Bead-pulling Measurement Principle and Technique Used for the SRF Cavities at JLab

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Thomas Jefferson Lab, Newport News, VA 23606

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Bead-pulling Measurement Curriculum

Bead-pull measurement

- Using a non-conducting wire (like a fishing line) to pull a bead (made of dielectric/metallic or ferromagnetic material) through a RF cavity to measure the electromagnetic fields distribution on resonance inside

Syllabus

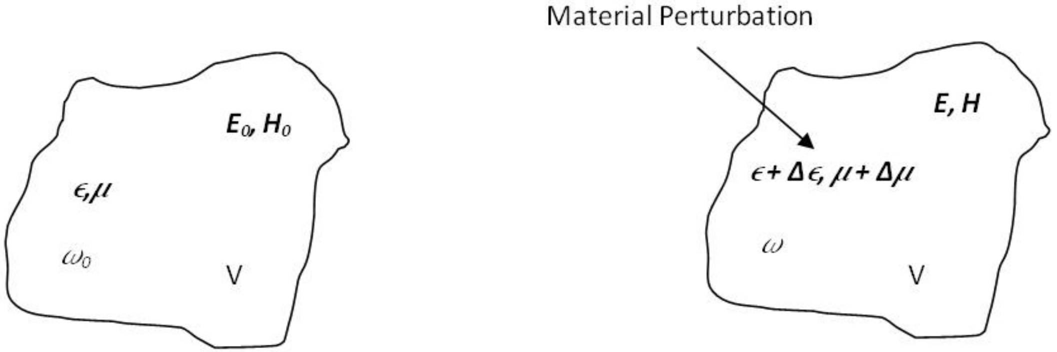
- **Principle of bead-pull**
  1. Perturbation theory
  2. Form factors of metallic and dielectric ellipsoids
  3. Calculations of EM field distribution and R/Q from SuperFish and bead-pulling data
- **Measurement techniques**
  1. Methods and tricks
  2. Bead-pull setup, bead selection, S parameters by VNA,
  3. Comparison to simulation and error analysis
  4. External Q measurement by S_{11} and S_{21} formula for FPC and FP setup
- **Advanced topics**
  1. Multiple modes, HOMs bead-pull
  2. Magnetic field extrapolation
- **Homework given here is due on next Monday in Q&A session (01/26)**

Perturbation of a Resonator – Material Medium Change

Cavity performance like resonance frequency ***f*** or quality factor ***Q*** can be changed by introducing a small perturbation to the resonance EM fields.

Bead-pull measurement involves two types of perturbations:

1. Small material perturbation, like a small dielectric bead enters a large volume of cavity. Although permittivity change from air to bead material could be large, the bead volume is small.

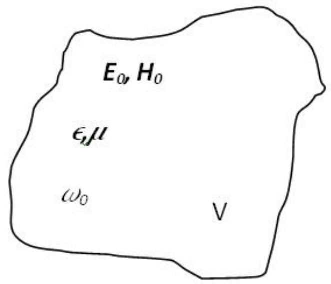


Original (unperturbed) cavity

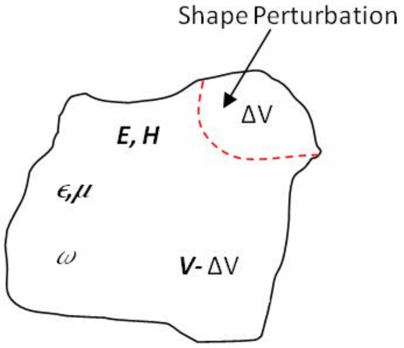
Perturbed cavity

Perturbation of a Resonator – Cavity Shape Change

2. Small cavity volume change, like a small metallic bead enters a large volume of cavity. Although cavity volume change may or my not be on the wall, the bead "repelling" volume is small.



Original (unperturbed) cavity



Perturbed cavity

3. Bead material could be either dielectric $\boldsymbol{\varepsilon_r>1}$, or metallic $\boldsymbol{\sigma\to\infty}$, or low σ ferromagnetic $\boldsymbol{\mu_r>1}$. A lossy material can be also used for the bead-pull as long as the cavity loaded $\boldsymbol{Q_L}$ is taking account into the frequency $\boldsymbol{f}$ change

Perturbation of a Resonator – Frequency Change

Before the perturbation, the EM field inside of cavity can be described as:

$$\boldsymbol{E} = \boldsymbol{E_0} e^{j\omega t} \qquad \boldsymbol{H} = \boldsymbol{H_0} e^{j\omega t} \tag{1}$$

Here $\boldsymbol{E_0}$ and $\boldsymbol{H_0}$ are function of position before perturbation. After the perturbation, the field changes are $\boldsymbol{E_1}$ and $\boldsymbol{H_1}$, frequency change is $\boldsymbol{\Delta\omega}$

$$\boldsymbol{E'} = (\boldsymbol{E_0} + \boldsymbol{E_1}) e^{j(\omega+\Delta\omega)t} \qquad \boldsymbol{H'} = (\boldsymbol{H_0} + \boldsymbol{H_1}) e^{j(\omega+\Delta\omega)t} \tag{2}$$

Here $\boldsymbol{E_1} \ll \boldsymbol{E_0}$, and $\boldsymbol{H}_1 \ll \boldsymbol{H_0}$ are result of small perturbation or small volume

1st and 2nd Maxwell's equation:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{3}$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} \tag{4}$$

Perturbation of a Resonator – Frequency Change

Apply (1) (2) to (3) :

$$\nabla \times \boldsymbol{E_0} = -j\omega \boldsymbol{B_0} \tag{5}$$

$$\nabla \times (\boldsymbol{E_0} + \boldsymbol{E_1}) = -j(\omega + \Delta\omega)(\boldsymbol{B_0} + \boldsymbol{B_1}) \tag{6}$$

Apply (6)-(5):

$$\nabla \times (\boldsymbol{E_1}) = -j\omega \boldsymbol{B_1} - j\Delta\omega(\boldsymbol{B_0} + \boldsymbol{B_1}) \tag{7}$$

Similarly apply (1) (2) to (4):

$$\nabla \times \boldsymbol{H_0} = j\omega \boldsymbol{D_0} \tag{8}$$

$$\nabla \times (\boldsymbol{H_0} + \boldsymbol{H_1}) = j(\omega + \Delta\omega)(\boldsymbol{D_0} + \boldsymbol{D_1}) \tag{9}$$

Apply (9)-(8):

$$\nabla \times (\boldsymbol{H_1}) = j\big(\omega \boldsymbol{D_1} + \Delta\omega(\boldsymbol{D_0} + \boldsymbol{D_1})\big) \tag{10}$$

Perturbation of a Resonator – Frequency Change

$\boldsymbol{H_0}^*$ conjugated $\boldsymbol{H_0}$ dot times (7):

$$\boldsymbol{H_0}^* \cdot \nabla \times (\boldsymbol{E_1}) = -j\left[\omega \boldsymbol{H_0}^* \cdot \boldsymbol{B_1} + \Delta\omega \boldsymbol{H_0}^* \cdot (\boldsymbol{B_0} + \boldsymbol{B_1})\right] \qquad (11)$$

$\boldsymbol{E_0}^*$ conjugated $\boldsymbol{E_0}$ dot times (10):

$$\boldsymbol{E_0}^* \cdot \nabla \times (\boldsymbol{H_1}) = j\left[\omega \boldsymbol{E_0}^* \cdot \boldsymbol{D_1} + \Delta\omega \boldsymbol{E_0}^* \cdot (\boldsymbol{D_0} + \boldsymbol{D_1})\right] \qquad (12)$$

Apply (12)-(11):

$$\begin{aligned}\boldsymbol{E_0}^* \cdot \nabla \times \boldsymbol{H_1} - \boldsymbol{H_0}^* \cdot \nabla \times \boldsymbol{E_1} &= j\omega\left(\boldsymbol{E_0}^* \cdot \boldsymbol{D_1} + \boldsymbol{H_0}^* \cdot \boldsymbol{B_1}\right) \\ &+ j\Delta\omega\left[\left(\boldsymbol{E_0}^* \cdot \boldsymbol{D_0} + \boldsymbol{H_0}^* \cdot \boldsymbol{B_0}\right) + \left(\boldsymbol{E_0}^* \cdot \boldsymbol{D_1} + \boldsymbol{H_0}^* \cdot \boldsymbol{B_1}\right)\right] \qquad (13)\end{aligned}$$

Use vector differential operation and (5) and (8) relationship:

$$\nabla \cdot \left(\boldsymbol{E_0}^* \times \boldsymbol{H_1} - \boldsymbol{H_0}^* \times \boldsymbol{E_1}\right) \equiv \boldsymbol{H_1} \cdot \nabla \times \boldsymbol{E_0}^* - \boldsymbol{E_0}^* \cdot \nabla \times \boldsymbol{H_1} + \boldsymbol{E_1} \cdot \nabla \times \boldsymbol{H_0}^* + \boldsymbol{H_0}^* \cdot \nabla \times \boldsymbol{E_1} = j\omega \boldsymbol{H_1} \cdot \boldsymbol{B_0}^* + j\omega \boldsymbol{E_1} \cdot \boldsymbol{D_0}^* - \left(\boldsymbol{E_0}^* \cdot \nabla \times \boldsymbol{H_1} - \boldsymbol{H_0}^* \cdot \nabla \times \boldsymbol{E_1}\right)$$

Perturbation of a Resonator – Frequency Change

This leads to:

$$\left(\boldsymbol{E_0}^* \cdot \nabla \times \boldsymbol{H_1} - \boldsymbol{H_0}^* \cdot \nabla \times \boldsymbol{E_1}\right) = j\omega\left(\boldsymbol{H_1} \cdot \boldsymbol{B_0}^* + \boldsymbol{E_1} \cdot \boldsymbol{D_0}^*\right) - \nabla \cdot \left(\boldsymbol{E_0}^* \times \boldsymbol{H_1} - \boldsymbol{H_0}^* \times \boldsymbol{E_1}\right) \qquad (14)$$

Apply (14) to (13):

$$j\omega\left(\boldsymbol{H_1} \cdot \boldsymbol{B_0}^* + \boldsymbol{E_1} \cdot \boldsymbol{D_0}^*\right) - \nabla \cdot \left(\boldsymbol{E_0}^* \times \boldsymbol{H_1} - \boldsymbol{H_0}^* \times \boldsymbol{E_1}\right) = j\omega\left(\boldsymbol{E_0}^* \cdot \boldsymbol{D_1} + \boldsymbol{H_0}^* \cdot \boldsymbol{B_1}\right) + j\Delta\omega\left[\left(\boldsymbol{E_0}^* \cdot \boldsymbol{D_0} + \boldsymbol{H_0}^* \cdot \boldsymbol{B_0}\right) + \left(\boldsymbol{E_0}^* \cdot \boldsymbol{D_1} + \boldsymbol{H_0}^* \cdot \boldsymbol{B_1}\right)\right] \qquad (15)$$

Integration over whole volume of cavity V_0 on both sides of (15) :

$$j\Delta\omega \iiint_{V_0} \left[\left(\boldsymbol{E_0}^* \cdot \boldsymbol{D_0} + \boldsymbol{H_0}^* \cdot \boldsymbol{B_0}\right) + \left(\boldsymbol{E_0}^* \cdot \boldsymbol{D_1} + \boldsymbol{H_0}^* \cdot \boldsymbol{B_1}\right)\right] dv$$

$$= j\omega \iiint_{V_0} \left[\left(\boldsymbol{E_1} \cdot \boldsymbol{D_0}^* - \boldsymbol{E_0}^* \cdot \boldsymbol{D_1}\right) + \left(\boldsymbol{H_1} \cdot \boldsymbol{B_0}^* - \boldsymbol{H_0}^* \cdot \boldsymbol{B_1}\right)\right] dv$$

$$- \iiint_{V_0} \nabla \cdot \left(\boldsymbol{E_0}^* \times \boldsymbol{H_1} - \boldsymbol{H_0}^* \times \boldsymbol{E_1}\right) dv \qquad (16)$$

