Duke

NNLO RC Correction in PRad2 with McMule and Status

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PRad-II/X17 Collaboration Meeting

Outline

- Introduction to McMule
- Implementation of NNLO RC corrections with McMule
- Interfaces built between PRad GEANT4 simulation and McMule
- Testing results
- Summary

About McMule

- McMule: Monte Carlo for MUons and other LEptons
  - a high-precision framework for theoretical calculations in QED
  - Core Function
    - Calculates physical cross-sections for various scattering processes
    - Provides predictions for direct comparison with experimental data
  - High Precision
  - Processes supported:
    - All leptonic $2 \to 2$ scattering processes
    - Other processes, i.e. particle decay

process	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$\ell p \to \ell p$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO
$eN \to eN$	PRad, ULQ2	background	+
$e^-e^- \to e^-e^-$	Prad 2	normalisation	NNLO
	MOLLER, ...	$\sin^2\theta_W$ at low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \ell\ell$	VEPP, BES, Daphne, ...	R-ratio	NNLO±
	Belle	τ properties	
$ee \to \gamma\gamma$	Daphne	dark searches	NNLO−
	any e^+e^- collider	luminosity measurement	
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO−
$\mu \to \nu\bar{\nu}e$	MEG	ALP searches	NNLO+
	DUNE	beam-line profiling	

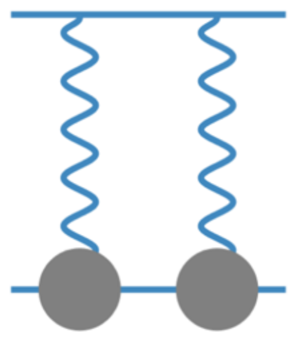
Part of processes supported by McMule

f.l.t.r.: S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), V.Sharkovska (Zurich & PSI), S.Gündogdu (Zurich & PSI), D. Moreno (PSI), A.Coutinho (IFIC), Y.Ulrich (Liverpool), D. Radic (Zurich & PSI), L.Naterop (Zurich & PSI), M.Rocco (Turin)
not shown: F.Hagelstein (Mainz), N.Schalch (Oxford), T.Engel (Freiburg), A.Gurgone (Pavia), P.Banerjee (Cosenza)

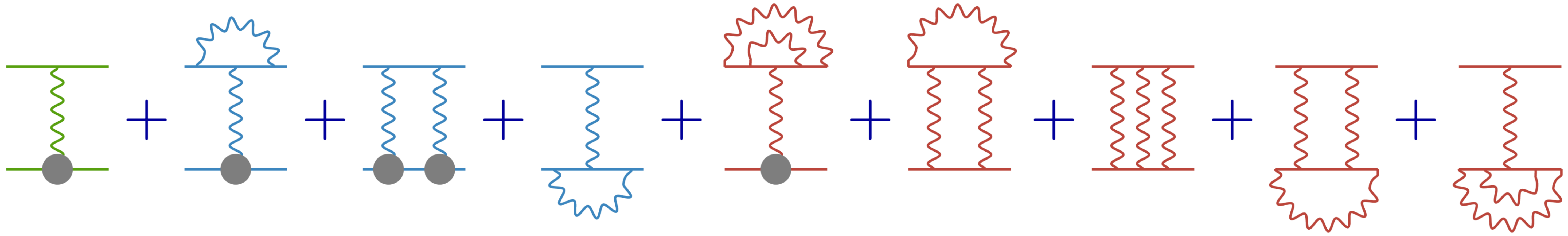
codes: https://mule-tools.gitlab.io/
docs: https://mcmule.readthedocs.io/

Processes relevant to PRad2

- Electron-proton elastic scattering
  - Precision: NNLO
  - Point-like proton assumed in some high order diagrams
  - Form factors: Support only dipole or monopole in TPE diagram
  - Form factors: rational(1, 1) is supported in others



$$e^- p \to e^- p : \quad \sigma \supset$$

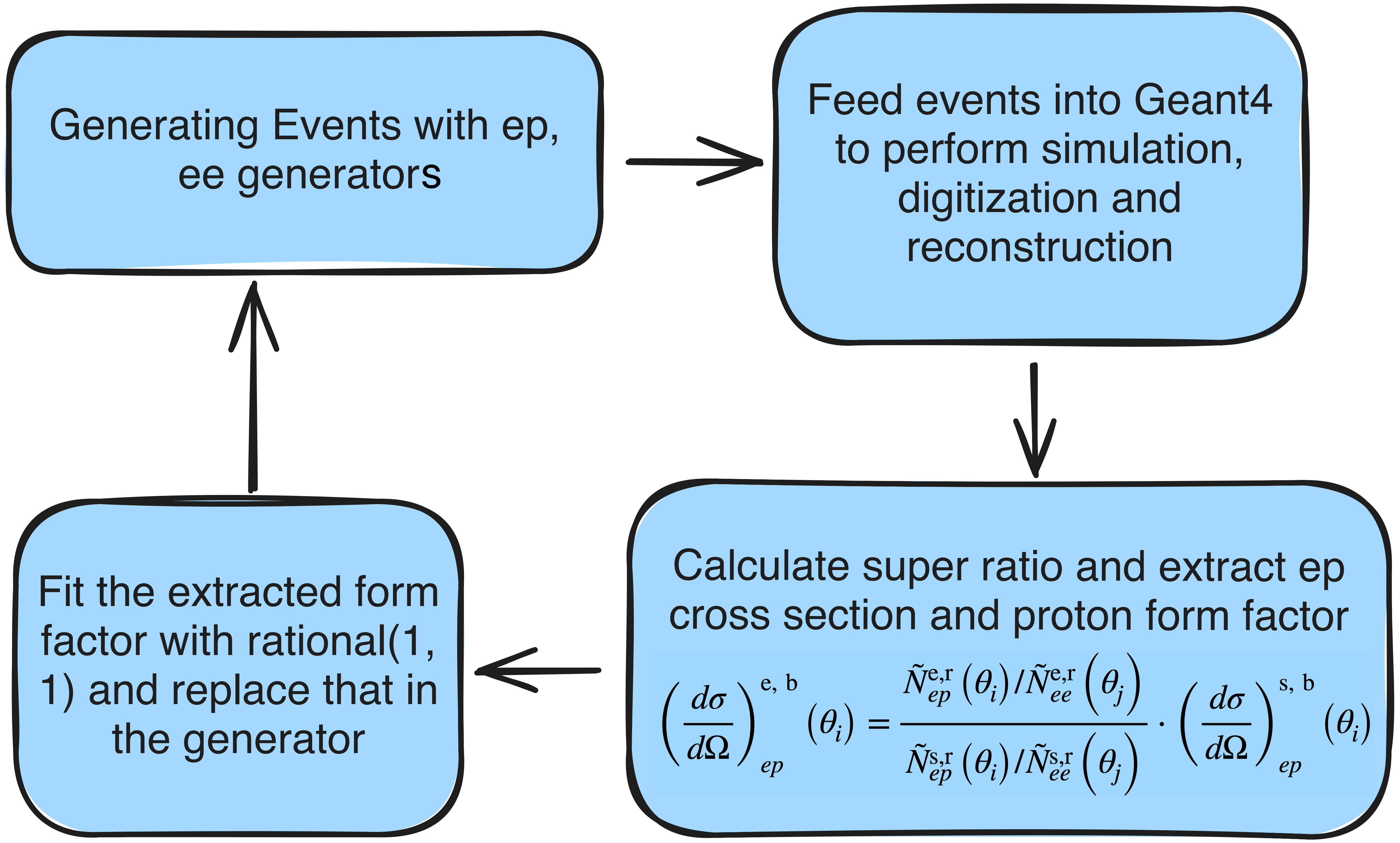


- Moller scattering
  - Precision: NNLO

Strategy to implement NNLO RC with McMule

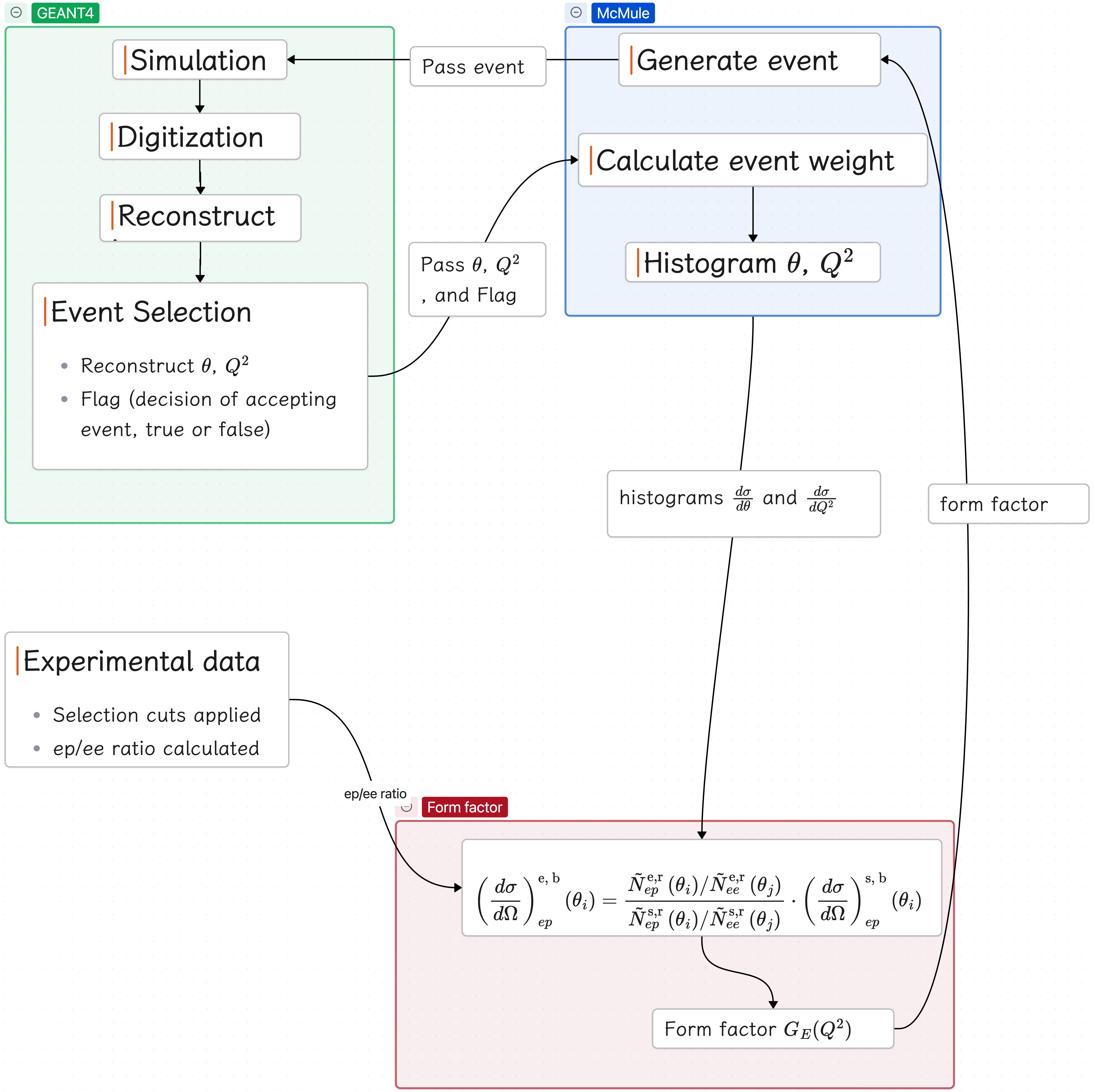
- A fast, O(10ms) per event GEANT 4 simulation should be built by PRad/PRad2
- The simulation can be arbitrary complex as long as it is fast
- The simulation tells McMule whether an event is accepted and what reconstructed kinematics are
- PRad/PRad2 will benefit from the best theory precision
- Interfaces between PRad/PRad2 simulation and McMule should be developed

