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Estimation of contributions from single pion electroproduction channel on a proton to inclusive observables

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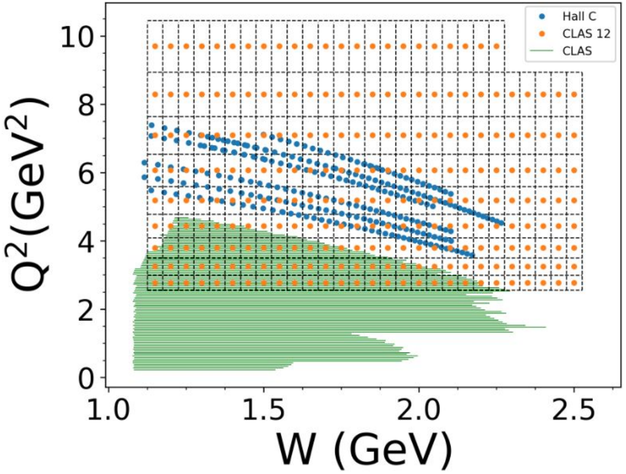
Contributions from Nπ Channels to (e,e'X) Observables- Physics Motivation

Kinematics coverage for (e.e'X) cross sections measured at Jlab in the resonance region



A unique opportunity to assess, from the experimental results, the contributions of exclusive meson electroproduction channels to (e,e'X) observables.

- Measurements at JLab provided information on (e,e'X) observables within the range of W<2.5 GeV for Q^2<10.0 GeV^2.
- Experiments with CLAS (M. Osipenko et al., Phys. Rev. D67, 092001 (2003)) and CLAS12 (V. Klimenko et al., accepted by PRC e-print: 2501-14996[hep-ex]) delivered results on (e,e'X) cross sections in each Q^2 bin within the W-range from the π-threshold to less than 2.5 GeV, thanks to the large detector acceptances.
- Most of the results on exclusive meson electroproduction in the resonance region have been obtained using the CLAS detector and are available in the CLAS Physics DB (https://clas.sinp.msu.ru/cgi-bin/jlab/db.cgi).

Observable

$$F_2\left(Q^2, W\right) = \frac{KW}{4\pi^2\alpha}\frac{2x}{1+\frac{Q^2}{\nu^2}}\left(\sigma_t\left(Q^2, W\right) + \sigma_l\left(Q^2, W\right)\right)$$

$$\sigma_u = \sigma_t + \varepsilon\sigma_l$$

$$R_{lt} = \frac{\sigma_l}{\sigma_t}$$

Reaction channel	Data source	Quantity
$e + p \to e' + X$	CLAS DB	F_2
$e + p \to e' + \pi^0 + p$		$\frac{d\sigma_{\gamma_v}}{d\Omega_\pi}$
$e + p \to e' + \pi^+ + n$		$\frac{d\sigma_{\gamma_v}}{d\Omega_\pi}$

$$K = \frac{W^2 - M_p^2}{2M_p} \qquad \nu = \frac{W^2 - M_p^2 + Q^2}{2M_p} \qquad x = \frac{Q^2}{2M_p\nu}$$

Evaluation of σ_u

$$\frac{d\sigma_{\gamma_v}}{d\Omega_\pi} = \frac{d\sigma_u}{d\Omega_\pi} + \varepsilon \frac{d\sigma_{tt}}{d\Omega_\pi} \cdot \cos 2\varphi + \sqrt{2\varepsilon(1+\varepsilon)} \frac{d\sigma_{lt}}{d\Omega_\pi} \cdot \cos\varphi$$

$$\frac{d\sigma_u}{d\Omega_\pi} = \frac{d\sigma_t}{d\Omega_\pi} + \varepsilon \frac{d\sigma_l}{d\Omega_\pi}$$

$\frac{d\sigma_u}{d\Omega_\pi}, \frac{d\sigma_t}{d\Omega_\pi}, \frac{d\sigma_l}{d\Omega_\pi}, \frac{d\sigma_{lt}}{d\Omega_\pi}, \frac{d\sigma_{tt}}{d\Omega_\pi}$ – unpolarized, transverse, longitudinal, longitudinal-transverse, transverse-transverse components

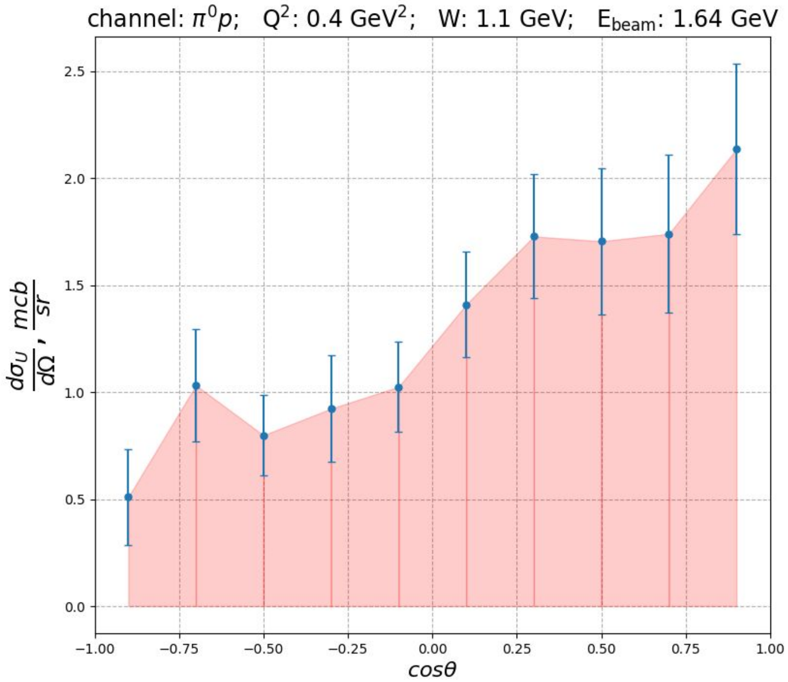
Evaluation of σ_u

$$\frac{d\sigma_{\gamma_v}}{d\Omega_\pi} = A + B \cdot \cos 2\varphi + C \cdot \cos\varphi$$

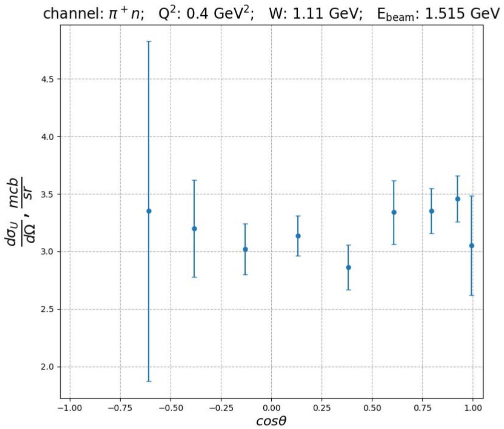
$$A = \frac{d\sigma_u}{d\Omega_\pi} \qquad \sigma_u = 2\pi \int_{\theta_{\min}}^{\theta_{\max}} \sin\theta d\theta \frac{d\sigma_u}{d\Omega_\pi}$$

$$(\sigma_u)_{\text{error}} = \sigma_u \cdot \left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{mean}\\\text{relative}\\\text{error}}}$$

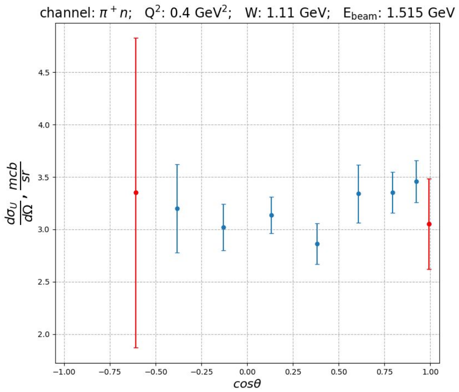
$$\left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{mean}\\\text{relative}\\\text{error}}} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left[\left(\frac{d\sigma_u}{d\Omega_\pi}\right)_i^{\text{error}} \Big/ \left(\frac{d\sigma_u}{d\Omega_\pi}\right)_i \right]$$



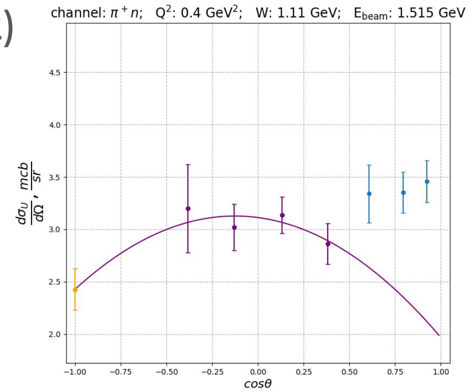
Evaluation of σ_u : $\pi^+ n$ channel case



1)

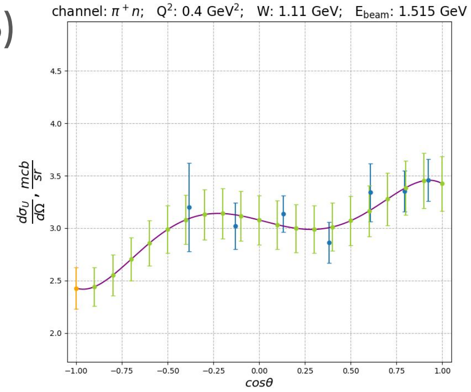


2)

