Vibration and Modal Analysis Basics

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USPAS Course:

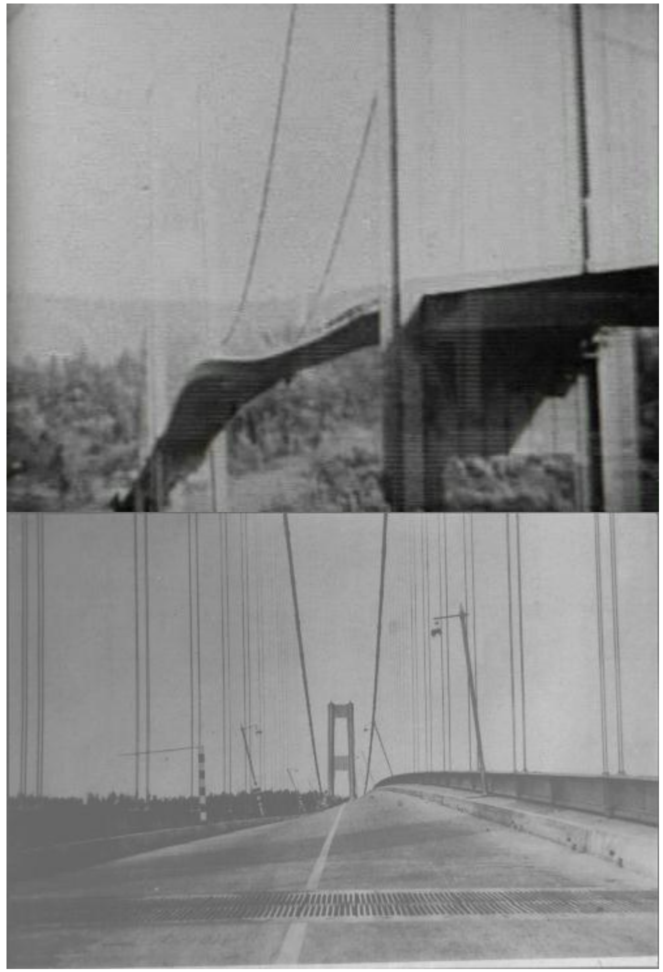
SRF Technology: Practices and Hands-On Measurements

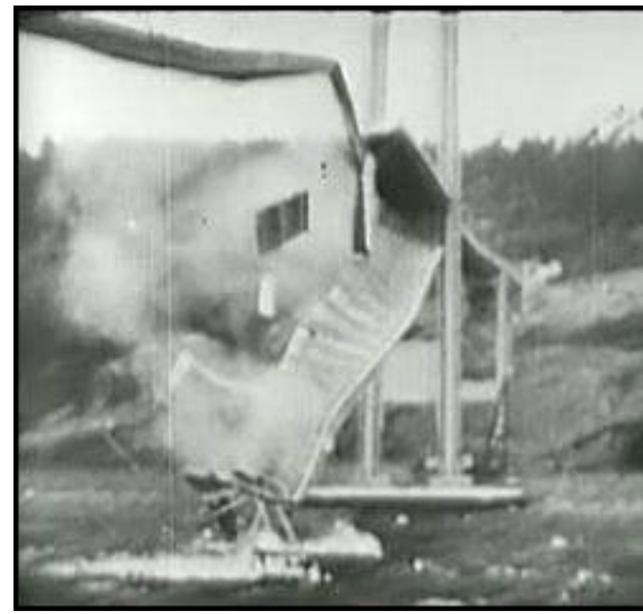
January 2015

Vibration and Modal Analysis Basics

Vibration and Modal Analysis Basics

April 1940





November 1940

Vibration and Modal Analysis Basics

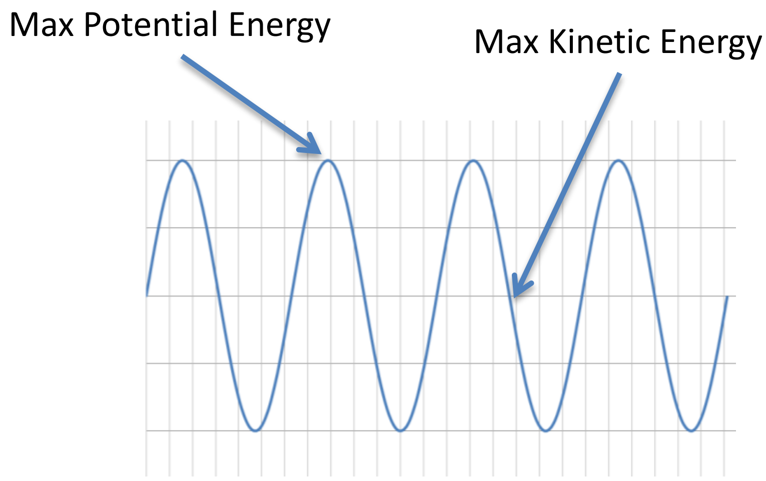
Objectives

- Why study vibrations?
  - – So my machine doesn't get screwed up
- What is modal analysis?
  - Analytical Modal Analysis (Calculations)
  - Experimental Modal Analysis (Measurements)
- How are real structures analyzed?

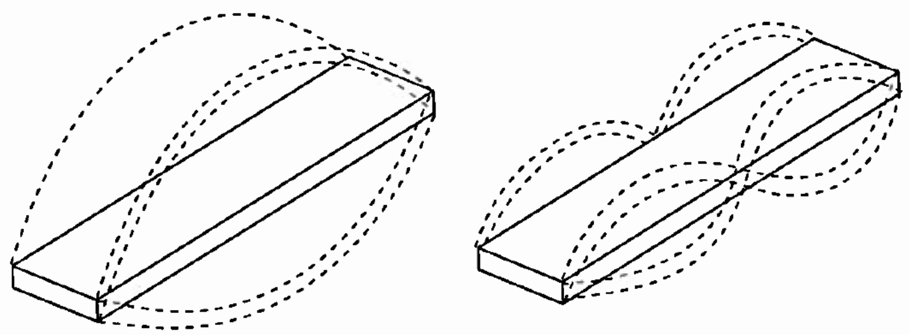
Vibration and Modal Analysis Basics

What is vibration?

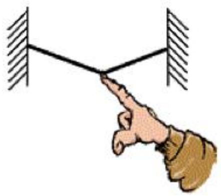
- Stored energy within a structure is transformed between potential (elastic deformation) and kinetic (moving mass) energy. The oscillatory motion is vibration.

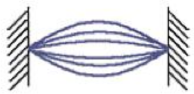


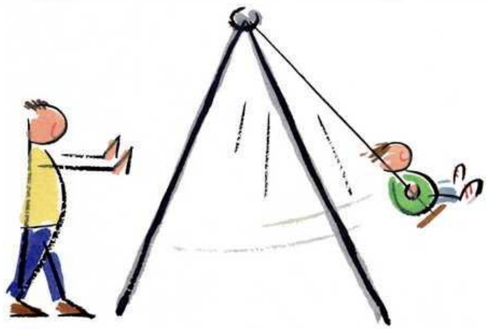
- The stored energy results in standing waves (modes) at inherent natural frequencies.



Types of Vibrations







- Vibrations
  - Free (natural)
    - Undamped
    - Damped
  - Forced
    - Undamped
    - Damped

$f(t) = F_0 \cos\omega_f t$

Undamped

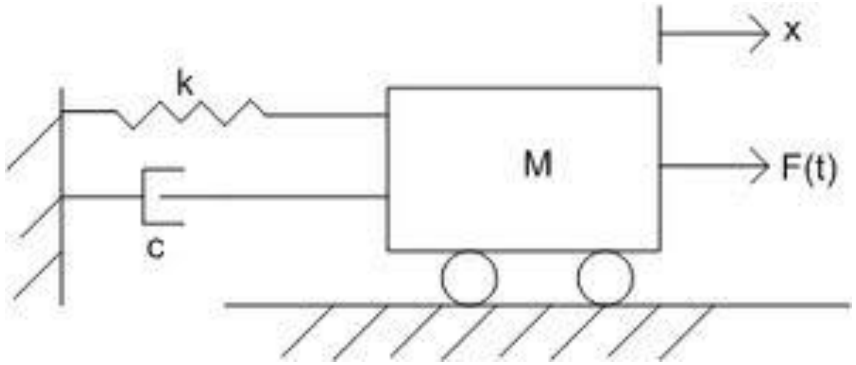
- Idealized
- No friction
- No energy dissipation
- Perpetual Motion

Damped

- Real Structures
- Energy is dissipated
- Viscous Damping (linear models)
  - Damping Force proportional to velocity

