Unveiling the 3D Structure of Deuterium: Focusing on the Spin-1 System through Transverse Momentum Distributions

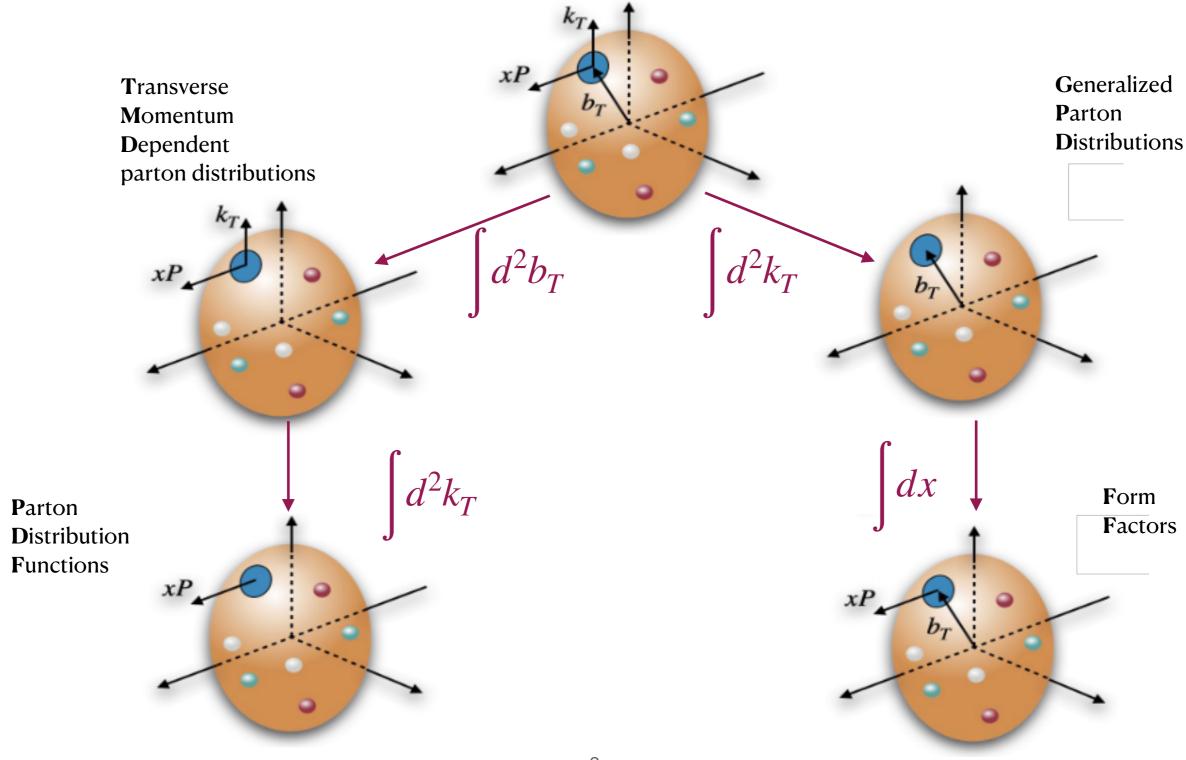
Nathaly Santiesteban
University of New Hampshire
October 14, 2025