RC Corrections in PRad Analysis



- Iteration procedure applied to perform RC correction
- Events generation and simulation are separated

Current strategy for the NNLO RC Correction



Event generation and processing at event level

1. Event generation and processing
   a. McMule passes an event to GEANT4 (Interfaces required)
   b. GEANT4 performs simulation, digitization, reconstruction, and event selection
   c. GEANT4 returns event accepting flag and reconstructed θ, Q^2 to McMule (Interfaces required)
   d. McMule fills histograms $d\sigma/d\theta$, $d\sigma/dQ^2$ with such info
   e. Repeat a→d for required statistics (event loop controlled by McMule)
2. Form super ratio and extract form factor
   a. McMule outputs the final histograms
   b. PRad2 calculates the super ratio and extract the form factor
   c. Update the form factor in McMule with the new fit (Interfaces required)
3. Repeat 1→2 for the next iteration

Interfaces and Status

- For testing purposes, interfaces are developed based on PRadSim used in the PRad analysis (easy to migrate into PRad2Sim)
- Interfaces developed
  - To update form factor used in McMule (developed by McMule team)
  - To accept events from McMule into PRadSim (developed by Duke)
  - To provide θ, Q^2, and accepting flag to McMule
    - Developed by Duke

Analysis chain in PRad GEANT4 simulation (PRadSim) → Digitization (PRadDig) → Event reconstruction (PRadRec) → Event Selection (SimYield)

  - Status:
    - Interfaces using PRadSim are done.
    - Migrate to PRad2Sim once the analysis chain is ready
- The McMule-PRadSim package can run standalone or with McMule

List of testings

- **Input:** Events from *newee* and *newep* generators
- **Test 1 (Done):** Compare the GEANT4 root tree from standalone PRadSim and McMule-PRadSim
  - Goal: Ensure new codes don't impact GEANT4 simulation
- **Test2 (Done):** Compare the output histograms used in the PRad analysis
  - Goal: Ensure event-level PRadSim->PRadDig->PRadRec->SimYield is consistent.
- **Test3 (Ongoing):** Test McMule-PRadSim with NLO McMule
  - Goal: Calculate super ratio and extract form factor and proton charge radius and compare with PRad results

