Cavity Testing Mathematics

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SRF Testing Course

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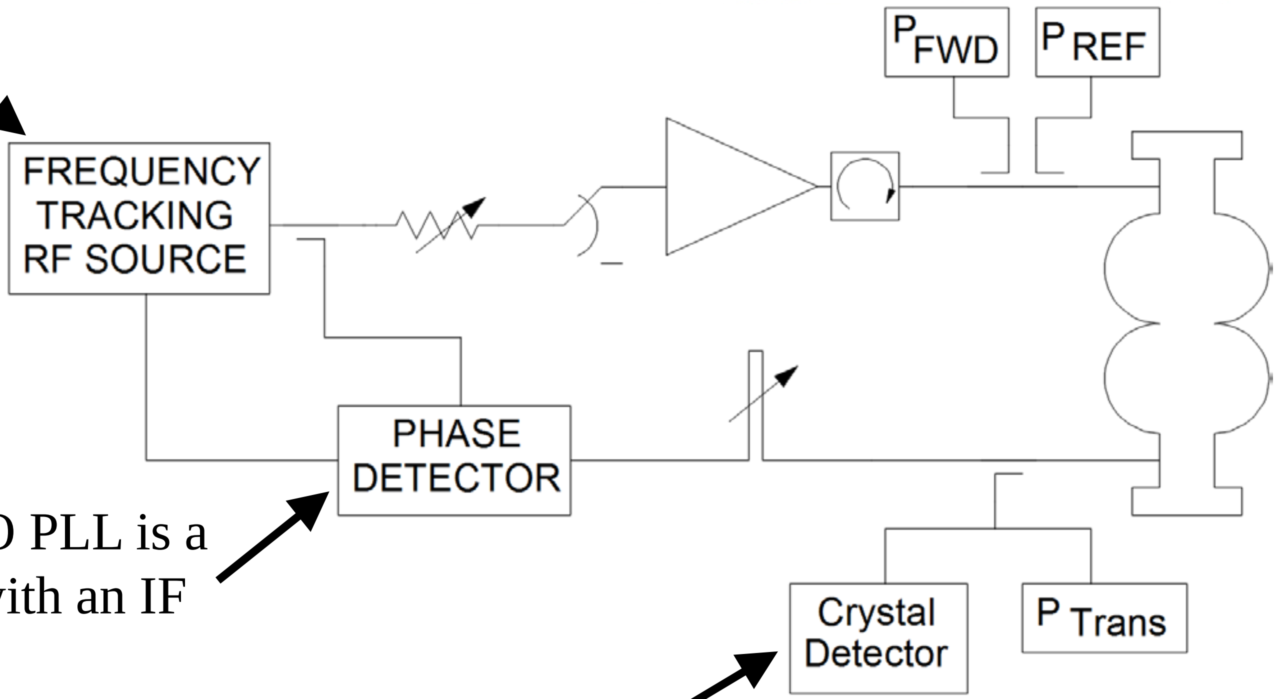
General Block Diagram for Vertical or Horizontal Test Stand

Frequency tracking source can be either a VCO-PLL based system or a self excited loop.

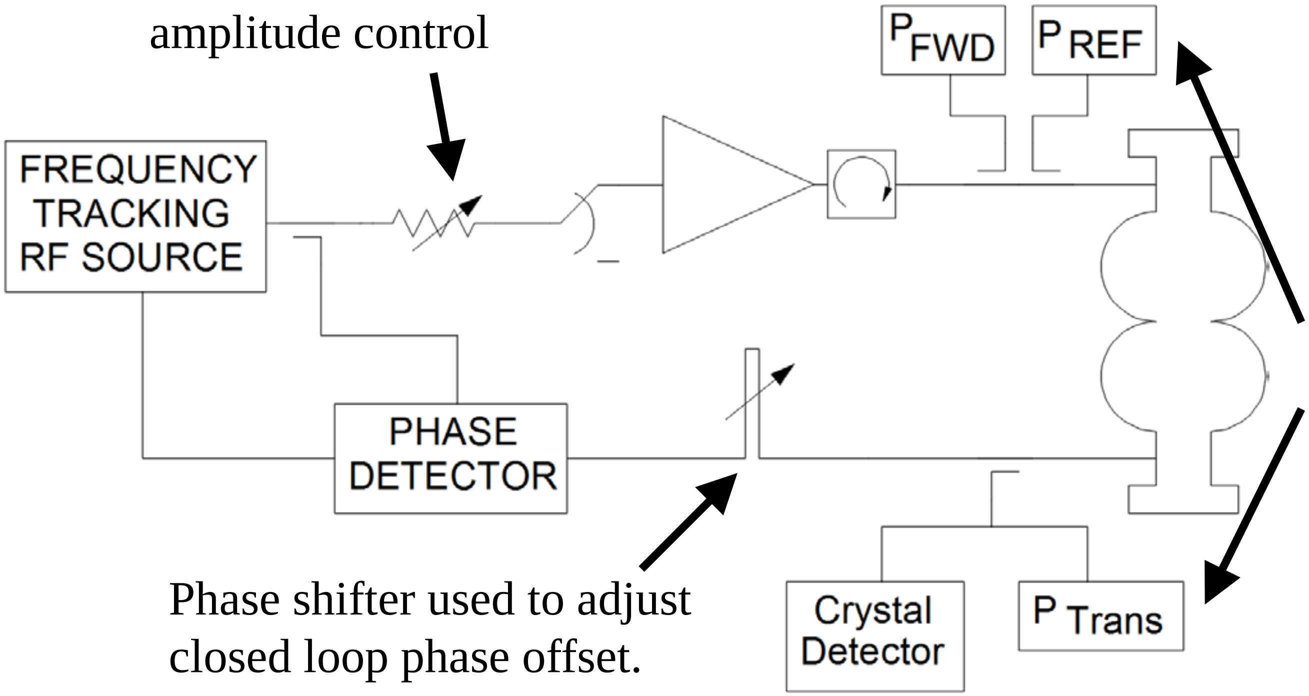


Phase detector for a VCO PLL is a double balanced mixer with an IF that is rated to DC.

Crystal detector used as a time domain RF power detector. Care must be taken that it is operated in the square law range with errors less than +/- 3%.

General Block Diagram for Vertical or Horizontal Test Stand

Variable Attenuator used for amplitude control



Phase shifter used to adjust closed loop phase offset.

RF power meters used for forward, reflected and transmitted power.

Meters can be CW or "pulsed" (time domain data record).

Wide dynamic range heads (down to -60 dB) are required for vertical testing because of dynamic range requirements for calibration.

What are we trying to calculate?

- Stored Energy (from this you can calculate gradient)
- Q_0 losses
- Coupling factor for the different ports (necessary for calculating U and Q_0)

What can we measure?

- Forward, or incident, RF power
- Reflected RF power
- RF power transmitted out of any of the other ports.
- Decay waveforms during a turn off transient.

VERTICAL AND HORIZONTAL TESTING

- During production cavities are generally tested using antenna inserted into the fundamental power couplers or one of the beam pipes. The goal is to have the cavity at or near critical coupling for these tests. In this way a minimum amount of power can be used to reach design gradient. Ideally this means just enough power to overcome the heat losses in the cavity and the power coupled out of the other ports. This has the advantage that the power lost to wall heating can be calculated based on RF measurements.
- In most labs these tests are done in vertical test dewars, hence they are commonly called vertical tests.
- Cavities in a cryomodule are typically tested using the production couplers that are strongly over coupled. This presents a problem as the errors in lost RF power get excessive when 95% to 99.9% of the incident power is reflected back out of the fundamental power coupler.
- During cryomodule tests the RF heat load is measured calorimetrically.

CW or Pulsed Measurements 0.1 < Beta < 10

- Do we have enough information to make all of the measurements in CW mode of operation . . . . **NO**

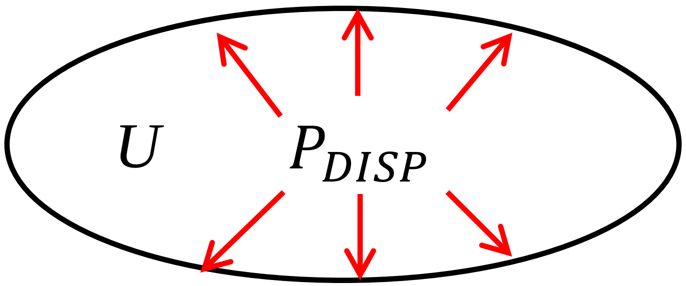
- When making CW measurements you only have forward, reflected and transmitted power. There is insufficient information to determine the system loaded-Q.

- You must do a decay measurement in order to determine the loaded-Q and to determine if you are over coupled or under coupled.

- Note: One can determine if you are over coupled or under coupled as well as the loaded-Q, using a network analyzer. However in vertical tests when you are near critical coupling the Q_0 value of the cavity is about the same as the coupling factor for the fundamental power coupler. What this means is that as Q_0 varies so does Q_L. Thus with an SRF cavity you can not do a low power Q_L determination and rely on it being a constant as a function of gradient.

Decay Measurement Derivation

- Consider a system that has contains a stored energy-U and wall losses P_{DISP}.
- Using the basic definition of Q_0



$$Q_0 = \frac{Stored\ Energy}{Energy\ Lost\ Per\ Cycle}$$

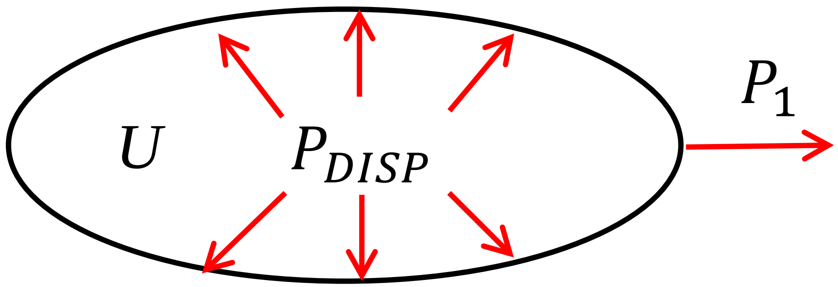
$$Q_0 = \frac{U}{P_{DISP} * T}$$

- Where U is the stored energy, P_{DISP} is the dissipated power in the walls and T is the period of the cycle.
- This can be rewritten as:

$$Q_0 = \frac{\omega U}{P_{DISP}}$$

Decay Measurement Derivation

- Now add a port through which power can leave the system.



- Now you have two paths that power may leave the system. These paths are wall losses and through an RF port on the cavity. Assuming that the RF power that leaves the system is proportional to the stored energy (e.g. the electromagnetic fields within the cavity)

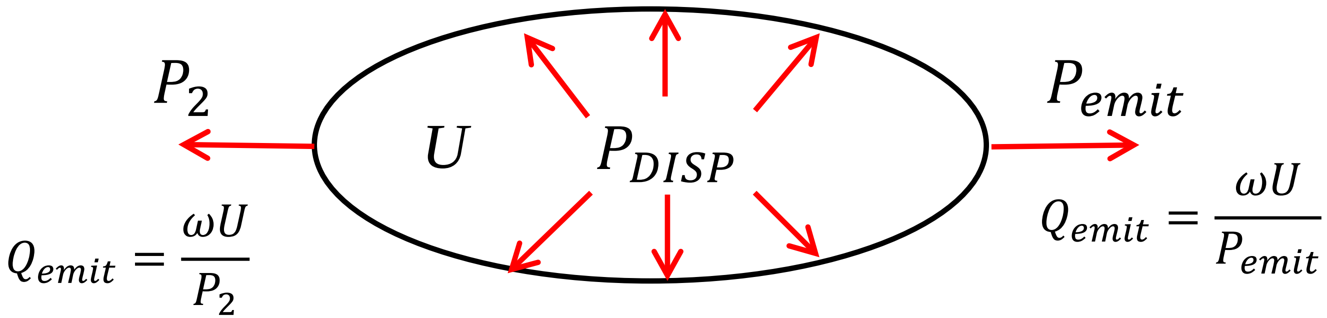
- We can DEFINE the Q for port 1 as:

$$Q_1 = \frac{\omega U}{P_1}$$

* *Note in Hassan's book P_1 is also called P**e** for emitted power.*

Decay Measurement Derivation

- Now add a second port through which power can leave the system.