SDOF System Model

Single Degree of Freedom (SDOF) Mechanical System Model



Newton

$$\sum f = ma$$

Equation of Motion

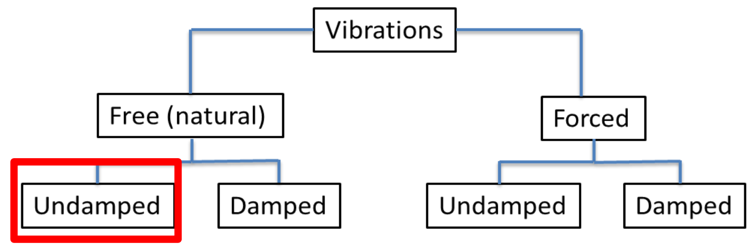
$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = f(t)$$

Acceleration   Velocity   Displacement

SDOF System Model

- Undamped Free Vibrations

$$c = 0 \qquad f(t) = 0$$

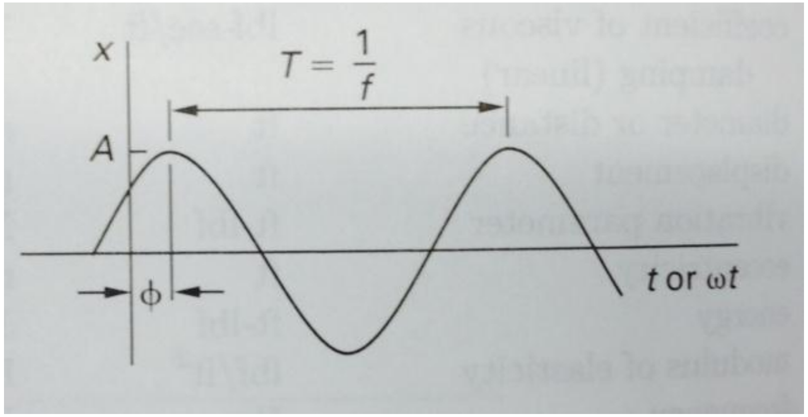


$$m\frac{d^2x}{dt^2} = -\mathrm{k}x$$

- Solution

$$x(t) = x_0 \cos \omega t + \left(\frac{v_0}{\omega}\right) \sin \omega t$$

$$x(t) = A\cos(\omega t - \phi)$$



$$A = \sqrt{{x_0}^2 + \left(\frac{v_0}{\omega x_0}\right)^2} \qquad \phi = \tan^{-1}\left(\frac{v_0}{\omega x_0}\right)$$

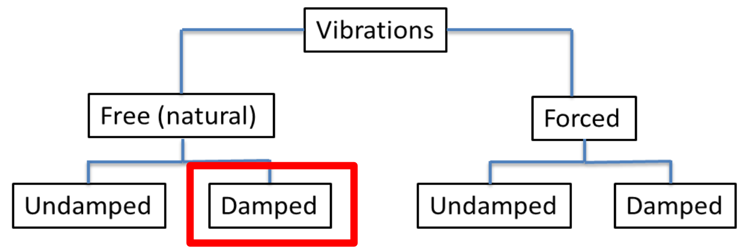
$$\omega = \sqrt{\frac{k}{m}}$$ Angular Natural Frequency $\left(\frac{rad}{sec}\right)$

$$f = \frac{\omega}{2\pi}$$ Linear Natural Frequency $(hertz)$

The initial conditions (displacement, velocity) do not affect the natural frequency. (Just the amplitude)

SDOF System Model

$f(t) = 0$



- Damped Free Vibrations

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

Linear, 2nd order differential equation

- homogeneous

- General Solution

$$x(t) = A\,e^{r_1 t} + B\,e^{r_2 t}$$

- A & B from initial conditions

$$r1, r2 = \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$$\zeta = \frac{c}{2m\omega} = \frac{c}{c_{critical}}$$ Damping Ratio

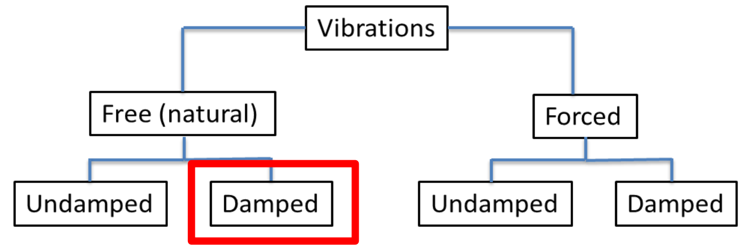
$$r1, r2 = -\zeta\omega \pm \omega\sqrt{\zeta^2 - 1}$$

- 3 Distinct Solution Sets Correspond to the Damping Ratio
  1. Underdamped $\zeta < 1$
  2. Overdamped $\zeta > 1$
  3. Critically Damped $\zeta = 1$

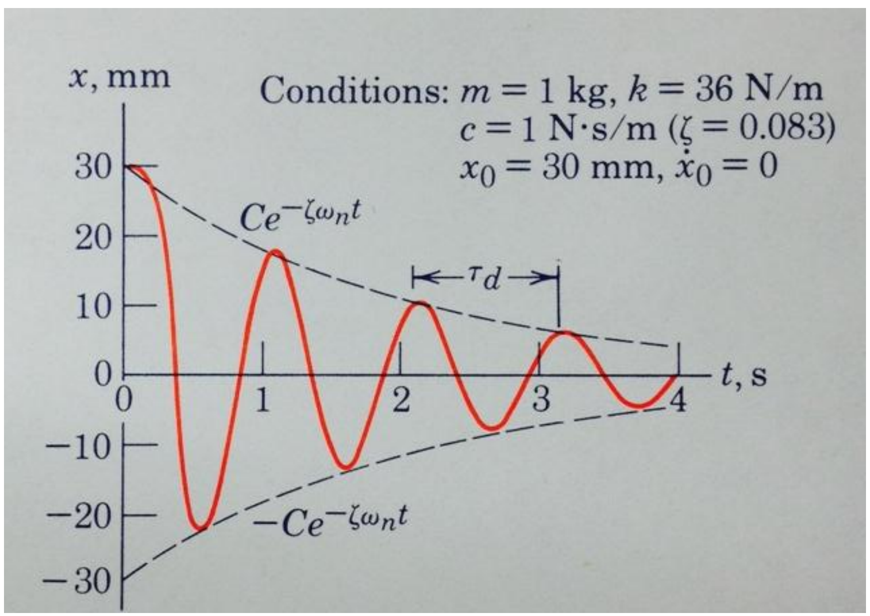
SDOF System Model

- Damped Free Vibrations
  - Underdamped $\zeta < 1$

$f(t) = 0$



- $x(t) = e^{-\zeta\omega t}\left[\frac{v_0+\zeta\omega x_0}{\omega\sqrt{1-\zeta^2}}\sin\left(\omega t\sqrt{1-\zeta^2}\right) + x_0\cos\left(\omega t\sqrt{1-\zeta^2}\right)\right]$



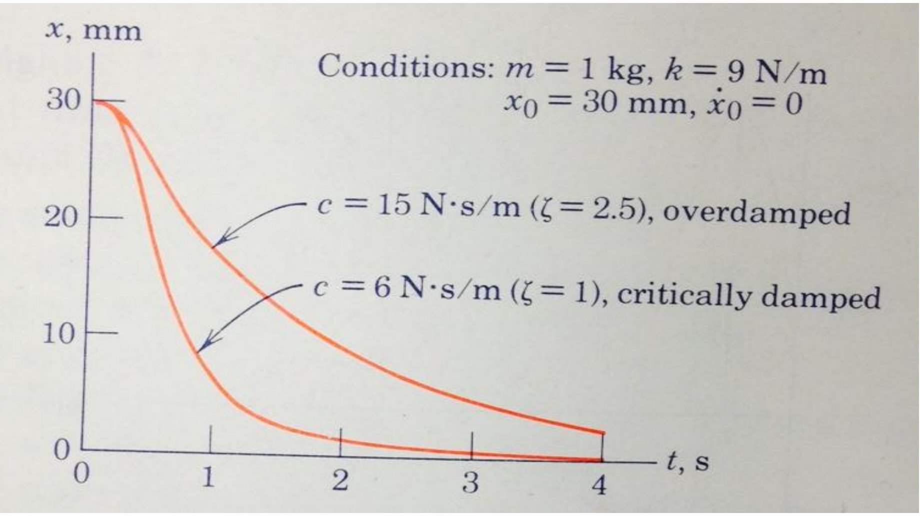
Damped Natural Frequency

$$\omega_d = \omega\sqrt{1-\zeta^2}$$

SDOF System Model

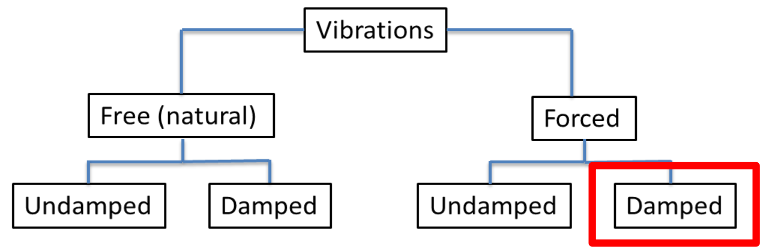
- Damped Free Vibrations
  - Critically Damped $\zeta = 1$ $x(t) = (v_0 + \omega x_0)te^{-\omega t} + x_0 e^{-\omega t}$
  - Overdamped $\zeta > 1$ $x(t) = Ae^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega t} + Be^{\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega t}$

$$A = \frac{v_0 + \left(\zeta + \sqrt{\zeta^2 - 1}\right)\omega x_0}{2\omega\sqrt{\zeta^2 - 1}} \qquad B = \frac{-v_0 - \left(\zeta - \sqrt{\zeta^2-1}\right)\omega x_0}{2\omega\sqrt{\zeta^2-1}}$$