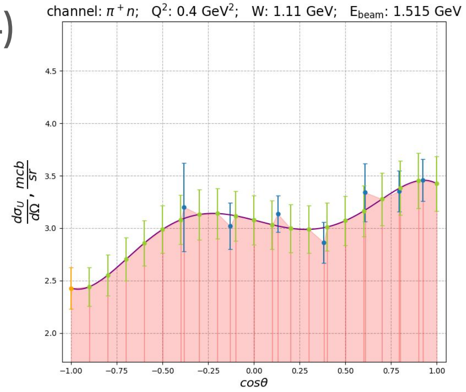


- lack of data at large θ angles
- we get underestimated values of σ_u

3)

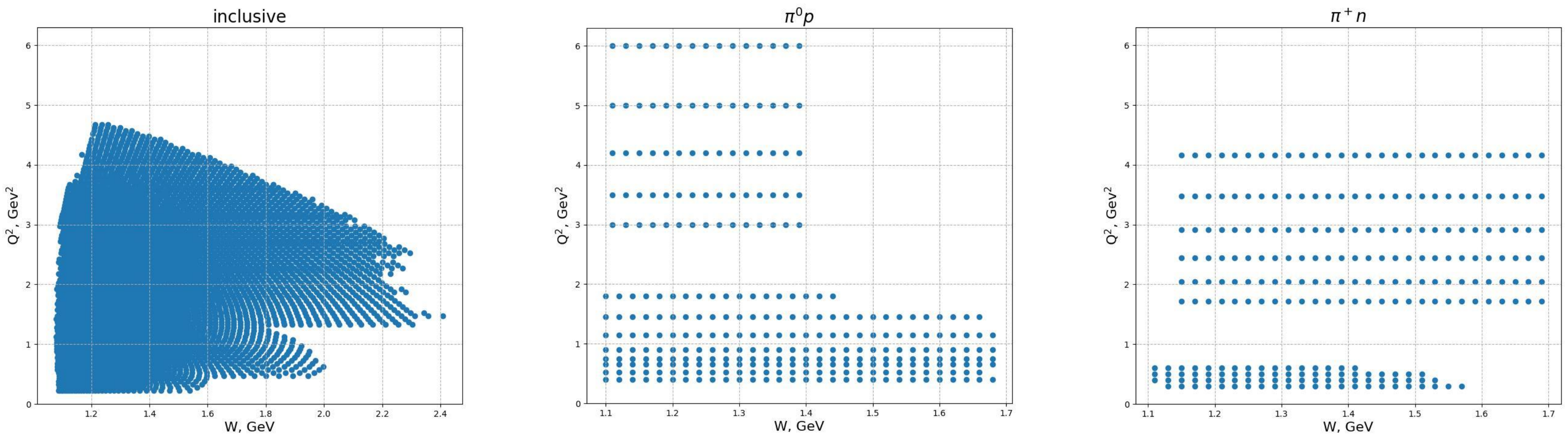


4)



<u>Solution:</u> extrapolation of data by Legendre polynomials

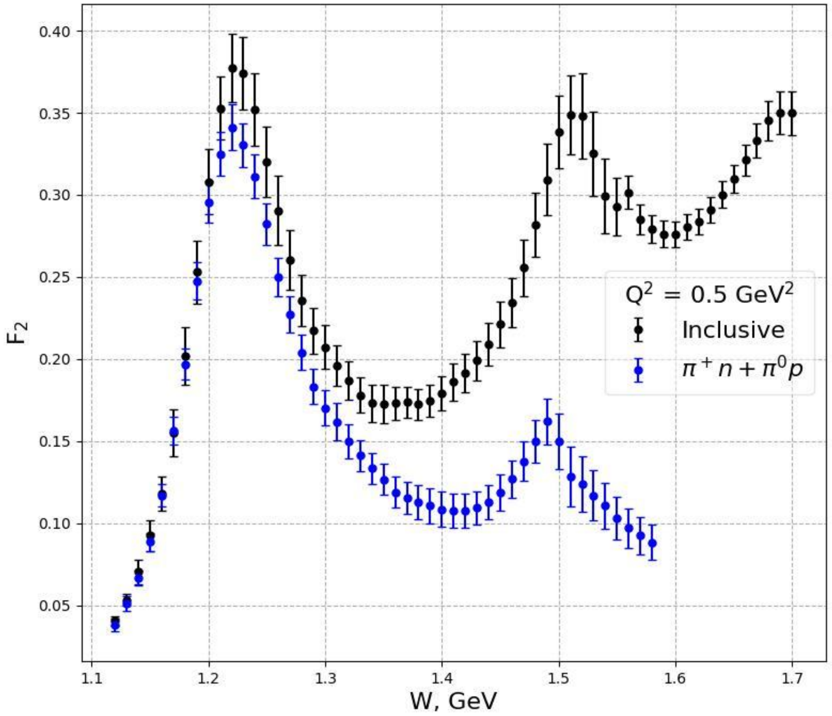
Data Maps



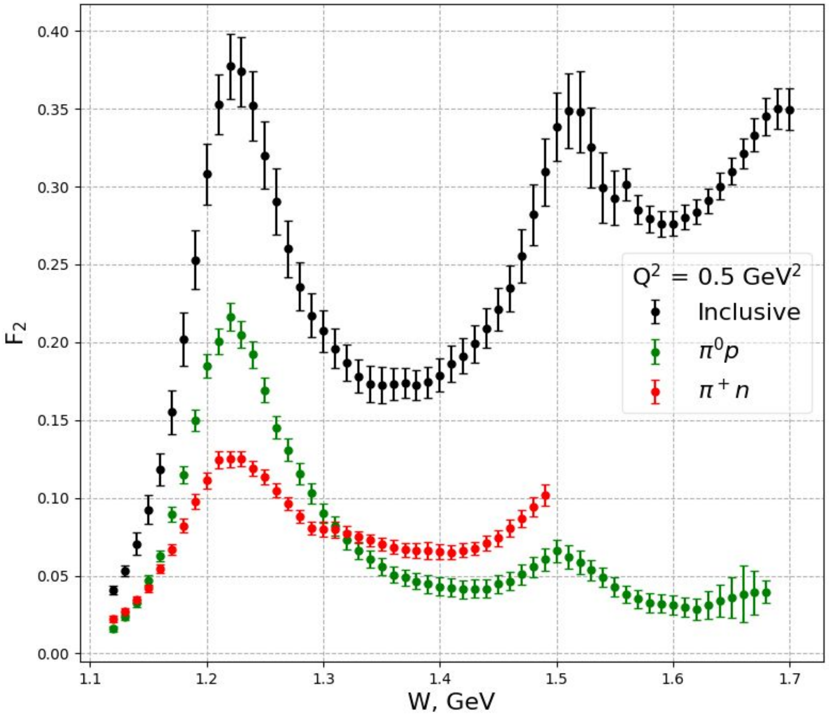
We will interpolate the data onto a common data grid using classical **linear** interpolation.

The dependence of F_2 on a W at $Q^2 = 0.5$ GeV^2

- The inclusive F_2 is completely determined by the contributions of *N*π channels in the 1st resonance region.
- The relative contributions of *N*π channels decreases when moving to the second resonance region as *N*ππ channels are open.