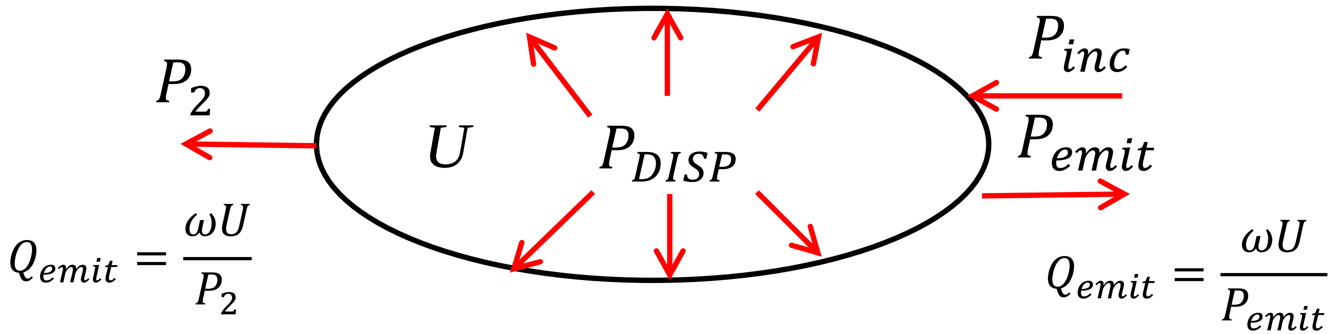
- Defining the Q-external of each port in the same way leads to and defining the total loaded-Q as seen by the stored energy in the cavity as the effective Q as determined by all of the "loss" mechanisms:

$$Q_L = \frac{\omega U}{P_{Disp} + P_{emit} + P_2}$$

$$\frac{1}{Q_L} = \frac{P_{Disp} + P_{emit} + P_2}{\omega U} = \frac{P_{Disp}}{\omega U} + \frac{P_{emit}}{\omega U} + \frac{P_2}{\omega U} = \frac{1}{Q_0} + \frac{1}{Q_{emit}} + \frac{1}{Q_2}$$

Decay Measurement Derivation

- Now lets add an incident power term to port 1. This power will be just enough to balance out all of the losses in the system.



- This gets tricky because of the concept of impedance miss-match and the fact that there will be a reflected power signal. Given that:

$$P_X = \frac{V_X^2}{Z_0} \quad or \quad V_X = \sqrt{P_X Z_0}$$

- **Assuming that the cavity is perfectly tuned** and if one looks at the RF voltage at the port

$$V_R = V_{emit} - V_{inc} \rightarrow P_R = \left(\sqrt{P_{emit}} - \sqrt{P_{Inc}}\right)^2$$

Decay Measurement Derivation

$$P_R = \left(\sqrt{P_{emit}} - \sqrt{P_{Inc}}\right)^2$$

- Taking the square root of both sides of the equation, which requires that one add the +/- operator.

$$\pm\sqrt{P_{REF}} = \sqrt{P_{emt}} - \sqrt{P_{FWD}}$$

or

$$\sqrt{P_{emt}} = \sqrt{P_{FWD}} \pm \sqrt{P_{REF}}$$

- Next we are going to define a variable that we will call the coupling coefficient represented by the variable β_X where:

$$\beta_X = \frac{Q_0}{Q_X} = \frac{P_X}{P_{DISP}}$$

- Where P_X is the power leaving the port because of the stored energy in the cavity.

Decay Measurement Derivation

- Starting with:

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{emit}} + \frac{1}{Q_2} \qquad \text{and} \qquad \sqrt{P_{emit}} = \sqrt{P_{FWD}} \pm \sqrt{P_{REF}}$$

- Using the previously defined coupling coefficient represented by the variable β_X where:

$$\beta_X = \frac{Q_0}{Q_X} = \frac{P_X}{P_{DISP}}$$

$$\beta_{FPC} = \frac{Q_0}{Q_{FPC}} = \frac{P_{emt}}{P_{disp}} = \frac{\left(\sqrt{P_{FWD}} \pm \sqrt{P_{REF}}\right)^2}{P_{disp}}$$

- For now we will define over coupling is when $\beta_{FPC} > 1$ and under coupling is when $\beta_{FPC} < 1$ and critically coupled when $\beta_{FPC} = 1$. Thus β_{FPC} can also be written as:

$$\beta_{FPC} = \frac{Q_0}{Q_{FPC}} = \frac{P_{emt}}{P_{disp}} = \frac{\left(\sqrt{P_{FWD}} + C_\beta\sqrt{P_{REF}}\right)^2}{P_{disp}}$$

- Using the same convention we can come up with β_{FP}.

$$\beta_{FP} = \frac{Q_0}{Q_{FP}} = \frac{P_{Trans}}{P_{disp}}$$

Decay Measurement Derivation

- Starting with.

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{FPC}} + \frac{1}{Q_{FP}}$$

- Multiplying both sides by Q_L/Q_0, and rewriting the equation:

$$Q_0 = \left(1 + \frac{Q_0}{Q_{FPC}} + \frac{Q_0}{Q_{FP}}\right) Q_L = (1 + \beta_{FPC} + \beta_{FP}) Q_L$$

$$Q_0 = \left(1 + \frac{\left(\sqrt{P_{FWD}} + C_\beta \sqrt{P_{REF}}\right)^2}{P_{FWD} - P_{REF} - P_{FP}} + \frac{P_{FP}}{P_{FWD} - P_{REF} - P_{FP}}\right) Q_L$$

$$Q_0 = \left(1 + \frac{\left(\sqrt{P_{FWD}} + C_\beta \sqrt{P_{REF}}\right)^2}{P_{FWD} - P_{REF} - P_{FP}} + \frac{P_{FP}}{P_{FWD} - P_{REF} - P_{FP}}\right) 2\pi f_0 \tau$$

Decay Measurement Derivation

- There are two ways to determine the loaded-Q of a cavity.
  - The first way is to measure the 1/e decay time constant, τ, for the reflected or transmitted power signal.
  - The second approach is to measure the bandwidth of the cavity transfer function (S21) and calculate the loaded-Q as the center frequency divided by the -3 dB full bandwidth.

$$Q_L = 2\pi f_0 \tau \qquad or \qquad Q_L = \frac{f_0}{BW}$$

- Starting with the last equation of the previous slide:

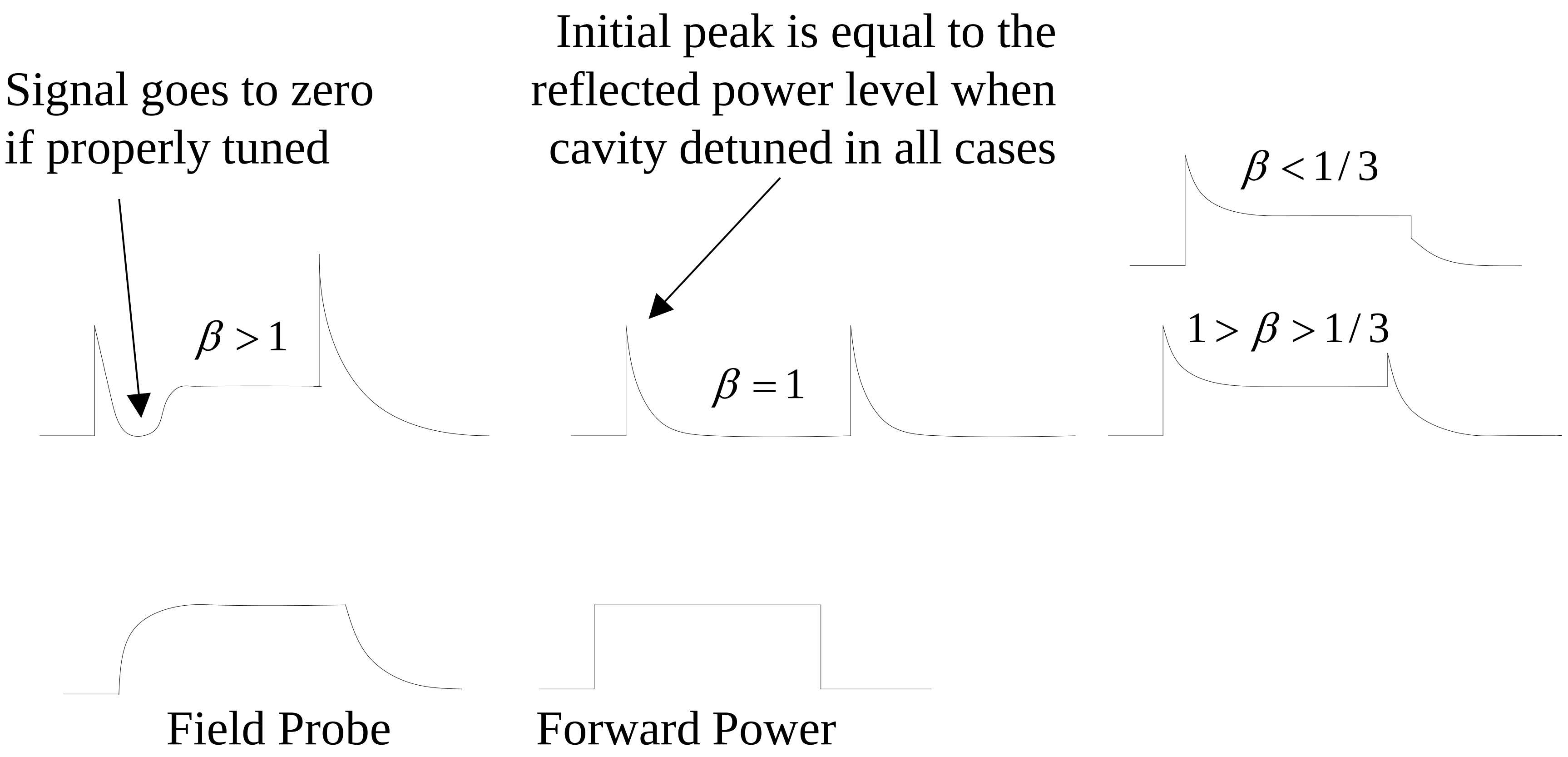
$$Q_0 = \left(1 + \frac{\left(\sqrt{P_{FWD}} + C_\beta \sqrt{P_{REF}}\right)^2}{P_{FWD} - P_{REF} - P_{FP}} + \frac{P_{FP}}{P_{FWD} - P_{REF} - P_{FP}}\right) 2\pi f_0 \tau$$

  - It can be shown that:

$$Q_0 = 4\pi f_0 \tau \frac{P_{FWD} + C_\beta \sqrt{P_{REF} P_{FWD}}}{P_{FWD} - P_{REF} - P_{FP}}$$

Determining If Cavity is Over or Under Coupled.

When operating cavities near critical coupling and preparing to make a decay measurement, one of the items that must be determined is the cavity is over coupled or under coupled. Typically a crystal detector is placed on the reflected power signal and the waveform is observed under pulsed conditions.



