U.S. DEPARTMENT OF ENERGY

U.S. Particle Accelerator School

Education in Beam Physics and Accelerator Technology

Higher Order Modes (HOMs)

SRF Hands-On Course at JLab
January 19-30, 2015
Newport News, VA

Frank Marhauser

Jefferson Lab
Thomas Jefferson National Accelerator Facility

This Lecture

- Introduction to HOMs
  - Excitation, wakefields and impedances, consequences
  - HOM nomenclature, examples of RF field configurations
- Figure of merit to characterize HOMs
  - R, R/Q, Quality factor, loss factor, stored energy, damping term
  - Bunch spectrum, modes in multi-cell cavities, dispersion relation
- Ways to damp HOMs
  - Coaxial couplers, waveguide dampers, beam tube absorbers, FPCs
  - Cutoff frequencies for round beam tubes, rectangular waveguides
  - Examples of SRF camped cavities (Pros and Cons)
- Absorbers Materials
  - Measurement of properties (S-Parameter)
- Preparation for experiments
- Backup Slides
  - Loss factor, Fundamental Theorem of Beam Loading, BBU, Examples of existing single-cell HOM-damped normal-conducting (NC) and superconducting RF (SRF) cavities, mode nomenclature, modes in pillbox cavities, cutoff frequencies (tube, waveguide)

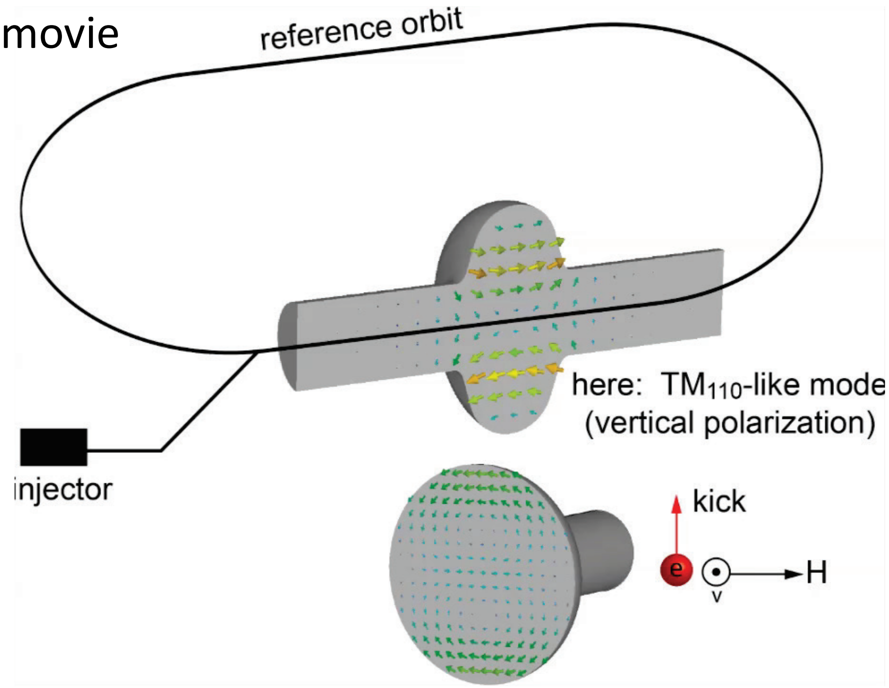
HOMs and Consequences

- HOMs (<u>H</u>igher <u>O</u>rder <u>M</u>odes) are Eigenmodes parasitically excited in a resonant (accelerating) RF cavity
  - Other than and with frequency greater than the operational mode
- HOMs are undesired since dangerous for beam dynamics
  - Transverse modes can deflect the beam from its reference orbit → Causes instable beam motion, transverse emittance growth or even beam loss
  - Monopole HOMs can lead to energy spread, timing/phase errors → Causes longitudinal emittance growth or instability
  - HOMs (dominantly monopole modes) also cause extra losses in cavity walls
    - Particularly crucial for SRF cavities since power is dissipated in Helium bath (every extra Watt matters)

HOMs and Consequences

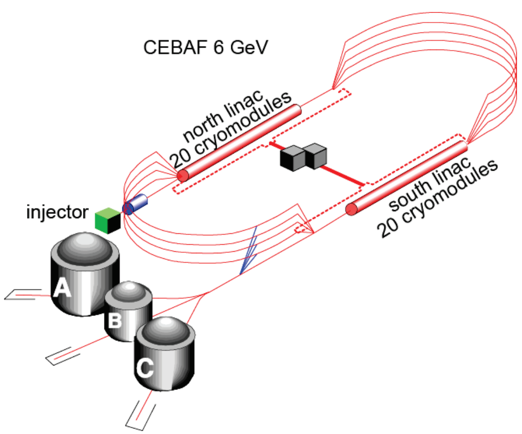
- Particularly in electron recirculating machines (e.g. CEBAF), ERLs or storage rings HOMs are a concern
  - HOMs can resonate for long time (many turns) depending on involved loss mechanisms (surface plus external losses)
  - Resonant (multi-bunch) beam break-up instabilities (BBU) can occur if HOMs are left undamped

movie



HOMs and Consequences

- **Real world example**: Single dipole-mode (TM_{110}-like) induced transverse BBU observed in CEBAF in 2007 caused by single cavity in then newly installed prototype cryomodule
  - f = 2.15 GHz, $\mathbf{Q_l = 1.1 \cdot 10^8 \rightarrow \tau = 8.1\ ms\ or\ c_0 . \tau = 2441\ km!}$
- **1st Consequence**: Limited threshold current for injected beam to as low as 40 µA (compare: CEBAF operational peak current is 200 µA peak, expected BBU threshold by design is 20 mA)
- **2nd Consequence**: Cryomodule prototype decommissioned



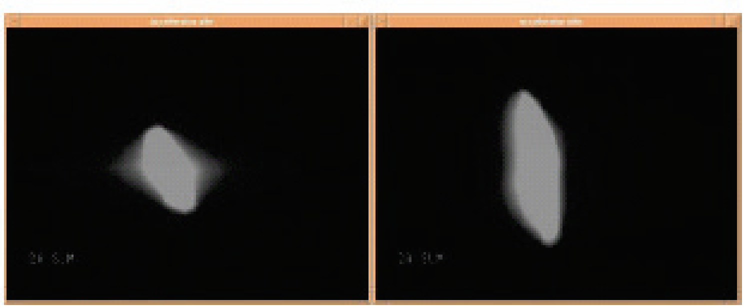
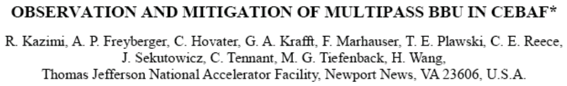


Figure 2: Beam spot well below BBU limit (left) and very close to BBU limit (right) on the SLM.

Proc. EPAC 2008

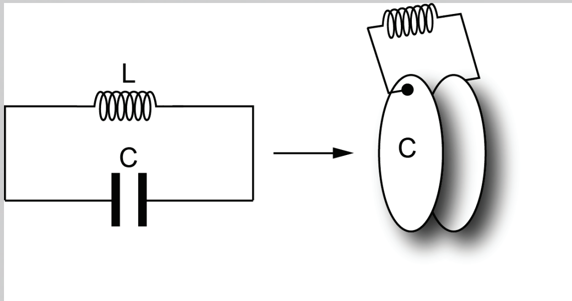


OBSERVATION AND MITIGATION OF MULTIPASS BBU IN CEBAF*

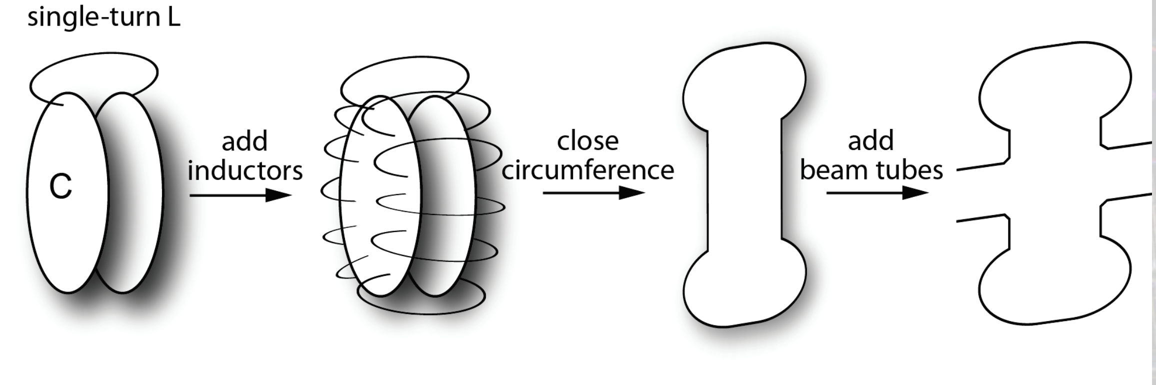
R. Kazimi, A. P. Freyberger, C. Hovater, G. A. Krafft, F. Marhauser, T. E. Plawski, C. E. Reece, J. Sekutowicz, C. Tennant, M. G. Tiefenback, H. Wang, Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, U.S.A.

Resonant RF Structure

- A resonant circuit can be represented by an inductance L and capacitance C



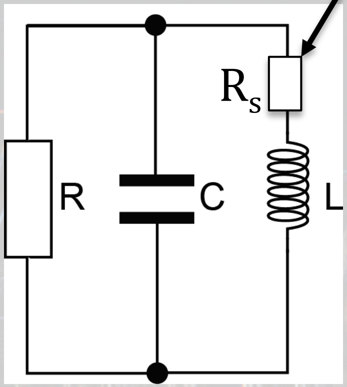
- The resonant behavior of an accelerating cavity can be well described by parallel resonant circuit with lumped elements
- consider two plates (capacitance) with parallel inductors



Resonant RF Structure

- In a cavity the beam excites a voltage along the so-called shunt impedance, this is the resistor added in parallel to L&C

$$P_{avg} = \frac{R_s}{2} \iint_A dA \cdot |\vec{H}|^2$$



complex impedance of the parallel resonant circuit is

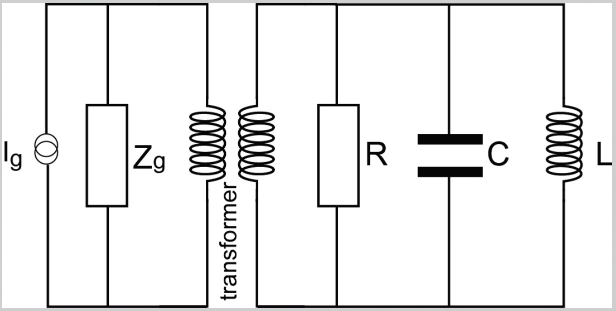
$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{Z_C} + \frac{1}{Z_L}$$

$$Z = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

- The impedance is real (Z=R), when $\omega C - \frac{1}{\omega L} = 0$ → $\omega = \omega_0 = \sqrt{\frac{1}{L \cdot C}}$
- This is the resonant frequency of the circuit
- The shunt impedance is maximized in this case

Resonant RF Structure

- To excite the circuit, we need a generator



- The impressed current shall oscillate at an arbitrary frequency ω and thus can have a phase difference to the circuit current
- The phase difference is $\boxed{\varphi = arctan\left(\frac{Im(Z)}{Re(Z)}\right) = arctan\left(R\cdot\left(\omega C - \frac{1}{\omega L}\right)\right)}$
- Inserting $\boxed{L = \frac{1}{{\omega_0}^2\cdot C}}$ yields $\boxed{\varphi = arctan\left(\frac{RC}{\omega}\cdot(\omega^2-\omega_0^2)\right)}$
- In resonance $\boxed{\varphi = 0}$, i.e. and V and I are in phase

Resonant RF Structure

- If the resonator is oscillating freely, the energy initially stored W(t) in the system decays as heat in the R
- The power dissipated is $P_{loss} = \frac{V_{eff}^2}{R} = \frac{V_0^2}{2 \cdot R}$
- The stored energy W(t) in the circuit continuously oscillates between L and C, e.g. periodically the energy can be fully stored in the capacitor or the inductor, respectively

$W_C(t_0) = \frac{1}{2} C \cdot V_0^2$ $\quad$ $W_L(t_1) = \frac{1}{2} L \cdot I_0$

- We now can define the **intrinsic (unloaded) quality factor**, e.g.:

$$Q_0 = \omega_0 \cdot \frac{W_C}{P_{loss}} = \omega_0 \frac{\frac{1}{2} C \cdot V_0^2}{\frac{1}{2} \frac{V_0^2}{R}} = \omega_0 RC = \frac{R}{\omega_0 \cdot L}$$

- This also provides us readily with the definition of the **characteristic shunt impedance** $\frac{R}{Q_0} = \omega_0 \cdot L = \frac{1}{\omega_0 \cdot C}$

Resonant RF Structure

- Important: **The R/Q value only depends on the cavity shape (L and C) but not losses (Q_0)**
- A large R/Q-value is desired for effective acceleration, but small R/Q-values for parasitic HOMs → no independent optimization
- We also want large Q_0 (small losses) for accelerating mode, but small Q's for HOMs

Damping Term

- From circuit theory we can derive the damping term of the resonant system

$$I(t) = I_c(t) + I_R(t) + I_L(t) = C \cdot \frac{dV(t)}{dt} + \frac{V(t)}{R} + \frac{1}{L}\int dt \cdot V(t)$$

- Yields differential equation:

$$\frac{d}{dt} I(t) = \left(C \frac{d^2}{dt^2} + \frac{1}{R}\frac{d}{dt} + \frac{1}{L} \right) V(t)$$

- Let's assume that at t=0 the energy is stored in the capacitor and the system if freely oscillating (no generator)

$$\left(\frac{d^2}{dt^2} + \frac{1}{RC}\frac{d}{dt} + \frac{1}{LC} \right) V(t) = 0$$

$$Ansatz: V(t) = V_0 \cdot e^{-\alpha t}$$

$$\alpha^2 - \frac{1}{RC}\alpha + \frac{1}{\mathrm{LC}} = 0$$

$$remember$$

$$Q_0 = \omega_0 RC; \omega_0 = \sqrt{\frac{1}{L \cdot C}}$$

$$\alpha^2 - \frac{\omega_0}{Q_0}\alpha + {\omega_0}^2 = 0$$

$$\alpha_{1,2} = \frac{\omega_0}{2Q_0} \pm \sqrt{\left(\frac{\omega_0}{2Q_0}\right)^2 - {\omega_0}^2}$$

$$solution$$

$$\alpha_{1,2} = \frac{\omega_0}{2Q_0} \mp j\omega_0 \sqrt{1 - \frac{1}{4Q_0^2}}$$

Damping Term

- Therefore the decay of voltage and energy, respectively, can be described by

$$V_c(t) = V_0 \cdot e^{-\frac{\omega_0}{2Q_0}\cdot t \pm j\omega_0 \cdot t \cdot \sqrt{1-\frac{1}{4Q_0^2}}}$$

$$W_C(t) = \frac{1}{2} C \cdot |V_c(t)|^2 = \frac{1}{2}\frac{Q_0}{\omega_0 R} V_0^2 \cdot e^{-\frac{\omega_0}{Q_0}\cdot t}$$

voltage damping term

this term ≈ 1 even at low Qs and may be neglected

energy damping term

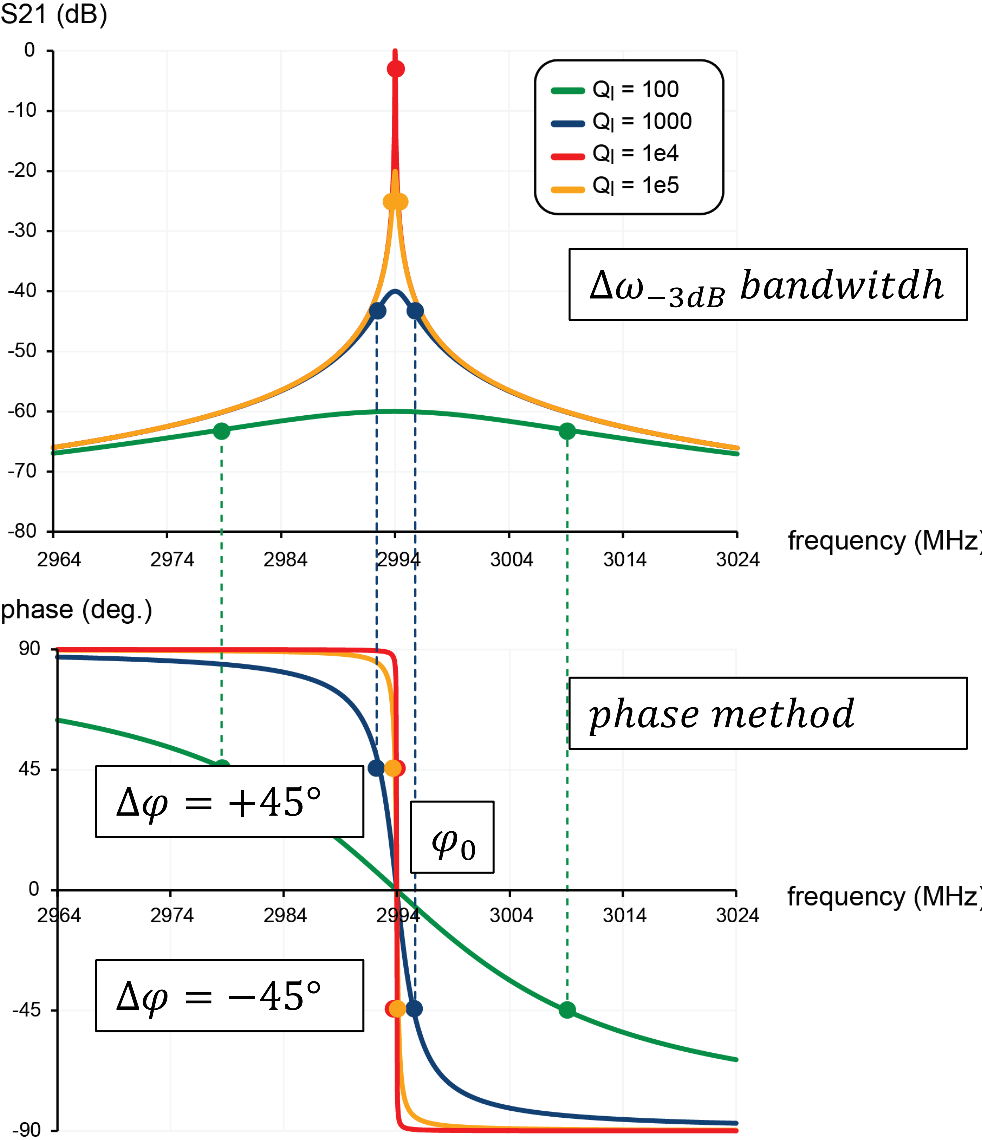
- Alternatively, the decay constant can be more readily derived utilizing the Q-definition

$$Q = \frac{\omega \cdot W}{P_{avg}} = \frac{\omega \cdot W}{dW/dt} \xrightarrow{\text{integration}} W(t) = W(t_0) e^{-\frac{\omega}{Q}\cdot t}$$

- The decay constant is $\tau = \frac{Q_0}{\omega_0}$
  - It is used for SRF cavities to measure the unloaded Q of the fundamental mode at cryogenic temperatures
  - Example: f = 1.5 GHz, Q_0 = 1e10 → $\tau \approx$ 1s
  - Decay method fine as long time constant in this order, so what other methods exist when Qs are lower (either warm or cold measurements)?

$$\frac{W(t_0)}{W(t_0+\tau)} = e^1 \approx 2.718$$

Measure Q-factor



- Convenient to use Vector Network Analyzer
  - Measures complex S-Parameter(in reflection or transmission)
- Note: S-Parameters signal ~ √ P
- To determine the Q, we search for the amplitude (S-Parameter) that is down by **1/√ 2** times the amplitude in resonance, which is equal to **1/2 the energy**
- Definition is equivalent to Q=ωW/P_{loss}
- Note that in logarithmic scale
  - dB = 10·Log(P_{out}/P_{in}) = 20·Log(V_{out}/V_{in})
- **dB=20Log(1/√ 2) =10Log(1/2)≈-3.0103 dB**
- Commonly one uses -3 dB points (though not truly exact) $Q = \dfrac{\omega}{\Delta\omega_{-3dB}}$
- **Equivalent to measure at phases +45° and -45°** apart from the resonance in the phase plot

Unloaded/Loaded/External Q

- Typically we measure the loaded Q (Q_l) of the system, since the all external couplers required for measurement extract power (P_{ext}), which add to the losses
- If we are interested in intrinsic cavity Q_0 all external losses need to be taken into account and thoroughly calibrated

$$\frac{1}{Q_l} = \frac{P_{loss}}{\omega W} = \frac{P_{cav} + P_{ext,1} + P_{ext,2} + \cdots}{\omega W}$$

$$\frac{1}{Q_l} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \quad with \quad \frac{1}{Q_{ext}} = \frac{1}{Q_{ext,1}} + \frac{1}{Q_{ext,2}} + \cdots$$

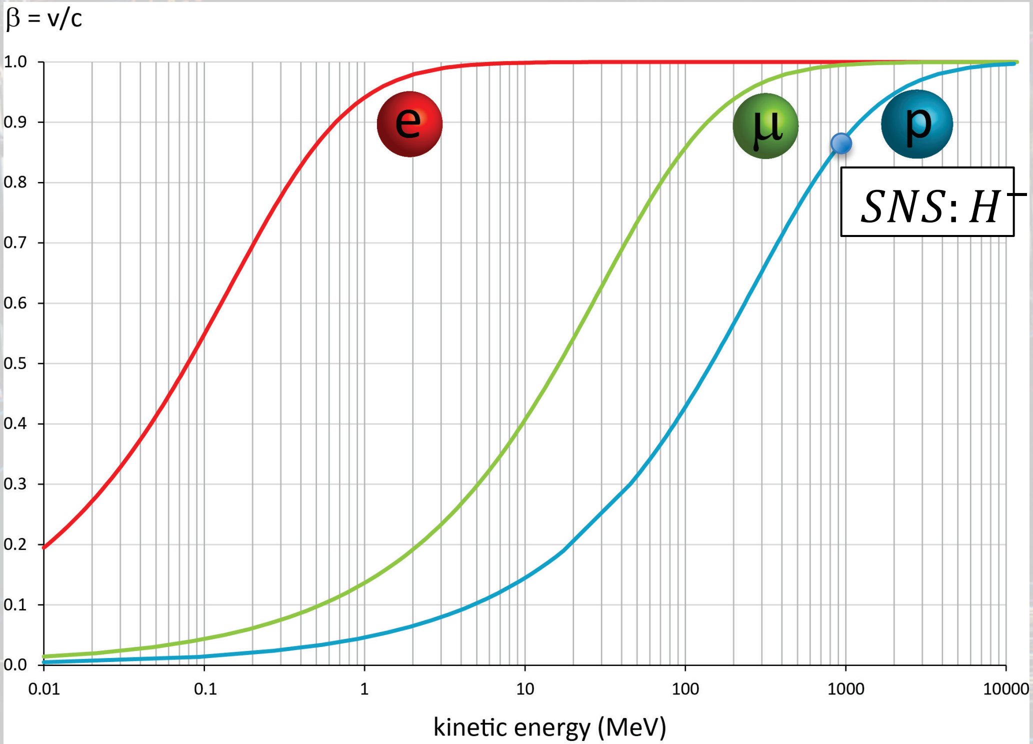
- Always: $Q_l < Q_0, Q_{ext}$
- When Q_0-value is large $Q_l \approx Q_{ext}$

Benefit of SRF Cavities and Drawback for HOMs

- What makes SRF cavities attractive compared to normal-conducting (NC) RF cavities is the million-fold lower surface resistance R_s
  - **nΩ (SRF) versus mΩ (NC) → power losses in cavity are few 10 Watts versus MW! for same operating field and duty cycle (SRF readily allows CW)**
  - **Q_0-values are $10^9<Q_0<10^{11}$ (SRF)** depending on temperature (and operating field) versus **$Q_0 \approx 10^4$-level (NC)**
- This yields a large shunt impedance (operating efficiency)
  - R = TΩ-level (SRF) vs. MΩ-level (NC)
- Unfortunately for the same reasons, HOMs will exhibit high shunt impedances in SRF cavities too
- Example: L-band cavity with HOM at f = 3 GHz, $Q_l = 10^5$ (damped already 1000-fold)
  - → HOM will oscillate for τ = 5.3 µs before energy is down to 1/exp(1)
  - → This still corresponds to a length ($c_0 \cdot \tau$) = 1.6 km, so many subsequent particles will experience the field the first bunch left behind

Mechanism of HOM Excitation

- So what is the mechanism that creates HOMs?
- For simplicity reasons assume $v/c_0 = \beta \approx 1$
- At given energy of particles, velocity depends on mass
  - Electrons travel with ultra-relativistic energies at rather low energies, e.g. β = 0.999 at only ~10.9 MeV kinetic energy

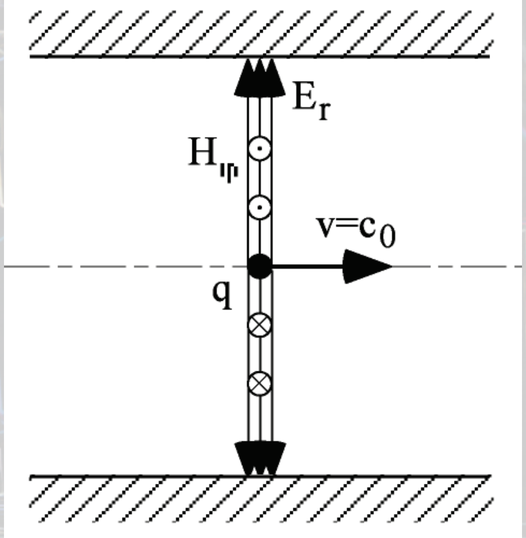


$$E_k = \gamma m_0 c^2 - m_0 c^2$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-(\beta)^2}}$$

Mechanism of HOM Excitation

- So what is the mechanism that creates HOMs?
- For simplicity reasons assume $v/c_0 = \beta \approx 1$
- At ultra-relativistic energies, the RF fields surrounding the particle in a beam tube are Lorentz-contracted
- It can be shown that particles within the bunch do not act on each other even when positioned in the same plane
  - Forces caused by E and H cancel out

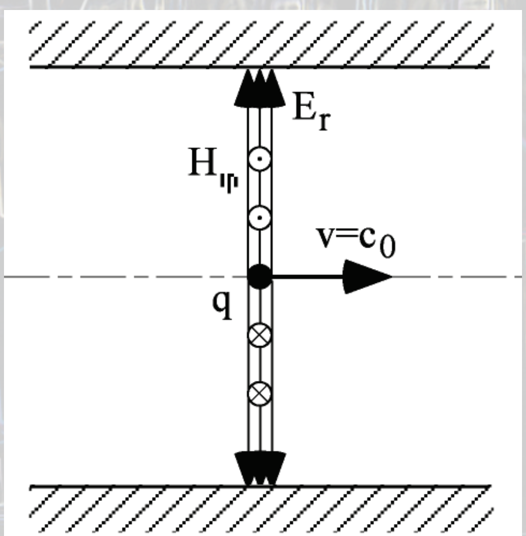


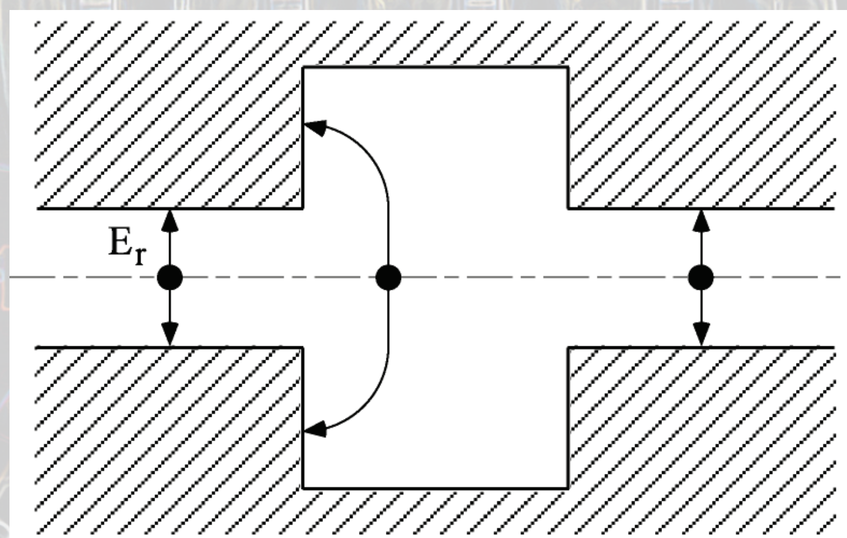
$$E_r = \frac{q}{2\pi\varepsilon_0}\frac{\delta(z-ct)}{r}, H_\varphi = \frac{qc}{2\pi}\frac{\delta(z-ct)}{r}, Z_0 = \frac{E_r}{H_\varphi}$$

$$F_r = q\left(E_r - Z_0 H_\varphi\right)$$
$$F_z = 0$$

Mechanism of HOM Excitation

- When particles enter a cavity the RF field expands as the cross-section changes
- Consider image charges at wall that cannot travel $> c_0$
- RF energy must be left behind the beam
- The energy left behind is called **wakefield**
- When the particles leave the cavity, the wakefields can be "stripped off", even without Ohmic wall losses taken into account





Mechanism of HOM Excitation

- Hence: The beam itself is the source of the mode excitation in a resonant cavity structure
- Particles carry kinetic energy and some portion of the kinetic energy is lost in each mode excitable
- We will determine the amount based on the figure of merits already discussed
- A resonant RF structure sustains only discrete Eigenmodes
- The wakefields (time domain) therefore carry all Eigenmodes that can be excited in the cavity (in frequency domain)
- For an HOM to be excitable, it must exhibit an RF electrical field along the beam path, which ideally is along the cavity axis
- So not all Eigenmodes are can be excitable, then a question is how many are?

Mode Spectrum

- The spectrum of the excitable Eigenmodes expands to a frequency determined by the bunch length (σ)
  - The long. bunch shape is typically approximated by a Gaussian curve
  - The Fourier Transform (FT) of a Gaussian function is a Gaussian function

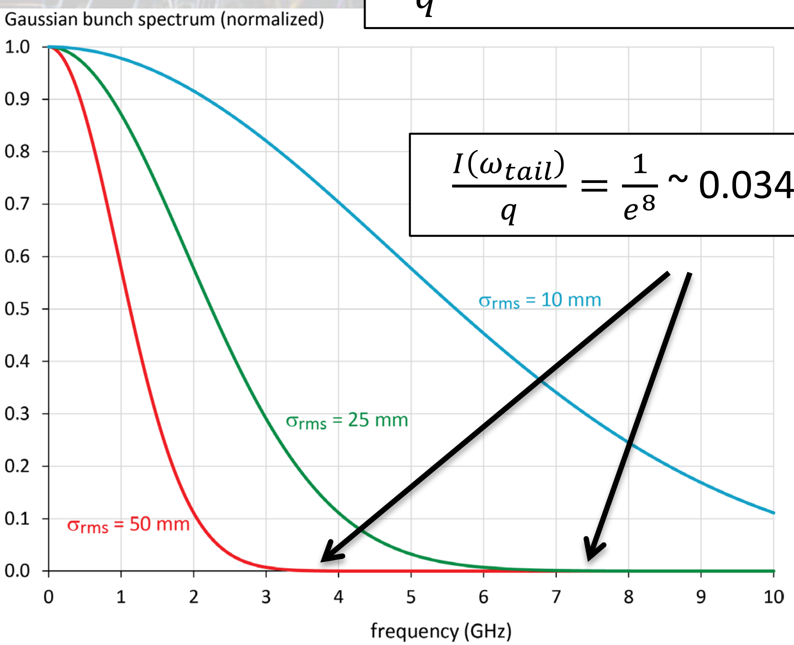
$$I(t) = \frac{c_0 \cdot q}{\sqrt{2\pi}\sigma_s} \cdot e^{-\frac{1}{2}\left(\frac{c_0 \cdot t}{\sigma_s}\right)^2} \overset{FT}{\Rightarrow} I(\omega) = q \cdot e^{-\frac{1}{2}\left(\frac{\sigma_s \cdot \omega}{c_0}\right)^2}$$

this is the current/bunch spectrum

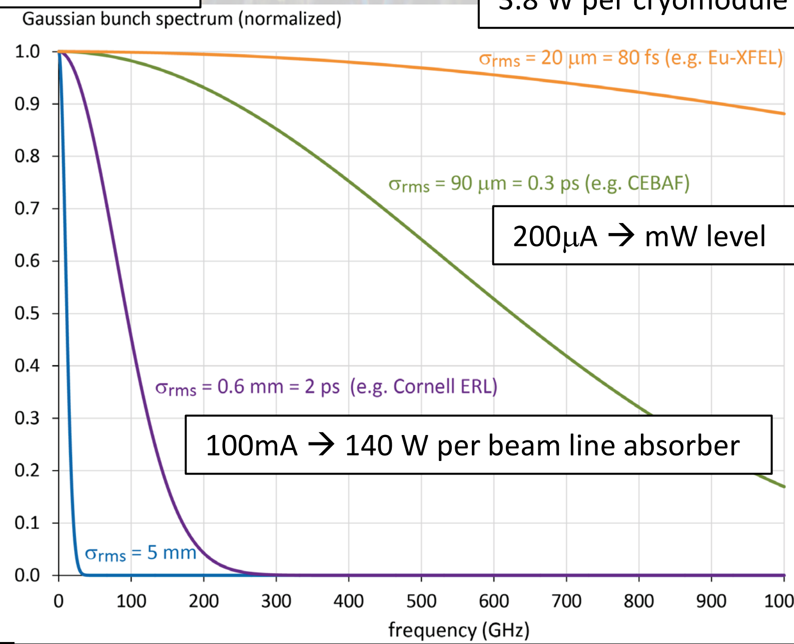
$$\frac{I(\omega)}{q} = e^{-\frac{1}{2}\left(\frac{\omega}{c} \cdot \sigma_{rms}\right)^2}$$

$$\omega_{tail} \sim \frac{4 \cdot c}{\sigma_{rms}}$$

3.8 W per cryomodule



$$\frac{I(\omega_{tail})}{q} = \frac{1}{e^8} \sim 0.034\%$$

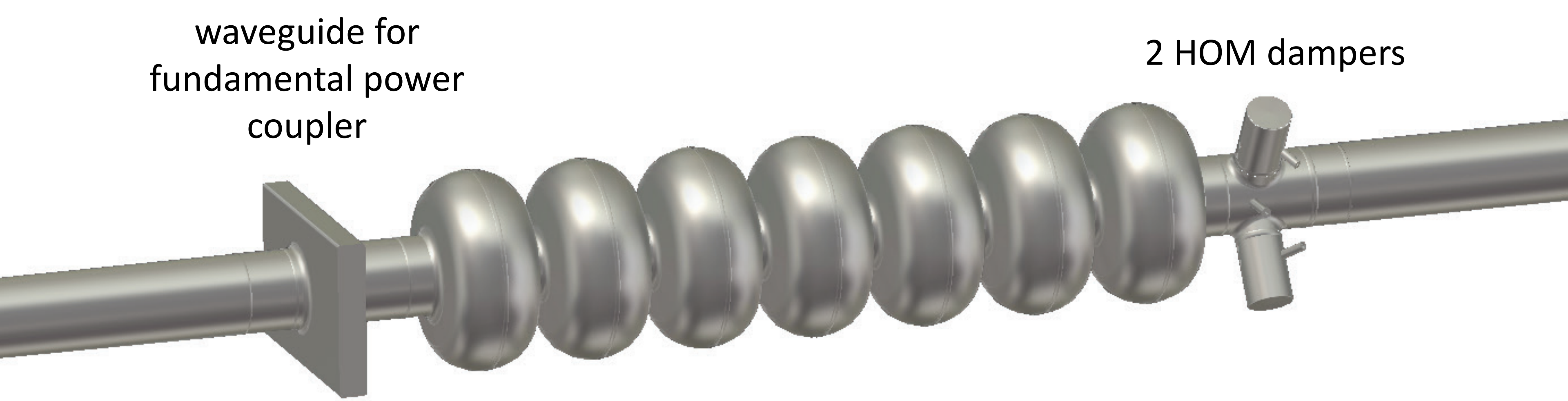


200μA → mW level

100mA → 140 W per beam line absorber

Wakefield

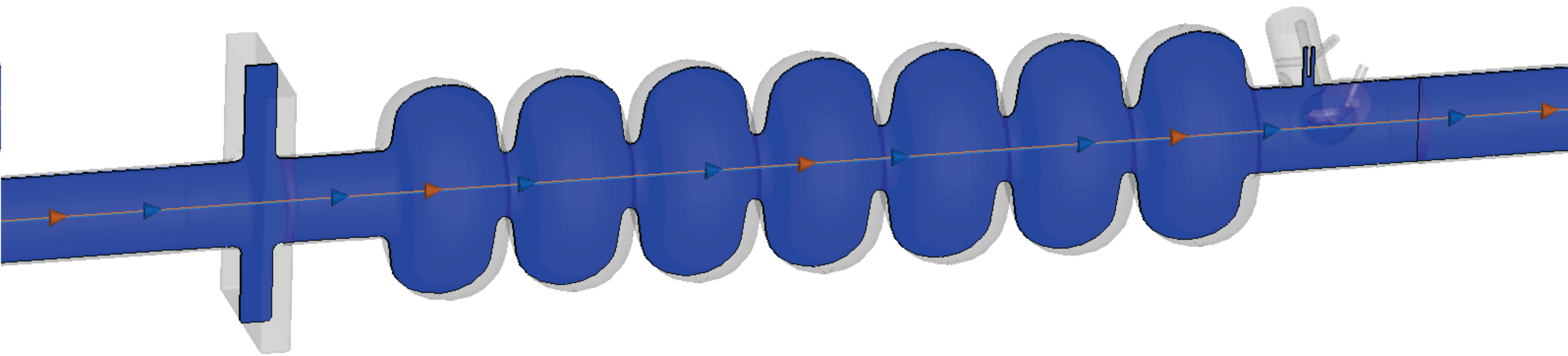
- Example: Wakefield in a JLab 1.5 GHz Low Loss (LL) seven-cell cavity (CEBAF ‘C100’ upgrade cavity)



Wakefields (on-axis excitation)

- Case 1: Gaussian bunch: σ_{rms} = 2”, ω_{tail}=3.76 GHz
- The wakefield (in time domain) consists of a superposition of all Eigenmodes that can be excited in a resonant cavity structure (in frequency domain)

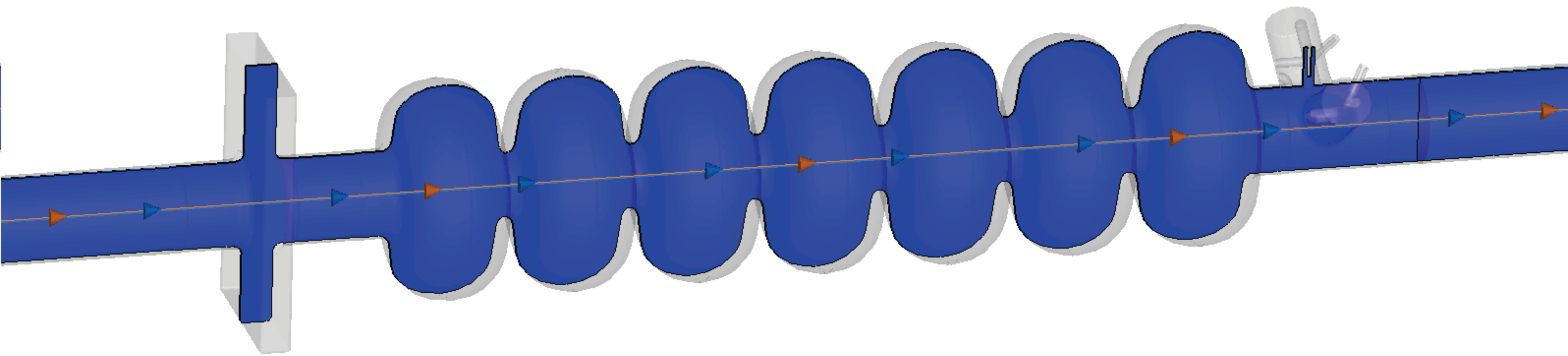
movie



Wakefields (on-axis excitation)