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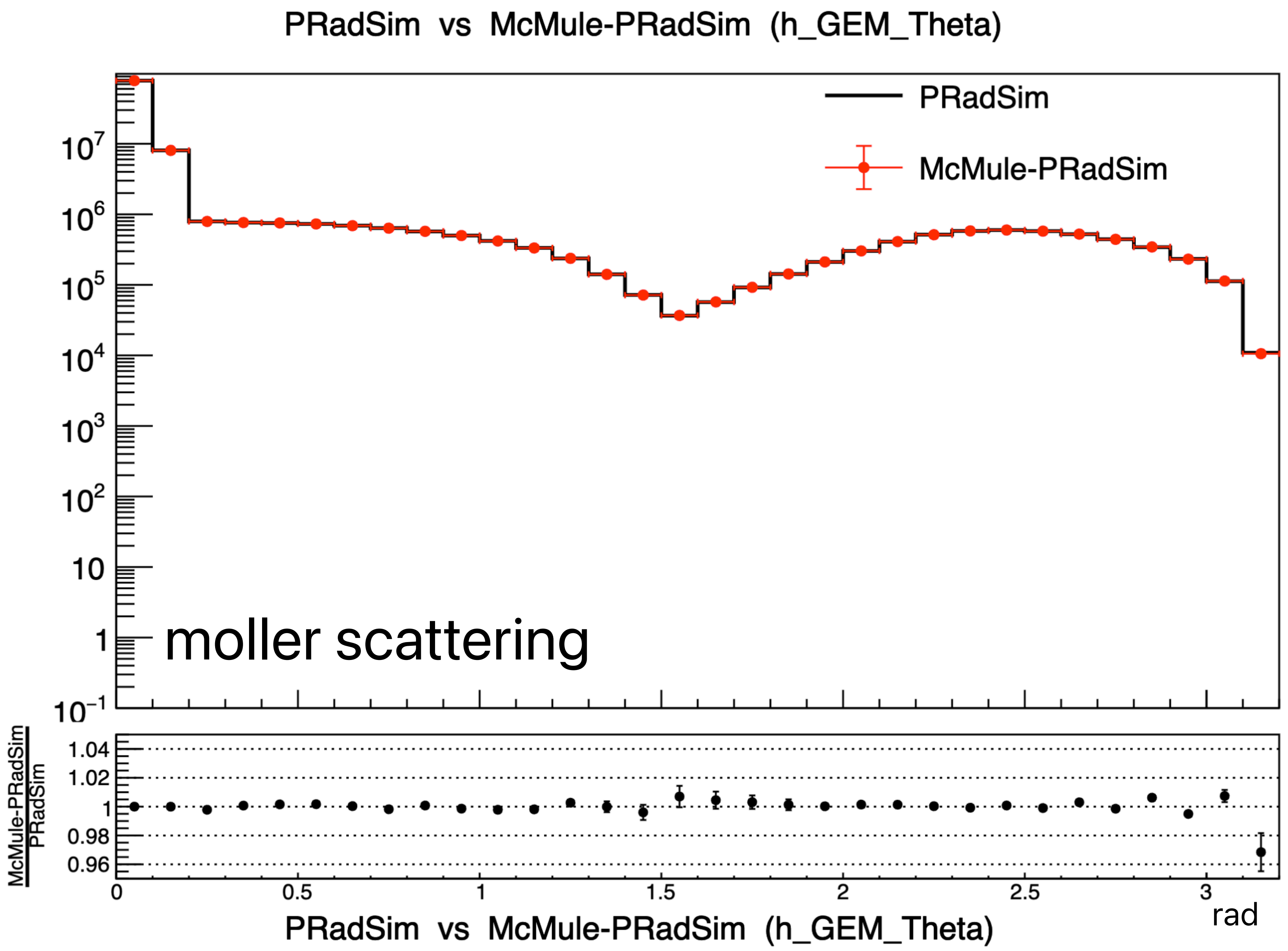
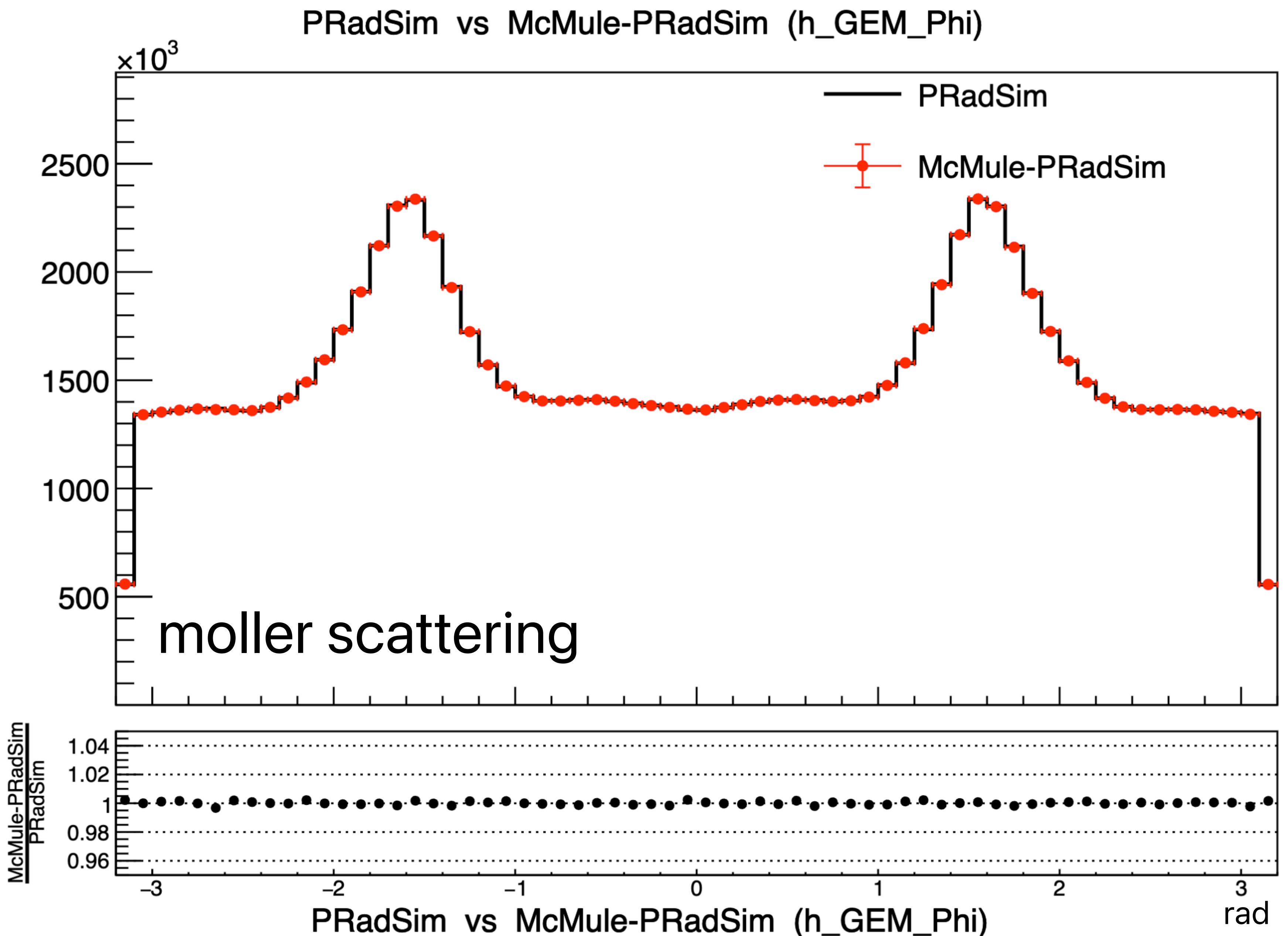
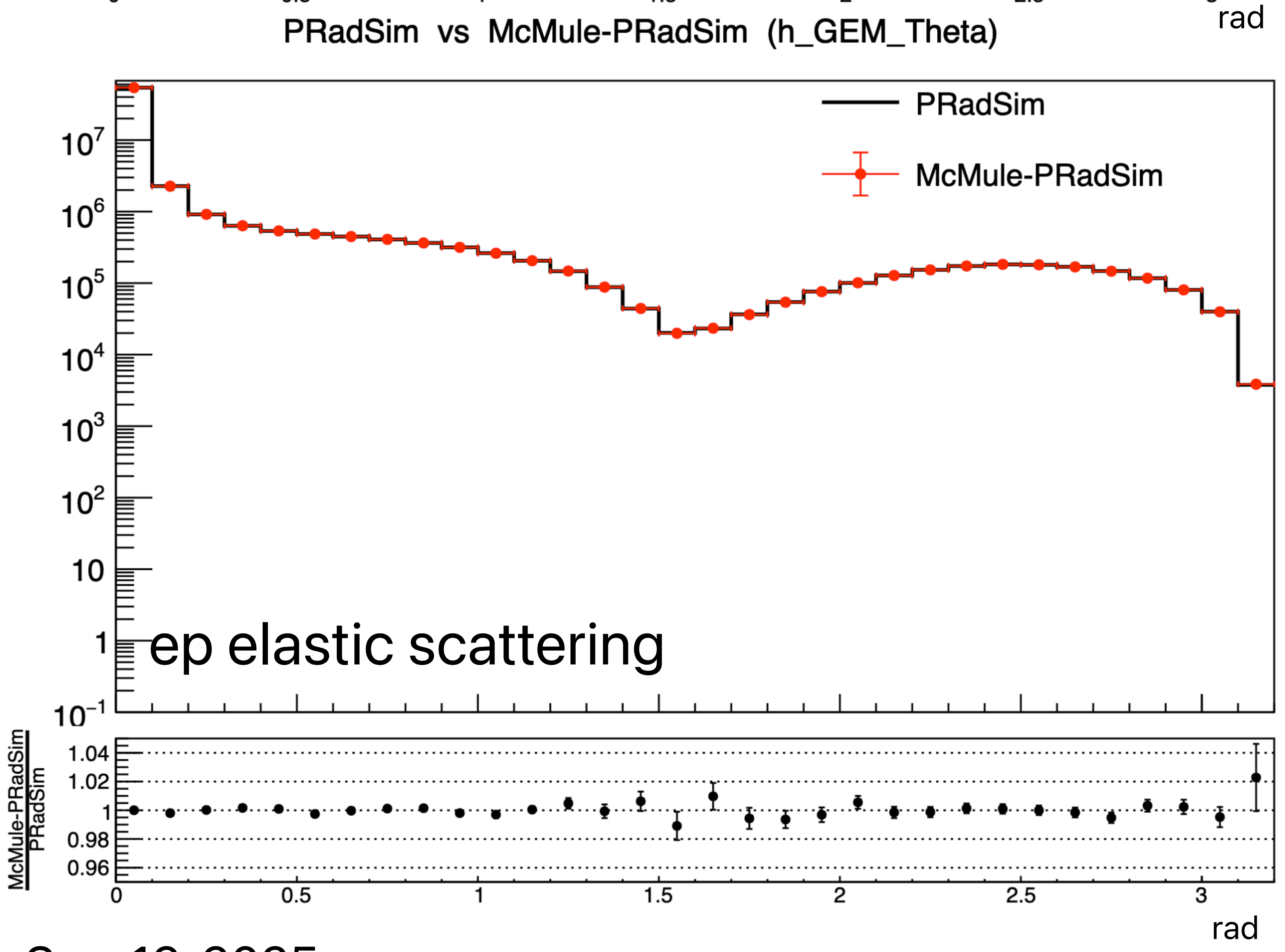
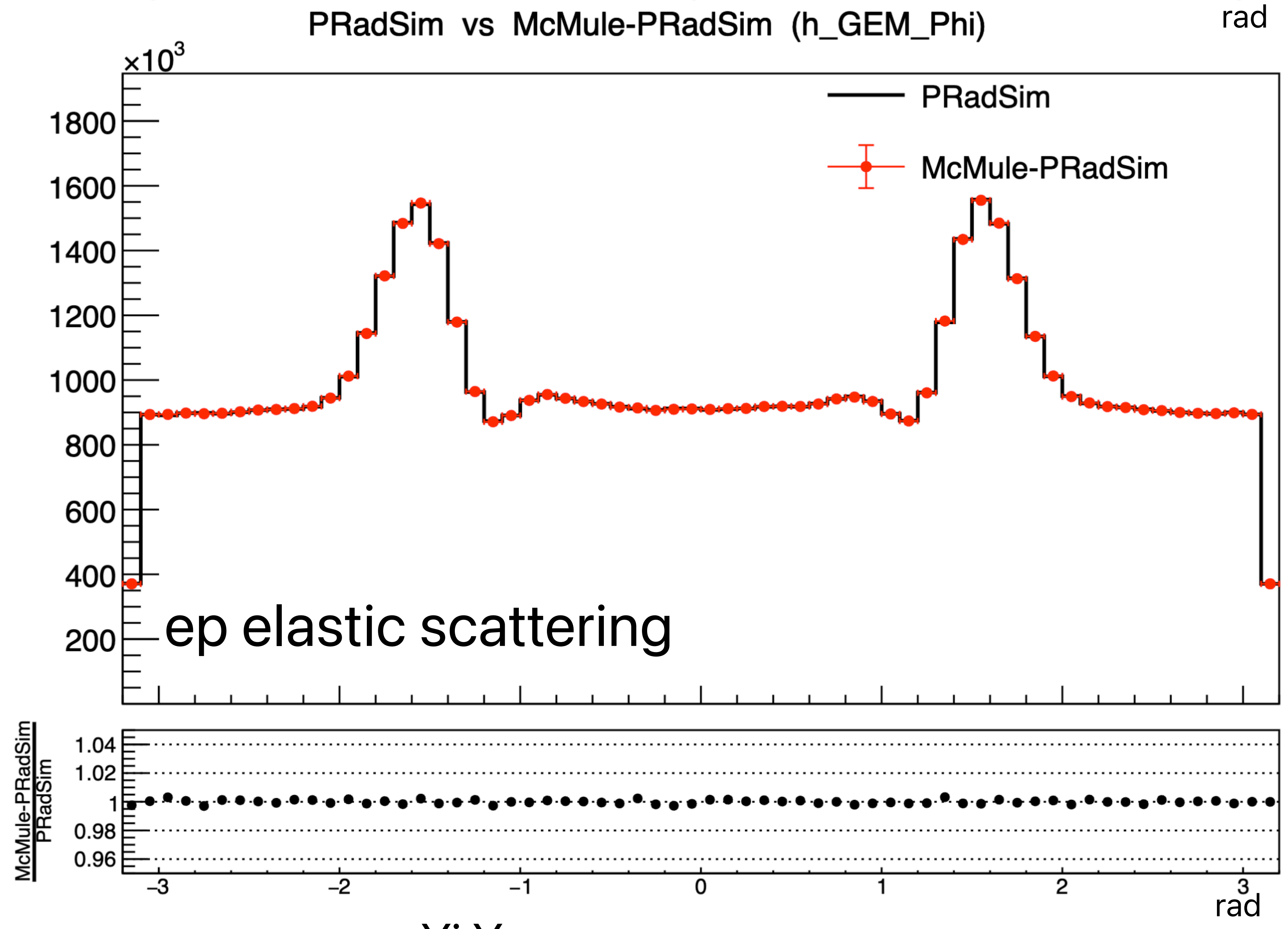
Test 1