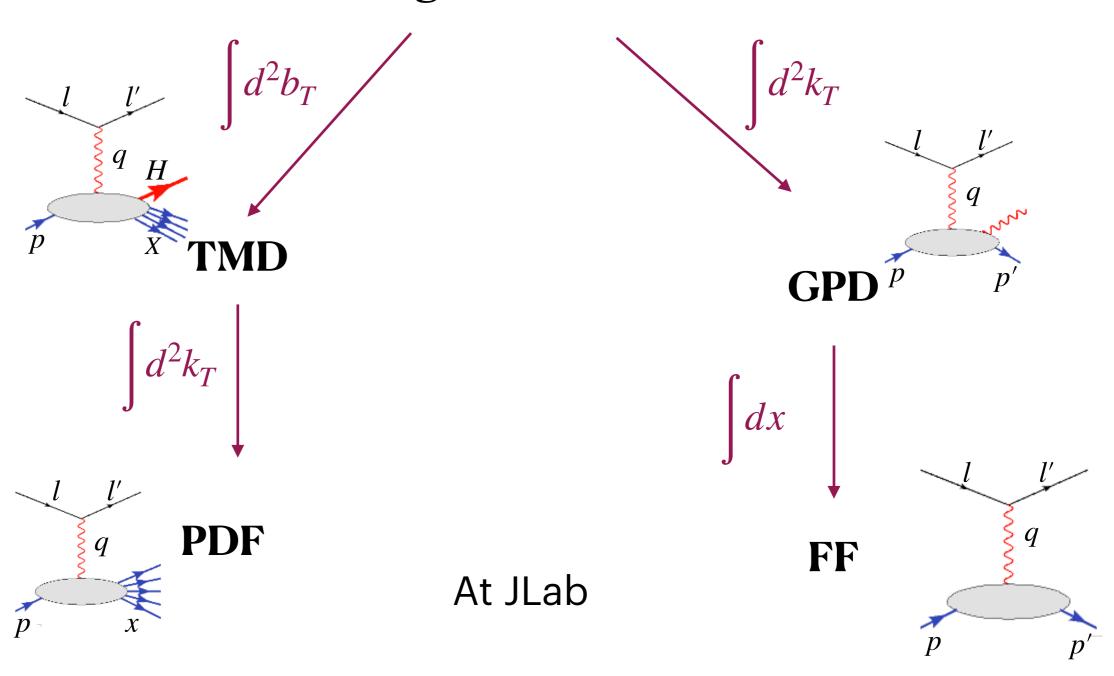
Unified View of Nucleon Structure





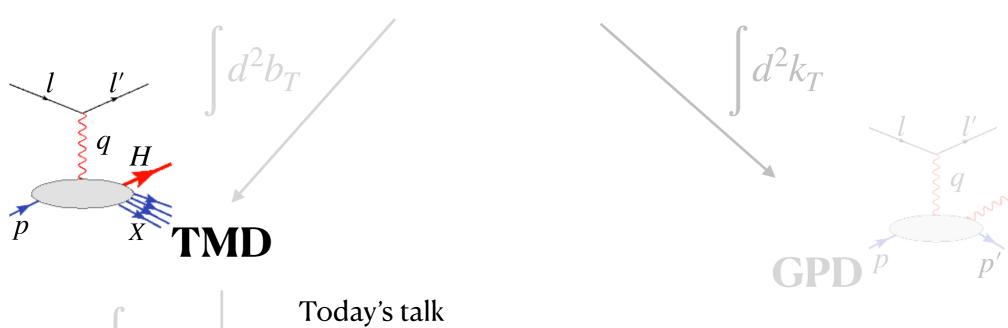
Unified View of Nucleon Structure

Wigner Function



Unified View of Nucleon Structure

Wigner Function



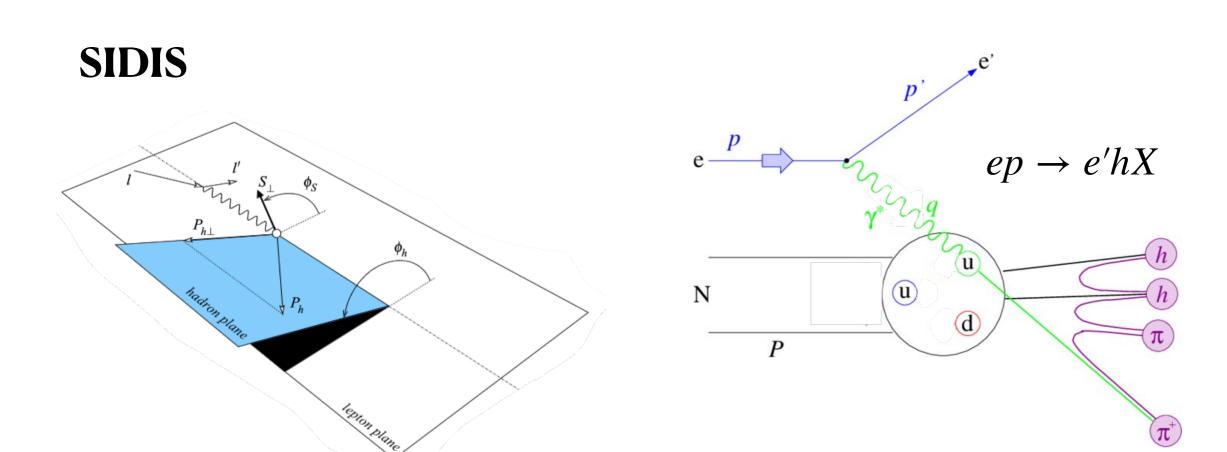
 $\int d^2k_T$

PDF

Describe the distribution of quarks and gluons in a nucleon with respect to x and k_T



Accessing TMDs at JLab

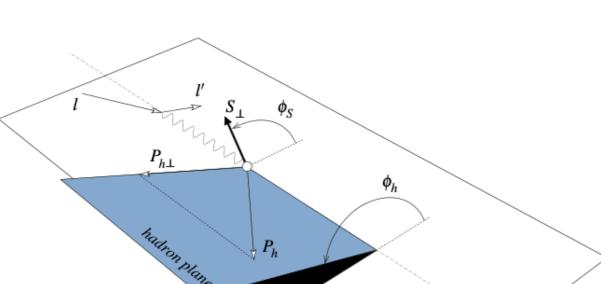


$$x Q^2 \phi_s z P_{h\perp} \phi_h$$

multi-dimensional observables

TMDs are NOT direct physical observables!

SIDIS Kinematics



 ϕ_h : Angle between lepton and hadron planes

 ϕ_S : Angle between lepton plane and nucleon spin

Momentum

$$Q^2 = -\left(l - l'\right)^2$$

transfer:

Center-of-mass energy: $s = (P + l)^2$

Invariant mass: $W^2 = (P + q)^2$

Missing mass: $W^2 = (P + q - P_h)^2$

$$y = \frac{P \cdot q}{P \cdot l}$$

$$x = \frac{Q^2}{2P \cdot q} \qquad \qquad z = \frac{P_h \cdot P}{P \cdot q}$$

$$\cos \phi_h = \frac{\hat{\mathbf{q}} \times \mathbf{l}}{|\hat{\mathbf{q}} \times \mathbf{l}|} \cdot \frac{\hat{\mathbf{q}} \times \mathbf{P}_h}{|\hat{\mathbf{q}} \times \mathbf{P}_h|}$$

$$\sin \phi_h = \frac{(\mathbf{l} \times \mathbf{P}_h) \cdot \hat{\mathbf{q}}}{|\hat{\mathbf{q}} \times \mathbf{l}| |\hat{\mathbf{q}} \times \mathbf{P}_h|}$$