SDOF System Model

- Damped Forced Vibrations



$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_0 \sin(\omega_f t)$$

- Solution consists of a complementary (transient) and a particular (steady state) solution
  - Complementary F_0=0; homogeneous DE

$$x(t) = x_c(t) + x_p(t)$$

$$x_p(t) = D\sin(\omega t - \phi)$$

$$D = \frac{F_0}{\sqrt{\left(k - m\omega_f{}^2\right)^2 + \left(\omega_f\right)^2}}$$

$$\phi = \tan^{-1}\left(\frac{c\omega_f}{k - m\omega_f{}^2}\right)$$

Damped Magnification Factor

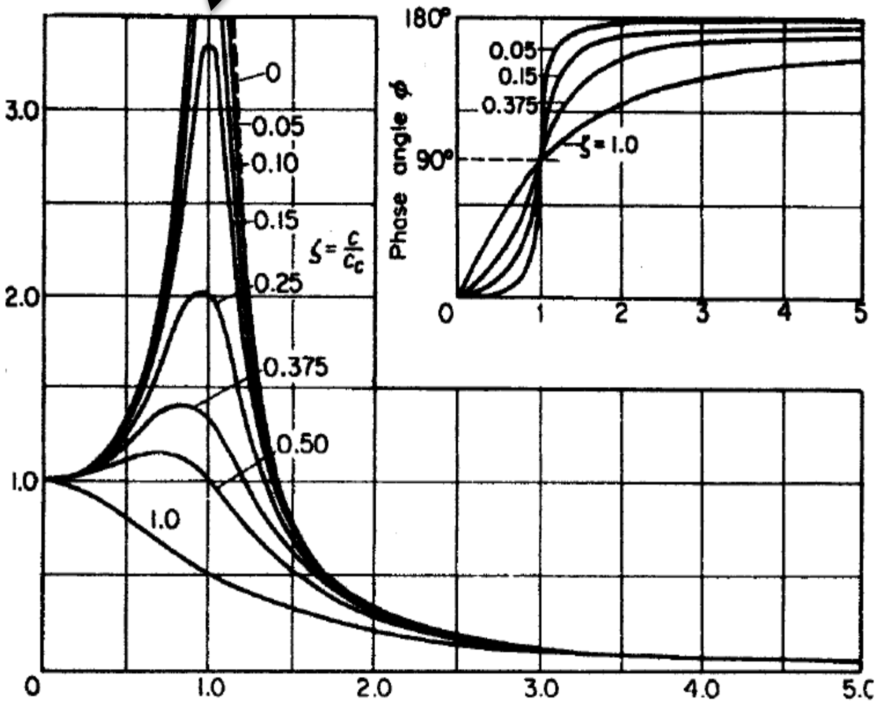
$$\beta_d = \left|\frac{D}{\frac{F_0}{k}}\right| = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega_f}{\omega}\right)^2\right)^2 + \left(2\zeta\frac{\omega_f}{\omega}\right)^2}}$$

SDOF System Model

- Forced Response of a SDOF System
  - How much energy is leaving before next force input?

RESONANCE

$\boldsymbol{\beta_d}$



Frequency Ratio $\frac{\omega_f}{\omega}$

Energy is readily absorbed by a system near its natural frequency

<u>Transmissibility</u>

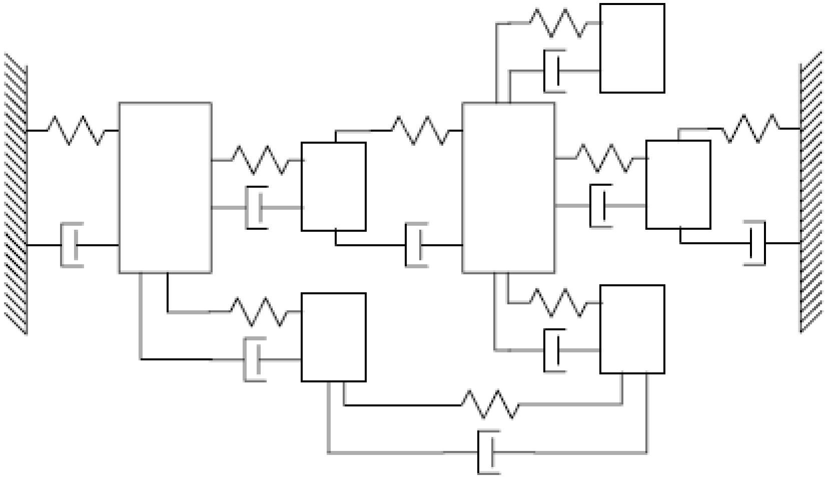
$$TR = \left|\frac{F_{transmitted}}{F_{applied}}\right| = \beta_d \sqrt{1 + \left(2\zeta \frac{\omega_f}{\omega}\right)^2}$$

<u>Amplification Ratio</u>

$$AR = \frac{displacement_{transmitted}}{displacement_{applied}} = \mathrm{TR}$$

MDOF System

Multiple Degree of Freedom (MDOF) Mechanical System Model



- Equation of Motion for 2 DOF system

$$\begin{bmatrix} m_1 & \\ & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix}$$

- Model Complex Systems
- Approximate Continuous Real Systems

- Matrix Formulation

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}$$

MDOF System

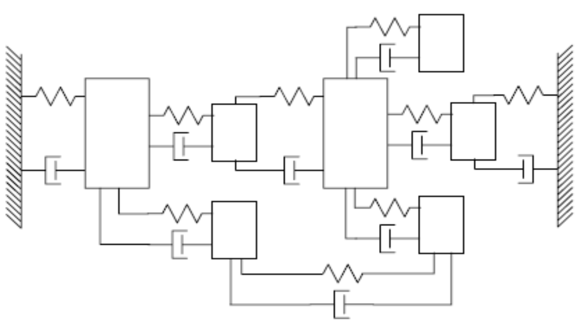
$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}$$

- An eigensolution yields eigenvalues (frequency) and eigenvectors (mode shapes) for each mode of the system.

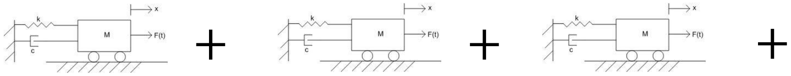
- Modal Transformation Equation is used to uncouple the set of highly coupled equations

$$\{x\} = [U]\{p\}$$

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \quad \Rightarrow \quad [\overline{M}]\{\ddot{p}\} + [\overline{C}]\{\dot{p}\} + [\overline{K}]\{p\} = [U]^T\{F\}$$



Independent SDOF Systems



1. Natural Frequency
2. Damping
3. Mode Shape

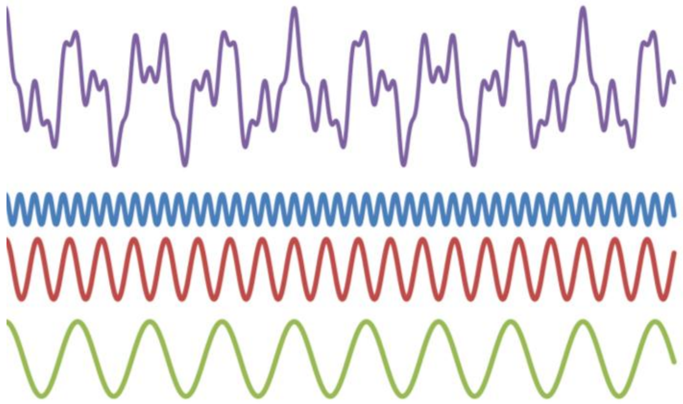
Vibration and Fourier Analogy

What is modal analysis?

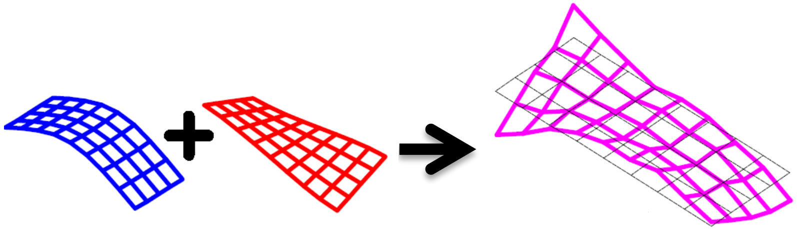
- The process of characterizing the dynamic response of a system in terms of its modes of vibration.