- Case 2: shorter bunch, σ_{rms} = ½ ", ω_{tail}=7.51 GHz
- The modes represent an orthogonal set of Eigenmodes (Superposition principle)
- Wakefields are therefore composed only of the sustained Eigenmodes that may resonate

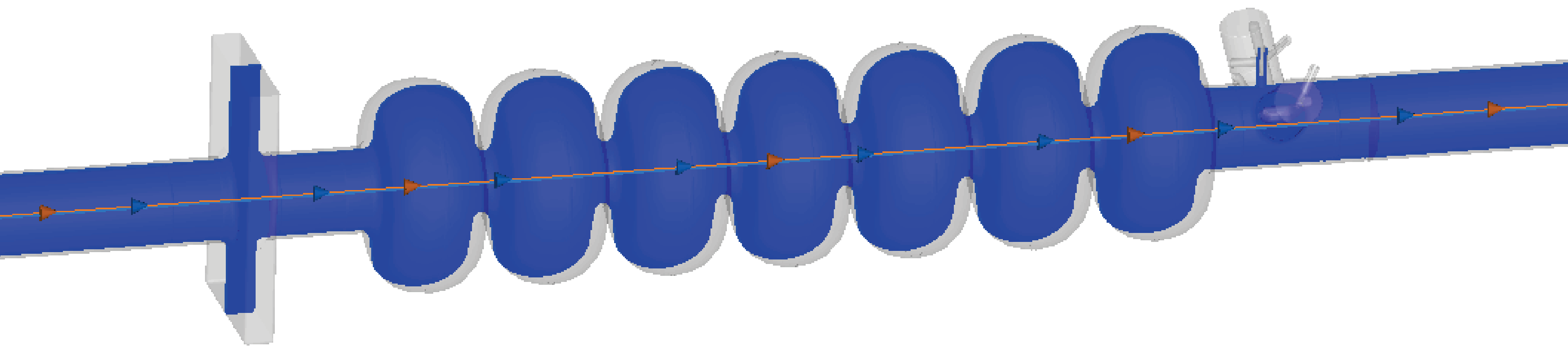
movie



Wakefields (on-axis excitation)

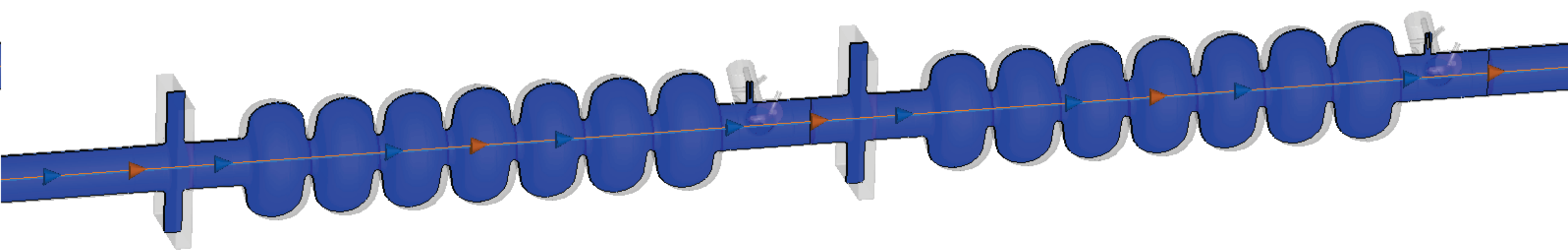
- Case 3: σ_{rms} = 12.5 mm, ω_{tail}=15.3 GHz

movie



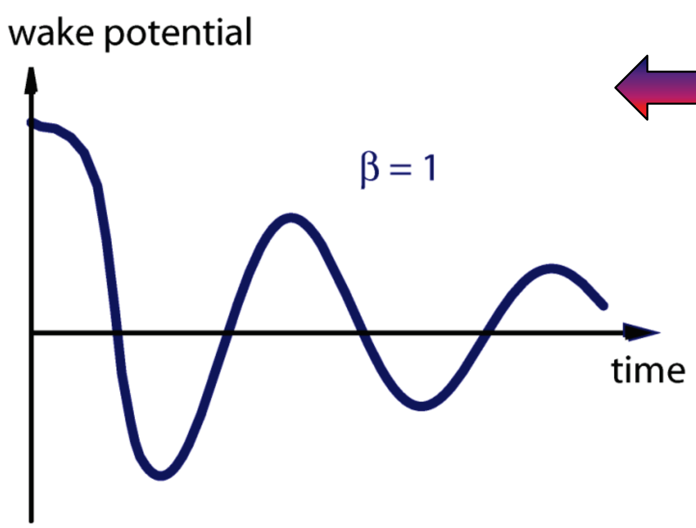
Wakefields

- Case 4: 2 cavities, bunch σ_{rms} = 25 mm, ω_{tail}=7.51 GHz



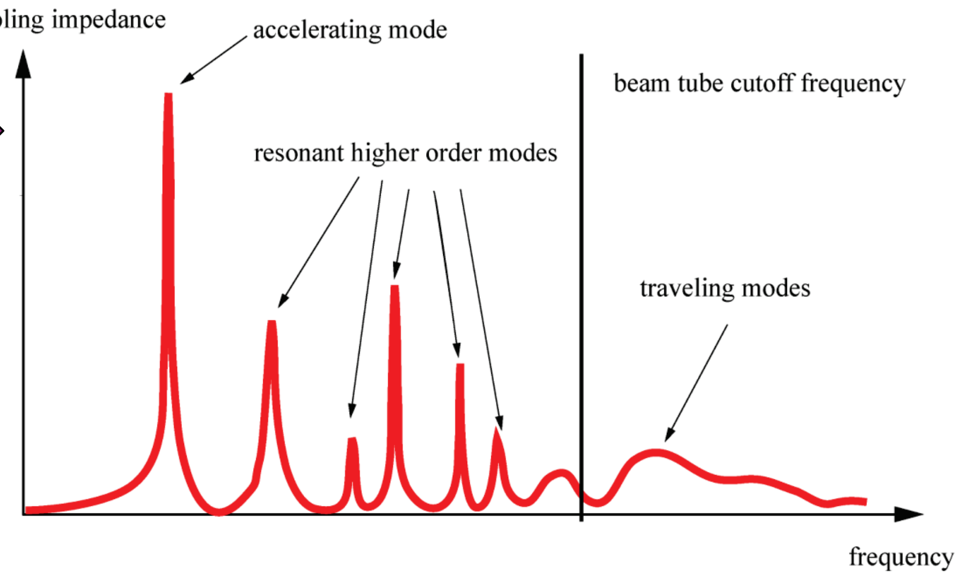
Wakefield and Impedance

- Wakefield and impedance are interchangeable by applying a Fourier Transform (FT)
- Computing the wakefield (wake potential) in time domain by an exciting line charge and applying the Fourier Transform is a very elegant way to calculate the full impedance spectrum at once (and compare results with shunt impedances calculated with Eigenmode solver)



wake potential
(time domain)

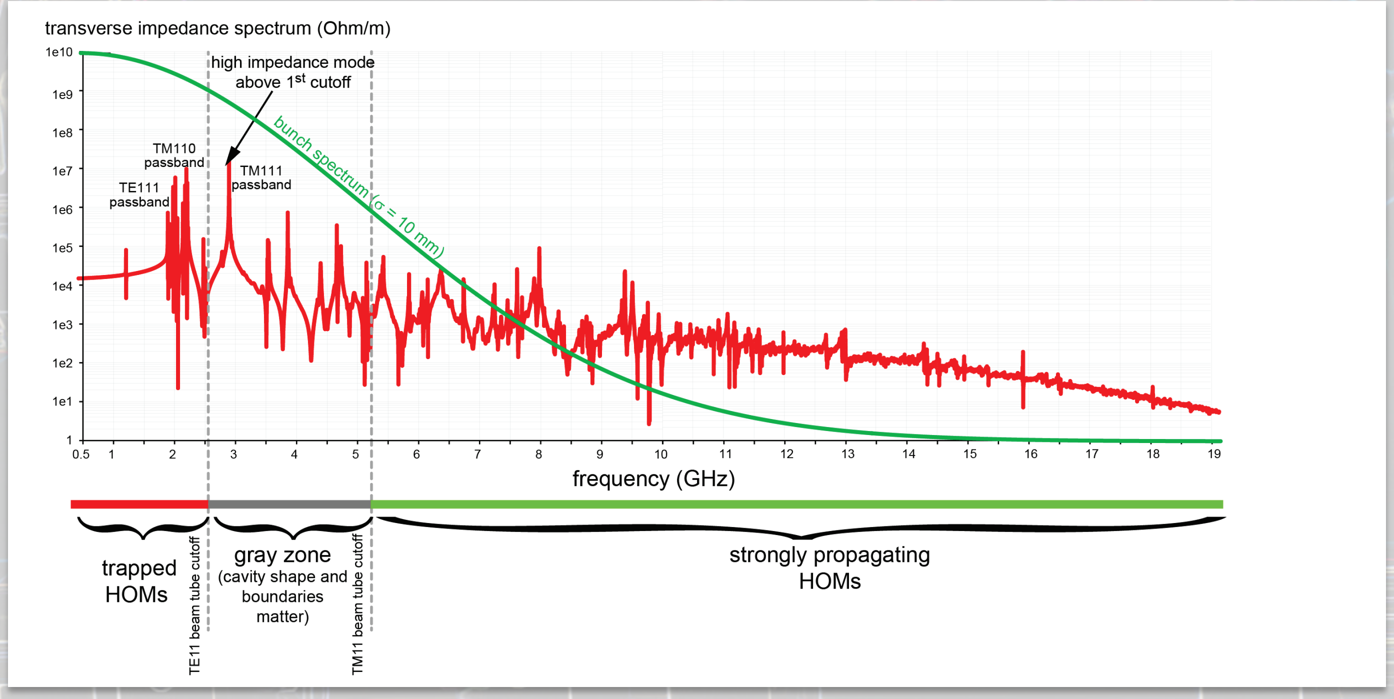
FT



cavity broadband coupling impedance
(frequency domain)

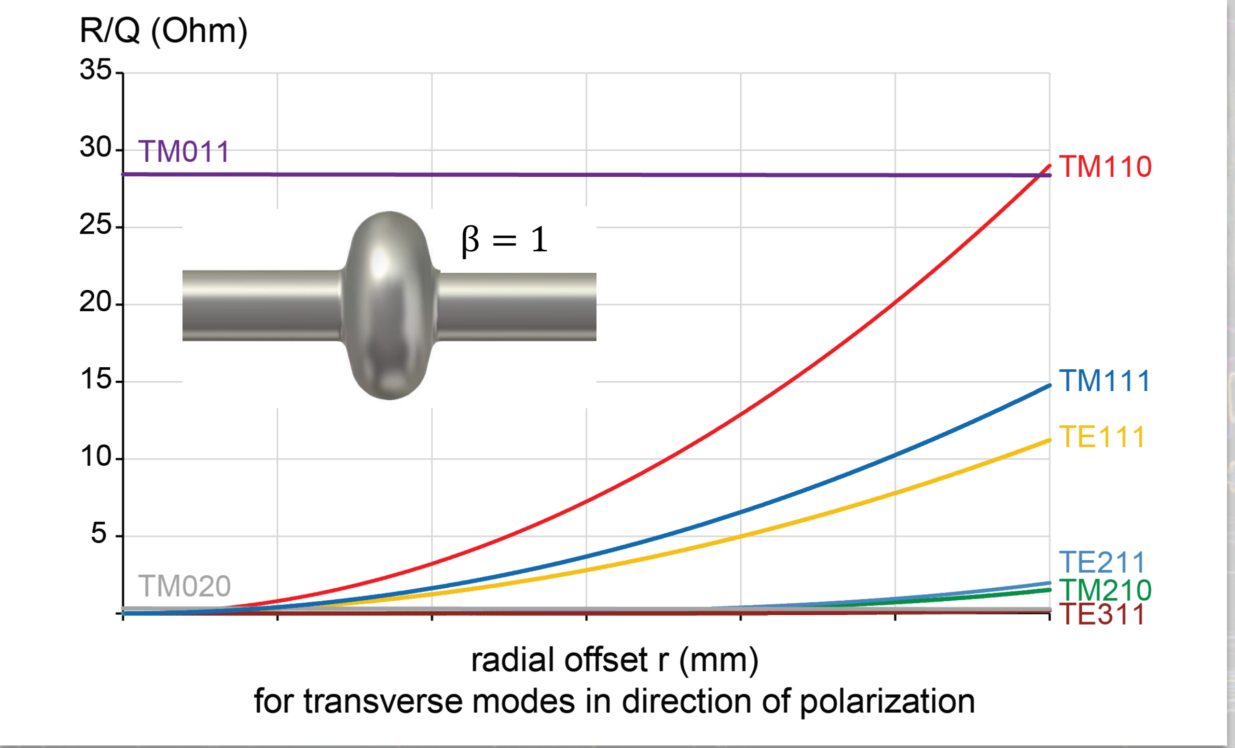
Which HOMs are dangerous?

- For many existing and planned machines the bunch spectrum covers several 100 GHz
  - 1000s of HOMs are excitable!
- Are all these HOMs problematic for beam dynamics? Short answer is: No
  - But one has to account for the power dissipated in cryogenic environment covering the full spectrum
- Usually the HOMs at the lower frequency end posses the largest R/Q-values (critical)
- Above 1st beam tube cutoff HOMs may travel out of beam tube, allows better damping (Q_l is reduced)
- However, there is “gray zone” with confined modes, i.e. boundaries between cavities may matter (reflections), (end) cell shapes too



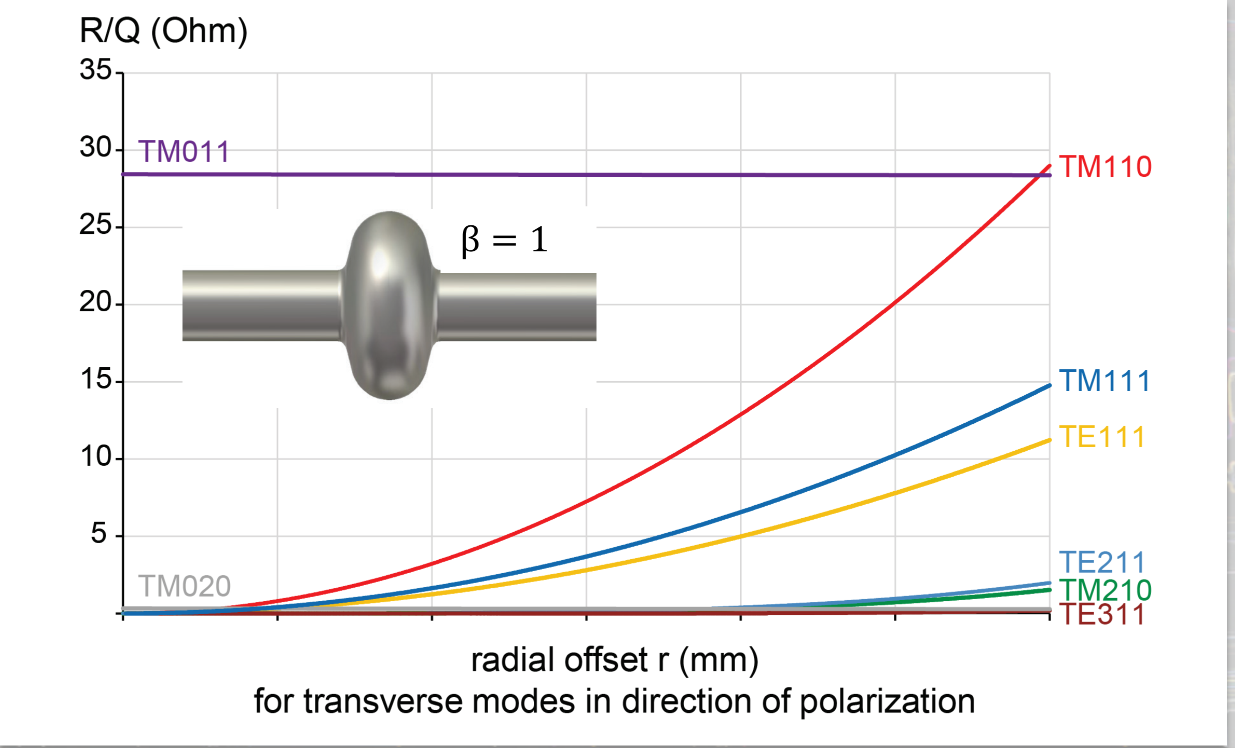
Which HOMs are dangerous?

- So which type of HOMs are the most troublesome?
- Problematic are typically all HOMs trapped inside the SRF cavity
  - i.e. all modes below the 1st (relevant, i.e. field depending) cut-off frequency of beam tubes
  - These (depending on design) are usually the TE_{111} and TM_{110} (dipole modes), the TM_{011} (monopole modes), while HOMs at higher frequencies (e.g. TM_{111}, TE_{121} , TM_{120}) can become dangerous even above the cutoff frequency



Which HOMs are dangerous?

- R/Q-value principally does not change for Monopole modes with radial offset to cavity/beam axis
- R/Q-values scale with r^2 for Dipole modes, with r^4 for Quadrupole modes asf.
  - This is the reason why Quadrupole and higher transverse HOMs are not considered dangerous with regard to beam deflection (may be left undamped)
  - The beam cannot effectively excite these modes (energy stored far off center)



Panofsky-Wenzel Theorem

- Remember parallel resonant circuit with $$P_{loss} = \frac{V_{eff}^2}{R}$$
- In turn this is the accelerator definition for the shunt impedance $$\Longrightarrow R = \frac{V_{eff}^2}{P_{loss}}$$
- For transverse modes (with ideally no fields on axis) a different definition is used due to the scaling of fields with radial offset (r)
- Generally $$R_z = \frac{V_{eff}^2}{P_{loss}} = TTF^2 \frac{\left(\int_0^{L_{cav}} dz \cdot E_z e^{-i\frac{\omega}{c}z}\right)^2}{P_{loss}}$$ $$R_\perp = \frac{\left(\frac{c\Delta p_\perp}{q}\right)^2}{P_{loss}}$$
    - The Transit Time Factor (TTF) takes into account the effective field the particle experience when traveling through the time-varying electrical field at given β
- The transverse momentum change depends on the change of the longitudinal field in radial direction $$\frac{\partial}{\partial t}\Delta\vec{p}_\perp = j\omega\Delta\vec{p}_\perp = -\vec{\nabla}_\perp(\Delta W_z)$$ $$\vec{\nabla}_\perp = \frac{\partial}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial\varphi}\vec{e}_r$$
- This is the **Panofsky-Wenzel Theorem, which states that a particle experiences a transverse momentum ($\Delta p_\perp$) only when the longitudinal change in energy (ΔW_z) depends on the radial position**

Panofsky-Wenzel Theorem

- The transverse momentum change ($\Delta p_\perp$) is 0 for TM-Monopole modes
- For TM-Transverse modes however

$$|\Delta\vec{p}_\perp|^{TM}_{mnp} \sim \frac{1}{\omega}\frac{1}{(m-1)!}\left(\frac{x_{mn}}{2r}\right)^m \cdot r^{m-1}$$

  - e.g. for Dipole modes (m=1) this term is constant
  - for Quadrupole modes (m-2) it scales linearly with distance
- For Dipole modes this allows to create a more practicable definition for the transverse shunt impedance, one can rewrite

$$R_\perp = \frac{\left(\frac{c\Delta p_\perp}{q}\right)^2}{P_{loss}} = \frac{\left(\frac{c}{\omega}\frac{\partial}{\partial r}\left(\Delta W_z = \frac{\Delta V_z}{q}\right)\right)^2}{P_{loss}}$$

$$\frac{\partial}{\partial t}\Delta\vec{p}_\perp = j\omega\Delta\vec{p}_\perp = -\vec{\nabla}_\perp(\Delta W_z)$$

- If one considers that the derivative of (ΔW_z) must yield a **$\Delta p_\perp$** =constant, then $\Delta W_z \sim r$
- In first approximation ($\delta/\delta r = 1/r$)

$$R_\perp = \frac{1}{kr}\cdot\frac{TTF^2\left(\int_0^{L_{cav}} dz\cdot E_z e^{-i\frac{\omega}{c}z}\right)^2}{P_{loss}}\ in\ \Omega$$

Thus: $R_\perp$ equals R_z at r=ω/c=1/k

$$R_\perp = R_z(r = \frac{1}{k})$$

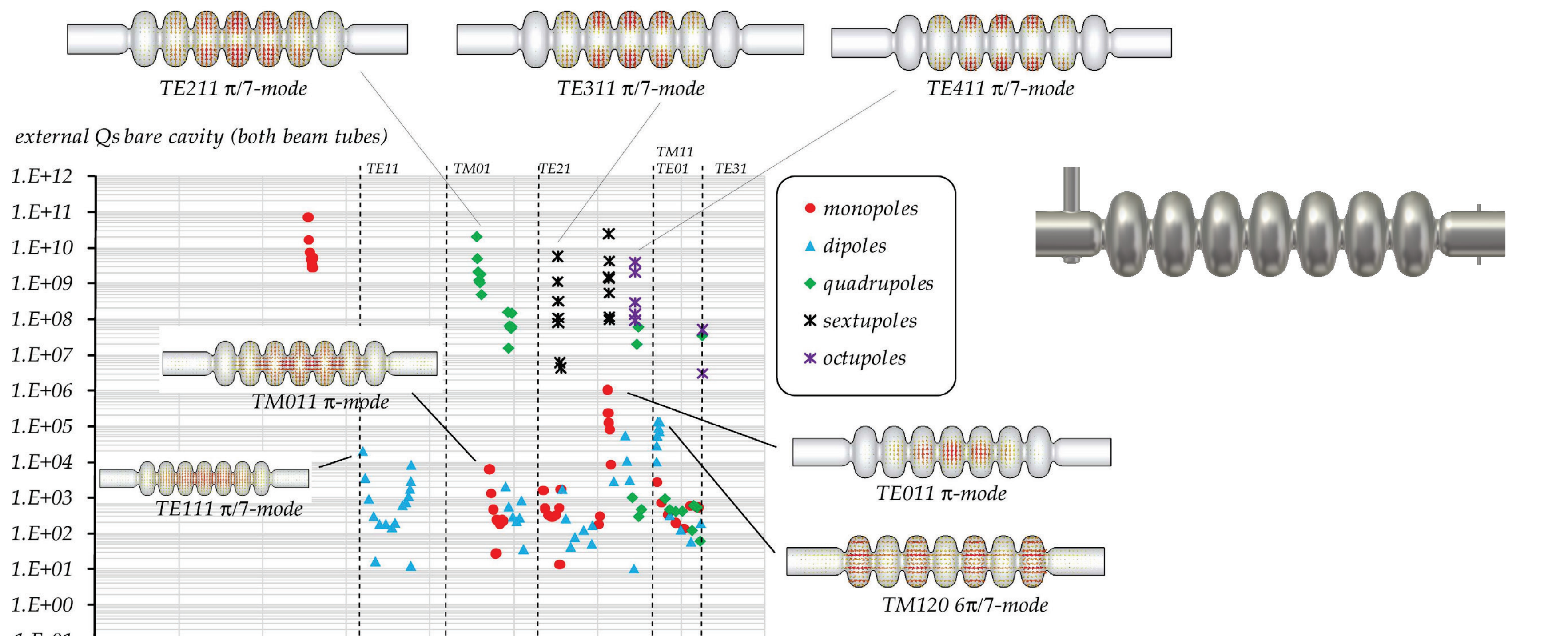
- Note: The following normalization is often employed, which considers that $R_\perp \sim r^2$

$$R_\perp = \frac{R_z}{k\cdot r^2}\quad in\ \Omega/m$$

this definition now provides a constant value for a given dipole mode (more convenient to use in BBU studies)

Large Beam Tubes?

- Though there is a benefit of large beam tubes to allow even TE111 HOMs propagate, there are other implications that not always allow to employ those
- Also: Some modes might still be very confined inside cavity (high Q_s)
- Example: Cornell ERL cavity (see TM_{120}, "gray zone")



Cornel 1.3 GHz ERL main linac cavity

Figure of Merit

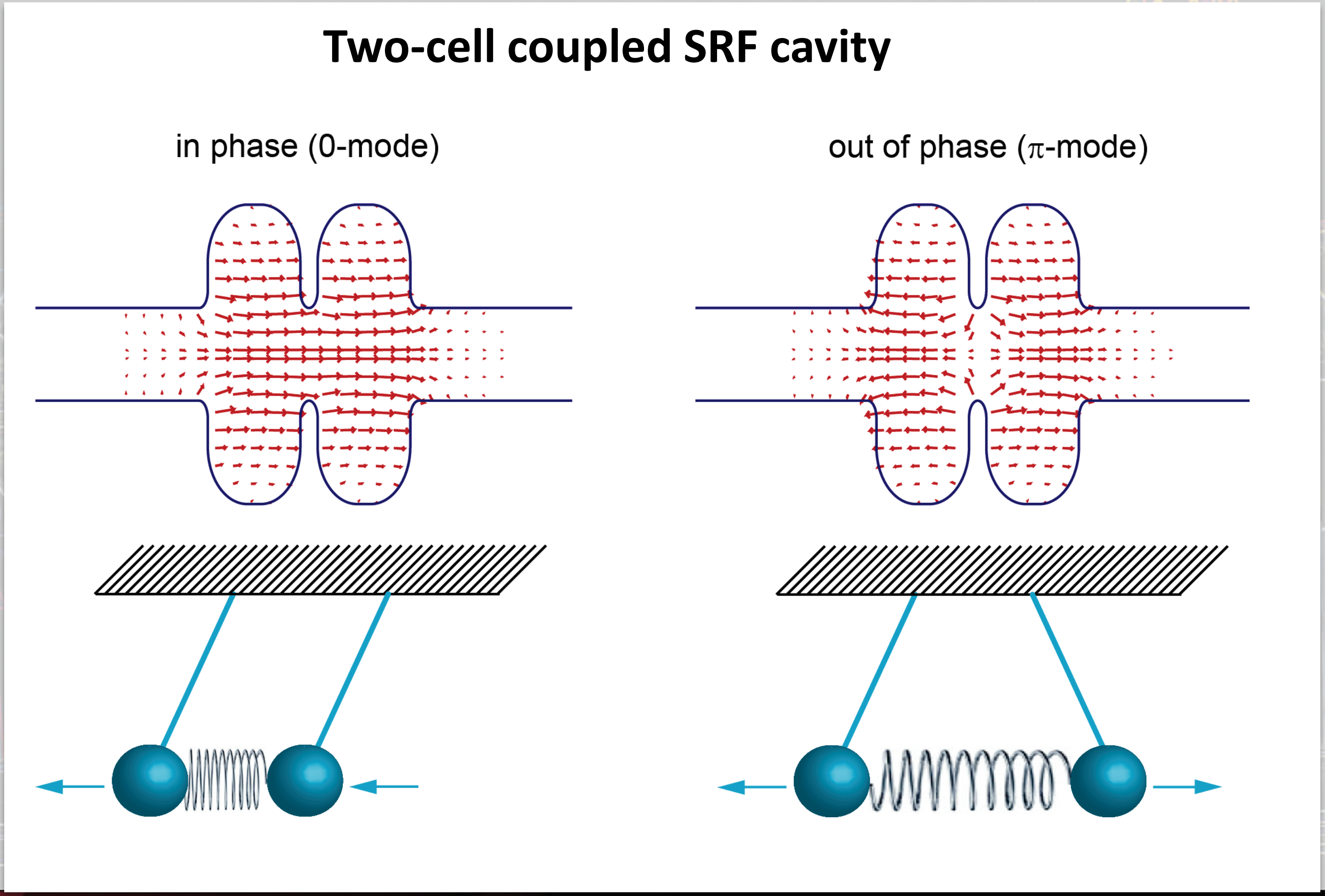
- Note that the external or loaded/external Q is not the only or main parameter to verify whether HOMs are critical for beam dynamic in a given machine
- The figure of merit is the shunt impedance

$$R_{z,\perp} = \left(\frac{R}{Q}\right)_{z,\perp} Q_l$$

- The R/Q is fix once cavity is designed, but yet the Transit Time Factor is important and it can be minimized to some extent by design for multi-cell cavities
- For this we need to understand the dispersion curve for coupled resonators

Coupled Oscillator

- Multi-cell cavities represent coupled oscillators
- Each single cell mode splits into N modes (N = number of cells)



Dispersion Relation

- Modes for each mode type (TE_{mnp}, TM_{mnp}) differ by phase advance (φ) per cavity cell period (half cell length)
- Lumped circuit analysis is complex, but yields a solution for identical cell-to-cell coupling factor (K) by Eigenwert solution (matrix formalism)
- The solution yields the **dispersion relation**
- **The dispersion relation accounts for the dependency of the Eigenfrequency with the phase advance**
- Various boundary condition at the end of the cavity can be accounted for, i.e. cavity can be open (magnetic boundary) or closed (electric boundary)
- Mixed boundaries are also possible

$$\omega_j = \frac{\omega_0}{\sqrt{1 \pm K \cos\varphi_j}} \quad \text{mit}$$

identical boundaries

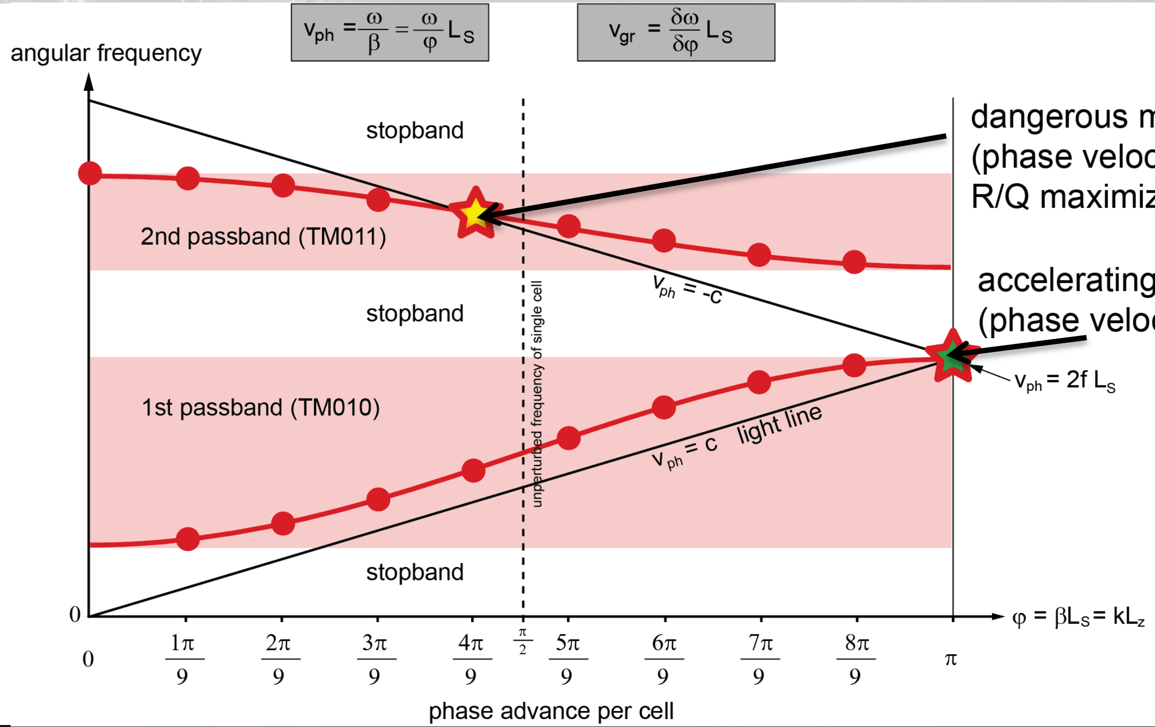
$$\varphi_j = j\frac{\pi}{N} \quad \text{mit} \quad j = 0, 1, \ldots, N_{ges}$$

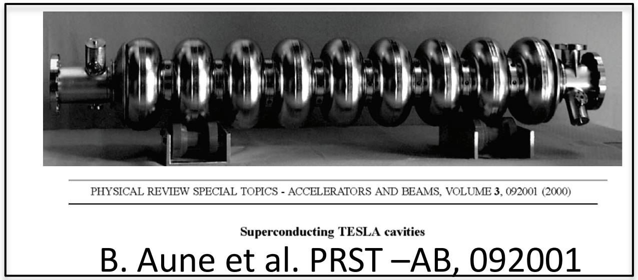
mixed boundaries

$$\varphi_j = \left(j - \frac{1}{2}\right)\frac{\pi}{N} \quad \text{mit} \quad j = 1, \ldots, N_{ges}.$$

Brillouin Diagram

- Dispersion relation can be best illustrated in Brillouin Diagram (frequency over phase advance)
- Example: Nine-cell cavity (red dots, e.g. TESLA cavity), curve is for infinite # of cells
- For operation mode one desires phase velocity = particle velocity
- For HOMs the intersection of the light line with HOM maximizes the R/Q-value, e.g. HOM has phase velocity that equals particle velocity (β=1 light line), TTF maximal





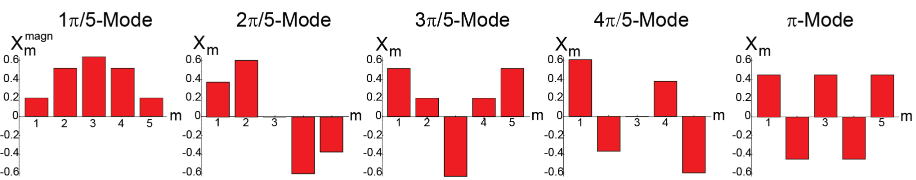
B. Aune et al. PRST –AB, 092001

Which HOMs are dangerous?