Gradient Calculation Decay Measurement

So far we have only been using stored energy and RF power flow and RF power measurements. What operations will want to know is the operating gradient for the cavity. The relationship between the stored energy and the cavity gradient is given by:

$$E_{acc}(V/m) = \sqrt{Q_0 P_{Disp} \frac{(r/Q)}{L}}$$

Where (r/Q) is the shunt impedance per unit length and L is the cavity length nominally from iris to iris. This can get confusing when folks start talking about quarter wave or half wave structures. Also there are many instances where you care about the peak surface field and not the accelerating gradient. The point being that the conversion between stored energy and gradient is somewhat arbitrary and should be discussed for each new cavity shape prior to performing the vertical test. Substituting the equation for Q_0 on the previous page one can reduce the accelerating gradient to the following.

$$E_{acc}(V/m) = \sqrt{4\pi f_o \tau \left(P_{FWD} + C_\beta \sqrt{P_{REF} P_{FWD}}\right) \frac{(r/Q)}{L}}$$

Q_{FP} calculation, Decay Measurement

Starting with: $$\beta_X = \frac{Q_0}{Q_X} = \frac{P_X}{P_{DISP}} \quad \rightarrow \quad \beta_X = \frac{P_{DisP} Q_0}{P_X}$$

$$Q_0 = 4\pi f_0 \tau \frac{P_{FWD} + C_\beta \sqrt{P_{REF} P_{FWD}}}{P_{FWD} - P_{REF} - P_{FP}}$$

$$Q_{FP} = 4\pi f_0 \tau \frac{P_{FWD} + C_\beta \sqrt{P_{REF} P_{FWD}}}{(P_{FWD} - P_{REF} - P_{FP})} \frac{(P_{FWD} - P_{REF} - P_{FP})}{P_{FP}}$$

$$Q_{FP} = 4\pi f_0 \tau \frac{P_{FWD} + C_\beta \sqrt{P_{REF} P_{FWD}}}{P_{FP}}$$

The Rest of the Decay Equations

$$U(Joules) = \frac{Q_0 P_{DISP}}{2\pi f_0} = 2\tau\left(P_{FWD} + C_\beta\sqrt{P_{FWD}P_{REF}}\right)$$

$$Q_1 = \frac{Q_0}{\beta_1} = 4\pi f_0\tau\frac{\sqrt{P_{FWD}}}{\sqrt{P_{FWD}} + C_\beta\sqrt{P_{REF}}}$$

$$Q_2 = \frac{Q_0}{\beta_2} = 4\pi f_0\tau\frac{P_{FWD} + C_\beta\sqrt{P_{REF}P_{FWD}}}{P_{Trans}}$$

- The reflection coefficient, Γ, is given by:

$$\Gamma = \sqrt{\frac{P_{REF}}{P_{FWD}}}$$

- And the overall coupler coupling coefficient β* is given by.

$$\beta^* = \frac{1 + C_\beta\sqrt{\frac{P_{REF}}{P_{FWD}}}}{1 - C_\beta\sqrt{\frac{P_{REF}}{P_{FWD}}}}$$

CW Measurements, 0.1 < β < 10

- The following equations are the basis of CW measurements. In this case one uses the field probe-Q (Q_{FP}), to determine the gradient.
- From the gradient and the dissipated power one can calculate Q_0.

$$E = \sqrt{Q_{FP} P_{FP} \frac{(r/Q)}{L}}$$

$$Q_0 = \frac{U\omega_0}{P_{DISP}} \qquad and \qquad U = \frac{E^2 L}{\omega_0 (r/Q)}$$

$$Q_0 = \frac{E^2 L}{P_{DISP}(r/Q)}$$

$$Q_0 = \frac{L}{P_{DISP}(r/Q)} \frac{Q_{FP} P_{FP} (r/Q)}{L}$$

$$Q_0 = \frac{Q_{FP} P_{FP}}{P_{DISP}}$$

CW Measurements, $0.1 < \beta < 10$

$$\beta_X = \frac{Q_0}{Q_X} = \frac{P_X}{P_{DISP}} \quad and \quad \frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{emit}} + \frac{1}{Q_2}$$

Multiply both sides by Q_0
$$\frac{1}{Q_L} = \frac{1}{Q_0}\left(1 + \frac{Q_0}{Q_{emit}} + \frac{Q_0}{Q_2}\right)$$

$$Q_0 = Q_L\left(1 + \frac{Q_0}{Q_{emit}} + \frac{Q_0}{Q_2}\right) = Q_L(1 + \beta_1 + \beta_2)$$

$$Q_L = \frac{Q_0}{(1 + \beta_1 + \beta_2)}$$

$$Q_L = \frac{Q_0}{\left(1 + \frac{\left(\sqrt{P_{FWD}} + C_\beta\sqrt{P_{REF}}\right)^2}{P_{DIsP}} + \frac{P_{FP}}{P_{DISP}}\right)}$$

$$Q_L = \frac{Q_{FP}P_{FP}}{2P_{FWD} + 2C_\beta\sqrt{P_{FWD}P_{REF}}}$$

CW Measurements, 0.1 < β < 10

$$\beta^* = \frac{1 + C_\beta \sqrt{\frac{P_{REF}}{P_{FWD}}}}{1 - C_\beta \sqrt{\frac{P_{REF}}{P_{FWD}}}}$$

$$\beta_{FPC} = \beta_1 = \frac{Q_0}{Q_{FPC}} = \frac{P_{emt}}{P_{disp}} = \frac{\left(\sqrt{P_{FWD}} + C_\beta \sqrt{P_{REF}}\right)^2}{P_{disp}}$$

It can be shown that: $\beta_1 = \beta^*(1 + \beta_2) = \dfrac{\left(\sqrt{P_f} + C_\beta \sqrt{P_r}\right)^2}{P_{Disp}}$

$$Q^* = \frac{Q_2 P_t \left(P_f\right)}{\left(2P_f + 2C_\beta \sqrt{P_f P_r} + P_r\right)^2}$$

$$Q_1 = \frac{Q_0}{\beta_1} = \frac{Q_2 P_t}{\left(\sqrt{P_f} + C_\beta \sqrt{P_r}\right)^2}$$

Error Calculations

Assuming all errors are not correlated and Gaussian in nature, given:

$$z = F(x, y)$$

$$\Delta z = \sqrt{\left(\frac{\partial F(x,y)}{\partial x}\Delta x\right)^2 + \left(\frac{\partial F(x,y)}{\partial y}\Delta y\right)^2}$$

Standard Solutions

$z = xy$ $$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

$z = x \pm x$ $$\frac{\Delta z}{z} = \frac{\sqrt{\Delta x^2 + \Delta y^2}}{x \pm y}$$

$z = x^A$ $$\frac{\Delta z}{z} = \mathrm{A}\frac{\Delta x}{x}$$

Decay Measurement Theoretical Errors

$$\Delta Q_0 = Q_0 \sqrt{\begin{array}{c} \left(\left(\frac{\left(2P_f + C_\beta\sqrt{P_f P_r}\right)}{2\left(P_f + C_\beta\sqrt{P_r P_f}\right)} - \frac{P_f}{P_{Disp}}\right)\frac{\Delta P_f}{P_f}\right)^2 + \\ \left(\left(\frac{\left(C_\beta\sqrt{P_r P_f}\right)}{2\left(P_f + C_\beta\sqrt{P_r P_f}\right)} + \frac{P_r}{P_{Disp}}\right)\frac{\Delta P_r}{P_r}\right)^2 + \\ \left(\frac{\Delta\tau}{\tau}\right)^2 + \left(\frac{\Delta P_t}{P_{Disp}}\right)^2 + \left(\frac{\Delta P_{HOMA}}{P_{Disp}}\right)^2 + \left(\frac{\Delta P_{tHOMB}}{P_{Disp}}\right)^2 \end{array}}$$

$$\Delta E = \frac{E}{2}\sqrt{\left(\frac{\left(2P_f + C_\beta\sqrt{P_f P_r}\right)}{2\left(P_f + C_\beta\sqrt{P_r P_f}\right)}\frac{\Delta P_f}{P_f}\right)^2 + \left(\frac{\sqrt{P_r}}{2\left(\sqrt{P_f} + C_\beta\sqrt{P_r}\right)}\frac{\Delta P_r}{P_r}\right)^2 + \left(\frac{\Delta\tau}{\tau}\right)^2}$$

Decay Measurement Theoretical Errors

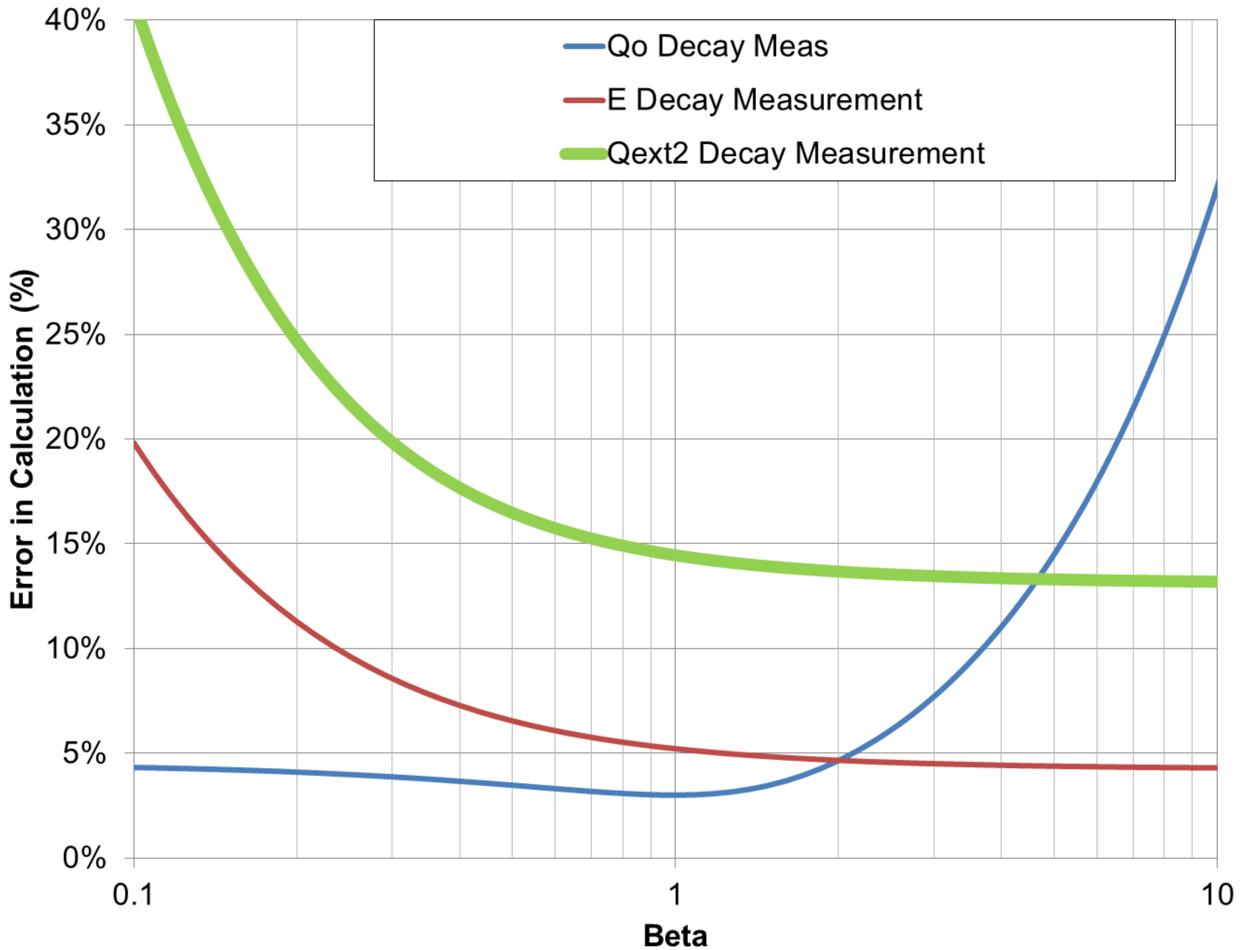
$$\Delta Q_2 = Q_2 \sqrt{\left(\frac{\left(2P_F + C_\beta\sqrt{P_F P_R}\right)}{\left(2P_F + C_\beta 2\sqrt{P_F P_R}\right)} \ \frac{\Delta P_F}{P_F}\right)^2 + \left(\frac{\left(\sqrt{P_F P_R}\right)}{\left(2P_F + C_\beta 2\sqrt{P_F P_R}\right)} \frac{\Delta P_R}{P_R}\right)^2 + \left(\frac{\Delta P_T}{P_T}\right)^2 + \left(\frac{\Delta \tau}{\tau}\right)^2}$$

Error Graphs, Decay Measurement

- RF Linearity applied to Field Probe signal during CW measurements.
- RF Power error of 10% applied to both CW measurements 5% used for CW measurements.

$$\left.\frac{\Delta P_{RF}}{P_{RF}}\right|_{Decay} = 10\%$$

$$\frac{\Delta\tau}{\tau} = 3\%$$



Error Propagation From Decay to CW Measurements

Gradient – We calculated a value for QFP during the decay measurement. In CW the gradient is calculated as:

$$E = Q_{FP} P_t \frac{(r/Q)}{L}$$

Using the standard error formula for multiplication.

$$\frac{\Delta E}{E} = \frac{1}{2}\sqrt{\left(\frac{\Delta Q_2}{Q_2}\right)^2 + \left(\frac{\Delta P_{tLin}}{P_t}\right)^2}$$

Where ΔP_{tLin} is the linearity of the transmitted power meter, which is nominally <2%.