Perturbation of a Resonator – Frequency Change

According to divergence theorem, second term of (16) can become the surface integration on the cavity inner wall S_0 where $\boldsymbol{E_0}$ and $\boldsymbol{H_0}$ vanish:

$$\iiint_{V_0} \nabla \cdot \left(\boldsymbol{E_0}^* \times \boldsymbol{H_1} - \boldsymbol{H_0}^* \times \boldsymbol{E_1}\right) dv = \oiint_{S_0} \left(\boldsymbol{E_0}^* \times \boldsymbol{H_1} - \boldsymbol{H_0}^* \times \boldsymbol{E_1}\right) \cdot d\boldsymbol{s} = 0 \quad (17)$$

So the (16) becomes:

$$\frac{\Delta\omega}{\omega} = \frac{\iiint_{V_0} \left[\left(\boldsymbol{E_1} \cdot \boldsymbol{D_0}^* - \boldsymbol{E_0}^* \cdot \boldsymbol{D_1}\right) + \left(\boldsymbol{H_1} \cdot \boldsymbol{B_0}^* - \boldsymbol{H_0}^* \cdot \boldsymbol{B_1}\right)\right] dv}{\iiint_{V_0} \left[\left(\boldsymbol{E_0}^* \cdot \boldsymbol{D_0} + \boldsymbol{H_0}^* \cdot \boldsymbol{B_0}\right) + \left(\boldsymbol{E_0}^* \cdot \boldsymbol{D_1} + \boldsymbol{H_0}^* \cdot \boldsymbol{B_1}\right)\right] dv} \quad (18)$$

Equation (18) is an exact expression of cavity relative frequency change due to the perturbations on cavity shape and medium. Its application to the bead-pull needs a further approximation. Since $|\boldsymbol{D_1}| \ll |\boldsymbol{D_0}|$ and $|\boldsymbol{B_1}| \ll |\boldsymbol{B_0}|$ or their contribution in V_1 to the cavity volume V_0 integration is small, so the second term in (18)'s denominator is approximately zero. Also E_1, D_1 and H_1, B_1 in volume of V_0-V_1 are nearly same to their values in volume V_0 as their subscripts change from 1 to 0. So in (18)'s numerator the volume integration can be approximately over volume V_1 only.

Perturbation of a Resonator – Frequency Change due to Shape Perturbation

$$\frac{\Delta f}{f} = \frac{\Delta\omega}{\omega} = \frac{\oiiint_{V_1}\left[\left(\boldsymbol{E_1}\cdot\boldsymbol{D_0}^* - \boldsymbol{E_0}^*\cdot\boldsymbol{D_1}\right) + \left(\boldsymbol{H_1}\cdot\boldsymbol{B_0}^* - \boldsymbol{H_0}^*\cdot\boldsymbol{B_1}\right)\right] dv}{\oiiint_{V_0}\left[\left(\boldsymbol{E_0}^*\cdot\boldsymbol{D_0} + \boldsymbol{H_0}^*\cdot\boldsymbol{B_0}\right)\right] dv} \qquad (19)$$

If V_1 is a metallic volume, inside of V_1:

$$E' = 0 \qquad D' = D_0 \qquad B' = 0 \qquad H' = H_0 \qquad (20)$$

Then field changes inside of V_1:

$$E_1 = E' - E_0 = -E_0 \qquad D_1 = D' - D_0 = 0 \qquad (21)$$

$$B_1 = B' - B_0 = -B_0 \qquad H_1 = H' - H_0 = 0 \qquad (22)$$

Apply (21) (22) to equation (19), the cavity frequency change due to a metallic boundary perturbation is:

$$\frac{\Delta f}{f} \cong \frac{\oiiint_{V_1}\left(\boldsymbol{H_0}^*\cdot\boldsymbol{B_0} - \boldsymbol{E_0}\cdot\boldsymbol{D_0}^*\right) dv}{\oiiint_{V_0}\left(\boldsymbol{E_0}^*\cdot\boldsymbol{D_0} + \boldsymbol{H_0}^*\cdot\boldsymbol{B_0}\right) dv} \qquad (23)$$

Perturbation of a Resonator – Frequency Change due to a Metallic Boundary Change

If V_1 is deformed in such a way that can be considered as a small perturbation, the new surface S_1 is parallel to the original surface S_0, then metallic boundary volume of δV:

$$\boldsymbol{B_0} = \mu \boldsymbol{H_0} \qquad \boldsymbol{D_0} = \varepsilon \boldsymbol{E_0} \tag{24}$$

$$\frac{\Delta f}{f} \cong \frac{\oiiint_{\delta V} \left(\mu|\boldsymbol{H_0}|^2 - \varepsilon|\boldsymbol{E_0}|^2\right) dv}{\oiiint_{V_0} \left(\mu|\boldsymbol{H_0}|^2 + \varepsilon|\boldsymbol{E_0}|^2\right) dv} \tag{25}$$

1/4 of denominator in (25) r.s. is the cavity stored energy before the perturbation.
1/4 of numerator in (25) r.s. is the time averaged force like LFD.

1. The frequency change due to a perturbation of E field is going down, and is up due to the H field.
2. Pulling a metallic bead is actually measuring both ***E*** and ***H*** fields inside of cavity
3. To get an independent ***E*** and ***H*** field, one needs two types of bead, metallic and dielectric, by pulling them separately (advanced topic)

Perturbation of a Resonator – Frequency Change due to a Dielectric Bead

Homework (HW)1: After equation (18) to derive a frequency change due to the material medium perturbation:

$$\frac{\Delta f}{f} \cong -\frac{\oiiint_{\delta V}\left[(\boldsymbol{\mu_r}-\mathbf{1})\boldsymbol{\mu_0}\boldsymbol{H_1}\cdot\boldsymbol{H_0}^{*}+(\boldsymbol{\varepsilon_r}-\mathbf{1})\boldsymbol{\varepsilon_0}\boldsymbol{E_1}\cdot\boldsymbol{E_0}^{*}\right]dv}{\oiiint_{V_0}\left(\boldsymbol{\mu}|\boldsymbol{H_0}|^2+\varepsilon|\boldsymbol{E_0}|^2\right)dv} \qquad (26)$$

1/4 of denominator in (26) r.s. is the cavity stored energy before the perturbation

1. Both ***E*** and ***H*** fields perturbation can cause the cavity frequency going down
2. Pulling a dielectric bead with $\boldsymbol{\varepsilon_r} > \mathbf{1},\ \boldsymbol{\mu_r} = \mathbf{1}$ is actually measuring ***E*** field only.
3. High $\boldsymbol{\mu_r}$ material can be used to make a bead to measure the ***H*** field, but it might be under preformed than a metallic bead
4. When $\boldsymbol{\varepsilon_r} \gg \mathbf{1}$ or $\boldsymbol{\mu_r} \gg \mathbf{1}$, $\boldsymbol{E_1}$ and $\boldsymbol{H_1}$ can not be approximated to zero

Sphere Shape Bead Perturbations

When a bead is introduced into cavity, the change in the stored energy will be equal to the work done in expanding the bead volume from zero to V_1 volume against the forces of the fields. A simple geometry of such bead is a sphere with a finite radius of **a**. If we assume the sphere to be small so that the fields can be regarded as uniform for an appreciable distance beyond the sphere, then we can use the potential function of a sphere introduced into an uniform $\boldsymbol{E_0}$ and $\boldsymbol{H_0}$ fields

HW2: Use static E/H potentials inside of metal sphere to derive the frequency change due to a metallic bead:

$$U\frac{\Delta f}{f} = \pi a^3 \left(\frac{\mu {H_0}^2}{2} - \varepsilon {E_0}^2 \right) \qquad (27)$$

HW3: Use static E/H potentials inside of dielectric sphere to derive the frequency change due to a dielectric bead:

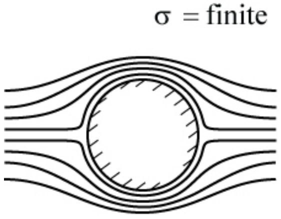
$$U\frac{\Delta f}{f} = -\pi a^3 \left(\frac{\mu_r - 1}{\mu_r + 2} \mu_0 {H_0}^2 + \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \varepsilon_0 {E_0}^2 \right) \qquad (28)$$

Ellipsoid Shape Perturbations

In order to obtain the orientation of the fields, an elongated body must be used for bead-pull.

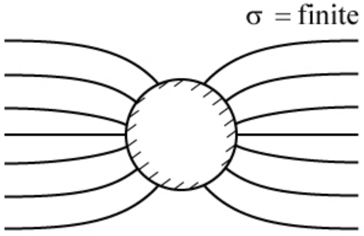
Metallic Ellipsoids:

The potential functions of metallic ellipsoids are derived and although ellipsoidal coordinates and elliptic integrations are involved [7], it is possible to calculate the work done when an ellipsoid is expanded from zero to a finite volume for the fields aligns its various axes.



field lines are parallel to an insulating surface at the wall

H field parallel to surface



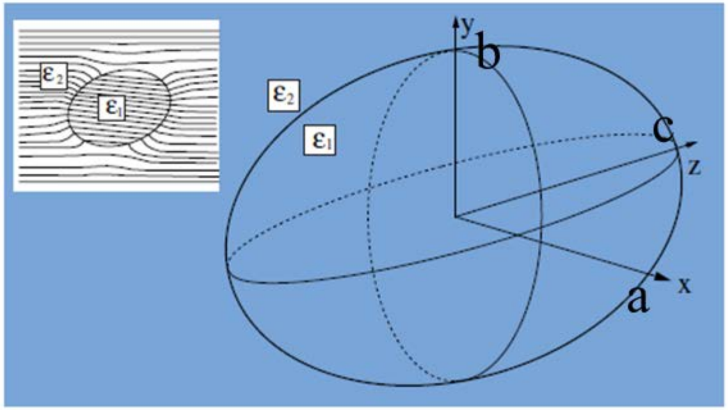
field lines are normal to a perfectly conducting surface at the wall

E field perpendicular to surface

Dielectric Ellipsoids:

The potential functions of both inside and outside sphere are known by solving Laplace's equation in ellipsoidal coordinates although such a solution needs elliptic integration. It is possible to calculate the work done in a similar way to the metallic.

The perturbation of an originally uniform electric field is shown due to the dielectric ellipsoid. The dielectric polarization inside the object is homogenous, however not parallel to the initial field.



