Vector & Beam-Target Asymmetries from b1/Azz Data

E. Long, EPJ A (in review)

Elena Long

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University of New Hampshire

Polarization Cross Section

- General electron-deuteron scattering cross section:

$$\sigma(h_e, P, Q) = \sigma_u\left[1 + h_e\left(A_e + PA_{ed}^V + QA_{ed}^T\right) + PA_d^V + QA_d^T\right]$$

$$A_d^T = \frac{1}{2}A_{zz}$$

- For b1/Azz, we ideally want $h_e = 0$ & $P = 0$
  - But we can't get $P = 0$ w/ large Q using a DNP target

$$\sigma(0, P, Q) = \sigma_u\left[1 + PA_d^V + QA_d^T\right]$$

Tensor Asymmetry: 2 States w/ $h_e = 0$

- We extract A_d^T (& thus A_{zz} & b_1) w/ tensor asymmetries
- For 2 arbitrary polarization states, we have

$$\sigma_1 = \sigma(0, P_1, Q_1) = \sigma_u \left(1 + P_1 A_d^V + Q_1 A_d^T\right)$$
$$\sigma_2 = \sigma(0, P_2, Q_2) = \sigma_u \left(1 + P_2 A_d^V + Q_2 A_d^T\right)$$

- Making a typical asymmetry gives

$$\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} = \frac{(P_1 - P_2)A_d^V + (Q_1 - Q_2)A_d^T}{2 + P_1 A_d^V + P_2 A_d^V + Q_1 A_d^T + Q_2 A_d^T}$$

- Solving for A_d^T gives

$$A_d^T = \left(\frac{\sigma_1 - \sigma_2}{Q_1\sigma_2 - Q_2\sigma_1}\right)\left[1 - A_d^V\left(\frac{P_1\sigma_2 - P_2\sigma_1}{\sigma_1 - \sigma_2}\right)\right]$$

Gives original proposal Eq. when 1 state unpolarized ($P_2 = Q_2 = 0$)

$$A_d^T = \left(\frac{\sigma_T - \sigma_u}{Q\sigma_u}\right)\left[1 - PA_d^V\left(\frac{\sigma_u}{\sigma_T - \sigma_u}\right)\right]$$

$$A_d^T = \left(\frac{\sigma_1 - \sigma_2}{Q_1\sigma_2 - Q_2\sigma_1}\right)\left[1 - A_d^V\left(\frac{P_1\sigma_2 - P_2\sigma_1}{\sigma_1 - \sigma_2}\right)\right]$$

Formalism

- We can generalize this equation
  - Tensor asymmetry defined by polarization configurations

$$A_d^T \begin{pmatrix} h_i \\ P_i \\ Q_i \end{pmatrix}_n$$

Beam Helicity States

Target Vector States

Target Tensor States

# of configs

Asym. to measure

$$A_d^T \begin{pmatrix} 0, 0 \\ P_1, P_2 \\ Q_1, Q_2 \end{pmatrix}_2 = R_2^Q \left(1 - \frac{A_d^V}{R_2^P}\right)$$

Primary Measurement

Contamination Term

with

$$R_2^Q = \frac{\sigma_1 - \sigma_2}{Q_1\sigma_2 - Q_2\sigma_1}$$

$$R_2^P = \frac{\sigma_1 - \sigma_2}{P_1\sigma_2 - P_2\sigma_1}$$

Formalism

- Same formalism holds for many different configurations!

$$A_d^T = R_n^Q \left(1 - \frac{A_d^V}{R_n^P} - \frac{A_e}{R_n^h} - \frac{A_{ed}^V}{R_n^{hP}} - \frac{A_{ed}^T}{R_n^{hQ}} \right)$$

- But will get different R terms for each
- Can rearrange to get vector & beam-target asymmetries