What this ends up meaning is that the error in E is close to constant and equal to the error in E during the decay measurement.

Error Propagation From Decay to CW Measurements

Qo gets more complicated because it is a function of P_{Fwd}, P_{Ref}, P_{Trnan} as well as Q_{FP} which was derived from the same signals potentially at exactly the same values as the CW measurements.

Starting out with a naive approach

$$Q_0 = \frac{Q_2 P_t}{P_{Fwd} - P_R - P_T}$$

$$\frac{\Delta Q_0}{Q_0} = \sqrt{\frac{\left(\Delta P_f^2 + \Delta P_r^2\right)}{(P_{Fwd} - P_R - P_T)^2}\frac{\Delta P_{RFCW}}{P_{RFCW}} + \left(\frac{\left(P_f - P_r\right)}{(P_{Fwd} - P_R - P_T)}\frac{\Delta P_{tLin}}{P_t}\right)^2 + \left(\frac{\Delta Q_2}{Q_2}\right)^2}$$

Example plots for this are shown in the next slides.

Unfortunately the calibrations (and associated errors) for all of the RF power readings are buried in Q_2.

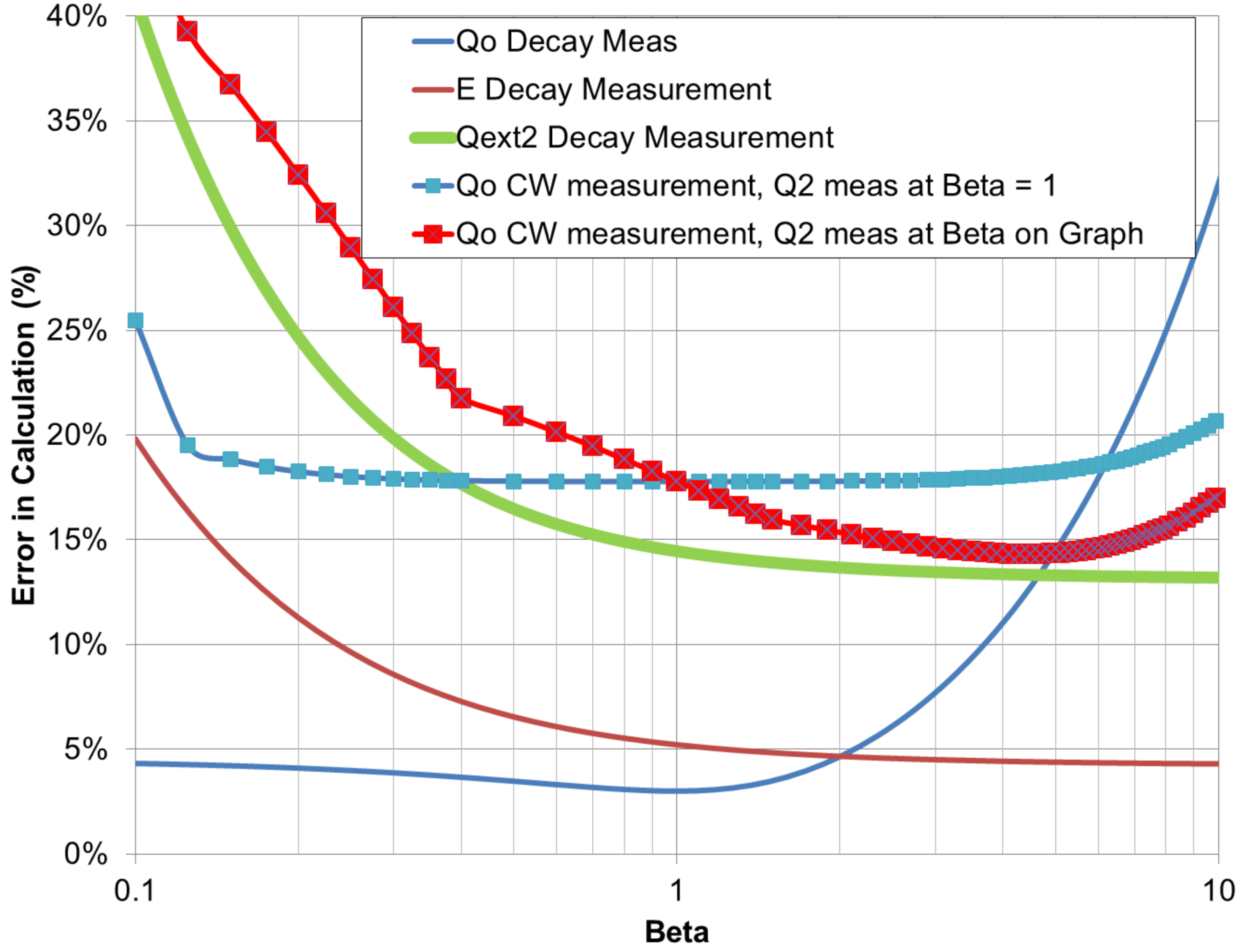
Error Graphs

- RF Linearity applied to Field Probe signal during CW measurements.
- RF Power error of 10% applied to decay measurements 5% used for CW measurements.

$$\left.\frac{\Delta P_{RF}}{P_{RF}}\right|_{Decay} = 10\%$$

$$\frac{\Delta\tau}{\tau} = 3\%$$

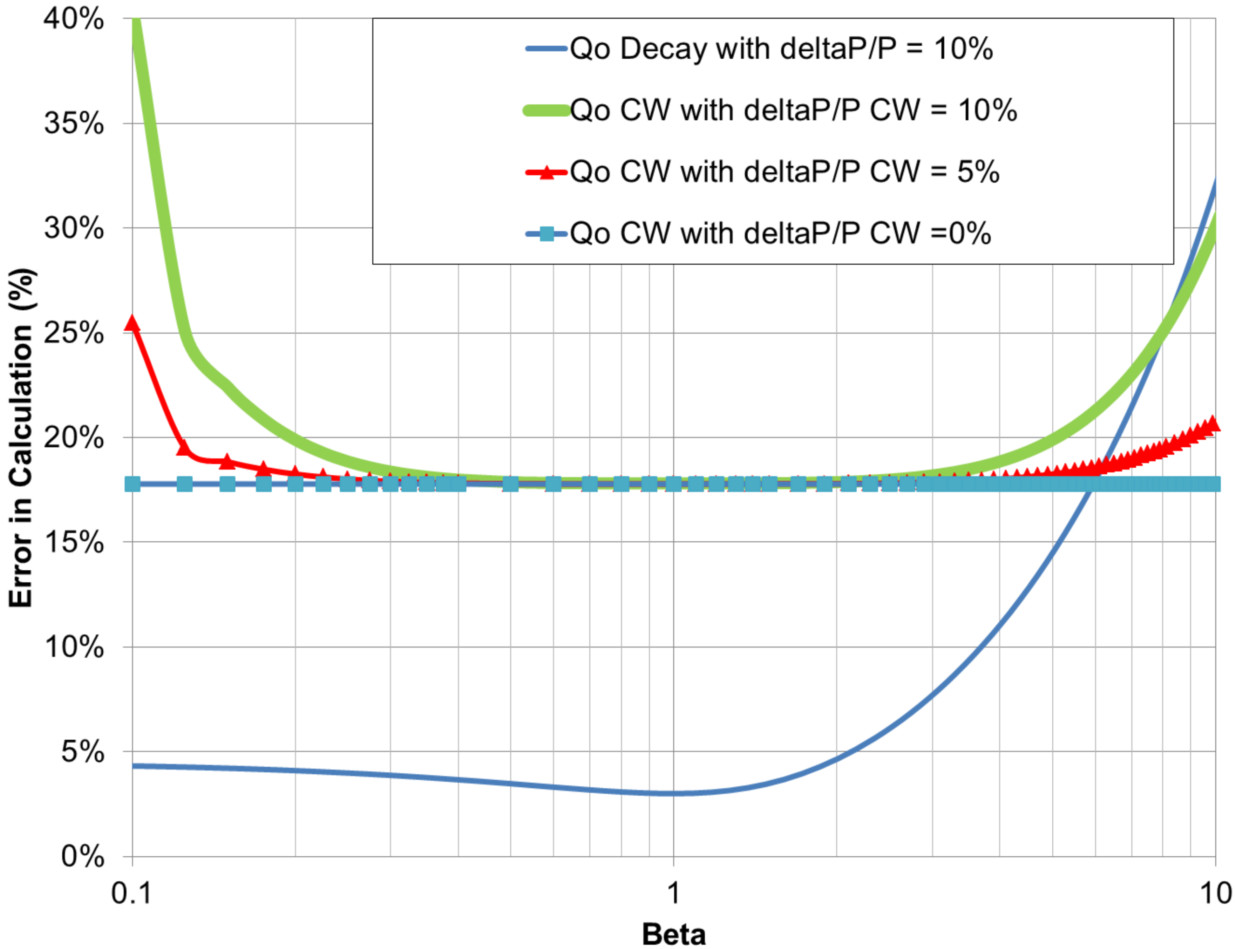
$$\left.\frac{\Delta P_{RF}}{P_{RF}}\right|_{CW,FWD,REF} = 5\%$$



$P_{FP,\,CW}$ $RF\ Linearity = 2\%$

Error Graphs, $\Delta Q_{2CW} = \Delta Q_{2DECAY}$@$\beta = 1$

- RF Linearity applied to Field Probe signal during CW measurements.
- RF Power error of 10% applied to decay measurements.
- X% applied to CW and decay measurements.



$$\left.\frac{\Delta P_{RF}}{P_{RF}}\right|_{Decay} = 10\%$$

$$\frac{\Delta\tau}{\tau} = 3\%$$

RF Linearity= 2%

$$\left.\frac{\Delta P_{RF}}{P_{RF}}\right|_{CW} = 0\%, 5\%, 10\%$$

Error Propagation From Decay to CW Measurements

Next we will assume that the errors in the power readings are all attributed to errors in their calibrations and that the raw power meter readings are perfect.

$$Q_0 = \left[\frac{C_t P_{tm}}{C_f P_{fm} - C_r P_{rm} - C_t P_{tm}}\right]\left[4\pi f_0 \tau \frac{C_f P_f' + C_\beta \sqrt{C_f C_r P_{fm}' P_{rm}'}}{C_t P_{tm}'}\right]$$

Where the values annotated with a prime (‘) symbol are the readings taken when performing the decay measurement and all of the power measurements are the actual readings from the instruments used for the CW measurement.

Taking the partial derivatives with respect to the calibration factors C_f, C_r, C_t as well as that for τ leads to the results shown on the following slide. Note that during the derivation I carried through all of the calibration factors C_x, after the mathematical manipulations each P_x was associated with its correction factor and the power values in the final equations are the calibrated values including the calibration factors.