Metallic Ellipsoid Shape Perturbation Formulae

$$\frac{\pi l^3 \varepsilon_0 E_0^{\ 2}}{8 ln\frac{l}{a} - 1}$$

More accurate E form factor for cylinder by ref. [8]. a is radius, l islength

E Form factor of syringe needle

$$U\frac{\Delta f}{f} =$$

H Form factor of syringe needle

sphere a=b=c		$\pi a^3\left[\frac{\mu_0 H_0^{\ 2}}{2} - \varepsilon_0 E_0^{\ 2}\right]$	
prolate spheroid like football a>b=c, β=b/a, e= $\sqrt{1-\beta^2}$		oblate spheroid like pill a=b>c, β=c/a, e= $\sqrt{1-\beta^2}$	
E_0 parallel to a	$-\frac{\frac{\pi a^3}{3} e^3 \varepsilon_0 E_0^{\ 2}}{\frac{1}{2} ln\frac{1+e}{1-e} - e}$	E_0 parallel to a or b	$-\frac{\frac{2\pi a^3}{3} e^3 \varepsilon_0 E_0^{\ 2}}{\frac{\pi}{2} - tan^{-1}\frac{\beta}{e} - \beta e}$
E_0 perpendicular to a	$-\frac{\frac{2\pi a^3}{3} e^3 \varepsilon_0 E_0^{\ 2}}{\frac{e}{\beta^2} - \frac{1}{2} ln\frac{1+e}{1-e}}$	E_0 perpendicular to a or b	$-\frac{\frac{\pi a^3}{3} e^3 \varepsilon_0 E_0^{\ 2}}{\frac{e}{\beta} + tan^{-1}\frac{\beta}{e} - \frac{\pi}{2}}$
H_0 parallel to a	$\frac{\frac{\pi a^3}{3} e^3 \mu_0 H_0^{\ 2}}{\frac{e}{\beta^2} - \frac{1}{2} ln\frac{1+e}{1-e}}$	H_0 parallel to a or b	$\frac{\frac{2\pi a^3}{3} e^3 \mu_0 H_0^{\ 2}}{tan^{-1}\frac{\beta}{e} + \frac{2-\beta^2}{\beta} e - \frac{\pi}{2}}$
H_0 perpendicular to a	$\frac{\frac{2\pi a^3}{3} e^3 \mu_0 H_0^{\ 2}}{\frac{2\beta^2-1}{\beta^2} e + \frac{1}{2} ln\frac{1+e}{1-e}}$	H_0 perpendicular to a or b	$\frac{\frac{\pi a^3}{3} e^3 \mu_0 H_0^{\ 2}}{\frac{\pi}{2} - tan^{-1}\frac{\beta}{e} - \beta e}$

Dielectric Ellipsoid Shape Perturbation Formulae

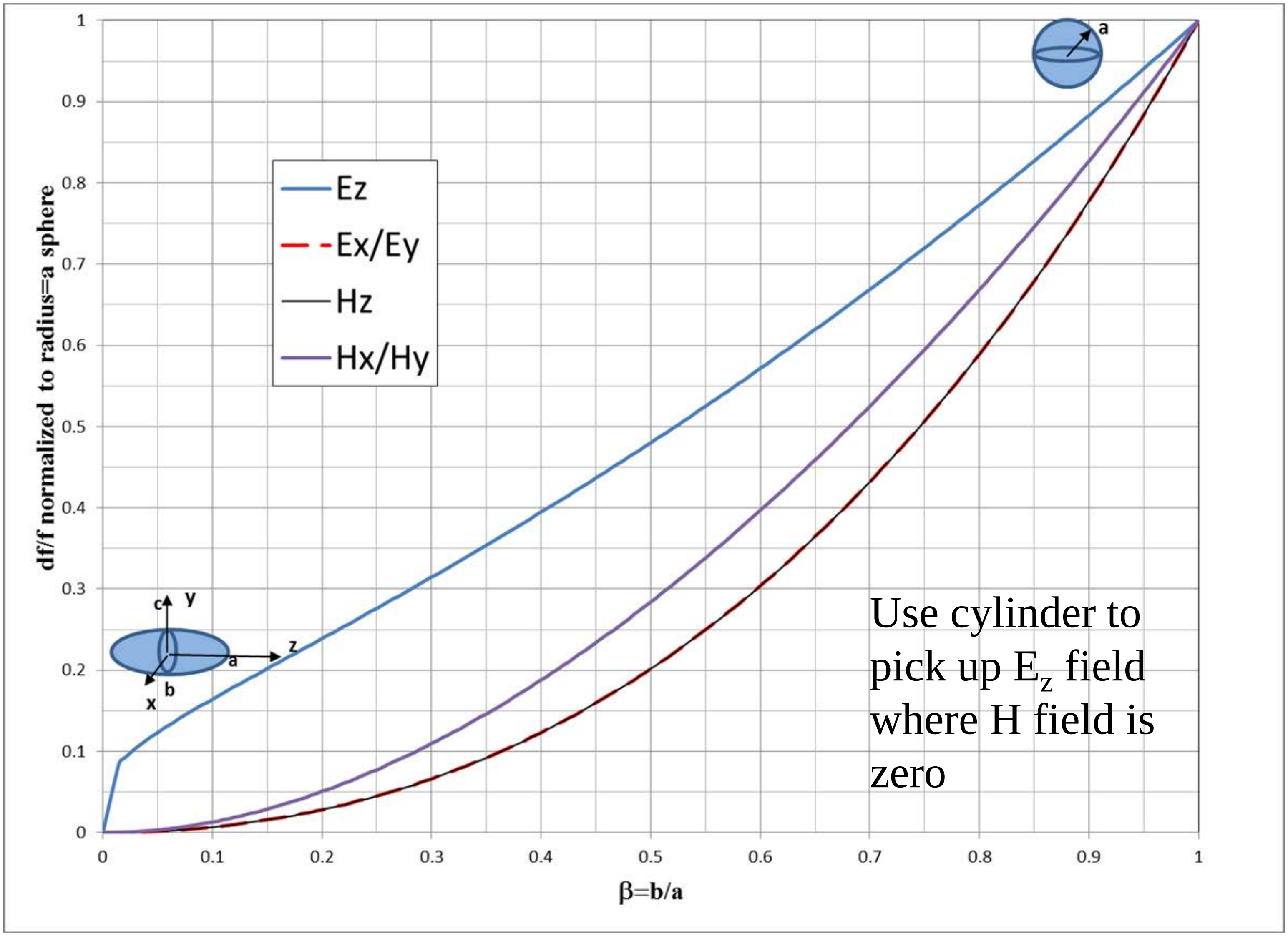
Form factor of cylinder for axial direction

$$U\frac{\Delta f}{f}=$$

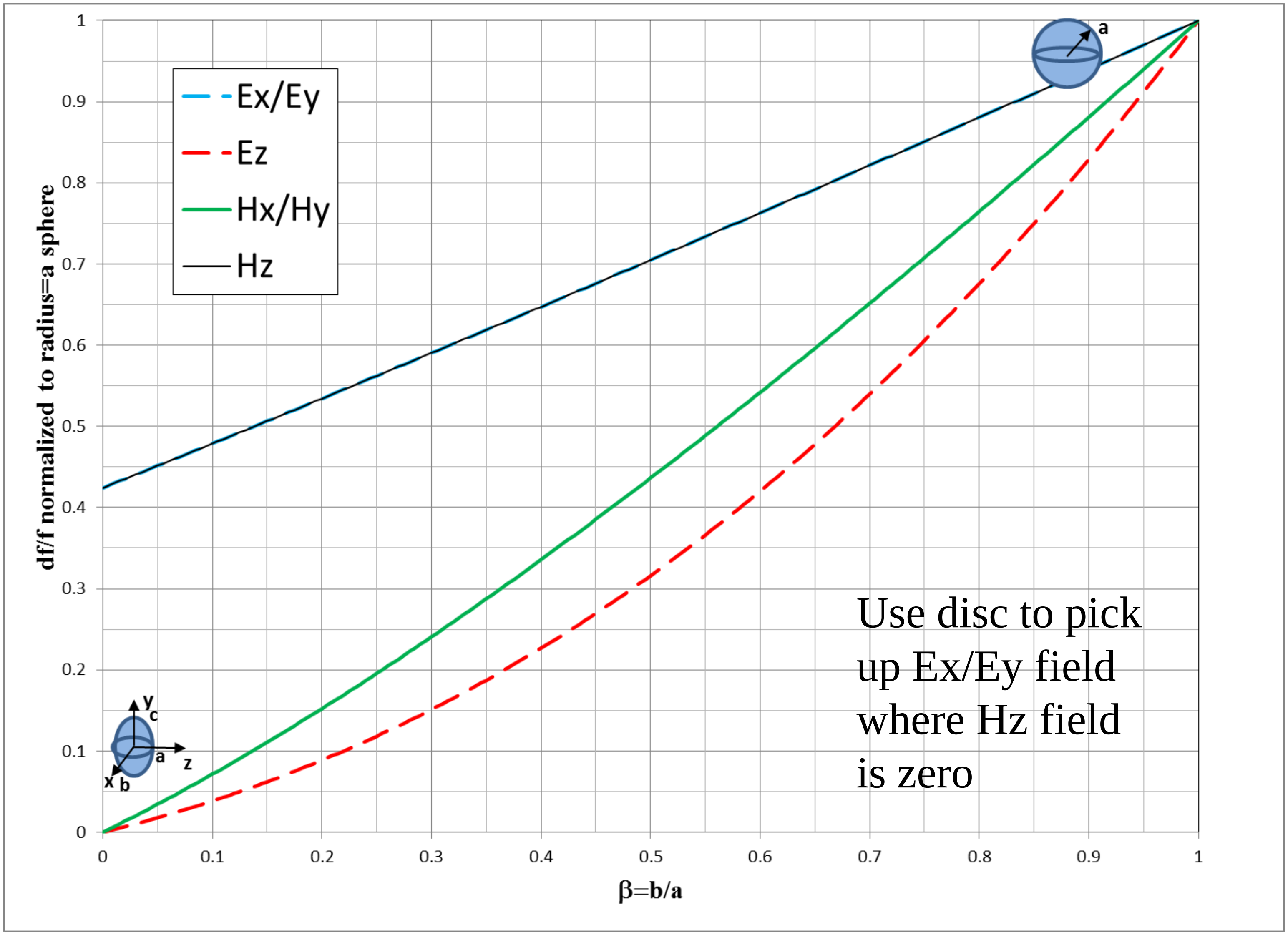
sphere a=b=c		$-\pi a^3\left[\frac{\varepsilon_r-1}{\varepsilon_r+2}\varepsilon_0{E_0}^2\right]$	
prolate spheroid like football a>b=c, β=b/a, e= $\sqrt{1-\beta^2}$		oblate spheroid like pill a=b>c, β=c/a, e= $\sqrt{1-\beta^2}$	
E_0 parallel to a	$-\frac{\frac{\pi a^3}{3}(\varepsilon_r-1)\varepsilon_0{E_0}^2}{\frac{\varepsilon_r-1}{e^3}\left[\frac{1}{2}ln\frac{1+e}{1-e}-e\right]+1}$	E_0 parallel to a or b	$-\frac{\frac{\pi a^3}{3}(\varepsilon_r-1)\varepsilon_0{E_0}^2}{\frac{\varepsilon_r-1}{e^3}\left[\frac{1}{2}ln\frac{1+e}{1-e}-e\right]+1}$
E_0 perpendicular to a	$-\frac{\frac{\pi a^3}{3}(\varepsilon_r-1)\varepsilon_0{E_0}^2}{\frac{\varepsilon_r-1}{2e^3}\left[\frac{e}{\beta^2}-\frac{1}{2}ln\frac{1+e}{1-e}\right]+1}$	E_0 perpendicular to a or b	$-\frac{\frac{\pi a^3}{3}(\varepsilon_r-1)\varepsilon_0{E_0}^2}{\frac{\varepsilon_r-1}{2e^3}\left[\frac{e}{\beta^2}-\frac{1}{2}ln\frac{1+e}{1-e}\right]+1}$