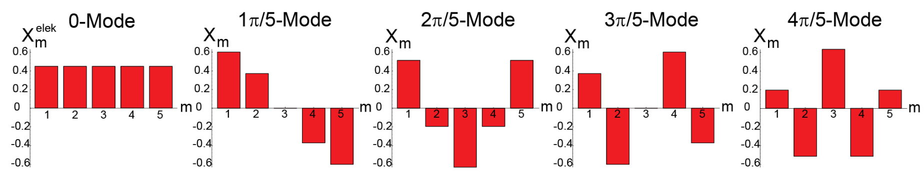
- Analytical formalism also allows to calculate relative amplitudes in cells

$$ {}^{\text{elek}}X_m^{\text{elek}} = \varepsilon_j \cos\left(\left(m-\frac{1}{2}\right)\varphi_j\right) \quad \begin{cases} \varepsilon_j = \dfrac{1}{\sqrt{N_{ges}}} & \text{für } j=0 \\ \varepsilon_j = \sqrt{\dfrac{2}{N_{ges}}} & \text{für } j \neq 0 \end{cases} ;\ \varphi_j = j\frac{\pi}{N_{ges}};\quad j = 0,1,\ldots,N_{ges}-1 $$

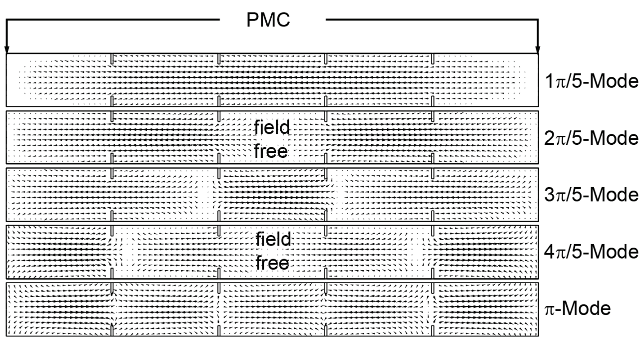
$$ {}^{\text{magn}}X_m^{\text{magn}} = -\varepsilon_j \sin\left(\left(m-\frac{1}{2}\right)\varphi_j\right) \quad \begin{cases} \varepsilon_j = \sqrt{\dfrac{2}{N_{ges}}} & \text{für } j \neq N_{ges} \\ \varepsilon_j = \dfrac{1}{\sqrt{N_{ges}}} & \text{für } j = N_{ges} \end{cases} ;\varphi_j = j\frac{\pi}{N_{ges}};\quad j=1,\ldots,N_{ges}. $$



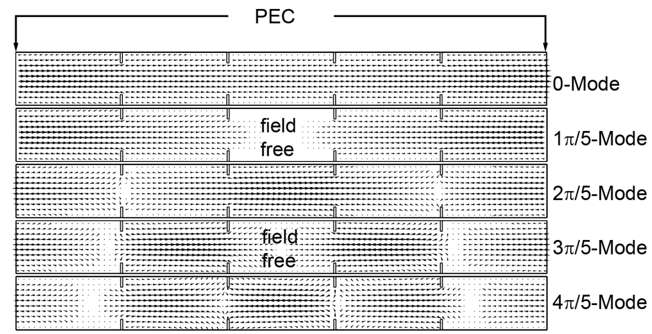
analytical solutions



analytical solutions



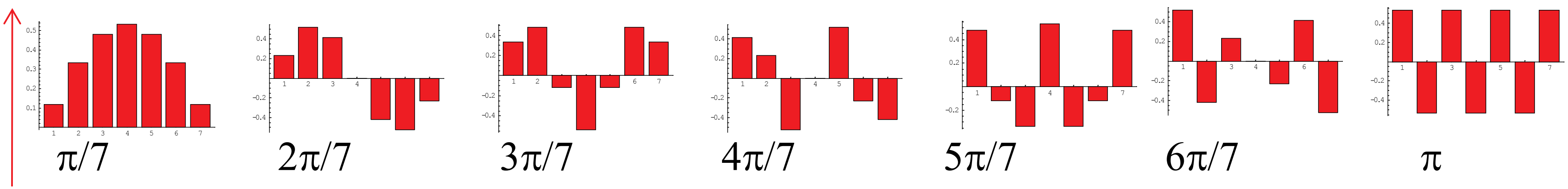
numerical solutions
TM_{010}-passband



numerical solutions
TM_{010}-passband

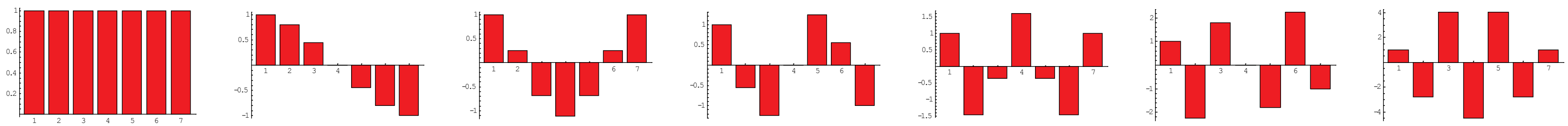
Field Amplitudes in a 7-cell Cavity

amplitude (a.u.) magnetic boundaries (left and right) → no 0-mode



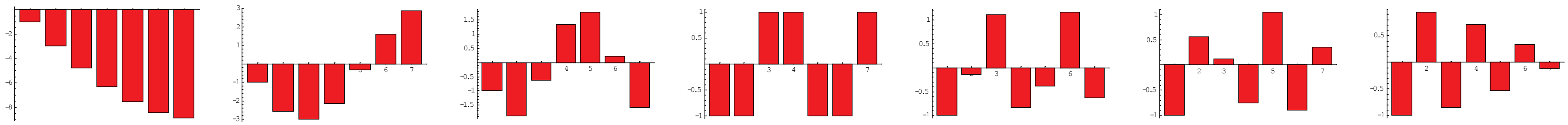
$\pi/7$ $2\pi/7$ $3\pi/7$ $4\pi/7$ $5\pi/7$ $6\pi/7$ π

electric boundaries (left and right) → no π-mode



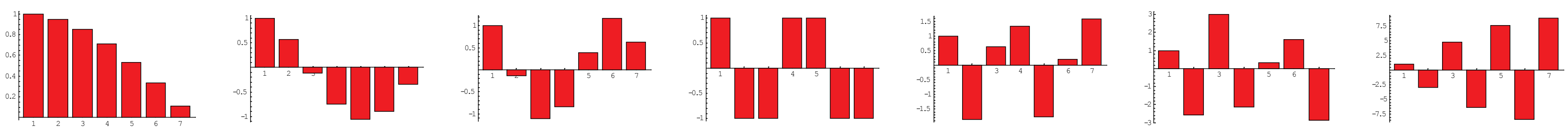
0 $1\pi/7$ $2\pi/7$ $3\pi/7$ $4\pi/7$ $5\pi/7$ $6\pi/7$

magnetic boundary (left) & electric boundary (right)



$0.5\pi/7$ $1.5\pi/7$ $2.5\pi/7$ $3.5\pi/7$ $4.5\pi/7$ $5.5\pi/7$ $6.5\pi/7$

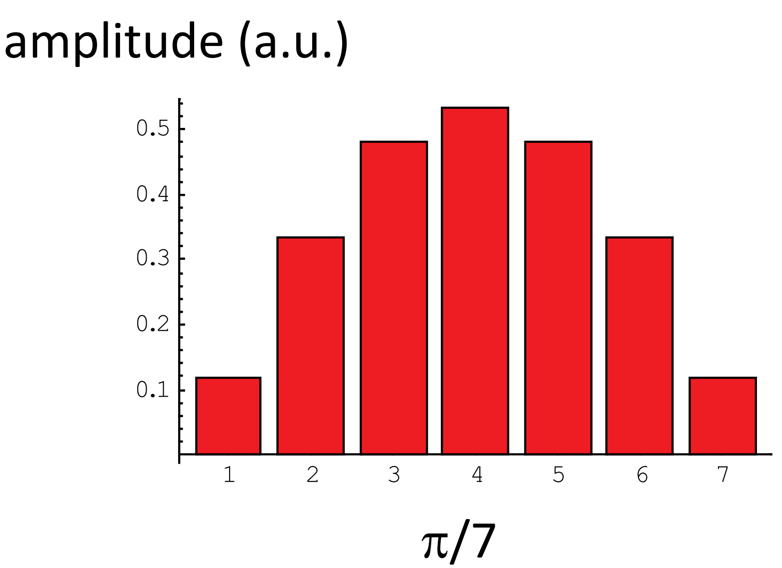
electric boundary (left) & magnetic boundary (right)

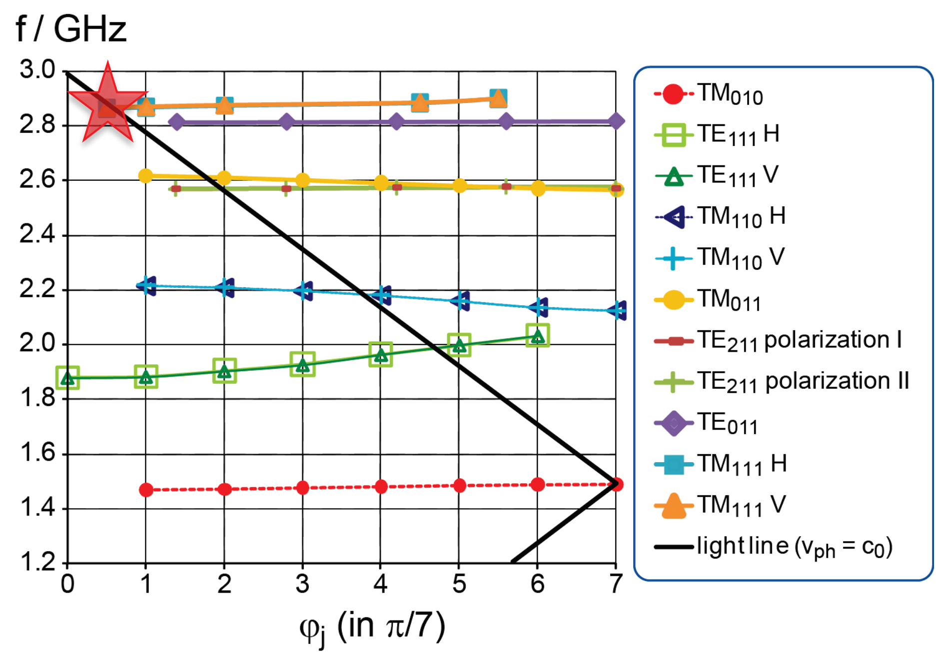


$0.5\pi/7$ $1.5\pi/7$ $2.5\pi/7$ $3.5\pi/7$ $4.5\pi/7$ $5.5\pi/7$ $6.5\pi/7$

Field Amplitudes in a 7-cell Cavity

- the 1π/N-modes looks similar in each multi-cell cavity
- It is by nature confined in cavity and it has a low group velocity (like π-mode), i.e. harder to coupler to
- If light-line falls together with the mode frequency, these types of HOMs become very dangerous





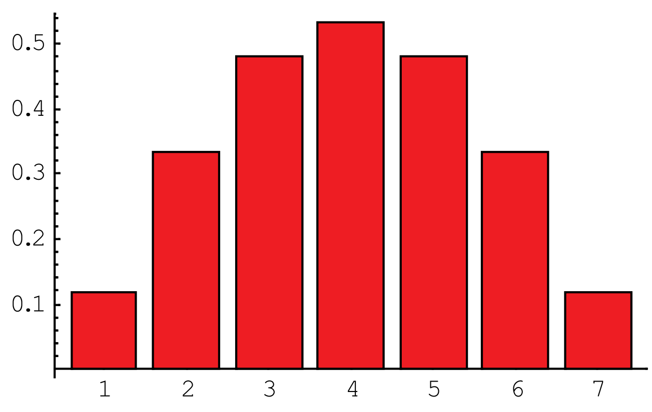
Field Amplitudes in a 7-cell Cavity

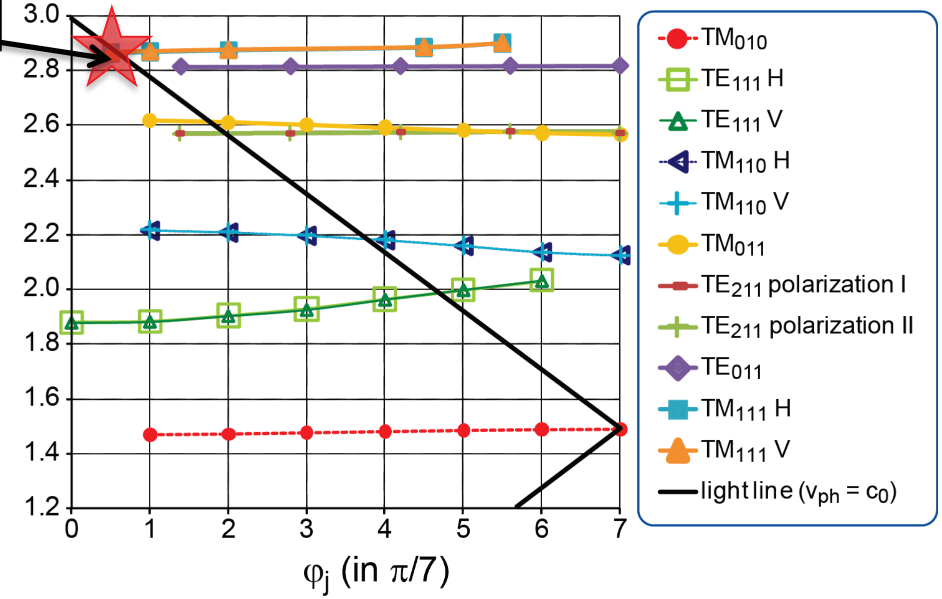
cavity #1
(slightly detuned)

cavity #2
(ideal shape)

cavity #3
(slightly detuned)

real problematic TM_{111}-mode
in CEBAF upgrade cavities





HOM Damping Technologies

How does one technically damp HOMs in SRF Cavities?

HOM Damping Technologies

- Principally 4 (+1) different damping methods are employed for SRF cavities:

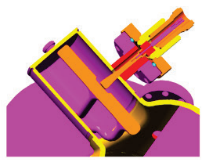
  1) **Coaxial dampers on beam tubes**
  2) **Waveguide dampers on beam tubes**
  3) **Absorbers in cavity-interconnecting beam tubes (or cryomodule-connecting beam tubes)**
  4) **Coupling through Fundamental Power Coupler (FPC)**
  5) **On-cell dampers (for special cavities only)**

- Or a combination of either of the techniques

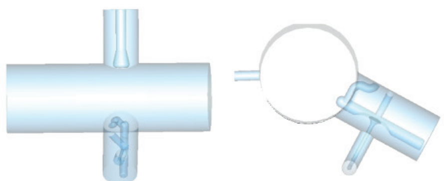
HOM Damping Technologies

Coaxial Couplers | Waveguide Couplers | Beam Line Absorbers

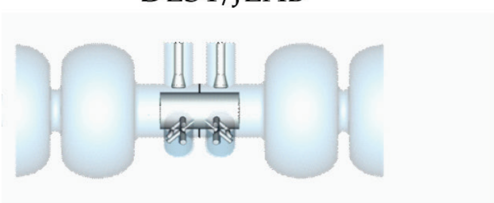
JLAB



TESLA/ILC

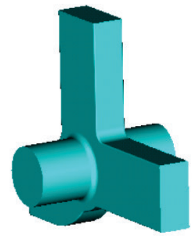


DESY/JLAB

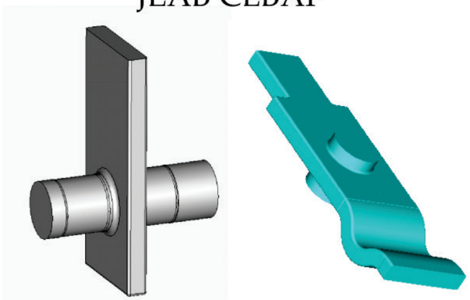


JLAB High Current

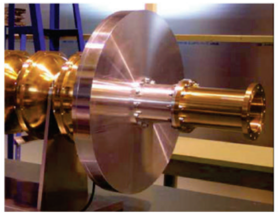
JLAB CEBAF



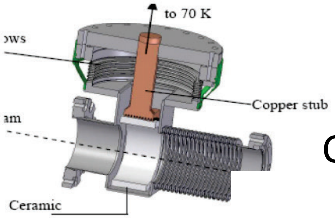
JLAB CEBAF



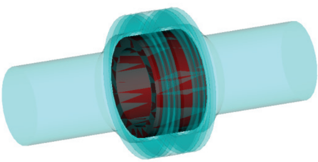
KEK



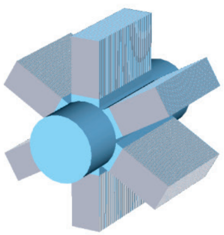
DESY FLASH/X-FEL

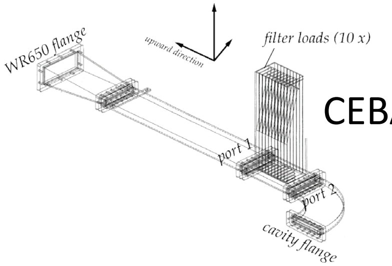


Cornell ERL



DESY





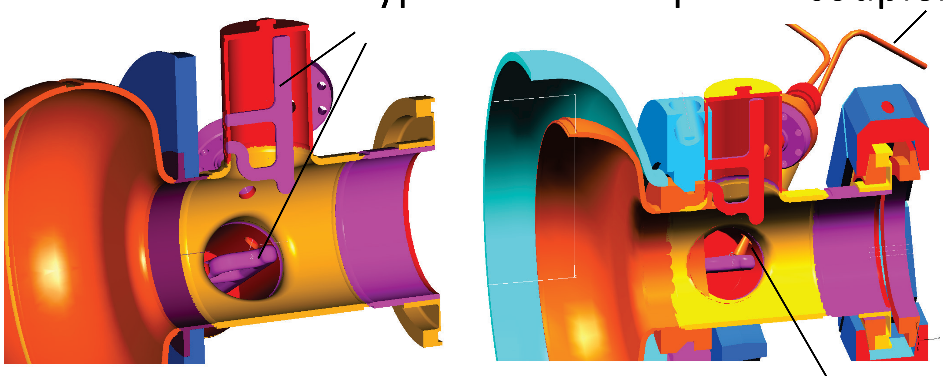
CEBAF filter

F. Marhauser, International Workshop on HOM Damping in SRF Cavities, Cornell University, Ithaca, 11-13. October 2010

HOM Damping Technologies

1) Coaxial dampers on beam tubes

- These HOM dampers generally extract HOM power via a hook coupler located in a can adjacent to the end cells
- The power is picked up by a TEM coaxial transmission line cable (separated by a RF vacuum feedthrough from the cavity vacuum)
- The power is transmitted to a standard (typically 50Ω load) outside the cryomodule (minimizes heat in cryogenic environment)
- The hook coupler features an antenna tip to couple capacitively to electrical fields and a loop structure to inductively couple to magnetic fields inductively



CEBAF upgrade cavity HOM coupler endgroup

HOM Damping Technologies

1) Coaxial dampers on beam tubes

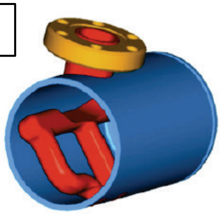
- Pros:
  - No vacuum-internal absorbers are used (clean environment)
- Cons:
  - The power capacity of the coaxial coupler depends critically on cooling layout (originally conceived for pulsed mode, but adapted for CW operation at CEBAF)
  - The coupling is not broadband, but can be optimized for crucial HOM passbands
  - A notch tuning filter is required to reject the power of the fundamental mode
  - Special tooling required for tuning the notch filter at room temperature
  - The notch frequency shifts upon cool-down (to be taken into account)
    - Periodically too much power is extracted from the fundamental mode as experienced during vertical tests
  - Complicated/costly design
  - RF vacuum window feedthrough also limits the frequency transmission
  - Vacuum leaks on feedthrough reported periodically (e.g. brazing of inner pin delicate, comes off sometimes)
  - Multipacting in the HOM cans is a frequently reported issues (e.g. SNS)
  - The antenna tip penetrates into the beam tube and creates field asymmetries
- Despite the potential issues, many projects rely on the coaxial couplers

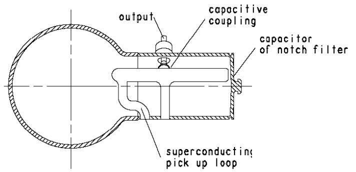
HOM Damping Technologies

- Examples:

1) Coaxial dampers on beam tubes

1.3 GHz TESLA/ILC/LCLS-II-type cavity



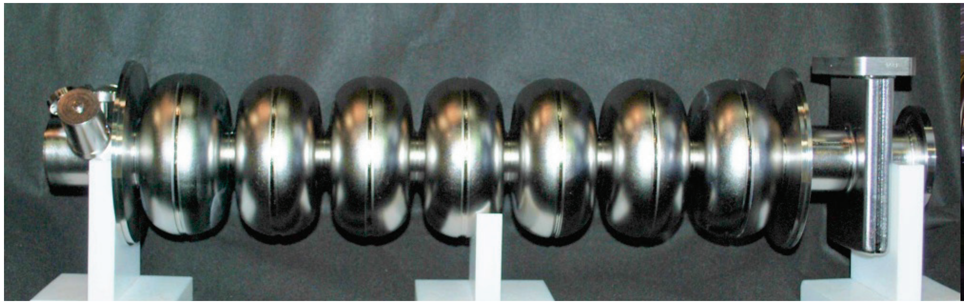


H.Edwards, D.Proch, K.Saito, ILC snowmass 05, Wg5

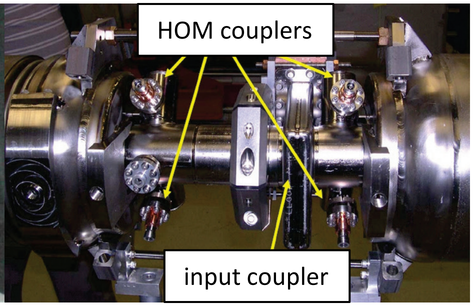
Proceedings of IPAC'10, Kyoto, Japan THXRA02

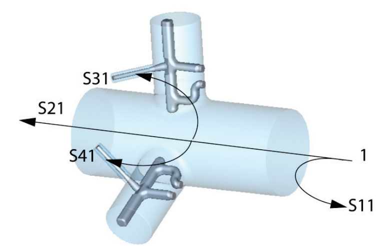
REVIEW OF SRF CAVITIES FOR ILC, XFEL, AND ERL APPLICATIONS

Hitoshi Hayano
High Energy Accelerator Research Organization (KEK), Tsukuba, Japan



1.5 GHz CEBAF upgrade cavity

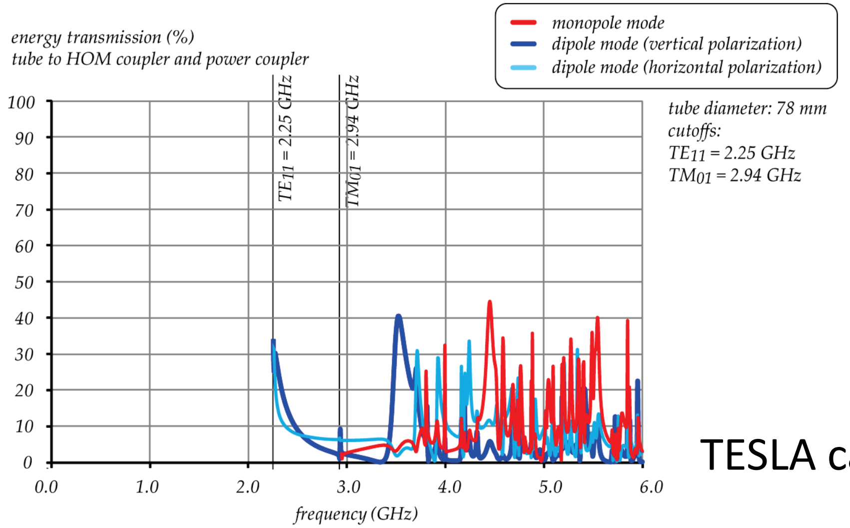




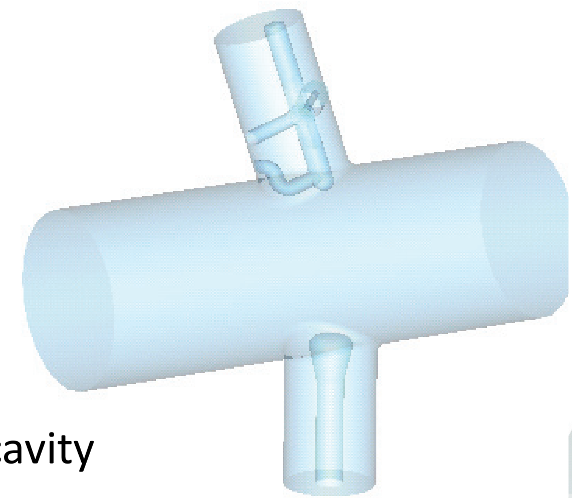
2 coaxial dampers on one side (115 deg. apart)

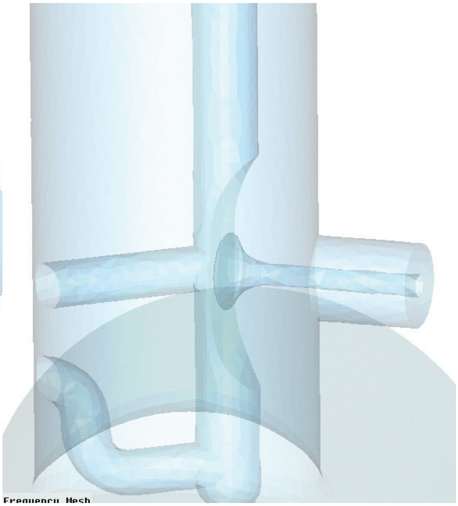
HOM Damping Technologies

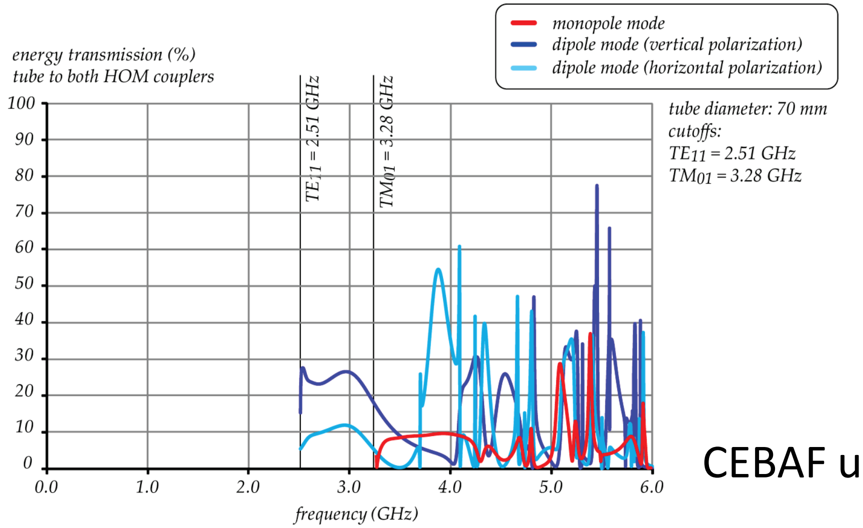
1) Coaxial dampers on beam tubes



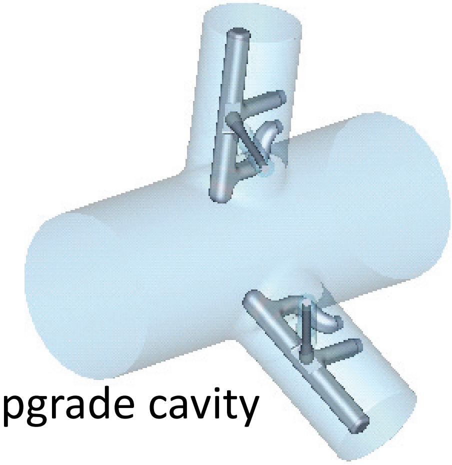
TESLA cavity





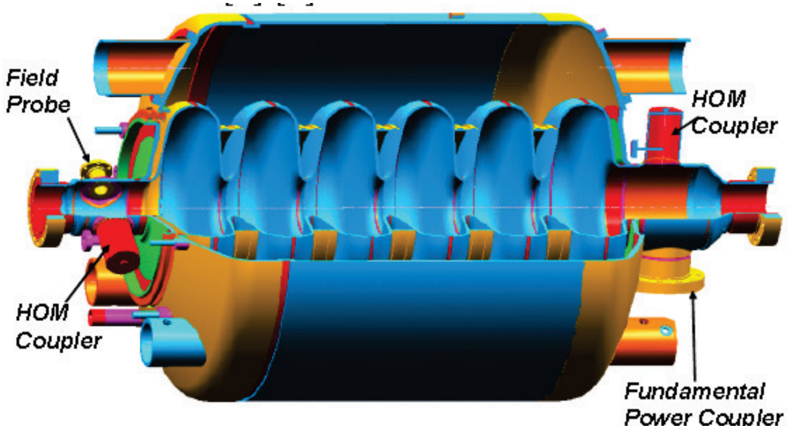


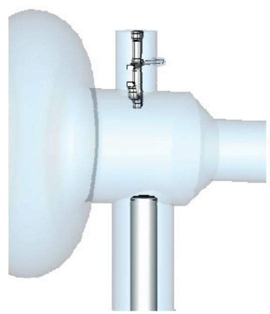
CEBAF upgrade cavity

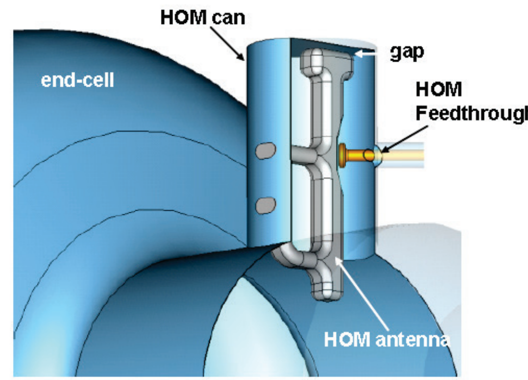


HOM Damping Technologies

- Examples:
  1) **Coaxial dampers on beam tubes**







Note: The HOM couplers are not necessarily required for SNS (H-) linac, some couplers are pulled off due to strong multipacting

THP081 Proceedings of LINAC 2006, Knoxville, Tennessee USA

STUDY ON FAULT SCENARIOS OF COAXIAL TYPE HOM COUPLERS IN SRF CAVITIES*

Sang-ho Kim, I. E. Campisi, D. Jeon, F. Casagrande, D. Stout, Y. Kang, M. Crofford, M. McCarthy and M. Champion, SNS/ORNL, Oak Ridge, TN 37831, U.S.A

SNS Medium- and High-Beta 805 MHz cavities

HOM Damping Technologies

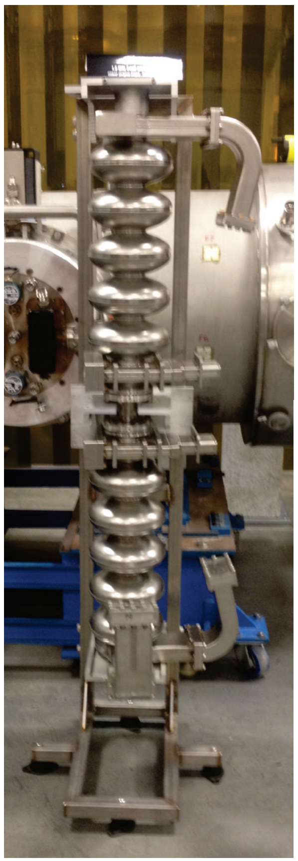
2) Waveguide dampers on beam tubes

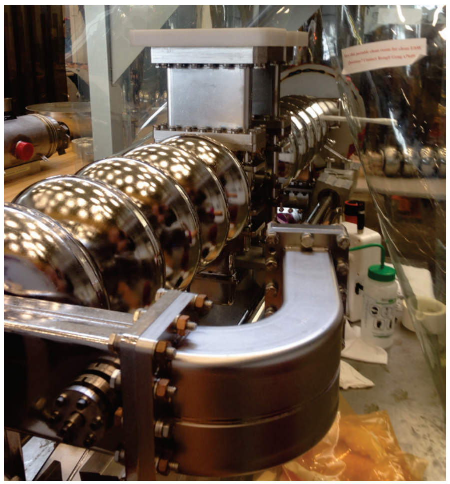
- Pros:
  - Provide natural cutoff filter for fundamental mode (no notch filter necessary)
  - More broadband damping than coaxial couplers possible depending on waveguide absorber material placed inside
  - Can be produced by deep-drawing and electron beam welding, i.e. less complex and costly than coaxial couplers
  - No RF vacuum window required
  - Power handling is enhanced (better cooling possibility) and Kilo-Watts can be handled with absorbers cooled at room temperature (standard water cooling)
  - Multipacting is better controllable by design (no issues reported at CEBAF with original cavities, i.e. 20+ operation)
  - Absorbers have no line-of-sight with beam, i.e. charge accumulation is not an issue
- Neutral:
  - May or may not occupy more beam line space than coaxial couplers (design-dependent)
  - Absorber material is usually in vacuum and particulates may be introduced, yet absorber is typically far away from beamline compared to beam line absorber
  - Long waveguides required to avoid residual damping to leaking fundamental fields
- Cons:
  - Damping efficiency is usually optimized for one (1st) waveguide mode (less efficient for another)
  - Static heat radiation from warm environment into cryogenic environment needs to be managed depending on power losses
  - For significantly large power losses, the damping has to be done outside the cryomodule and requires a

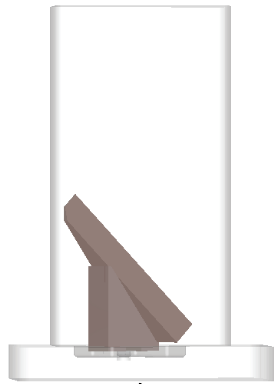
HOM Damping Technologies

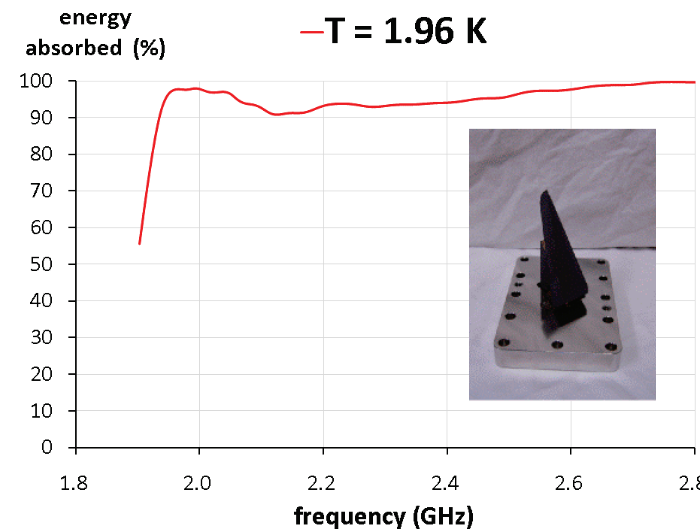
- Examples:

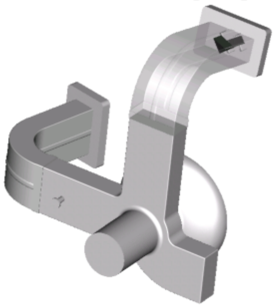
2) Waveguide dampers on beam tubes



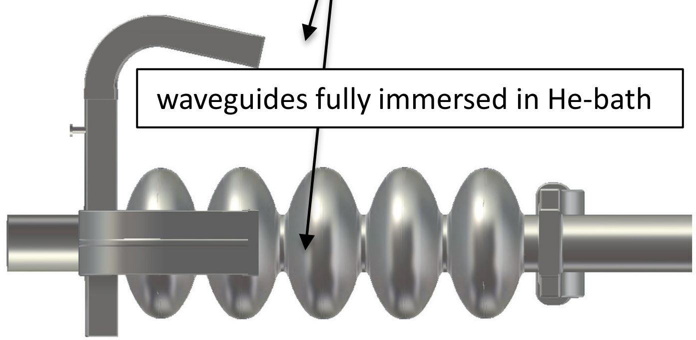








waveguides fully immersed in He-bath



CEBAF 2K HOM absorber made from AlN-7% glassy carbon ceramic material (few tens of mW power dissipation in He-bath)

Proc. SRF 1993

Artificial Dielectric Ceramics for CEBAF's Higher-Order-Mode Loads *

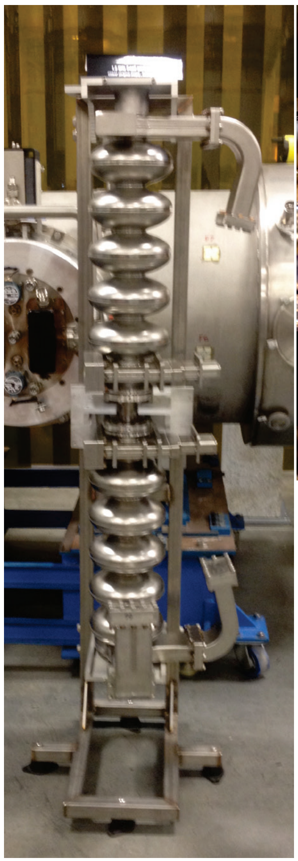
Isidoro E. Campisi, Lynda K. Summers, Aldo Betto, and Ben H. Branson
Continuous Electron Beam Accelerator Facility
12000 Jefferson Avenue, Newport News, VA 23606-1909 USA

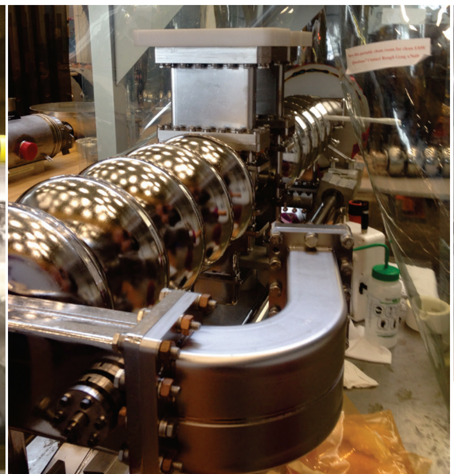
Original CEBAF (OC) cavity with 2 waveguide coupler terminated with cold absorbers

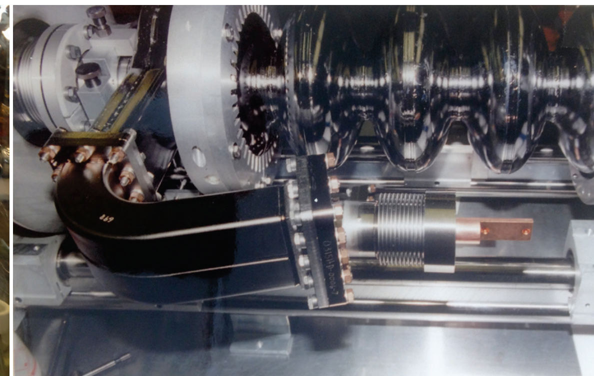
HOM Damping Technologies

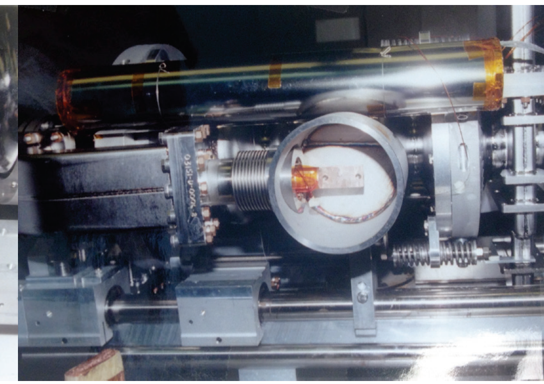
- Examples:

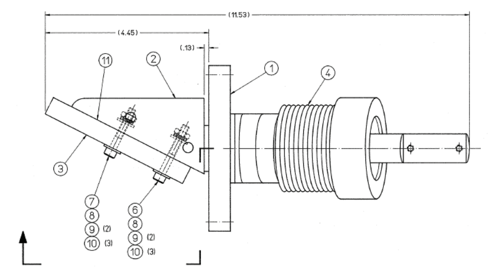
  2) Waveguide dampers on beam tubes



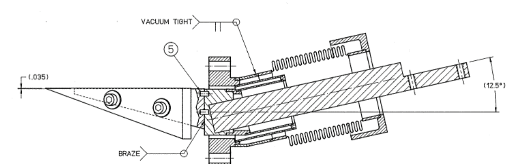








For JLab FEL the HOM absorber shape and cooling path (adding conduction cooling) has been altered to cope with higher power levels



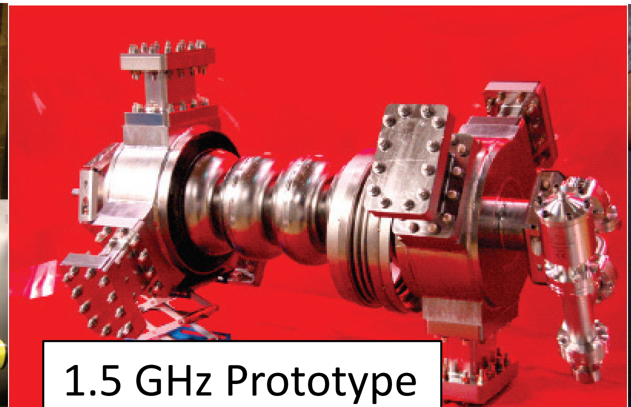
Original CEBAF (OC) cavity with altered absorbers for JLab's FEL operation

HOM Damping Technologies

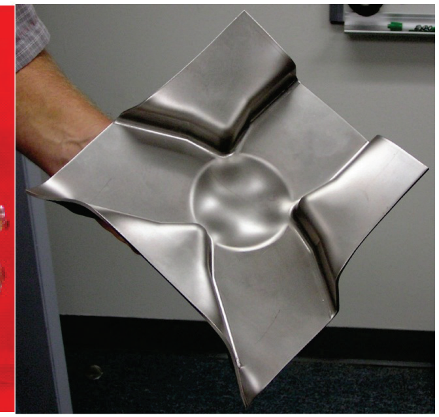
- Examples:

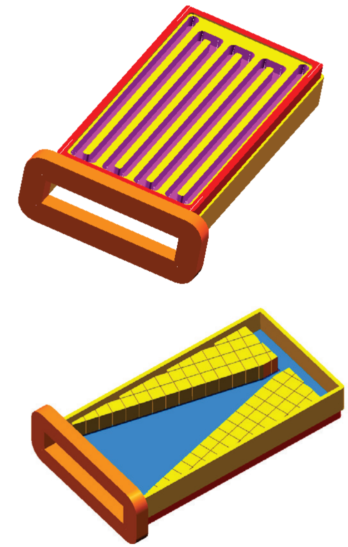
2) Waveguide dampers on beam tubes

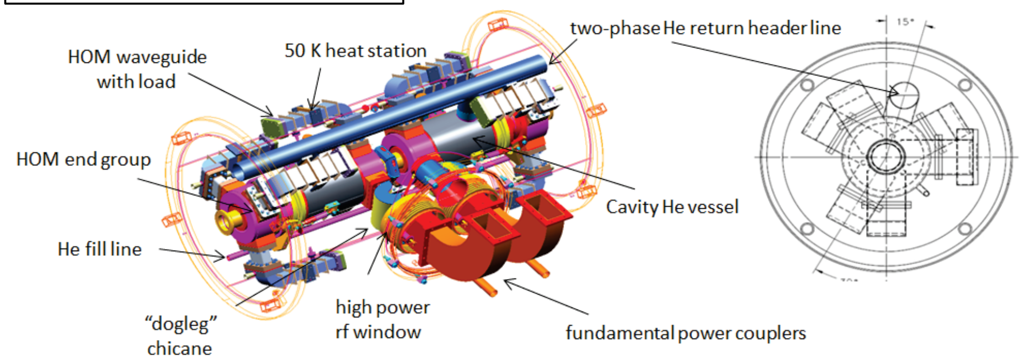
750 MHz Prototype



1.5 GHz Prototype





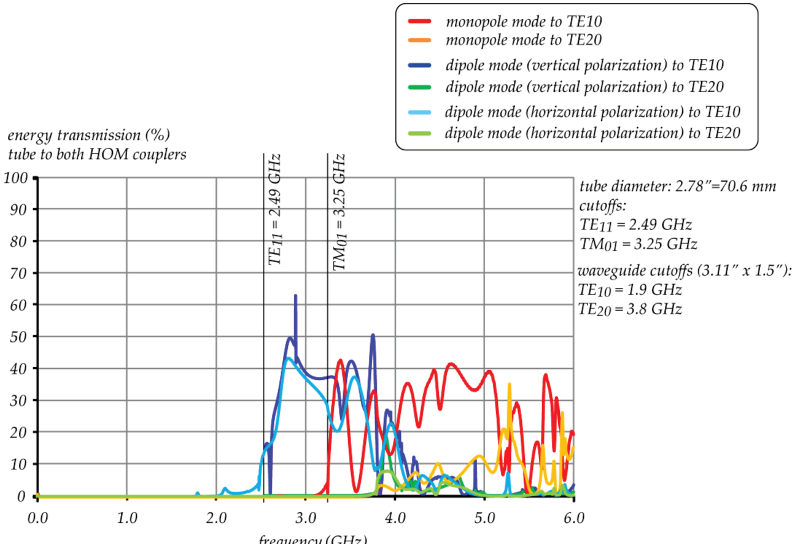


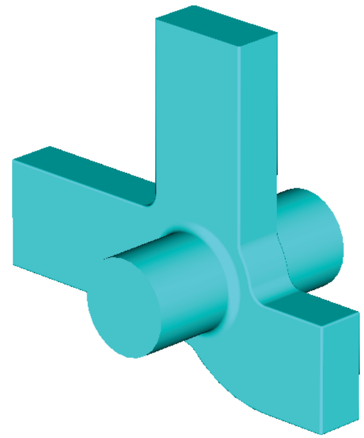
Water-cooled SiC absorbers at room temperature, several kW power capability