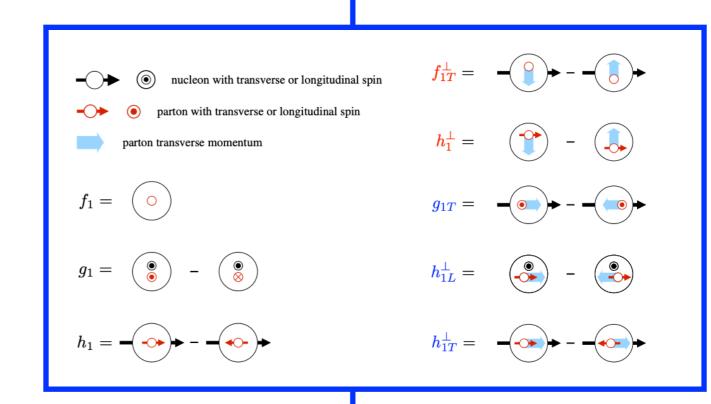
Leading twist distribution functions

Quark	U (γ ⁺)		$L(\gamma^+\gamma_5)$		$T(i\sigma^{i+}\gamma_5/\sigma^{i+})$	
Hadron	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1					$[h_1^{\perp}]$
L			$g_{1 m L}$		$[h_{1 ext{L}}^{ot}]$	
Т		$f_{ m IT}^{\scriptscriptstyle \perp}$	$g_{1\mathrm{T}}$		$[h_1],[h_{1\mathrm{T}}^\perp]$	

After integrating over the transverse momentum:

Quark	U (γ ⁺)		$L(\gamma^+\gamma_5)$		$T(i\sigma^{i+}\gamma_5/\sigma^{i+})$	
Hadron	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	f_1			1 1 1 1 1		
L			$g_{1L}(g_1)$			
Т				1 1 1 1 1 1	$[h_1]$	

Spin 1/2



Phys. Rev. D 62 (2000)

Leading twist distribution functions

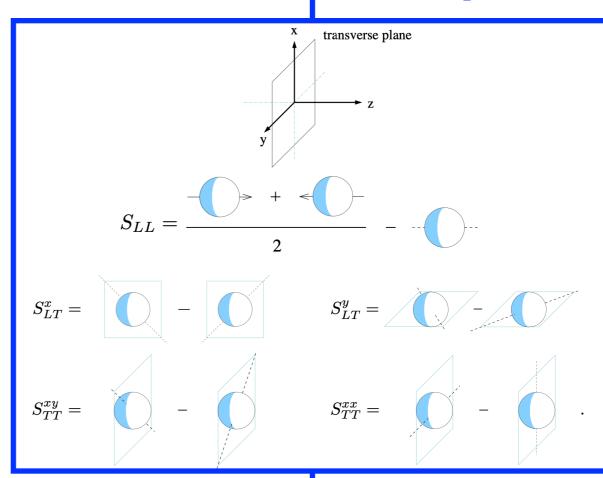
Quark	U (γ ⁺)		$L(\gamma^+\gamma_5)$		$T(i\sigma^{i+}\gamma_5/\sigma^{i+})$	
Hadron	T-even	T-odd	T-even	T-odd	T-even	T-odd
LL	$f_{ m 1LL}$					$[h_{1 ext{LL}}^{ot}]$
LT	$f_{1 m LT}$			g_{1LT}		$[h_{1\mathrm{LT}}],[h_{1\mathrm{LT}}^{\perp}]$
ТТ	$f_{ m 1TT}$			$g_{1\mathrm{TT}}$		$[h_{1 ext{TT}}], [h_{1 ext{TT}}^{\perp}]$

Add 10 leading functions completely unexplored

After integrating over the transverse momentum:

Quark	U (γ ⁺)		$L(\gamma^+\gamma_5)$		$T(i\sigma^{i+}\gamma_5/\sigma^{i+})$	
Hadron	T-even	T-odd	T-even	T-odd	T-even	T-odd
LL	$f_{1LL}(b_1)$					
LT						*1
TT						

Spin 1



$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\,\left(1+\frac{\gamma^2}{2x}\right) \\ \left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \right. \\ \left. + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos\,2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \right. \\ \left. + S_{\parallel}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin\,2\phi_h}\right] \right. \\ \left. + S_{\parallel}\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \right. \\ \left. + T_{\parallel\parallel}\left[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{U(LL)}^{\cos\phi_h}\right] \right. \\ \left. + \varepsilon\cos(2\phi_h)\,F_{U(LL)}^{\cos\,2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{L(LL)}^{\sin\phi_h}\right] \right\}.$$

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\,\left(1+\frac{\gamma^2}{2x}\right) \\ \left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \right. \\ \left. + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos\,2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \right. \\ \left. + S_{\parallel}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin\,2\phi_h}\right] \right. \\ \left. + S_{\parallel}\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \right. \\ \left. + T_{\parallel\parallel}\left[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{U(LL)}^{\cos\phi_h}\right] \right. \\ \left. + T_{\parallel\parallel}\left[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{U(LL)}^{\cos\phi_h}\right] \right\}.$$

