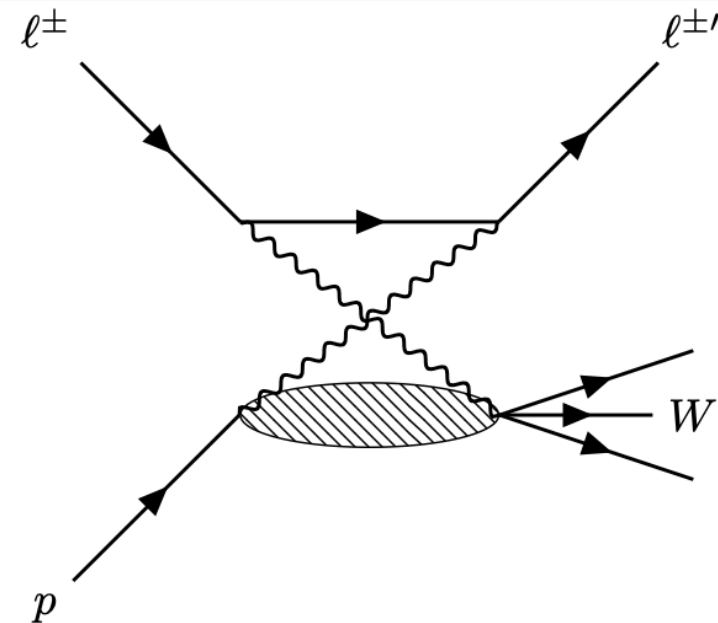
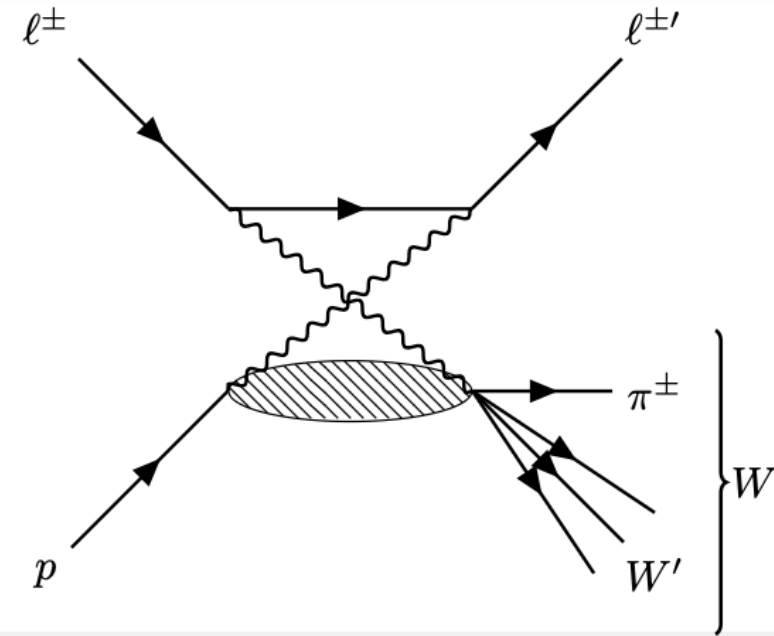


Multi-Photon Effects in Inclusive and Semi-Inclusive Deep Inelastic Scattering

(A positron proposal)

June 18, 2025

Dave Gaskell, Tyler Hague, Mike Nycz

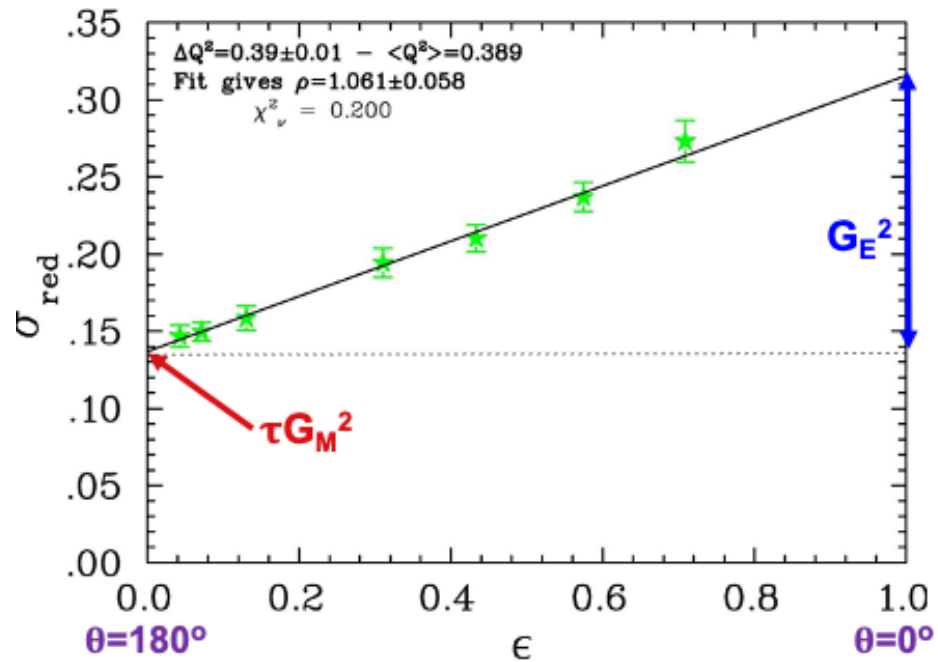


Two Photon Exchange

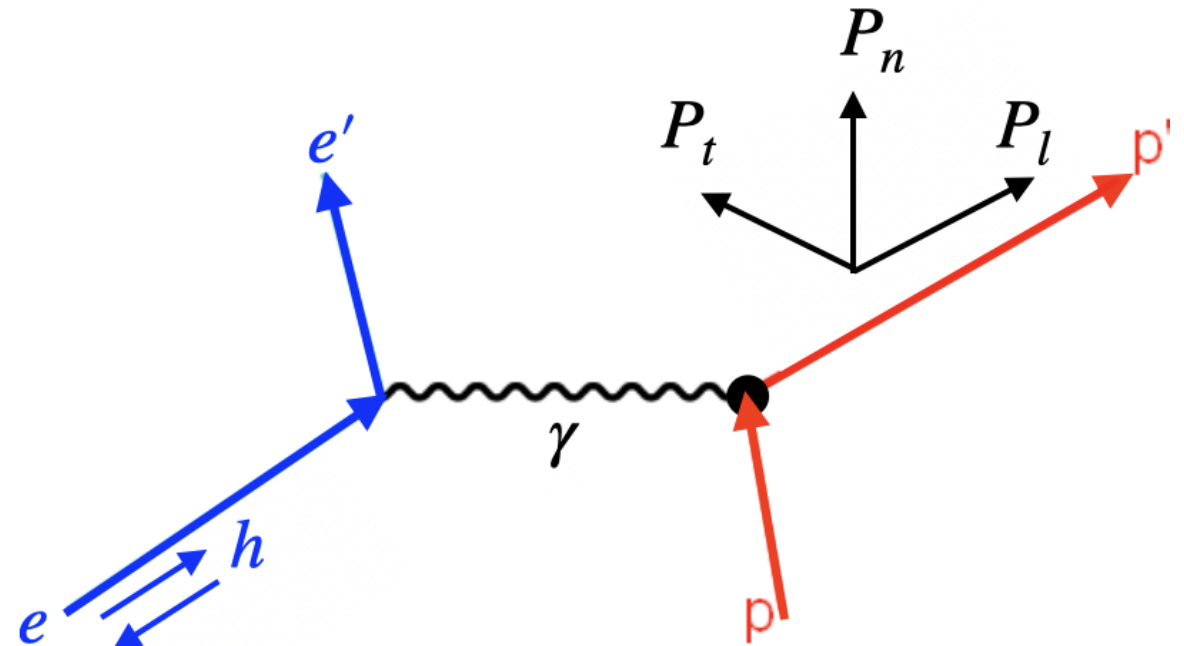
G_E/G_M

$$\sigma_R = d\sigma/d\Omega[\varepsilon(1 + \tau)/\sigma_{Mott}]$$

$$\sigma_R = \tau G_M^2(Q^2) + \varepsilon G_E^2(Q^2)$$



$$\frac{G_E}{G_M} = -\frac{P_t}{P_l} \frac{E + E'}{2M} \tan\left(\theta_e/2\right)$$