Error Propagation From Decay to CW Measurements

Starting with the equation on the previous slide and applying the following:

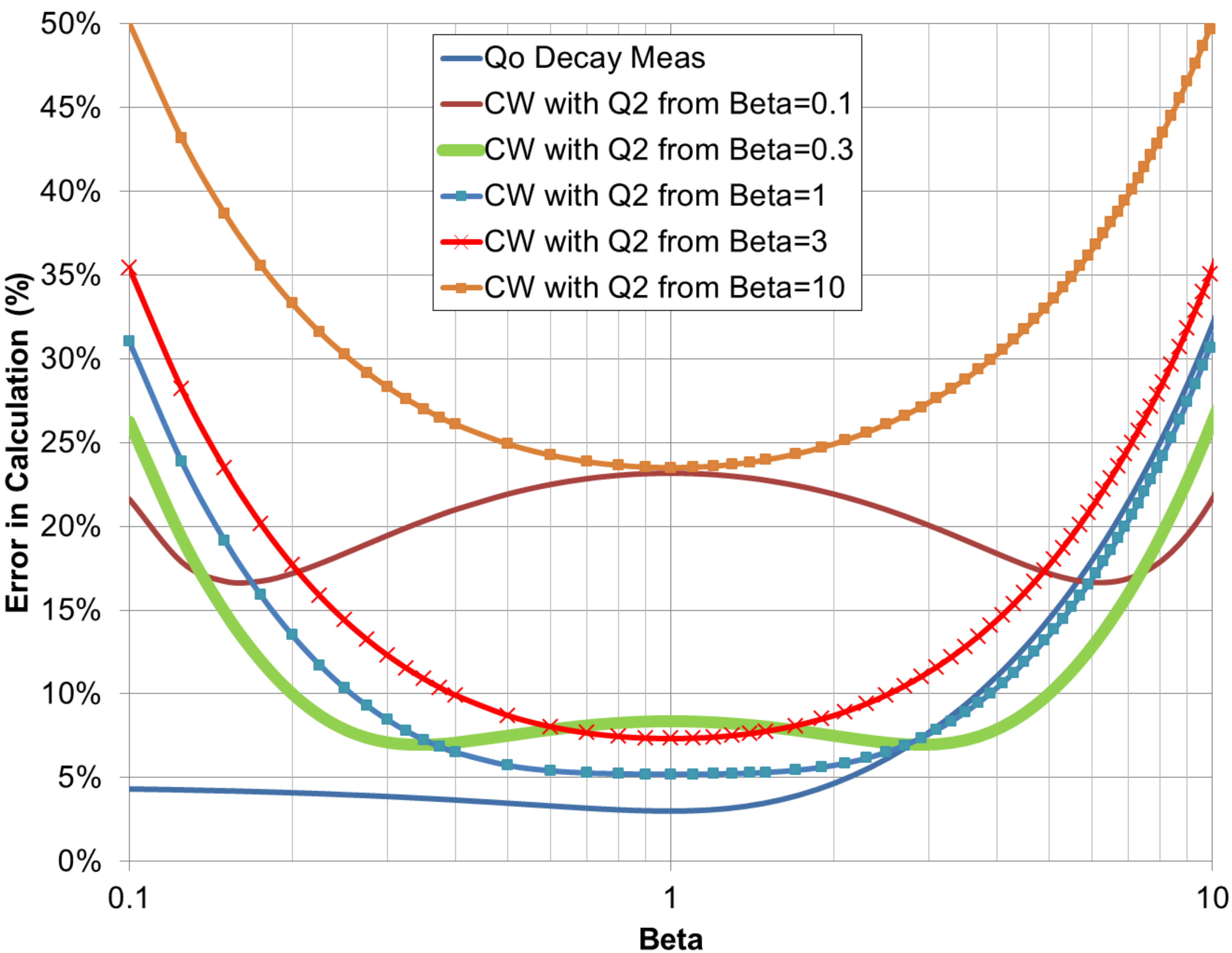
$$(\Delta Q_0)^2 = \left(\left(\frac{\partial Q_0}{\partial C_f}\right)\Delta C_f\right)^2 + \left(\left(\frac{\partial Q_0}{\partial C_r}\right)\Delta C_r\right)^2 + \left(\left(\frac{\partial Q_0}{\partial C_t}\right)\Delta C_t\right)^2 + \left(\left(\frac{\partial Q_0}{\partial \tau}\right)\Delta \tau\right)^2 + \left(\left(\frac{\partial Q_0}{\partial P_f}\right)\Delta P_f\right)^2 + \left(\left(\frac{\partial Q_0}{\partial P_r}\right)\Delta P_r\right)^2 + \left(\left(\frac{\partial Q_0}{\partial P_t}\right)\Delta P_t\right)^2$$

Leads to:

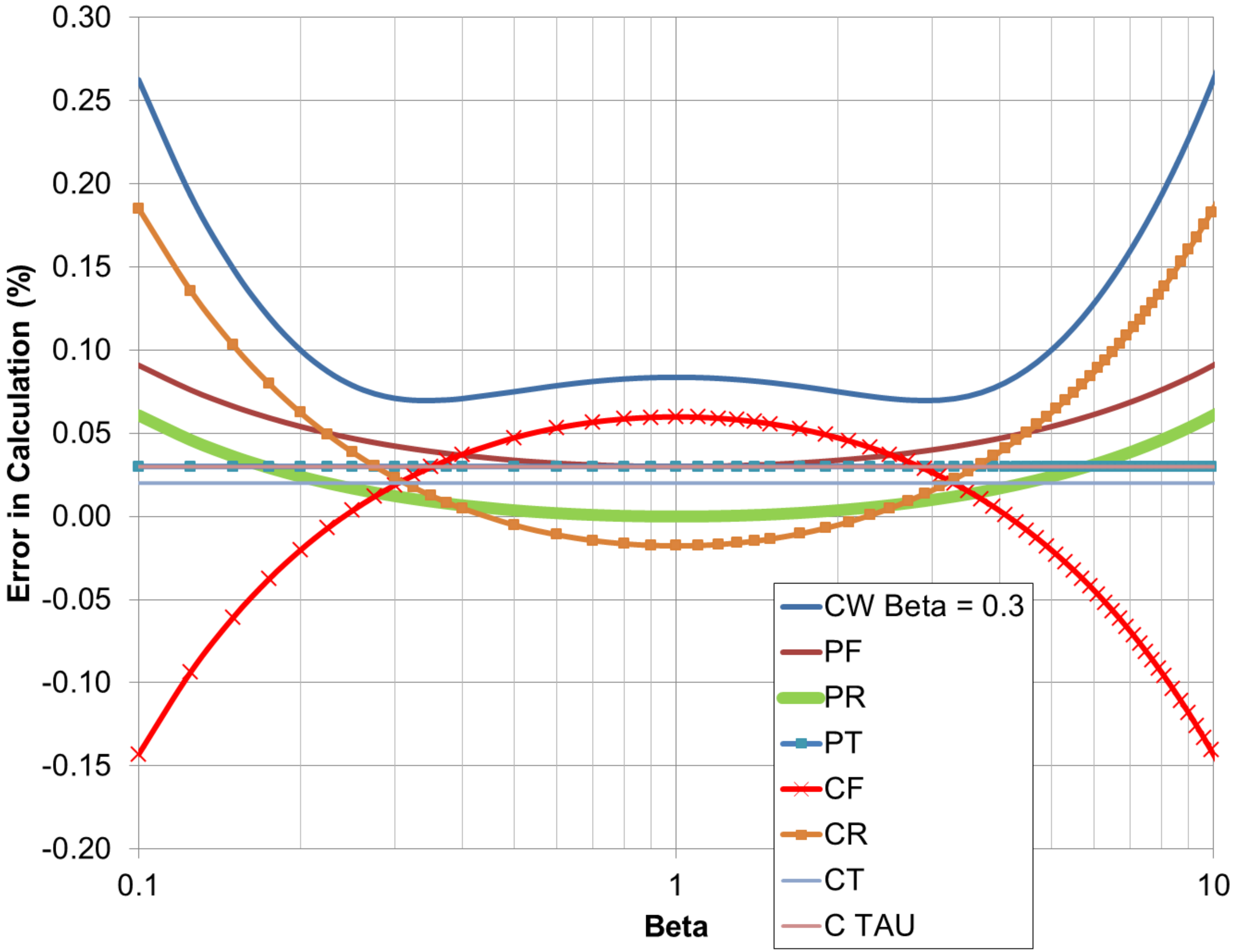
Q0 Errors for CW Measurements

$$\Delta Q_0 = Q_0 \sqrt{\begin{aligned}&\left(\left[\frac{2P_f' + C_\beta\sqrt{P_f'P_r'}}{2\left(P_f' + C_\beta\sqrt{P_f'P_r'}\right)} - \frac{P_f}{P_f - P_r - P_t}\right]\frac{\Delta C_f}{C_f}\right)^2 \\ &+ \left(\left[\frac{C_\beta\sqrt{P_f'P_r'}}{2\left(P_f' + C_\beta\sqrt{P_f'P_r'}\right)} + \frac{P_r}{P_f - P_r - P_t}\right]\frac{\Delta C_r}{C_r}\right)^2 + \left(Q_0\frac{\Delta\tau}{\tau}\right)^2 \\ &+ \left(\left[\frac{C_f P_f}{(C_f P_f - C_r P_r - C_t P_t)}\right]\frac{\Delta P_f}{P_f}\right)^2 + \left(\left[\frac{C_f P_r}{(C_f P_f - C_r P_r - C_t P_t)}\right]\frac{\Delta P_r}{P_r}\right)^2 \\ &+ \left(\frac{P_t}{P_f - P_r - P_t}\frac{\Delta C_t}{C_t}\right)^2 + \left(\left[1 + \frac{P_t C_t}{(C_f P_f - C_r P_r - C_t P_t)}\right]\frac{\Delta P_t}{P_t}\right)^2\end{aligned}}$$

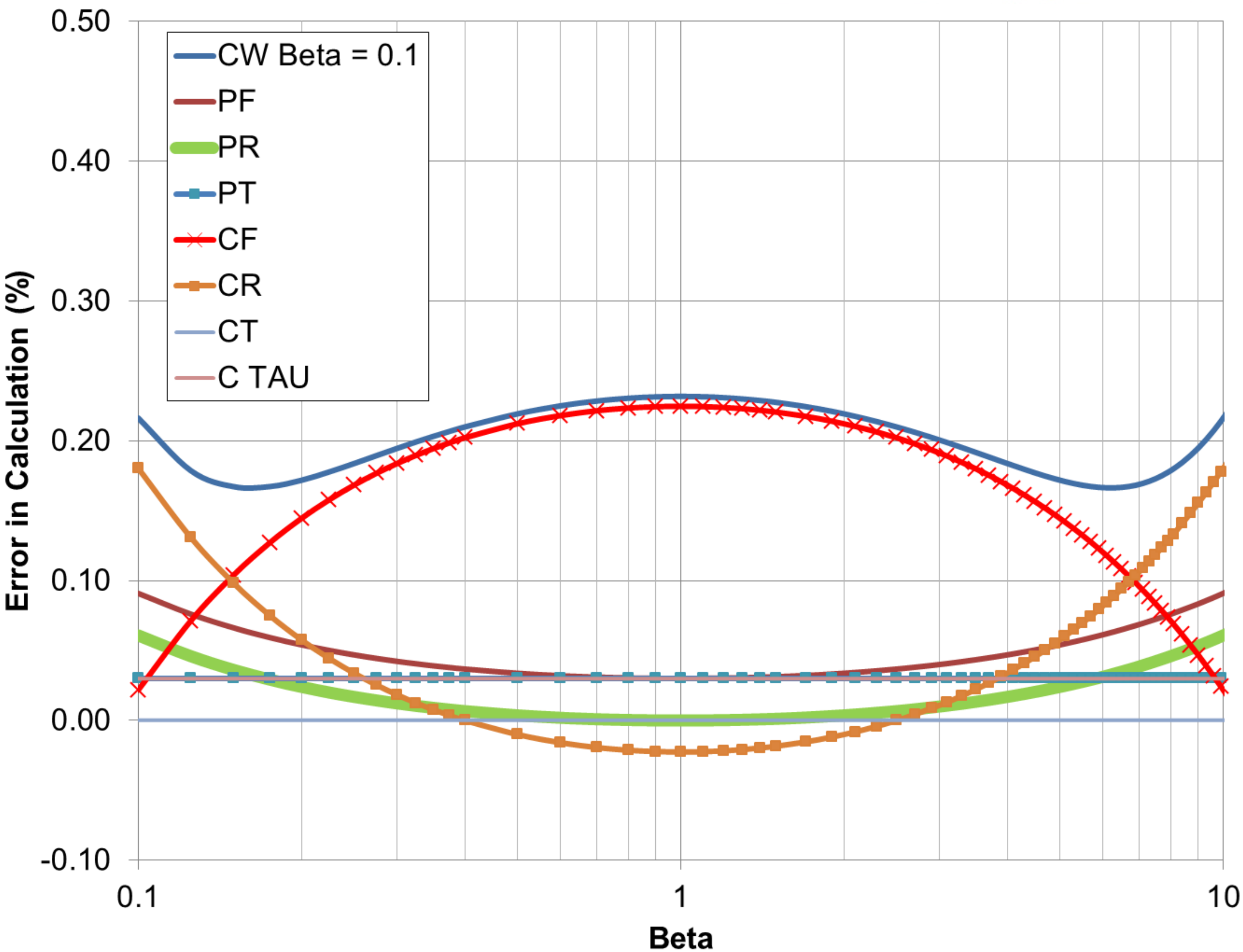
Errors in Q_0 for Decay and CW for Various Starting Points for Q_{FP}