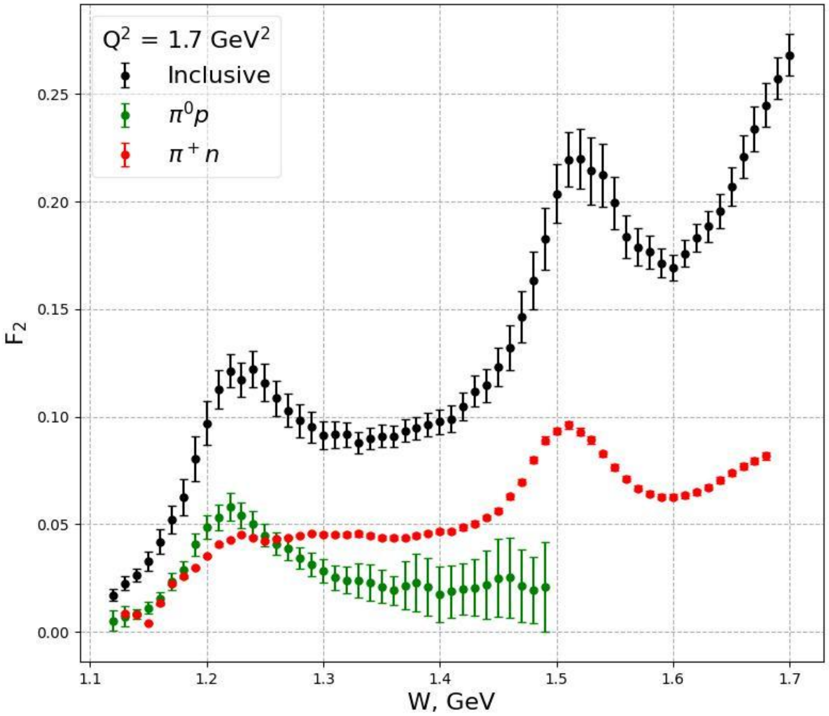


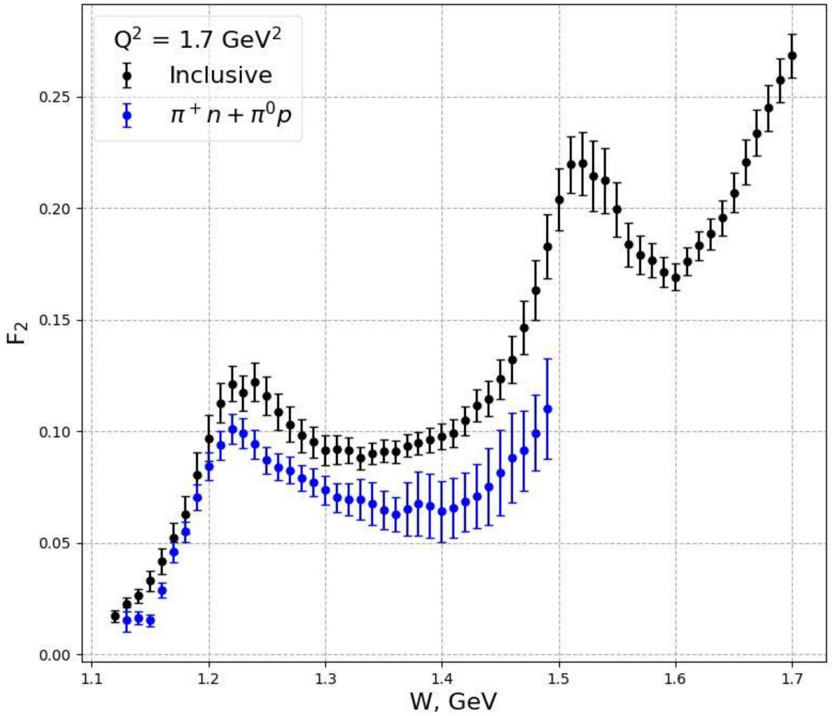
The dependence of F_2 on a W at Q^2 = 0.5 GeV^2



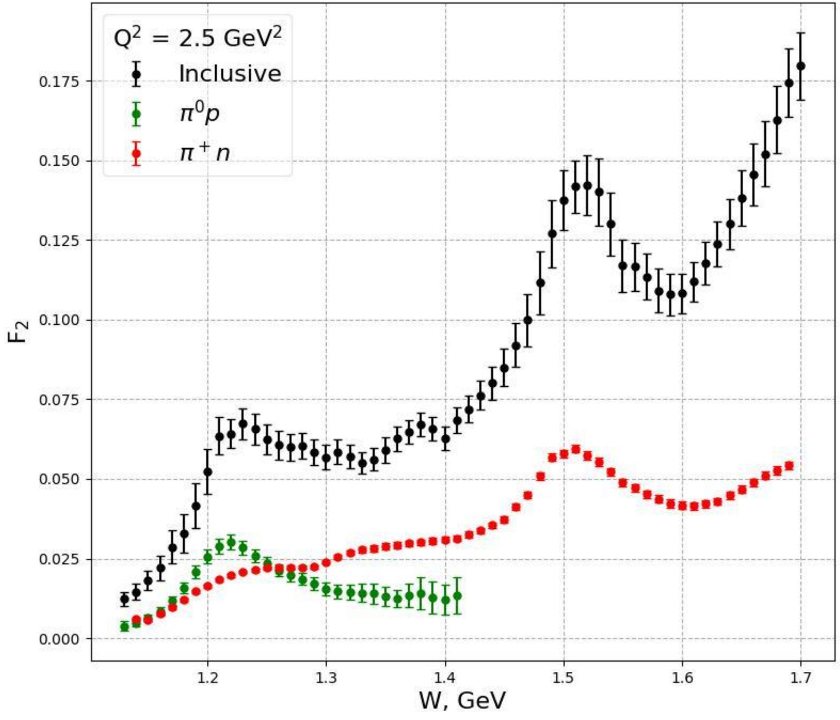
- The contribution of the π^0p channel dominates the contribution of the π^+n channel in the 1st resonance region. This is because the decay amplitude of the $\Delta(1232)\frac{3}{2}^+$ resonance with isospin 3/2 to the π^0p final state is greater than the decay amplitude to the π^+n final state according to the Clebsch-Gordan coefficients.
- The situation changes in the 2nd resonance region as it is defined by the $N(1440)\frac{1}{2}^+$, $N(1520)\frac{3}{2}^-$ and $N(1535)\frac{1}{2}^-$ resonances with isospin 1/2.

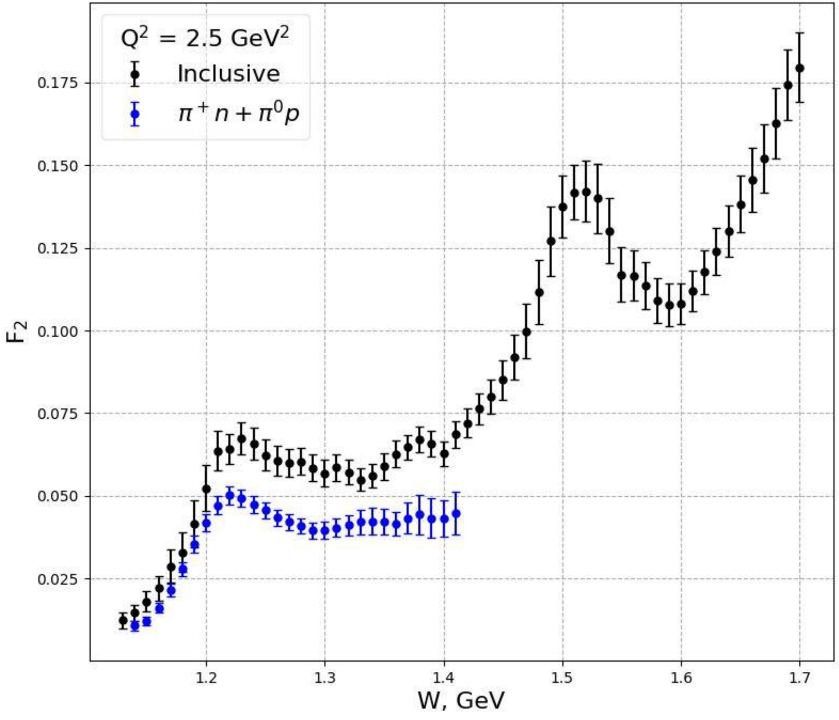
The dependence of F_2 on a W at $Q^2 = 1.7$ GeV^2



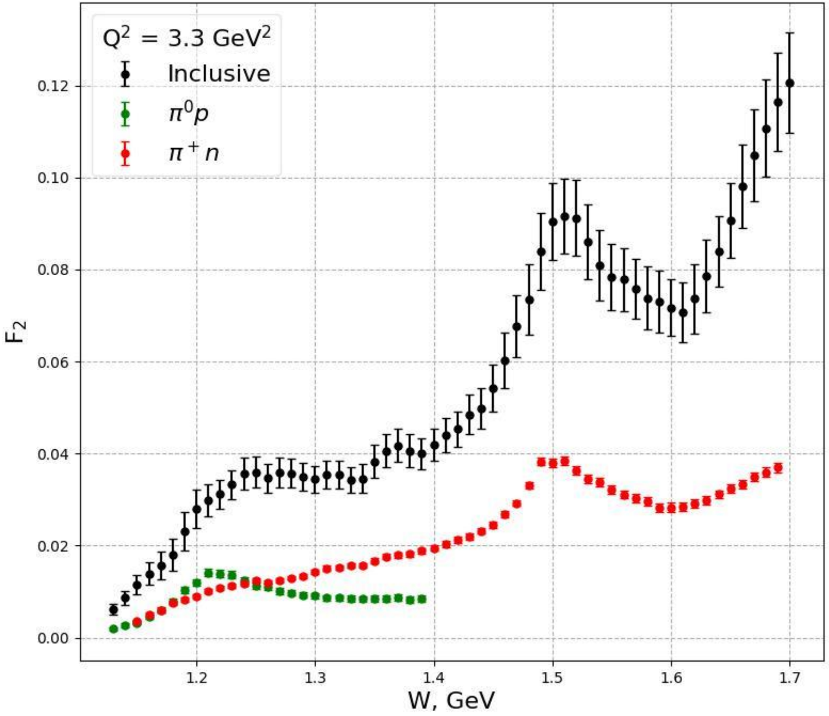


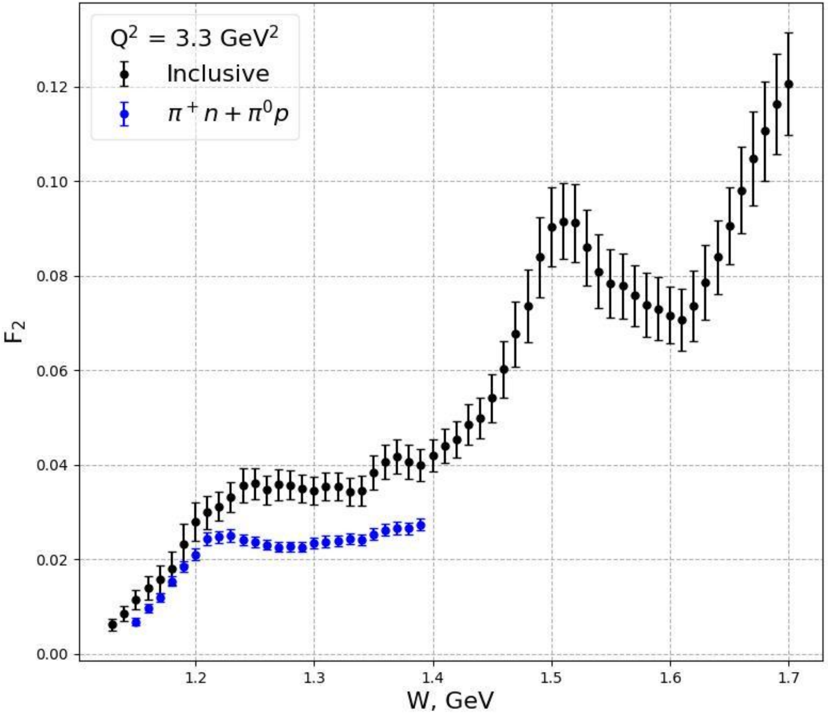
The dependence of F_2 on a W at $Q^2 = 2.5\ GeV^2$