Slide from Mike Nycz

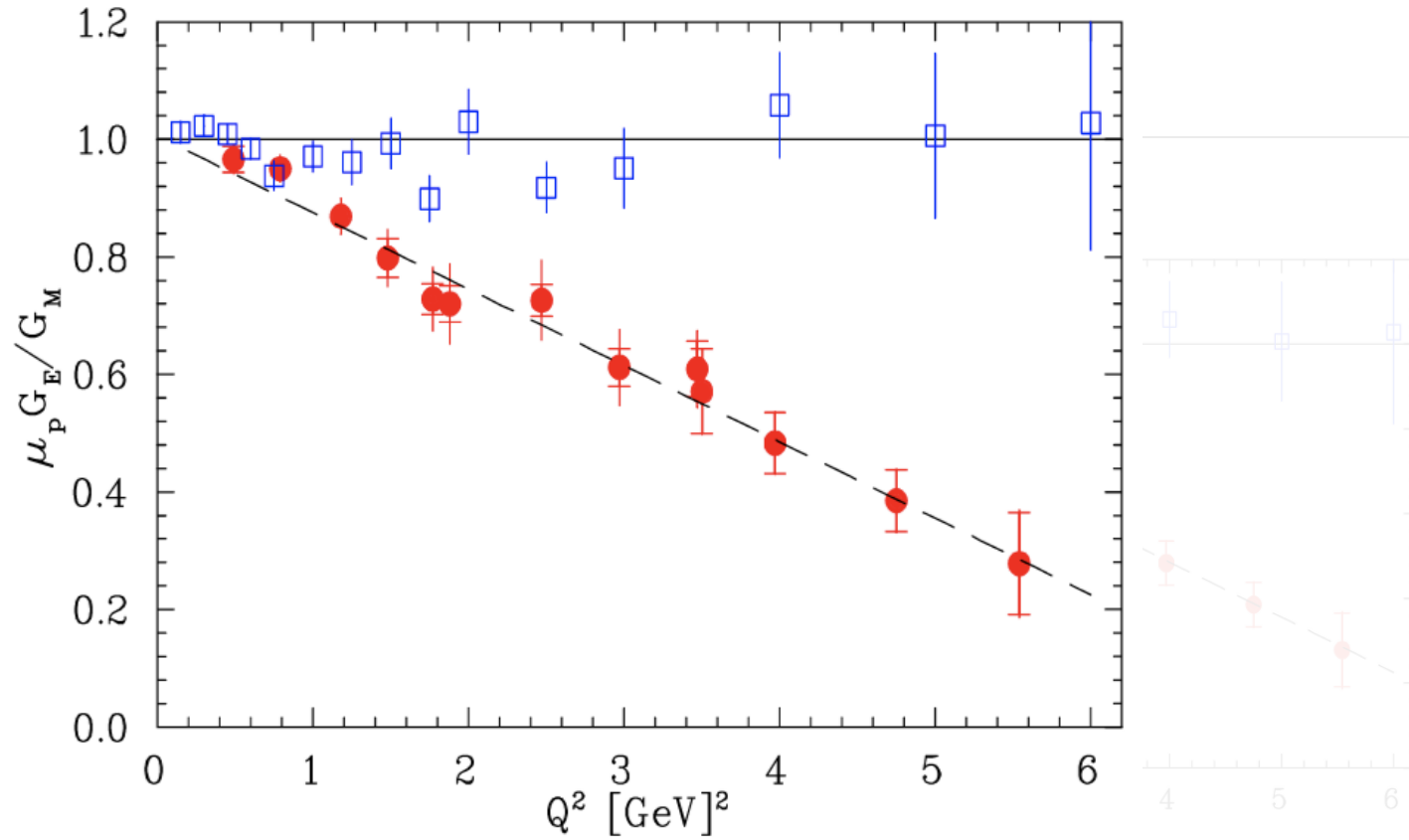
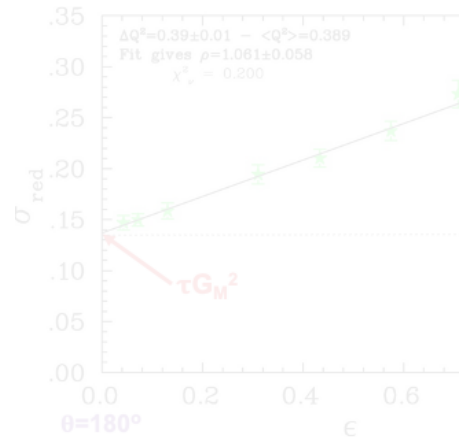
G_E/G_M

VOLUME 84, NUMBER 7

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14 FEBRUARY 2000

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Two-Photon Exchange in DIS & SIDIS

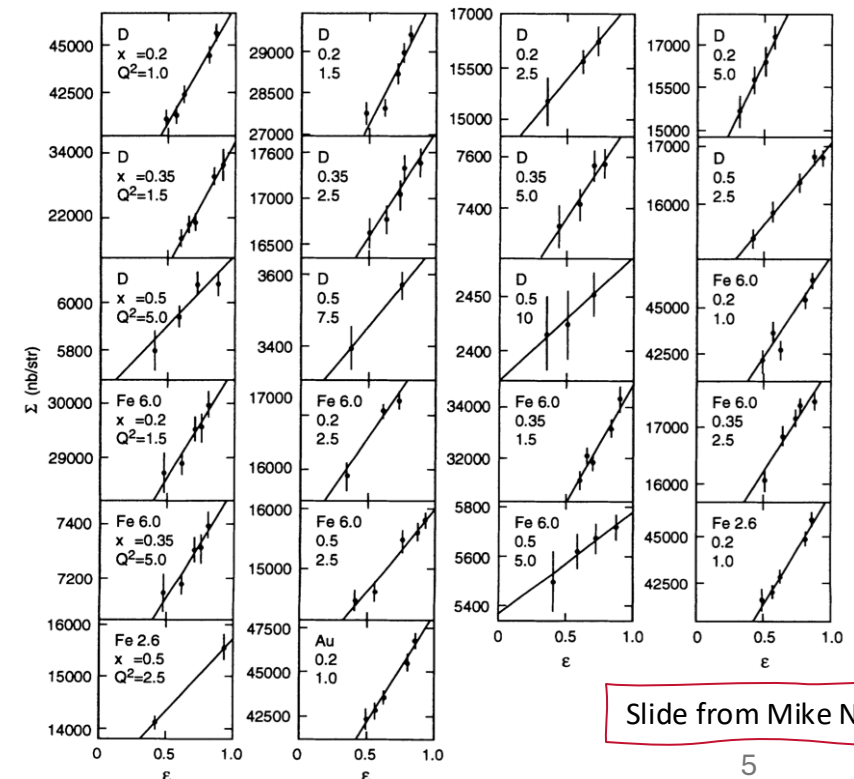
- No corollary to polarization transfer in DIS or SIDIS
- $\frac{G_E}{G_M}$ highlights the importance of understanding TPE effects in DIS
 - Small effect can possibly have a large impact on
- Impact of TPE
 - L/T separations (Constant Q^2 and x bins)

$$\frac{d^2\sigma}{d\Omega dE'} = \Gamma[\sigma_T(x, Q^2) + \varepsilon\sigma_L(x, Q^2)]$$

$$R = \frac{\sigma_L(x, Q^2)}{\sigma_T(x, Q^2)}$$

TPE effects

- ε dependent ?
- At large Q^2 ?



Slide from Mike Nycz

Two-Photon Exchange in DIS & SIDIS

Slide from Mike Nycz

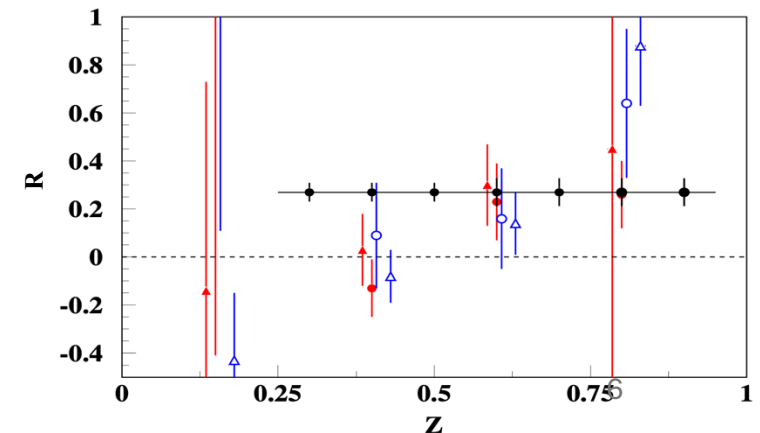
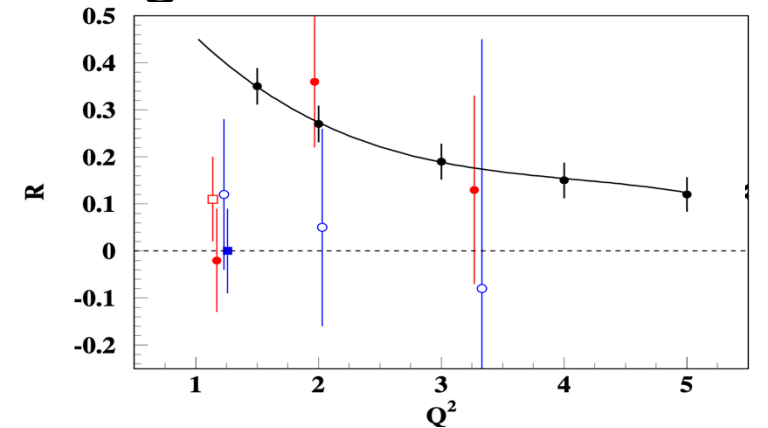
- No corollary to polarization transfer in DIS or SIDIS
- Elastic measurements highlight importance of understanding TPE effects in both
 - Small effect can possibly have a large impact on
- Impact of TPE
 - L/T separations: Pion electroproduction cross section

$$\frac{d^2\sigma}{d\Omega_\pi dM_x} = \frac{d\sigma_T}{d\Omega_\pi dM_x} + \varepsilon \frac{d\sigma_L}{d\Omega_\pi dM_x} + \varepsilon \frac{d\sigma_{TT}}{d\Omega_\pi dM_x} \cos 2\varphi_{pq} + \sqrt{2\varepsilon(1+\varepsilon)} \frac{d\sigma_{LT}}{d\Omega_\pi dM_x} \cos \varphi_{pq}$$