Component parts of $\Delta Q_0/Q_0$ with Beta=0.3 during decay measurement

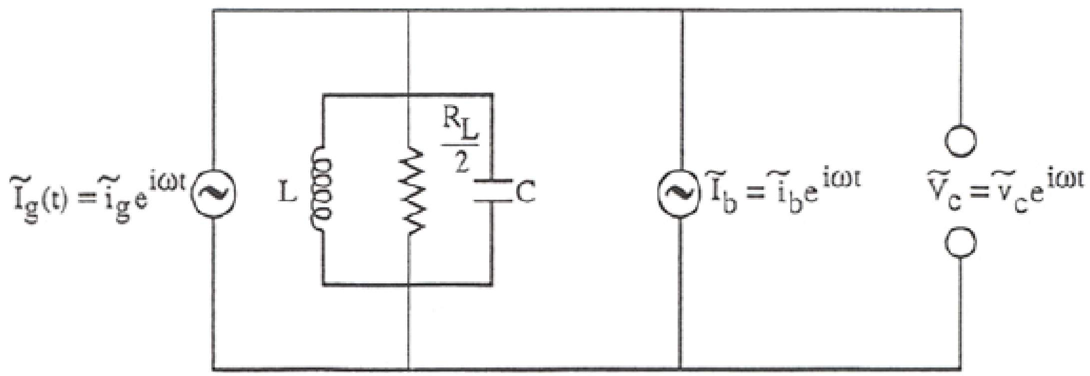


Component parts of $\Delta Q_0/Q_0$ with Beta=0.3 during decay measurement



Equivalent Circuit for a Cavity with Beam

- Beam in the rf cavity is represented by a current generator.
- Equivalent circuit:



$$R_L = \frac{R_{sh}}{(1+\beta)}$$

$\tilde{i}_b$ produces $\tilde{V}_b$ with phase ψ (detuning angle)

$\tilde{i}_g$ produces $\tilde{V}_g$ with phase ψ

$$\tilde{V}_c = \tilde{V}_g - \tilde{V}_b$$

$$\tan \psi = -2 \frac{Q_0}{1+\beta} \frac{\Delta\omega}{\omega_0}$$

Delayen USPAS 2008.

RF Power With Beam Loading

Based on the previous model with the addition of an RF "transformer" called a fundamental power coupler.

$$P_{FWD} = \frac{(\beta+1)L}{4\beta Q_L(r/Q)}\left\{\left(E + I_0 Q_L (r/Q)cos(\varphi_B)\right)^2 + \left(2Q_L\frac{\delta f}{f_0}E + I_0 Q_L (r/Q) sin(\varphi_B)\right)^2\right\}$$

Where is defined as:

$$\beta = \frac{Q_0}{Q_L} - 1 = \frac{Q_0 - Q_L}{Q_L}$$

Because the field probe Q is several orders of magnitude above the Q_0 losses and the loaded-Q.

Although using the forward power to calculate gradient is a reasonable technique, practical experience says that there can easily be as much as 25% difference between the gradient measured using this technique as compared to the that measured using the emitted power technique or using a well calibrated field probe measurement. This difference can be reduced by properly tuning the phase locked loop, for a variable frequency system or the cavity for a fixed frequency system.

For details on derivation see Merminga JLAB TN-95-019

EMITTED POWER MEASUREMENT

THE REFERENCE MEASUREMENT FOR STRONGLY OVER COUPLED CAVITIES

Consider what happens when you suddenly remove the incident RF power from a cavity that has the stored energy U. This stored energy leaves the system through dissipation due to wall losses, i.e. Q_0 losses, and as RF power that is emitted from all of the RF ports in the system. Since $Q_L << Q_{FP}$ and $Q_L << Q_0$ in a strongly over coupled superconducting cavity the stored energy can be calculated as:

$$U = \int_{t_0}^{\infty} P_{emitted}(t)dt \approx \int_{t_0}^{\infty} P_{reflected}(t)dt$$

Historically value of U was measured using a gating circuit and an RMS power meter. In a sampled system, such as can be done with a Boonton 4532 pulsed power meter, the stored energy can be approximated by:

$$U \approx \sum_{m}^{N} \left(P_{reflected}\right)_i \Delta t$$

Where m is the sample point where the incident power is removed and N is the total number of sample points. In addition to the errors associated with the power measurement, there are errors in this measurement which are introduced by the sampling system that can be reduced by proper choice of system parameters.

EMITTED POWER MEASUREMENT UNCERTAINTY

The uncertainty in the stored energy is given by the following:

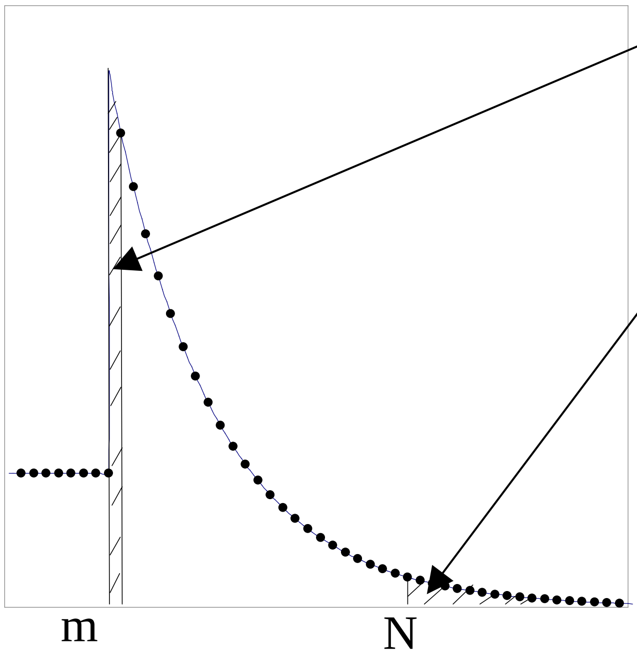
$$\Delta U = U\sqrt{\Delta C_R^2 + \Delta P_{CAL}^2} + \Delta t(N-m)C_R P_{\min} + (\Delta P_{emitted})_m \Delta t + \tau (P_{emitted})_N$$

Where :

ΔC_R is the percentage error in the power reading due to the cable calibration errors and

ΔP_{CAL} is the error in the power meter calibration.

$\Delta t(N-m)C_R P_{\min}$ is the contribution of the power meter noise floor during the integration.



$(\Delta P_{emitted})_m \Delta t$ is due to the jitter in the start of the integration and the peak of the emitted power transient

$\tau (P_{emitted})_N$ is the error introduced because you only summed the series to N and not to ∞

The last two errors can be minimized by sampling the system at a high sample rate compared to the decay time and insuring that that *(m-N)Δt* is greater than 4 decay time constants.

FIELD PROBE CALIBRATION

Once the stored energy has been determined the gradient can be calculated by using the following:

$$E_{Emitted} = \sqrt{2\pi f_0 * U * \frac{r/Q}{L}}$$

Where the emitted subscript is just an indicator of method used to determine the value. The filed probe coupling factor, Q_{FP} can the be calculated using:

$$Q_{FP} = \frac{E_{Emitted}^2}{\left(P_{Transmitted}\right)_{m-1}} * \frac{L}{r/Q}$$

Where $P_{Transmitted}$ is sampled just prior to removal of the incident power signal. Normally an average of several points just prior to m is used for this value.

With good calibrations and proper sample rates the gradient, E, can be measured with an accuracy of 5% to 7% and Q of the field probe to about 10% to 12%.

Qo MEASUREMENTS STRONGLY OVER COUPLED

When making a Q_0 measurement on a cavity that is strongly over coupled the dissipated power must be measured calorimetrically. To do this:

- The inlet and outlet values on the helium vessel are closed
- The rate of rise of the helium pressure is measured under static heat load.
- The rate of rise of the helium pressure is measured under a heat load of static plus known resistive power.
- The rate of rise of the helium pressure is measured under a heat load of static plus unknown cavity dissipated power.
- The following equation is used to calculate the unknown cavity dissipated power.

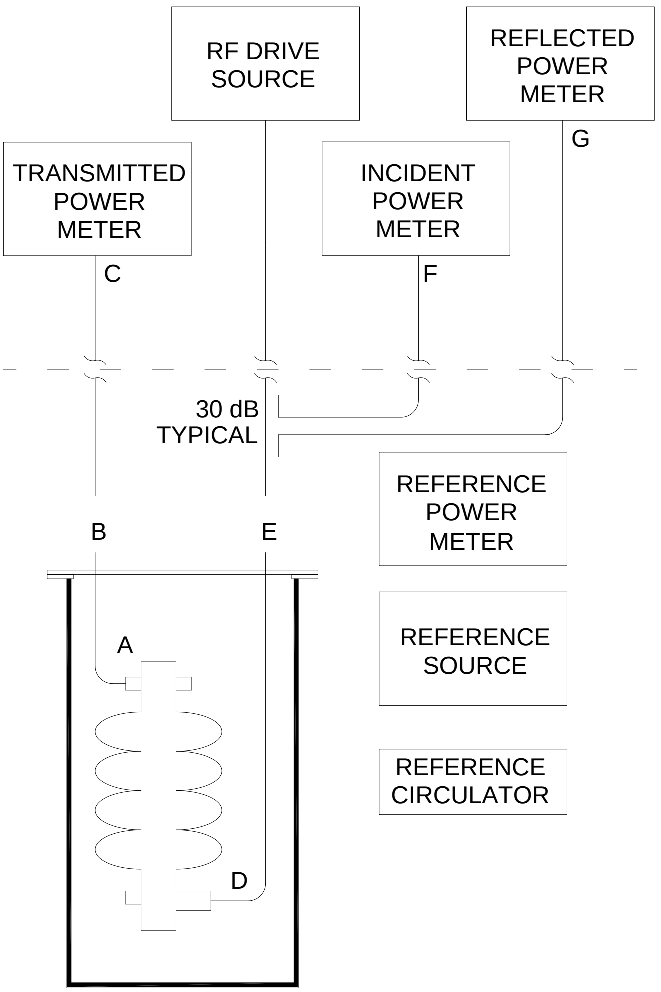
$$P_{DISSIPATED} = \left(\frac{\left(\frac{dP}{dt}\right)_{RF-ON} - \left(\frac{dP}{dt}\right)_{STATIC}}{\left(\frac{dP}{dt}\right)_{HEATER-ON} - \left(\frac{dP}{dt}\right)_{STATIC}} \right) P_{HEATER}$$

where $\left(\frac{dP}{dt}\right)$ is the rate of rise of the pressure under the different conditions.

