frac{C_F f_\phi}{N_c Q}.$$



“Threshold Approximation” – Keep only $j = 1$

$$\begin{pmatrix} \mathcal{H}(\xi, t) \\ \mathcal{E}(\xi, t) \end{pmatrix} \approx \frac{2\kappa}{\xi^2} \sum_a \begin{pmatrix} H_1^a(\xi, t) \\ E_1^a(\xi, t) \end{pmatrix} T_{10}^a(\xi, \mu^2).$$

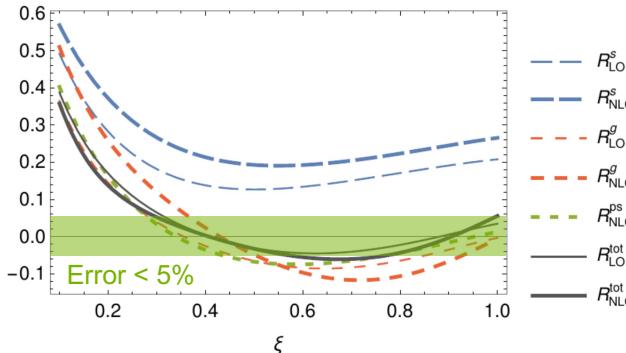
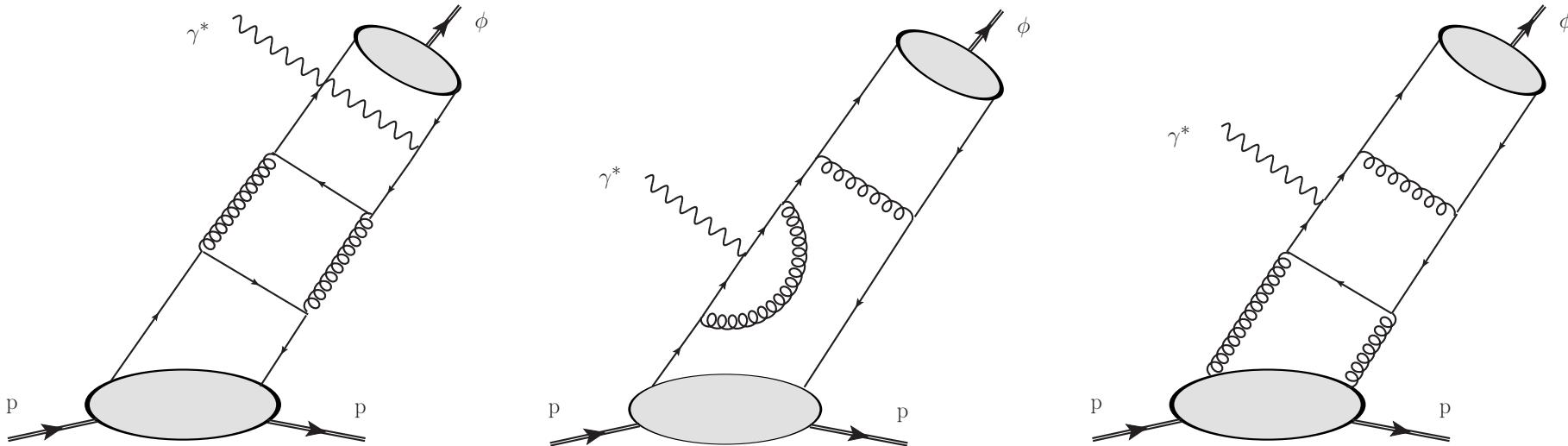


FIG. 4: Relative error for the amplitude \mathcal{H} from truncating the conformal partial wave expansion after the first term. Plotted quantities are defined in (40). The subscript denotes whether the leading order (LO) or next-to-leading order (NLO) coefficient function has been used. In this and the next plots, we have set $t = t_{\min}(\xi)$, $\alpha_s = 0.3$ and $\kappa = 1$.

THEORY PREDICTIONS

Hatta, HK, Passek, Schoenleber (2501.12343)



THEORY PREDICTIONS

D_s is a non-negligible contributor to the total D -term, and thus necessary for a full extraction of many of the mechanical properties of the proton!