- R_{SIDIS} is assumed to be the same as R_{DIS} ($R_{\text{SIDIS}} = R_{\text{DIS}}$)
- Possible z & p_t -dependence of R_{SIDIS} ?
- $R_{\text{SIDIS}}^{\pi^+} = R_{\text{SIDIS}}^{\pi^-}$?

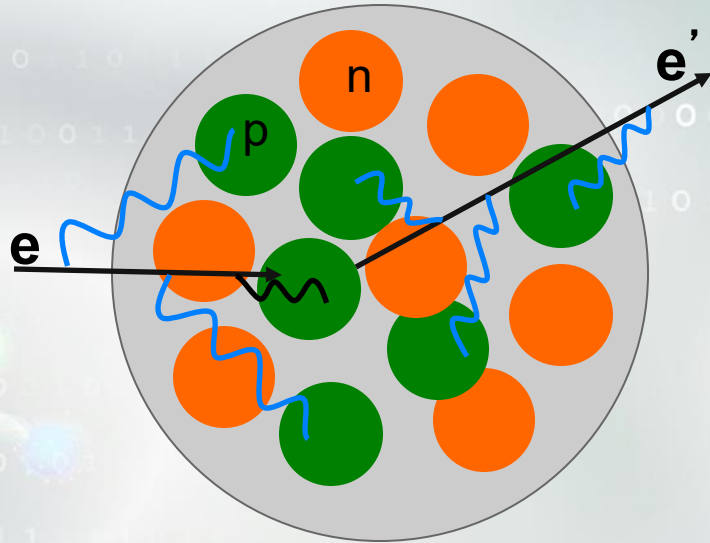
Impact of TPE?

Multi-Photon Effects in DIS and SIDIS



Coulomb Corrections

Coulomb Distortion in Heavy Nuclei



Electrons scattering from nuclei can be accelerated/decelerated in the Coulomb field of the nucleus
 → This effect is in general **NOT** included in most radiative corrections procedures
 → *Coulomb Corrections are perhaps more appropriately described in terms of multi-photon exchange, but Coulomb Corrections provide convenient shorthand*

- Well-known effect in QE scattering - relevant particularly for Coulomb sum rule
- Can be calculated in QE using DWBA → experimentalists use Effective Momentum Approximation (EMA) to apply corrections to data
- Comparisons of EMA with detailed DWBA calculations resulted in “improved EMA”

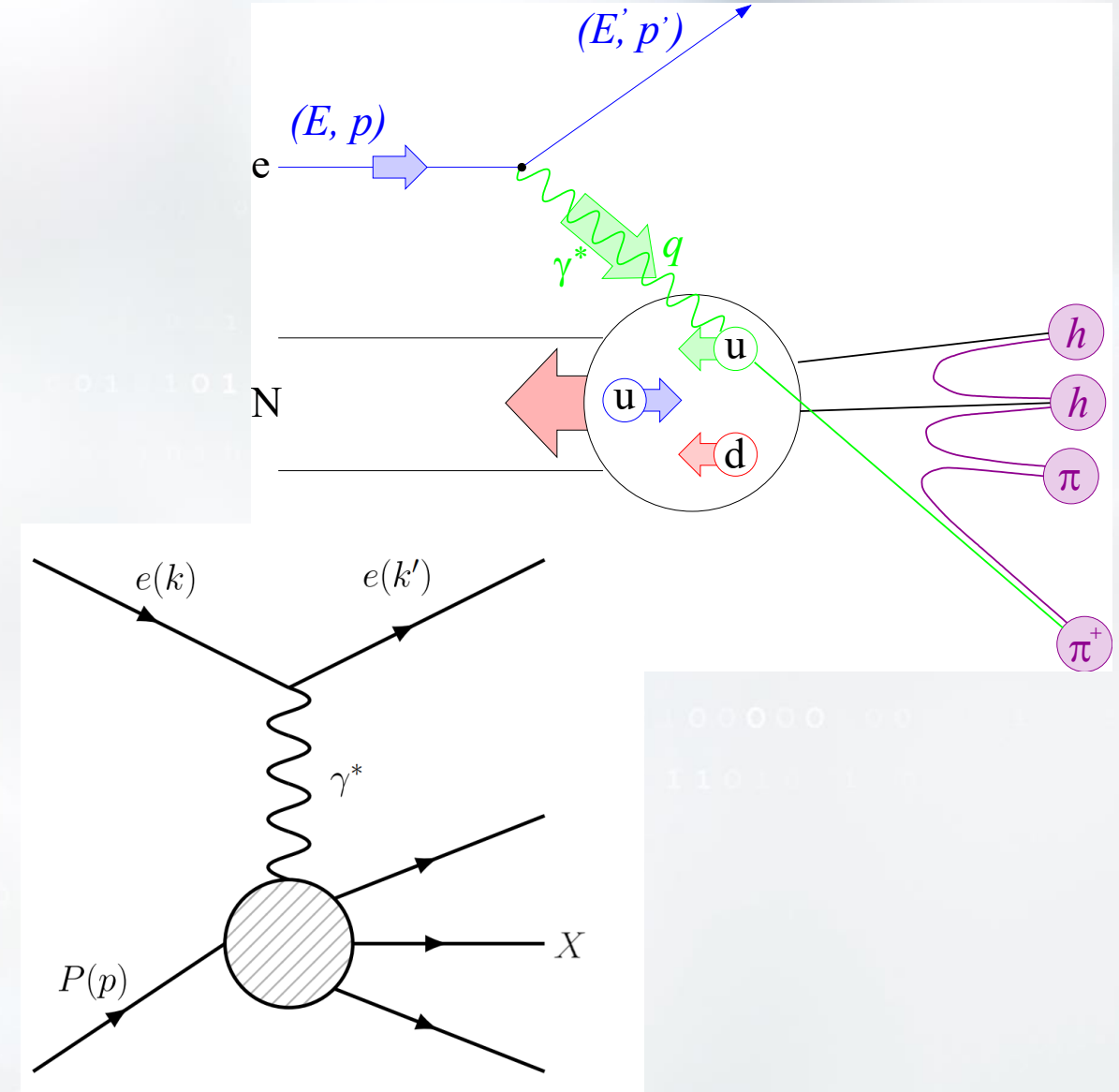
$$E_e \rightarrow E_e + V_0 \quad E_e' \rightarrow E_e' + V_0 \quad \text{with “focusing factor” } F^2 = (1 + V_0/E_e)^2$$