- Any periodic function can be represented as a series of sinusoidal functions.
- Each individual sinusoid is define by its amplitude, frequency and phase.

$$\sum_{n=1}^{N} A_n \sin(2\pi f_n t + \phi_n)$$



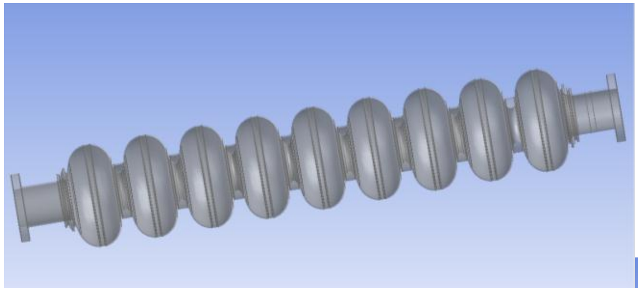
- Vibration of a real structure can be represented as a series of modal contributions.
- Each mode is defined by its natural frequency, damping, and mode shape.



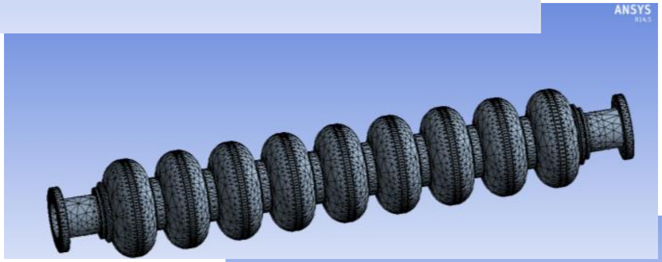
Analytical Modal Analysis

- Modal Analysis is the process of characterizing the dynamic response of a system in terms of its modes of vibration.

- Analytical Modal Analysis depends on the generation of the equations of motion of a system through a finite element model.

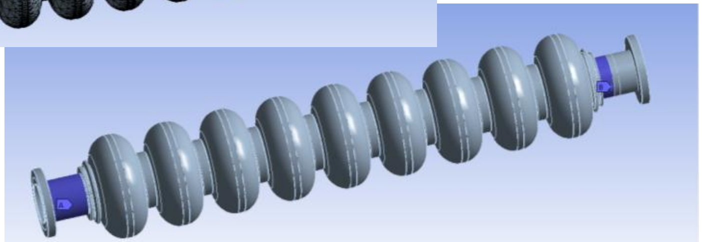


- 3D model typically generated with CAD tool



- Import & mesh with FEA tool

- Requires good material property info



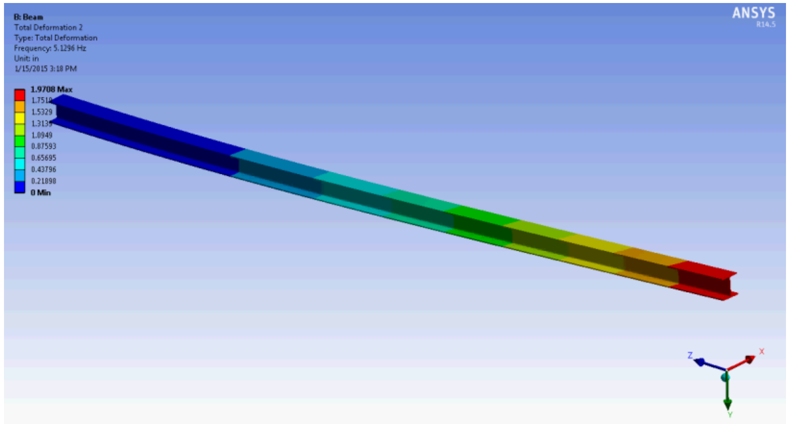
- Application of accurate boundary conditions is vitally important for reasonable results