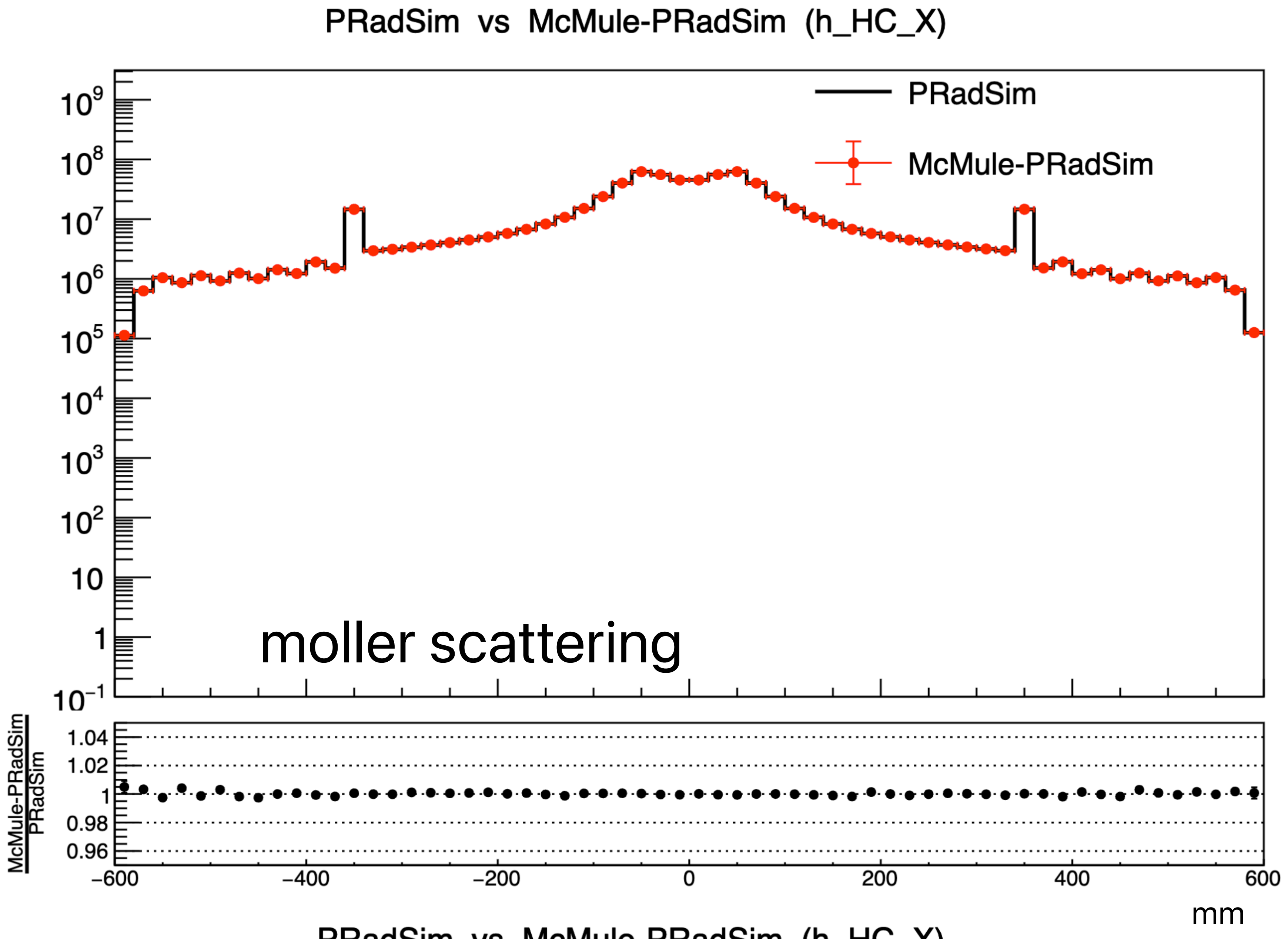


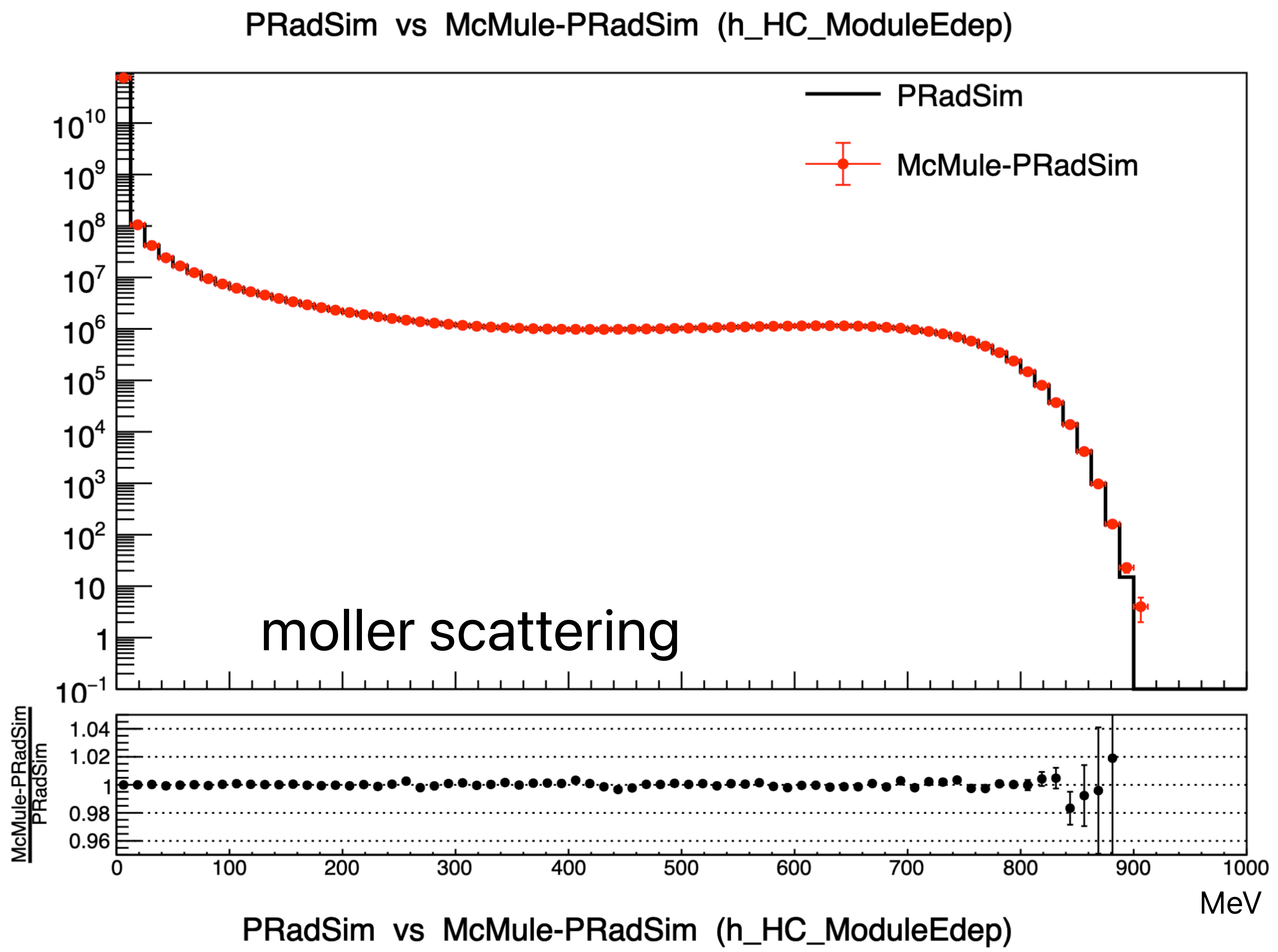


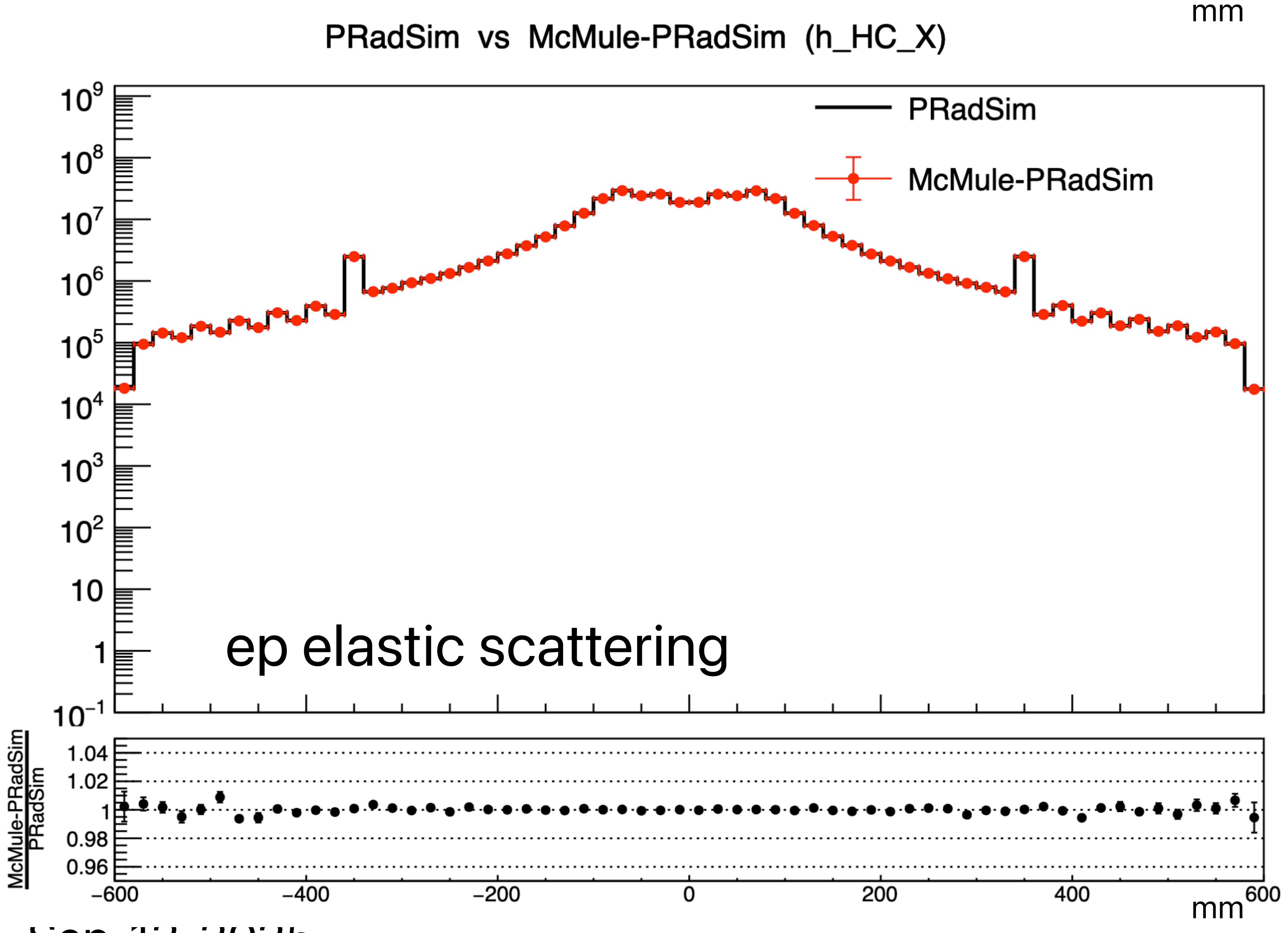


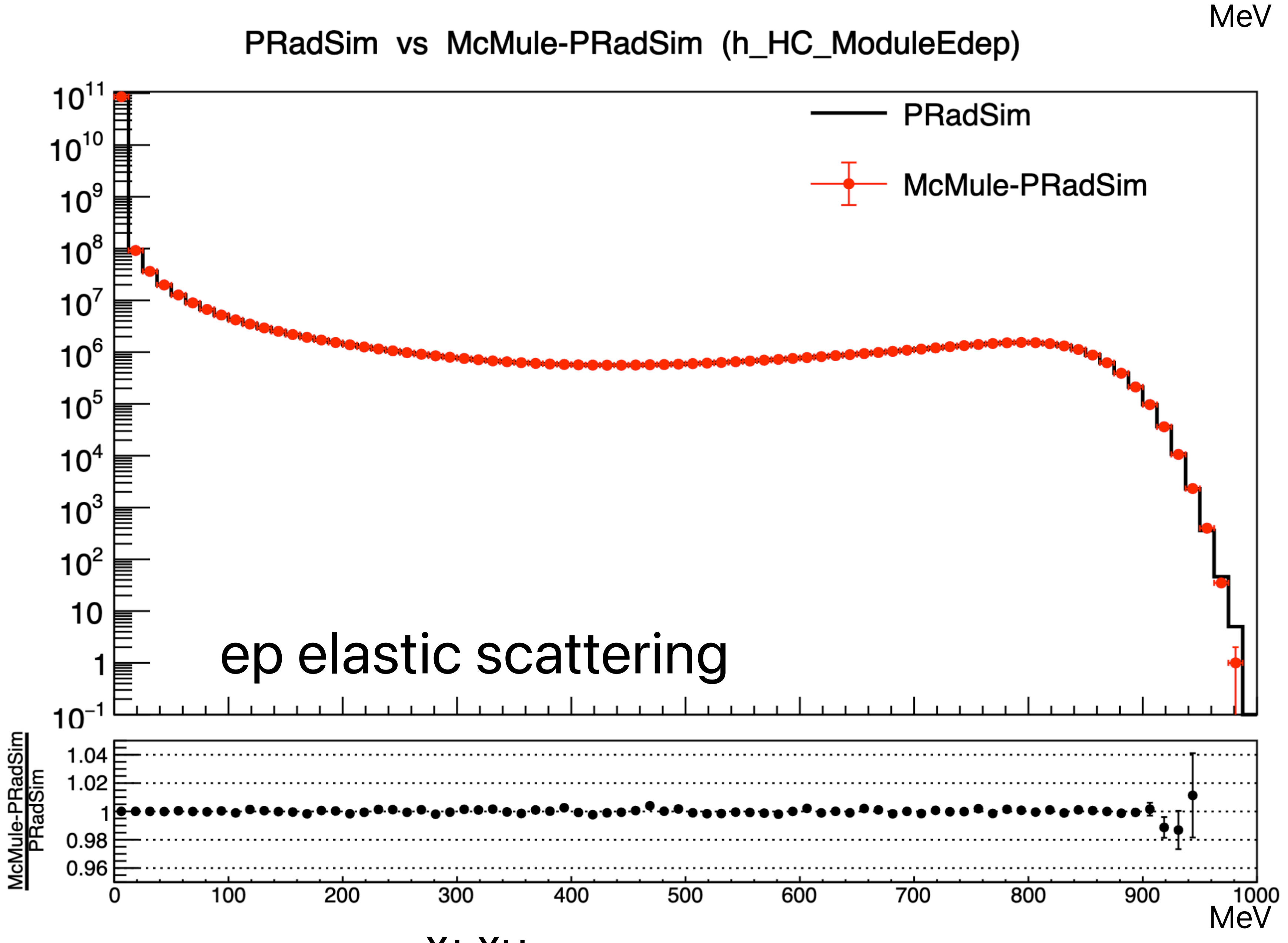
- **Inputs:** 50M events
- **Comparisons:** GEM hits θ and ϕ
- Results for beam energy 1101 MeV are shown as examples