Form factor of disc for radial direction

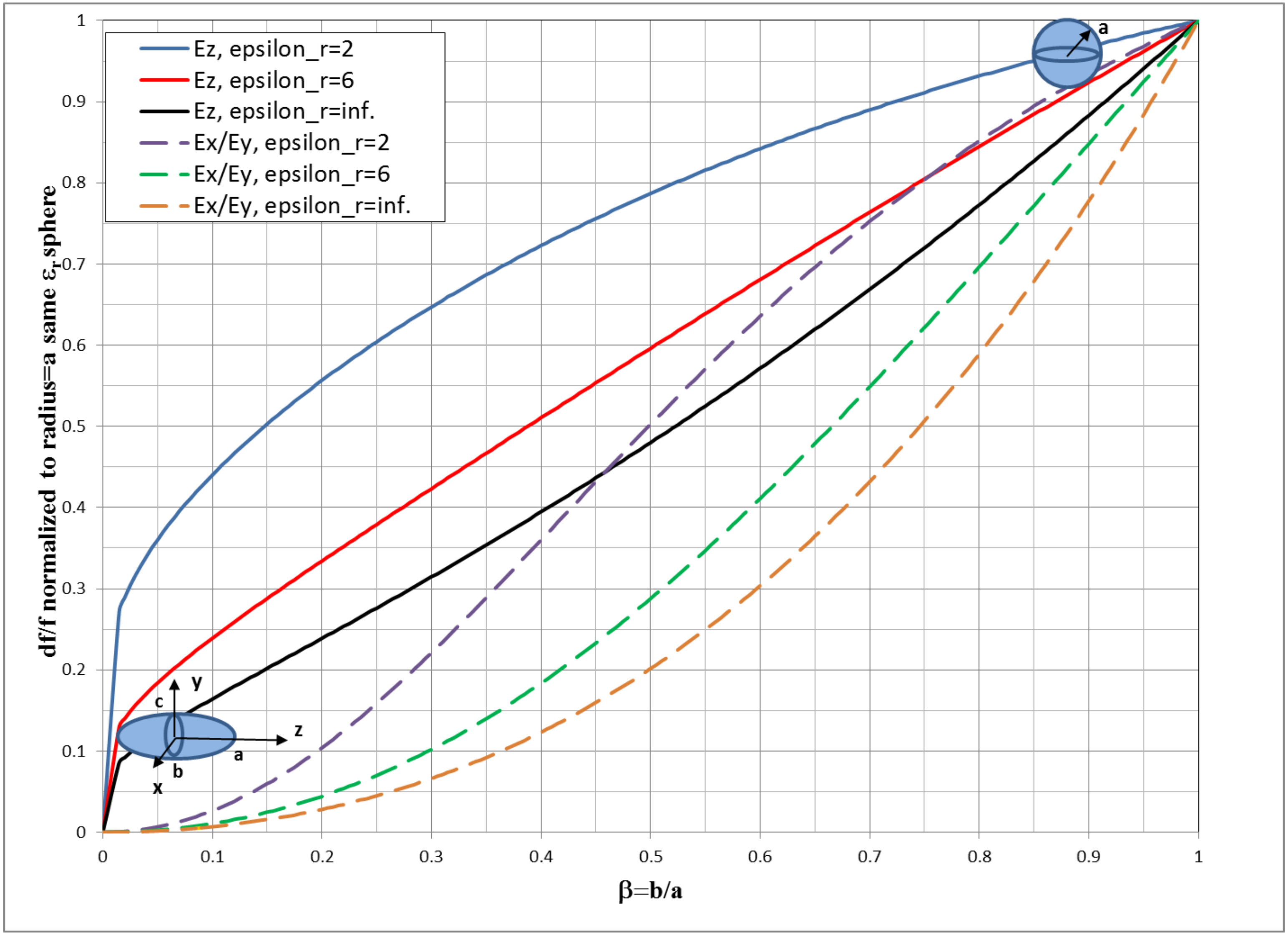
Perturbation due to Metallic Prolate Spheroids



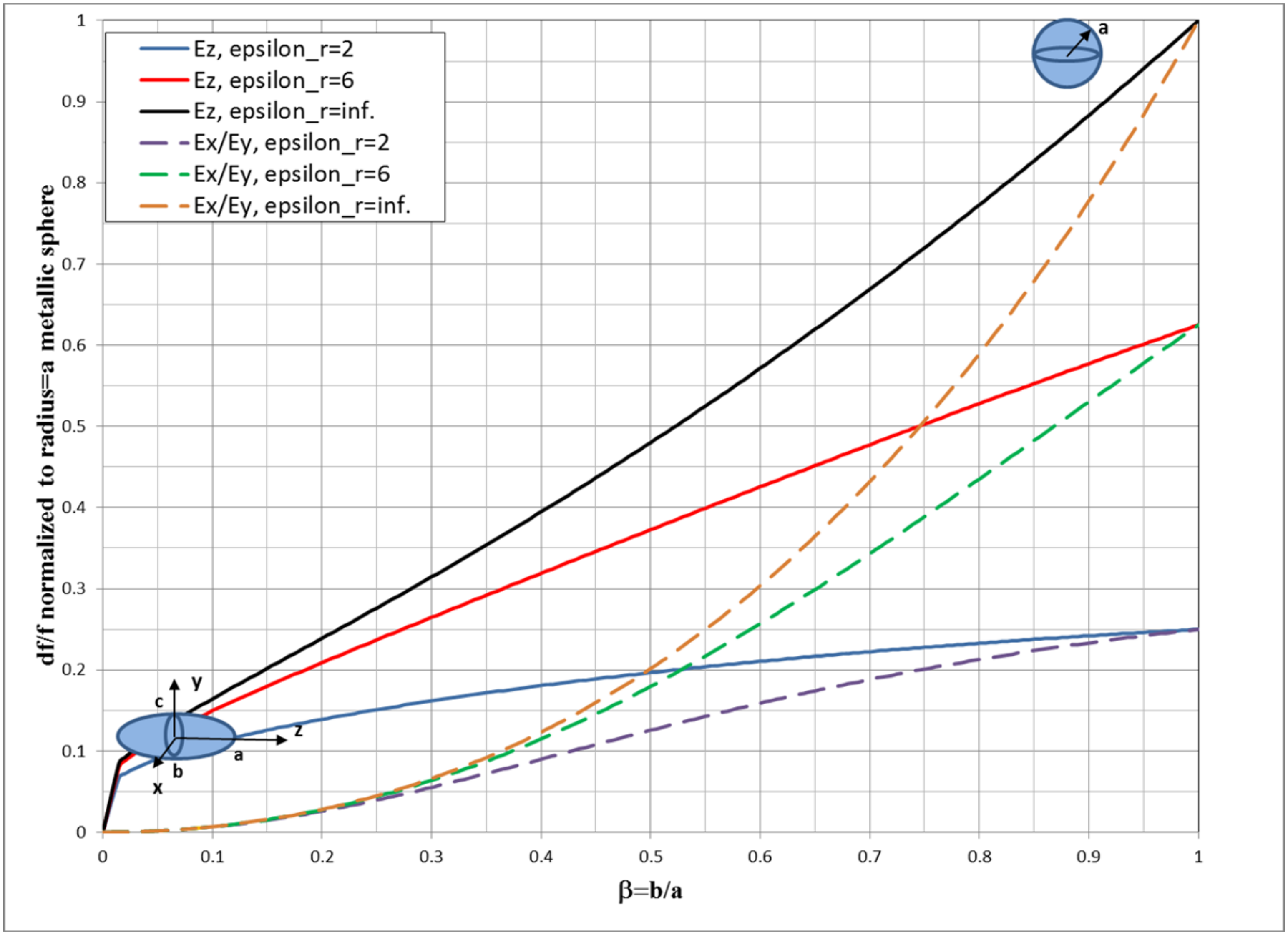
Perturbation due to Metallic Oblate Spheroids



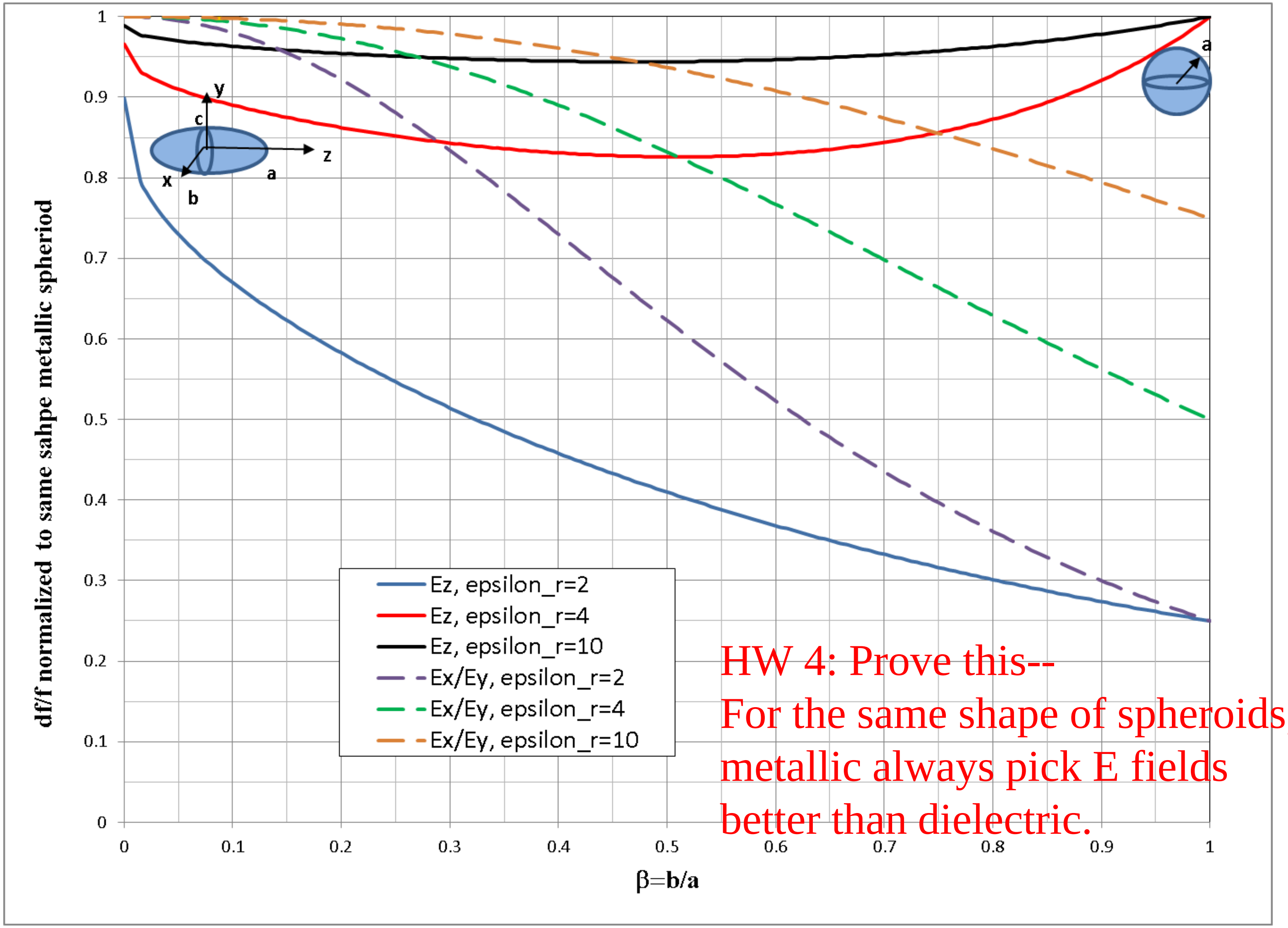
Perturbation due to Dielectric Prolate Spheroids