$$V_0 \rightarrow (0.7-0.8)V_0, \quad V_0 = 3a(Z-1)/2R$$

Slide from Dave Gaskell

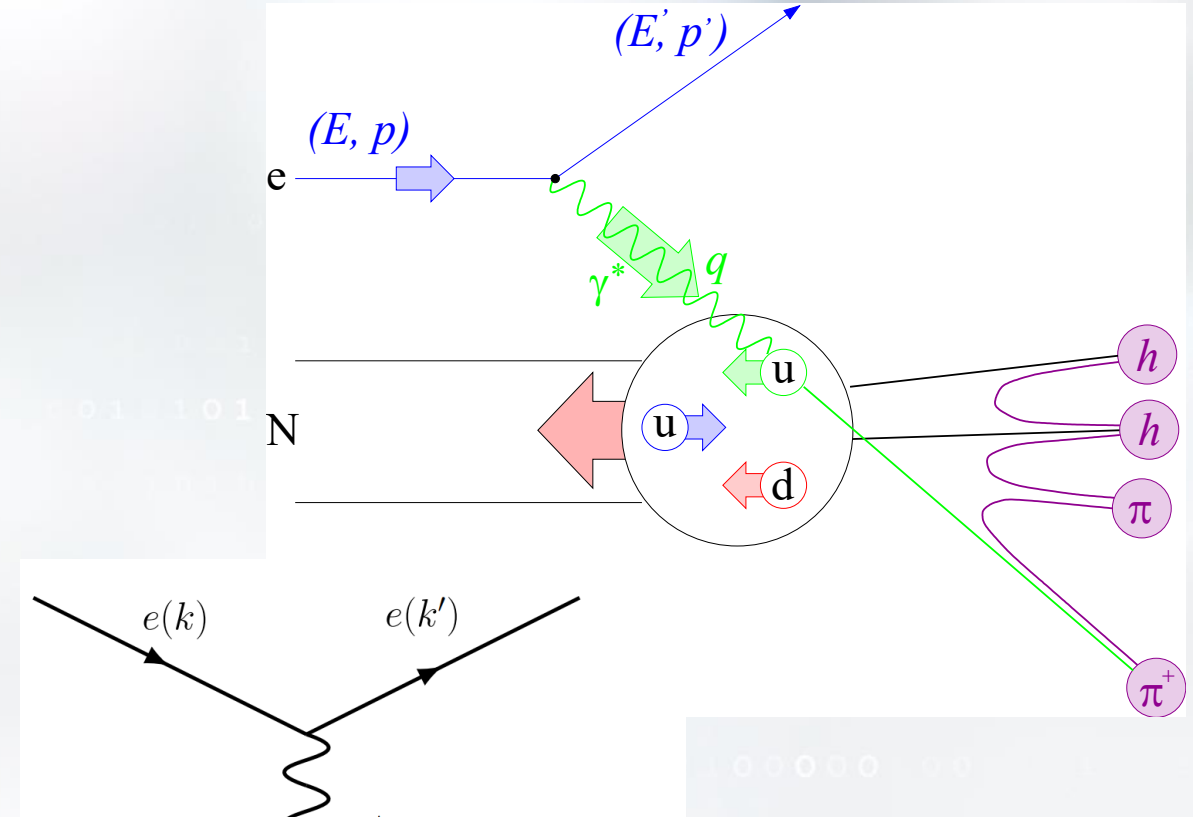
DIS and SIDIS

- It is unclear if the EMA is applicable at DIS kinematics
 - This will be tested by E12+23-003!
- A natural extension of this question is if CCs in SIDIS are the same as DIS
 - DIS integrates over all hadronic final states
 - SIDIS selects for a subset of these for specific kinematics of a *piece* of the final state
- Does this selection impact CCs to the process?

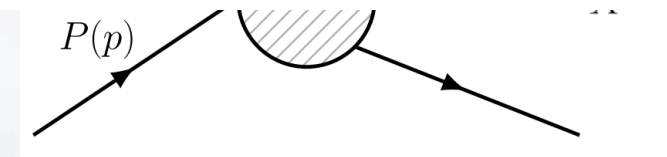


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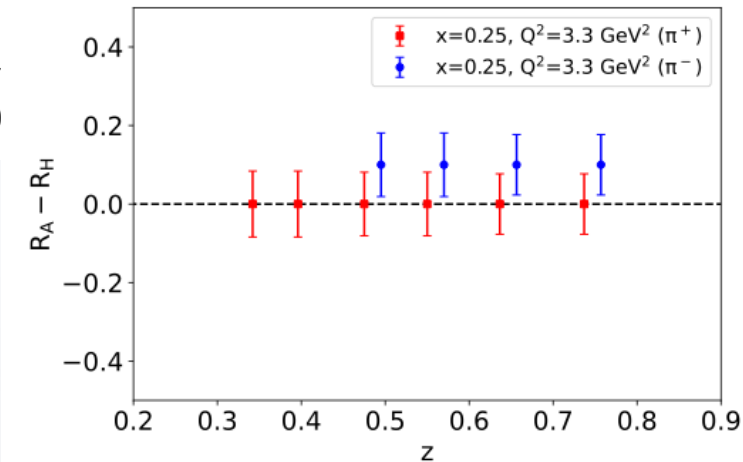
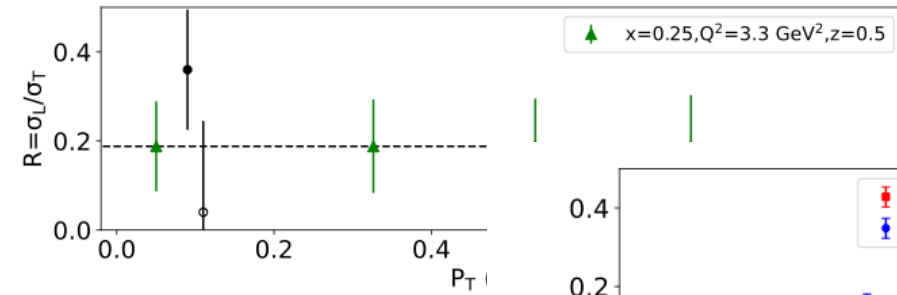
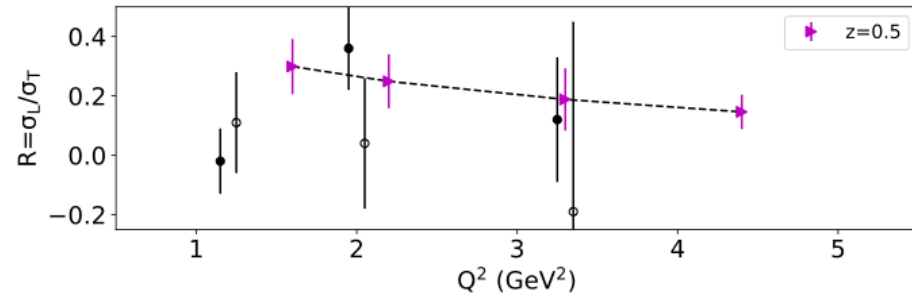
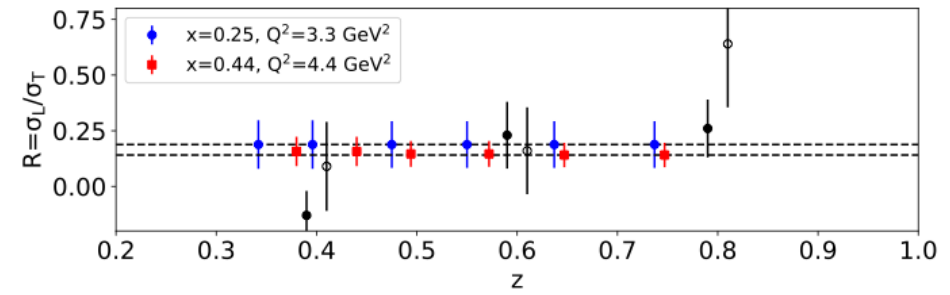


This is untested! The applicability of the EMA to SIDIS is an assumption that we can and should test!



Impact on SIDIS Data

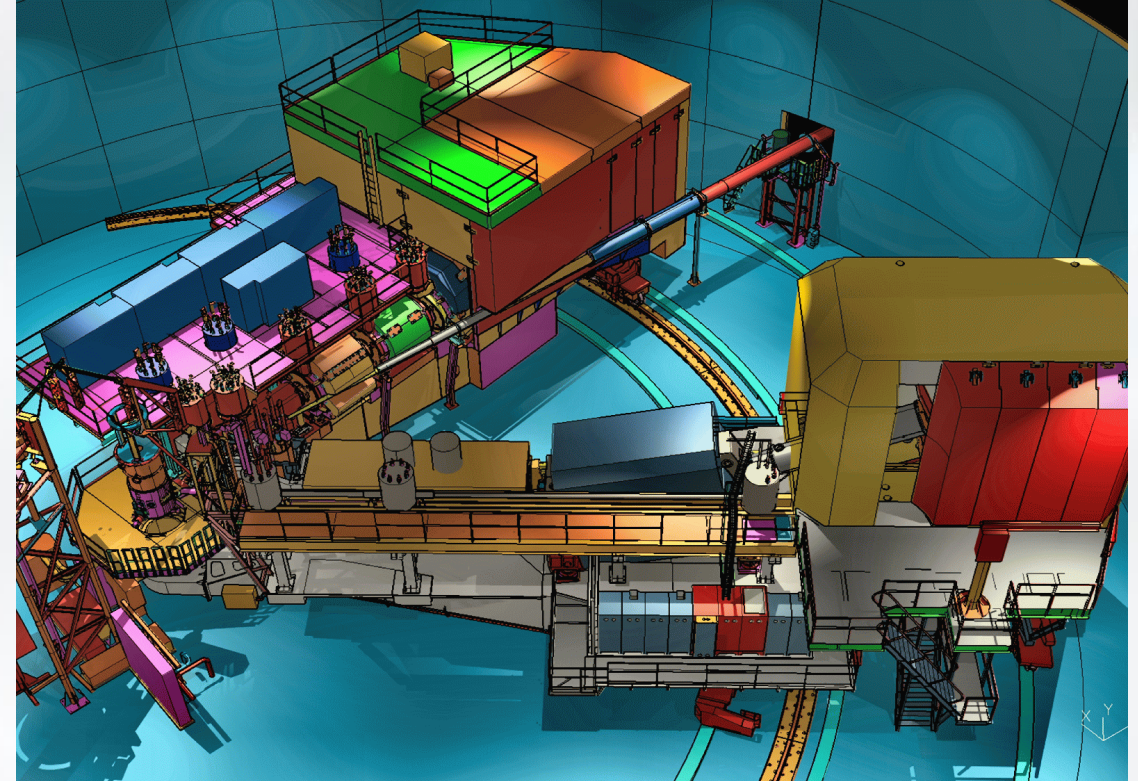
- E12-06-104 (**this year!**) will extract R_{LT} in SIDIS data!
 - Two-photon exchange, an ε -dependent effect, will clearly impact this
- E12-24-001 formed a run group to extract R_{LT} in *nuclear* SIDIS data
 - Exploratory study of if R_{LT} is the same in SIDIS as in DIS
 - Carbon and Copper targets
 - Unclear if Coulomb Corrections will be the same in DIS and SIDIS – *No Data!*
 - Coulomb Corrections are also an ε -dependent effect, will directly impact the results
- SIDIS is a key part of JLab program
 - Important to understand and constrain corrections