- Conclusion: Very good agreements

Test 1



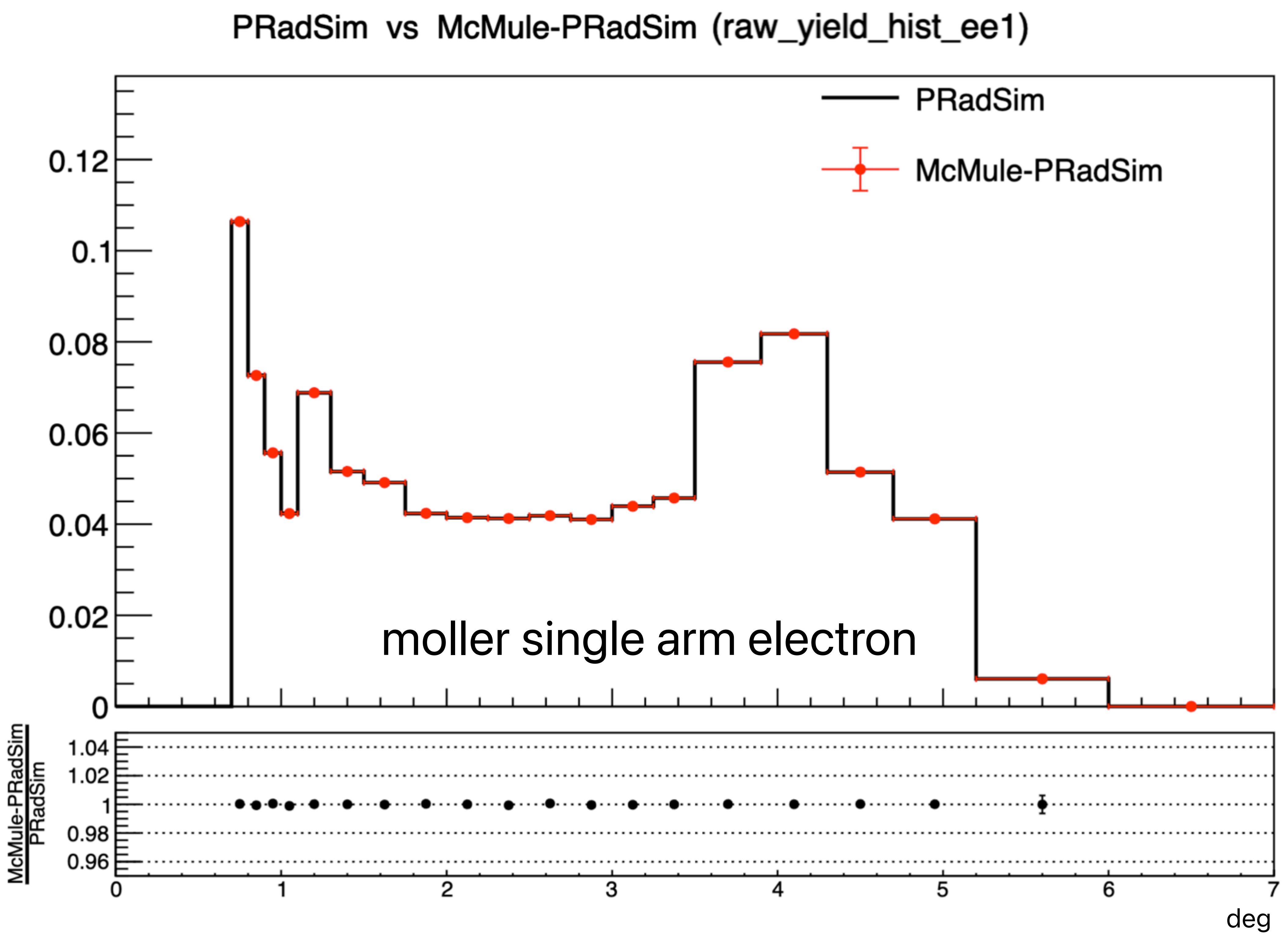


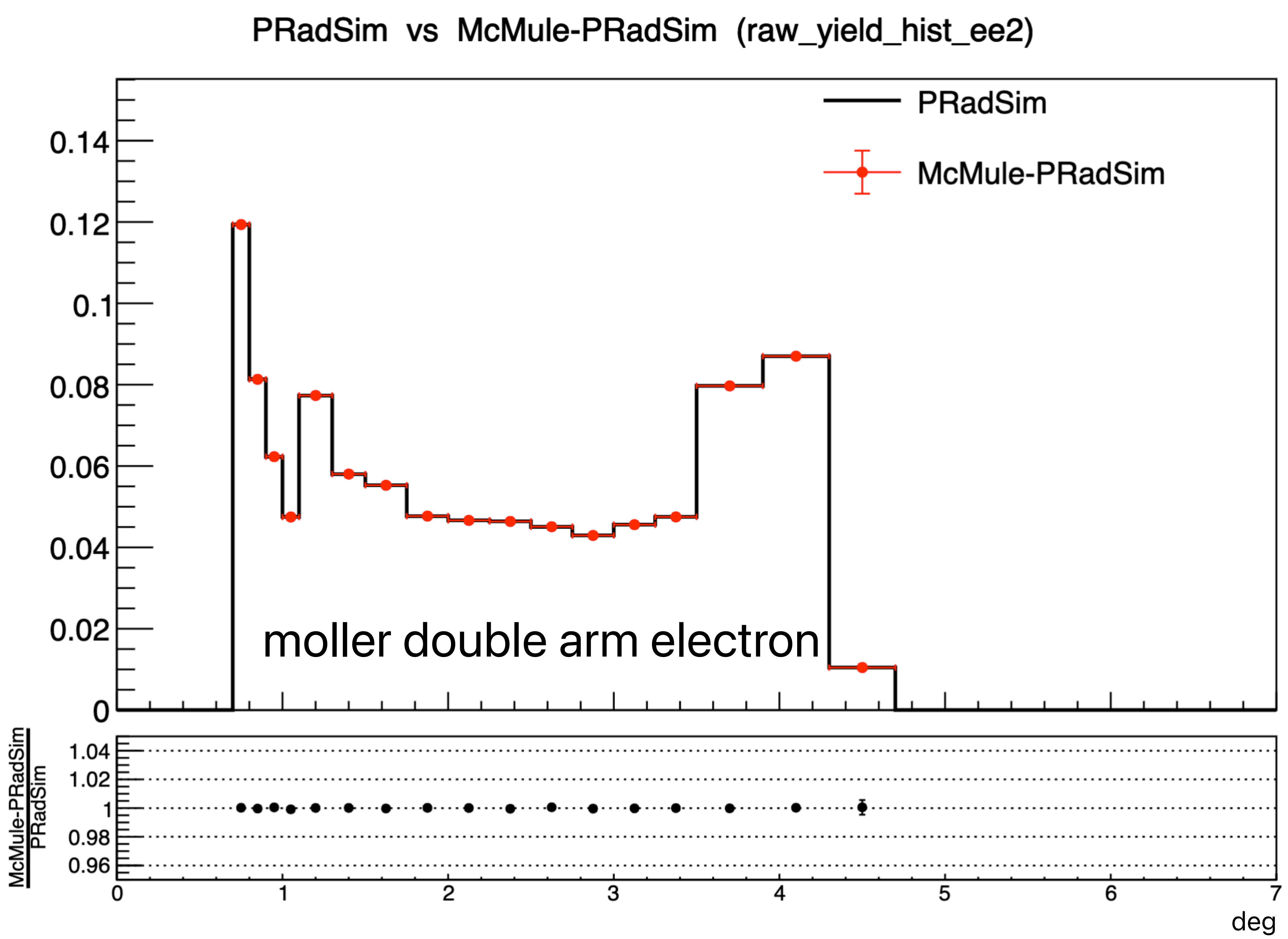




- **Inputs:** 50M events
- **Comparisons:** hycal hits x and module energy deposition
- Results for beam energy 1101 MeV are shown as examples
- Conclusion: Very good agreements