1000’s of simultaneous equations are common for FEA modal models

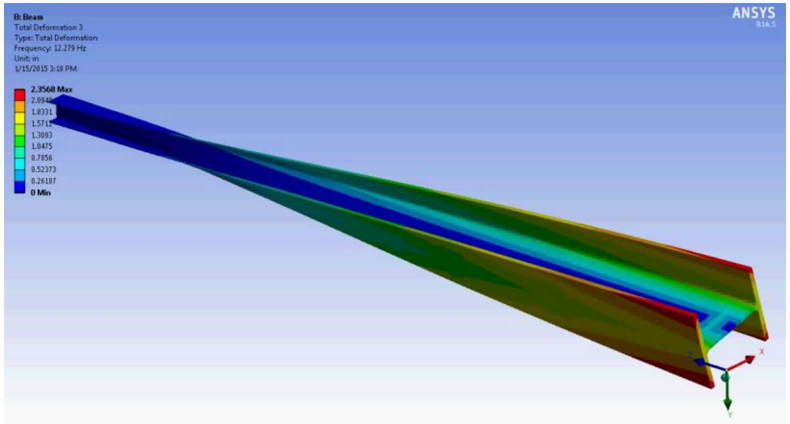
Analytical Modal Analysis

- Output is an ordered list of frequencies and the corresponding mode shapes

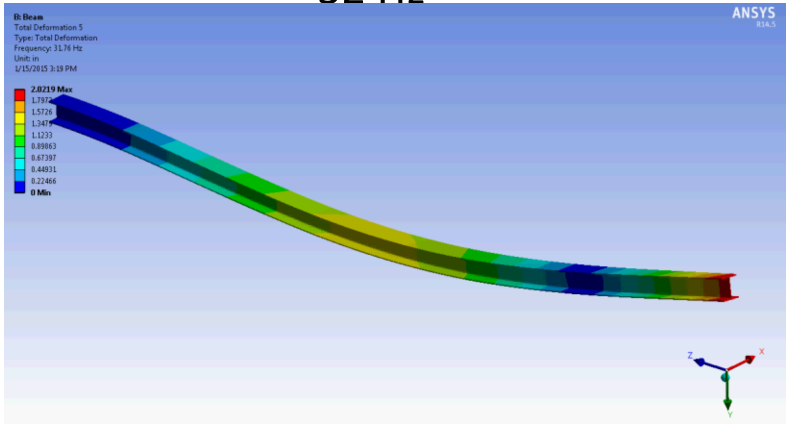
5 Hz



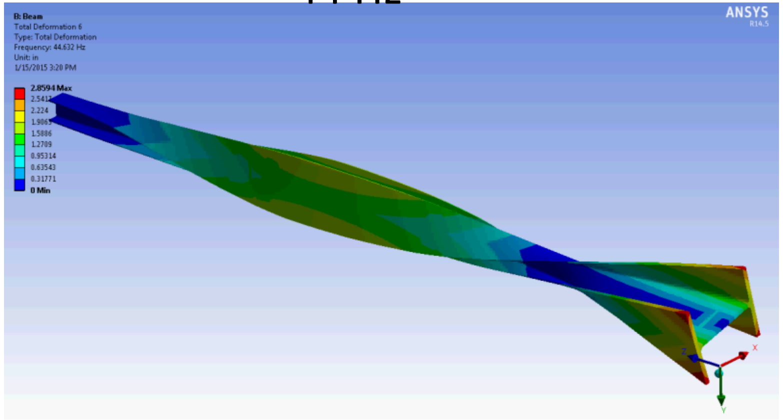
12 Hz



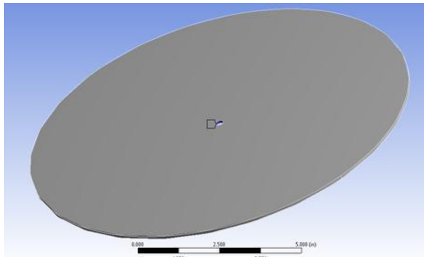
32 Hz

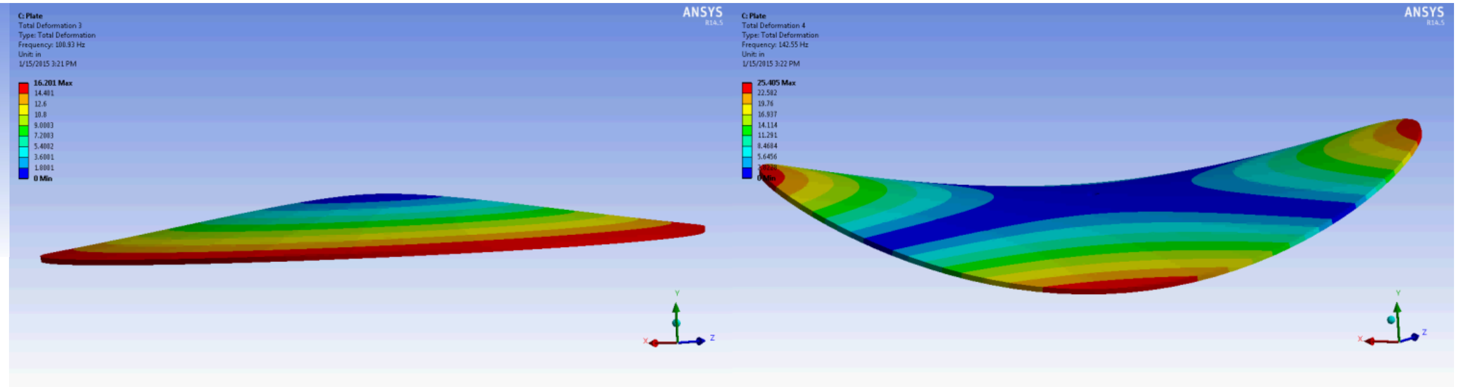


44 Hz

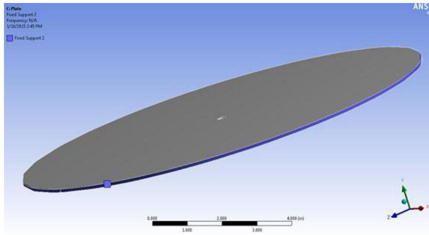


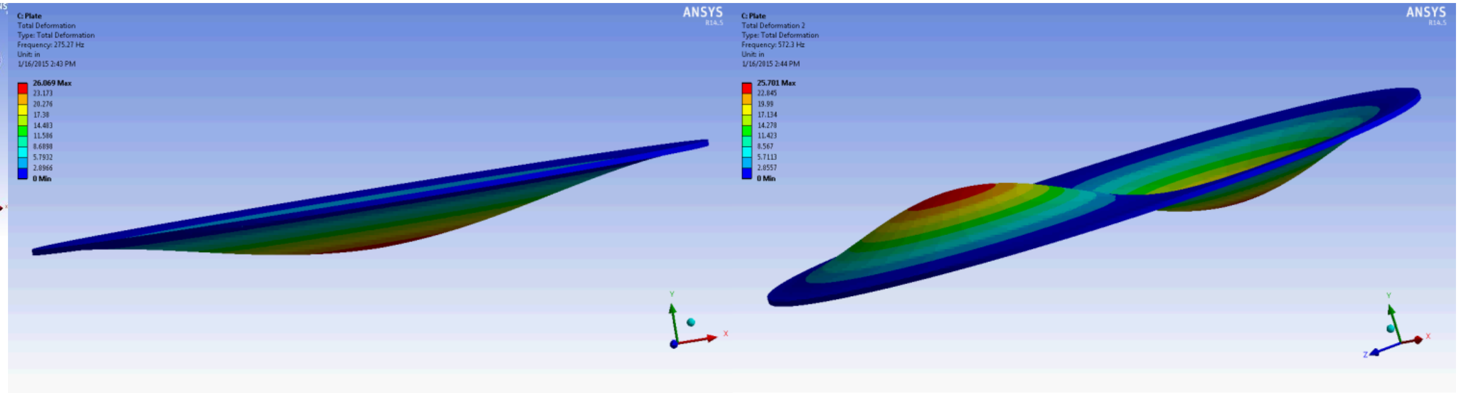
Analytical Modal Analysis





100 Hz & 142 Hz





275 Hz & 572 Hz

- Note,
  - The individual mode animations do NOT reflect an expected deflection shape
  - The modes are a function of the inherent mass and stiffness of the structure (no loads are applied)

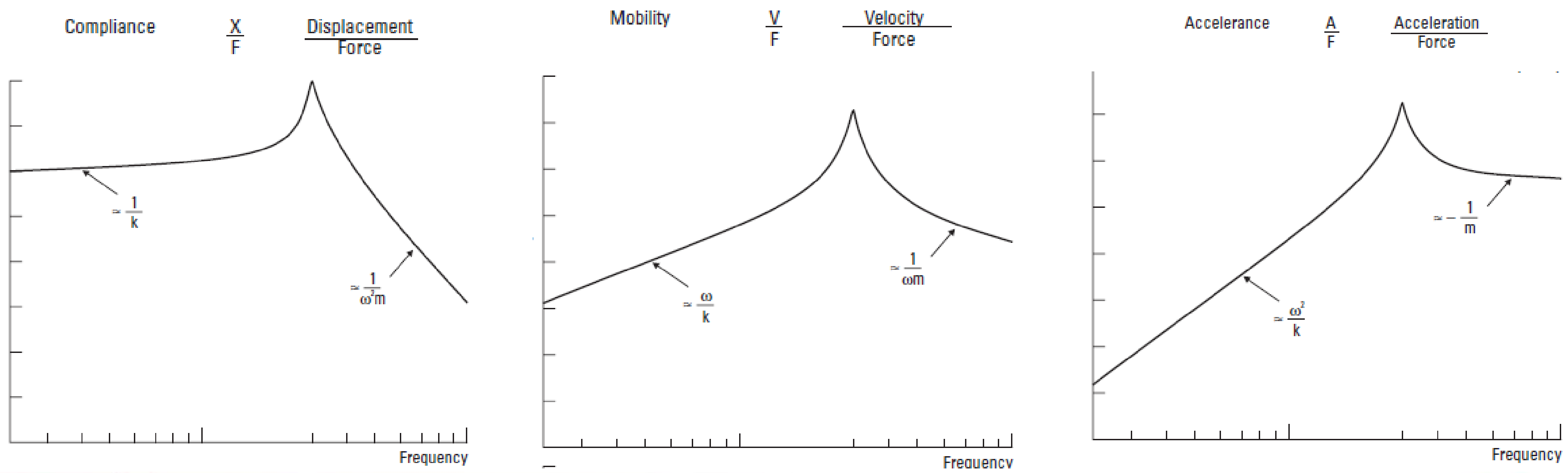
Experimental Modal Analysis

- Modal Analysis is the process of characterizing the dynamic response of a system in terms of its modes of vibration.

- Experimental Modal Analysis depends on parameter estimation techniques to extract modal information from experimental data.

Frequency Response Function (FRF)

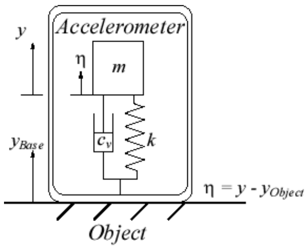
- Ratio of the output response of a structure to the applied force
- The applied force and structure response are measured simultaneously
- Time domain data is transformed to frequency domain (FFT)

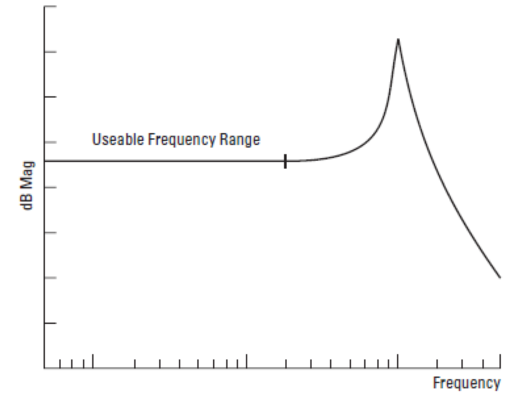


Experimental Modal Analysis

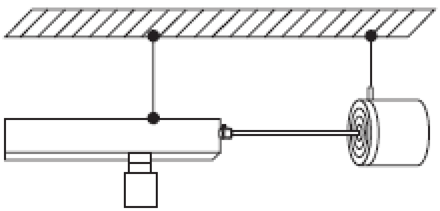
- Components used to take measurements
  - Want to accurately measure force and response simultaneously

Accelerometer - a(t)

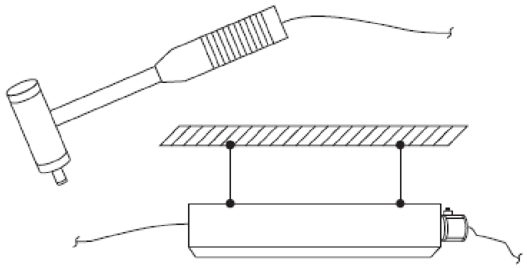




Shaker - F(t)

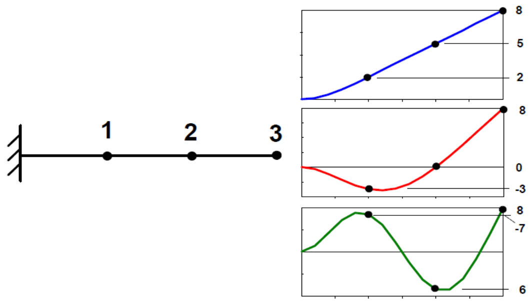


Hammer - F(t)