Equipment and Measurement

Required Equipment

- High Momentum Spectrometer for leptons
- Super High Momentum Spectrometer for pions
- Beam Energies
 - 4.4 GeV
 - 5.5 GeV
 - 6.5 GeV
 - 8.6 GeV
 - 10.7 GeV
- Positron beam (assuming 1 μA)
- Targets:
 - 10 cm Hydrogen
 - Empty target cell
 - 6% RL Copper Foil
- 25 μA Electron beam



Electron Data

- These measurements focus on positron to electron cross section ratios
- It is necessary to record electron data in the same run period (that is on the same targets) for the experiment to succeed
 - Dominated by target thickness but cancels in ratio if using the same targets for all data
 - TPE will have 1.7% normalization uncertainty if using past data, 0.5% with new electron data
 - CC will have 2.3% normalization uncertainty if using past data, 0.5% with new electron data
- Using data from upcoming E12-06-104 (RSIDIS), E12-24-001 (Nuclear RSIDIS), and E12-14-002 (A Dependence of R) instead of recording in the same run period dramatically reduces usefulness of data
- We assume a 25 μ A electron beam – only adds an additional $\sim 15\%$ to the beam time request

CC in SIDIS

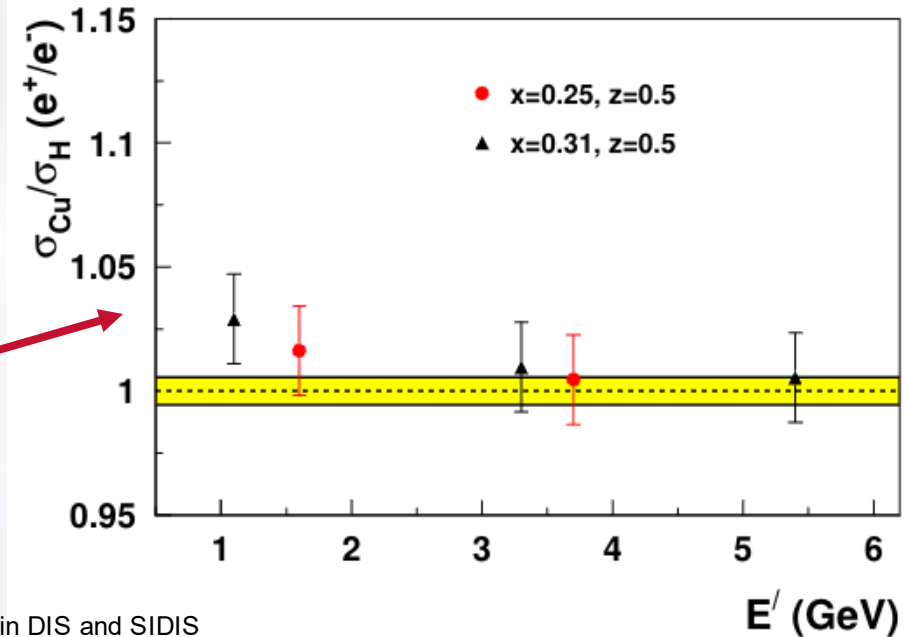
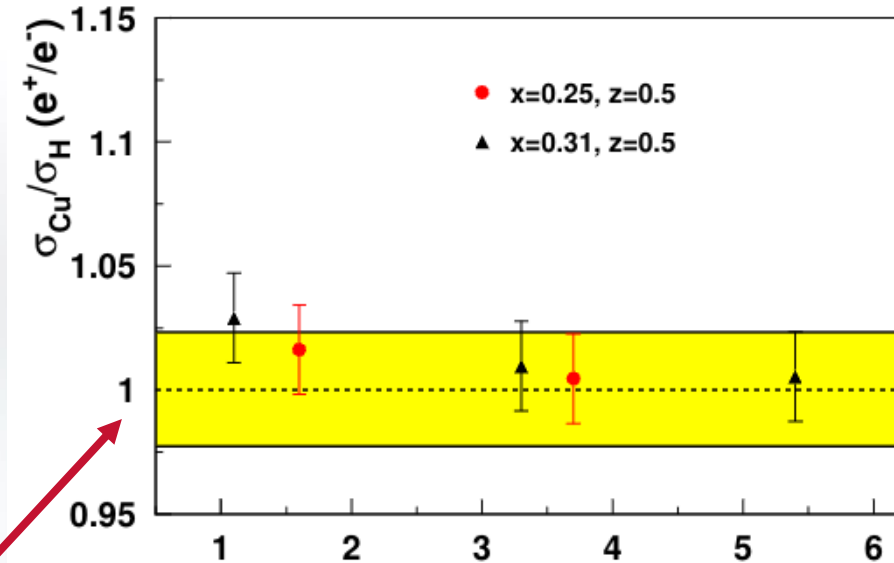
- Complementary to E12+23-003 (CC in DIS)
- Uses double ratio to extract corrections

$$R_{CC} = \frac{\frac{\sigma_A}{\sigma_H}|_{e^+}}{\frac{\sigma_A}{\sigma_H}|_{e^-}}$$

- Projections are based on EMA calculations

Electron data in different run period
(2.3% normalization uncertainty)

Electron data in same run period
(0.5% normalization uncertainty)



CC in SIDIS Kinematics

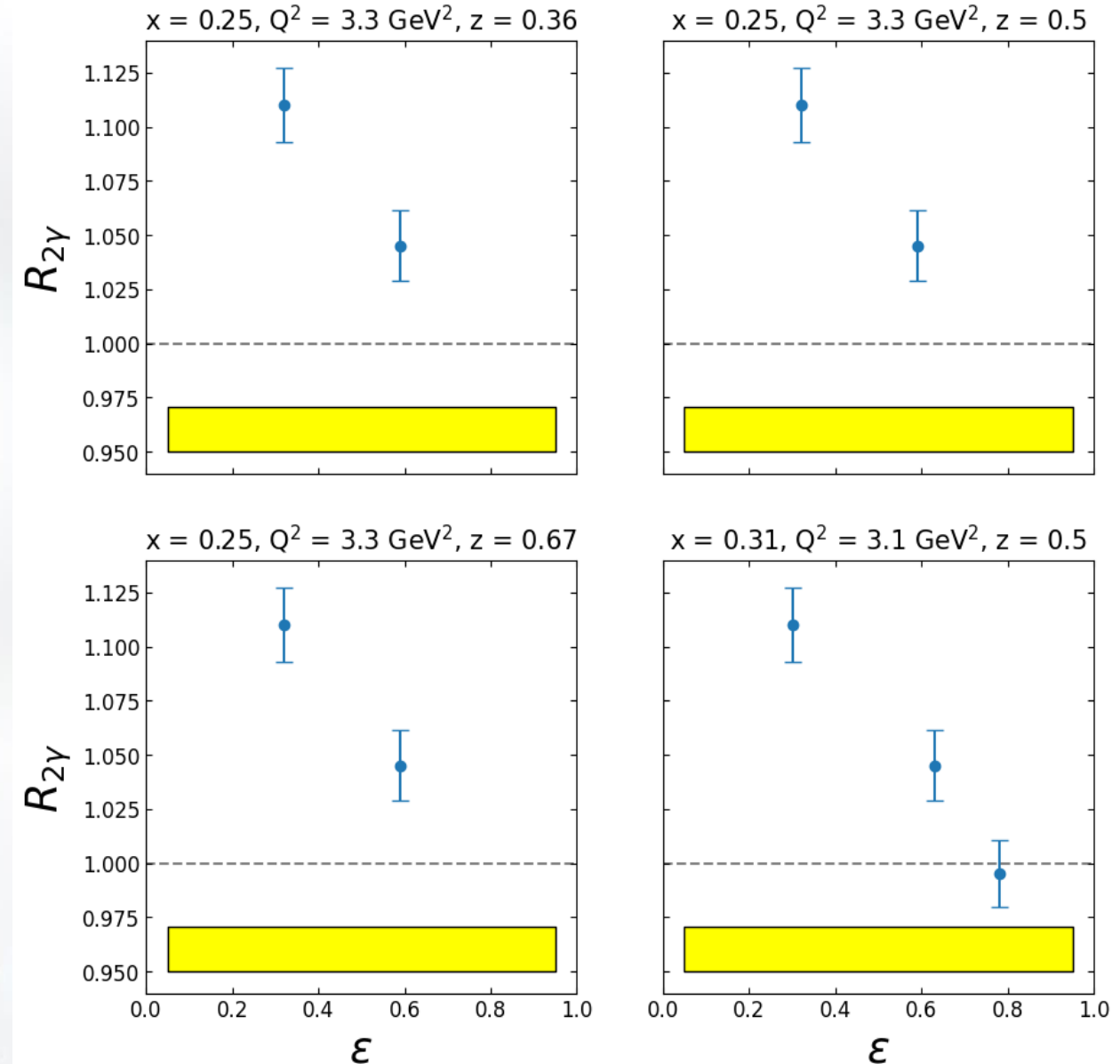
- A small number of kinematics to assess the validity of the IEMA in SIDIS
- Data recorded with π^+ final state at $z=0.5$
 - Higher rate than π^-
 - With no model of CC specific to SIDIS available, it is unclear how to disentangle effects from pion kinematics. Data is needed.
- Using 6% R.L. copper target
 - Though it is lower Z, it maximizes luminosity/R.L. in a rate limited scenario
 - 6% R.L. chosen to balance luminosity and radiative correction uncertainties
- Double ratio will be formed with hydrogen as opposed to deuterium
 - Make use of data already being recorded for TPE studies rather than increasing time to include another target