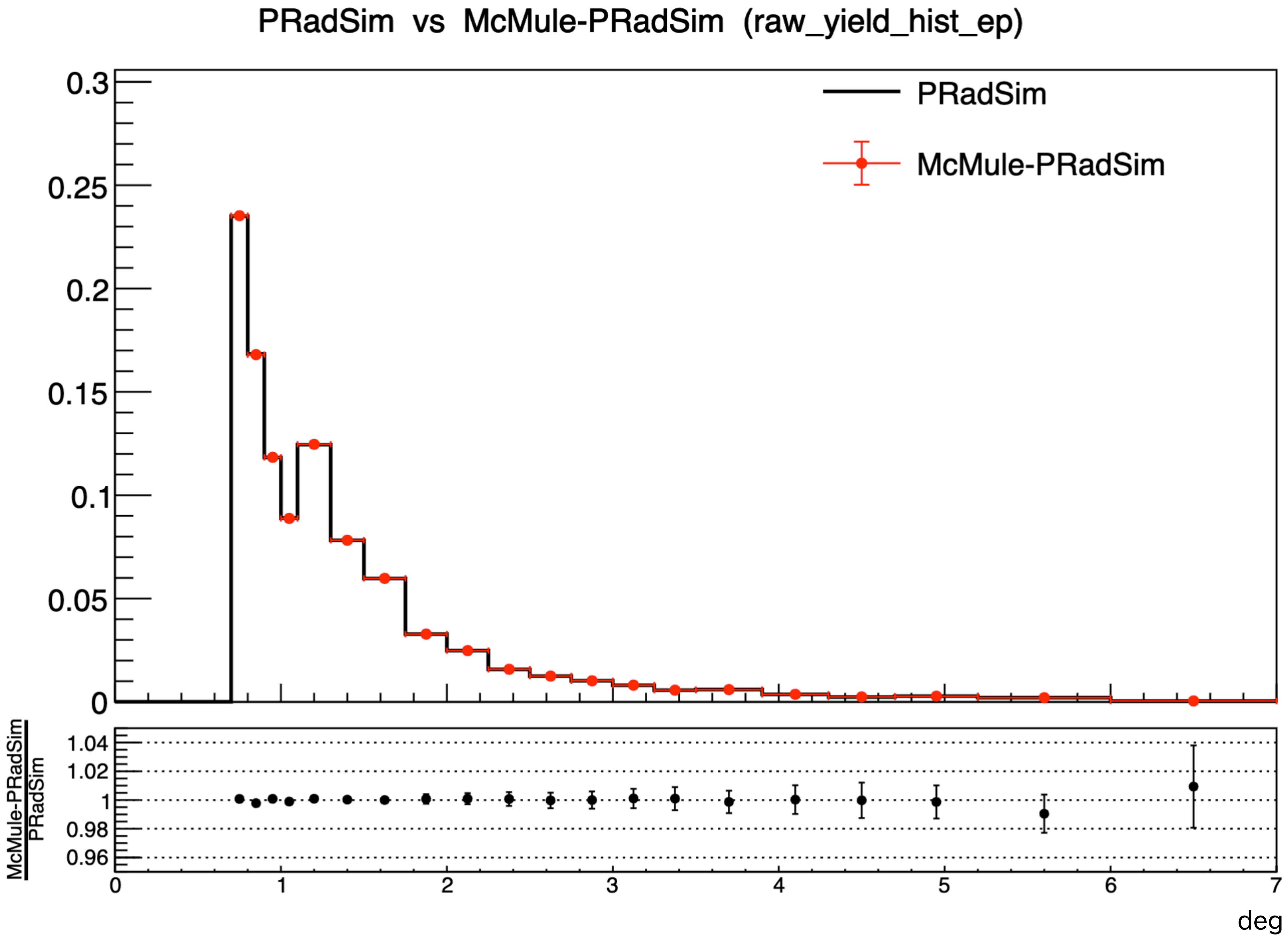
Test 2

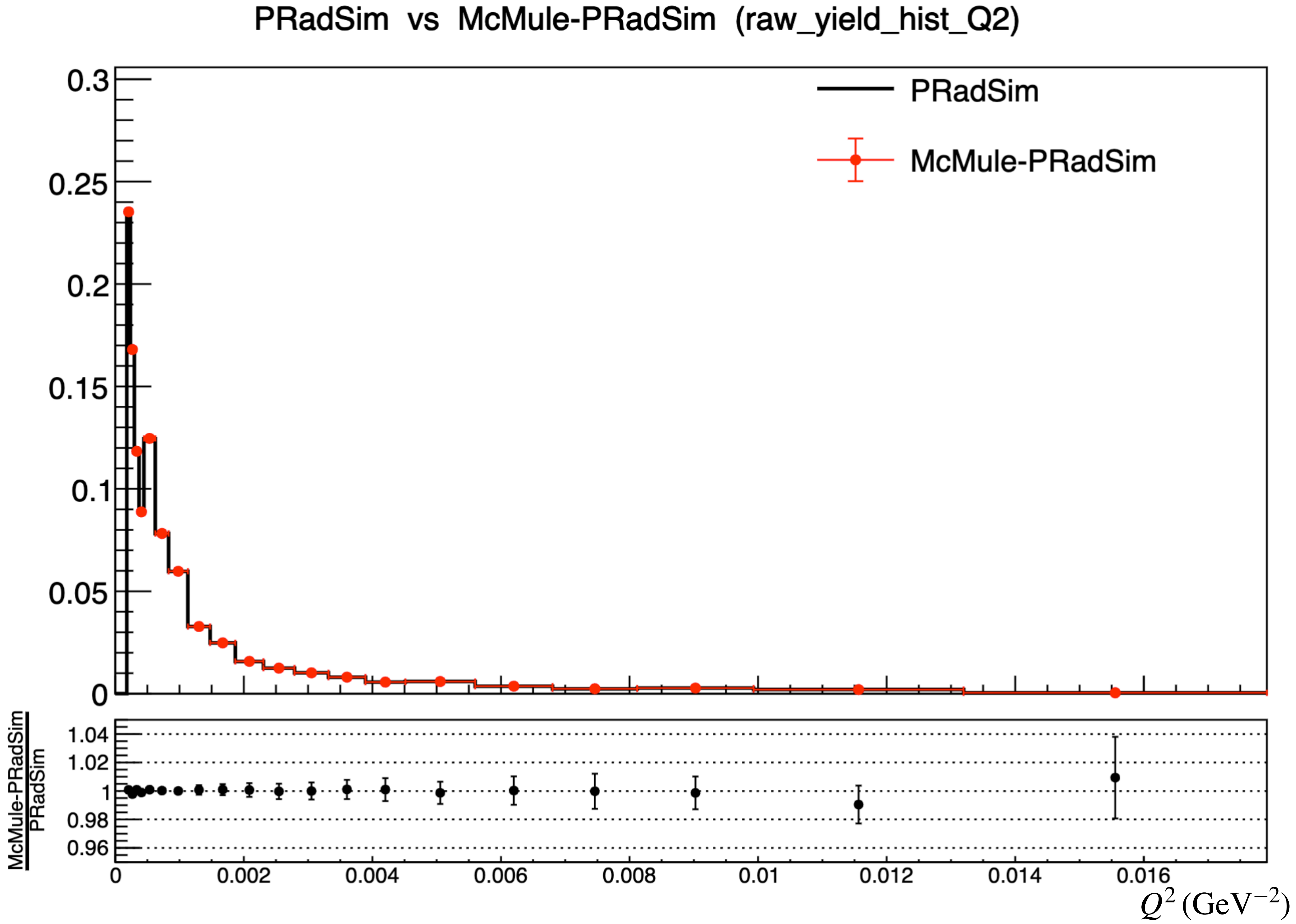




- Moller scattering with Ebeam 1101 MeV
- Normalized yield vs scattering angle for both single arm electron and double arm electrons
- Results are in very good agreement

Test 2





- e-p elastic scattering with beam energy 1101 MeV
- Normalized yield vs scattering angle and Q^2
- Results are in very good agreement

Summary and Todo

- Summary
  - Interfaces between McMule and PRadSim are ready (McMule-PRadSim)
  - Standalone running of McMule-PRadSim gives consistent results with the default analysis
- Todo
  - Test3: Complete McMule-PRadSim test with NLO McMule, and compare with PRad result
  - Run McMule-PRadSim with NNLO McMule, and update proton charge radius for PRad
  - Migrate interfaces to PRad2 GEANT4 simulation
    - Migration is expected to be straightforward
    - Requires updates based on PRad2 analysis (digitization, reconstruction, and event selection)
  - Future studies (for McMule team)
    - Impact of the point-like proton assumption used in higher order diagrams
    - TPE with other form factor functional forms (e.g. rational(1, 1))
    - Systematic uncertainties of the NNLO RC correction

Backups

Possible ways to implement NNLO calculation in RC

current plan

Stream T(heory):
PRad build a fast, O(10ms) per event version of the detector simulation (for example by fitting the GEANT4 response). This will allow **PRad to benefit from the best theory** available in McMule, now and in the future. This model needn't be analytically simple and can involve arbitrary computer code as long as it's fast.

Too late when it comes to available to PRad2

Stream E(xperiment):
Once McMule's generator is fully ready, PRad will be able to use it with their full detector simulation to get the best experimental precision at the cost of delayed theoretical improvements.