* Along the *q*-vector

J. Zhao et al., arXiv:2508.06134 (2025).

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right) \\ \left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}\right. \\ \left. + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \frac{\lambda_e}{\lambda_e}\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}\right. \\ \left. + S_{\parallel}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin2\phi_h}\right] \\ + S_{\parallel}\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\ \left. + T_{\parallel\parallel}\left[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{U(LL)}^{\cos\phi_h}\right. \\ \left. + \varepsilon\cos(2\phi_h)\,F_{U(LL)}^{\cos2\phi_h} + \frac{\lambda_e}{\lambda_e}\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{L(LL)}^{\sin\phi_h}\right]\right\}.$$

J. Zhao et al., arXiv:2508.06134 (2025).

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

tensor
$$+ T_{\parallel\parallel} \left[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2 \varepsilon (1+\varepsilon)} \cos \phi_h \, F_{U(LL)}^{\cos \phi_h} \right.$$
$$\left. + \varepsilon \cos(2\phi_h) \, F_{U(LL)}^{\cos 2\phi_h} + \lambda_e \, \sqrt{2 \varepsilon (1-\varepsilon)} \, \sin \phi_h \, F_{L(LL)}^{\sin \phi_h} \right] \right\}.$$

5 structure functions analogous to the unpolarized

J. Zhao et al., arXiv:2508.06134 (2025).

Tensor-polarized structure functions

tensor
$$+ T_{\parallel\parallel} \left[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2 \varepsilon (1+\varepsilon)} \cos \phi_h F_{U(LL)}^{\cos \phi_h} + \varepsilon \cos(2\phi_h) F_{U(LL)}^{\cos 2\phi_h} + \lambda_e \sqrt{2 \varepsilon (1-\varepsilon)} \sin \phi_h F_{L(LL)}^{\sin \phi_h} \right] \right\}.$$

$$F_{U(LL),T} = \mathcal{C}[f_{1LL}D_1],$$

$$\begin{split} F_{U(LL),L} &= 0, \\ F_{U(LL)}^{\cos\phi_h} &= \frac{2M}{Q} \, \mathcal{C} \bigg[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T}{M_h} \bigg(x h_{LL} \, H_1^\perp + \frac{M_h}{M} \, f_{1LL} \frac{\tilde{D}^\perp}{z} \bigg) - \frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T}{M} \bigg(x f_{LL}^\perp D_1 + \frac{M_h}{M} \, h_{1LL}^\perp \frac{\tilde{H}}{z} \bigg) \bigg], \\ F_{U(LL)}^{\cos 2\phi_h} &= \mathcal{C} \bigg[-\frac{2 \, \big(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T \big) \, \big(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T \big) - \boldsymbol{k}_T \cdot \boldsymbol{p}_T}{M M_h} h_{1LL}^\perp H_1^\perp \bigg], \\ F_{L(LL)}^{\sin\phi_h} &= \frac{2M}{Q} \, \mathcal{C} \bigg[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_T}{M_h} \bigg(x e_{LL} \, H_1^\perp + \frac{M_h}{M} \, f_{1LL} \frac{\tilde{G}^\perp}{z} \bigg) + \frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_T}{M} \bigg(x g_{LL}^\perp D_1 + \frac{M_h}{M} \, h_{1LL}^\perp \frac{\tilde{E}}{z} \bigg) \bigg]. \end{split}$$

Tensor-polarized structure functions

tensor
$$+ T_{\parallel\parallel} \left[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2 \varepsilon (1+\varepsilon)} \cos \phi_h \, F_{U(LL)}^{\cos \phi_h} \right.$$

$$\left. + \varepsilon \cos(2\phi_h) \, F_{U(LL)}^{\cos 2\phi_h} + \lambda_e \, \sqrt{2 \varepsilon (1-\varepsilon)} \, \sin \phi_h \, F_{L(LL)}^{\sin \phi_h} \right] \right\}.$$

$$F_{U(LL),T} = \mathcal{C}[f_{1LL}D_1],$$

$$F_{U(LL),L}=0,$$

$$\begin{split} F_{U(LL)}^{\cos\phi_{h}} &= \frac{2M}{Q} \, \mathcal{C} \bigg[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_{T}}{M_{h}} \bigg(x h_{LL} \, H_{1}^{\perp} + \frac{M_{h}}{M} \underbrace{f_{1LL}}_{z} \underbrace{\tilde{D}^{\perp}}_{z} \bigg) - \frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T}}{M} \bigg(x f_{LL}^{\perp} D_{1} + \frac{M_{h}}{M} \underbrace{h_{1LL}^{\perp}}_{z} \underbrace{\tilde{H}}_{z} \bigg) \bigg], \\ F_{U(LL)}^{\cos 2\phi_{h}} &= \mathcal{C} \bigg[-\frac{2 \, \left(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_{T} \right) \left(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T} \right) - \boldsymbol{k}_{T} \cdot \boldsymbol{p}_{T}}{M M_{h}} \underbrace{h_{1LL}^{\perp}}_{z} H_{1}^{\perp} \bigg], \\ F_{L(LL)}^{\sin\phi_{h}} &= \frac{2M}{Q} \, \mathcal{C} \bigg[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_{T}}{M_{h}} \bigg(x e_{LL} \, H_{1}^{\perp} + \frac{M_{h}}{M} \underbrace{f_{1LL}}_{z} \underbrace{\tilde{G}^{\perp}}_{z} \bigg) + \frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T}}{M} \bigg(x g_{LL}^{\perp} D_{1} + \frac{M_{h}}{M} \underbrace{h_{1LL}^{\perp}}_{z} \underbrace{\tilde{E}}_{z} \bigg) \bigg]. \end{split}$$