				Times (hrs)	
x	Q^2 (GeV ²)	E_0 (GeV)	ε	e^+	e^-
0.25	3.3	8.6	0.60	88.5	3.5
		10.7	0.33	11.5	1
0.31	3.1	6.5	0.30	252.6	10.1
		8.6	0.63	21.8	1
		10.7	0.78	5.8	1
Totals				380.2	16.6

TPE in SIDIS

- Coincident leptons in the HMS and pions in the SHMS
- We use the positron to electron ratio to extract the TPE contribution $\delta_{2\gamma}$

$$R_{2\gamma} = \frac{\sigma_{e^+}}{\sigma_{e^-}} = 1 - 2\delta_{2\gamma}$$



TPE in SIDIS Kinematics

					Times (hrs)						
x	Q^2	E_0	ε	z	π^+, e^+	π^+, e^-	Hz/ μ A	π^-, e^+	π^-, e^-	Hz/ μ A	DIS Hz/ μ A
0.25	3.3	8.6	0.32	0.36	49.4	2	0.056				6.11
				0.5	36.4	1.5	0.076				
				0.67	37.8	1.5	0.073				
0.25	3.3	10.7	0.59	0.36	6.3	1	3.22				19.99
				0.5	4.7	1	0.592				
				0.67	5	1	0.56				
0.31	3.1	6.5	0.3	0.5	105.8	4.2	0.026	252.5	10.1	0.011	4.47
		8.6	0.63	0.5	8.6	1	0.322	20.1	1	0.138	19.58
		10.7	0.78	0.5	2.3	1	1.23	5.3	1	0.527	44.49
Totals					256.3	14.2		277.9	12.1		

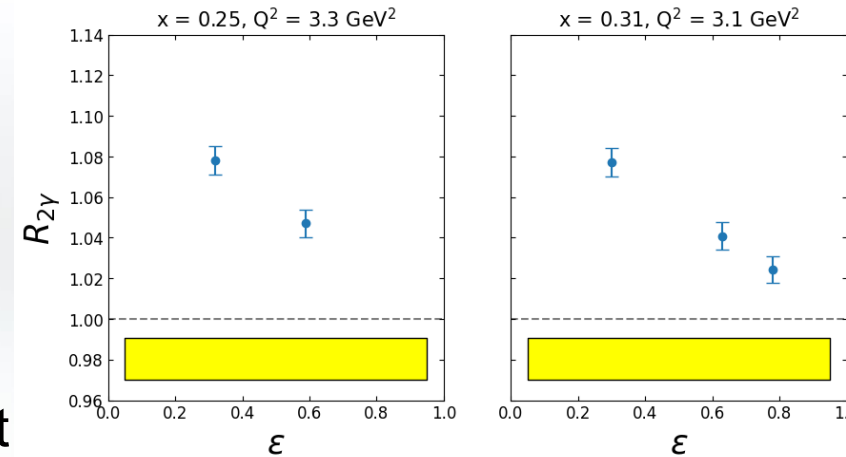
Many kinematics to test z , ε , x , and pion charge dependence
 DIS rates are low enough for concurrent measurement (maxing out at ~1.1 kHz)

TPE in DIS

- Inclusive lepton measurements in HMS
- Including additional inclusive-only kinematics as it is higher rate
- We use the positron to electron ratio to extract the TPE contribution $\delta_{2\gamma}$

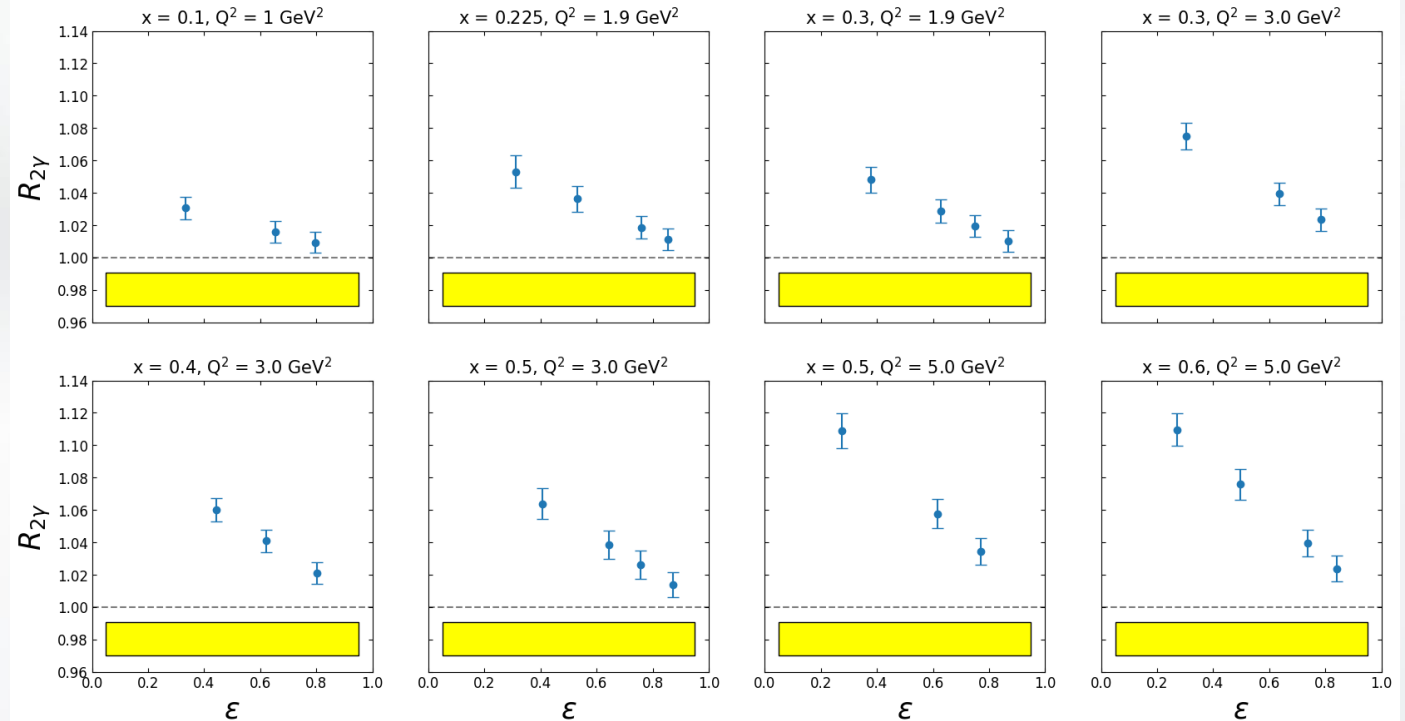
$$R_{2\gamma} = \frac{\sigma_{e^+}}{\sigma_{e^-}} = 1 - 2\delta_{2\gamma}$$