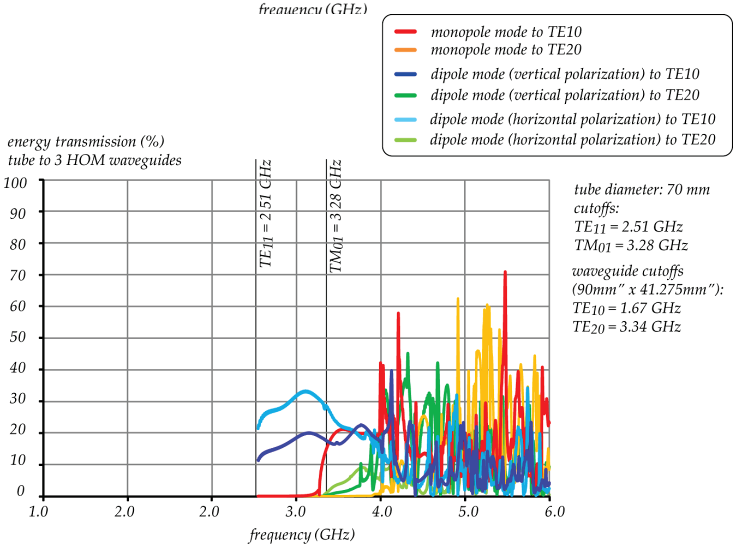
JLab High Current (HC) (Ampere-class FEL/ERL) conceptual cavity design employing 6 waveguides for heavy HOM-damping

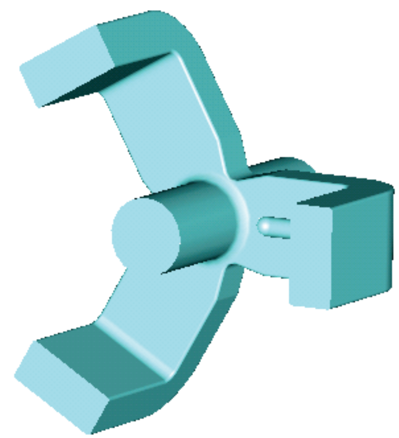
HOM Damping Technologies

2) Waveguide dampers on beam tubes









HOM Damping Technologies

3) Absorbers in cavity-interconnecting beam tubes

- Pros:
  - Intercept power traveling between cavities, becomes a requirement for high average current, short bunches (producing hundred of Watts) to minimize cryogenic losses
  - Absorbers are efficient to cover a wide bandwidth (might require different load materials though to cover various spectral ranges adequately)
  - Damps all HOM polarization equally due to symmetric absorber arrangement around beam tube
  - Can be placed in warm beam line sections between cryomodules (if existing), which mitigates cooling effort (does not require cryomodule beam line space and can be used in conjunction with already installed HOM dampers (least risk of contamination)
- Cons:
  - Absorber material is directly placed in beamline vacuum, high risk of contamination of SRF surfaces, might require shielding (e.g. by additional lossless ceramic)
  - Occupies additional beam line space
  - Requires adequate thermal management, which adds complexity (typically cooled at LN temperature to cope with high power levels)
  - Brazing of tiles to metal delicate and adds to cost
  - Charge accumulation can arise on absorber surfaces (deflects beam), i.e. absorber material needs some finite DC conductivity to allow charge drainage (thin conducting surface coating of conductive material might be necessary)

HOM Damping Technologies

- Examples: **Hybrid**
  - **2) Waveguide dampers on beam tubes**
  - **3) Absorbers in cavity-interconnecting beam tube**

radial transmission line

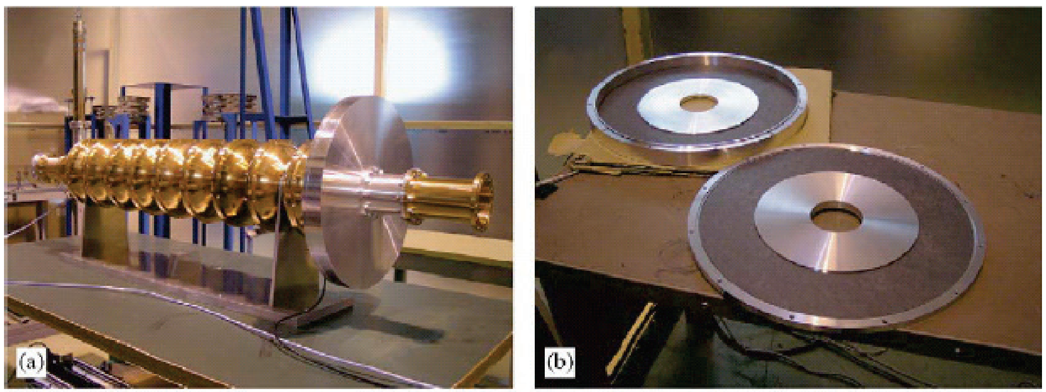


Fig. 2. (a) Low-power model of a 9-cell TESLA type cavity equipped with a radial-line HOM damper and (b) inside the radial-line HOM damper.

ELSEVIER

Available online at www.sciencedirect.com

SCIENCE DIRECT

Nuclear Instruments and Methods in Physics Research A 557 (2006) 272–275

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

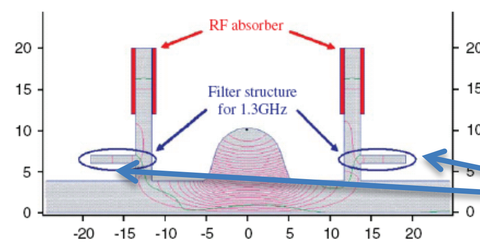
www.elsevier.com/locate/nima

New higher-order-mode damping scheme for L-band superconducting cavities using a radial transmission line☆

K. Umemori[a,*], M. Izawa[a], K. Saito[b], S. Sakanaka[a]

[a]Institute of Materials Structure Science, High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan
[b]Accelerator Laboratory, High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

Available online 16 November 2005



notch filter required

Fig. 1. Conceptual design of the radial-line HOM damper for a single-cell TESLA-type cavity.

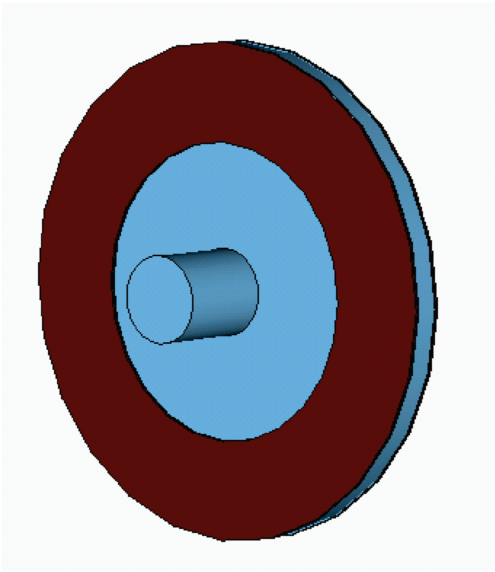
HOM Damping Technologies

- Examples: **Hybrid**

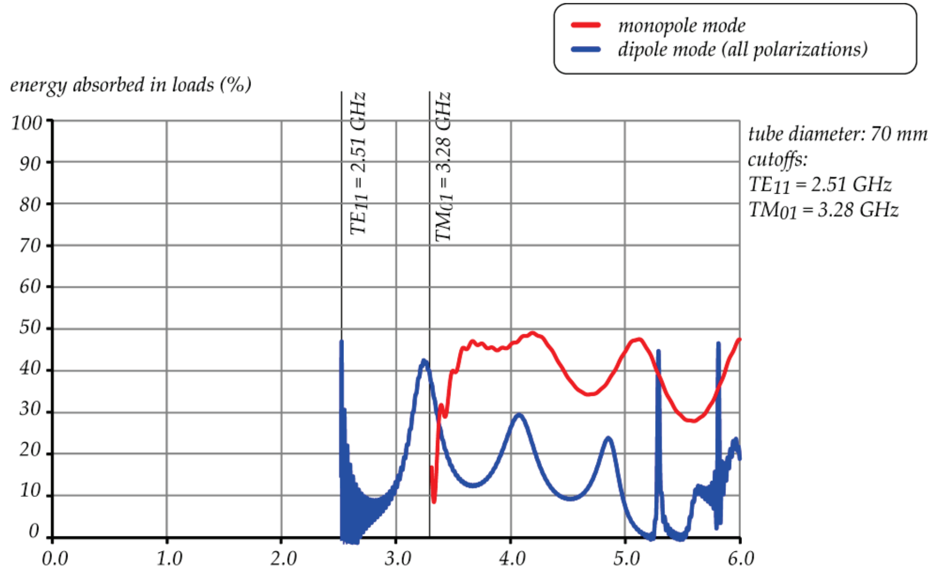
  2) Waveguide dampers on beam tubes

  3) Absorbers in cavity-interconnecting beam tube

radial transmission line



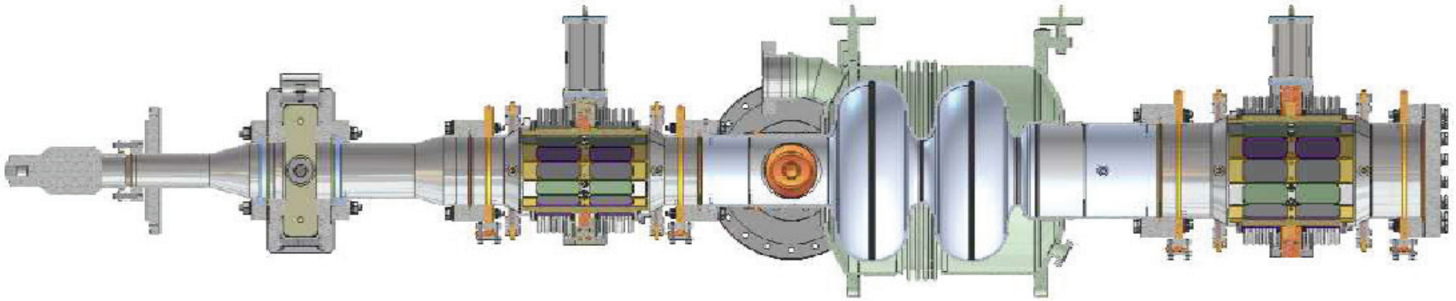
absorption does not depend on mode polarization



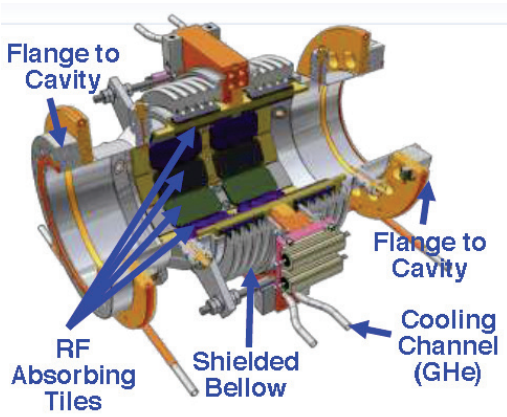
HOM Damping Technologies

- Examples:

3) Absorbers in cavity-interconnecting beam tubes



Cornell ERL 1.3 GHz injector cavity



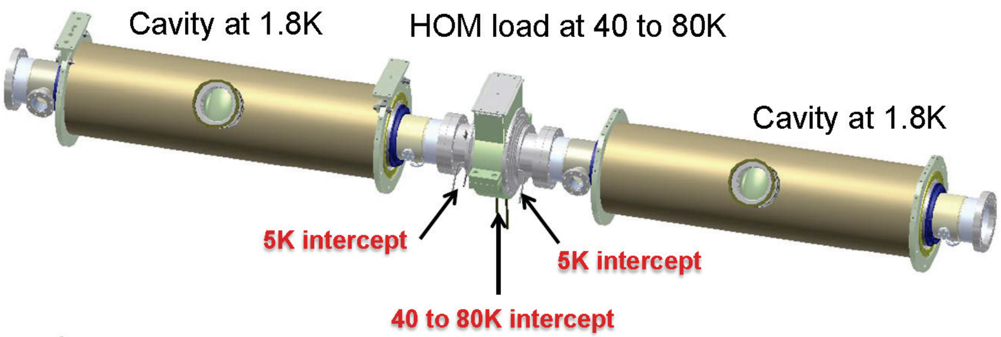
Broadband damping (1.5-100 GHz)
> 200 W power handling

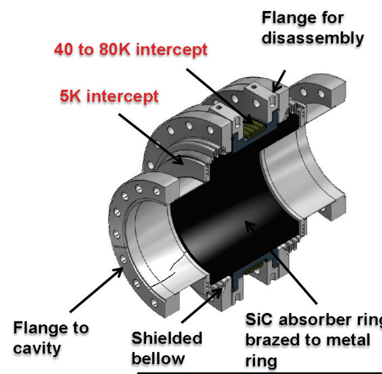
Matthias Liepe, TTC Meeting, February 28-March 3 2011, Milano, Italy

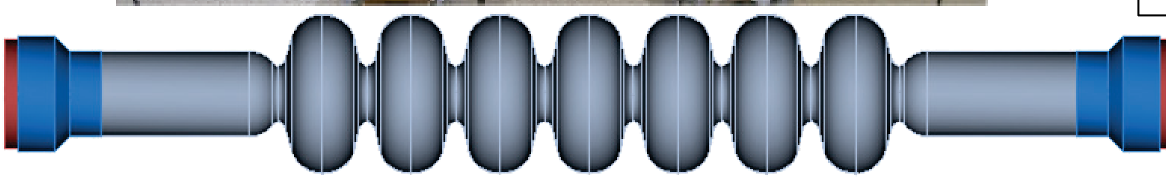
HOM Damping Technologies

- Examples:

3) Absorbers in cavity-interconnecting beam tubes







Silicon Carbide (SiC) or Aluminum Nitride (AlN) absorber

> 500 W power handling @ 80 K

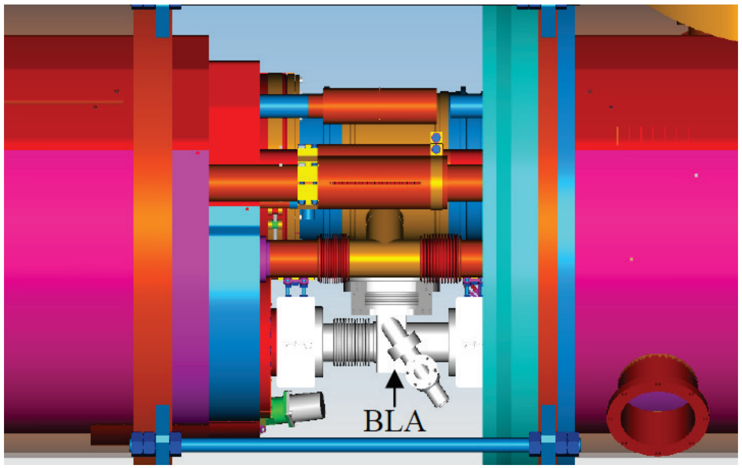
Cornel 1.3 GHz ERL main linac cavity

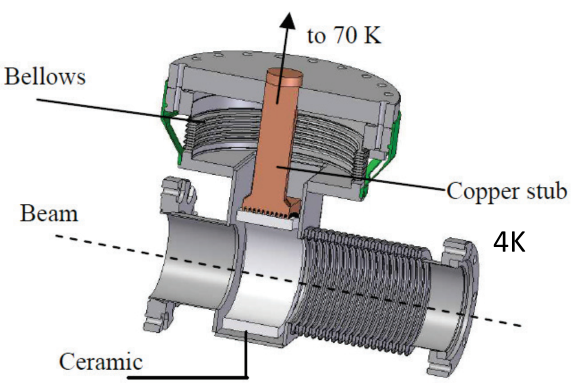
Matthias Liepe, TTC Meeting, February 28-March 3 2011, Milano, Italy

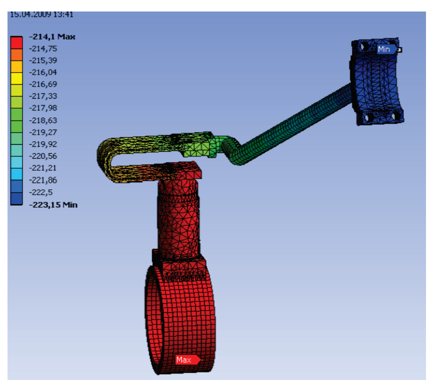
HOM Damping Technologies

- Examples:

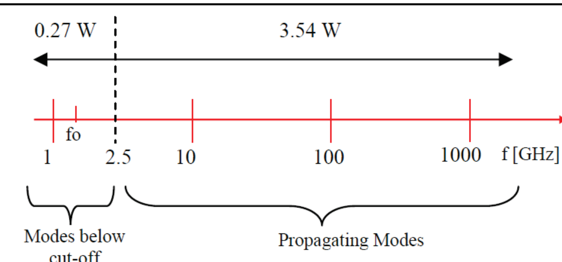
3) Absorbers in cavity-interconnecting beam tubes







Ceralloy CA137 absorber ring,
up to 100 W power capability @ 70 K



Proceedings of IPAC'10, Kyoto, Japan

BEAM TESTS OF HOM ABSORBER AT FLASH

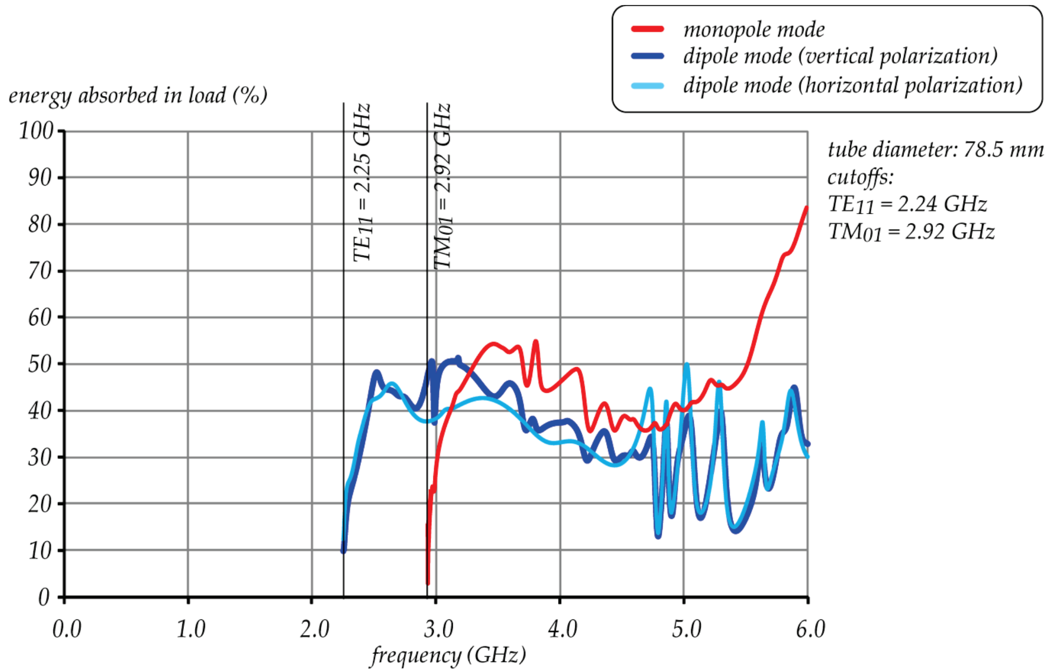
J. Sekutowicz, A. Gössel, N. Mildner, M. Dohlus
DESY, Notkestrasse 85, 22607 Hamburg, Germany

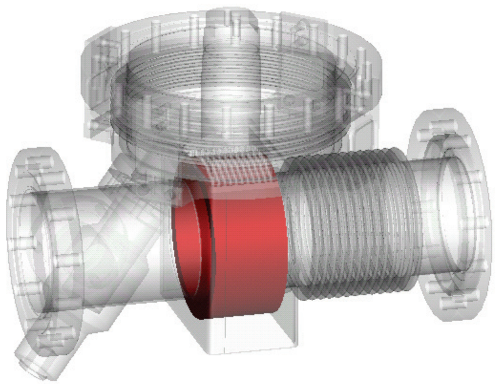
DESY FLASH/EU-XFEL beam line absorber between cryomodules

HOM Damping Technologies

3) Absorbers in cavity-interconnecting beam tubes

absorption does principally not depend on mode polarization (symmetry breaking (housing with ports) induces slight differences)



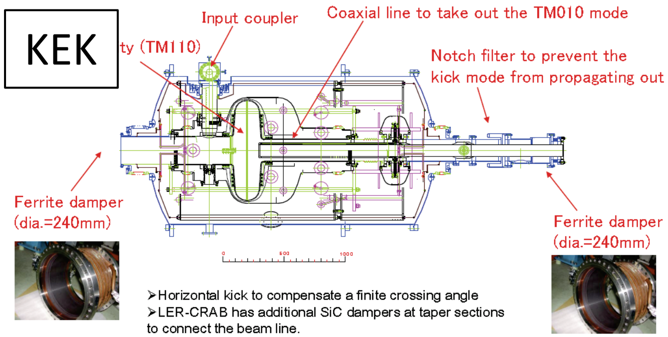


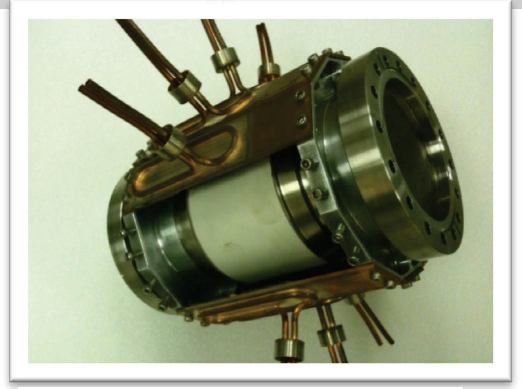
HOM Damping Technologies

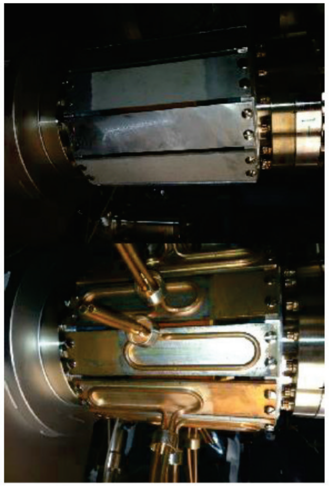
- Examples:

3) Absorbers in cavity-interconnecting beam tubes

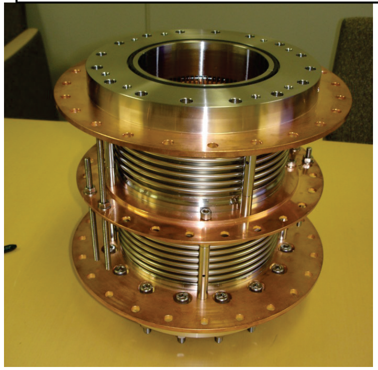
KEK



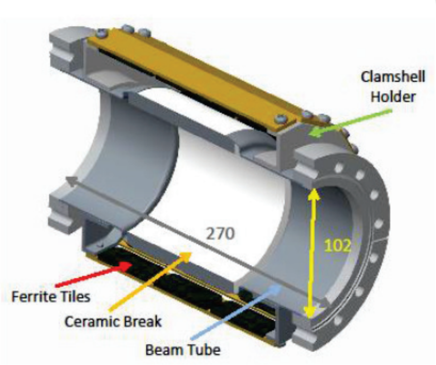




KEK ERL HOM load



HOM'10 at Cornell U.
2010/Oct/11-14
T. Furuya, KEK



BNL-106323-2014-CP

Ceramic-Ferrite damper for the BNL superconducting RF electron gun cavity

H. Hahn[1], I. Ben-Zvi[1,2], S. Belomestnykh[1,2], L. Hammons[1], V. Litvinenko[1,2], Y. R. Than[1], R. Todd[1], D. Weiss[1], W. Xu[1],

BNL RF gun (704=MHz) absorber (ferrites surrounding ceramic)

HOM Damping Technologies

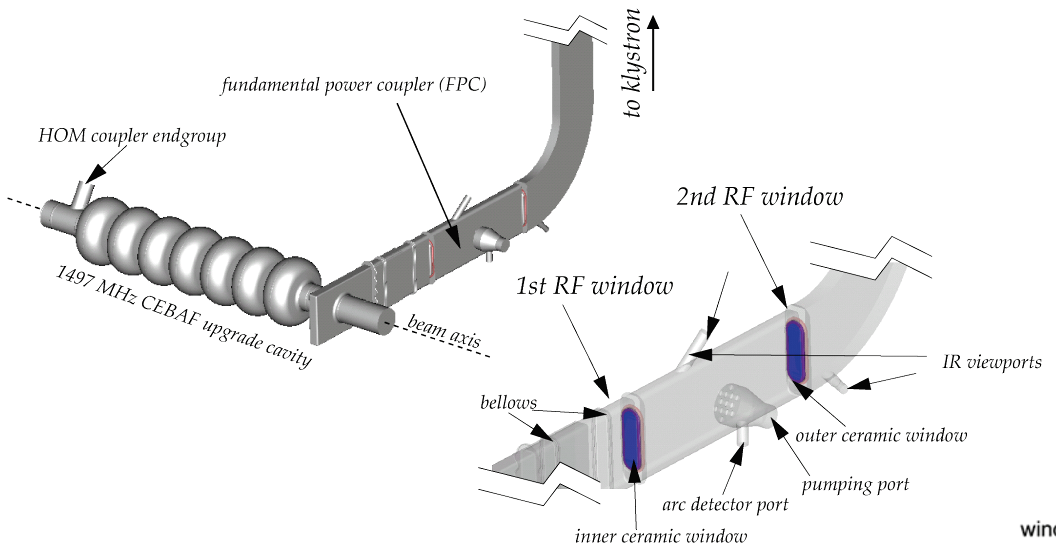
4) Coupling through Fundamental Power Coupler (FPC)

- Pros:
  - Uses existing input fundamental coupler (no extra costs) to extract power
  - The power extracted will propagate to the circulator load
- Neutral:
  - The circulator load is not a broadband device, RF power can be reflected, however a waveguide absorber can be placed outside the cryomodule (no particulate issues). The filter however can be complex and costly since it must not damp the incoming fundamental mode. The damping could also be waveguide mode-dependent based on design.
- Cons:
  - Damping might prefer only one of two dipole mode polarizations and thus depends on direction of input coupler (this can be mitigated to some extent by design)
  - The transmission of other than the fundamental mode is limited by the RF vacuum window(s) and may require some optimization to be useful
  - Transmission through RF window(s) is also waveguide mode-dependent. Only problem if more than one waveguide mode is required to damp critical modes (tradeoff possible by design)
  - When external waveguide filter is used, it needs to be optimized to minimize insertion losses

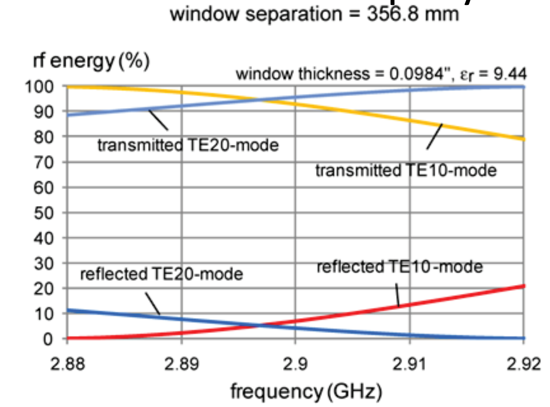
HOM Damping Technologies

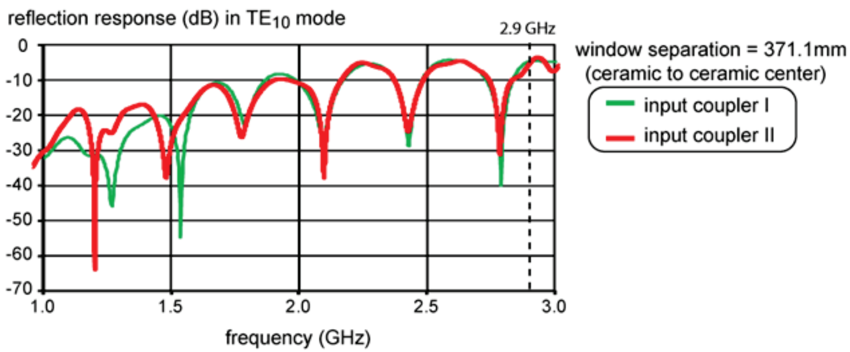
- Examples:

4) Coupling through Fundamental Power Coupler (FPC)

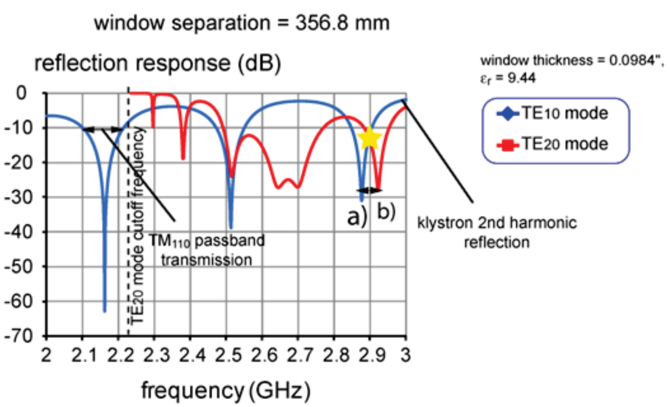


trade-off between TE10 and TE20-mode transmission at crucial mode frequency





original

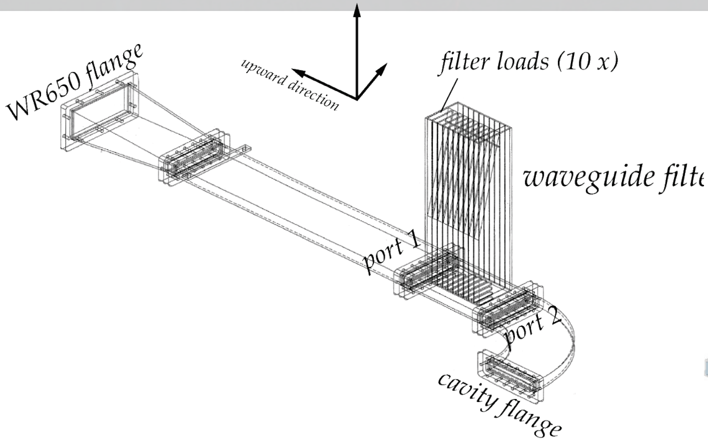


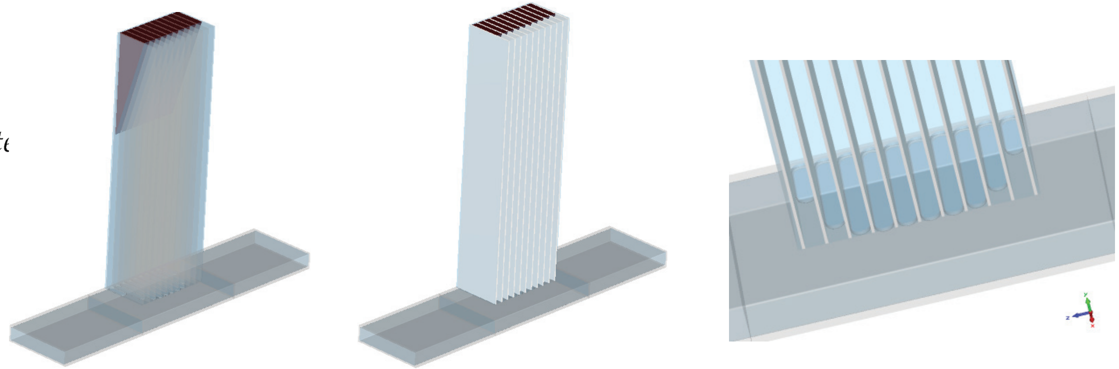
optimized double window configuration

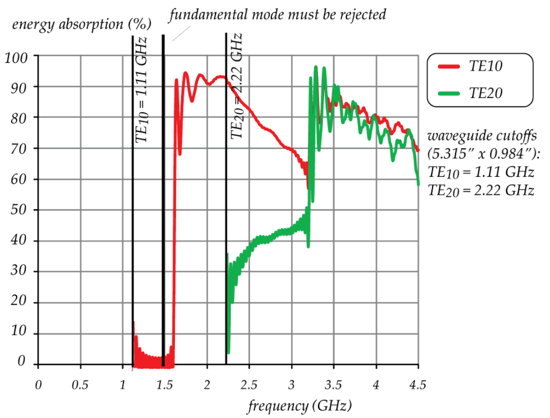
HOM Damping Technologies

- Examples:

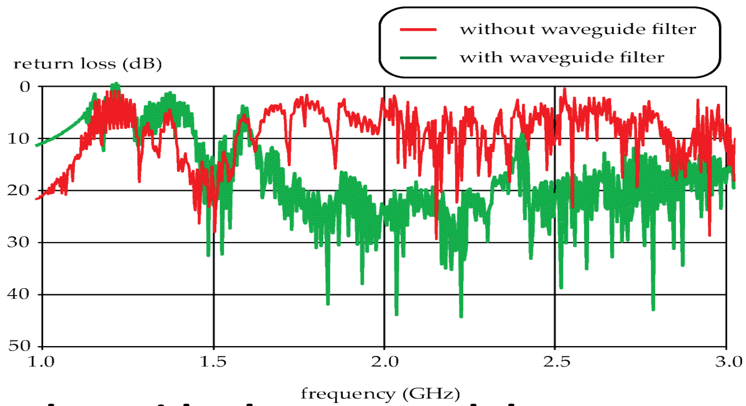
4) Coupling through Fundamental Power Coupler (FPC)







measured return loss in TE10 mode



CEBAF FPC Waveguide Filter placed outside the cryomodule
(originally conceived for intercepting klystron harmonic signals)

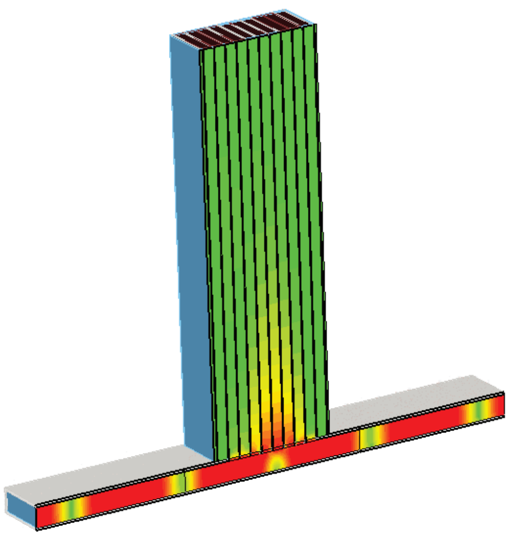
HOM Damping Technologies

- Examples:

4) Coupling through Fundamental Power Coupler (FPC)

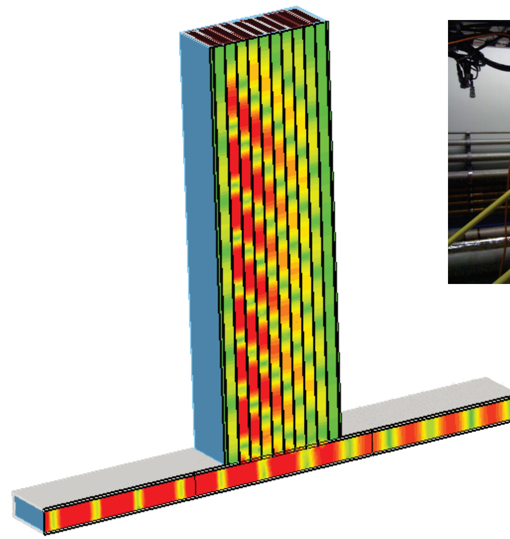
No coupling to fundamental mode at 1497 MHz (TE10-mode)

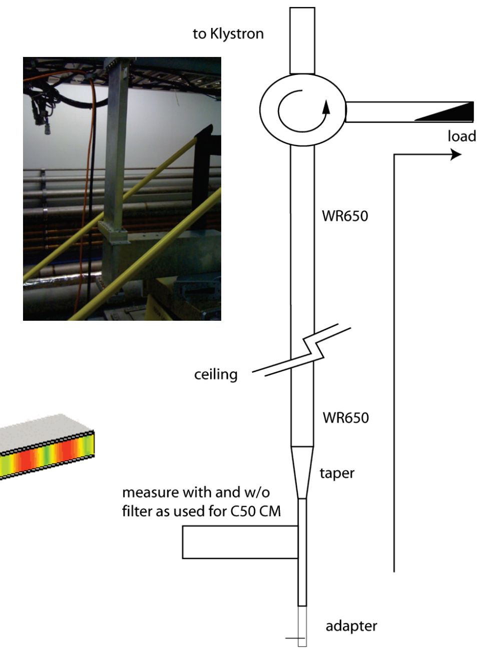
movie



Strong coupling at 2.9 GHz (TE10-mode)

movie





CEBAF FPC HOM waveguide Filter placed outside the cryomodule (originally conceived for intercepting klystron harmonic signals)

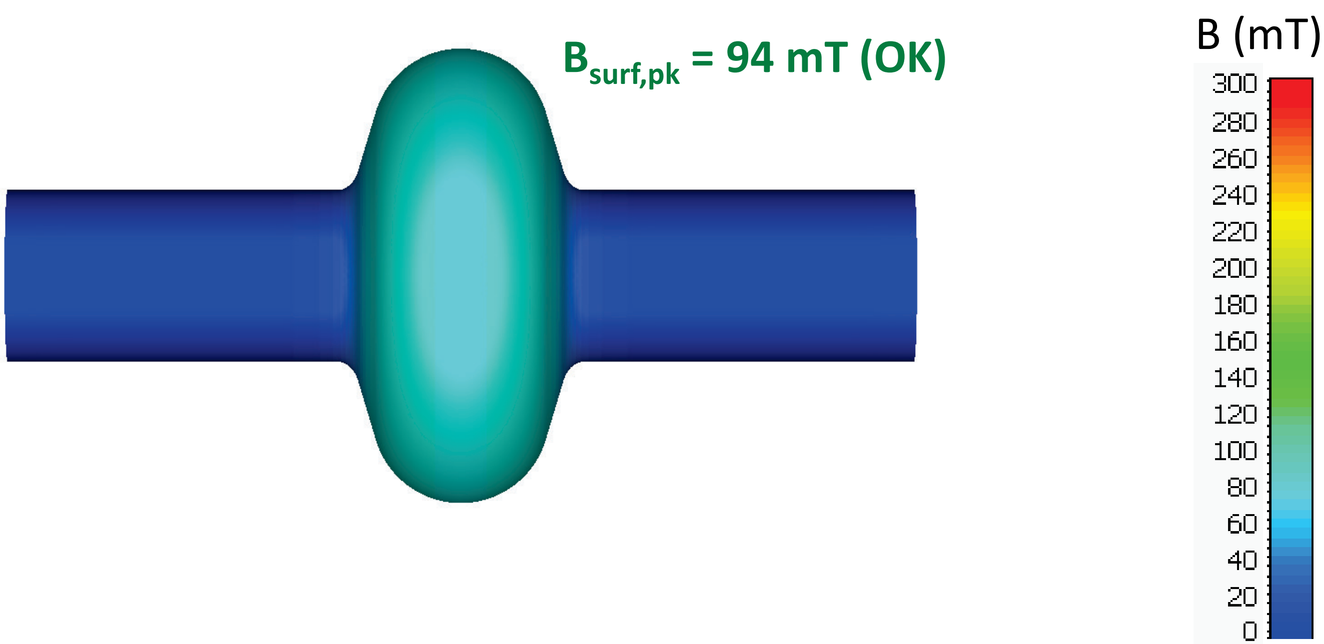
HOM Damping Technologies

5) On-cell dampers (for special cavities only)

- **Using HOM-dampers positioned on cavity dells is the most efficient way to damp HOMs directly**
- Pros and Cons would apply as detailed depending on the used damper technology
- Works well for NC cavities
- This technique can only be used for cavities that do not use the TM_{010}-mode as operating mode
- **The technique should not be used for SRF accelerating (TM_{010}) cavities, when aiming for the typically high operating efficiency attractive for the usage of the cavities**
- **Why not? Next slides will show**

On Cell Damper ?