CABLE CALIBRATIONS

- **Accurate consistent cable calibrations can make or break a test program.**
- **VSWR mismatches in the RF circuits will cause errors to "appear" when the frequency is shifted or the load mismatch changes.**
- **Cable calibrations for cavity testing are complicated by the fact that one or more of the cables are only accessible from one end.**
  - **In a vertical test the incident power cable, the field probe cable, as well as any HOM cables all have sections that are in the helium bath.**
  - **In cryomodule testing the field probe cable and any HOM cables have sections of cable that are within the cryomodule.**
- **When possible cables should be calibrated using signal injection and measurement at the other end using either a source and power meter combination; or a network analyzer.**
- **Cables should be measured at or near the frequency of the test.**
- **The only way to measure the losses of a cable within a cryostat is to do a two way loss measurement either with a calibrated network analyzer or a source, a circulator and a power meter.**

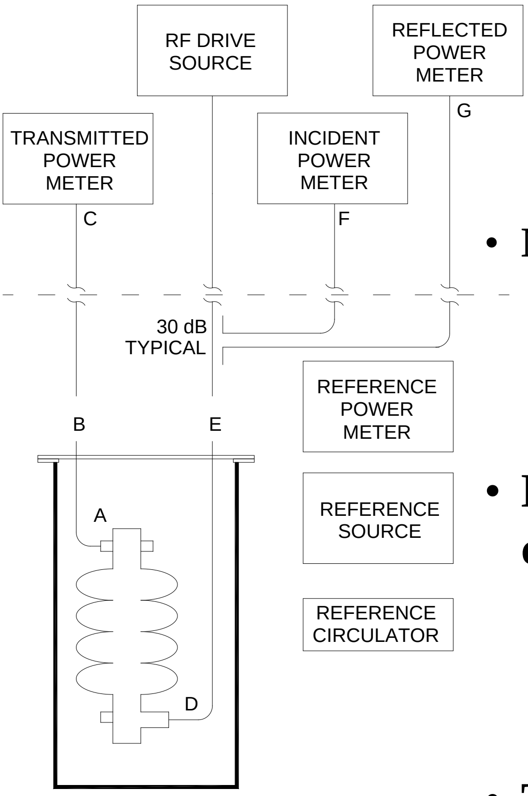
ONE WAY CABLE CALIBRATION



- **To calibrate the cable from point A to point C.**
- **Measure the one way loss of cable B-C.**
  - **Measure the reference source power level with the reference power meter. (P1)**
  - **Connect the reference source to point B of cable B-C.**
  - **Measure the power level with the transmitted power meter. (P2)**
  - **The one way loss is P1-P2 (dB)**
- **Measure the two way return loss of cable A-B**
  - **Connect the reference source to the input terminal of the circulator.**
  - **Connect the reference power meter to the load port on the circulator.**
  - **Record the reading on the reference power meter with the output port of the circulator open.* (P3)**
  - **Connect the output port of the circulator to port B of cable A-B and record the reading on the reference power meter. (P4)**
  - **The two way return loss is P3-P4 (dB)**
- **The cable calibration between for the A-C path is**

$$C_{AC} = (P1-P2) + (P3-P4)/2.$$

TWO WAY CABLE CALIBRATION



- **To calibrate the cable from point D to F and D to G**
- **Measure the forward power calibration from E to F**
  - **Connect the reference power meter to point E of the cable from the RF drive source.**
  - **Turn on the RF drive source and increase the power until the power level on the reference power meter is about 2/3 of the maximum allowed.**
  - **Record the power levels on the reference meter (P5) and the incident meter (P6)**
- **Measure the reflected power calibration from E to G**
  - **Turn off the RF source drive**
  - **Measure the reference source power level with the reference power meter. (P7)**
  - **Connect the reference source to point E of the path E-G.**
  - **Measure the power level with the reflected power meter. (P8)**
- **Measure the two way loss for the cable D-E with a detuned cavity.**
  - **Connect the RF drive source to the cavity at point E.**
  - **Turn on the RF drive source and apply power to the cavity at a frequency about 10 to 20 kHz higher or lower than the cavity's resonant frequency.**
  - **Measure the incident (P9) and reflected power (P10) with the respective meters.**
- **The cable calibration are:**

  Incident $C_{D\text{-}F}$ = (P5 – P6 + P7 – P8 – P9 + P10)/2 (dB)

  Reflected $C_{D\text{-}G}$ = (– P5 + P6 + 3*P7 – 3*P8 – P9 + P10)/2 (dB)

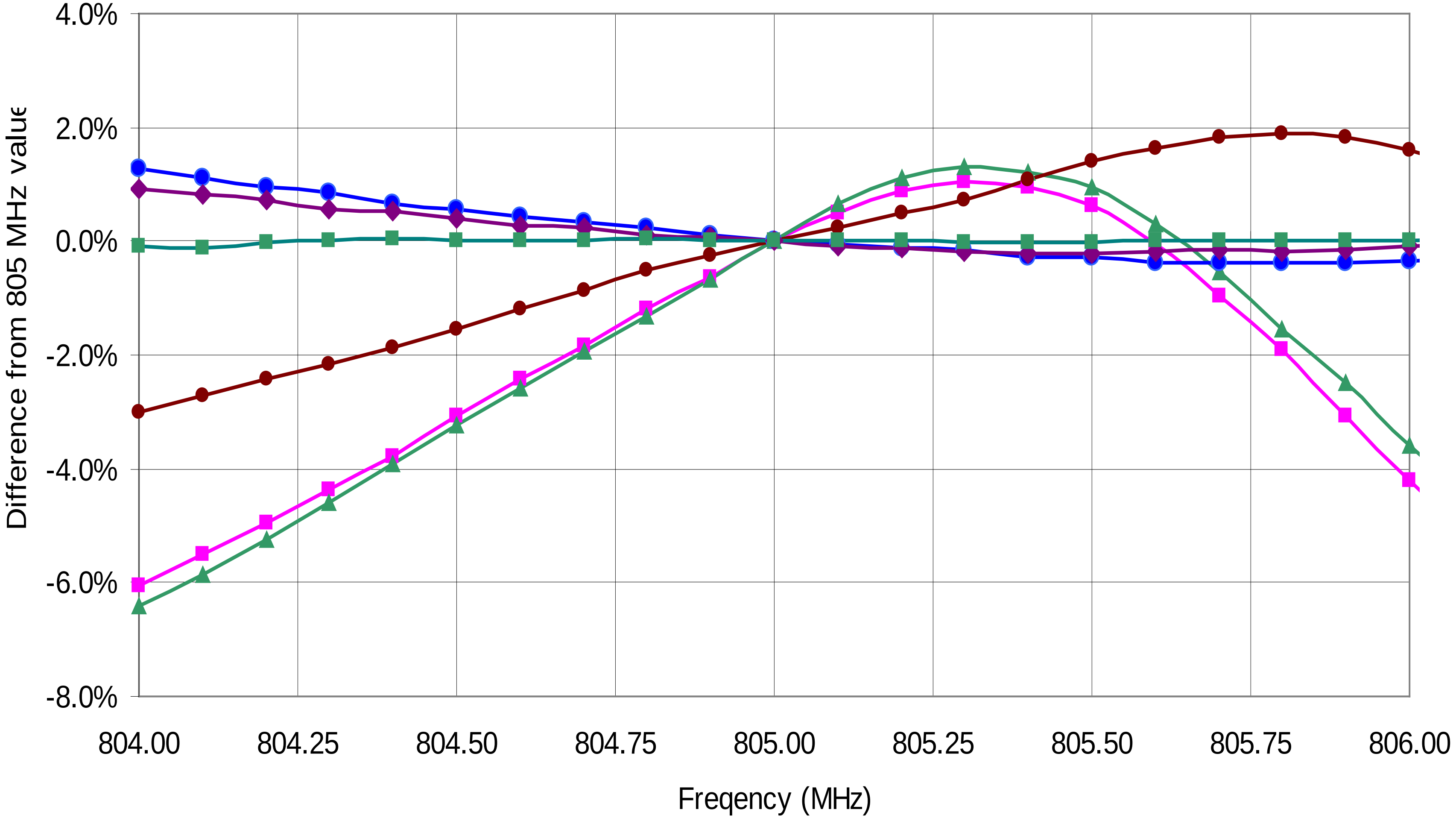
CALIBRATION VERIFICATION

- **Two ways that I use to verify calibration procedures are to:**
- **Calibrate the system using an external cable rather than a cable within the dewar then:**
  - **For field probe power and reflected power inject a known signal level into the external cable and measure the power using the calibrated meter.**
  - **For the forward power connect the external cable to a remote power meter and measure the power using the remote power meter and the system power meter.**
- **In both cases it can be a useful exercise to vary the frequency over a 1 MHz to 2 MHz range and compare the values over the range.**

CALIBRATION VERIFICATION

- **A third way to verify the calibration and look for VSWR problems in the incident power cable is to:**
  - **Use the RF drive source to apply power to either an open test cable that has been calibrated or a detuned cavity.**
  - **Measure the calibrated forward and reflected power. They should be equal.**
  - **Vary the RF frequency by +/- 1MHz in 100 kHz increments.**
- **Variations in the ratio of forward to reflected power indicate a VSWR problem within the cabling system.**

MEASUREMENT OF VSWR INDUCED ERRORS



Difference between RF readings calibrated at 805 MHz and those taken at nearby frequencies for several different signal paths. The paths with smaller errors had attenuators distributed throughout the signal path.

Lorentz Force Detuning Coefficient

- The RF magnetic fields in a cavity interact with the RF will currents resulting in a Lorentz force that is proportional to E^2.
- These forces cause the cavity to deform and shift in frequency. For an electrical cavity the walls of the cavity near the iris are bent inward and the walls at the equator are bent outwards.
- Putting the dimension of the deflections in perspective. 50 Hz is the typical 3 dB full bandwidth of a C100 cavity. It takes 24 nm of change in length to change the frequency of a C100 cavity by 50 Hz.

$$K = \frac{f_1 - f_2}{{E_1}^2 - {E_2}^2}$$

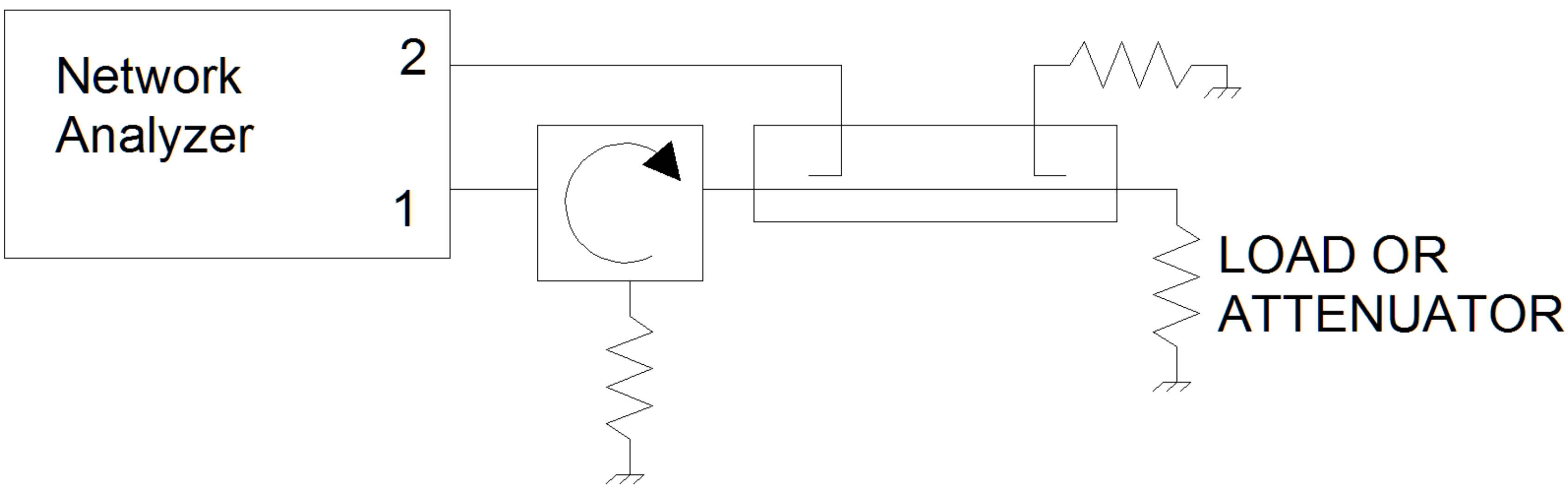
Here F_1 and F_2 are the cavity resonant frequency at E_1 and E_2 respectively. For this calculation E is in MV/m and K is in $Hz/(MV/m)^2$