Quark
 U
$$(\gamma^+)$$
 L $(\gamma^+\gamma_5)$
 T $(i\sigma^{i+}\gamma_5/\sigma^{i+})$

 Hadron
 T-even
 T-odd
 T-even
 T-odd

 LL
 f_{1LL}
 $[h_{1LL}^{\perp}]$

Let's start simple:

tensor
$$+ T_{\parallel\parallel} \left[F_{U(LL),T} + \varepsilon F_{U(LL),L} + \sqrt{2 \varepsilon (1+\varepsilon)} \cos \phi_h F_{U(LL)}^{\cos \phi_h} + \varepsilon \cos(2\phi_h) F_{U(LL)}^{\cos 2\phi_h} + \lambda_e \sqrt{2 \varepsilon (1-\varepsilon)} \sin \phi_h F_{L(LL)}^{\sin \phi_h} \right] \right\}.$$

$$F_{U(LL),T} = \mathcal{C}[f_{1LL}D_1],$$

How do we approach this structure function?

1. Understand the size of these contribution to propose dedicated experiments

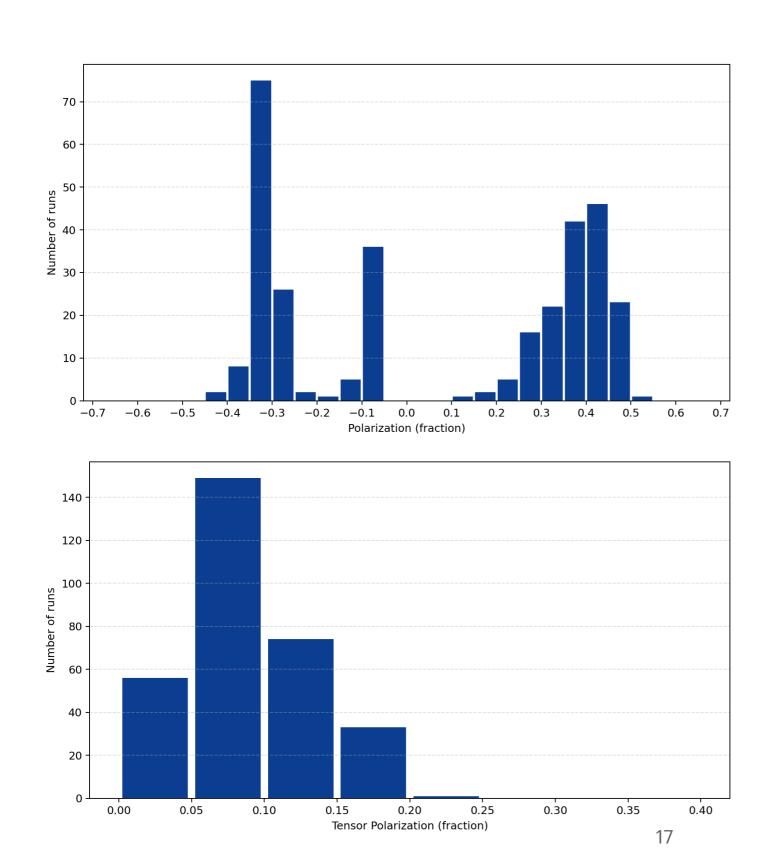
"Spin 1 Transverse Momentum Dependent Tensor Structure Functions in CLAS12"

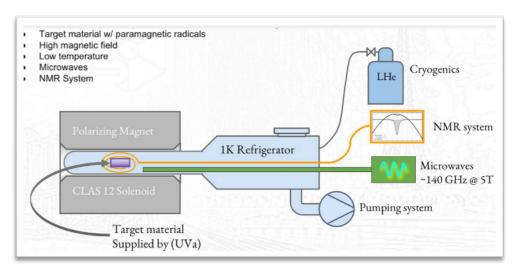
CLAS12 Approved Analysis (CAA approved in Fall 2024)

Data: Polarized Deuterium (ND₃ via DNP). Q_{max} ~20%

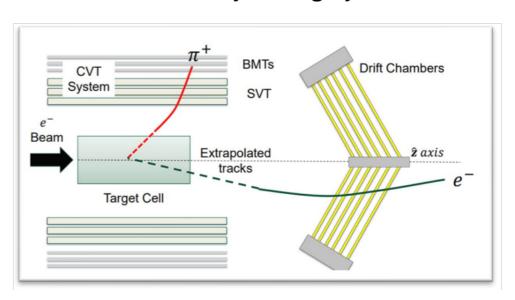
Goal: understand the size of the tensor contribution to the SIDIS processes $(eD \rightarrow e'\pi^{\pm}X)$

Run group C data (ND₃ via DNP)

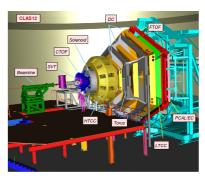




Courtesy of Gregory Matousek



Although small, the tensor polarization is non-negligible.