Experimental Modal Analysis

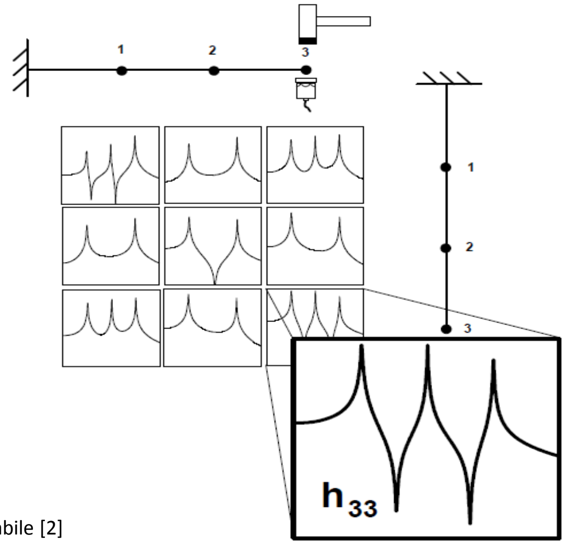
- FRFs are used to generate modal data
  - Consider Simple 3DOF beam model



- 3 possible locations for force application
- 3 locations for response measurement
- 9 possible FRFs; organized in matrix form
- Notation convention
  - $h_{row,column}$ $h_{output,input}$

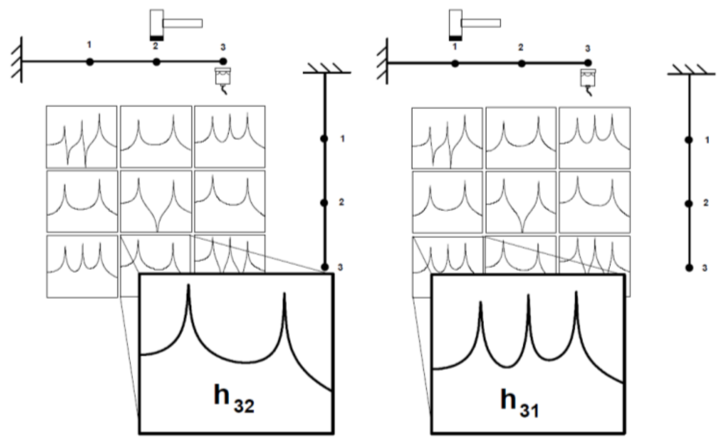
Drive Point FRF

-Excitation and measurement at same location



Cross FRF

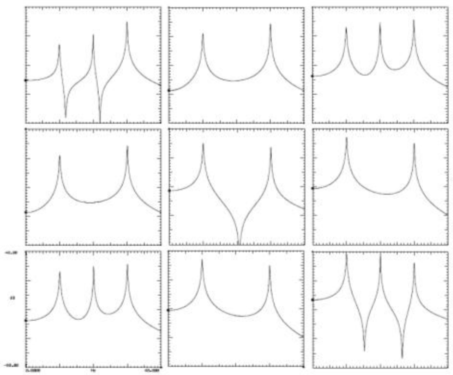
-Excitation and measurement at different location



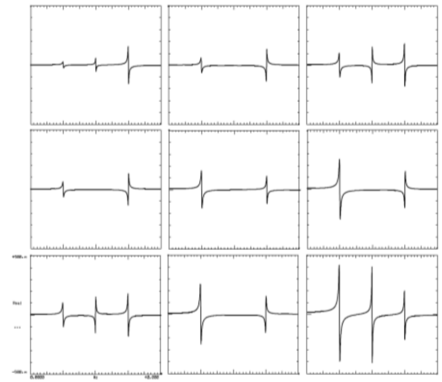
Images from Peter Avitabile [2]

Experimental Modal Analysis

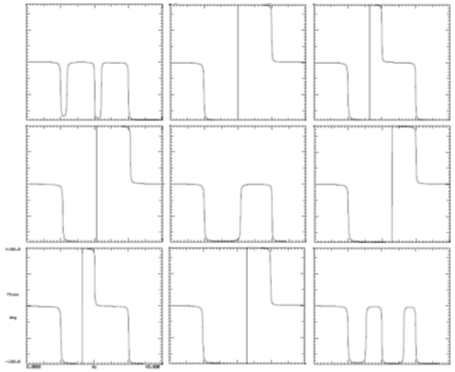
- Due to the Fast Fourier Transformation the FRFs are complex valued quantities
  - Magnitude & Phase or Real & Imaginary



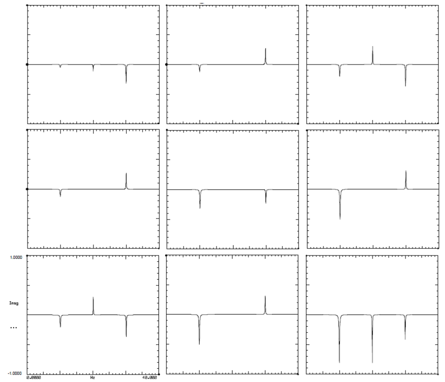
Magnitude



Real



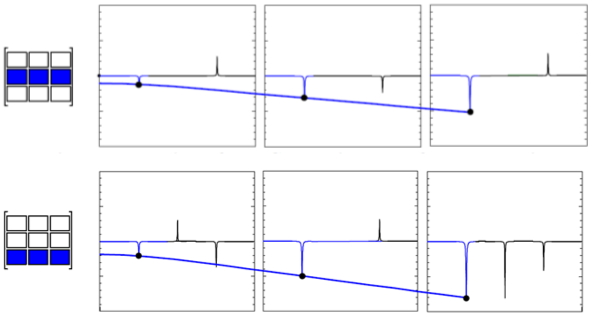
Phase



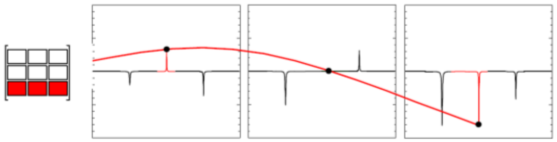
Imaginary

- FRFs matrix used to determine
  1. Natural Frequency
  2. Damping
  3. Mode Shape

1st Mode



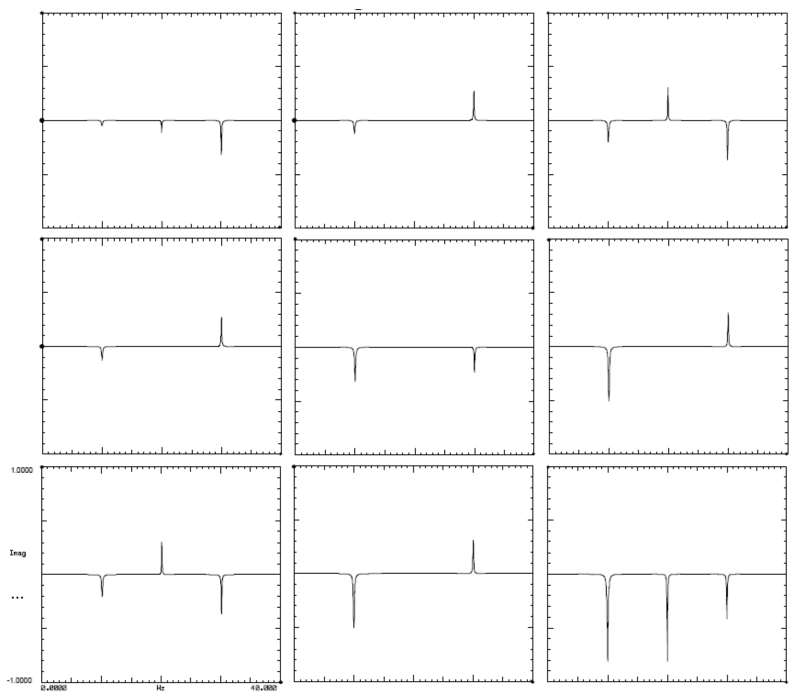
2nd Mode



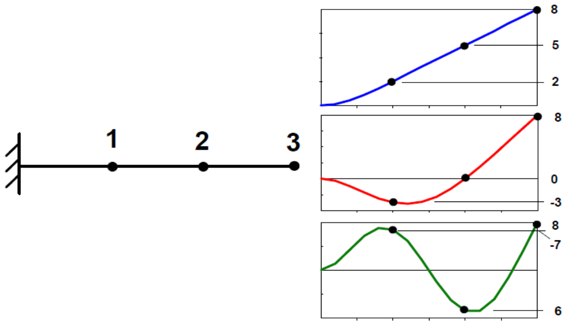
Images from Peter Avitabile [2]

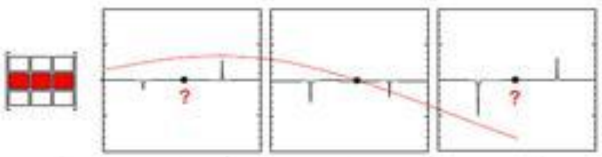
Experimental Modal Analysis

- FRFs
  - Reference points cannot be located at the node of a mode.