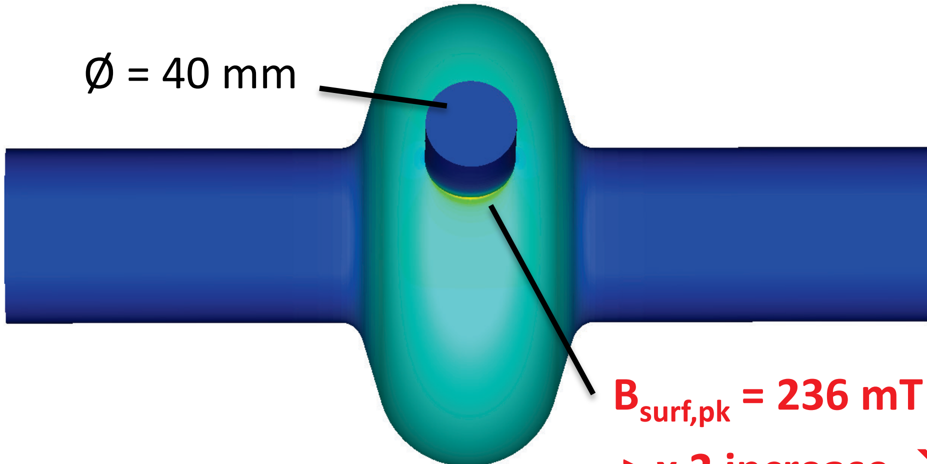
- Consider operating at 20 MV/m



On Cell Damper ?

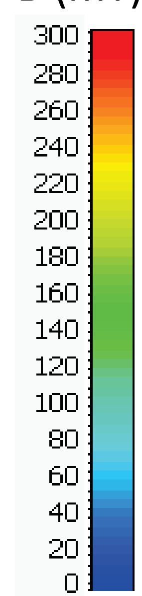
- How strong is surface magnetic flux density enhanced?

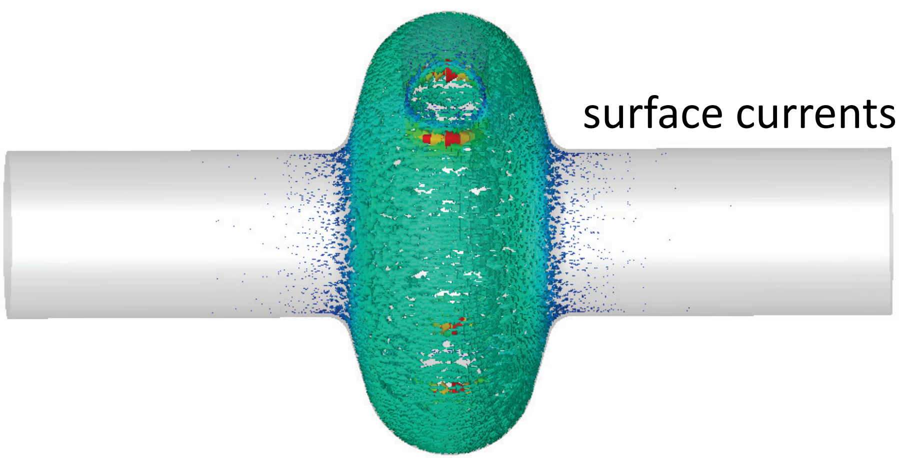
Ø = 40 mm



$B_{surf,pk}$ = 236 mT @ 20 MV/m

> x 2 increase → quench!





surface currents

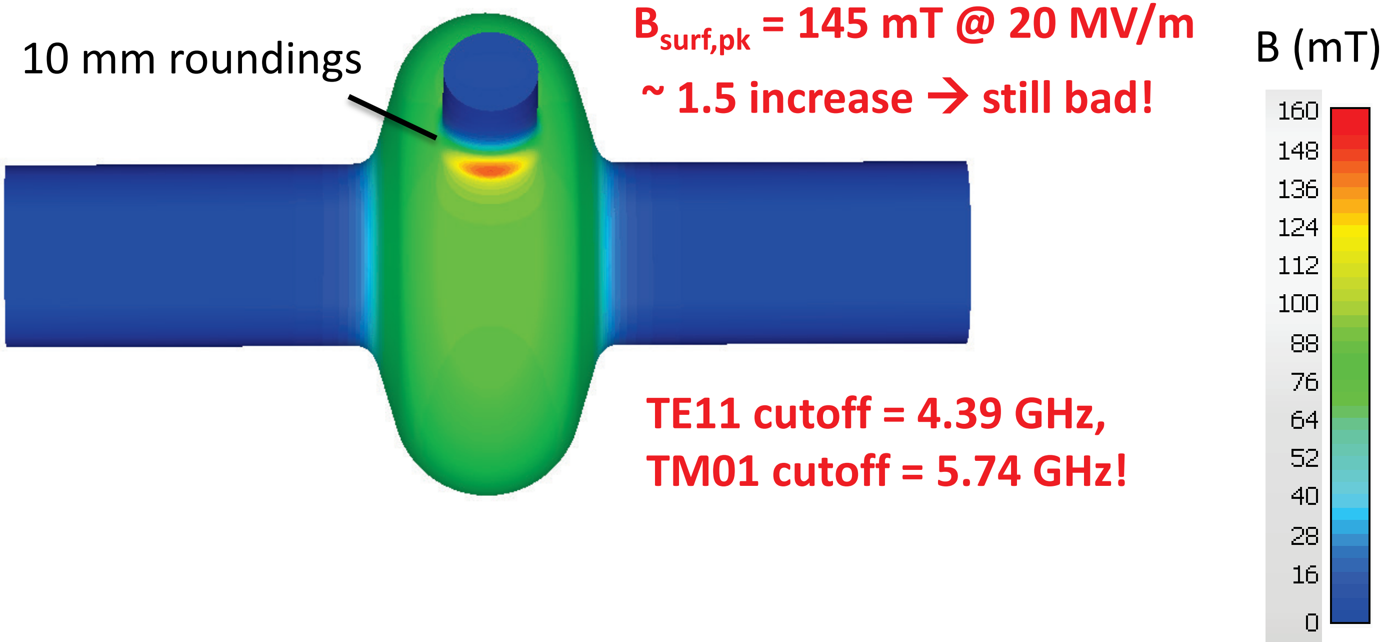
On Cell Damper ?

- Rounding edges mitigates field enhancement

10 mm roundings

$B_{surf,pk}$ = 145 mT @ 20 MV/m

~ 1.5 increase → still bad!

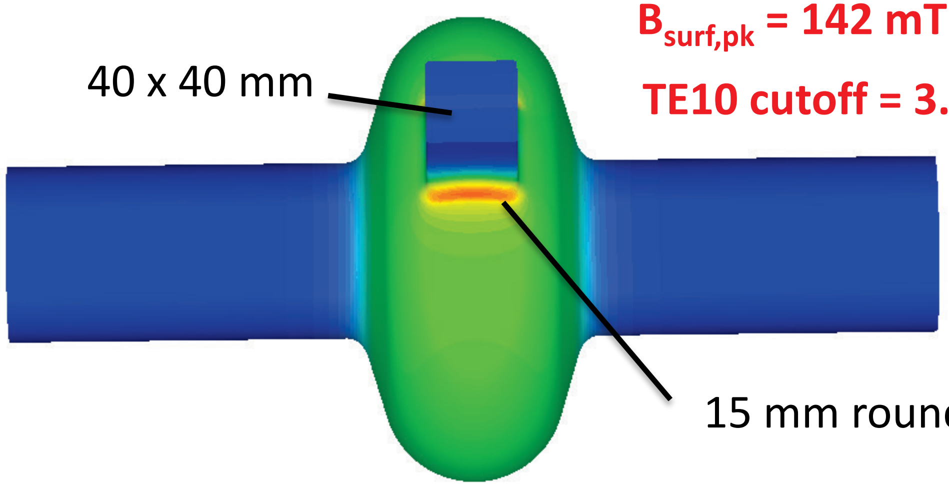


TE11 cutoff = 4.39 GHz,
TM01 cutoff = 5.74 GHz!

- Yet, to let lower HOMs propagate we still require a damper with ID > tube ID

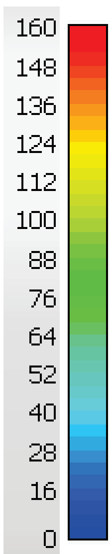
On Cell Damper ?

- Square Waveguide



$B_{surf,pk}$ = 142 mT @ 20 MV/m

TE10 cutoff = 3.75 GHz!

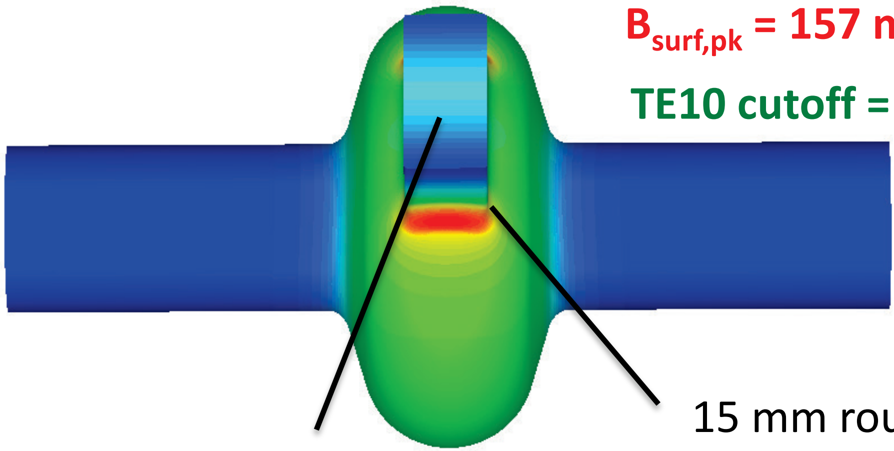


On Cell Damper ?

- Rectangular Waveguide

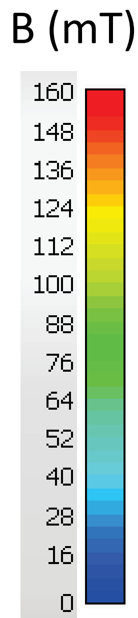
$B_{surf,pk}$ = 157 mT @ 20 MV/m

TE10 cutoff = 1.9 GHz!



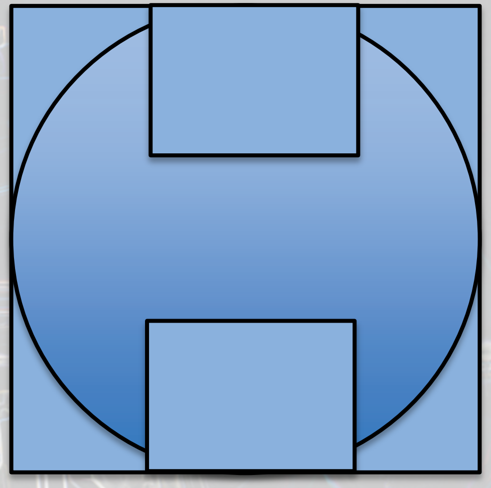
15 mm roundings

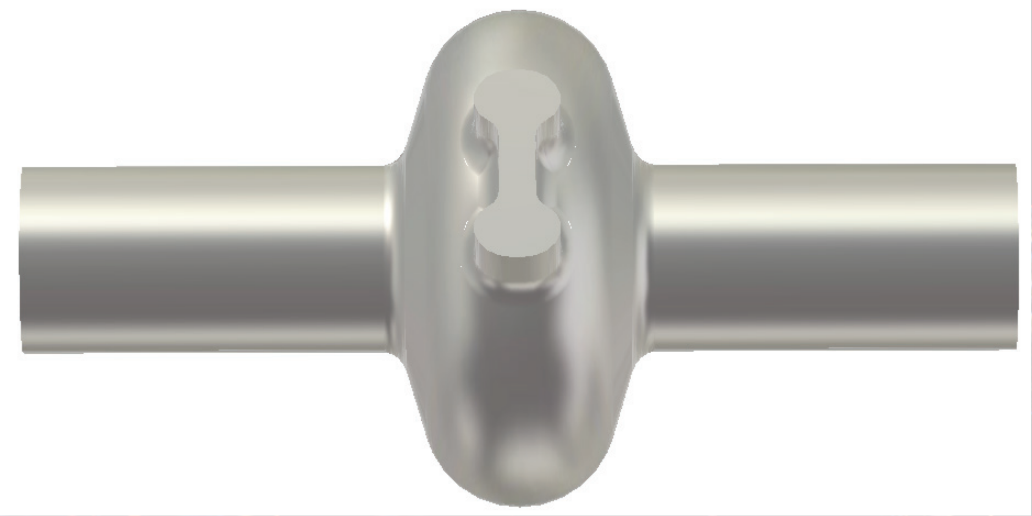
38.1 x 79 mm
(CEBAF OC HOM damping waveguide size)



Ways to Reduce Cutoff Frequency ?

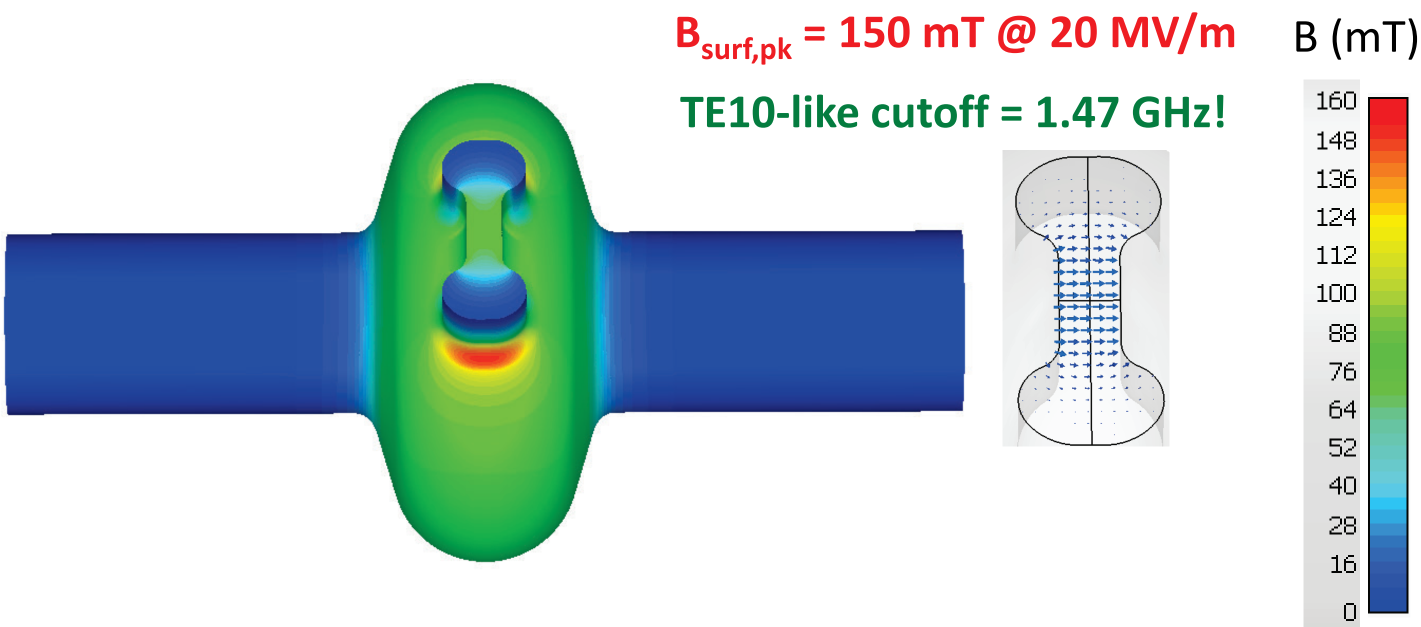
- Introduce ridge(s) at given lateral size
  - increases capacitance → lowers cutoff frequency





On Cell Damper ?

- Double-ridged waveguide (dogbone → edges rounded)



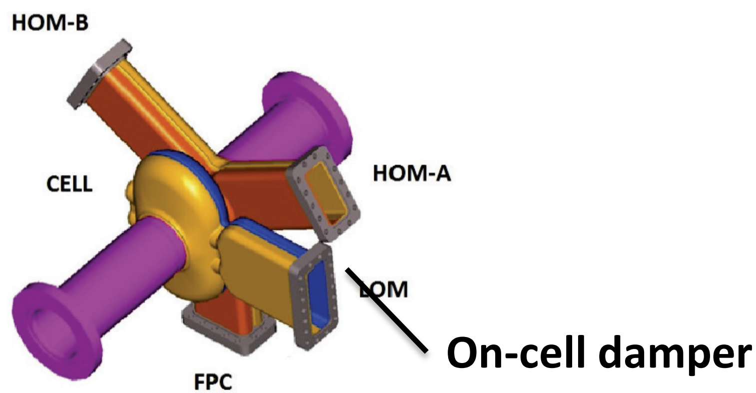
- HOM waveguide can be tailored to reduce cutoff frequency and let HOMs propagate
- However, there is a considerable magnetic field enhancement on cavity cell

HOM Damping Technologies

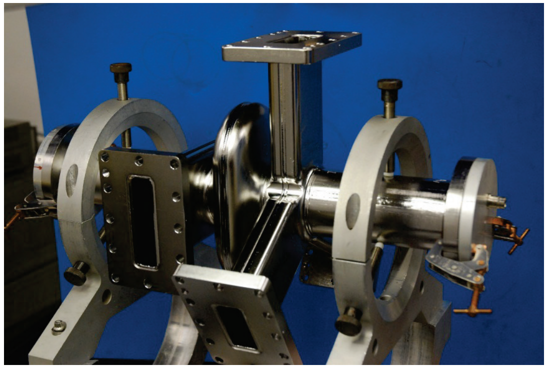
- Examples:

  5) On-cell dampers what work (for special cavities only)

- Crab cavities are not meant for acceleration and are employed for beam manipulation (luminosity enhancement) close to interaction point of colliding beams by providing transverse deflecting fields
- On-cell dampers in fact are required for trapped Lower Order Modes (LOMs), not HOMs, which exist at lower frequencies than the operating mode
- The operating dipole mode used allows to place the damper on the cell



APS crab (deflecting) cavity



FRIOA03 Proceedings of SRF2013, Paris, France

FABRICATION AND TESTING OF DEFLECTING CAVITIES FOR APS*

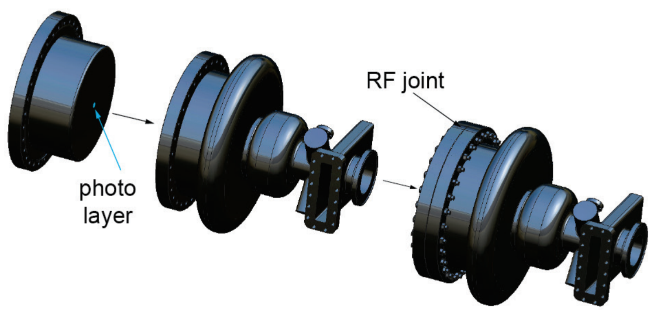
J. Mammosser[†], H. Wang, R. Rimmer, J. Henry, K. Wilson, P. Dhakal, A. Nassiri[‡], J. Kerby[‡], J. Holzbauer[‡], G Wu[‡], J. Fuerst[‡], Y. Yang[‡], G. Waldschmidt[‡], Z. Li[||], F. He[♠]
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA
[‡]Argonne National Laboratory, Argonne, IL 60439, USA
[||]SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA
[♠]Peking University, Bejing, China

HOM Damping Technologies

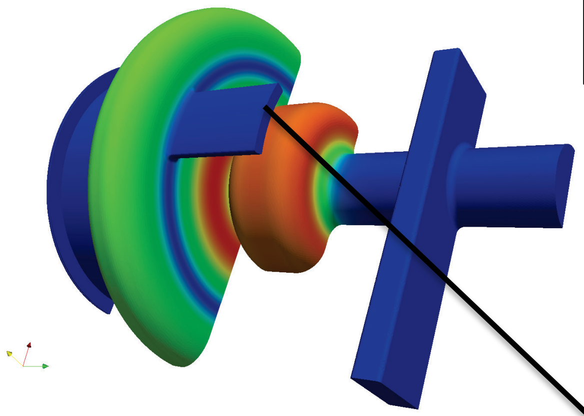
- Examples:

5) On-cell dampers what work (for special cavities only)

- A high brightness SRF Photoinjector gun cavity has been designed using a TM_{020}-mode in the first half-cell, but typical cells in the following full cells for efficient TM_{010} acceleration
- The TM_{020}-mode allows to place dampers on the cell, where the RF magnetic fields vanish, thus allowing high-field operation



contours for surface magnetic flux density at 50 MV/m at cathode



On-cell waveguide damper – no field enhancement

Muons, Inc. SRF photoinjector gun development

NOVEL SRF GUN DESIGN* Proc. SRF, 2013

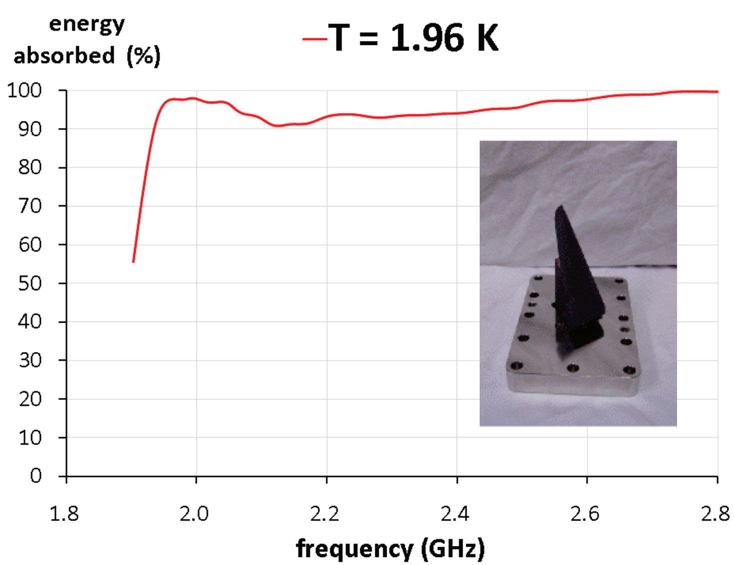
F. Marhauser#, MuPlus, Inc., Newport News, VA, USA
Z. Li, K. Lee, SLAC, Menlo Park, CA, USA

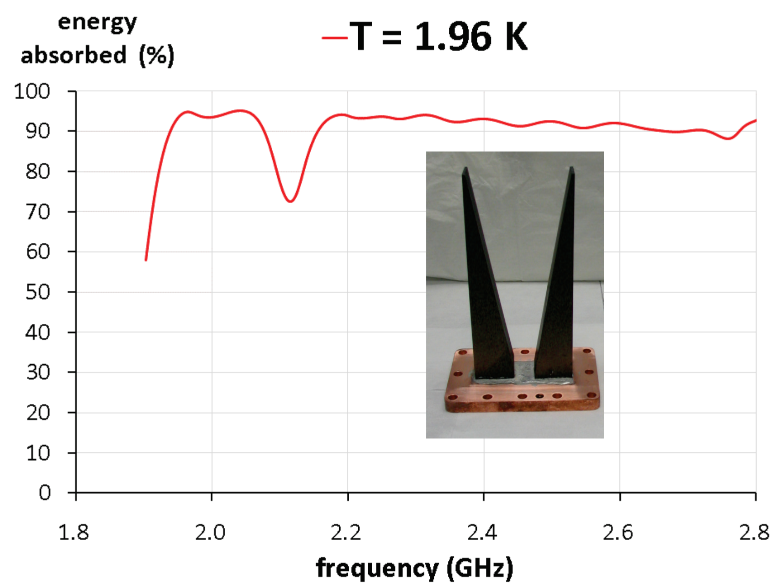
Damping Materials

- Absorber material need to be able to absorb and terminate the RF incoming wave in a broadband manner
- The figure of merit is the loss tangent given by the complex permittivity and or permeability of the material
- Ceramic-based absorbers (e.g. Carbon- or SiC loaded Aluminum Nitride) can be tailored with high electric loss tangent but usually do not provide efficient damping to magnetic fields
- Ferrites (iron oxide FeO3) can also be combined with other material compounds (e.g. NiZn) to provide a lossy ceramic that provides damping to the magnetic field as well
- The materials can be sintered or hot pressed and thereby tailored to provide efficient RF absorption, which also must include proper loading of the lossy ingredients (by percentage and distribution) to the base material
- E.g. the latter aspect was crucial to yield absorbers that do not lose their RF loss characteristics even at cryogenic temperatures (2K), i.e. the CEBAF MilliWatt-range absorber

Damping Materials

- Absorber material providing high RF losses even at 2 Kelvin !



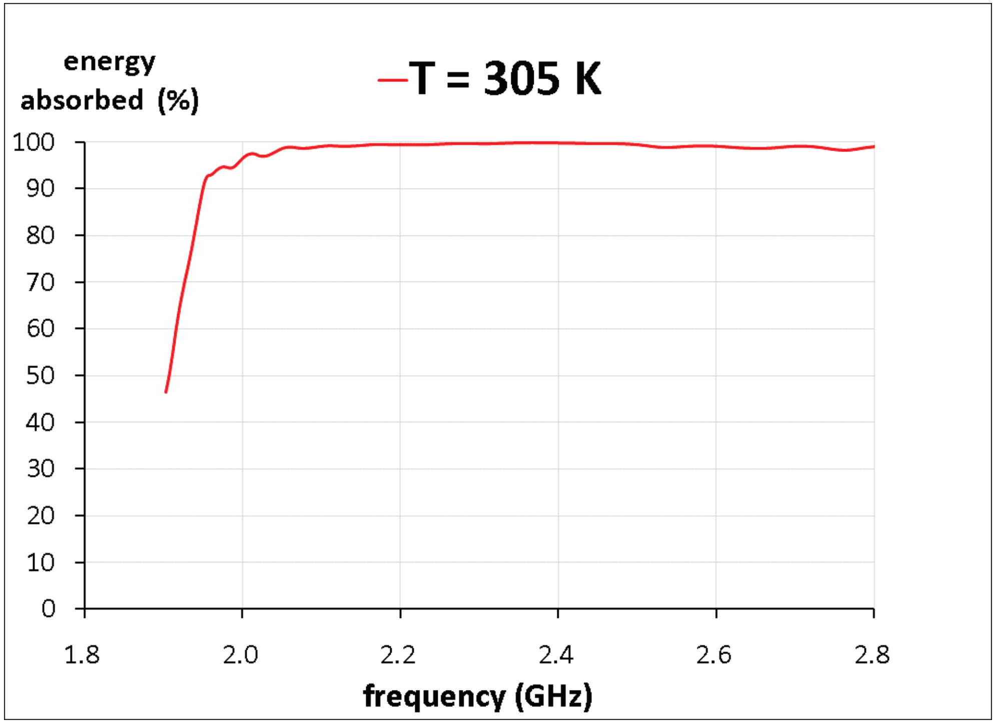


Original CEBAF absorber (AlN with glassy carbon, left) and commercially available absorber (AlN-SiC, Ceradyne 137A) measured in reflection-response at 2K

Damping Materials

- Other AlN-compounds typically lose their excellent damping performance at room temperature upon cool-down

movie

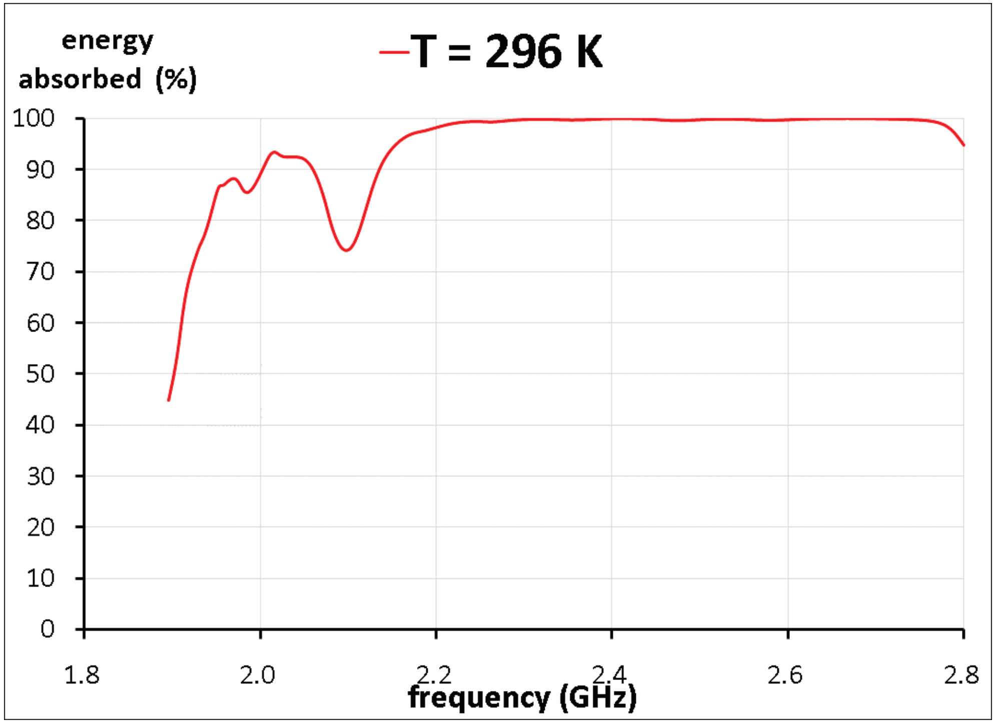


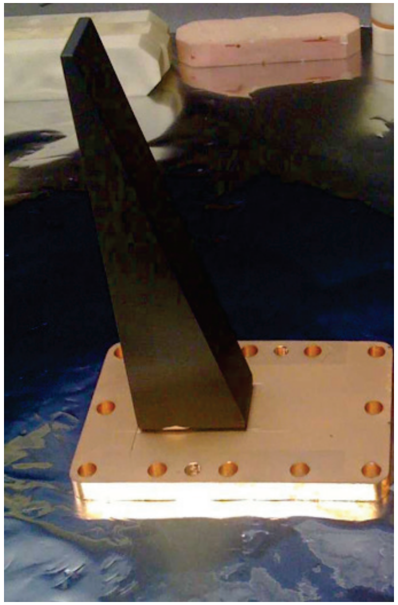
Special R&D absorber load as tested at JLab (STL-100 HP-179, Sienna Technologies, Inc.)

Damping Materials

- Another material that loses some of its absorption properties upon cool-down

movie



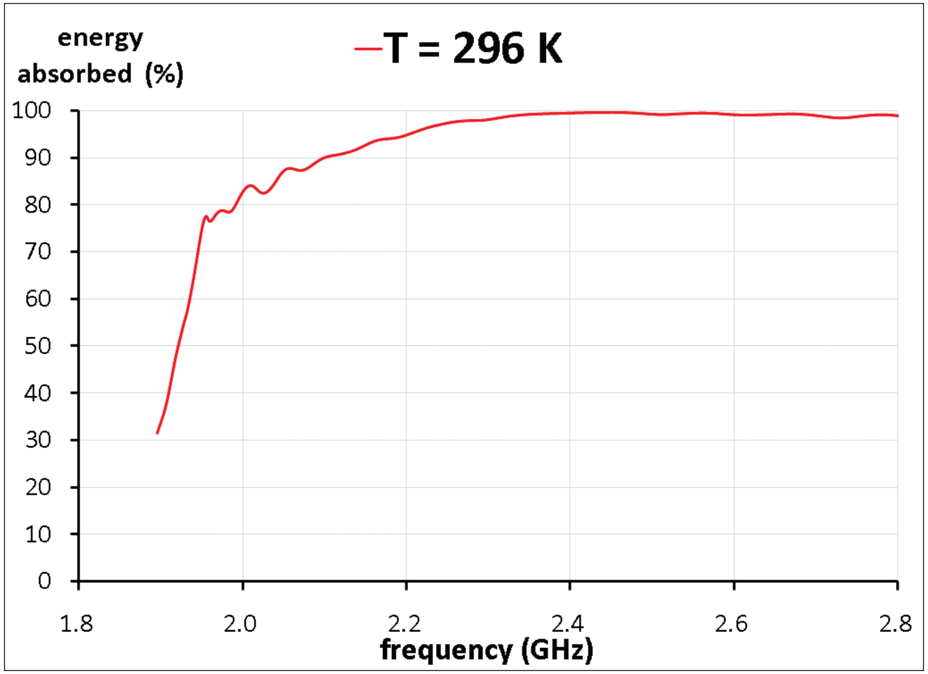


Commercially available SiC absorber (SC-30, CoorsTeK)

Damping Materials

- One of the few absorbers that does **not** lose its absorption properties at cryogenic temperatures

movie



Commercially available SiC absorber (SC-35 Graphite-loaded, CoorsTeK)

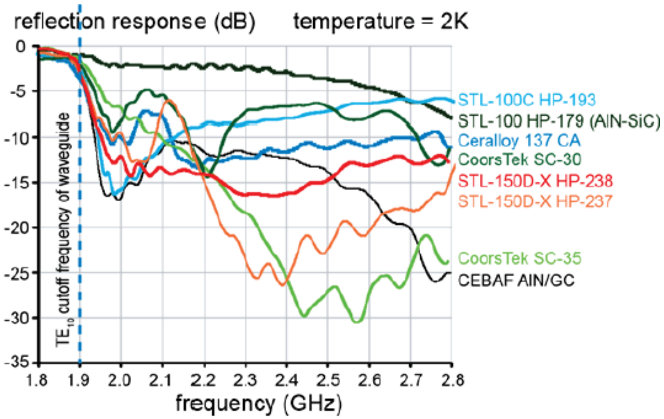
Damping Materials

- Two other cold absorbers have been tailored for JLab in collaboration with Sienna Technologies, Inc. (STL-150D-X-HP237 and HP238) making use of nanoparticle conductors that are not subject to freeze out at lower temperatures

Table 1: Material properties of lossy dielectric for HOM absorbers (ρ = density, CTE = coefficient of thermal expansion, κ = thermal conductivity, ε_r = relative permittivity, tan δ = loss tangent, σ = flexural strength).

Parameter	ρ	CTE (25-800°C)	κ	ε_r	tan δ	σ
Unit	g/cm^3	10^{-6}/°C	W/m•K			MPa
CEBAF AlN/GC [7]	3.0	4.7	55	20	> 0.1	300
Ceralloy 137 CA [5]	2.99	5.0	85	28^a 18^b 11^c	0.2^a 0.2^b 0.2^c	-
CoorsTek SC DS (SC-30) [9][11]	3.15	4.4	150	14^a 11^b 11^c 12^d	0.46^a 0.18^b 0.18^c 0.15^d	480
CoorsTek SC-DSG (SC-35) [9][11]	2.8	4.4	125	70^a 37^b 36^c 33^d	0.71^a 0.58^b 0.57^c 0.57^d	220
Sienna STL-100 AlN-SiC	3.26	5.1	115	38^b 36^c 33^d	0.27^b 0.33^c 0.36^d	590
Sienna STL-150D-X doped AlN	3.21	5.1	130	26^b 26^c 25^d	0.69^b 0.54^c 0.53^d	350

a = at 1 GHz, b = at 8 GHz, c = at 10 GHz, d = at 12 GHz



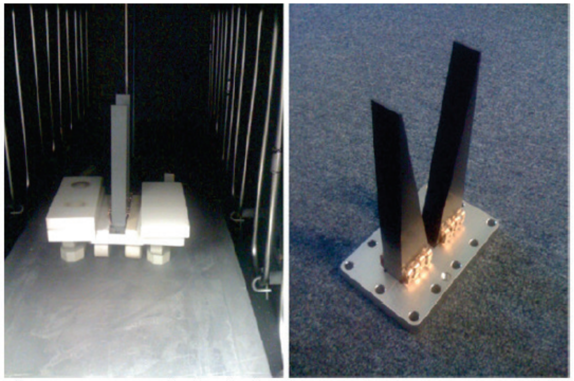


Figure 2: Ceramic loads in vacuum brazing furnace (left) and after brazing (right) to stainless steel flange.

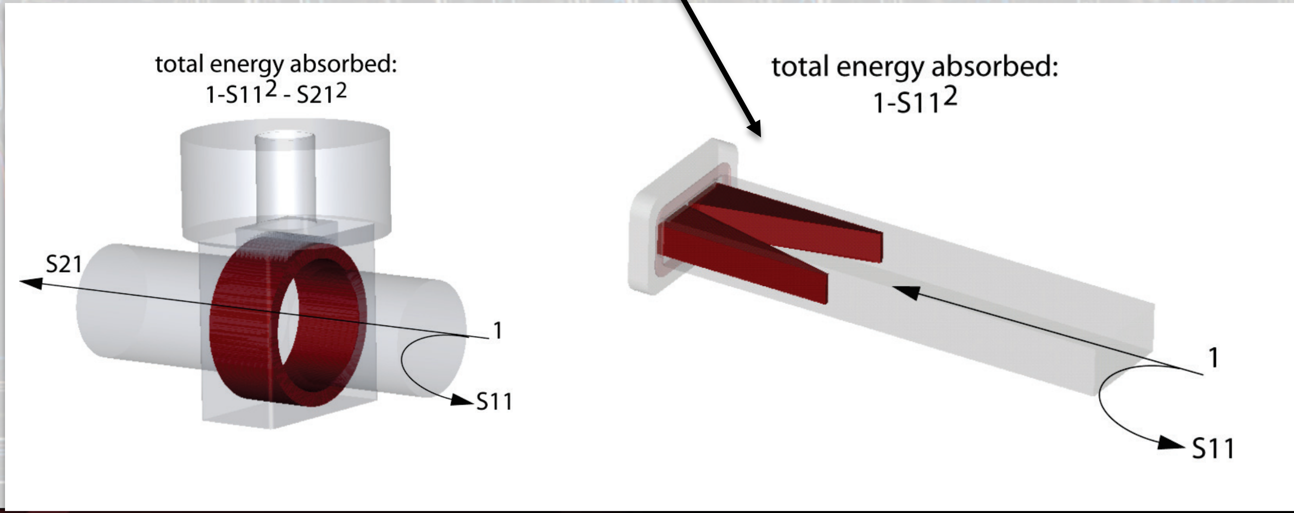
Proc. IPAC 2011

ABSORBER MATERIALS AT ROOM AND CRYOGENIC TEMPERATURES*

F. Marhauser#, T. Elliot, A.T. Wu, JLab, Newport News, VA 23606, U.S.A.
E. Savrun, Sienna Technologies Inc., Woodinville, WA 98072, U.S.A.
E. Chojnacki, Cornell University, Ithaca NY 14853, U.S.A.