Perturbation due to Dielectric Prolate Spheroids

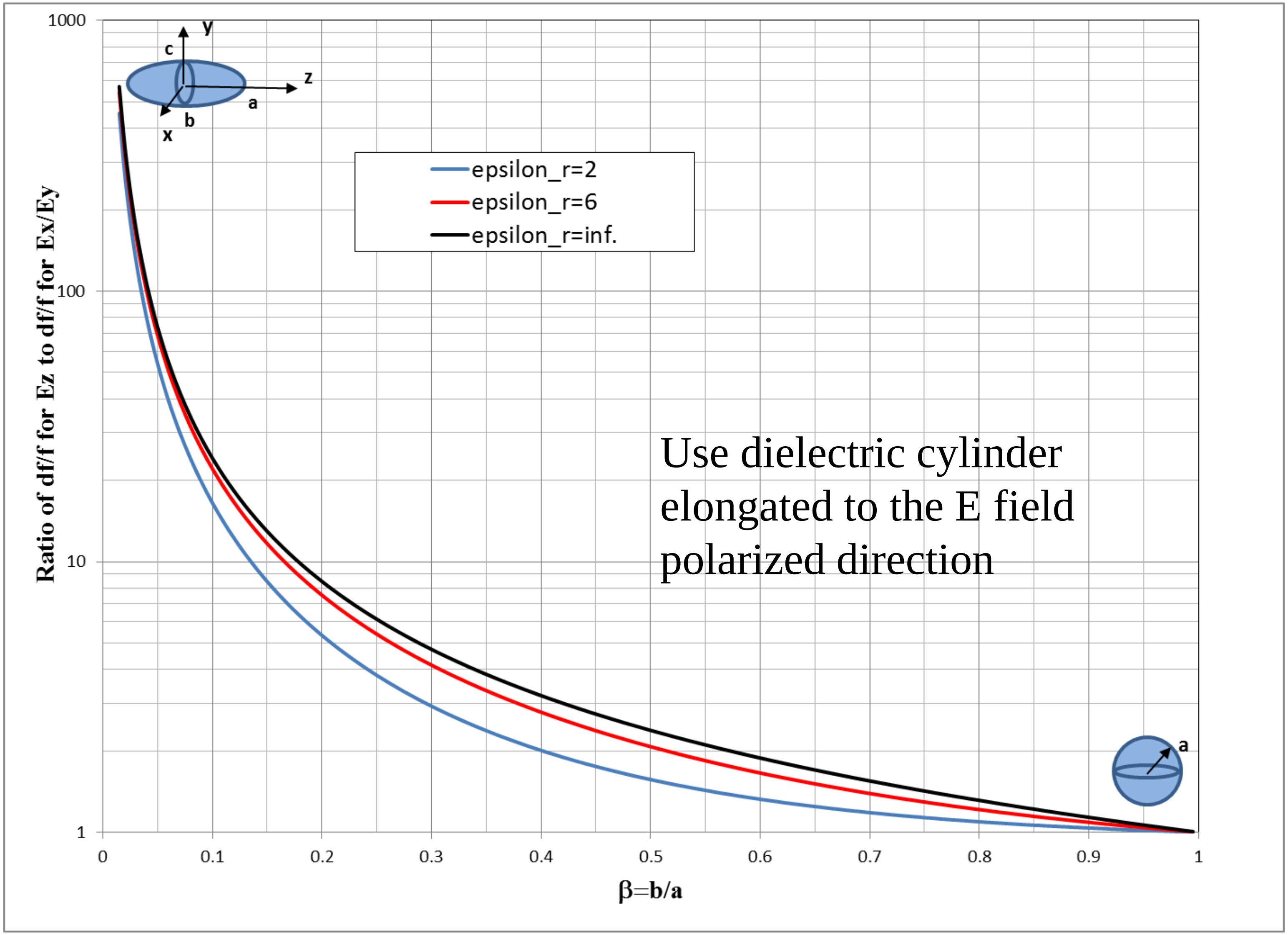


Perturbation due to Dielectric Prolate Spheroids



HW 4: Prove this--
For the same shape of spheroids, metallic always pick E fields better than dielectric.

Perturbation due to Dielectric Prolate Spheroids



Use dielectric cylinder elongated to the E field polarized direction

Cavity Acceleration Voltage and R/Q Definition

RF cavity acceleration voltage [6]:

$$V_c = \left| \int_{-L/2}^{L/2} E_z(r=0,z) exp\left(\frac{i\omega z}{\beta c}\right) dz \right| \qquad \text{Volt} \qquad (29)$$

Cavity R/Q, also called shunt impedance to quality factor ratio in unit of Ohm:

$$\frac{R}{Q} = \frac{{V_c}^2/P}{\omega U/P} = \frac{{V_c}^2}{\omega U} \qquad \Omega \qquad (30)$$

R/Q is cavity geometry shape dependent only, acceleration voltage $\boldsymbol{V_c}$ is normalized to cavity stored energy $\boldsymbol{U}$, $\boldsymbol{\omega=2\pi f}$, $\boldsymbol{f}$ is cavity resonance frequency in Hz.

This definition is from Accelerator Physics. Some Electrical Engineering folks use a definition of R/Q with an extra factor of **2** in denominator. Make sure to use corresponding R/Q value for your design when reading literatures and tech notes.

R/Q for CEBAF C100 7-cell, 1497MHz SRF cavity is 871.5Ω.

HW5: use SuperFish (SF,2D) code to calculate R/Q value to confirm this. (download SF and SF input file with cavity geometry is provided separately)

Cavity R/Q Calculation from Bead-pull Measurement

From (29) (30):

$$\frac{R}{Q} = \frac{\left| \int_{-L/2}^{L/2} E_z(r=0,z) exp\left(\frac{i\omega z}{\beta c}\right) dz \right|^2}{\omega U} \quad \Omega \qquad (31)$$

From (19) pulling a bead to measure the cavity **Δf(z)/f** can measure the **sqrt[Ez(r=0,z)]/U** :

$$\frac{\Delta f(z)}{f} = \frac{\Delta W_m(z) - \Delta W_e(z)}{W_m + W_e} = \frac{\Delta W_m(z) - \Delta W_e(z)}{U} \qquad (32)$$

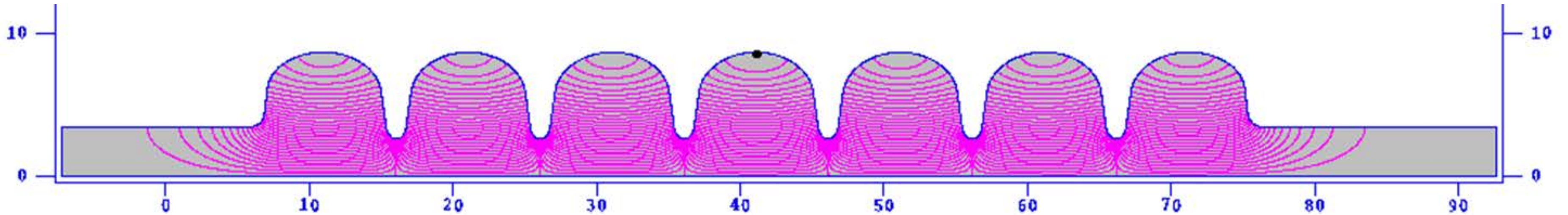
For example, using a Teflon sphere bead or syringe needle, ***Hz(r=0,z)***≈0 in this case :

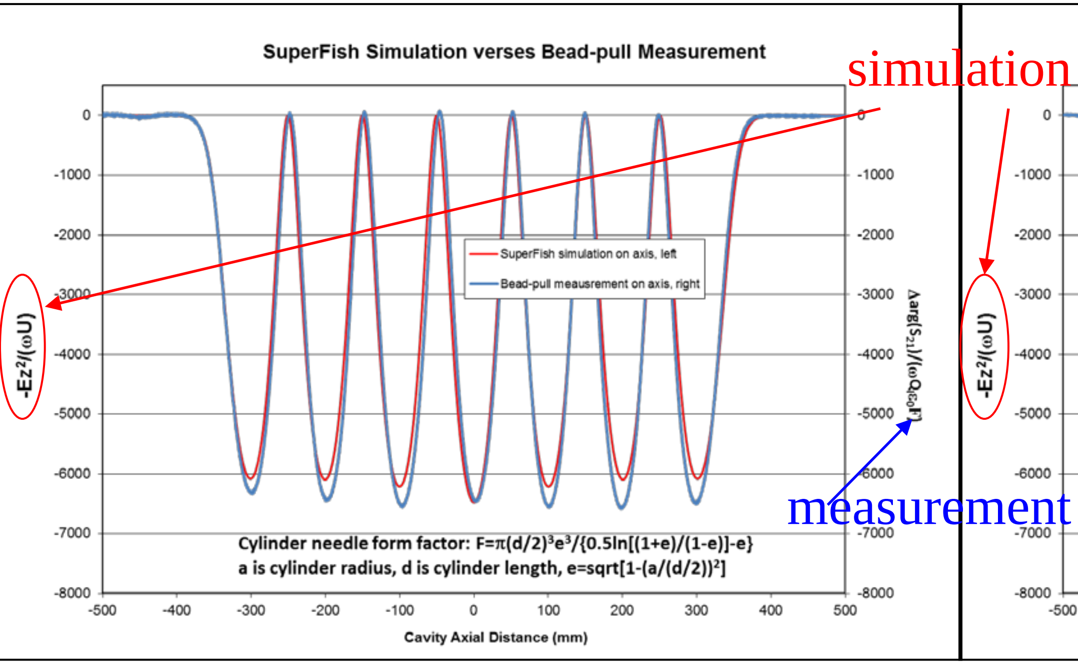
$$\frac{\Delta f(z)}{f} \frac{F_{bead}}{\omega} = \frac{|Ez(z)|^2}{\omega U} \qquad (33)$$

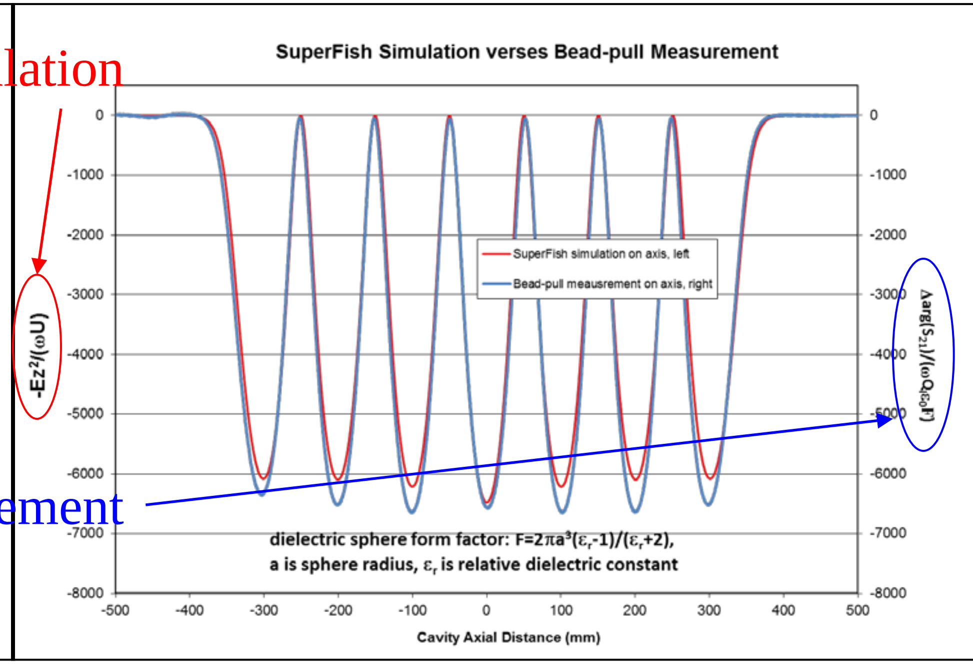
Here, F_{bead} or its alternative factor with multiplication to other constants is called **bead form factor** which is only defined by the bead geometry and material which we have found early. So the bead-pull measurement can measure the normalized EM field distribution along the pulling direction which is **not** cavity frequency and drive power dependent parameter.

Cavity R/Q Calculation from Bead-pull Measurement

HW6: Use SF to calculate $Ez(r=0,z)^2/(\omega U)$ field in Excel format to be ready before the hands-on class next week for simulation to measurement comparison.