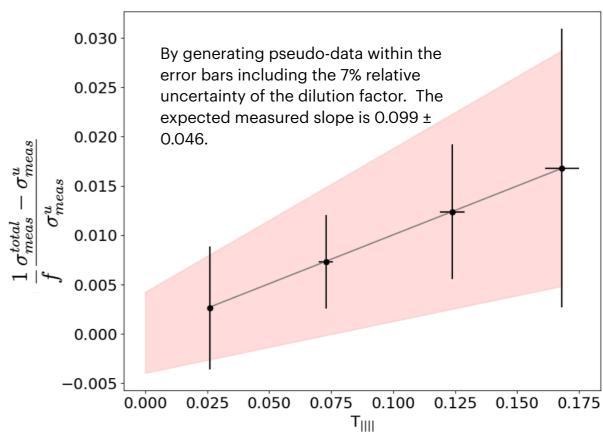
General Idea

Assuming vector + and - are about the same size:

$$\begin{split} \sigma_{meas}^{+} &= \frac{1}{2} \bigg(\sigma_{u}^{D} + S_{\parallel}^{+} \sigma_{S} + T_{\parallel \parallel} \sigma_{T} + \sum_{i} \sigma^{i} \bigg) \\ \sigma_{meas}^{-} &= \frac{1}{2} \bigg(\sigma_{u}^{D} + S_{\parallel}^{-} \sigma_{S} + T_{\parallel \parallel} \sigma_{T} + \sum_{i} \sigma^{i} \bigg) \\ \sigma_{meas}^{total} &= \sigma_{meas}^{+} + \sigma_{meas}^{-} = \sigma_{u}^{D} + T_{\parallel \parallel} \sigma_{T} + \sum_{i} \sigma^{i} \end{split}$$

Subtracting the unpolarized cross-section and rearranging terms:

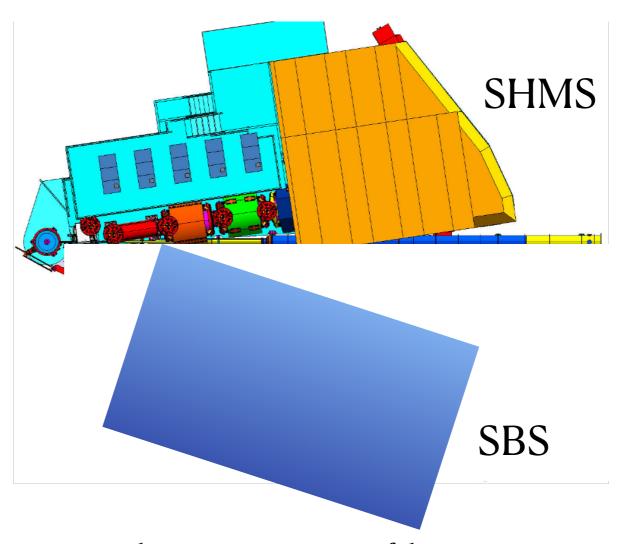
$$T_{\parallel\parallel}rac{\sigma_T}{\sigma_u^D} = rac{1}{f}rac{\sigma_{meas}^{total} - \sigma_{meas}^u}{\sigma_{meas}^u}$$



CAA: arXiv:2502.20044, 2025

^{*} This 10% estimate comes from the HERMES measurement of b1 (the collinear structure function), which is the only available data to date. Phys.Rev.Lett.95 (2005)

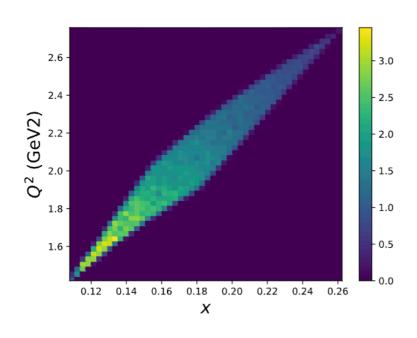
2. Dedicated experiment

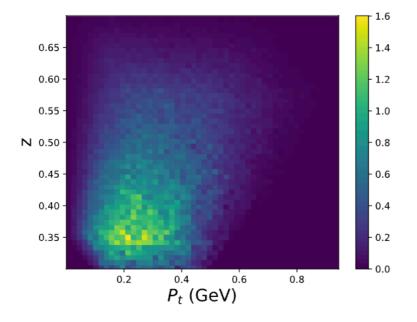


"Spin-1 TMDs and Structure Functions of the Deuteron" at Hall C

Letter of Intent (LOI12-24-002 PAC 52, 2024)

Goal: Dedicated Measurement in Hall C ($eD \rightarrow e'\pi^{\pm}X$)





Kinematic Settings

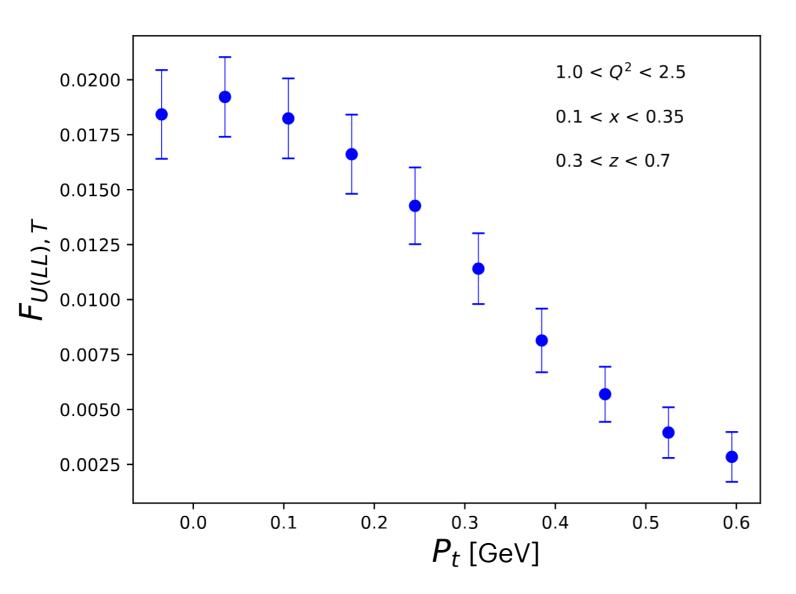
	θ (deg.)	ϕ (deg.)	P (GeV)
Electron	10.3 - 12.4	-2.87 - 2.87	4.0 - 5.4
Hadron	5.0 - 15.0	167 - 193	2.0 - 4.0