4 State Tensor Asymmetry w/ $h_e = 0$

Target advances allow us to maintain large P while enhancing or suppressing Q

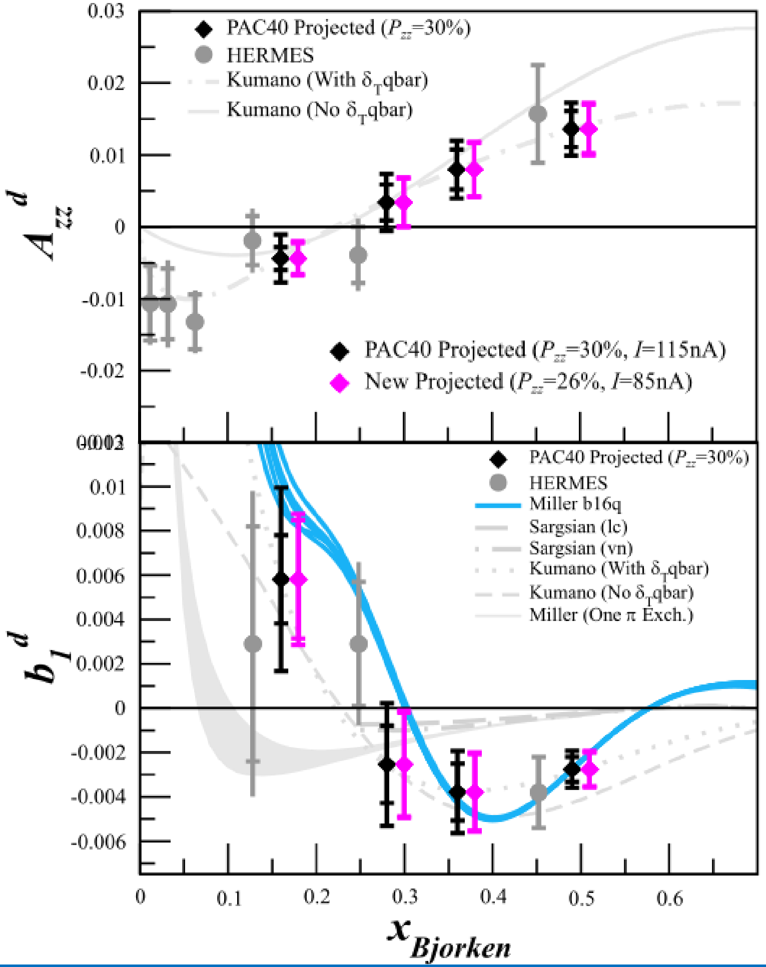
- Rapid tensor enhancement/suppression decreases systematic uncertainty

$$A_d^T \begin{pmatrix} 0,0,0,0 \\ P,-P,P,-P \\ Q,Q,0,0 \end{pmatrix}_4 = R_4^Q \left(1 - \frac{A_d^V}{R_4^P}\right)$$

with

$$R_4^Q(Q,Q,0,0) = \left(\frac{1}{f}\right)\left(\frac{(Y_1+Y_2)-(Y_3+Y_4)}{Q(Y_3+Y_4)}\right),$$

$$\frac{1}{R_4^P(P,-P,P,-P)} = 0.$$



Parasitic Vector Asymmetry (2 State)

With b1/Azz data, we can extract the vector asymmetry in two different 2-state configurations

$$A_d^V \begin{pmatrix} 0,0 \\ P,-P \\ 0,0 \end{pmatrix}_2 = R_2^P \left(1 - \frac{A_d^T}{R_2^Q}\right)$$

$$A_d^V \begin{pmatrix} 0,0 \\ P,-P \\ Q,Q \end{pmatrix}_2 = R_2^P \left(1 - \frac{A_d^T}{R_2^Q}\right)$$

or

$$R_2^P(P,-P) = \frac{Y_1 - Y_2}{fP(Y_1 + Y_2)},$$

$$\frac{1}{R_2^Q(0,0)} = 0.$$

Statistics ~ the ½ that of Azz/b1

$$R_2^P(P,-P) = \frac{Y_1 - Y_2}{fP(Y_1 + Y_2)},$$

$$\frac{1}{R_2^Q(Q,Q)} = -fQ$$

Statistics ~ the ½ that of Azz/b1

Parasitic Vector Asymmetry (4 State)

Can combine into a single 4-state measurement, increasing statistics but including tensor term

$$A_d^V \begin{pmatrix} 0,0,0,0 \\ P,P,-P,-P \\ Q,0,Q,0 \end{pmatrix}_4 = R_4^P \left(1 - \frac{A_d^T}{R_4^Q}\right)$$

$$R_4^P(P,P,-P,-P) = \left(\frac{1}{fP}\right)\left(\frac{(Y_1+Y_2)-(Y_3+Y_4)}{(Y_1+Y_2)+(Y_3+Y_4)}\right),$$

$$\frac{1}{R_4^Q(Q,0,Q,0)} = -\frac{1}{2}fQ,$$

Parasitic Beam-Target Double Spin Asymmetries (8 states)

Since the JLab beam is polarized, we can split the beam helicities from our 4 state measurement to get 8 state measurements – Can extract vector & tensor DSA!