DIRECTIONAL COUPLERS ARE NOT CREATED EQUAL

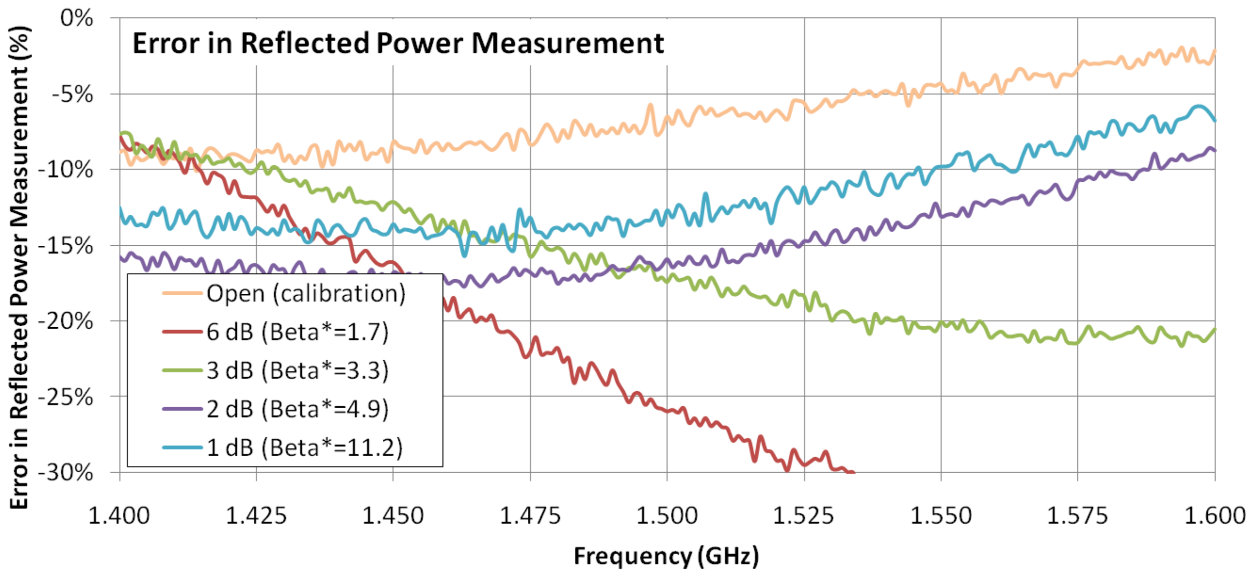
- **Frequently directional couplers are used to make measurements for which the load is not matched at 50 Ohms**
- **One example is the final step on the incident and reflected power calibrations where the reflected power is some 3 to 6 dB below the forward power**
- **Another example is when measuring a cavity that is not quite matched, i.e.** $\beta \neq 1$.

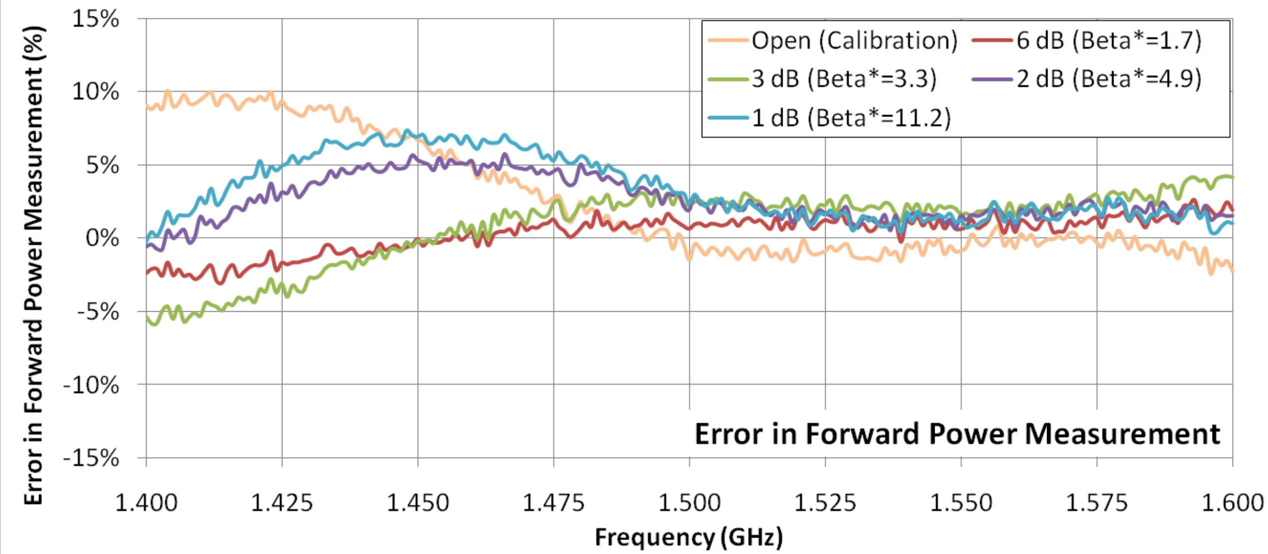
MEASUREMENT TECHNIQUES

- **Perform S21 measurements of different ports with all of the other ports terminated at 50 Ohms or with a broad band miss-matched load.**
- **One critical item is that there is a significant error introduced due to S11 of the output port on the network analyzer. To remedy this one must insert a circulator between port 1 and the unit under test.**
- **A good broad band miss match is an unterminated attenuator.**



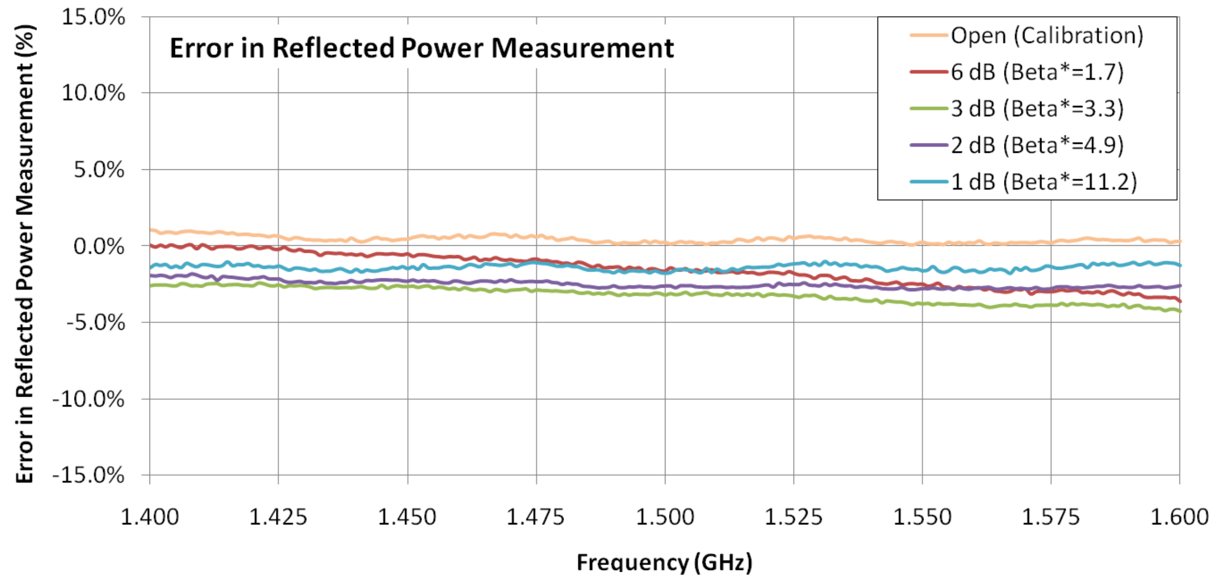
NARDA 20 dB COUPLER

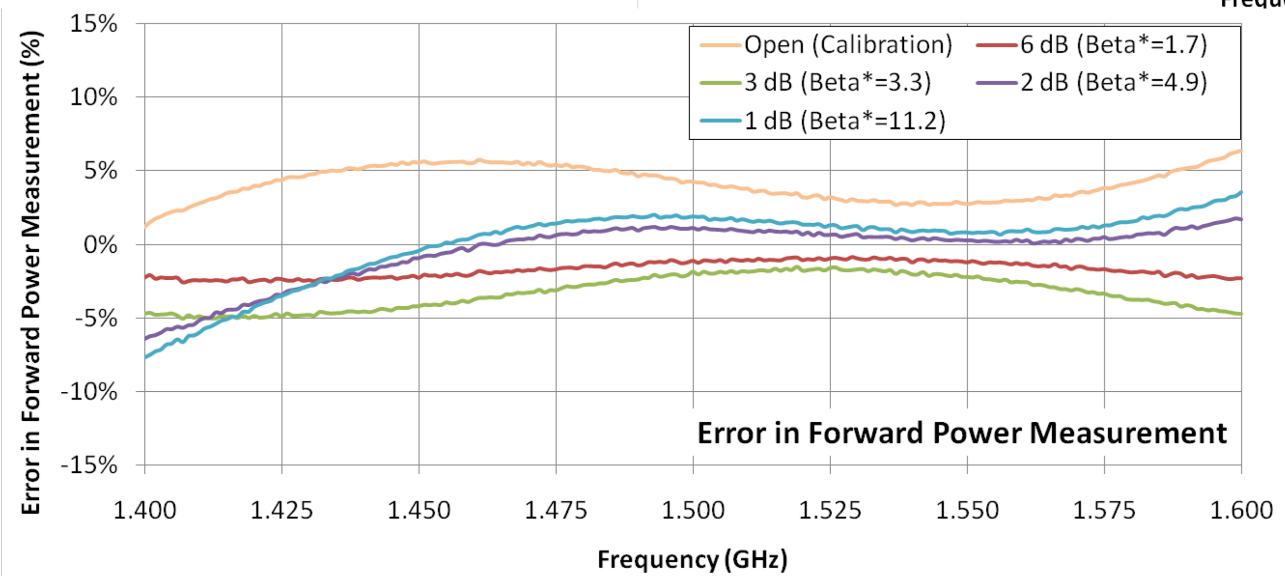




Error in power measurement with different loads on the output of the directional coupler (i.e. different beta*) Narda 3320 Serial 73091

CT MICROWAVE 30 dB COUPLER





Error in power measurement with different loads on the output of the directional coupler (i.e. different beta*) CT Microwave 441433, serial 73091

MEASUREMENT CONCLUSIONS

- Quality measurements necessary to qualify superconducting cavities require quality equipment designs, careful measurement techniques and well characterized calibrations processes.
- Errors for the standard measurements are calculable. However, the are a function of the measurement equipment, the quality of the calibration and the specific conditions of each data point. As such they should be included in the measurement system not as an afterthought.
- In addition to the slides presented, I have included a handout of the equations for both the cavity measurements and the associated errors.
- I want to thank all of the folks in the SRF Institute at Jefferson Lab for their constant patience in helping me put this presentation together.

BACKUP SLIDES

$$Q_L = \frac{Q_0}{\left(1 + \frac{\left(\sqrt{P_{FWD}} + C_\beta \sqrt{P_{REF}}\right)^2}{P_{DIsP}} + \frac{P_{FP}}{P_{DISP}}\right)}$$

$$Q_L = \frac{\frac{\mathrm{Q_{FP}P_{FP}}}{\mathrm{P_{DISP}}}}{\left(1 + \frac{\left(\sqrt{P_{FWD}} + C_\beta \sqrt{P_{REF}}\right)^2}{P_{DIsP}} + \frac{P_{FP}}{P_{DISP}}\right)}$$

$$Q_L = \frac{\frac{\mathrm{Q_{FP}P_{FP}}}{\mathrm{P_{DISP}}}}{\left(\frac{P_{DISP}}{P_{DISP}} + \frac{\left(\sqrt{P_{FWD}} + C_\beta \sqrt{P_{REF}}\right)^2}{P_{DIsP}} + \frac{P_{FP}}{P_{DISP}}\right)}$$

$$Q_L = \frac{Q_{FP} P_{FP}}{P_{FWD} - P_{REF} - P_{FP} + \left(\sqrt{P_{FWD}} + C_\beta \sqrt{P_{REF}}\right)^2 + P_{FP}}$$

$$Q_L = \frac{Q_{FP} P_{FP}}{P_{FWD} - P_{REF} - P_{FP} + \left(\sqrt{P_{FWD}} + C_\beta \sqrt{P_{REF}}\right)^2 + P_{FP}}$$

$$Q_L = \frac{Q_{FP} P_{FP}}{P_{FWD} - P_{REF} - P_{FP} + P_{FWD} + 2C_\beta \sqrt{P_{FWD} P_{REF}} + +P_{REF} + P_{FP}}$$

$$Q_L = \frac{Q_{FP} P_{FP}}{2P_{FWD} + 2C_\beta \sqrt{P_{FWD} P_{REF}}}$$