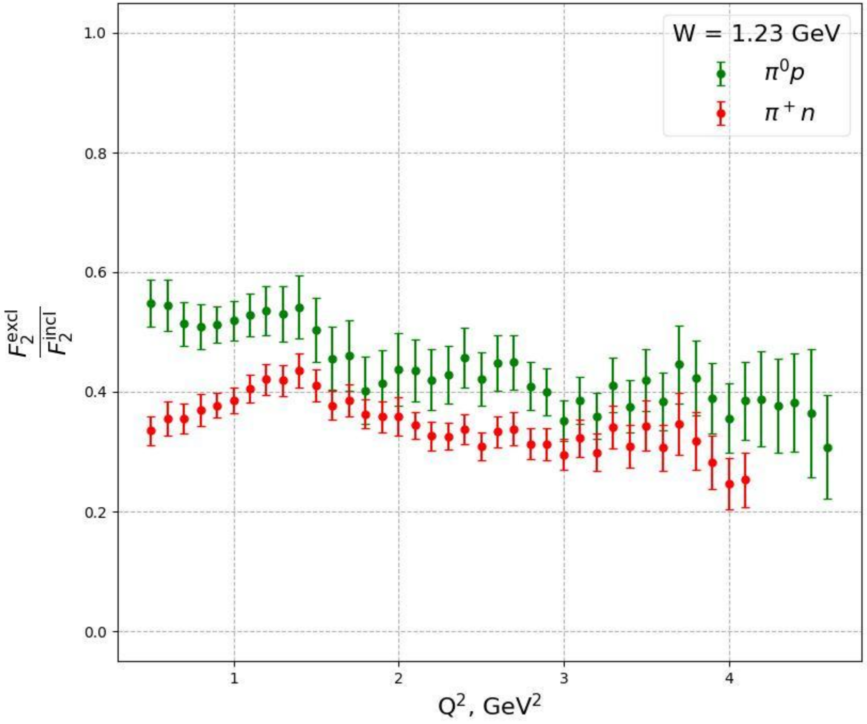


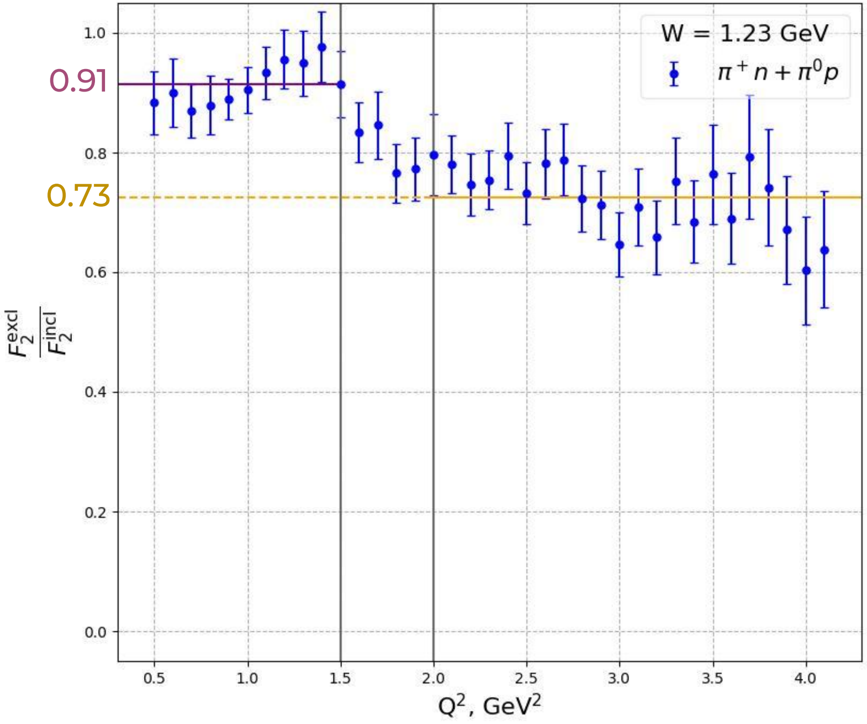
The dependence of F_2 on a W at Q^2 = 3.3 GeV^2



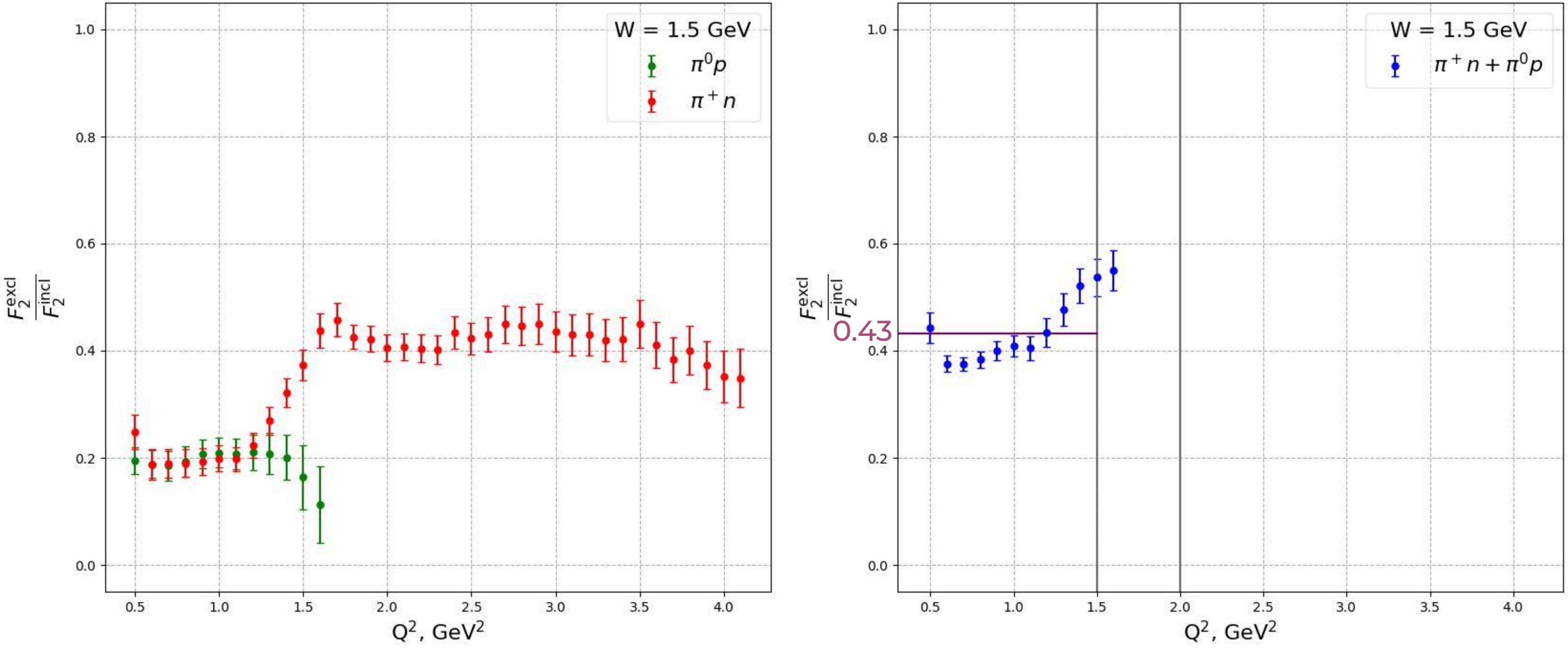


Multiplicity





Multiplicity



Conclusion

- For the first time, the contributions from the exclusive $\pi^0 p$ and $\pi^+ n$ channels to the inclusive structure function F_2 have become available from the experimental results measured with CLAS.
- Below the $N\pi\pi$ threshold, the contribution from the sum of the $\pi^0 p$ and $\pi^+ n$ channel is consistent with the measured inclusive structure function F_2. It is a compelling evidence for the reliable extraction of the $N\pi$ cross section from the experimental data measured with CLAS.
- The results on the multiplicity for the $\pi^0 p$ and $\pi^+ n$ channels have become available, and they show evidence for a transition, presumably from a complex interplay between quark core and meson-baryon cloud at $Q^2 < 1.5$ GeV^2 to a dominant contribution from quark core at $Q^2 > 2$ GeV^2, confirming observations made previously in studies of excited nucleon state structure.