Imaginary



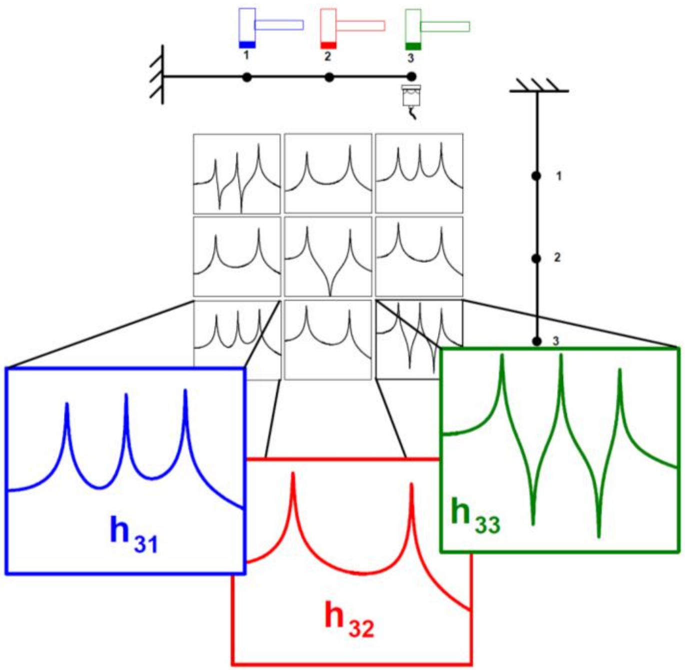


Images from Peter Avitabile [2]

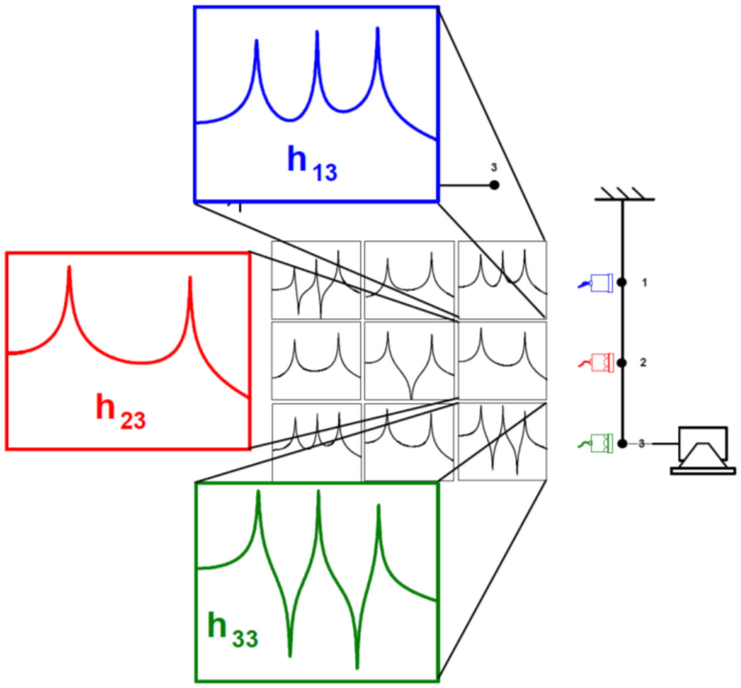
Experimental Modal Analysis

➢ FRFs

- Roving Impact
  - Force input is moved
  - Transducer stationary



- Roving Response
  - Force input is stationary
  - Transducer is moved



Images from Peter Avitabile [2]

Experimental Modal Analysis

- Practical Considerations
  - Theoretically no difference between shaker test and impact (hammer) test
  - Ideal
    - No interaction between applied force and the structure
    - Massless transducer
  - Reality
    - Collecting data on the structure plus all the measurement apparatus
      - Structure supports
      - Mass of transducers
      - Stiffening effects of shaker attachment
  - Impact tests
    - Typically faster, lower cost, and easier to implement
    - Hammer tip hardness must be matched to the frequency range of modes desired
    - S/N ratio may be poor
    - Windowing required (less accuracy in predicting damping)
  - Shaker tests
    - Better precision, enables frequency sweep (targeted investigations)
    - Setup is timely and can be difficult

Experimental Modal Analysis

- Actual FRF measurements

DAQ

Remove high frequency signals

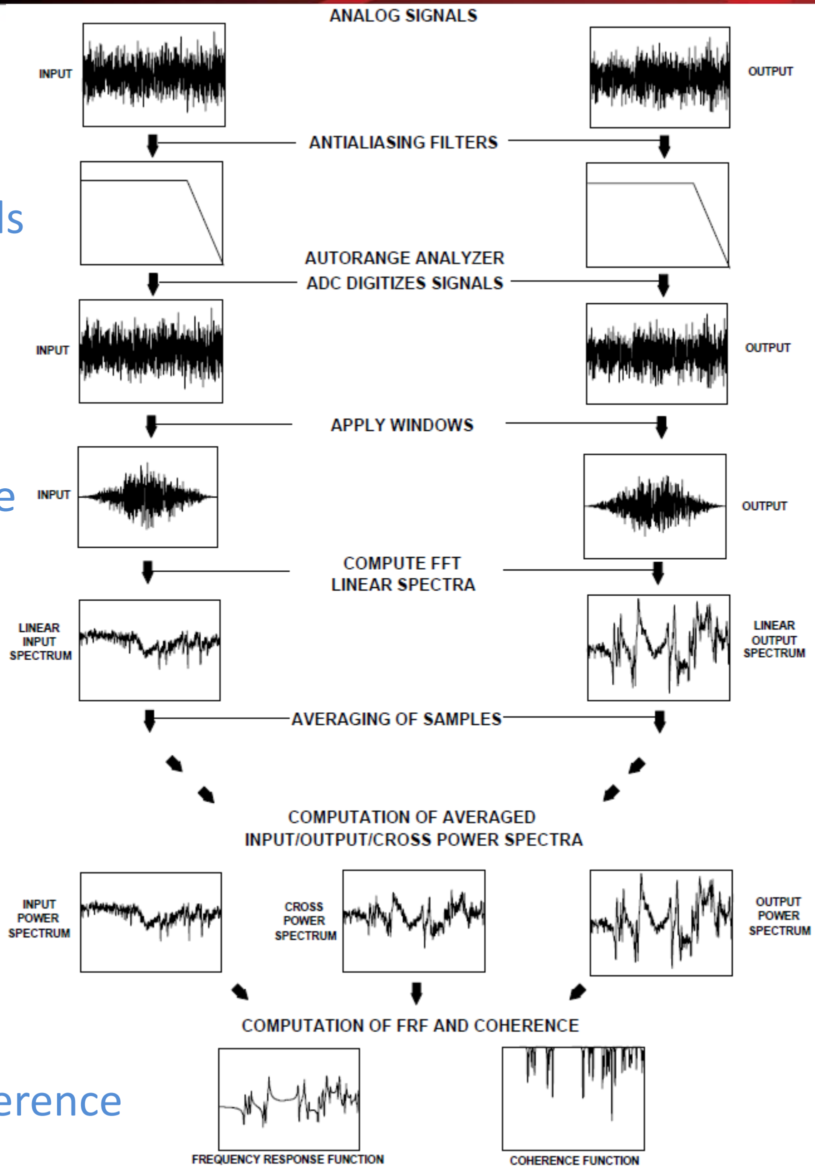
More bits, better resolution

Minimize leakage

FFT

Averaging & Math

FRF & Coherence



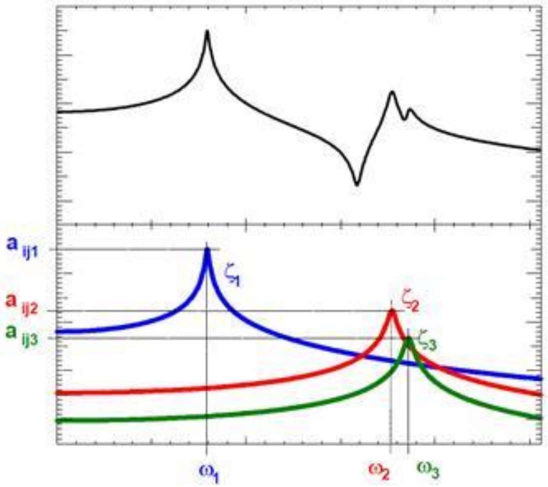
FFT process requires the sampled data consist of a complete representation of the data for all time or contain a periodic repetition of the measured data. When this is not satisfied (leakage) a serious distortion of the data in the frequency domain is the result.

Coherence function is used as a data quality assessment tool. It identifies how much of the output signal is related to the input signal.