How to measure HOM damping performance

- S-Parameter calculations (model shown below) can be readily done, but measurements require special hardware (adapter, waveguides)
- The load material is yet unknown and the shape of the load depends on it
- Use Vector Network Analyzer to measure the reflection response (S11) into a waveguide terminated with the HOM load
- All adapters are bandwidth-limited and allow only for a certain frequency range to be covered unless various sets are used
- The energy absorbed is $1-S11^2$
- A special calibration technique employed (in time domain, not in frequency domain) to de-embed the device under test (DUT), which is the load material – from the adapter



SRF Hands-On Course at JLab

- **Goal of this course:**
  - Hands-on experiments related to HOMs
  - Learn how to perform measurements utilizing a Vector Network Analyzer (VNA)

http://www.agilent.com/

- **Two Tasks:**
  1) **Conduct an HOM survey for a multi-cell Superconducting Radio Frequency (SRF) cavity immersed in a Helium bath at 2 Kelvin**
     - Experiment will be done in JLab's Vertical Test Area (VTA)
  2) **Measure the damping performance of various HOM absorber materials at room temperature**
     - Experiment will be done in JLab's RF structure lab

SRF Hands-On Course at JLab

- **Requirements and Leaning Experience:**
  - Basic understanding of the functions a VNA and S-Parameters
  - Use of calibration techniques (incl. using time-domain option)
  - Understanding of other RF equipment, e.g. calibration standards, waveguides and adapters
  - Record/analyze/report/discuss measurement data, e.g. ask yourself whether measurement are accurate enough? How to improve accuracy? What limits the accuracy?
- **Teamwork:**
  - We split up in 2 groups à 2 students
  - Swap experiment during afternoon depending on speed
  - There might be students with previous experience, particularly how to use a VNA, some others may not have any experience

Backup Slides and Further Information

Fundamental Theorem of Beam Loading

- When a particle with given charge q and energy traverses cavity (cavity may or may not be powered) it excites a voltage with magnitude V_j along its path (j denotes any specific mode)
- Note that a mode with an electrical field pointing along the beam path is required to excite the mode
- Imagine that particle represents a current (I=dq/dt) which excites this voltage along a cavity impedance (resembles Ohm's law: V = R·I)
- R represents the so-called shunt impedance: $R_{sh,j} = \frac{V_j}{I_{avg}} = \frac{V_j^2}{P_{avg,j}}$
- $P_{avg\,j}$ is the average power dissipated in the cavity in mode j
- The power dissipated, resp. the associated energy lost ΔW_j can only originate from the particle itself (no driving source)
  - → The particle must be decelerated by a voltage $V_{j,dec}$
- The energy lost is proportional to the bunch charge: $\Delta W_j = W_j(0) = q \cdot V_{j,dec}$
  - This energy is stored in the cavity when the bunch just left the cavity (t = 0)
    - For now we omit the damping term and that the voltage oscillates at the mode frequency for simplicity

Fundamental Theorem of Beam Loading

- Question is: How large is the decelerating voltage $\boxed{V_{j,dec}}$ that acts on the beam itself when traversing the cavity, i.e. it is not clear yet whether the particle experiences the full voltage $\boxed{V_j}$ it excited
- What is the fraction $\boxed{f = \frac{V_{j,dec}}{V_j}}$?
- The short answer is: $\boxed{V_{j,dec} = \frac{1}{2} \cdot V_j}$
- This is called the **Fundamental Theorem of Beam Loading (FTBL)**
  - One derivation is given in Padamsee's book (p. 333)
- **The beam experiences exactly half the voltage it has excited in the cavity**
- True for any given mode j (superposition principle applies)

Fundamental Theorem of Beam Loading

- As a consequence, the stored energy in the cavity is

$$\boxed{\Delta W_j = W_j(0) = \frac{1}{2} q \cdot V_j}$$

- We also know that the energy stored in a cavity is proportional to the voltage squared (since ~E² at times energy is in electric field) $\boxed{W_j(0) \sim V_j^2}$
- The proportionality factor per definition is

$$\boxed{W_j(0) = \frac{1}{\omega_j \cdot \left({}^{R}/_{Q}\right)_j} \cdot V_j^2}$$ or $$\boxed{\left(\frac{R}{Q}\right)_j \equiv \frac{V_j^2}{\omega_j \cdot W_j(0)}}$$ $\boxed{in\ \Omega}$ (accelerator definition)

- Herein R/Q denotes the characteristic shunt impedance
- **R/Q only depends on cavity geometry (no surface losses involved yet)**

Loss Factor

- Combining $W_j(0) = \frac{1}{2} q \cdot V_j$ & $W_j(0) = \frac{1}{\omega_j \cdot \left({}^{R}/_{Q}\right)_j} \cdot V_j^2$
- Yields $V_j = q \cdot \frac{\omega_j}{2} \left(\frac{R}{Q}\right)_j$
- **This means that the excitation strength at given bunch charge only depends on the mode frequency and R/Q (given by cavity shape)**
- This expression is also used to define the loss factor for a certain mode j

$$V_j = q \cdot \frac{\omega_j}{2} \left(\frac{R}{Q}\right)_j \equiv 2 \cdot q \cdot \kappa_{j,z}$$

or

$$\kappa_{j,z} \equiv \frac{\omega_j}{4} \left(\frac{R}{Q}\right)_j$$

$in\ V/C$

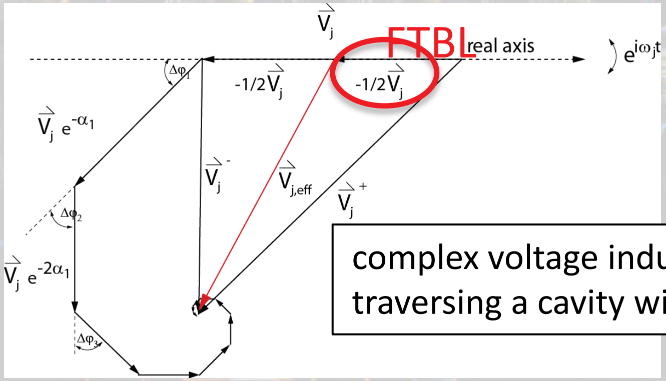
- This is the **longitudinal loss factor**
  - Can be readily computed numerically for any cavity shape
- The energy in turn is then $W_j(0) = q^2 \cdot \kappa_{j,z}$
- Very important:
  Dangerous modes are those with high R/Q-values, respectively loss factors

Which HOMs are dangerous?

- To excite an HOM an electrical field along the beam path is required
- The beam excites a voltage
- With a beam traversing on the axis of a perfect cylindrically-symmetrical cavity only Monopole modes are excited
- However this is the ideal world, i.e.
    - Beam has finite radial size
    - Cavity may not be aligned with ideal orbit
    - Cavity fabrication imperfections (e.g. cell eccentricities) and symmetry-breaking elements (couplers) cause RF field asymmetries
- Transverse mode (dominantly dipole modes) can be excited from noise
- Once excited, subsequent particles will experience transverse force
- This will excite transverse HOM even stronger (R/Q increases with r)
    - Can lead to resonant buildup, i.e. same and/or subsequent particle bunches can add resonantly to energy lost in the same cavity, while experiencing the accumulated voltage

Complex Voltage Summation

- After the bunch left the cavity, the voltage/energy oscillates freely with HOM frequency with ω_j and is damped by means of the decay time $\tau_j = Q_l/\omega_j$ as derived above
- This creates a complex voltage $$V(0)_{j,eff} = (2q\kappa_{j,z}) \cdot e^{-\frac{\omega_j}{2Q_l} \cdot t} \cdot e^{-i\omega_j \cdot t}$$
    - Factor 2 in exponent of damping term due to voltage (□ energy)
- Subsequent bunches may add in complex way depending on phase slippage $\Delta\phi$, this can be illustrated by voltage phasor diagram



complex voltage induced by a finite chain of particles traversing a cavity with phase slippages $\Delta\phi$

- When $\Delta\phi \neq 0$, the voltage phasor rotates by $\Delta\phi$ given by the bunch repetition rate and the HOM frequency, the real part of the voltage sum is what affects the particle
- When $\Delta\phi=0$, this voltage constantly increases (resonance) and is real throughout → HOM coincides with beam spectral line

HOMs and Consequences

- Threshold current (I_{th}) for BBU is defined by beam current above which BBU-causing HOM produces a net gain
  - or at threshold the power fed into the HOM equals the power dissipated
- 2-pass BBU threshold:

$$I_{th} = \frac{-2p_1 c}{e \cdot k (R/Q) \cdot Q_l \cdot e^{-\frac{\omega}{2Q_l} \cdot T_r} M \cdot \sin(\omega T_r + l \cdot \pi/2)}$$

p_1 = 1st pass beam momentum
M = optical transfer matrix element from cavity back to itself (1,2 – hor., 3,4 – vert., 5,6 – long.)
l = 1 for longitudinal modes, l = 0 for all other
T_r = recirculation time
k = ω/c
$R_{sh} = (R/Q)*Q_l$ = HOM shunt impedance

- Multipass BBU:
  - All higher passes may contribute to BBU, replace

$$M \sin(\omega T_r) \rightarrow \sum_j^N \sum_{i<j} M^{ij} \frac{p_1}{p_i} sin(\omega T_r^{ij})$$

N = number of passes
M^{ij} = transfer matrix element form pass i to pass j
T^{ij}_r = transit time from pass i to pass j

- By measuring I_{th}, M, f and Q one may deduce R/Q of BBU mode
  - Identification of BBU mode possible in-situ by comparing with simulations

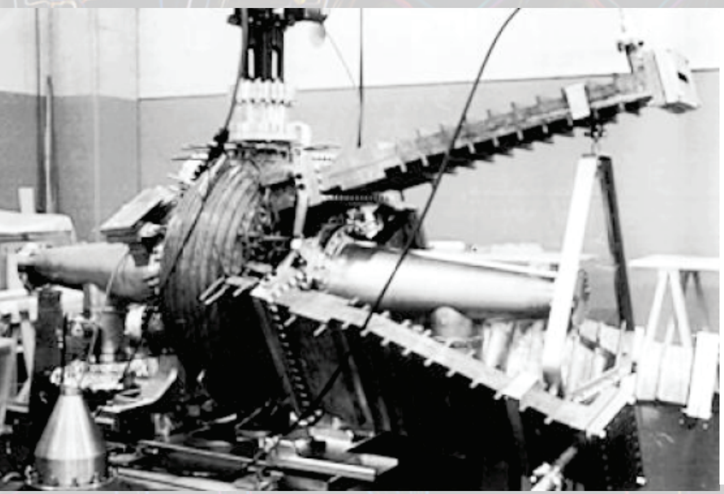
Cures

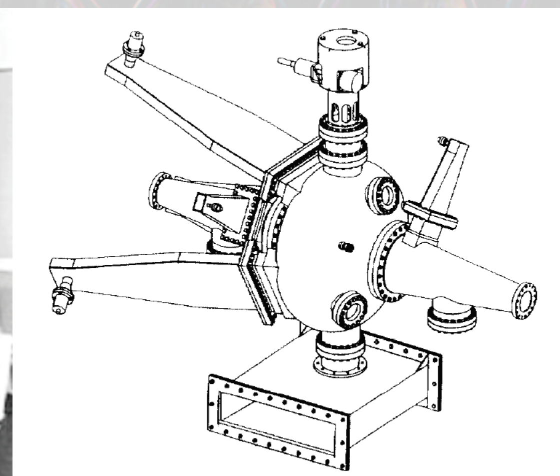
- ‘Tricks’ are possible to increase I_{th}, e.g. changing optics (M) or implement phase space rotators to decouple resonant motion
- Or beam-based feedback system to kick beam back to ideal reference orbit (complex systems)
- Primary goal however: Damp HOMs ↔ Reduce Q_l

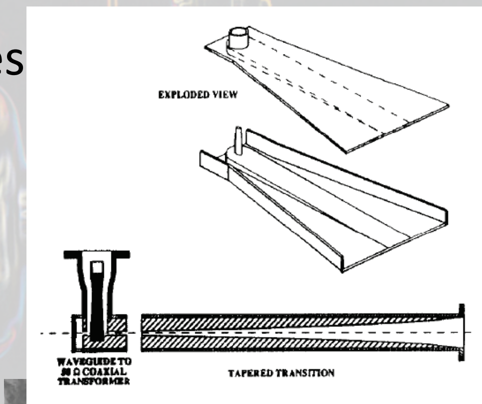
$$I_{th} = \frac{-2p_1 c}{e \cdot k (R/Q) \cdot Q_l \cdot e^{-\frac{\omega}{2Q_l} \cdot T_r} M \cdot \sin(\omega T_r + l \cdot \pi/2)}$$

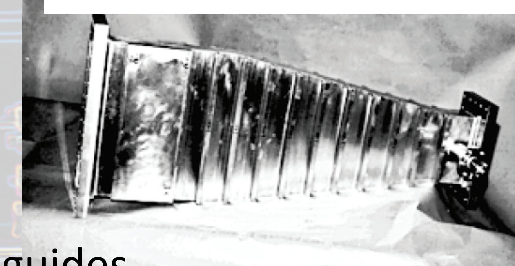
Examples: NC HOM-Damped Cavities

- 368 MHz DAΦNE Cavity
  - Employs rectangular waveguides-to-coaxial line transitions for HOM damping
  - Avoids under vacuum RF absorbers, i.e. power dissipated in external standard 50 Ω loads through vacuum RF window
  - 3 dampers on cavity cell, 2 on tapered beam tubes





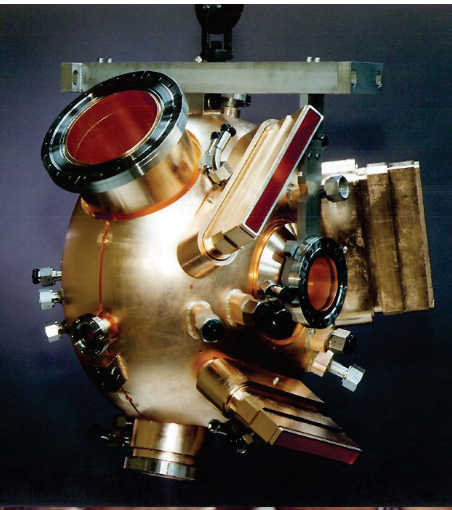


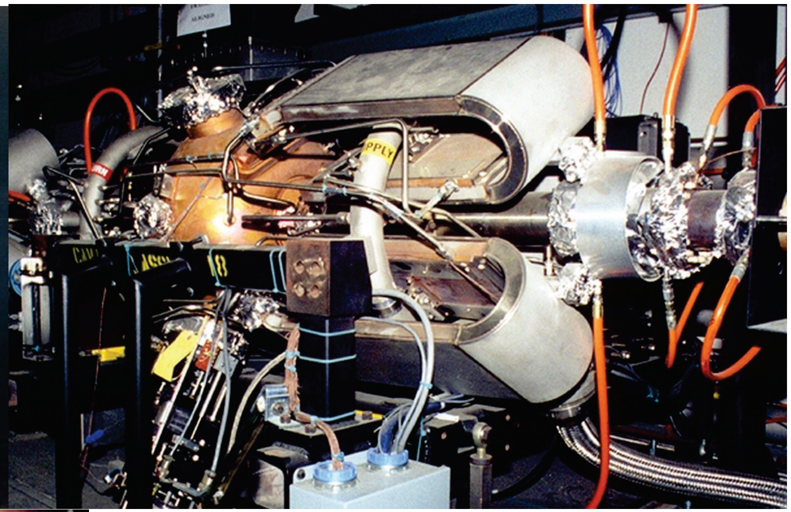


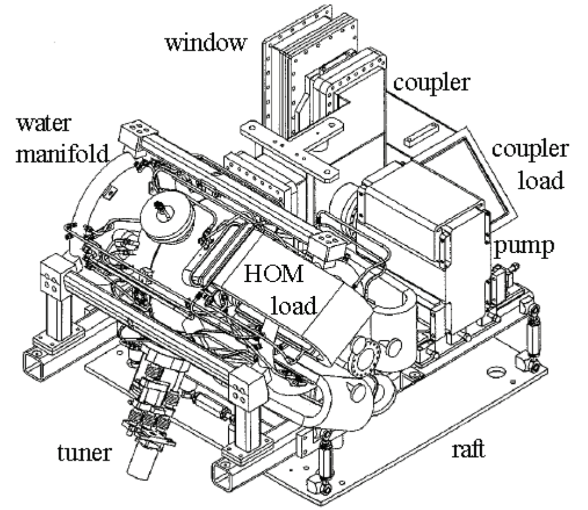
$R = 4\ \mathrm{M\Omega}$, $R/Q = 121\ \Omega$, $Q_0 = 33000$ with waveguides,
($R/Q = 32\ \Omega$ for TM_{011}-mode)

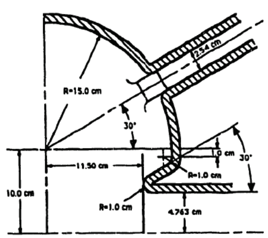
Examples: NC HOM-Damped Cavities

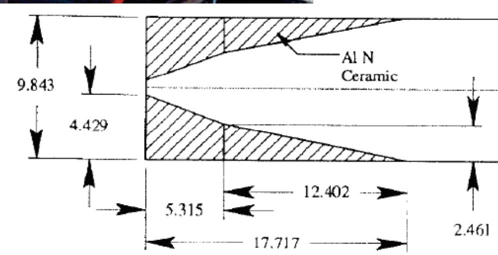
- 476 MHz PEP-II Cavity
  - Employs 3 rectangular (folded) waveguides for HOM damping
  - Using in-vacuum RF absorbers (AlN-SiC), 3 dampers on cavity cell

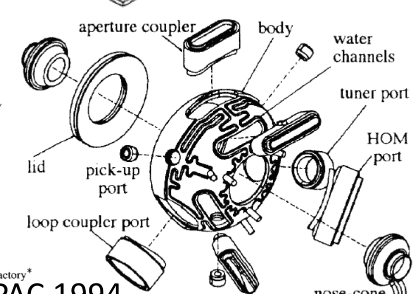












R = 7.6 MΩ, R/Q = 117.3 Ω, Q_0 = 32469,
Cutoff of waveguides (250 x 25.1 mm) is 600 MHz, 10 Kil0Watt HOM absorber loads
R/Q = 44.6 Ω for TM_{011} at 760 MHz)

High-Power RF Cavity R&D for the PEP-II B Factory*

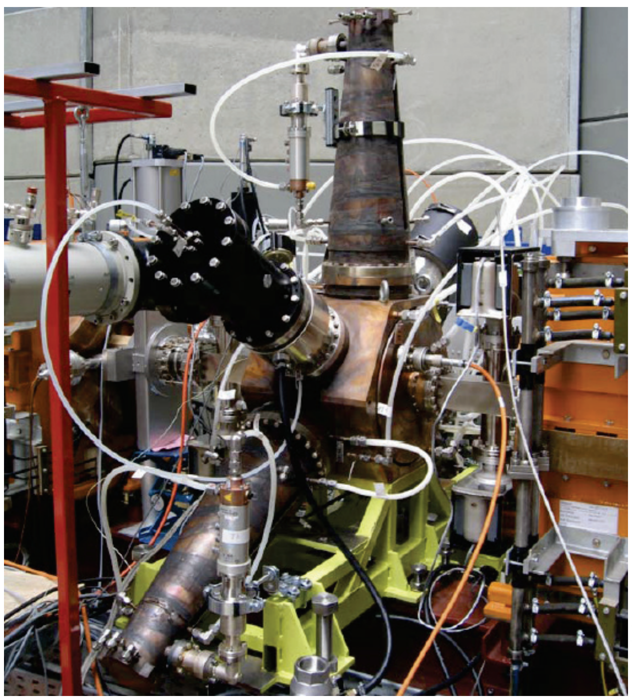
R. Rimmer, G. Lambertson,#
LBL, 1 Cyclotron Road, Berkeley, CA 94720, USA
J. Hodgson, K. Ko, C. Ng, R. Pendleton, E. Reuter, H. Schwarz, K. Skarpaas, ‡
SLAC, 2575 Sand Hill Road, Menlo Park, CA 94025, USA
F. Adams, B. Burton, M. DeJong, M. Lipsett, †
AECL Chalk River Laboratories, Chalk River, Ontario, Canada, K0J 1J0
C. Belser, M. Franks, T. McCarville,**
LLNL, Livermore, CA 94551, USA

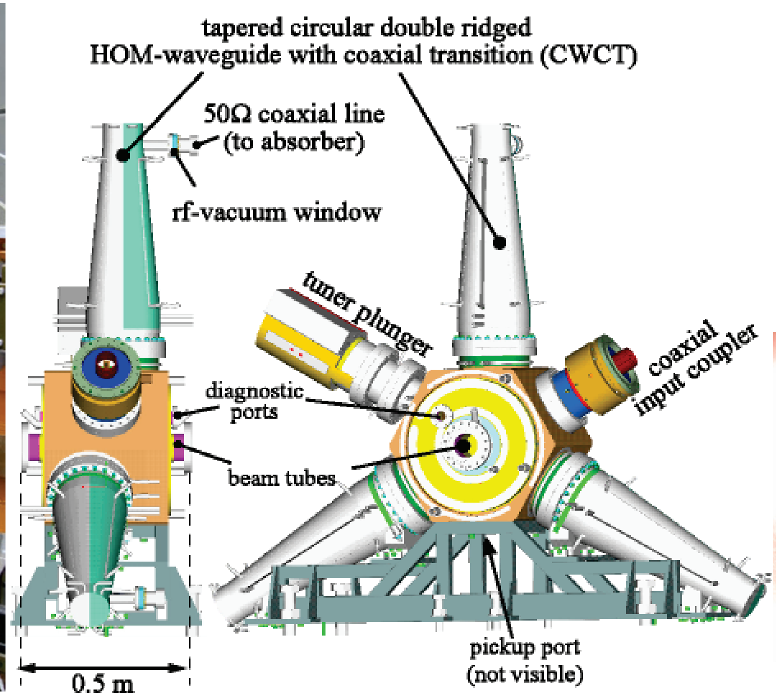
EPAC 1994

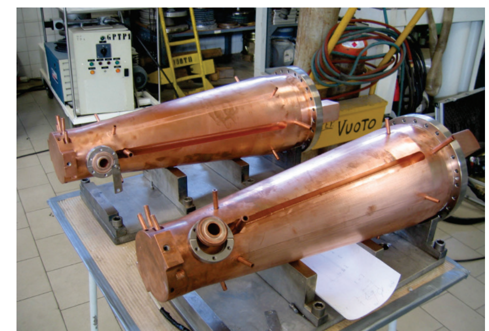
http://www.slac.stanford.edu/BFROOT/www/doc/public/babar_plots/PEPII/photographs/pep_ii_photographs.htm

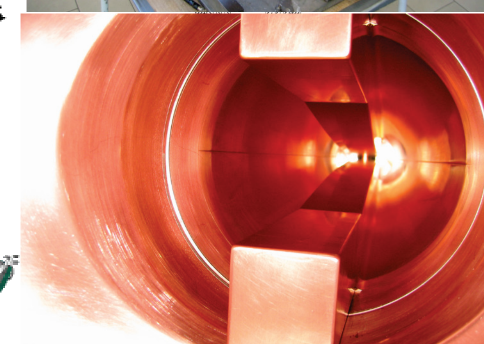
Examples: NC HOM-Damped Cavities

- 500 MHz BESSY (European) HOM-Damped Cavity
  - Uses double-ridged round waveguides
  - First prototype with tapered waveguide to coax line transition
  - Power dissipated in water-cooled standard 50 Ohm loads









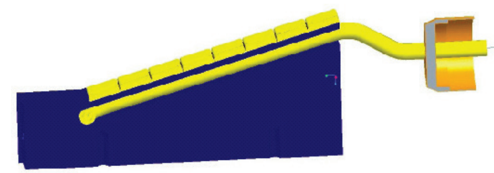
Proceedings of EPAC 2004, Lucerne, Switzerland

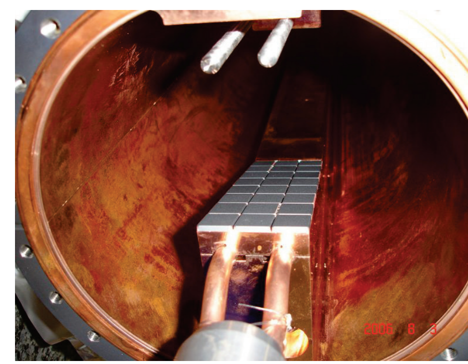
FIRST TESTS OF A
HOM-DAMPED HIGH POWER 500 MHZ CAVITY*

F. Marhauser#, E. Weihreter, BESSY II, Berlin, Germany

Examples: NC HOM-Damped Cavities

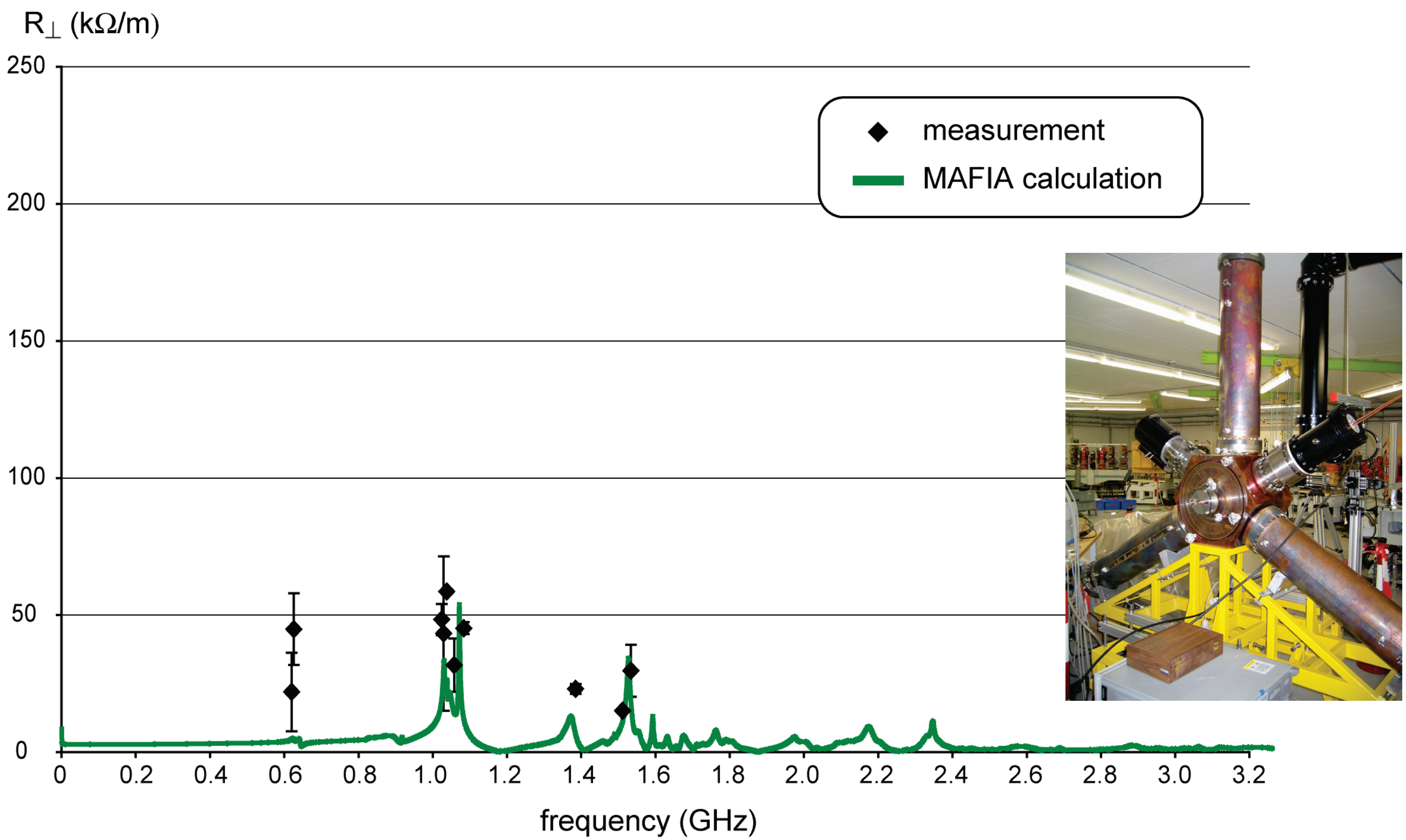
- 500 MHz BESSY (European) HOM-Damped Cavity
  - Series production with homogeneous double-ridged waveguides and in-vacuum water-cooled ferrite (NiZn) absorber tiles
  - Used successfully at BESSY, MLS, ALBA and ESRF storage rings





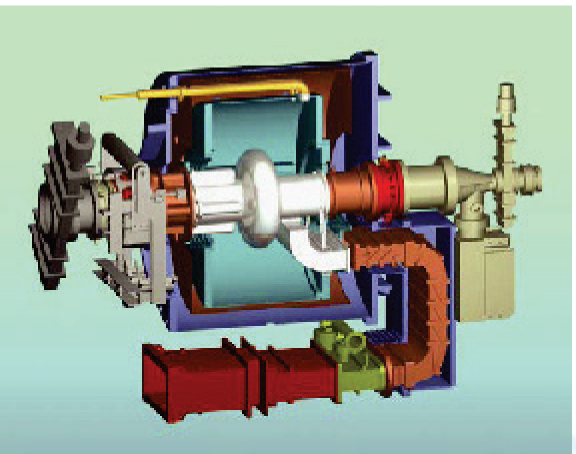
Examples: NC HOM-Damped Cavities

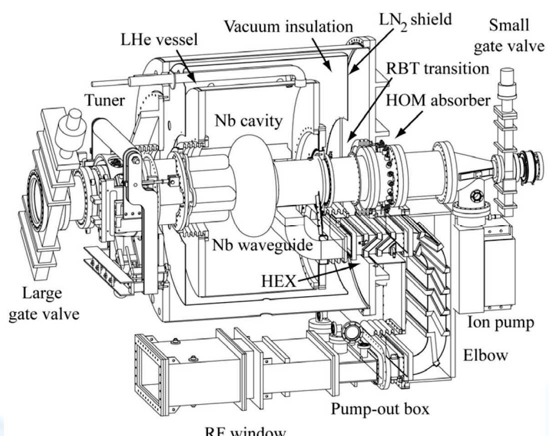
- 500 MHz BESSY (European) HOM-Damped Cavity
  - Excellent damping (< 60kΩ/m for dipole modes)

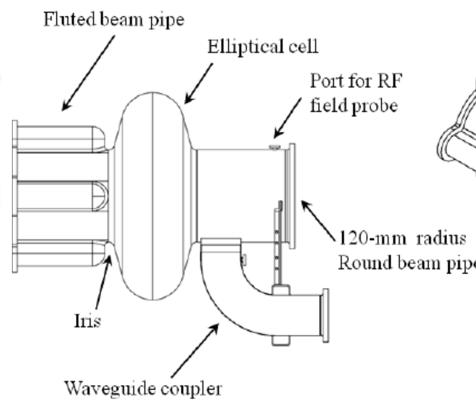


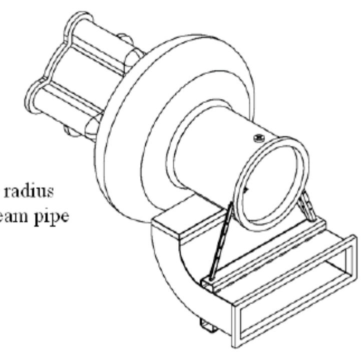
Examples: SRF HOM-Damped Cavities

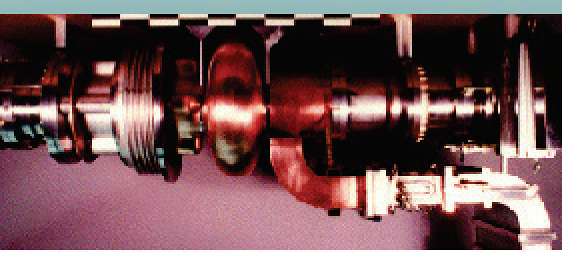
- 500 MHz CESR Cavity

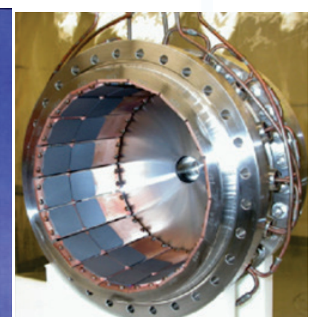












Presented at the 9-th Workshop on RF Superconductivity, Santa Fe, NM, November 1-5, 1999 SRF 000202-03

OPERATING EXPERIENCE WITH SUPERCONDUCTING RF AT CESR AND OVERVIEW OF OTHER SRF RELATED ACTIVITIES AT CORNELL UNIVERSITY*

S. Belomestnykh+, P. Barnes, E. Chojnacki, R. Ehrlich, R. Geng#, D. Hartill, R. Kaplan, J. Knobloch, H. Padamsee, S. Peck, P. Quigley, J. Reilly, D. Rubin, D. Sabol, J. Sears, V. Veshcherevich, G. Werner,
Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, U.S.A.

Examples: SRF HOM-Damped Cavities

- 500 MHz CESR Cavity
  - Fluted beam tube to untrap TE_{111} and TM_{110} modes without affecting fundamental mode

tube ID = 240 mm

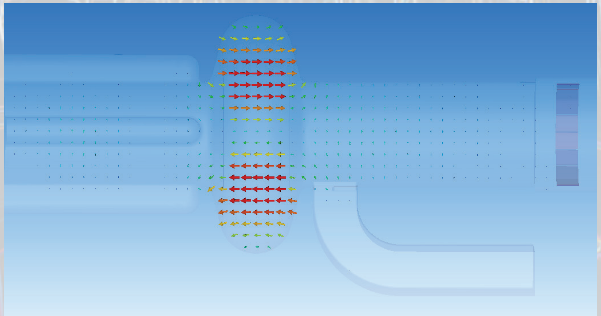
$f_{TE11}^{cutoff} = 732\ MHz$

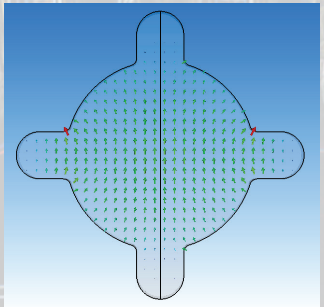
$f_{TM01}^{cutoff} = 956\ MHz$

fluted tube with inner ID 240 mm

$f_{TE11-like}^{cutoff} = 571\ MHz$

$f_{TM01-like}^{cutoff} = 928\ MHz$





tube ID = 300 mm (KEK)

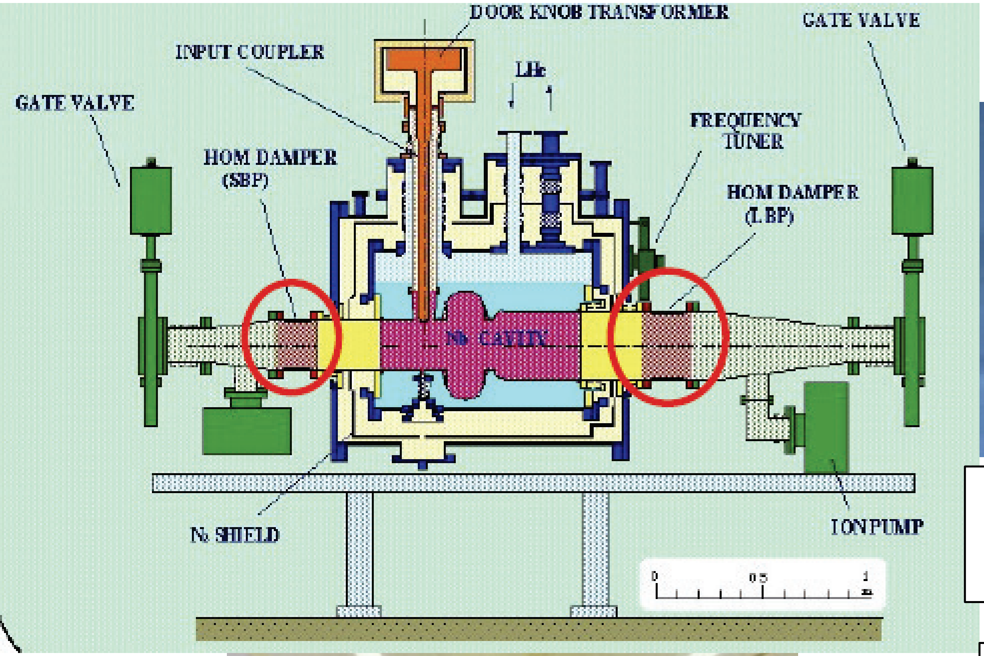
$f_{TE11}^{cutoff} = 586\ MHz$

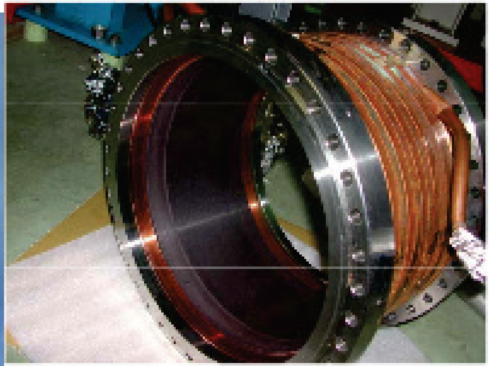
$f_{TM01}^{cutoff} = 765\ MHz$

this will impact TM_{010} more

Examples: SRF HOM-Damped Cavities

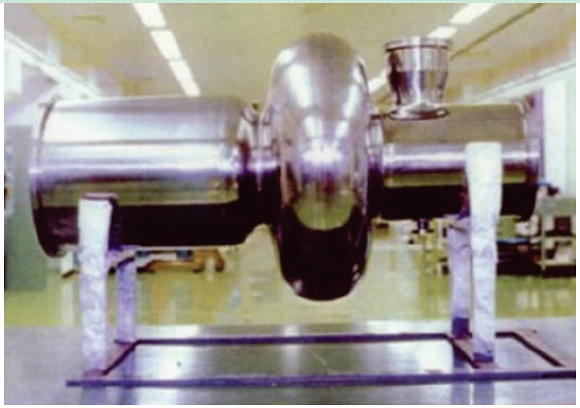
- 509 MHz KEKB SRF cavity





14 kW HOM power handling per cavity, water-cooled ferrite ring

IB0004 ferrite absorbers

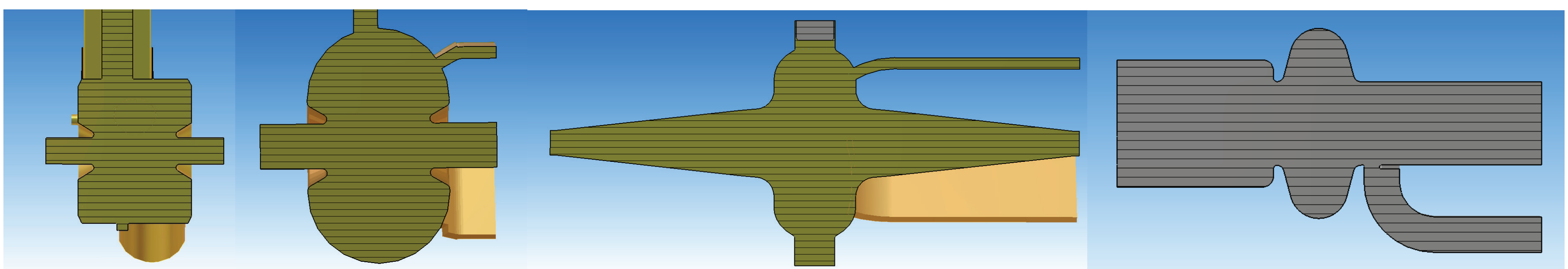


HOM'10 at Cornell U.
2010/Oct/11-14
T. Furuya, KEK

Examples: NC HOM-Damped Cavities

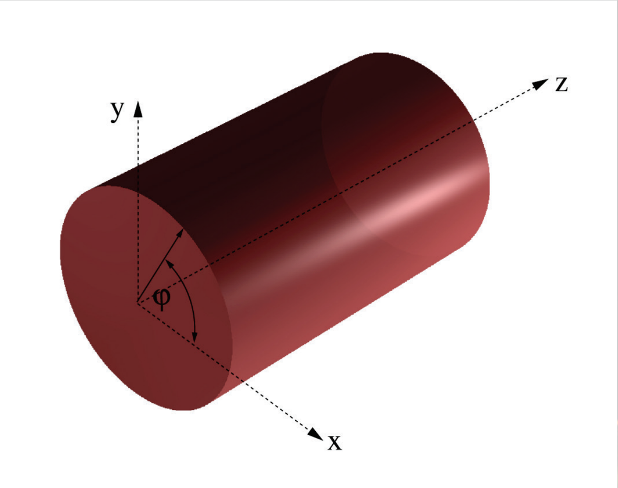
- Comparison of storage ring, e^+e^- collider cavities

Cavity	f_0 (MHz)	1st f_{cutoff} of damping waveguide (MHz)	f_{cutoff}/f_0	R/Q (Ω)	Q_0	R_{sh} (MΩ)	Beam tube ID (mm)
DAϕNE	368.3	491.5	1.33	121.2	33000	4.0	242.8 - 88
PEP-II	476	600	1.26	234.6 (meas.)	32469 (meas.)	7.62	95.25
BESSY	499.96	625	1.25	234.6 (meas.)	29626 (meas.)	6.95	74.0
CESR	499.77	n/a	n/a	89	$>10^9$ @ 4.5 K	> 89000	240 (fluted)
KEK-B	500	n/a	n/a		$>10^9$ @ 4.5 K		300



RF Mode Nomenclature for Cylindrically-Symmetrical Structures

- Nomenclatures for RF modes:
  - TE = Transverse Electric, $E_z = 0 \ \forall\ z$
  - TM = Transverse Magnetic, $H_z = 0 \ \forall\ z$
  - Index m = azimuthal (φ) field dependenc
  - Index n = radial (r) field dependency
  - Index p = longitudinal (z) field dependenc
- Types of RF fields determined by m
  - m = 0 is monopole mode
  - m = 1 is dipole mode
  - m = 2 is quadrupole
  - m = 3 is sextupole
  - m = 4 octupole asf.