A longitudinally tensor polarized deuteron target

The Super BigBite (SBS) Spectrometer, which will be used to measure the produced hadron

The Super High Momentum Spectrometer (SHMS) will be used to measure the scattered electron

Current estimate



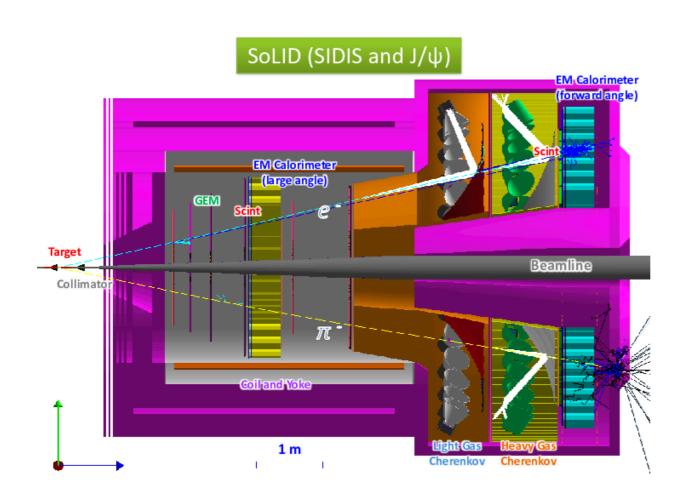
This estimate was done using the event generator for SoLID: https://github.com/TianboLiu/LiuSIDIS

Current plans:

- Perform full simulation (Jan Vanek)
- Full proposal (2026)

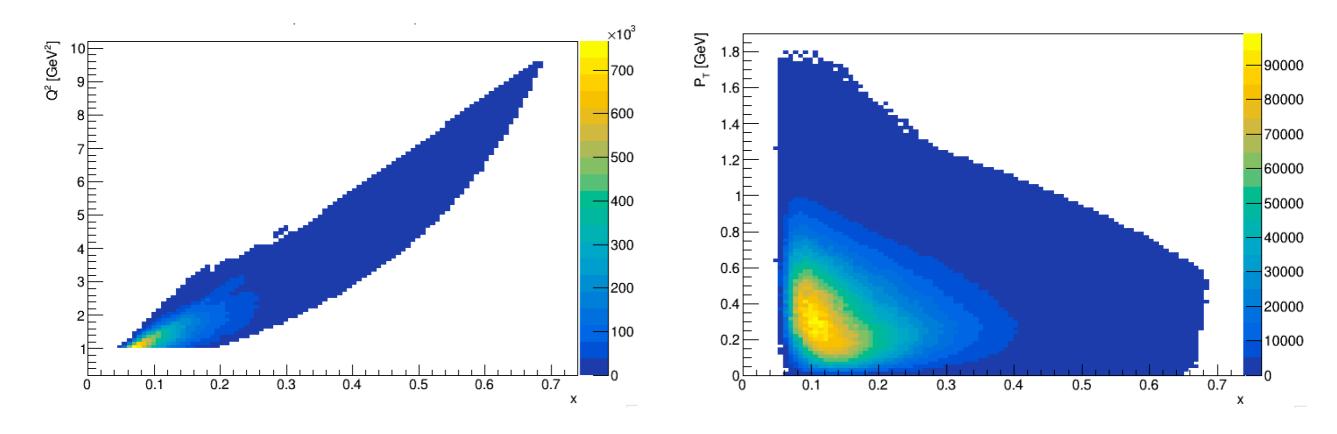
This 10% estimate comes from the HERMES measurement of b1 (the collinear structure function), which is the only available data to date. Phys.Rev.Lett.95 (2005)

3. Full mapping in SoLID



"Future Proposals" 0.3 < z < 0.7 $Q^2 > 1.0 \text{ GeV}^2$ W > 2.3 GeVW' > 1.6 GeV