Evaluation of σ_t and σ_l

$\sigma_u = \sigma_t + \varepsilon \sigma_l$ – unpolarized cross section

$\varepsilon = \left(1 + 2\left(1 + \frac{\nu^2}{Q^2}\right) tg^2 \frac{\theta_e}{2}\right)^{-1}$ – polarization of a virtual photon

$Q^2 = -q^2 = -(P_{e'} - P_e)^2$ – virtuality of photon

$W = \sqrt{(q + P_p)^2}$ – invariant mass of the final hadronic system

$$\nu = \frac{W^2 - M_p^2 + Q^2}{2M_p} \qquad tg^2 \frac{\theta_e}{2} = \frac{Q^2}{4E_{beam}(E_{beam} - \nu) - Q^2}$$

Evaluation of σ_t and σ_l

$\sigma_u = \sigma_t + \varepsilon\sigma_l$ – unpolarized cross section

$$R_{lt} = \frac{\sigma_l}{\sigma_t} \qquad \sigma_t = \frac{\sigma_u}{1+\varepsilon R_{lt}} \qquad \sigma_l = \frac{R_{lt}\sigma_u}{1+\varepsilon R_{lt}}$$

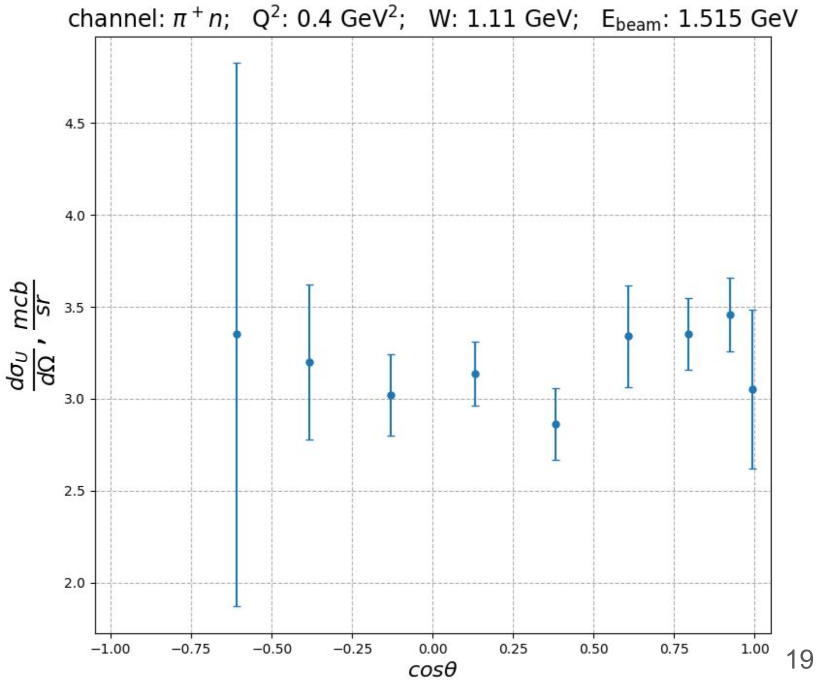
$$R_{lt} = \begin{cases} \frac{(1-x)^3}{(1-x_{th})^3}\left[\frac{0.041\xi_{th}}{\zeta} + \frac{0.592}{Q^2} - \frac{0.331}{0.09+Q^4}\right] & W < 2.5\ \text{GeV}, \\ \frac{0.041\xi}{\zeta} + \frac{0.592}{Q^2} - \frac{0.331}{0.09+Q^4} & W > 2.5\ \text{GeV} \end{cases}$$

$$\zeta = \log\frac{Q^2}{0.04} \qquad \xi = 1 + 12\cdot\frac{Q^2}{1+Q^2}\cdot\frac{0.015625}{0.015625+x^2} \qquad \xi_{th} = \xi(W=2.5) \qquad x_{th} = x(W=2.5) \qquad x = \frac{Q^2}{2M_p\nu}$$

Evaluation of σ_u
$\pi^+ n$ channel case

- lack of data at large θ angles
- we get underestimated values of σ_u

Solution: extrapolation of data by Legendre polynomials



Evaluation of σ_u
$\pi^+ n$ channel case

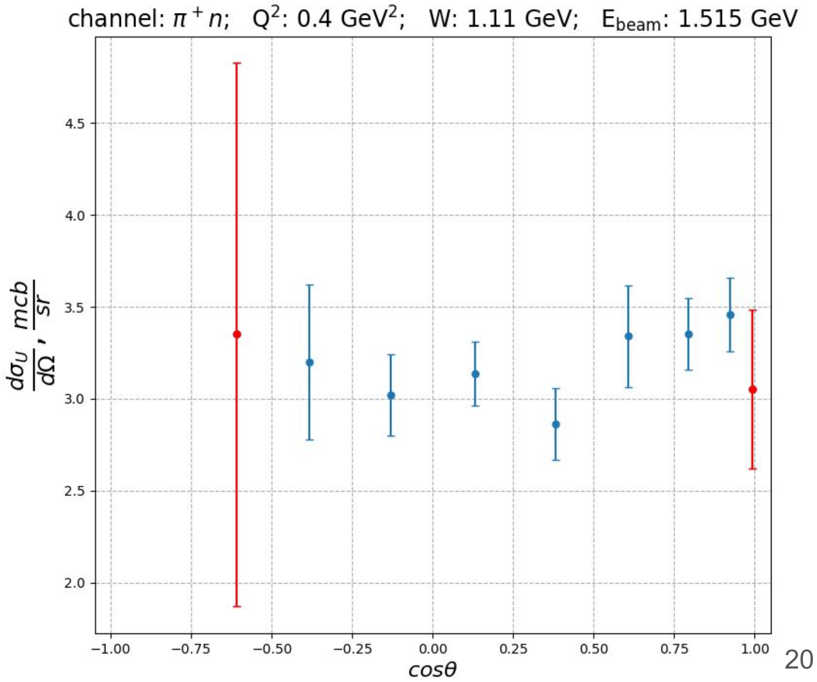
<u>1st step:</u> remove points with large errors

Criterion for removing *i* point:

$$\frac{\delta_i}{\delta_{i+1}} > 2 \text{ or } \frac{\delta_i}{\delta_{i-1}} > 2$$

δ_i – error of *i* point

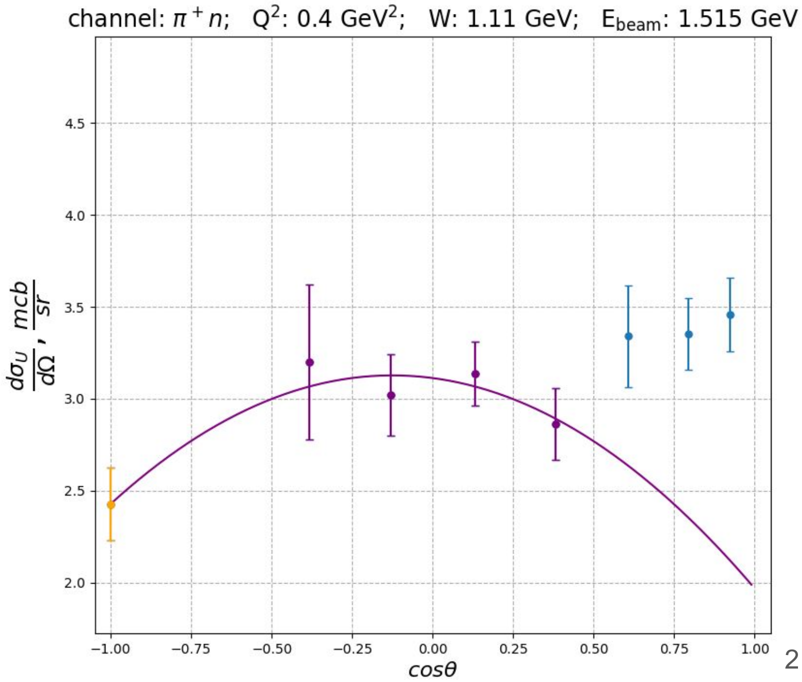
The data points to be removed on the graph are marked in **red**.



Evaluation of σ_u

$\pi^+ n$ channel case

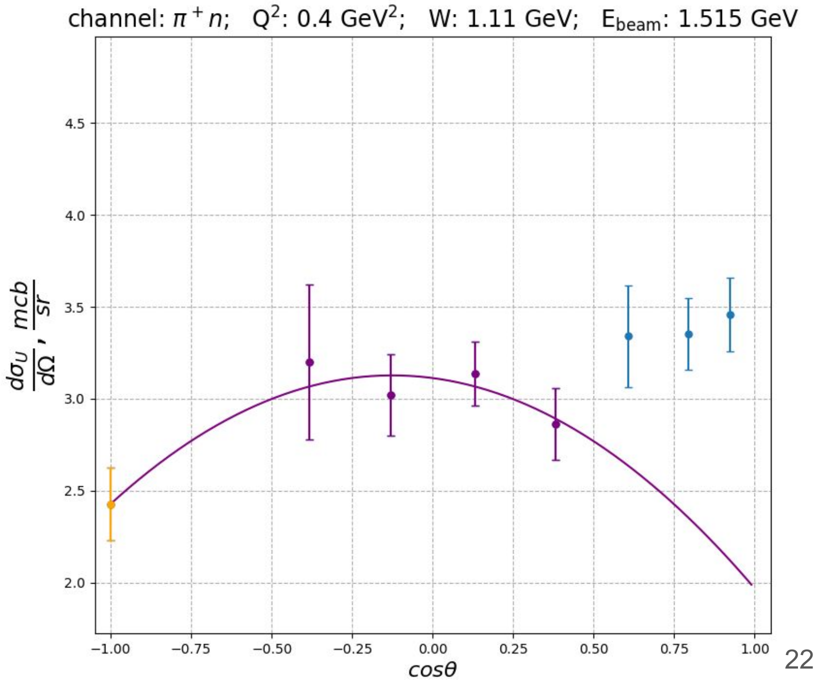
2nd step: extrapolate the data by at least 4 points corresponding to the minimum values of $\cos\theta$ (marked in **purple**) to a point with $\cos\theta = -1$ (marked in **yellow**) using **Legendre polynomials of the 2nd degree**, if possible and all values are greater than 0.