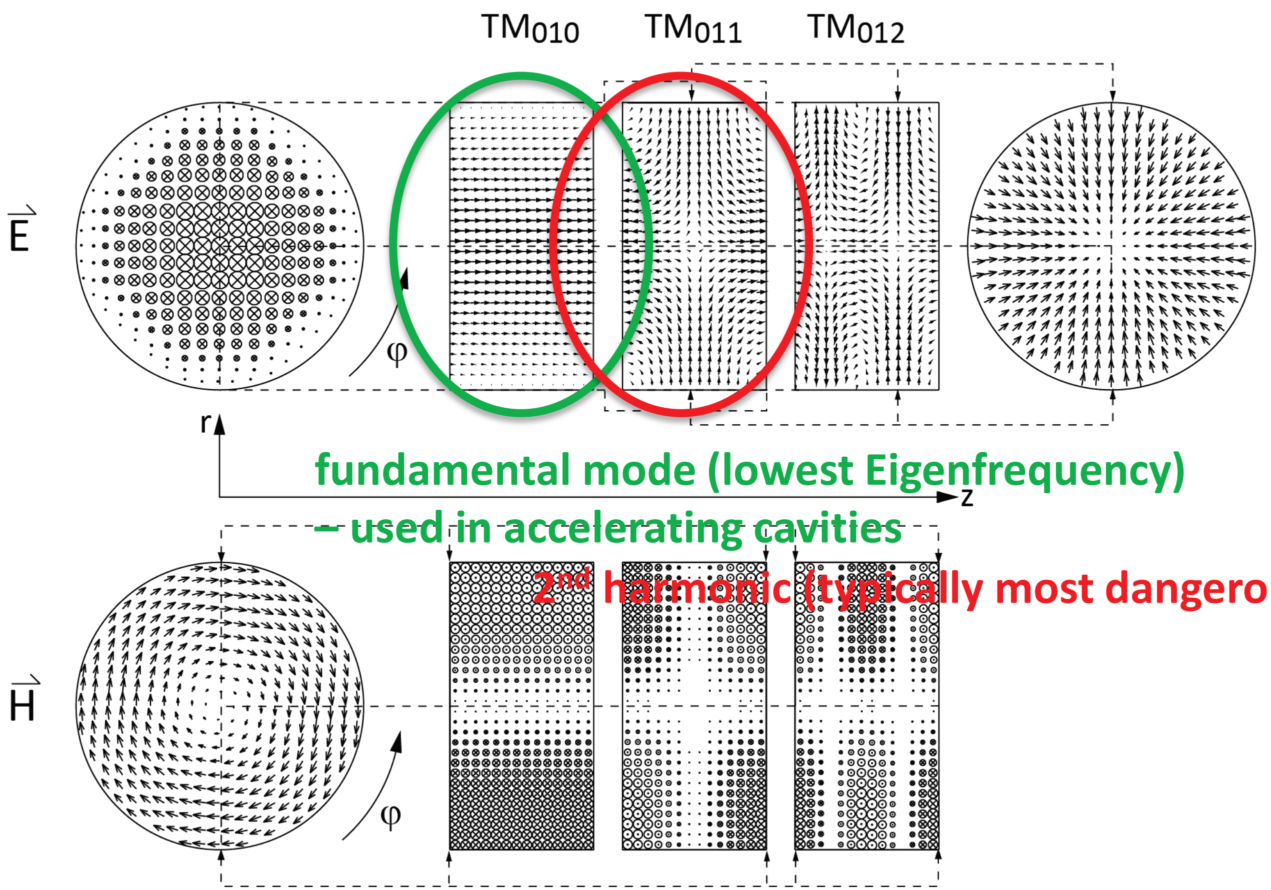


$$\omega_{mnp}^{TM} = 2\pi f_{mnp}^{TM} = 2\pi \frac{c}{\lambda_{mnp}^{TM}} = \frac{1}{\sqrt{\varepsilon\mu}} \sqrt{\left(\frac{p\pi}{L}\right)^2 + \left(\frac{x_{mn}}{R}\right)^2}$$

$$\omega_{mnp}^{TE} = 2\pi f_{mnp}^{TE} = 2\pi \frac{c}{\lambda_{mnp}^{TE}} = \frac{1}{\sqrt{\varepsilon\mu}} \sqrt{\left(\frac{p\pi}{L}\right)^2 + \left(\frac{x'_{mn}}{R}\right)^2}$$

Cylindrical Cavity (Pillbox)

- TM-monopole modes
  - These modes posses longitudinal electric field components along cavity axis → can be readily excited by beam traversing cavity



fundamental mode (lowest Eigenfrequency) – used in accelerating cavities

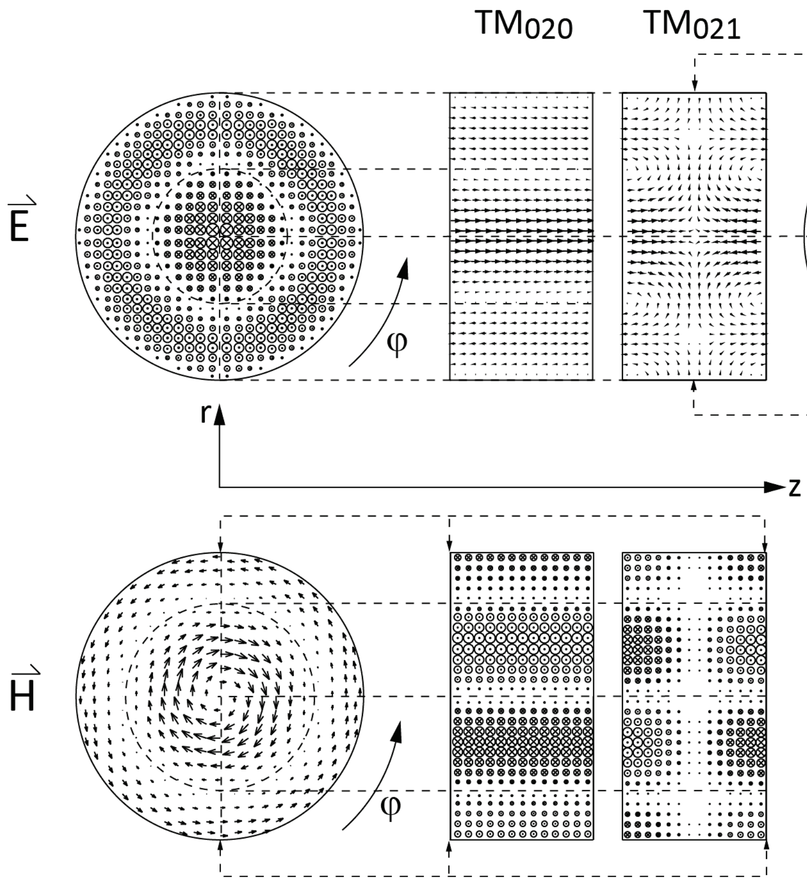
2nd harmonic (typically most dangerous monopole HOM)

$\mathbf{m = 0}$
$\mathbf{n = 1}$
$\mathbf{p = 0, 1, 2}$

Note: $n \geq 1$

Cylindrical Cavity (Pillbox)

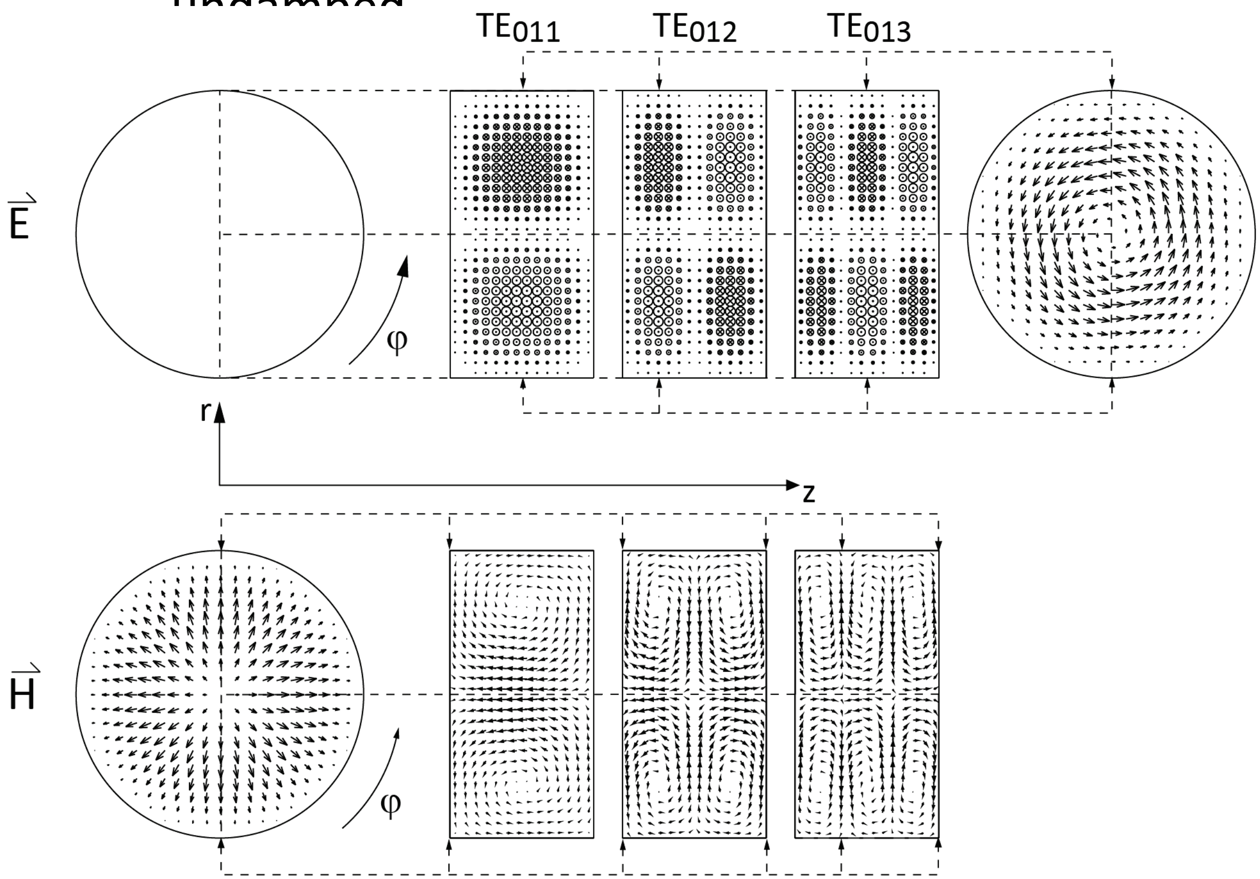
- TM-monopole modes
  - These modes posses longitudinal electric field components along cavity axis → can be readily excited by beam traversing cavity



$$\mathbf{m} = 0$$
$$\mathbf{n} = 2$$
$$\mathbf{p} = 0, 1$$

Cylindrical Cavity (Pillbox) Modes

- TE-monopole modes
  - These modes do not posses long. electric field components along cavity axis → typically harmless for beam even when left undamped

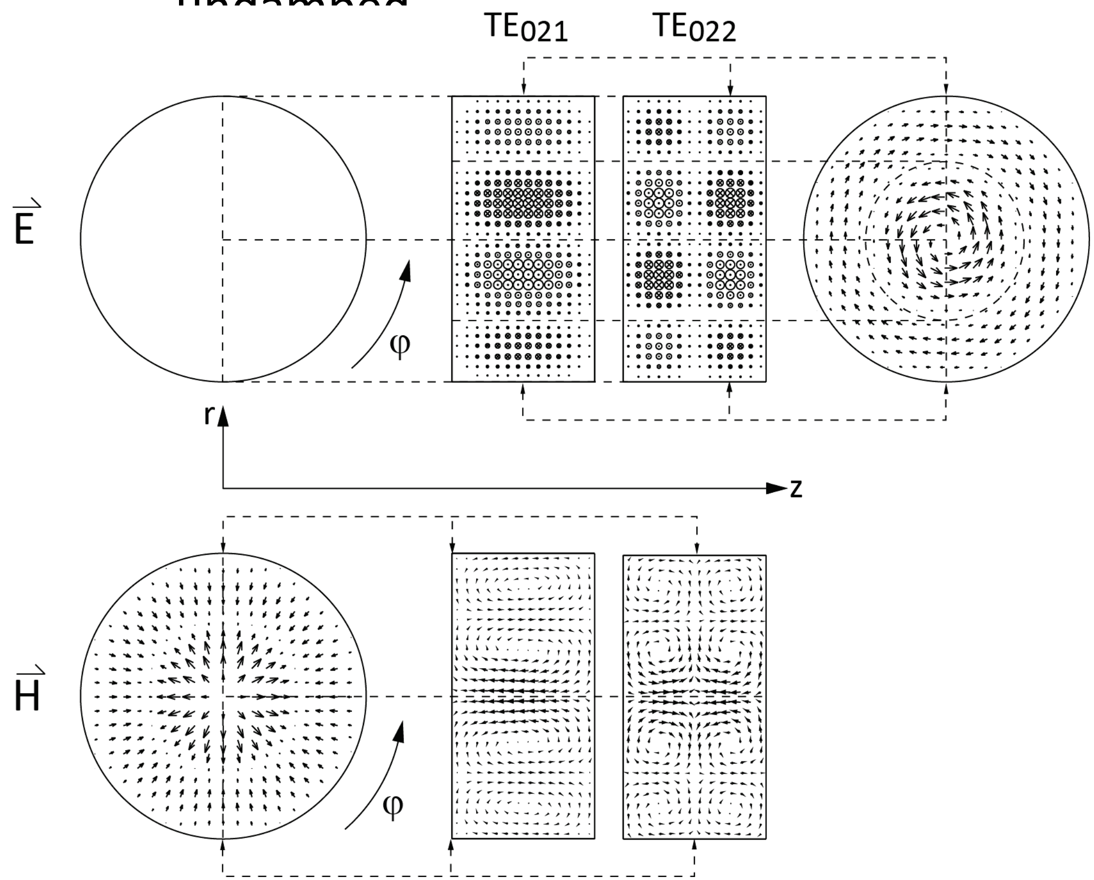


$$\mathbf{m = 0}$$
$$\mathbf{n = 1}$$
$$\mathbf{p = 1, 2, 3}$$

Note:
p = 0 not possible for TE-modes

Cylindrical Cavity (Pillbox)

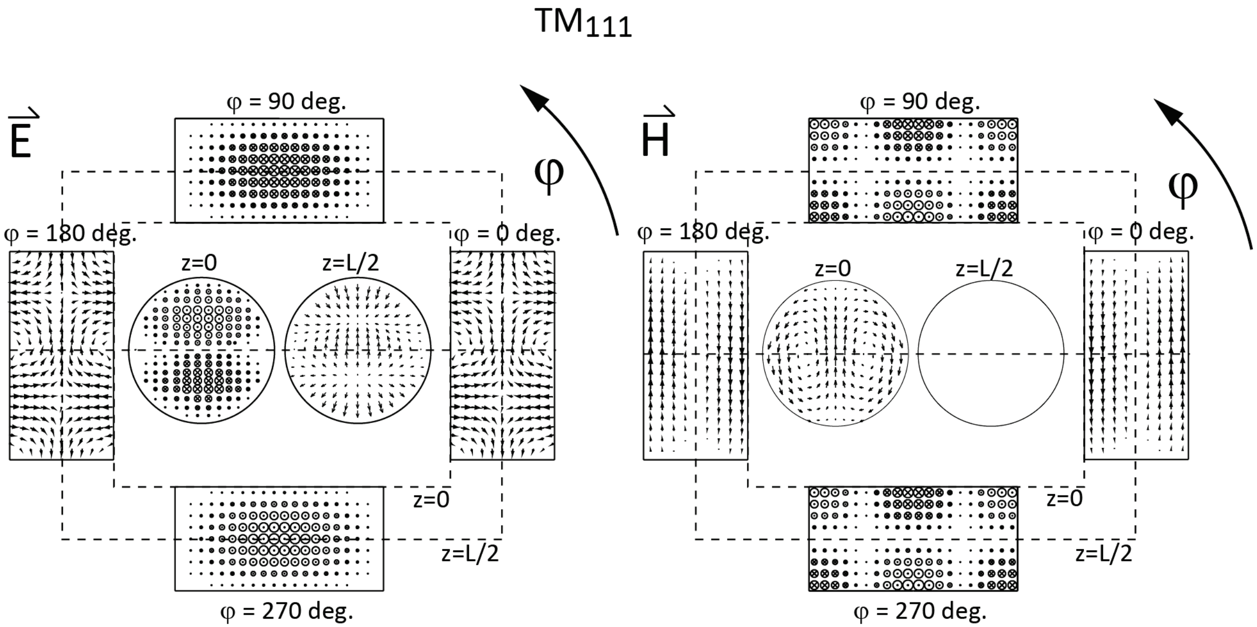
- TE-monopole modes
  - These modes do not posses long. electric field components along cavity axis → typically harmless for beam even when left undamped



$$\mathbf{m = 0}$$
$$\mathbf{n = 2}$$
$$\mathbf{p = 1, 2}$$

Cylindrical Cavity (Pillbox)

- TM-dipole modes
  - These modes posses strong long. electric field components off axis (no long. field on axis theoretically) → once beam is off axis deflection possible -> subsequent beam resonant effect

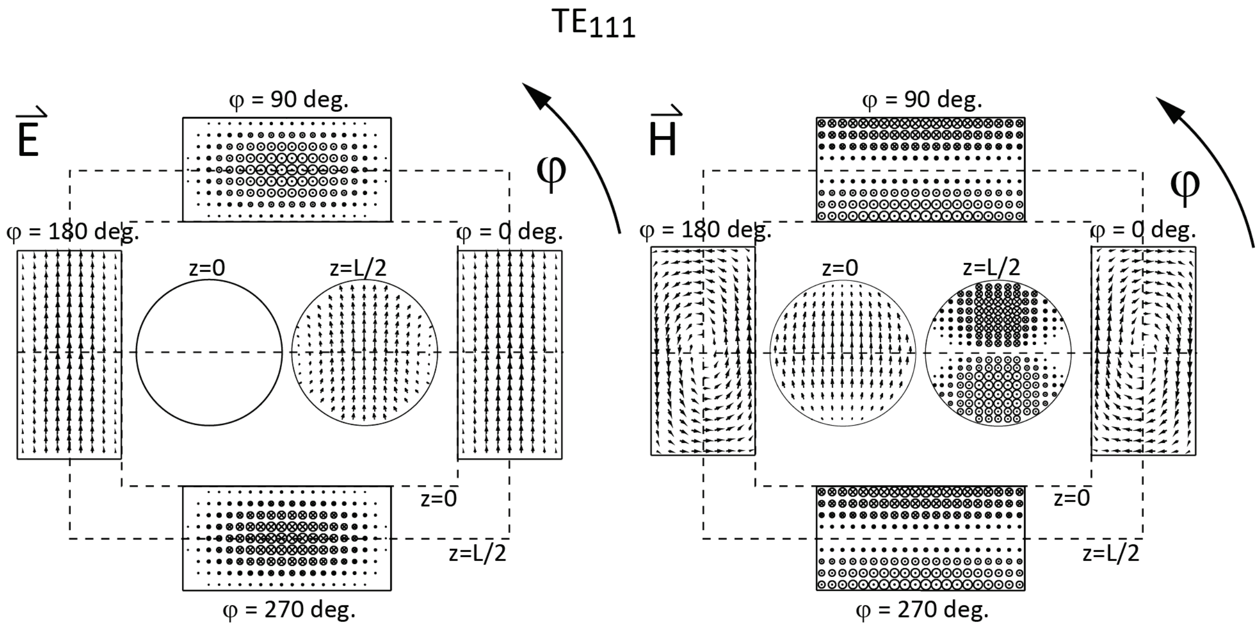


m = 1
n = 1
p = 1

Note:
p = 0 is possible
(not shown here)

Cylindrical Cavity (Pillbox)

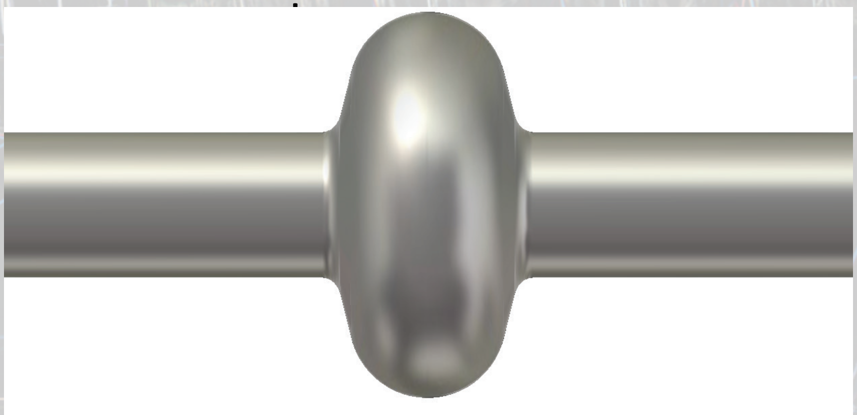
- TE-dipole modes
  - These modes do theoretically not posses long. electric field components on and off axis

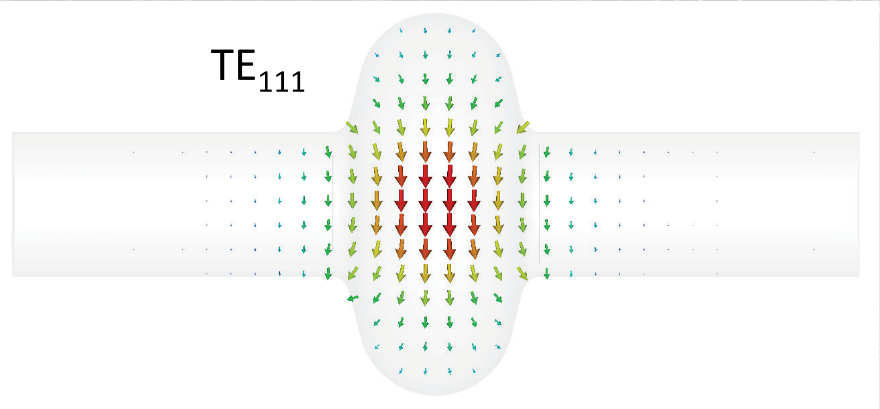


$$\begin{aligned} m &= 1 \\ n &= 1 \\ p &= 1 \end{aligned}$$

Elliptical Cavity

- SRF accelerating cavities exhibit elliptical cell contour
- Beam tubes need to be added
- As a consequence RF fields deviate from pure pillbox fields
  - No closed analytical expression of fields possible
  - Numerical codes required for RF field computations
- TM-modes may have longitudinal magnetic field components
- TE-modes may have longitudinal electric field component
  - Can be significant such that e.g. TE-dipole HOMs become

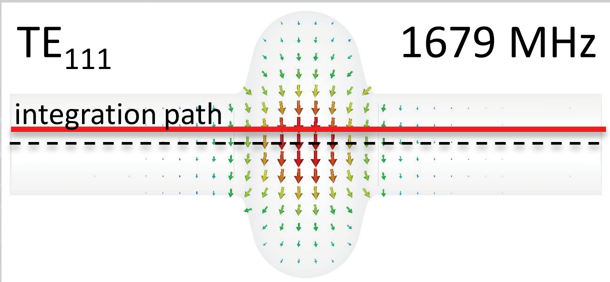


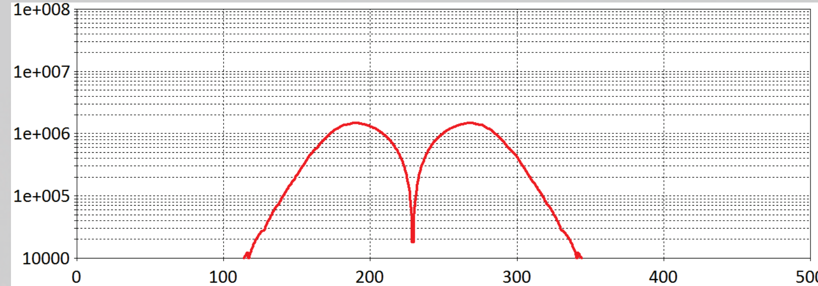


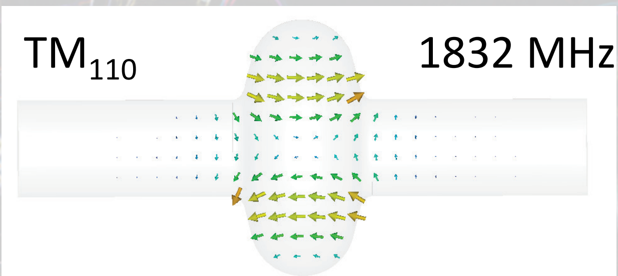
Elliptical Cavity (1-cell from TESLA cavity endcups) Dipole HOMs

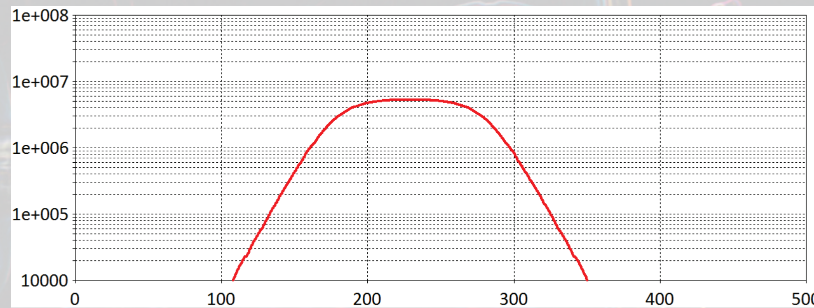
shorted beam tubes throughout

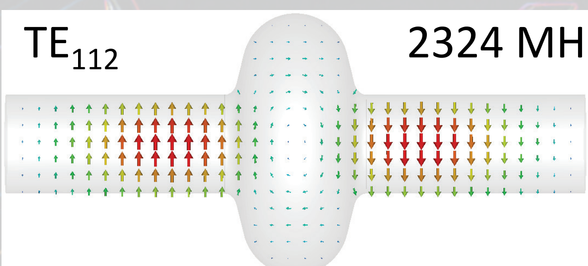
$|E|_{long}$ (V/m @ 1 Joule stored energy)

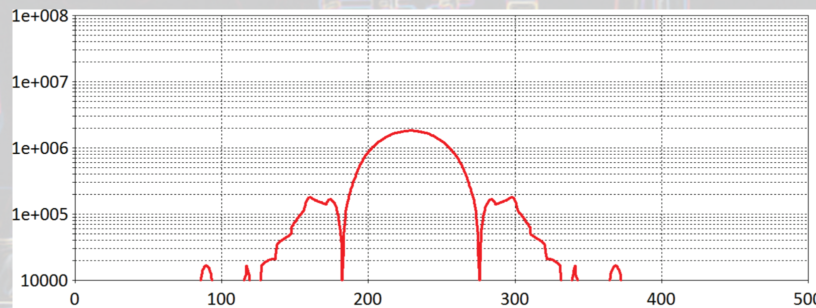








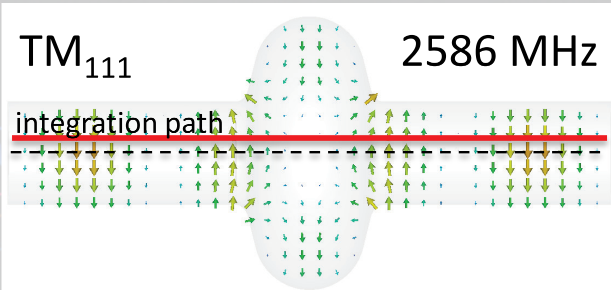


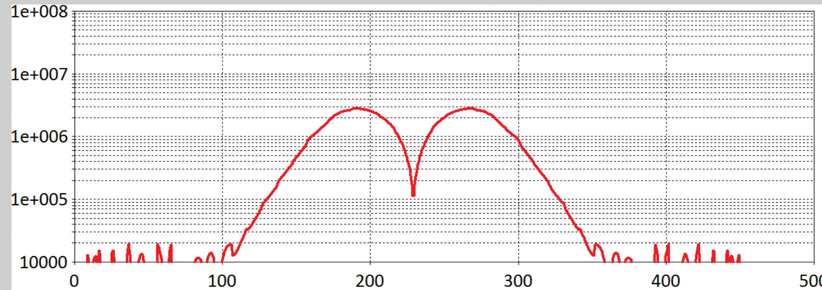


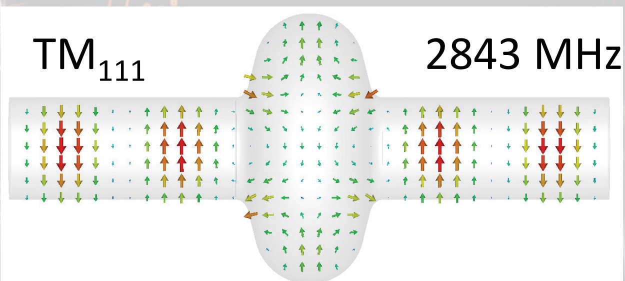
position (mm)

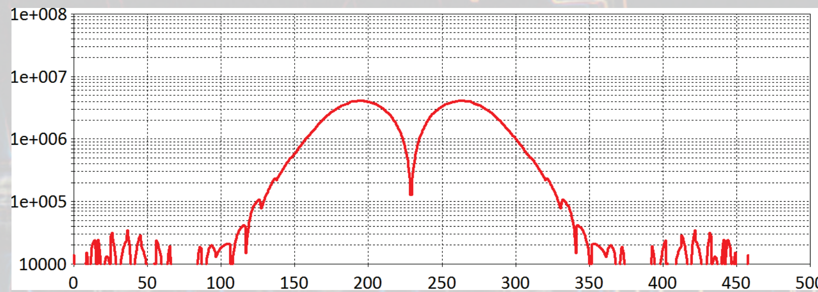
Elliptical Cavity (1-cell from TESLA cavity endcups) Dipole HOMs

$|E|_{long}$ (V/m @ 1 Joule stored energy)

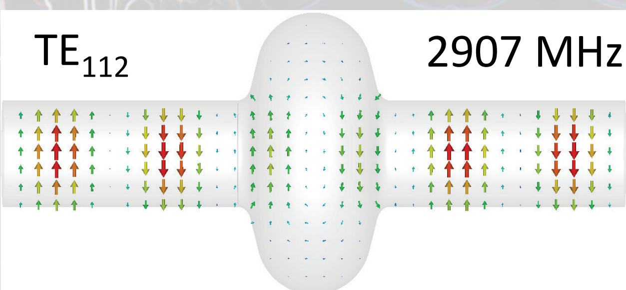


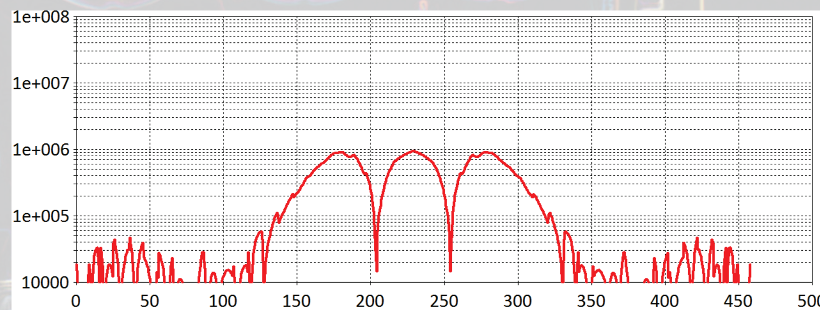






same type of mode as previous, propagating mode, i.e. reflecting boundaries matters!



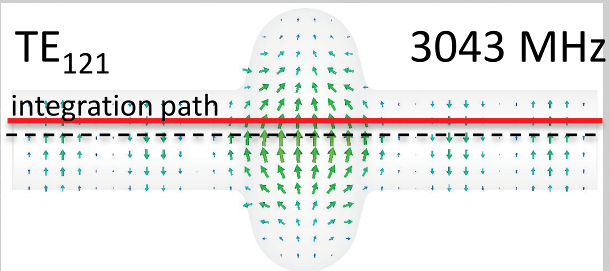


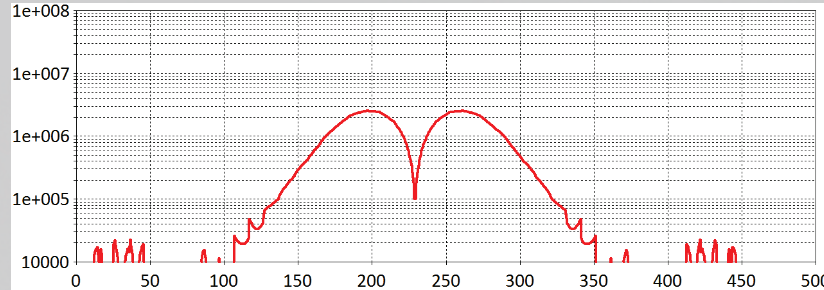
same type of mode as before, propagating mode,
i.e. reflecting boundaries matters. Beam tube modes appear!

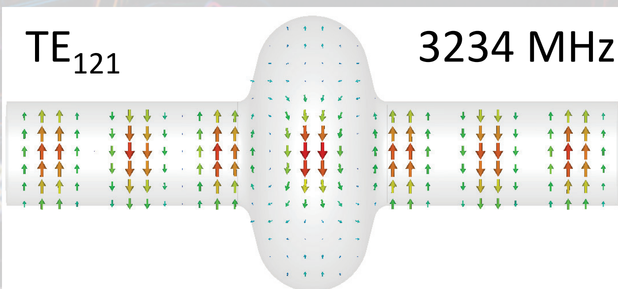
position (mm)

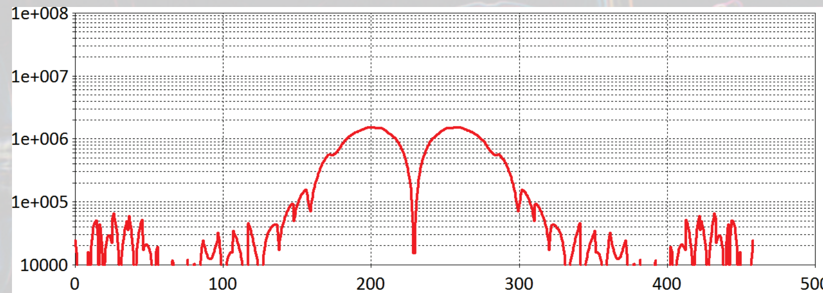
Elliptical Cavity (1-cell from TESLA cavity endcups) Dipole HOMs

$|E|_{long}$ (V/m @ 1 Joule stored energy)

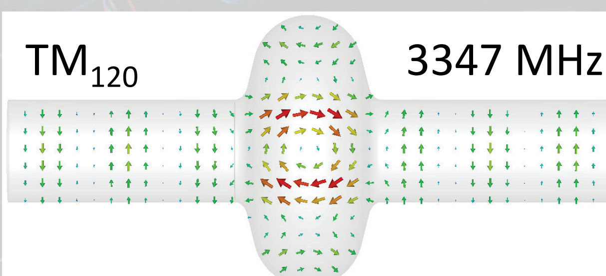


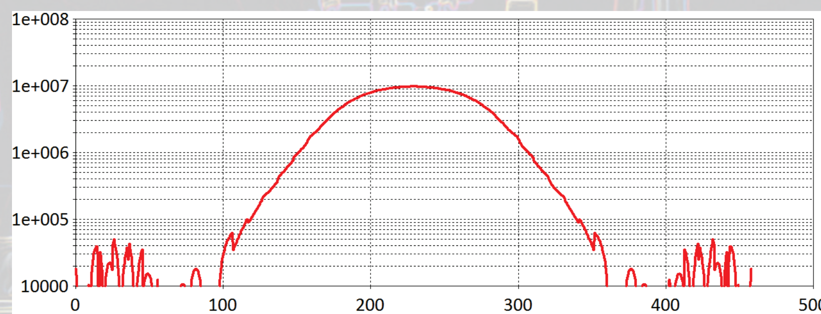






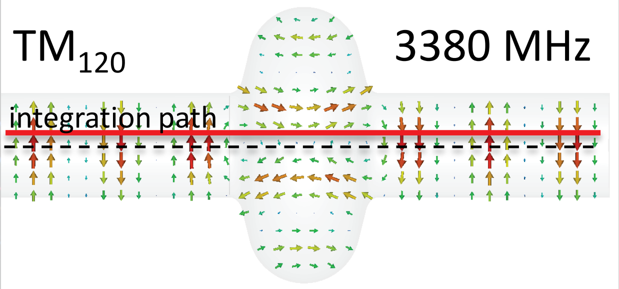
same type of mode as previous, propagating mode, i.e. reflecting boundaries matters!