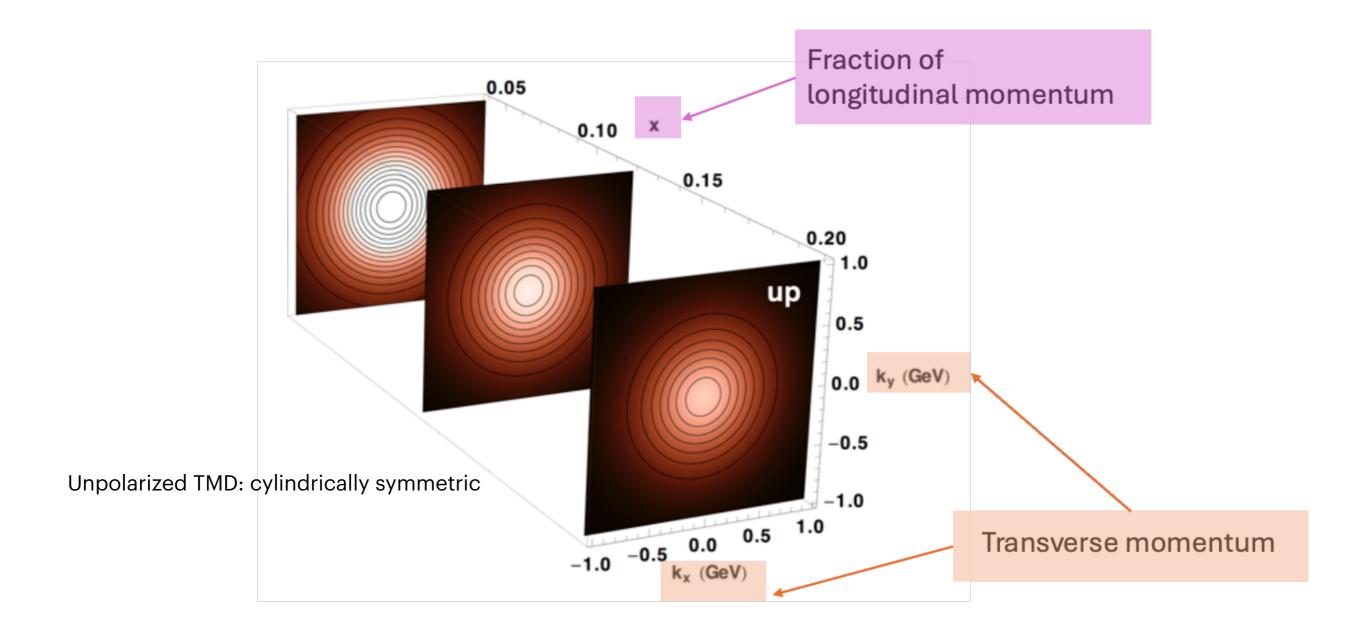
Unpolarized rates for π^-



"Future Proposals"

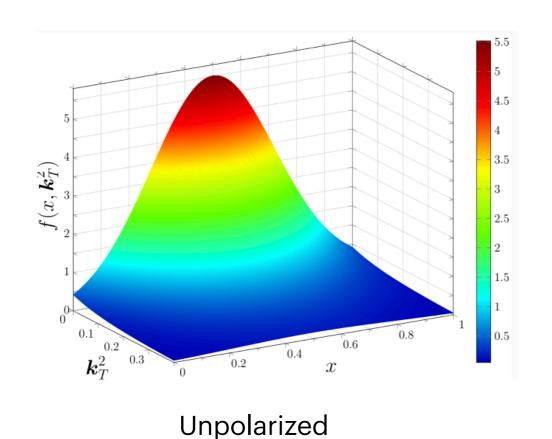
$$Q^2 > 1.0 \text{ GeV}^2$$

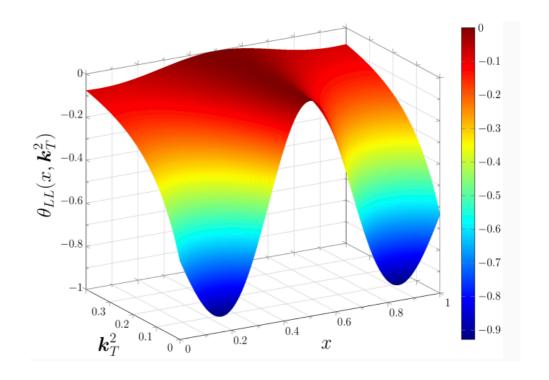
Transverse Momentum Distributions (TMDs) 3D structure of the nucleon



Work in Progress

Theory predictions for the spin-1 tensor st ρ predictions





Tensor

Maybe cylindrical is not a right approximation

Phys.Rev.C 96 (2017) 4, 045206





Nathaly Santiesteban (UNH)

Jean-Ping Chen (Jlab)



Karl Slifer (UNH)





(UNH)



Dustin Keller (Uva)



David Ruth (NMSU) Jiwan Poudel (JLab)



Postdocs



Jan Vanek (UNH)



Ishara Fernando (Uva)

Theory Support

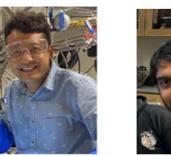


Alessandro Bacchetta (Pavia U.)



Ian Cloet (Argonne Lab)

Students



Chetra Lama UNH

QΕ



Muhammad Farooq (UNH)



Anchit Arora (UNH)



Hector Chinchay-Espino (UNH)

SIDIS

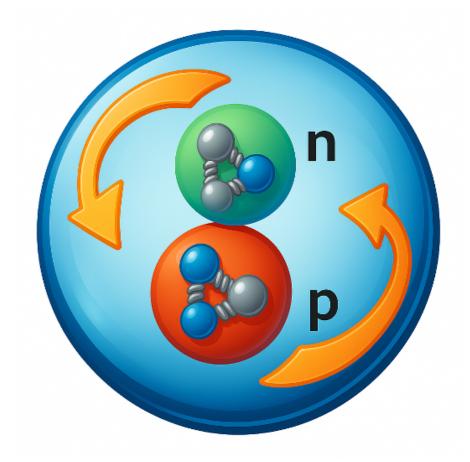
DIS

Polarimetry

26

Summary

- SIDIS spin 1 measurements open the door to a complete new set of observables.
- Theory efforts are being made to provide predictions.



These observables enable more refined studies of QCD and provide new insights into deuteron, illuminating the interplay between QCD dynamics and nuclear structure