Tensor DSA

$$A^T_{ed}\begin{pmatrix} \pm h, \pm h, \pm h, \pm h \\ P, -P, P, -P \\ Q, Q, 0, 0 \end{pmatrix}_{8'} = R^{hQ}_{8'}\left(1 - \frac{A^T_d}{R^Q_{8'}} - \frac{A^V_d}{R^P_{8'}} - \frac{A_e}{R^h_{8'}} - \frac{A^V_{ed}}{R^{hP}_{8'}}\right)$$

$$R^{hQ}_{8'}\begin{pmatrix} \pm h, \pm h, \pm h, \pm h \\ Q, Q, 0, 0 \end{pmatrix} = \left(\frac{2}{fQh}\right)\left(\frac{Y^{+12}_{-34} - Y^{-12}_{+34}}{Y^{+12}_{-34} + Y^{-12}_{+34}}\right),$$

$$\frac{1}{R^{hP}_{8'}\begin{pmatrix} \pm h, \pm h, \pm h, \pm h \\ P, -P, P, -P \end{pmatrix}} = 0,$$

$$\frac{1}{R^h_{8'}(\pm h, \pm h, \pm h, \pm h)} = 0,$$

$$\frac{1}{R^Q_{8'}(Q, Q, 0, 0)} = -\frac{1}{2}fQ,$$

$$\frac{1}{R^P_{8'}(P, -P, P, -P)} = 0,$$

$$Y^{+12}_{-34} = Y_{+1} + Y_{+2} + Y_{-3} + Y_{-4},$$
$$Y^{-12}_{+34} = Y_{-1} + Y_{-2} + Y_{+3} + Y_{+4},$$

Statistics roughly the same as Azz/b1, but scaled by beam helicity

Parasitic Beam-Target Double Spin Asymmetries (8 states)

Since the JLab beam is polarized, we can split the beam helicities from our 4 state measurement to get 8 state measurements – Can extract vector & tensor DSA!

Vector DSA

$$A_{ed}^{V}\begin{pmatrix}\pm h,\pm h,\pm h,\pm h\\ P,P,-P,-P\\ Q,0,Q,0\end{pmatrix}_{8'}=R_{8'}^{hP}\left(1-\frac{A_d^T}{R_{8'}^{Q}}-\frac{A_d^V}{R_{8'}^{P}}-\frac{A_e}{R_{8'}^{h}}-\frac{A_{ed}^T}{R_{8'}^{hQ}}\right)$$

$$R_{8'}^{hP}\begin{pmatrix}\pm h,\pm h,\pm h,\pm h\\ P,P,-P,-P\end{pmatrix}=\left(\frac{2}{fPh}\right)\left(\frac{Y_{-34}^{+12}-Y_{+34}^{-12}}{Y_{-34}^{+12}+Y_{+34}^{-12}}\right),$$

$$\frac{1}{R_{8'}^{hQ}\begin{pmatrix}\pm h,\pm h,\pm h,\pm h\\ Q,0,Q,0\end{pmatrix}}=0,$$

$$\frac{1}{R_{8'}^{h}(\pm h,\pm h,\pm h,\pm h)}=0,$$

$$\frac{1}{R_{8'}^{Q}(Q,0,Q,0)}=-\frac{1}{2}fQ,$$

$$\frac{1}{R_{8'}^{P}(P,P,-P,-P)}=0.$$

$$Y_{-34}^{+12}=Y_{+1}+Y_{+2}+Y_{-3}+Y_{-4},$$
$$Y_{+34}^{-12}=Y_{-1}+Y_{-2}+Y_{+3}+Y_{+4},$$

Statistics roughly the same as Azz/b1, but scaled by beam helicity

Complications

Things get messier when polarizations aren't exactly equal, but the algebra has all been worked out in upcoming EPJA paper

Also includes:

- Configurations better suited for atomic beam source targets/collider experiments
  - 3 state configurations are very useful w/ pure polarization states
- Full uncertainty equations