Evaluation of σ_u
$\pi^+ n$ channel case

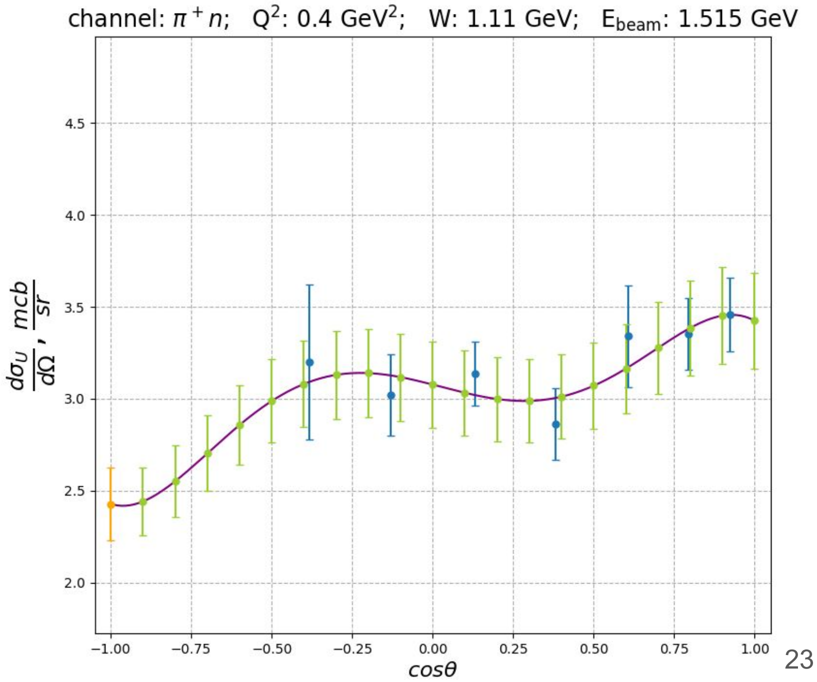
$$\left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{extrap.}\\ \text{error}}} = \left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\text{extrap.}} \cdot \left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{mean}\\ \text{relative}\\ \text{error}}}$$

$$\left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{mean}\\ \text{relative}\\ \text{error}}} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left[\left(\frac{d\sigma_u}{d\Omega_\pi}\right)_i^{\text{error}} \Big/ \left(\frac{d\sigma_u}{d\Omega_\pi}\right)_i \right]$$



Evaluation of σ_u
$\pi^+ n$ channel case

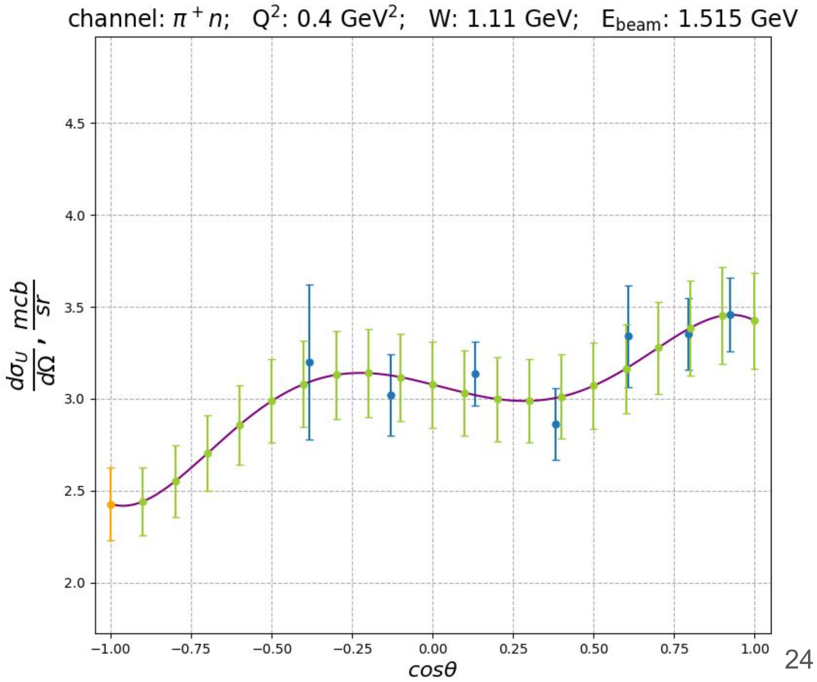
<u>3rd step:</u> interpolate data (on the **blue** points and **yellow** point) with higher degree Legendre polynomials (5th degree or less) to $\cos\theta$ fill the points with increments of 0.1 (marked in **green**), if possible and all values are greater than 0.



Evaluation of σ_u
$\pi^+ n$ channel case

$$\left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{interp.}\\ \text{error}\\ i}} = \left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{interp.}\\ i}} \cdot \left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{mean}\\ \text{relative}\\ \text{error}}}$$

$$\left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{mean}\\ \text{relative}\\ \text{error}}} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left[\left(\frac{d\sigma_u}{d\Omega_\pi}\right)_i^{\text{error}} \Big/ \left(\frac{d\sigma_u}{d\Omega_\pi}\right)_i \right]$$



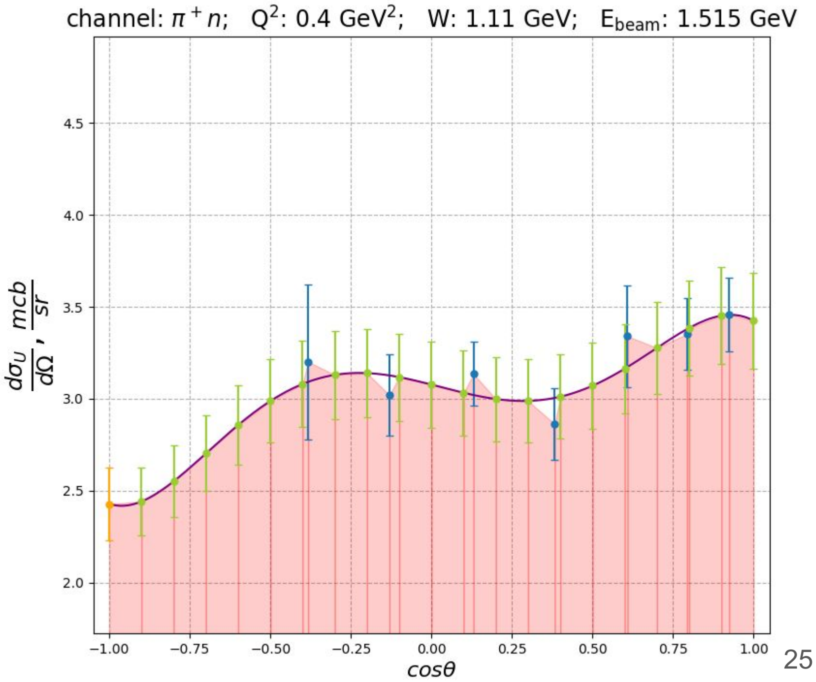
Evaluation of σ_u
$\pi^+ n$ channel case

4th step: now we can get

$$\sigma_u = 2\pi \int_{\theta_{\min}}^{\theta_{\max}} \sin\theta d\theta \frac{d\sigma_u}{d\Omega_\pi}$$

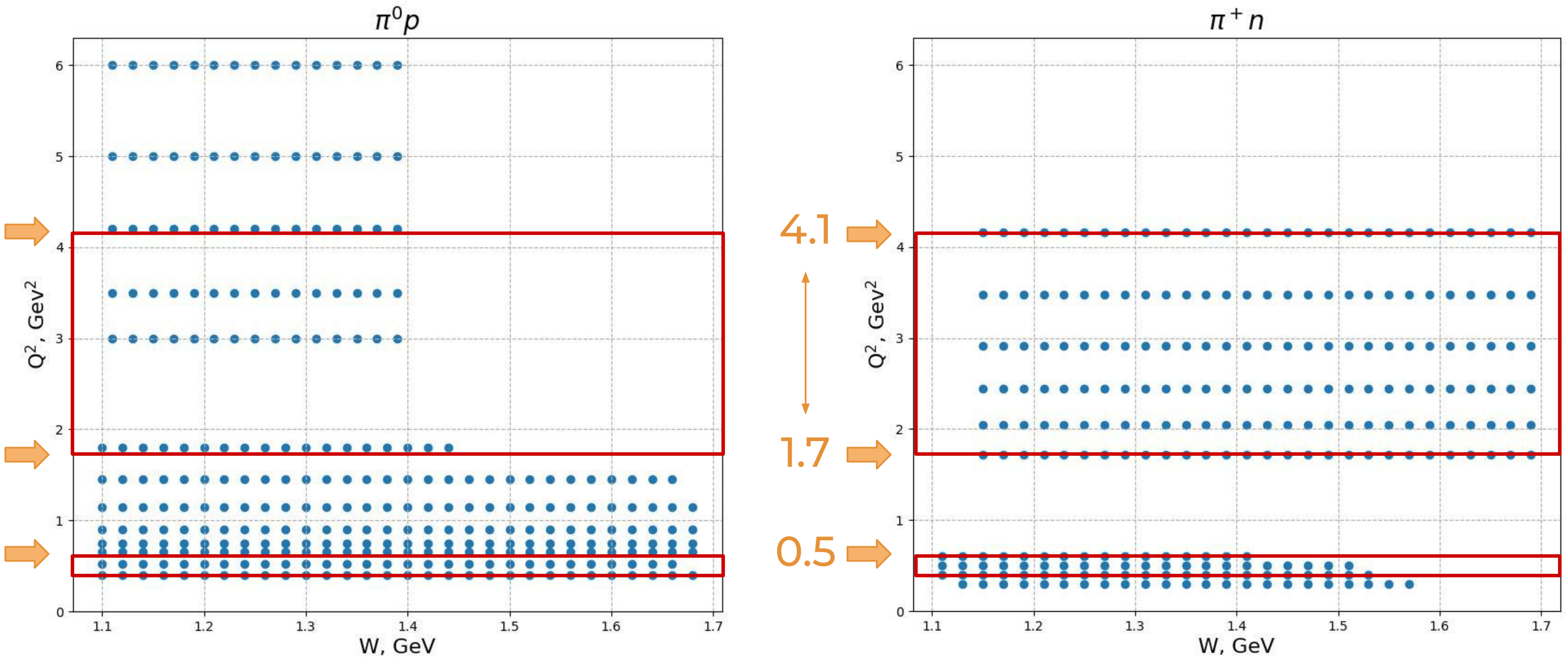
$$(\sigma_u)_{\text{error}} = \sigma_u \cdot \left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{mean}\\\text{relative}\\\text{error}}}$$