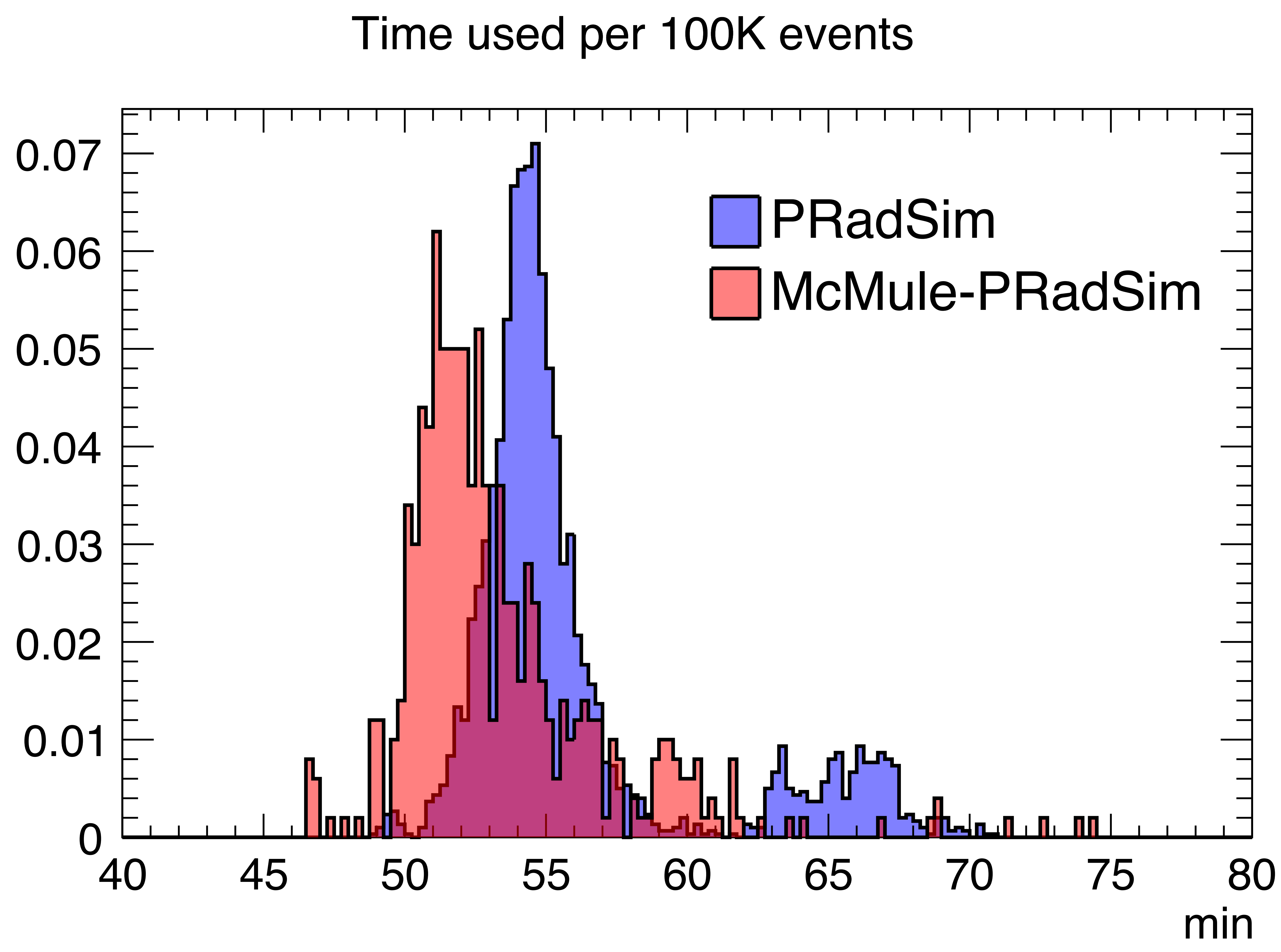
McMule can only provide up to 3 dimensional differential cross section
It is not sufficient to determine kinematic for all particles when photon emitted

Stream I(nterpolation):
related to Stream K, we will investigate whether we can provide interpolations of multi-differential cross sections for PRad to sample from.

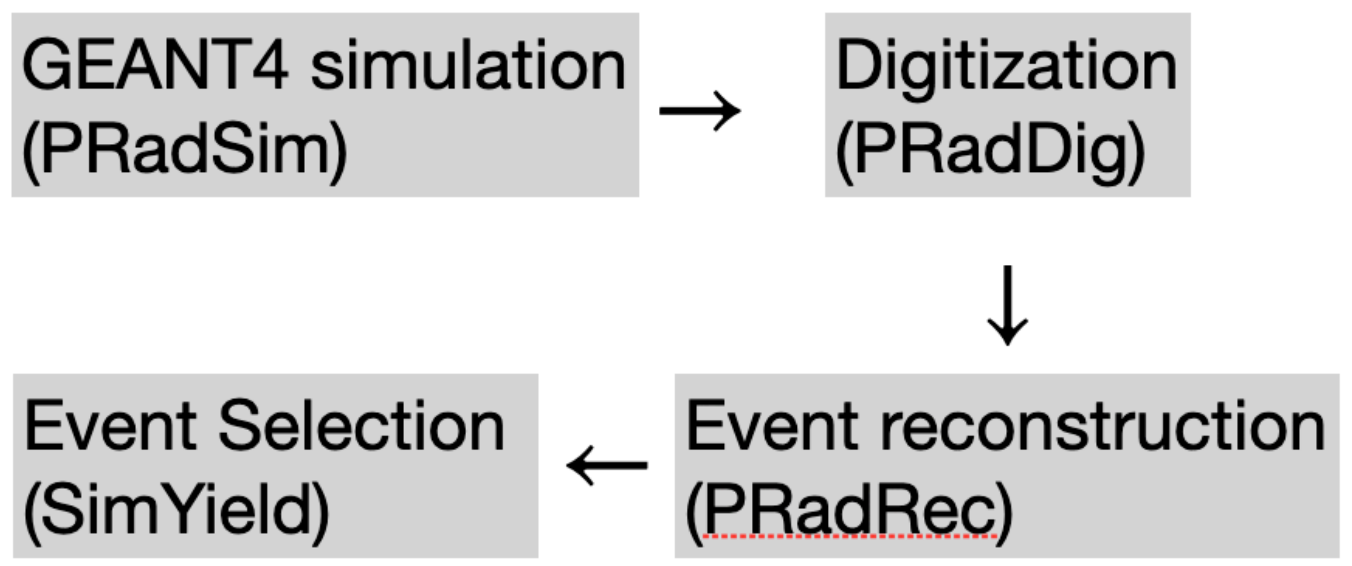
McMule can only provide up to 3 dimensional differential cross section
It is not sufficient to determine kinematic for all particles when photon emitted

Stream K(inematics):
PRad provides McMule with realistic kinematic and geometric cuts and histograms, similar to slide 15 of the Argonne talk for all four processes ep->ep, ee->ee, ep->ep\gamma, ee->ee\gamma. Especially the handling of photons is important here. McMule will use these to tune the code and make sure it performs.