Uncertainties

$$\delta A_d^T = \Bigg[\left(\left(\left(1 - \frac{A_d^V}{R_n^P} - \frac{A_e}{R_n^h} - \frac{A_{ed}^V}{R_n^{hP}} - \frac{A_{ed}^T}{R_n^{hQ}}\right)\delta R_n^Q\right)^2 + \left(A_d^V \frac{R_n^Q}{(R_n^P)^2}\delta R_n^P\right)^2 + \left(\frac{R_n^Q}{R_n^P}\delta A_d^V\right)^2 + \left(A_e \frac{R_n^Q}{(R_n^h)^2}\delta R_n^h\right)^2 + \left(\frac{R_n^Q}{R_n^h}\delta A_e\right)^2 + \left(A_{ed}^V \frac{R_n^Q}{(R_n^{hP})^2}\delta R_n^{hP}\right)^2 + \left(\frac{R_n^Q}{R_n^{hP}}\delta A_{ed}^V\right)^2 + \left(A_{ed}^T \frac{R_n^Q}{\left(R_n^{hQ}\right)^2}\delta R_n^{hQ}\right)^2 + \left(\frac{R_n^Q}{R_n^{hQ}}\delta A_{ed}^T\right)^2\Bigg]^{(1/2)}$$

$$\frac{R_2^Q}{R_2^P} = \frac{P_1Y_1 - P_2Y_2}{Q_1Y_1 - Q_2Y_2},$$

$$\frac{R_3^Q}{R_3^P} = \frac{(P_1 + P_2)Y_0 - P_0(Y_1 + Y_2)}{(Q_1 + Q_2)Y_0 - Q_0(Y_1 + Y_2)},$$

$$\frac{R_{4'}^Q}{R_{4'}^P} = \frac{(P_1 + P_2 + P_3)Y_0 - P_0(Y_1 + Y_2 + Y_3)}{(Q_1 + Q_2 + Q_3)Y_0 - Q_0(Y_1 + Y_2 + Y_3)},$$

$$\frac{R_4^Q}{R_4^P} = \frac{(P_1 + P_2)(Y_3 + Y_4) - (P_3 + P_4)(Y_1 + Y_2)}{(Q_1 + Q_2)(Y_3 + Y_4) - (Q_3 + Q_4)(Y_1 + Y_2)}.$$

Polarization Choice Can Reduce Uncertainty Terms

Existing & Upcoming Tensor Measurements

Observable	Method	Target Configuration	Target Type	Location and Reaction
A_d^T, $T_{20} \approx \sqrt{2}\frac{A_d^T}{P_2(\cos\theta)}$	R_2^Q	$\begin{pmatrix} h_1, h_2 \\ P_1, P_2 \\ Q_1, Q_2 \end{pmatrix} = \begin{pmatrix} 0, 0 \\ 0, 0 \\ +Q, -2Q \end{pmatrix}$	Atomic Beam Source	NIKHEF $d(e, e'd')$
A_d^T	R_2^Q	$\begin{pmatrix} h_1, h_2 \\ P_1, P_2 \\ Q_1, Q_2 \end{pmatrix} = \begin{pmatrix} 0, 0 \\ -P, 0 \\ +Q_{eq}, 0 \end{pmatrix}$	Solid ND_3	Bonn $d(e, e'd')$
$T_{20} \approx \sqrt{2}\frac{A_d^T}{P_2(\cos\theta)}$	R_2^Q	$\begin{pmatrix} h_1, h_2 \\ P_1, P_2 \\ Q_1, Q_2 \end{pmatrix} = \begin{pmatrix} 0, 0 \\ P, 0 \\ +Q, 0 \end{pmatrix}$	Solid ND_3	TRIUMF $d(\pi, 2p)$
$T_{20} \approx \sqrt{2}\frac{A_d^T}{P_2(\cos\theta)}$	R_2^Q	$\begin{pmatrix} h_1, h_2 \\ P_1, P_2 \\ Q_1, Q_2 \end{pmatrix} = \begin{pmatrix} 0, 0 \\ 0, 0 \\ +Q, -Q \end{pmatrix}$	Atomic Beam & Storage Cell	VEPP-2, VEPP-3 $d(e, e'd')$
A_d^T	R_3^Q	$\begin{pmatrix} h_1, h_2, h_0 \\ P_1, P_2, P_0 \\ Q_1, Q_2, Q_0 \end{pmatrix} = \begin{pmatrix} 0, 0, 0 \\ P, -P, 0 \\ +Q_{eq}, +Q_{eq}, -2Q \end{pmatrix}$	Atomic Beam Source	MIT-Bates $d(e, e'd')$, $d(e, e'p)$
$2A_d^T = A_{zz}$	$R_{4'}^Q$	$\begin{pmatrix} h_1, h_2, h_3, h_0 \\ P_1, P_2, P_3, P_0 \\ Q_1, Q_2, Q_3, Q_0 \end{pmatrix} = \begin{pmatrix} 0, 0, 0, 0 \\ P, -P, 0, 0 \\ Q, Q, Q, -2Q \end{pmatrix}$	Atomic Beam Source	DESY HERA $d(e^+, e^{+'})$
$2A_d^T = A_{zz}$	R_4^Q	$\begin{pmatrix} h_1, h_2, h_3, h_4 \\ P_1, P_2, P_3, P_4 \\ Q_1, Q_2, Q_3, Q_4 \end{pmatrix} = \begin{pmatrix} 0, 0, 0, 0 \\ P, -P, P, -P \\ Q, Q, 0, 0 \end{pmatrix}$	Solid ND_3	JLab $d(e, e')$