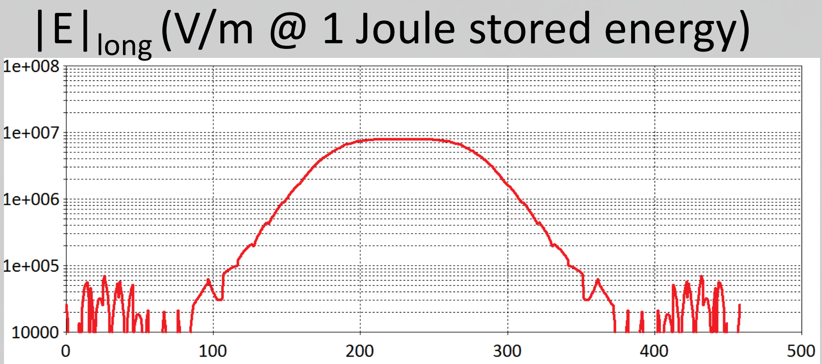




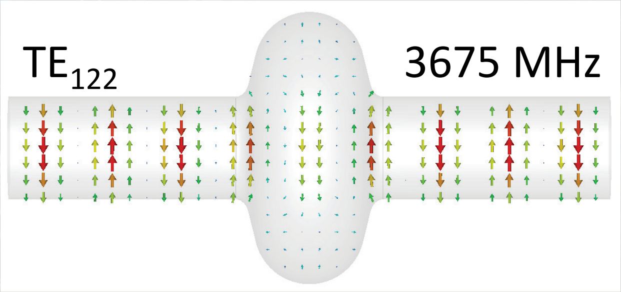
position (mm)

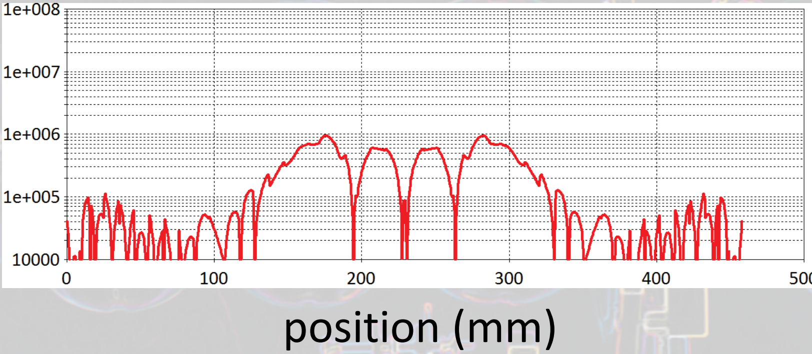
Elliptical Cavity (1-cell from TESLA cavity endcups) Dipole HOMs



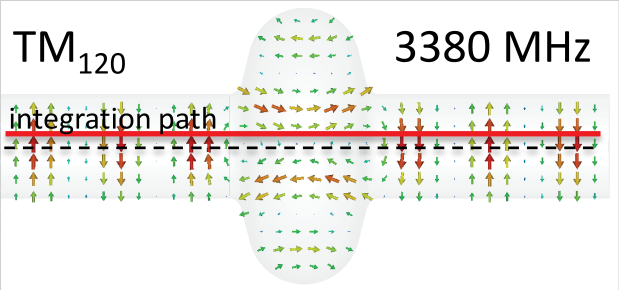


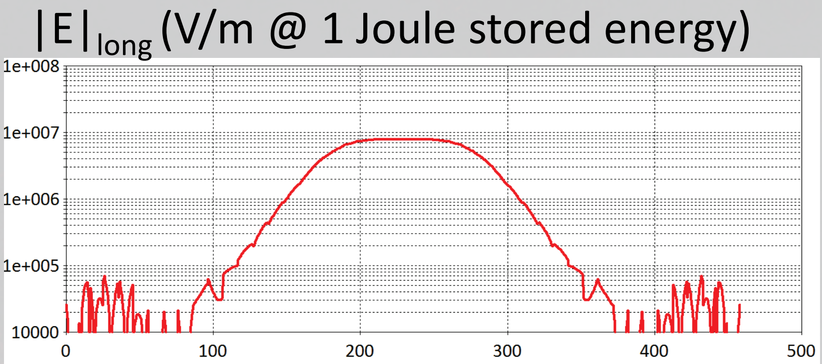
same type of mode as previous, propagating mode, i.e. reflecting boundaries matters!



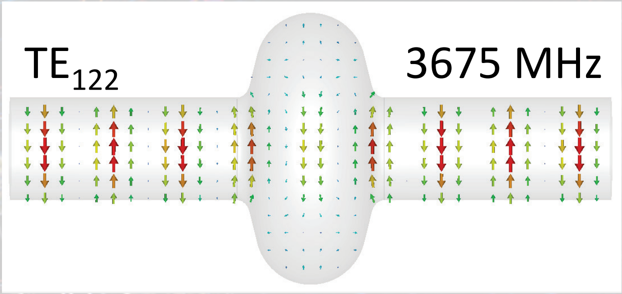


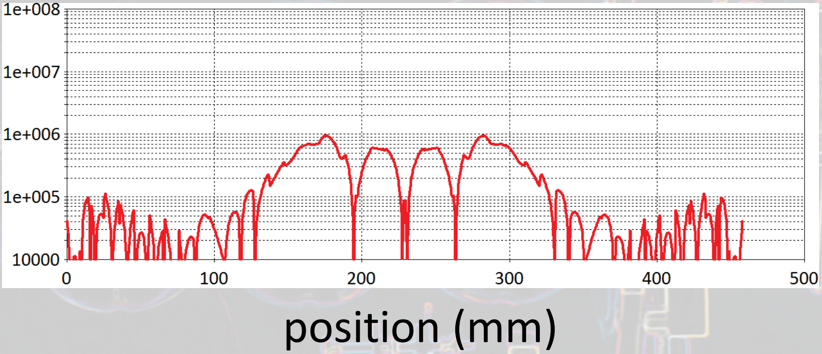
Elliptical Cavity (1-cell from TESLA cavity endcups) Dipole HOMs





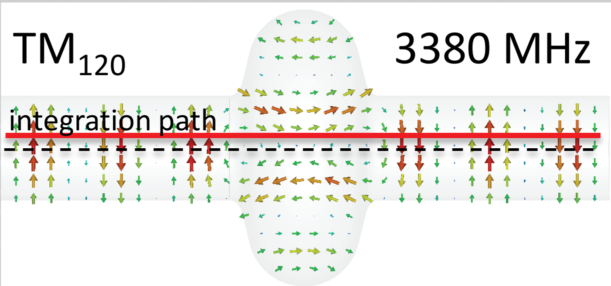
same type of mode as previous, propagating mode, i.e. reflecting boundaries matters!



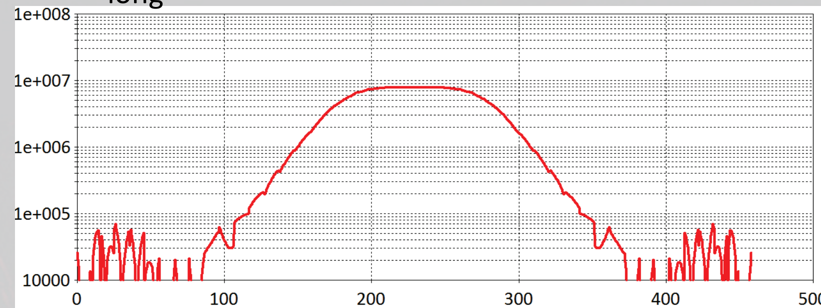


- Letting HOMs propagate is beneficial for damping
  - This is matter of beam tube size
- Yet, reflections from neighboring cavities can still cause standing waves for a traveling mode
  - Boundaries still may matter

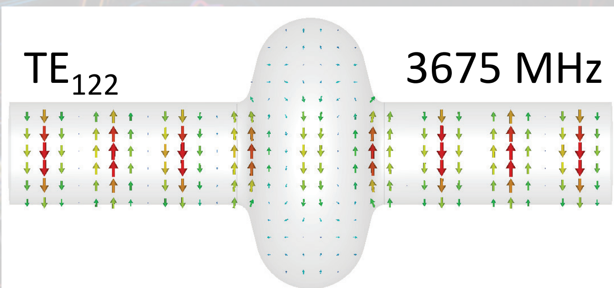
Benefit of SRF Cavities

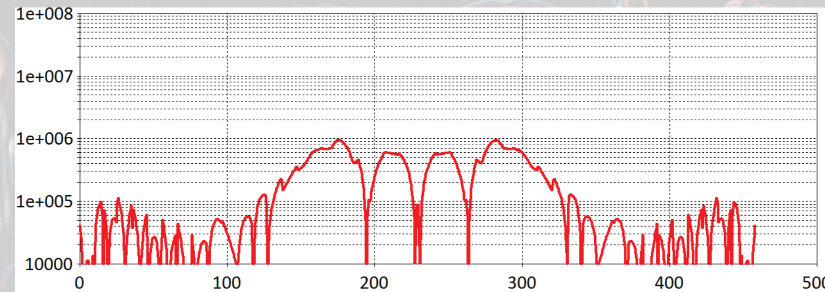


$|E|_{long}$ (V/m @ 1 Joule stored energy)



same type of mode as previous, propagating mode, i.e. reflecting boundaries matters!





position (mm)

- Letting HOMs propagate is beneficial for damping
  - This is matter of beam tube size
- Yet, reflections from neighboring cavities can still cause standing waves for a traveling mode
  - Boundaries still may matter

Cutoff frequencies of beam tubes

- Cutoff frequency determined by:
  - Interior medium (permittivity, permeability)
  - Tube radius (R)
  - Roots (x_{mn}, x'_{mn}) of Bessel function (TM-modes) or its derivative (TE-

x_{mn}

m \ n	1	2	3	4
0	2.40483	5.52008	8.65373	11.7915
1	3.83171	7.01559	10.1735	13.3237
2	5.13562	8.41724	11.61980	14.79600
3	6.38016	9.76102	13.01520	16.22350

6 significant figures

x'_{mn}

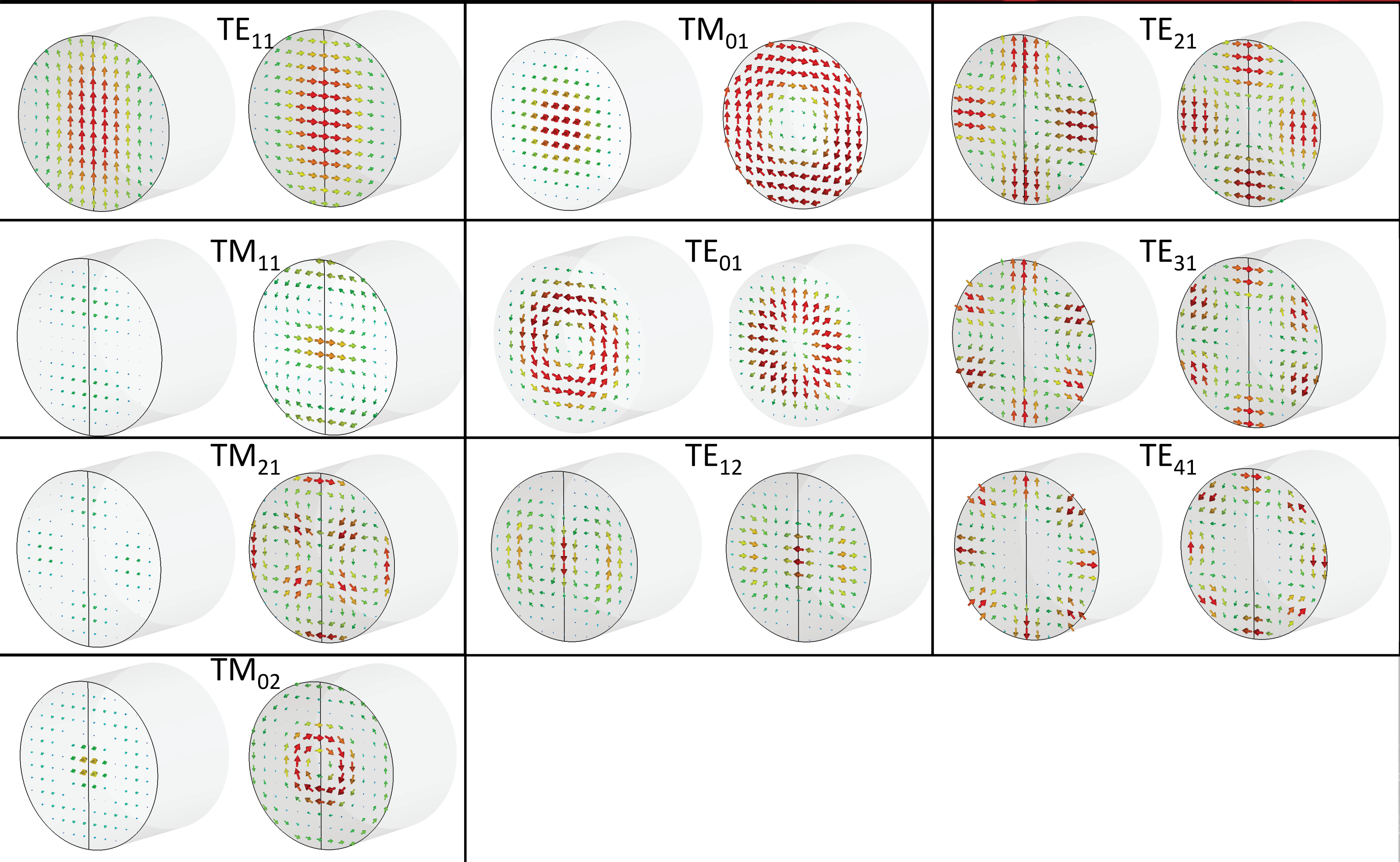
m \ n	1	2	3	4
0	3.83171	7.01559	10.1735	13.3237
1	1.84118	5.33144	8.53632	11.7060
2	3.05424	6.70613	9.96947	13.17040
3	4.20119	8.01524	11.34590	14.58580

$$TM\ modes:\ f_{mn}^{cutoff} = \frac{c}{\lambda_{mn}^{cutoff}} = \frac{c}{2\pi\sqrt{\varepsilon_r \cdot \mu_r}} \cdot \frac{\boldsymbol{x_{mn}}}{\boldsymbol{R}}$$

$$TE\ modes:\ f_{mn}^{cutoff} = \frac{c}{\lambda_{mn}^{cutoff}} = \frac{c}{2\pi\sqrt{\varepsilon_r \cdot \mu_r}} \cdot \frac{\boldsymbol{x'_{mn}}}{\boldsymbol{R}}$$

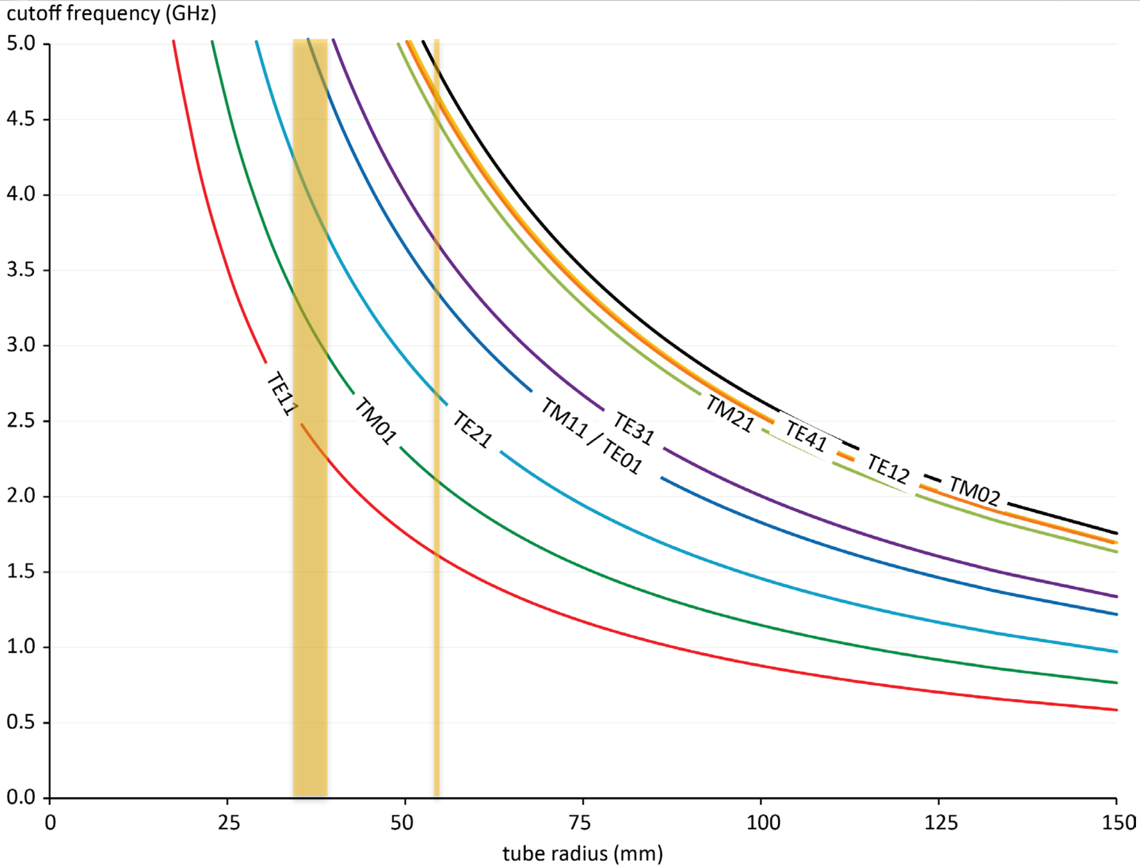
- Note:
  - 1st cutoff is dipole TE_{11} (**x'_{mn} = 1.84118**)
  - 2nd cutoff is monopole TM_{01} (**x_{mn} = 2.40483**)
  - Degeneracy may occur, e.g. [illegible]

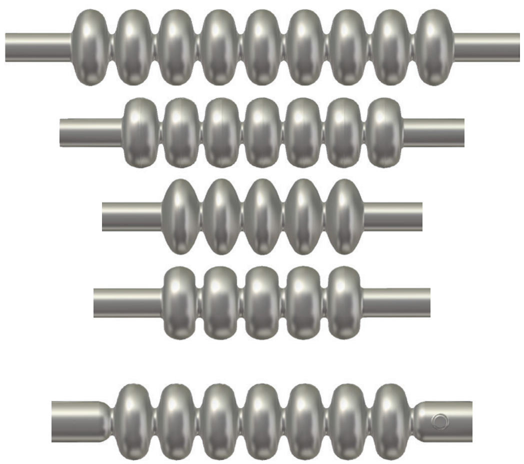
First 10 Beam Tube Modes (E/H)



Cutoff frequencies of beam tubes

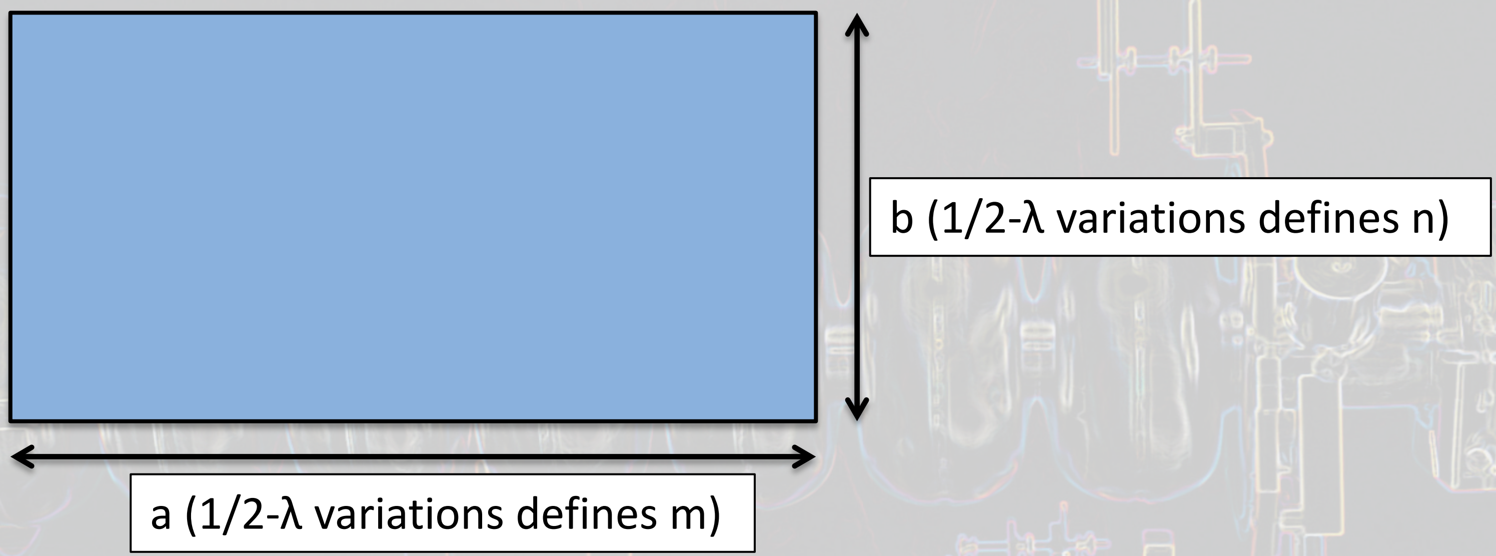
- 35-39 mm are typical tube radii for 1.3-1.5 GHz (L-band) SRF cavities , e.g. EU-XFEL/ILC/LCLS-II TESLA-type cavities or JLab's CEBAF/FEL cavities
- Larger tube radii are considered for high-current heavily HOM-damped cavities, which lets HOMs propagate out of cavity at lower frequencies, e.g. 55 mm for Cornell ERL cavity





Cutoff frequencies of rectangular waveguides

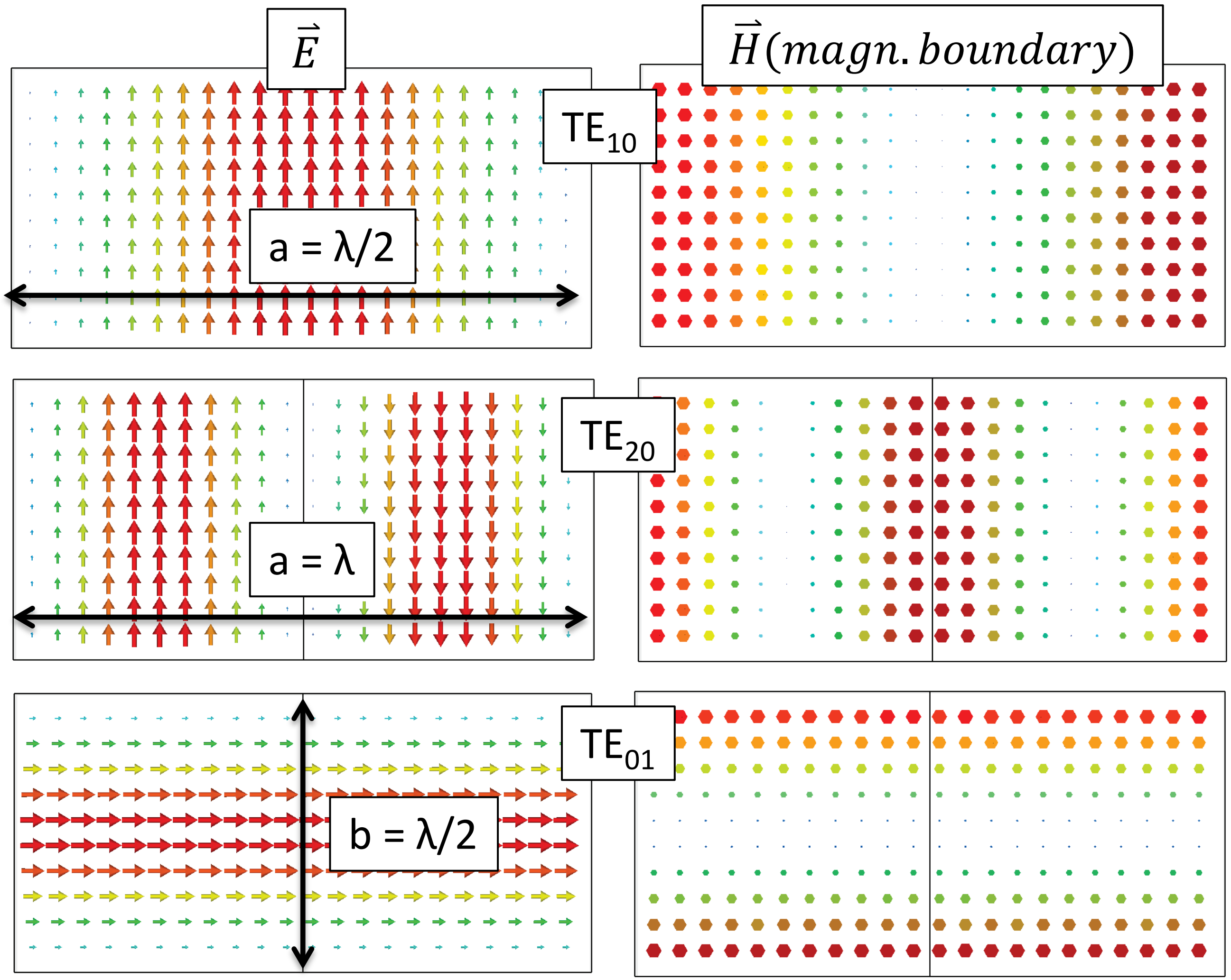
- Cutoff frequency determined by:
  - Interior medium (permittivity, permeability)
  - Waveguide broad (a) and short (b) inner dimension



$$TM/TE\ modes\!:\ f_{mn}^{cutoff} = \frac{c}{\lambda_{mn}^{cutoff}} = \frac{c}{2\sqrt{\varepsilon_r \cdot \mu_r}} \cdot \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{a}\right)^2}$$

Cutoff frequencies of rectangular waveguides

- Formula for cutoff frequencies simple to memorize



$$\lambda_{TE10,cutoff} = 2 \cdot a$$

$$f_{TE10,cutoff} = \frac{c}{2 \cdot a}$$

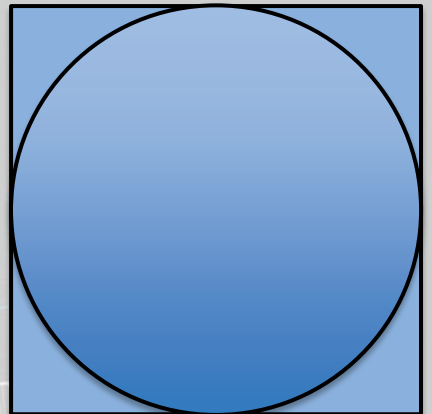
$$f_{TE20,cutoff} = \frac{c}{a}$$

$$f_{TE01,cutoff} = \frac{c}{b}$$

Waveguide Dimensions

- CEBAF HOM damping waveguide dimensions
  - H x B = 0.71” x 6.3” = 18 mm x 160 mm
- Standard WR430 adapter
  - H x B = 1.875” x 4.3” = (good for 1.7 GHz to 2.6 GHz)

Round vs. Rectangular Waveguides



a = b = 2R

- 1st cutoff frequencies

round

$$f_{TE11}^{cutoff} = \frac{c}{\sqrt{\varepsilon_r \cdot \mu_r}} \cdot \frac{1.84118}{\pi \cdot a}$$

rectangular

$$f_{TE10}^{cutoff} = \frac{c}{\sqrt{\varepsilon_r \cdot \mu_r}} \cdot \frac{1}{2 \cdot a}$$

- round/rectangular

$$\frac{f_{TE11}^{cutoff}}{f_{TE10}^{cutoff}} = \frac{2 \cdot 1.84118}{\pi} \sim 1.17$$

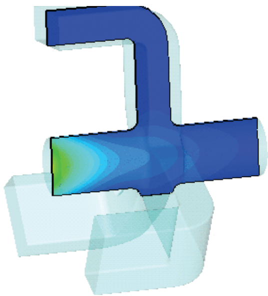
  - round waveguide has 17% higher cutoff frequency than rectangular waveguide for same lateral dimension

Waveguide Damping End-Group

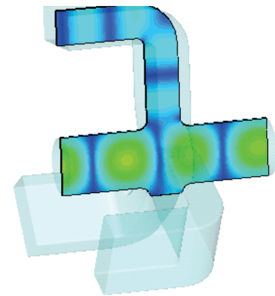
- Movies shall illustrate why damping through waveguide is not perfectly broadband, but depends on the frequency/wavelength of the mode

movies

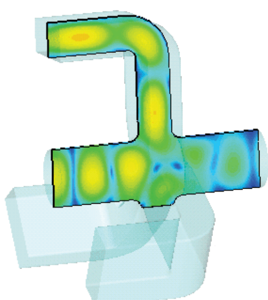
@ 2 GHz (below cutoff)



@ 3 GHz

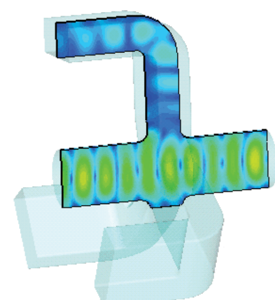


@ 4 GHz



@ 5 GHz

@ 6 GHz

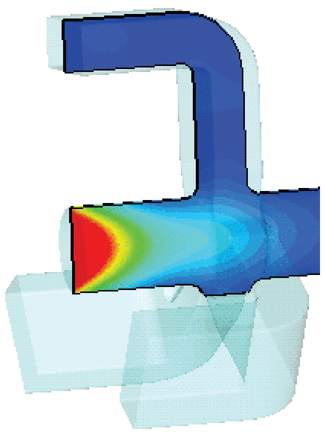


Waveguide Damping End-Group

- Movies shall illustrate why the damping waveguide needs to be placed as close as practicably possible to the end cells to facilitate damping to the trapped modes below the beam tube cutoff frequency (TE111, TM110) and how an evanescent field turn into a propagating field once (principle also used for fundamental power couplers)

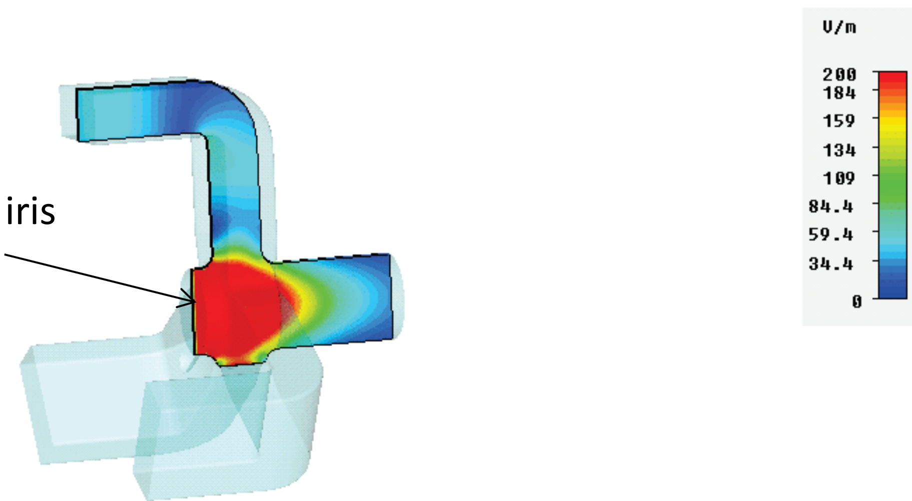
movies

- [ ] 2 GHz (below cutoff)



TE10 cutoff = 1.67 GHz

cavity iris

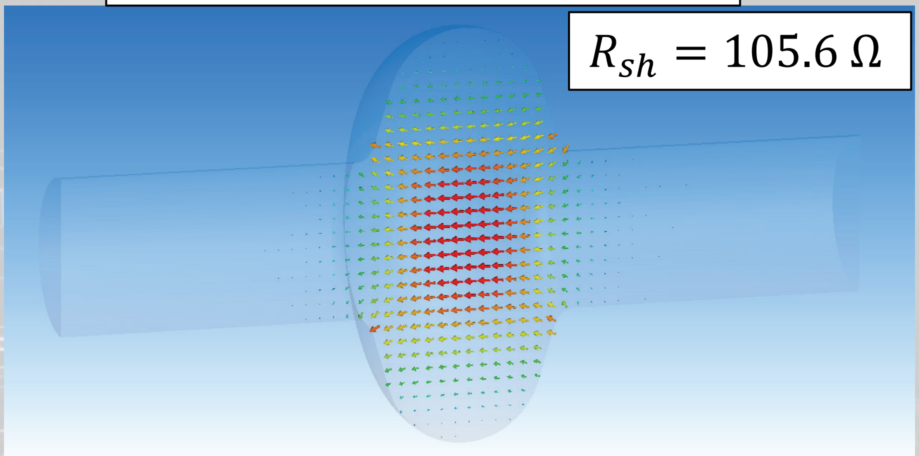


TE10 cutoff = 1.67 GHz

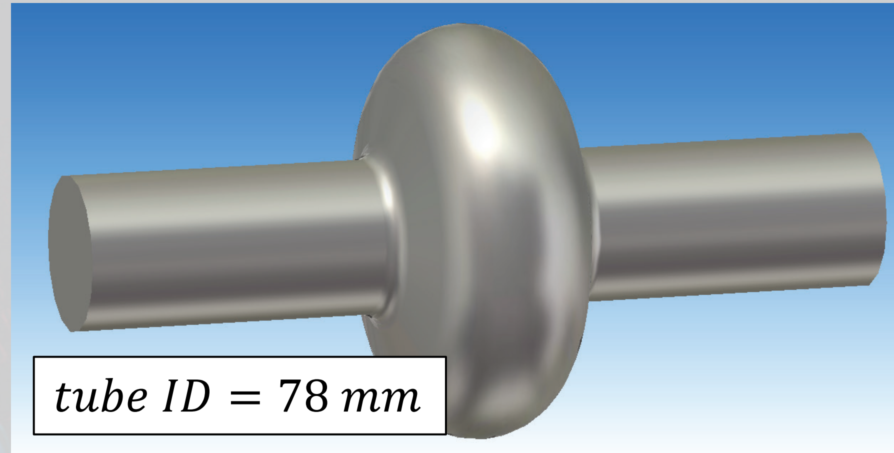
Two distances of HOM damping end-group with respect to the cavity end cell iris (one further away (left), one very close (right))

Example: TESLA 1-cell cavity

$TM010\!: \; f = 1301.97\ MHz$



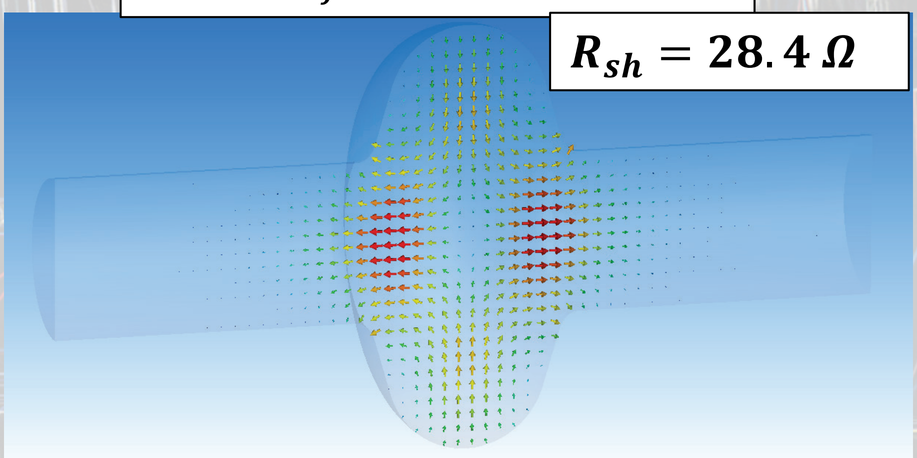
$R_{sh} = 105.6\ \Omega$



$tube\ ID = 78\ mm$

- First monopole harmonic of fundamental mode (TM011)

$TM011\!: \; f = 2451\ MHz$



$\boldsymbol{R_{sh} = 28.4\ \Omega}$

- **relevant** first monopole **cutoff** frequency **is TM$_{01}$** (not TE$_{11}$)

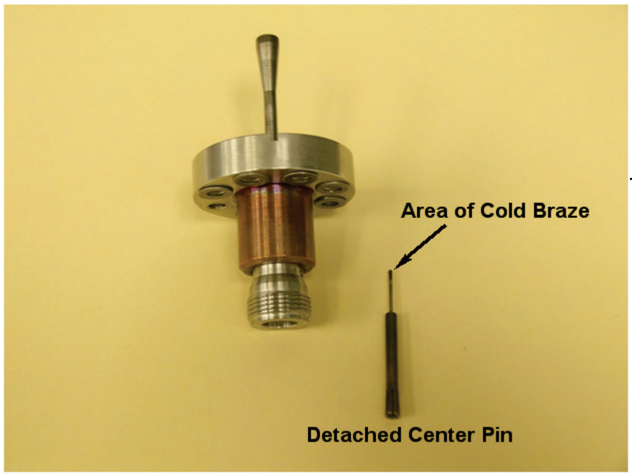
$$f_{TM01}^{cutoff} = \frac{c}{2\pi} \cdot \frac{\boldsymbol{x_{mn}}}{\boldsymbol{R}} = \mathbf{2.94\ GHz}$$

→ TM$_{011}$ is trapped inside cavity

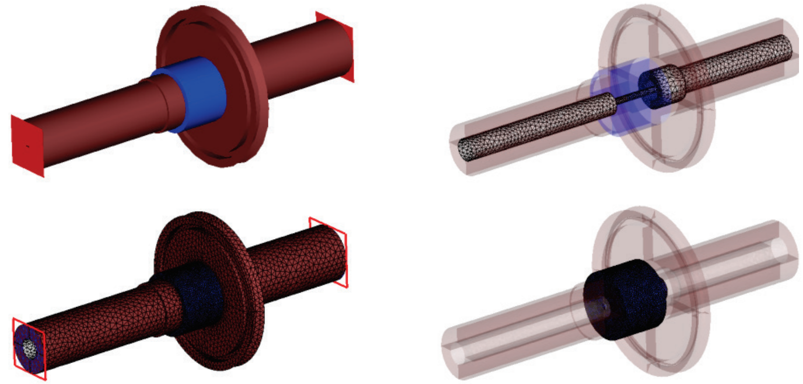
HOM Damping Technologies

- Examples:

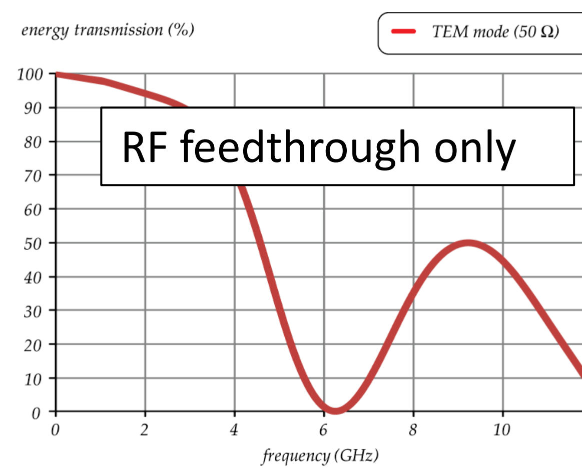
  1) Coaxial dampers on beam tubes



TESLA-style coaxial coupler with sapphire feedthrough



anisotropy of Sapphire taken into account



RF feedthrough only

Disclaimer

- All technical figures in this lecture have been prepared by the author of this lecture, who reserves all copyrights
- Photographies/figures/charts shown that do not originate from prior work and involvement at JLab but from other institutions are referenced to the paper published by the corresponding author(s). If by any chance a reference is missing, please let me know so I will immediately revise the document to include the proper reference/acknowledgment.