Performance of the new McMule-PRadSim



- Time used to finish the analysis chain



- Time estimated on ifarm
- McMule-PRadSim is even faster

PRad analysis flow

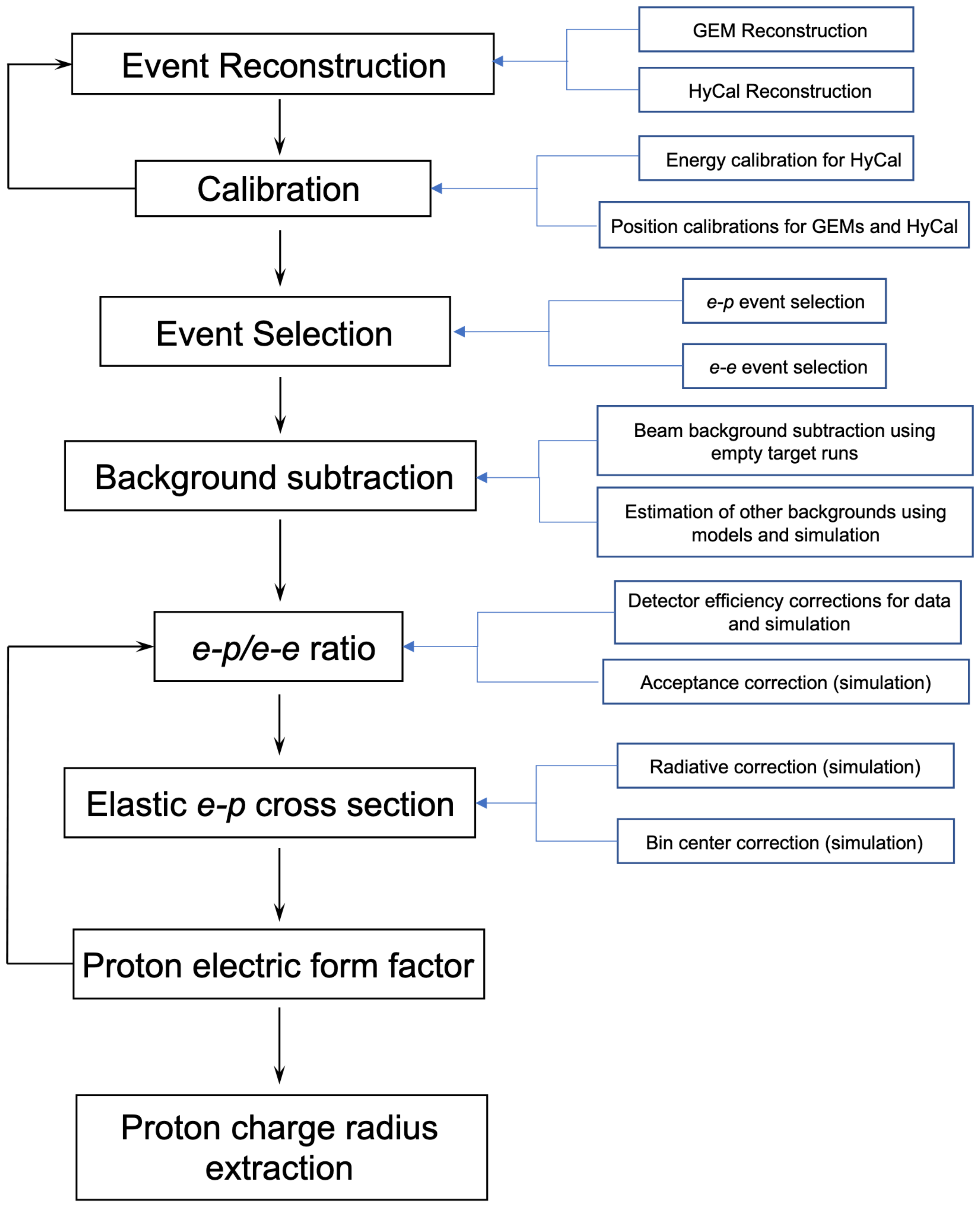
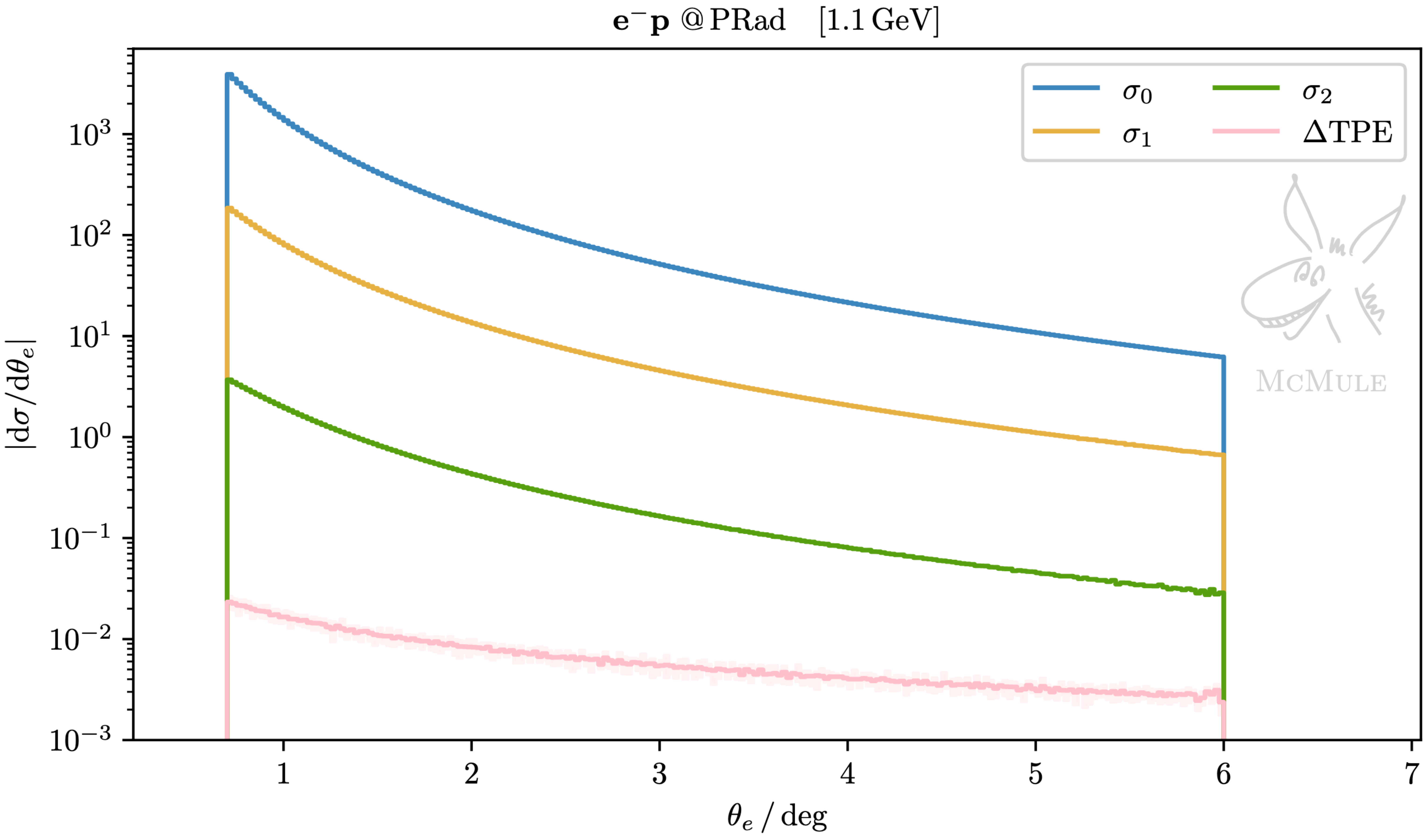
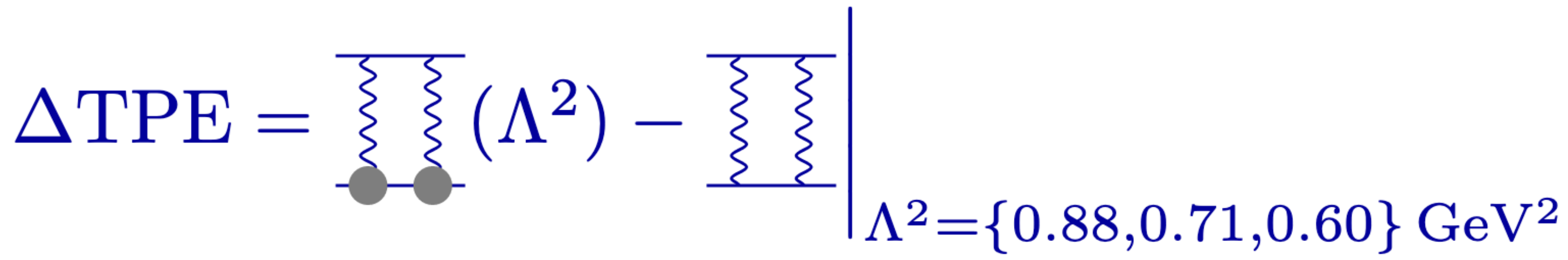


Figure 4.1: The flow chart for the data analysis procedure.

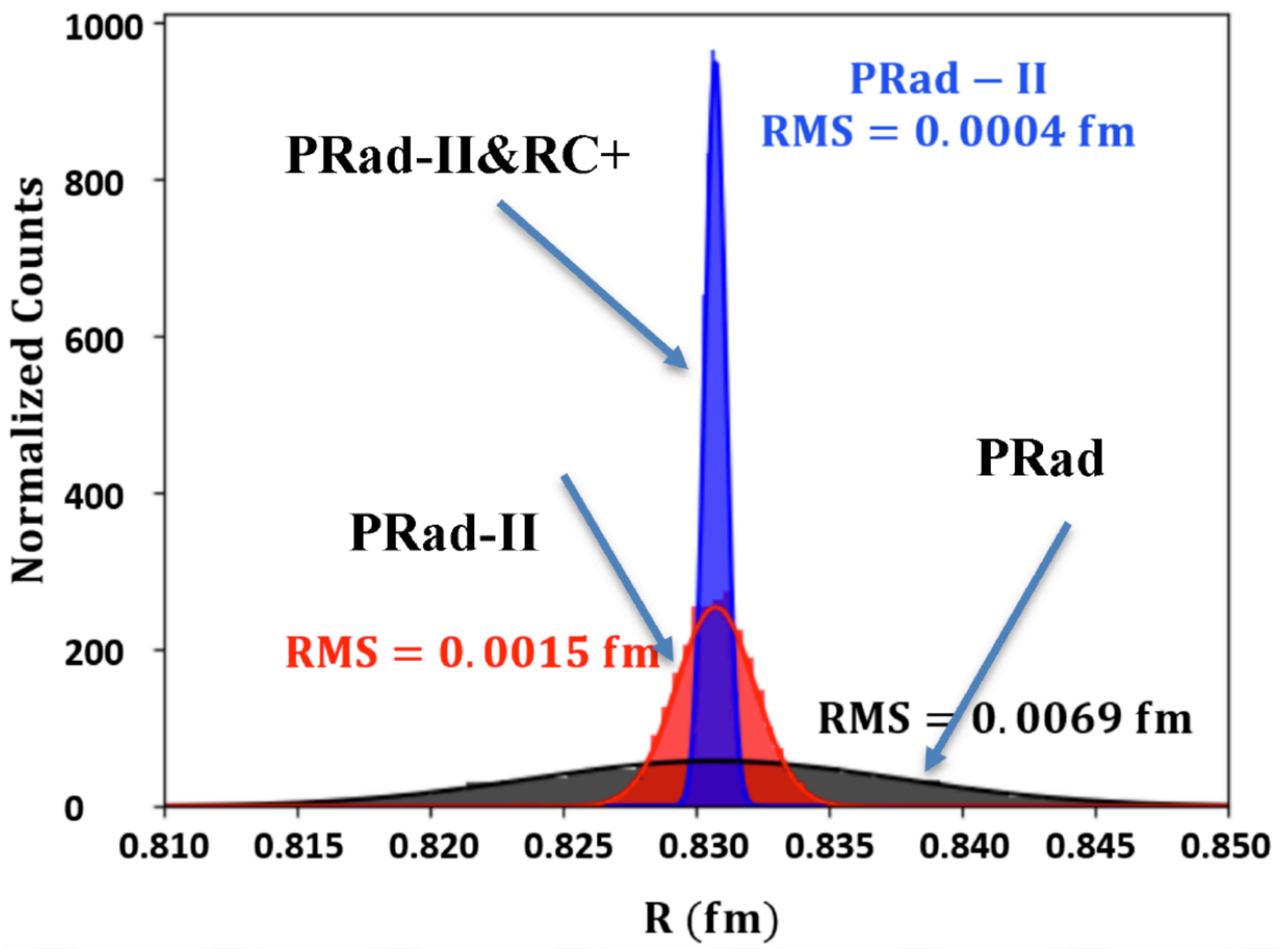


$$G_E = \frac{G_M}{1+\kappa} = \left(1 + \frac{Q^2}{\Lambda^2}\right)^{-2}$$



PRad r_p result and its syst. uncertainty on RC

- PRad measured radius: Nature 575, 147 (2019)
  - $r_p = (0.831 \pm 0.007_{\text{stat}} \pm 0.012_{\text{syst}})$ fm
  - RC is one of the largest syst. uncertainties
- Significant reduction of syst. uncertainty of RC from PRad to PRad-II&RC+
- Extremely important to implement NNLO RC calculation



- RC δr_p for PRad
- Projected RC δr_p with planes of coordinate tracking detectors plus current RC calculations
- Projected RC δr_p with two planes of coordinate tracking detectors plus improved RC calculation at NNLO