$$\left(\frac{d\sigma_u}{d\Omega_\pi}\right)_{\substack{\text{mean}\\\text{relative}\\\text{error}}} = \frac{1}{n} \cdot \sum_{i=1}^{n} \left[\left(\frac{d\sigma_u}{d\Omega_\pi}\right)_i^{\text{error}} \Big/ \left(\frac{d\sigma_u}{d\Omega_\pi}\right)_i \right]$$



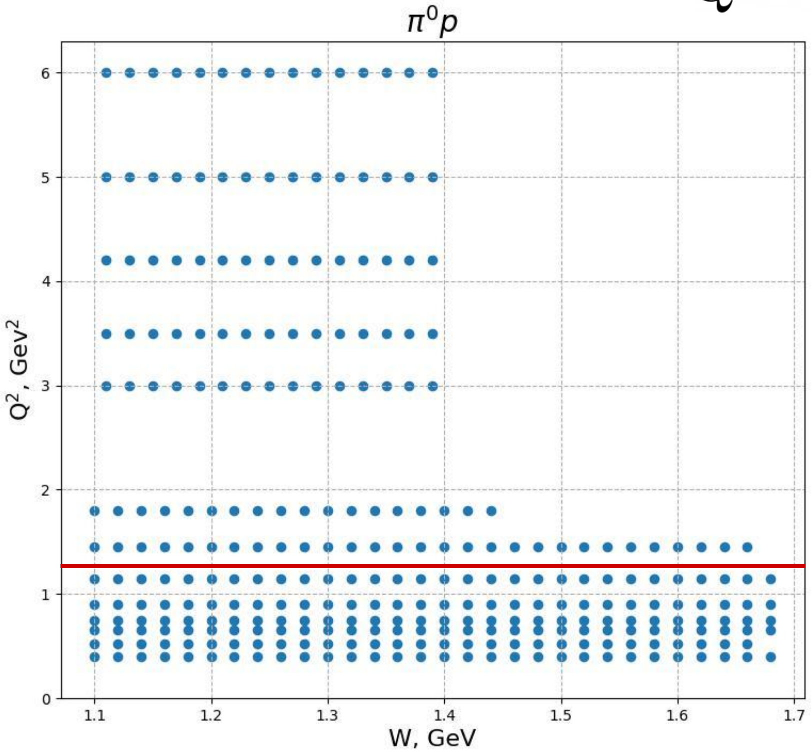
Data Maps

Areas of correct linear interpolation

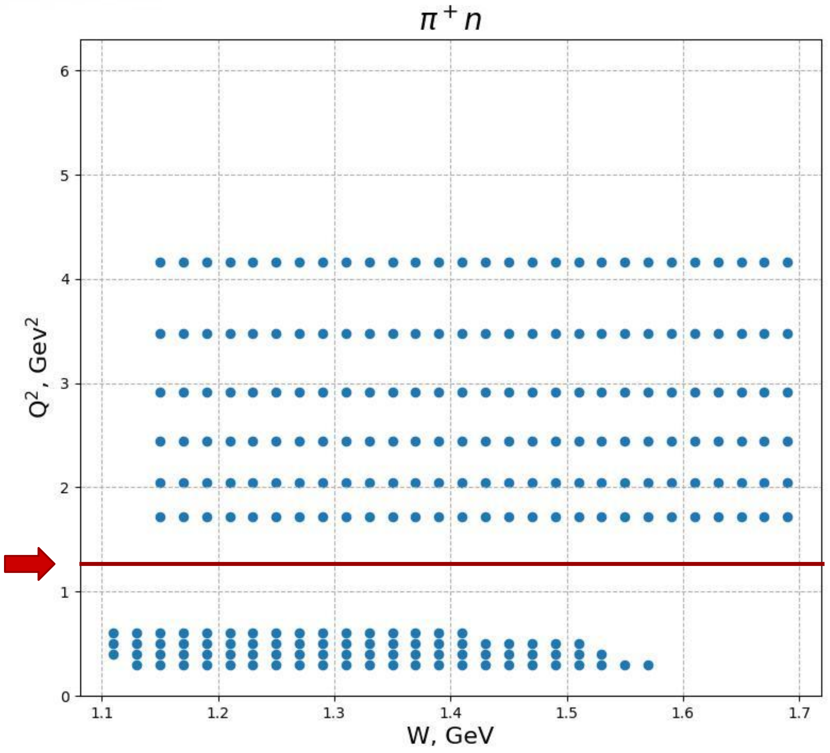


Data Maps

Example of incorrect linear interpolation at $Q^2 = 1.3\ \mathrm{GeV}^2$

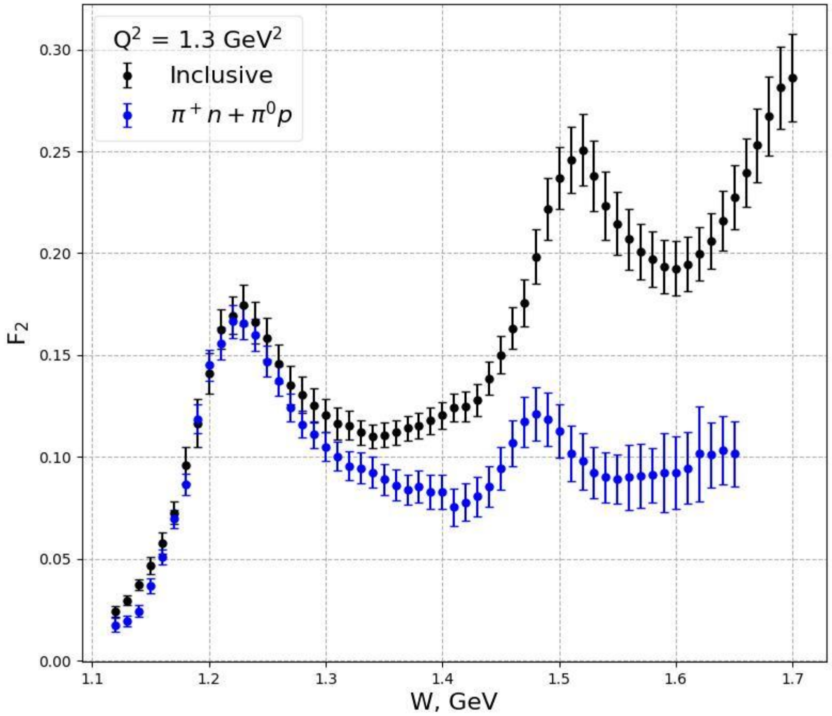


1.3

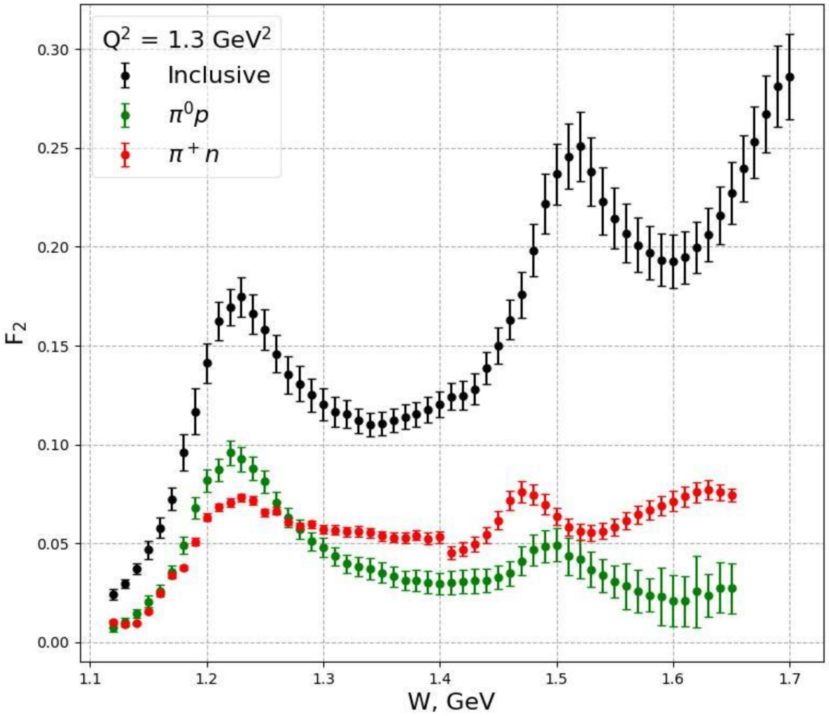


The dependence of F_2 on a W at Q^2 = 1.3 GeV^2

- We can notice a significant shift of the peak of the 2nd resonance region towards lower W.