- Mapping large x range



Concurrent kinematics

Independent kinematics



TPE in DIS Kinematics

- Kinematics largely mirror E12-14-002 “A dependence of R” experiment
- These kinematics were selected to cover a large range in x to map any effect and focus on high Q² data where the effect is expected to be largest
- All settings aim to collect at least 10k events and were then rounded up to a minimum of 1 hour per setting
- Settings are divided between SHMS and HMS such that kinematics that will be compared most directly to each other are recorded in the same spectrometer

				SHMS Time (hrs)		HMS Time (hrs)	
x	Q^2	E_0	ε	e^+	e^-	e^+	e^-
0.1	1	6.5	0.34	1	1		
		8.6	0.66	1	1		
		10.7	0.8	1	1		
0.225	1.9	5.5	0.33	1	1		
		6.5	0.55	1	1		
		8.6	0.77	1	1		
		10.7	0.87	1	1		
0.3	1.9	4.4	0.38	1	1		
		5.5	0.63	1	1		
		6.5	0.76	1	1		
		8.6	0.88	1	1		
0.3	3	6.5	0.33	1	1		
		8.6	0.66	1	1		
		10.7	0.8	1	1		
0.4	3	5.5	0.44	2	1		
		6.6	0.63	1	1		
		8.6	0.81	1	1		
		10.7	0.89	1	1		
0.5	3	4.4	0.47			5	1
		5.5	0.7			2	1
		6.6	0.8			1	1
		8.8	0.9			1	1
0.5	5	6.6	0.3			16	1
		8.8	0.64			4	1
		11	0.79			3	1
0.6	5	5.5	0.27			20	1
		6.6	0.51			6	1
		8.8	0.75			3	1
		10.7	0.85			2	1
Totals				19	18	63	11

Systematics

Source	$\delta R/R$ (%) point-to-point	$\delta R/R$ (%) scale
Spectrometer momentum	-	< 0.1%
Beam energy	-	< 0.1%
θ_{spec}	-	< 0.1%
Charge	0.35%	-
Target Boiling	-	0.1%
Total dead time	0.15%	0.14%
Detector efficiency	0.11%	-
Charge Symmetric Background	-	-
Pion background	0.2%	-
Radiative Corrections	-	0.5%
Acceptance	0.2%	-
Cryotarget wall subtraction	-	-
Total	0.49%	0.56%

Coulomb Correction Double Ratio

Lower uncertainties due to cancellations in double ratio

Source	$\delta R/R$ (%) point-to-point	$\delta R/R$ (%) scale
Spectrometer momentum	-	< 0.1%
Beam energy	-	< 0.1%
θ_{spec}	0.15-0.46%	< 0.1%
Charge	0.35%	2%
Target Boiling	-	0.1%
Total dead time	0.15%	0.14%
Detector efficiency	0.11%	-
Charge Symmetric Background	-	-
Pion background	0.2%	-
Radiative Corrections	-	0.5%
Acceptance	0.1-0.14%	-
Cryotarget wall subtraction	-	-
Total	0.48-0.65%	2.07%

TPE Ratios

Scale uncertainty dominated by total charge measurement

Unser does not work for low currents

2% based on past injector faraday cup measurements

Discussion of Specific Questions from the PAC

E12-06-104, E12-24-001, and E12-14-002 Running Beam time overhead requests

- *“Have these experiments run or been scheduled to run?”*
 - E12-06-104 and E12-24-001 will be starting imminently
 - E12-14-002 has not been scheduled yet (originally anticipated to run FY26). Beam time request was submitted and approved in September 2024
- *“What is the impact on the proposed measurement if only a fraction of these other experiments produce useful data?”*
 - The readers correctly point out that we state that the data collected in these experiments are necessary for parameterizing the charge symmetric background (CSB) for this proposal
 - If the data for CSB is not adequately recorded in these experiments, we will need to record CSB data in this experiment
 - While we certainly hope for the success of these experiments beyond just our need for their CSB data, we will be able to know of any issues well in advance of the planned positron program (Expected in ~10 years?) in order to adjust our plans accordingly
- *“There are assumptions that the time estimated for polarity change and beam energy changes are underestimated”*
 - We have 5 pass changes for each beam polarity and we have assumed 8 hours per pass change to mirror the conditionally approved experiment E12+23-003 (CC in DIS)
 - If we modify this to 1 PAC day per pass change, our total beam time request would increase to 1560.6 hours (65 days)
 - We have not included the time for a beam polarity change in our request as there has been no guidance for what to assume in the Ce+BAF era. If we assume 1 PAC day, in addition to the other modifications our request would be 1584.6 hours (66 days)

Pion Rates

- Pion Rates for the dedicated DIS running is to the right. For the SIDIS and concurrent DIS is below.
- The HMS and SHMS Cherenkov efficiencies are typically greater than 99%
- The very high pion rates at some kinematics will necessitate the inclusion of the Cherenkov in the trigger
- Electron/Positron rates are all below the ideal 5kHz threshold, pre-scaling will be used should the Cherenkov not sufficiently reduce the trigger rate

x	Q^2 GeV ²	E_0 GeV	e^+ Rate Hz at 1 μ A	π^+ Rate Hz at 1 μ A	e^- Rate Hz at 25 μ A	π^- Rate Hz at 25 μ A	π/e
0.1	1	6.5	5.28e+01	1.68e+04	1.32e+03	4.20e+05	319
		8.6	2.33e+02	1.19e+04	5.81e+03	2.98e+05	52
		10.7	5.38e+02	8.58e+03	1.34e+04	2.14e+05	16
0.225	2	5.5	5.90e+00	4.03e+03	1.48e+02	1.01e+05	683
		6.5	1.65e+01	2.64e+03	4.13e+02	6.60e+04	160
		8.6	5.26e+01	1.46e+03	1.32e+03	3.65e+04	28
		10.7	1.10e+02	1.03e+03	2.75e+03	2.58e+04	10
0.3	2	4.4	4.33e+01	2.11e+03	1.08e+03	5.28e+04	49
		5.5	1.24e+02	1.17e+03	3.11e+03	2.92e+04	10
		6.5	2.54e+02	8.44e+02	6.36e+03	2.11e+04	4
		8.6	6.45e+02	5.56e+02	1.61e+04	1.39e+04	<1
0.3	3	6.5	2.05e+01	1.57e+03	5.12e+02	3.92e+04	77
		8.6	8.65e+01	6.04e+02	2.16e+03	1.51e+04	7
		10.7	1.97e+02	3.67e+02	4.92e+03	9.18e+03	2
0.4	3	5.5	2.21e+01	5.46e+02	5.53e+02	1.36e+04	25
		6.5	5.07e+01	3.17e+02	1.27e+03	7.92e+03	7
		8.6	1.42e+02	1.71e+02	3.56e+03	4.28e+03	2
		10.7	2.83e+02	1.26e+02	7.07e+03	3.15e+03	<1
0.5	3	4.4	1.87e+00	7.37e+01	4.68e+01	1.84e+03	40
		5.5	4.96e+00	2.86e+01	1.24e+02	7.15e+02	6
		6.5	8.71e+00	1.79e+01	2.18e+02	4.47e+02	3
		8.6	2.00e+01	9.29e+00	5.00e+02	2.32e+02	<1
		10.7	4.09e+01	4.06e+00	1.02e+03	1.01e+02	<1
0.5	5	6.5	5.51e-01	5.70e+01	1.38e+01	1.42e+03	104
		8.6	2.28e+00	2.10e+00	5.69e+01	5.25e+01	<1
		10.7	5.13e+00	1.16e+00	1.28e+02	2.90e+01	<1
0.6	5	5.5	5.98e-01	2.34e+01	1.49e+01	5.85e+02	40
		6.5	1.58e+00	7.11e+00	3.94e+01	1.78e+02	5
		8.6	4.81e+00	2.10e+00	1.20e+02	5.25e+01	<1
		10.7	9.88e+00	1.16e+00	2.47e+02	2.90e+01	<1

x	Q^2 GeV ²	E_0 GeV	e^+ Rate Hz at 1 μ A	π^+ Rate Hz at 1 μ A	e^- Rate Hz at 25 μ A	π^- Rate Hz at 25 μ A	π/e
0.25	3	8.6	6.11e+00	5.79e+02	1.53e+02	1.45e+04	95
		10.7	2.00e+01	2.18e+02	5.00e+02	5.45e+03	11
0.31	3	6.5	4.47e+00	5.39e+02	1.12e+02	1.35e+04	121
		8.6	1.96e+01	1.12e+02	4.90e+02	2.80e+03	6
		10.7	4.45e+01	5.65e+01	1.11e+03	1.41e+03	2