RF Measurement Basics-S parameters

HW 7: Refer to [6] pp149-151 to prove following two-port S parameter formulae (34)-(37) (38) no mater of $\beta_1<1$ or $\beta_1>1$ or $\beta_2<1$ or $\beta_2>1$. The conditions of approximation in (39) and the relative error for a 10^o of phase change:

$$S_{21}(f)=\frac{2\sqrt{\beta_1\beta_2}}{(1+\beta_1+\beta_2)+i\Omega(f)} \quad (34)$$

$$\Omega(f)=Q_0\left(\frac{f}{f_0}-\frac{f_0}{f}\right) \qquad Q_L=\frac{Q_0}{1+\beta_1+\beta_2} \quad (35)$$

$$\beta_1=\frac{Q_0}{Q_{e1}} \quad (36)$$

Q_{e1} is input coupler external Q

$$\beta_2=\frac{Q_0}{Q_{e2}} \quad (37)$$

Q_{e2} is output coupler external Q

When $f=f_0$ or $\Omega(f_0)=0$: $S_{21}(f_0)=10\log\left[\frac{4\beta_1\beta_2}{(1+\beta_1+\beta_2)^2}\right]$ dB (38)

$$\Delta ang(S_{21})=\tan^{-1}\left(\frac{-\Omega(f)}{1+\beta_1+\beta_2}\right)\approx -2Q_L\frac{\Delta f}{f_0}$$

Radian and $\Delta f=f-f_0$ (39)

RF Measurement Basics-S parameters

HW 8: Refer to [6] p153 to prove following S parameter formulae (40)-(44):

Reflection:

$$\Gamma(f)=\frac{\beta_1-1-i\Omega(f)}{\beta_1+1+i\Omega(f)} \quad (40)$$

$$-1\le\Gamma(f_0)=\frac{\beta_1-1}{\beta_1+1}\le 1 \quad (41)$$

$$\beta_1(f_0)=\frac{1\pm|S_{11}(f_0)|}{1\mp|S_{11}(f_0)|} \quad (42)$$

Upper sign is for $\beta_1>1$ (over-coupled)
Lower sign is for $\beta_1<1$ (under-coupled)

In the case of $\beta_1<<1$, $S_{11}\sim 1$, (41) gives a large error. We used (42) instead of (41) in the bench measurement:

$$\beta_1\approx\frac{1-\left|S_{11}(f_0)/S_{11}(|f|>>f_0)\right|}{1+\left|S_{11}(f_0)/S_{11}(|f|>>f_0)\right|} \quad (43)$$

$|f|>>f_0$ is off resonance condition
$f=f_0$ is on resonance

No cable calibration is needed

In the case of $\beta_2<<1$, $S_{22}\sim 1$, we measure S_{21} instead of S_{22}:

Transmission:

$$Q_{e2}(f_0)=\frac{4\beta_1}{1+\beta_1+\beta_2}Q_L\cdot 10^{\frac{|S_{21}(f_0)|(dB)}{10}} \quad (44)$$

cable calibration is needed

Field Flatness Measurement from Bead-pull Data

The original definition of the N-cell cavity field flatness is

$$\eta_{ff} = \left(1 - \frac{V_{c\max} - V_{c\min}}{\frac{1}{N}\sum_{i=1}^{N} V_{ci}} \right) \times 100\% \qquad (45)$$

Here V_{ci} is the accelerating voltage of the *i*th cell. $V_{c\max}$ and $V_{c\min}$ is the maximum and minimum cell voltage in the cavity, respectively.

For a fixed $\beta_g = v/c$ structure, all the cells in the cavity have an equal accelerating gap $d = \beta c/(2f_0)$. The cell's accelerating voltage is

$$V_c = E_c \int_{-L/2}^{L/2} f(z)\cos(\pi \frac{z}{L} + \phi_0)dz = E_c T(d) \qquad (46)$$

Here E_c is the peak axial electric field along the beam axis *z*. The $f(z)$ is the field distribution function. As long as the cavity shape and the gap *L* don't change from cell to cell, or a small perturbation change which only changes the E_c but not $f(z)$ and *d*. So the transit time factor $T(d)$ will be a constant. Then the equation (46) becomes:

$$\eta_{ff} = \left(1 - \frac{E_{c\max} - E_{c\min}}{\frac{1}{N}\sum_{i=1}^{N} E_{ci}} \right) = \left(1 - \frac{\sqrt{\Delta\phi_{c\max}} - \sqrt{\Delta\phi_{c\min}}}{\frac{1}{N}\sum_{i=1}^{N} \sqrt{\Delta\phi_{ci}}} \right) \times 100\% \qquad (47)$$

Here E_{ci} is the peak axial electric field in the *i*th cell. $\Delta\phi_{ci}$ is S_{21} phase change in bead-pull data after correction of slop and offset

Bead-pull Formula for Dielectric Sphere and its Error Analysis

From page 17 and equation [39], we can derive:

$$-\frac{{E_z}^2}{\omega U}=\frac{tan(\Delta angS_{21})}{\omega Q_L \varepsilon_0 F_{di-sphere}} \quad (48)$$

Here $F_{di\text{-}sphere}$ is dielectric sphere form factor:

$$\mathrm{F_{di-sphere}}=2\pi a^3\left(\frac{\varepsilon_r-1}{\varepsilon_r+2}\right) \quad (49)$$

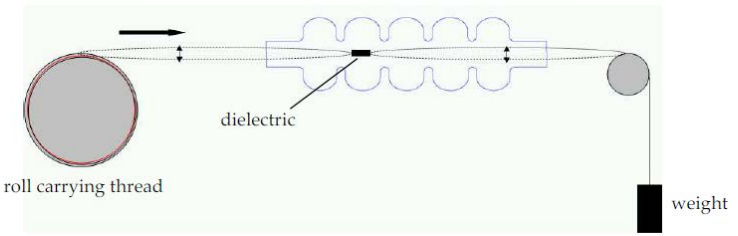
Except the approximation in [39], additional relative errors caused by measurement parameters are:

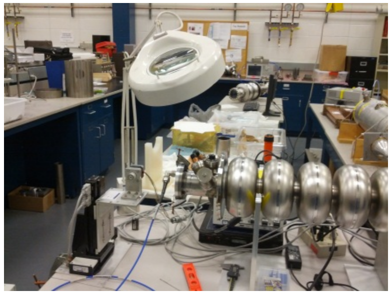
$$2\left|\frac{\Delta E_z}{E_z}\right|=\left|\frac{\Delta\phi}{\phi}\right|+\left|\frac{\Delta Q_L}{Q_L}\right|+3\left|\frac{\Delta a}{a}\right|+3\left|\frac{\Delta\varepsilon_r}{(\varepsilon_r-1)(\varepsilon_r+2)}\right| \quad (50)$$

For a teflon sphere $|\Delta E_z/E_z|$~(1%+1%+3*0.2%+3*1%)/2~5%

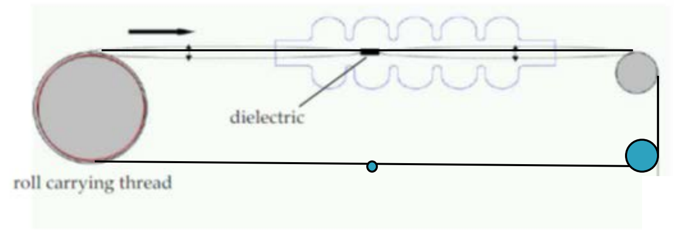
HW9 optional: Try to estimate cavity frequency shift due to a nylon (ε_r=4) fishing wire (dia.=0.019 inch) stretching through the axis of C100 cavity. When using ΔF method to measure the E field, should you consider this frequency shift? Try to confirm your estimate in your experiment.

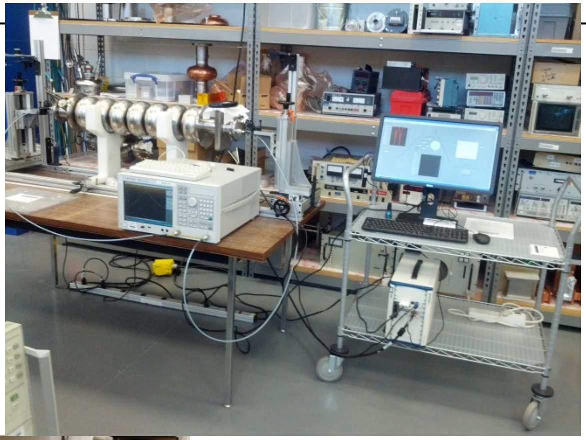
Bead-pull Measurement Systems





JLab system with wire holders on x-y stages





ODU system with wire loop frame

Bead-pull Measurement Techniques

System	ODU	JLab
wire pulling device	close loop with spring tension on pulleys	weight drop with multi-loop travel on pulleys
step-motor-drive	1D in Z w/ Labview	3D w/ X-Y stages
automation control	Labview 2010	Labview 2014
measure with VNA	search S_{21} peak f to find $\Delta f/f_0=(f_{peak}-f_0)/f_0$	S_{21} phase $\Delta f/f_0=\tan(\Delta \arg(S_{21}))/(2Q_l)$
pros	no wire over latch on reel; no bead vibration issue; no Q_l measurement; change bead w/o rewiring	fast; can be manual operation for quick setup; multi-mode measure in one pull is possible
cons	slow, frequency drift; need automation; suitable for short cavity	possible bead vibration; wire over latch caused pulling speed change; phase lock drift; bead change needs rewiring

Bead-pull Measurement Common Errors and Remedy Tricks

Common Errors	Most Possible Causes	Remedy Tricks
large noise small signal	Too weak in couplings, small drive power, high IF frequency, drive frequency off resonance in phase measurement	P=0-10dBm, IF<1kHz, S_{21}_loss >-60dB $0.01<S_{11}\sim S_{22}$. Use RF amplifier is needed. Scan for new peak drive frequency
Bead vibration	String suspension structure resonance with motor step frequency	Increase string tension or hanging weight, use damper on wire eye
Field tilts to one side	Not a real tilt, too strong coupling on weak peak side	Control $\beta_1\sim\beta_2<0.1$. Large β causes $f_0\downarrow$ Antenna not perturb f_0 by <20kHz
Positive phase or frequency change	off-axis pulling, picking up H field Wrong mode	Re-align the wire Change drive frequency
Phase display jump during time sweep	Phase near ±180°, normal in VNA phase lock sweep	Use phase delay on the VNA display screen
Too large phase change	Too large bead size or wrong shape	Change bead
Signal base line tilt	Cavity frequency/VNA phase drift	Correct slop in data process Faster pulling speed Use small IF<500Hz
Large error on R/Q calculation	Bead size measurement error ε_r error, wrong calculation formula Misalignment of Ez field versus cavity position z	Use caliper to measure bead size Calibrate bead-pulling speed/distance with stop watch Calibrate ε_r with a pillbox cavity