Images from Peter Avitabile [2]

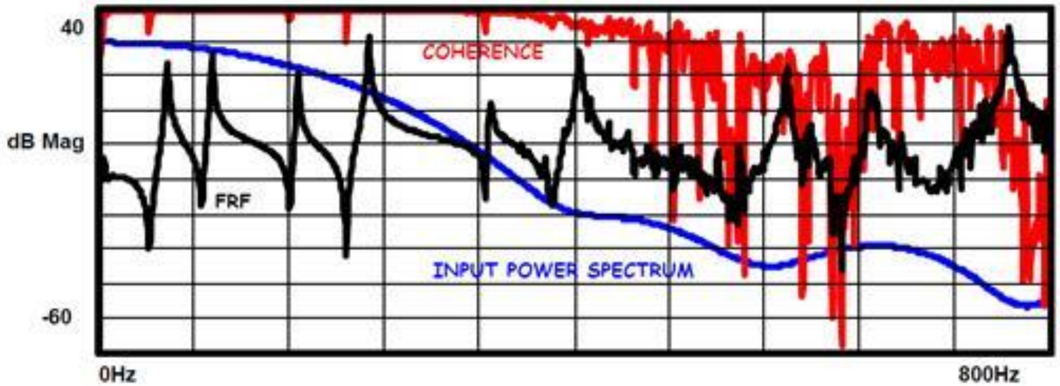
Experimental Modal Analysis

- Actual FRF measurements



<u>Modal Parameter estimation (Curve Fitting)</u>

- FRF Broken down into multiple SDOF systems
- Determine frequency, damping, and mode shape
- Multiple techniques & automated algorithms are utilized to extract data.



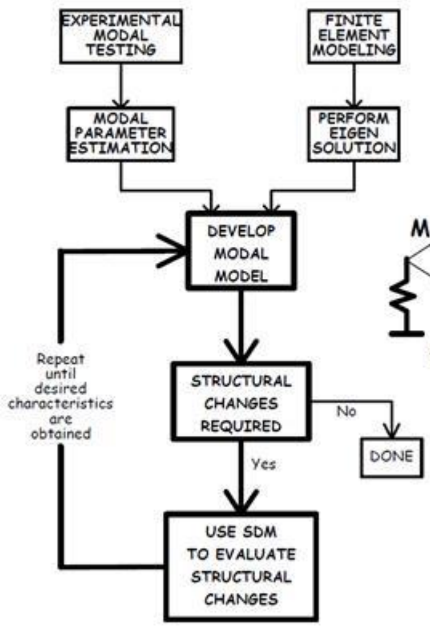
- Insufficient input power at higher frequencies
- Coherence good at low frequencies, poor at high frequencies.

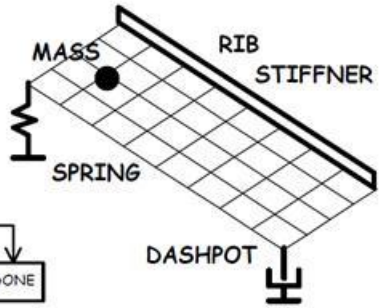
Images from Peter Avitabile [2]

Experimental Modal Analysis

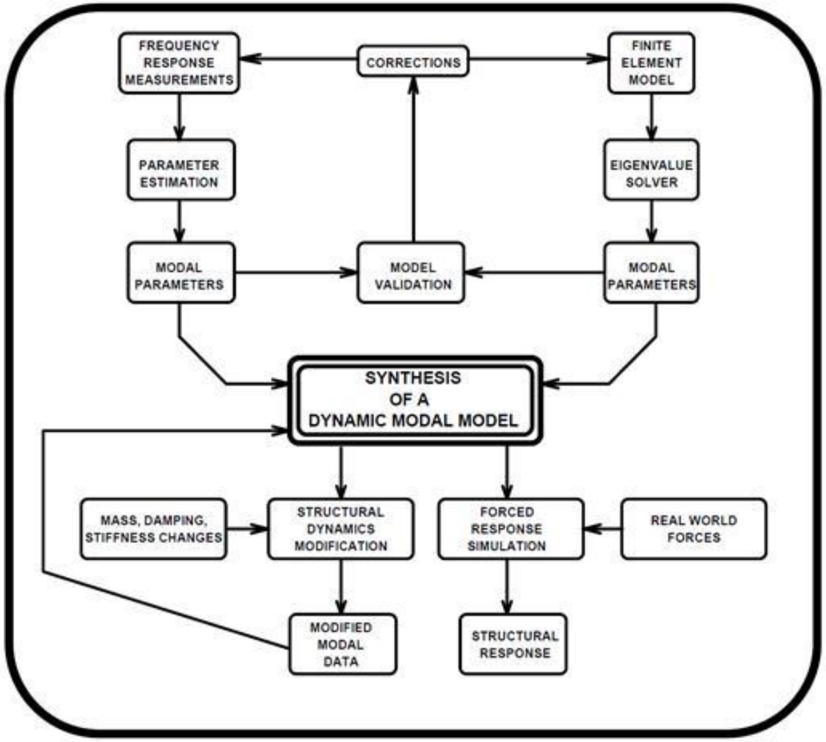
OK, I've got my frequencies and mode shapes. Now what?

- Visualization of mode shapes is invaluable in the design (& redesign) process.
- Evaluate the most cost effective design modifications.
- Identifies areas of weakness in a system or structure.
- Predict system response to proposed loads or operating conditions.
- Update / correlate FEA models.





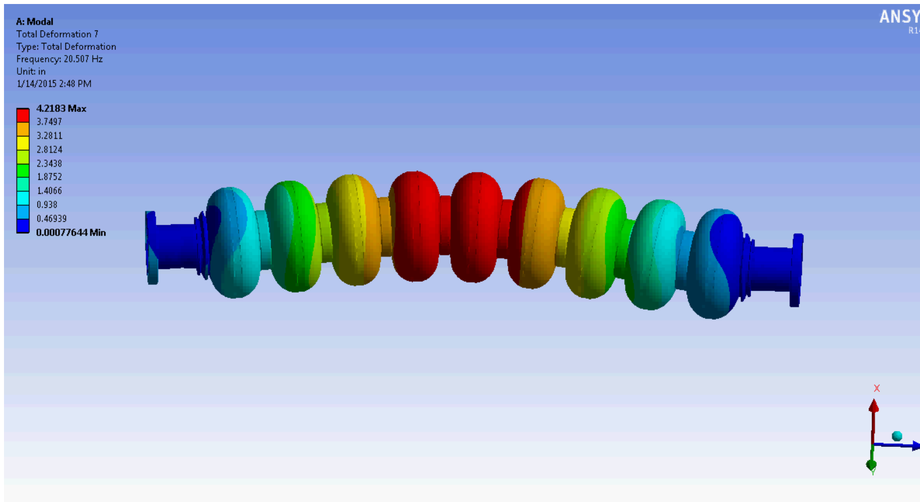
STRUCTURAL DYNAMIC MODIFICATIONS

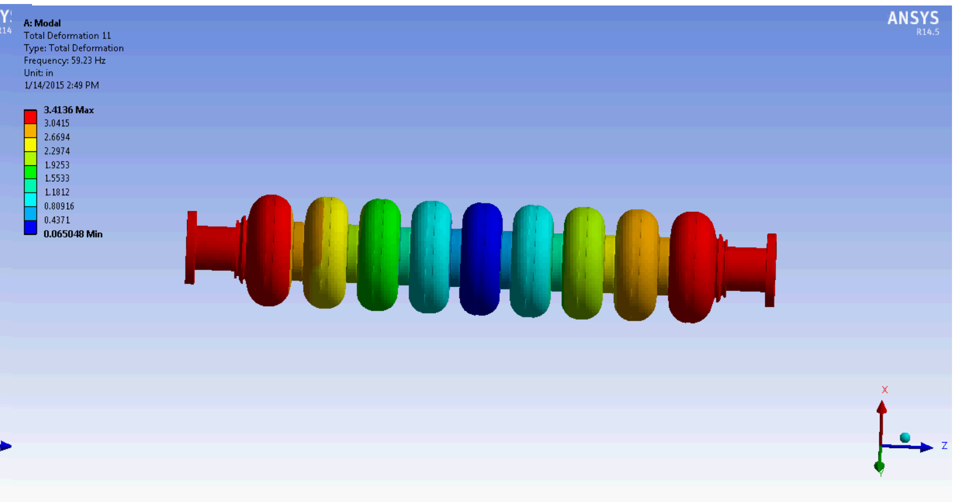


Images from Peter Avitabile [2]

Vibration and Modal Analysis Basics

OK, fix your beams, buildings, & bridges. Why do I care?





- SRF cavities have mechanical modes too !
  - Example: JLAB 12GeV cavities tuning sensitivity = 300 Hz / micron
  - Low frequency oscillations cause cavity target frequency to vary (1497.000… MHz)
  - Accelerating gradient per supplied RF power degraded

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