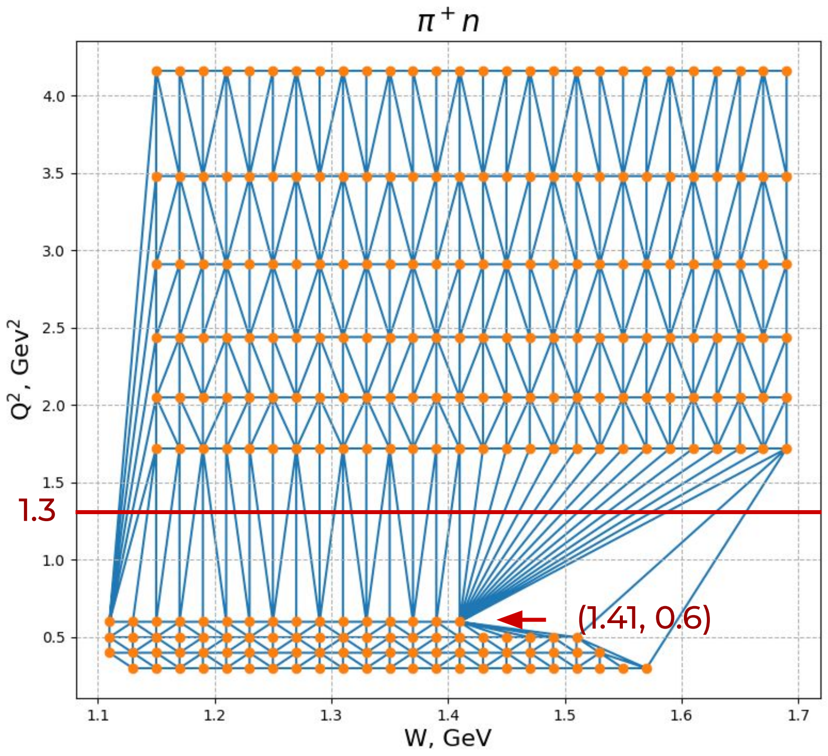


The dependence of F_2 on a W at Q^2 = 1.3 GeV^2



- The problem is incorrect linear interpolation of π^+ channel data.

Delaunay triangulation and determination of barycentric coordinates



- Linear interpolation of a function of two variables is achieved using the Delaunay triangulation (shown in the graph) and determination of barycentric coordinates.
- The available data have a distribution such that in the region 0.5 GeV^2 < Q^2 < 1.7 GeV^2 we have a bias of the F_2 interpolation values towards smaller W in the range 1.41 GeV < W < 1.65* GeV due to point (W, Q^2) = (1.41, 0.6).

* depends on Q^2

Delaunay triangulation

- Connects points into triangles such that no point lies inside the circumcircle of any triangle.
- Maximizes minimum angles in triangles, avoiding overly skewed shapes.

Barycentric coordinates

- For a triangle ABC any interior point P can be expressed as:

$$P = uA + vB + wC \quad \text{with} \quad u + v + w = 1$$

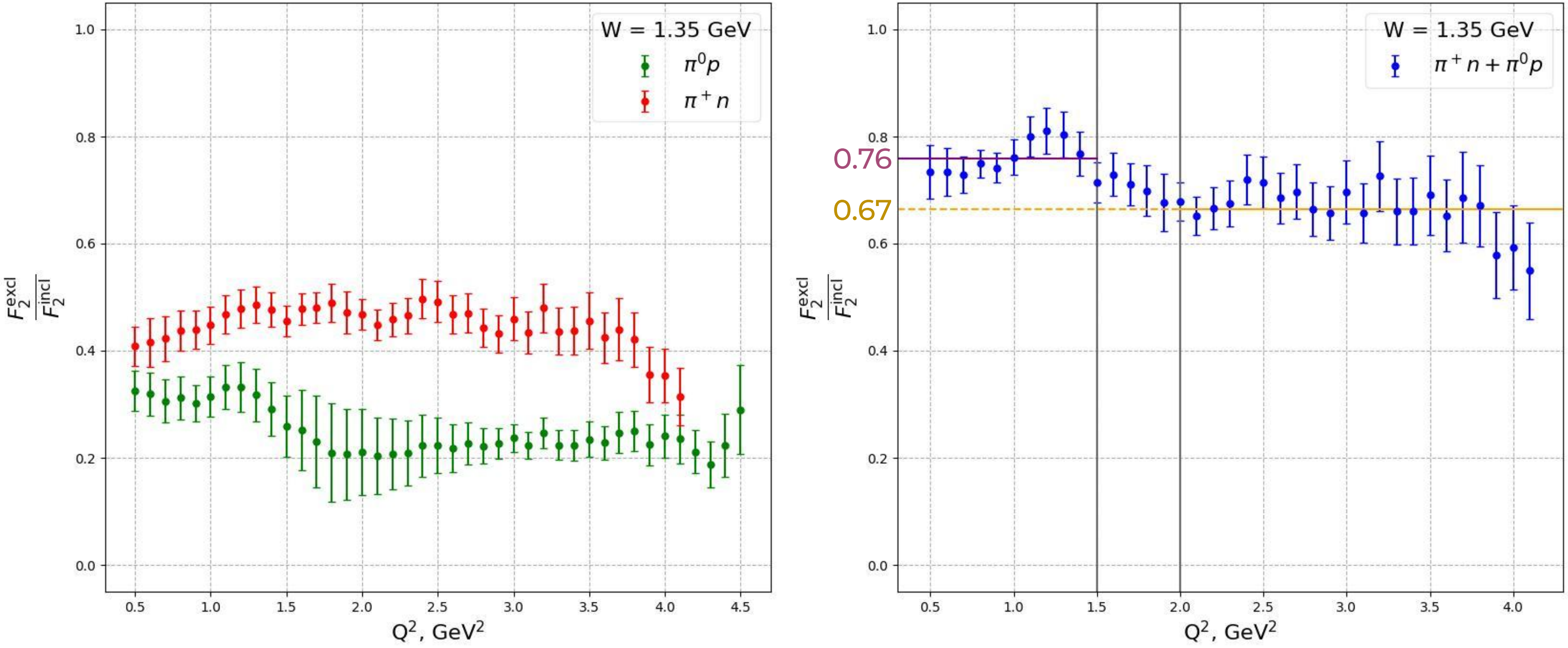
- Coordinates correspond to area ratios:

$$u = Area(BCP)/Area(ABC), \quad v = Area(CAP)/Area(ABC), \quad w = 1 - u - v$$

- For function values V_A, V_B, V_C at vertices:

$$VP = uV_A + vV_B + wV_C$$

Multiplicity



Multiplicity errors

In general, if we have values x_i with their errors σ_{xi}

$$f = f\left(x_1, x_2, \ldots, x_n\right)$$

$$\sigma_f = \sqrt{\sum_{i=1}^{n}\left[\left(\frac{\partial f}{\partial x_i}\right)^2 \cdot \sigma^2\left(x_i\right)\right]}$$

Multiplicity errors

$$f = \frac{F_2^{\pi^+ n(\pi^0 p)}}{F_2^{inclusive}}$$

$$\sigma_f = \sqrt{\left(\frac{1}{F_2^{inclusive}}\right)^2 \cdot \sigma^2\left(F_2^{\pi^+ n(\pi^0 p)}\right) + \left(-\frac{F_2^{\pi^+ n(\pi^0 p)}}{\left(F_2^{inclusive}\right)^2}\right)^2 \cdot \sigma^2\left(F_2^{inclusive}\right)}$$

$$= \frac{\sqrt{\left(F_2^{inclusive} \cdot \sigma\left(F_2^{\pi^+ n(\pi^0 p)}\right)\right)^2 + \left(F_2^{\pi^+ n(\pi^0 p)} \cdot \sigma\left(F_2^{inclusive}\right)\right)^2}}{\left(F_2^{inclusive}\right)^2}$$

Multiplicity errors

$$f = \frac{F_2^{\pi^+ n} + F_2^{\pi^0 p}}{F_2^{inclusive}}$$

$$\sigma_f = \sqrt{\left(\frac{F_2^{\pi^0 p}}{F_2^{inclusive}}\right)^2 \cdot \sigma^2\left(F_2^{\pi^+ n}\right) + \left(\frac{F_2^{\pi^+ n}}{F_2^{inclusive}}\right)^2 \cdot \sigma^2\left(F_2^{\pi^0 p}\right) + \left(-\frac{F_2^{\pi^+ n} + F_2^{\pi^0 p}}{\left(F_2^{inclusive}\right)^2}\right)^2 \cdot \sigma^2\left(F_2^{inclusive}\right)}$$

$$= \frac{\sqrt{\left(F_2^{\pi^0 p} \cdot F_2^{inclusive} \cdot \sigma\left(F_2^{\pi^+ n}\right)\right)^2 + \left(F_2^{\pi^+ n} \cdot F_2^{inclusive} \cdot \sigma\left(F_2^{\pi^0 p}\right)\right)^2 + \left(\left(F_2^{\pi^+ n} + F_2^{\pi^0 p}\right) \cdot \sigma\left(F_2^{inclusive}\right)\right)^2}}{\left(F_2^{inclusive}\right)^2}$$