Optimal Experimental Configurations

Target Type	Observable	Method	Target Configuration
ABS, DNP with H,D Molecule	A_d^T	R_2^Q Eqs. (58-60, 72-73)	$\begin{pmatrix} h_1, h_2 \\ P_1, P_2 \\ Q_1, Q_2 \end{pmatrix} = \begin{pmatrix} 0, 0 \\ P, 0 \\ +Q, 0 \end{pmatrix}$
ABS, DNP + ssRF	A_d^V	R_2^P Eqs. (83-85)	$\begin{pmatrix} h_1, h_2 \\ P_1, P_2 \\ Q_1, Q_2 \end{pmatrix} = \begin{pmatrix} 0, 0 \\ P, -P \\ 0, 0 \end{pmatrix}$
Dual-Cell DNP	A_d^T	R_3^Q Eqs. (76-78)	$\begin{pmatrix} h_1, h_2, h_0 \\ P_1, P_2, P_0 \\ Q_1, Q_2, Q_0 \end{pmatrix} = \begin{pmatrix} 0, 0, 0 \\ P, -P, 0 \\ Q, Q, 0 \end{pmatrix}$
ABS	A_d^T	R_3^Q Eqs. (64-66)	$\begin{pmatrix} h_1, h_2, h_0 \\ P_1, P_2, P_0 \\ Q_1, Q_2, Q_0 \end{pmatrix} = \begin{pmatrix} 0, 0, 0 \\ P, -P, 0 \\ Q, Q, -2Q \end{pmatrix}$
ABS	A_d^T	$R_{4'}^Q$ Eqs. (67-71)	$\begin{pmatrix} h_1, h_2, h_3, h_0 \\ P_1, P_2, P_3, P_0 \\ Q_1, Q_2, Q_3, Q_0 \end{pmatrix} = \begin{pmatrix} 0, 0, 0, 0 \\ P, -P, 0, 0 \\ Q, Q, Q, -2Q \end{pmatrix}$
DNP + ssRF	A_d^T	R_4^Q Eqs. (80-82)	$\begin{pmatrix} h_1, h_2, h_3, h_4 \\ P_1, P_2, P_3, P_4 \\ Q_1, Q_2, Q_3, Q_4 \end{pmatrix} = \begin{pmatrix} 0, 0, 0, 0 \\ P, -P, P, -P \\ Q, Q, 0, 0 \end{pmatrix}$
DNP + ssRF	A_d^V	R_4^P Eqs. (86-87)	$\begin{pmatrix} h_1, h_2, h_3, h_4 \\ P_1, P_2, P_3, P_4 \\ Q_1, Q_2, Q_3, Q_4 \end{pmatrix} = \begin{pmatrix} 0, 0, 0, 0 \\ P, P, -P, -P \\ Q, 0, Q, 0 \end{pmatrix}$
DNP + ssRF	A_{ed}^T	$R_{8'}^{hQ}$ Eqs. (88-91)	$\begin{pmatrix} h_1^\pm, h_2^\pm, h_3^\pm, h_4^\pm \\ P_1, P_2, P_3, P_4 \\ Q_1, Q_2, Q_3, Q_4 \end{pmatrix} = \begin{pmatrix} \pm h, \pm h, \pm h, \pm h \\ P, -P, P, -P \\ Q, Q, 0, 0 \end{pmatrix}$
DNP + ssRF	A_{ed}^V	$R_{8'}^{hP}$ Eqs. (92-95)	$\begin{pmatrix} h_1^\pm, h_2^\pm, h_3^\pm, h_4^\pm \\ P_1, P_2, P_3, P_4 \\ Q_1, Q_2, Q_3, Q_4 \end{pmatrix} = \begin{pmatrix} \pm h, \pm h, \pm h, \pm h \\ P, -P, P, -P \\ Q, Q, 0, 0 \end{pmatrix}$

Thank you!