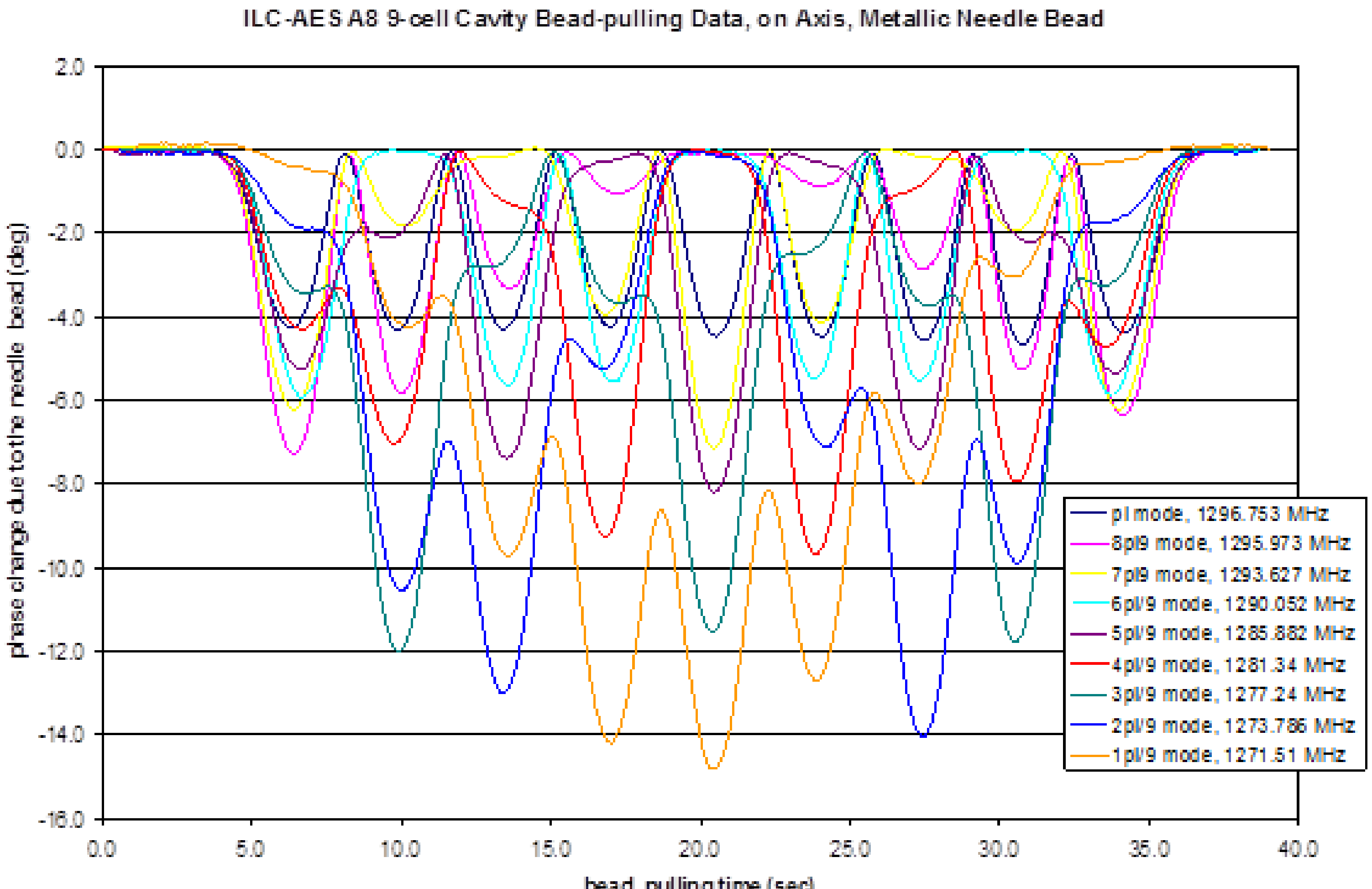
Qext Measurement Common Errors and Remedy Tricks

Common Errors	Most Possible Causes	Remedy Tricks
Calibration error	Changed cable, Cal Kit standards are misused	Recalibrated cable Follow calibration procedure
Input beam pipe coupler is too strong	Cavity field perturbed and tilted by input antenna	Reduce the $\beta_1<0.1$
Bad cable	Extra loss on bad connector	Change the cable
Using mismatched top hat	Extra loss on top hat	Use Adaptor Removal Calibration Procedure
Nb top hat on C50 cavity waveguide FPC	Extra waveguide mode build up	Avoid assembly structure with waveguide build up
antenna perpendicular to E field or loop plane parallel to H field	Very sensitive to coupler tilting angle or rotation	Use "L" shape rod antenna, mark the orientation of rotation for reassembly
Large Qext difference between bench and cold measurements	Third port or add-on structure affect S_{21} transmission coupling	Measure the S_{21} with full assembly, avoid any ghost mode nearby, use simulation to understand the coupling

Bead-pull Measurement-Advanced Topics

Multi-mode measurement with one bead-pull:

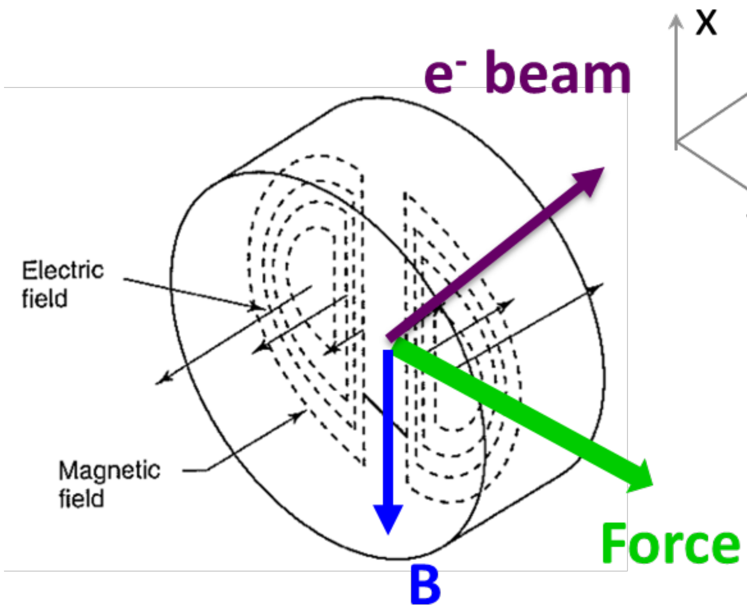
- Example: TM010 passband modes of ILC (TESLA) 9-cell Cavity

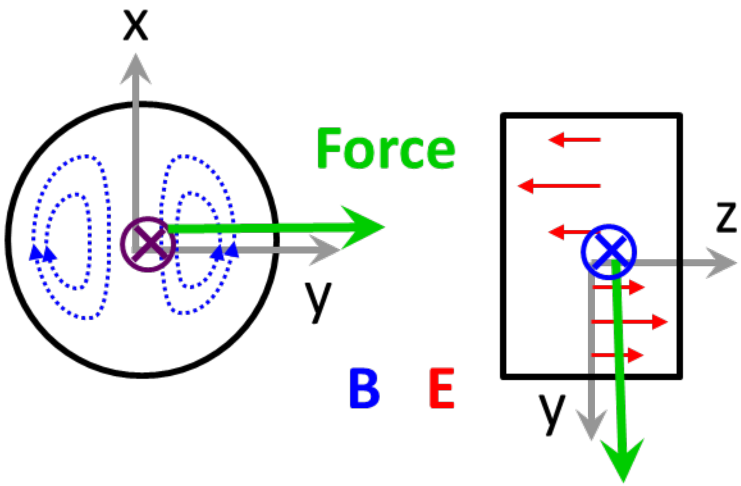


Bead-pull Measurement-Advanced Topics

Bead-pull for HOMs

- Example: TM110 π modes





- Use analytical or 3D simulation to understand EM field pattern
- Use metallic spherical bead to pick up on-axis Bx field
- Use metallic disc or sphere to pick up off-axis Bx and Ez fields
- Use ceramic spherical or needle bead to pick up off-axis Ez field
- Rotate ceramic sphere around beam axis to measure the dipole polarization

Bead-pull Measurement Tasks

- **Perform two-port VNA cable calibration procedure**
- **Practice VNA measurement key menu**
- **β_1 and β_2 measurement by S_{11} and S_{21} for electric antenna setup on beam pipes**
- **Change antenna length for appropriate input and output couplings β_1 and β_2**
- **External Q measurement for C100 cavity's fundamental Power Coupler (FPC) and Field Probe FP**
- **Practice 1D wire pulling by manual control of step motor through NI's IMAX**
- **Measure cavity resonance frequency f_0, Q_L by -3dB bandwidth**
- **1D bead-pull on C100 cavity using either ODU or JLab system with student's choice of bead**
- **Download bead-pull data in csv format for HW10**
- **HW10: compare bead-pull data with SF simulation on $Ez(r=0,z)^2/(\omega U)$ distribution and calculate the R/Q value from the bead-pull data to compare it with your SF result. Calculated cavity's field flatness. (due in the morning of the last school day (01/23) for grade)**
- **If an extra time is allowed, we will do advanced topics**
  1. **TM010 passband modes bead-pull demonstration by Labview automation**
  2. **TM110 π mode rotation bead-pull to measure mode polarization (on JLab system)**
  3. **On-axis bead-pull for H field, off-axis bead-pull for both E and H fields**

References

1. Wikipedia page: http://en.wikipedia.org/wiki/Cavity_perturbation_theory
2. David M. Pozar, Microwave Engineering, 2nd edition, Wiley, New York, NY, 1998, Section 6.8 Cavity Perturbation, pages 340-344
3. J. D. Jackson, Classical Electrodynamics, John Wiley and Sons, Inc.
4. L. B. Mullett, Perturbation of a Resonator, UK Atomic Energy Research Establishment, Harwell, Berkshire, 1957, Unclassified A.E.R.E. G/R 853
5. R. G. Carter, Engineering Department, Lancaster University, Lancaster LA14YR, U.K., School Teaching Note, year?
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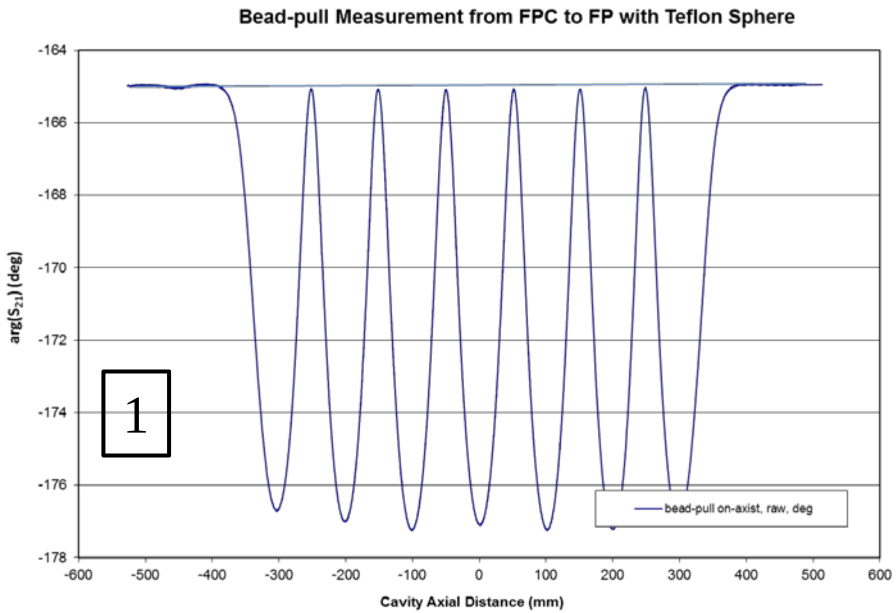
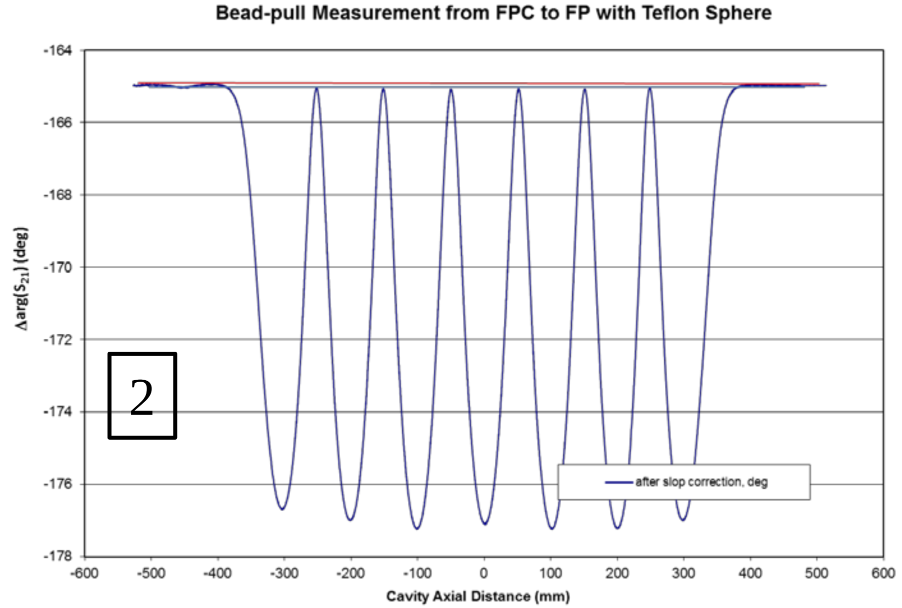
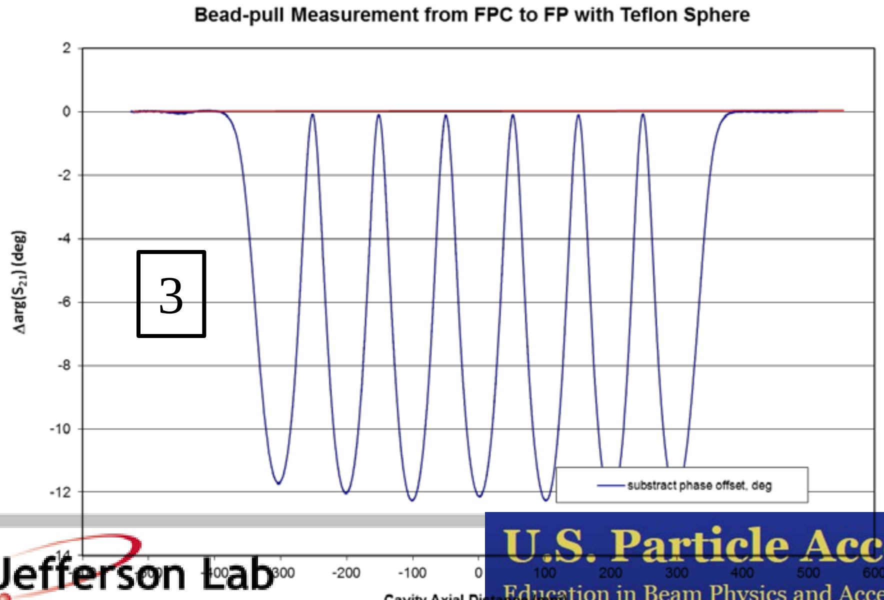
7. L. C. Maier Jr. and J. C. Slater, Field Strength Measurements in Resonator Cavities, Journal of Applied Physics Vol. 23, No. 1, Jan. 1952
8. H. Hahn and H.J. Halama, Pertubation measurements of transverse R/Q in irisi-loaded waveguides, IEEE Trans. MTT-16, no.1, p. 20 (January 1968)

Backup Slides

R/Q Calculation from Bead-pull Data Procedure

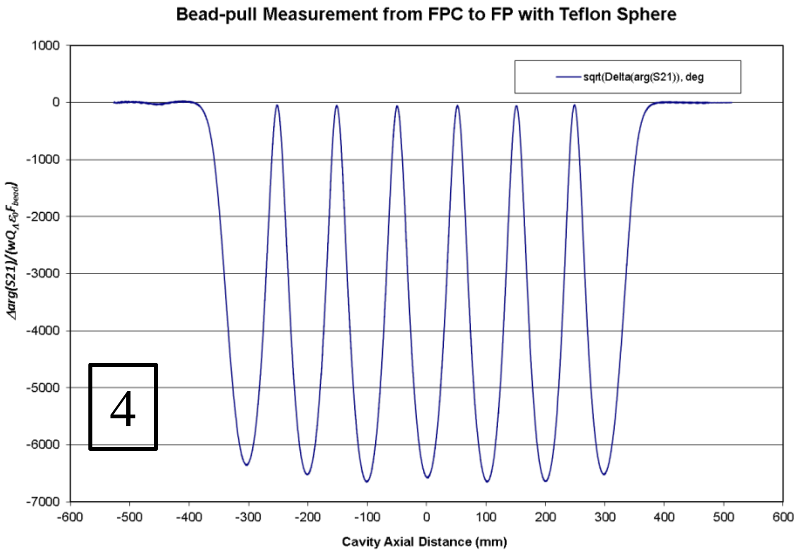


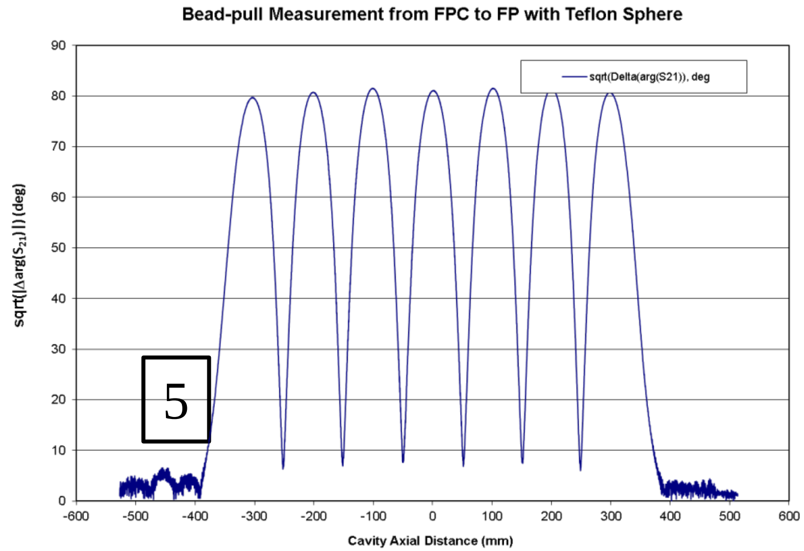


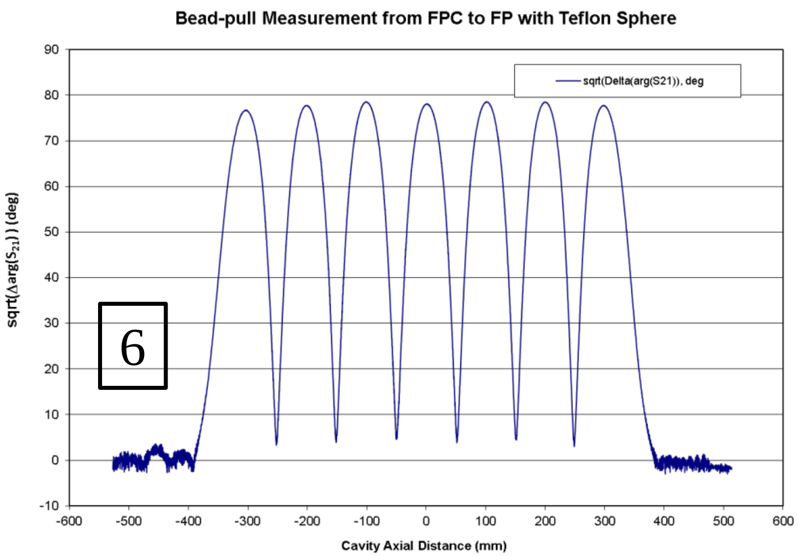


1. Take phase change (deg) raw data
2. Correct baseline slop
3. Subtract baseline offset to zero

R/Q Calculation from Bead-pull Data Procedure

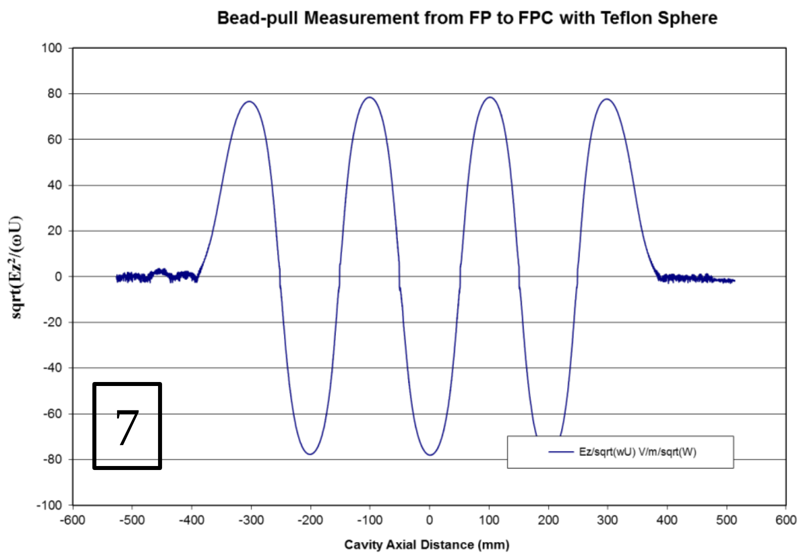


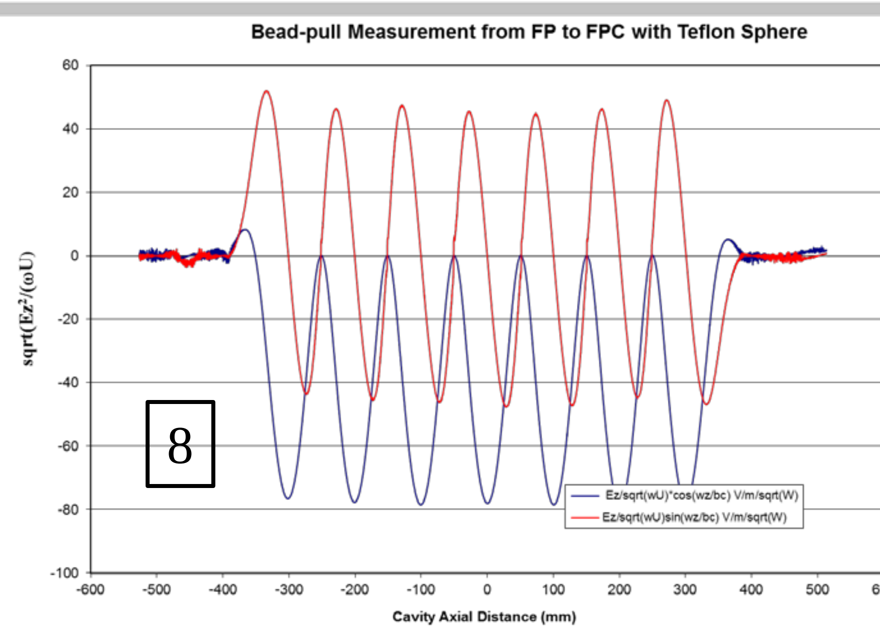


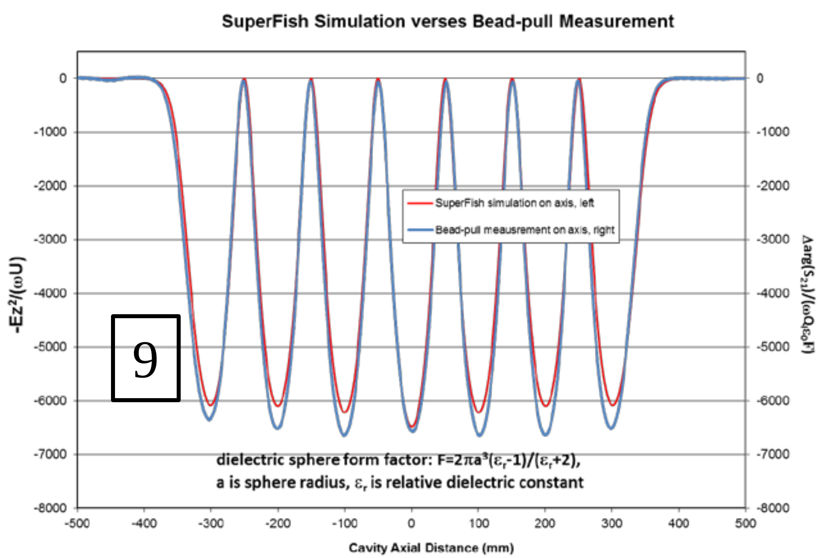


4. ***$-Ez^2/(\omega U)=\Delta arg(S21)/(\omega Q_L \varepsilon_0 F_{bead})$***
5. ***$sqrt(|Ez^2/(\omega U)|)$***
6. Subtract offset ***$sqrt(Ez^2/\omega U)$***

R/Q Calculation from Bead-pull Data Procedure







7. **Manually flip field amplitudes according cosine distribution, pay attention to the sign near zero crossing to avoid any large kink**
8. **Check *sine and cosine* multiplication terms for odd and even function and values of integration**
9. **Check F_{beand} with $Ez^2/(\omega U)$ *simulation* amplitudes**
10. **Sums of *sine and cosine* terms**
11. **Sums of sine*dz and cosine*dz terms**
12. **RTTQ=(sumsine*dz)^2+(sumcosine*dz)^2**