Positron Beam Current Measurement

- As it stands, the Hall C user cannot measure the currents that will be available for positrons
- In our proposal, we have assumed a 2% normalization uncertainty in line with past experiments that measured the beam current with the injector Faraday cup
- Dave Mack has outlined a Faraday cup design that could be built for the Hall C positron era
 - 30 kW max continuous power load
 - Insertable
 - Theoretically capable of 0.1% precision on the *ratio* of currents, even with significant time separation
 - Continuous power load limit leads to a current limit of $3\mu\text{A} \cdot 11\text{GeV}/E_{\text{beam}}$
 - For currents greater than this, the Faraday cup could not provide continuous monitoring but could record a ~ 10 second beam in order to calibrate the BCMs
 - Would yield an uncertainty of $\sim 0.5\%$ in the ratio, limited by BCM stability
- To our knowledge, the design is very preliminary
- We are hopeful that such a device will be created, but do not believe it would be fair to assume these dramatically improved uncertainties in our projections at this point

Summary

- Using positron and electron beams we will measure two-photon exchange effects in DIS and SIDIS and coulomb corrections in SIDIS
- These are critical to the interpretation and analysis of past and future JLab data

Activity	e^+ Time (hrs)	e^- Time (hrs)
TPE π^+ Production	256.3	14.2
TPE π^- Production	277.9	12.1
CC Production	380.2	16.6
Dedicated DIS Production	63	18
Pass Changes	40	40
Kinematic Changes	38	38
Target Changes	21	21
Empty Target	119.4	8.9
Target Boiling Studies	4	4
BCM Calibrations	8	8
Detector Checkout	12	
Subtotals	1219.8	180.8
Total	1400.6 hours	(58.4 days)

QUESTIONS?

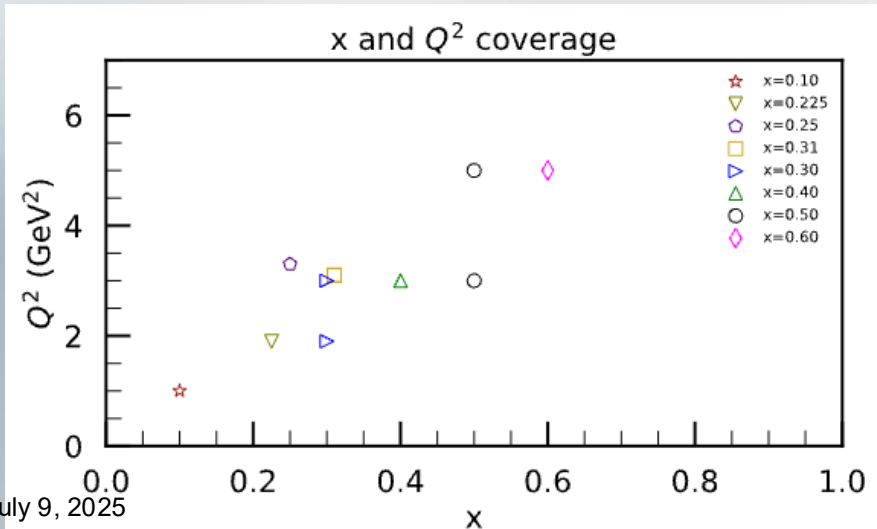
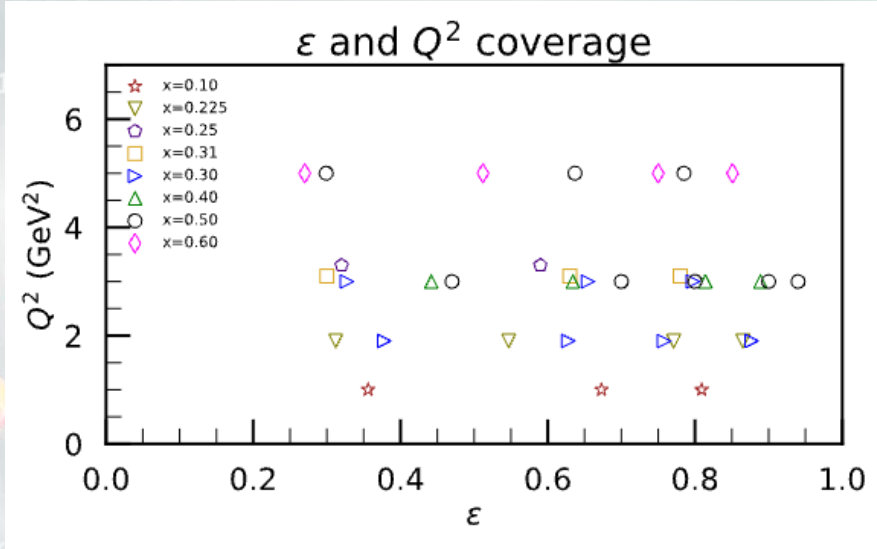
Backup

More Kinematic Details – SIDIS TPE

x	Q^2	E_0	z	HMS		SHMS	
				$E_{e'}$	$\theta_{e'}$	E_{π}	θ_{π}
0.25	3.3	8.6	0.36	1.6	28.7°	2.6	7.9°
			0.5	1.6	28.7°	3.6	7.9°
			0.67	1.6	28.7°	4.9	7.9°
0.25	3.3	10.7	0.36	3.7	16.7°	2.6	10.3°
			0.5	3.7	16.7°	3.6	10.3°
			0.67	3.7	16.7°	4.9	10.3°
0.31	3.1	6.5	0.5	1.1	37.7°	2.8	9.2°
		8.6	0.5	3.3	19.2°	2.8	13°
		10.7	0.5	5.4	13.4°	2.8	14.7°

Table 3: The kinematic and spectrometer settings for SIDIS TPE studies. The HMS will be set to detect the scattered beam lepton. The SHMS will be set to detect the final hadronic state pion. For all kinematics, $\theta_{\pi q} = 2^\circ$. All kinematics will be recorded with a π^+ in the final state. The three data points at $x = 0.31$ and $Q^2 = 3.1 \text{ GeV}^2$ will also be recorded for a π^- final state. All lepton kinematics will be simultaneously recorded with an inclusive trigger in the HMS for the DIS TPE studies.

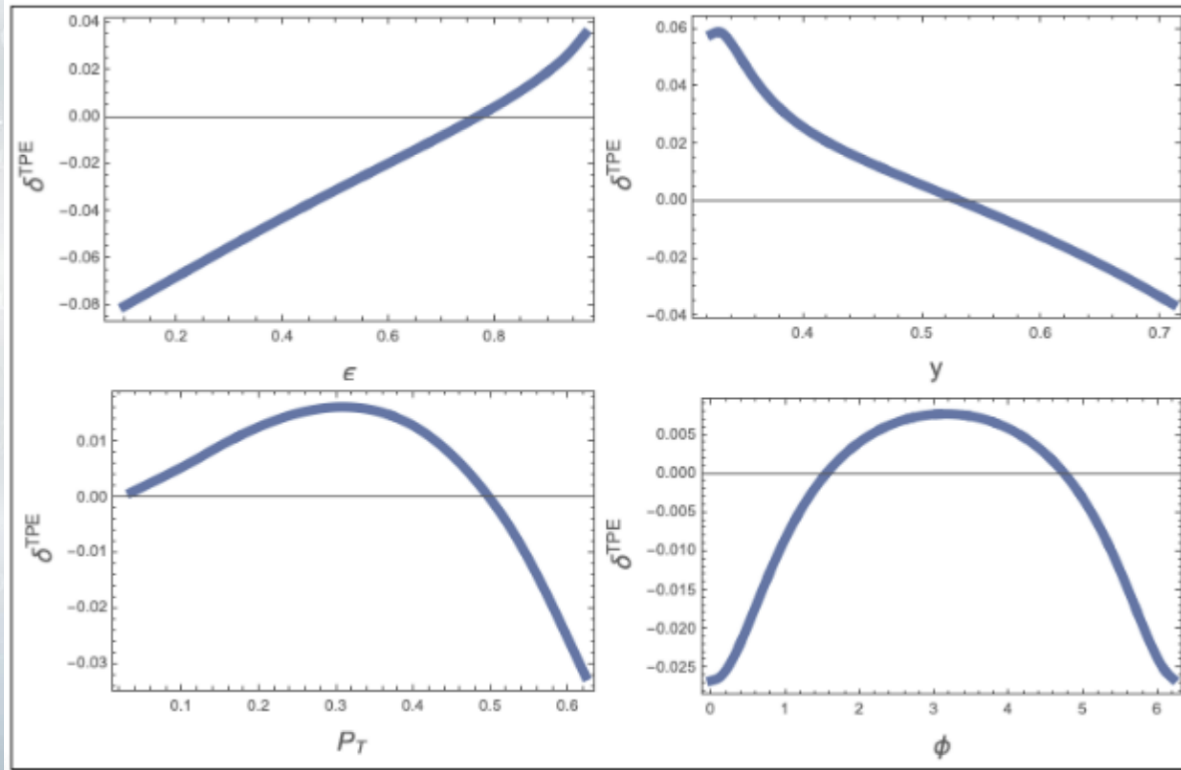
More Kinematic Details – DIS TPE



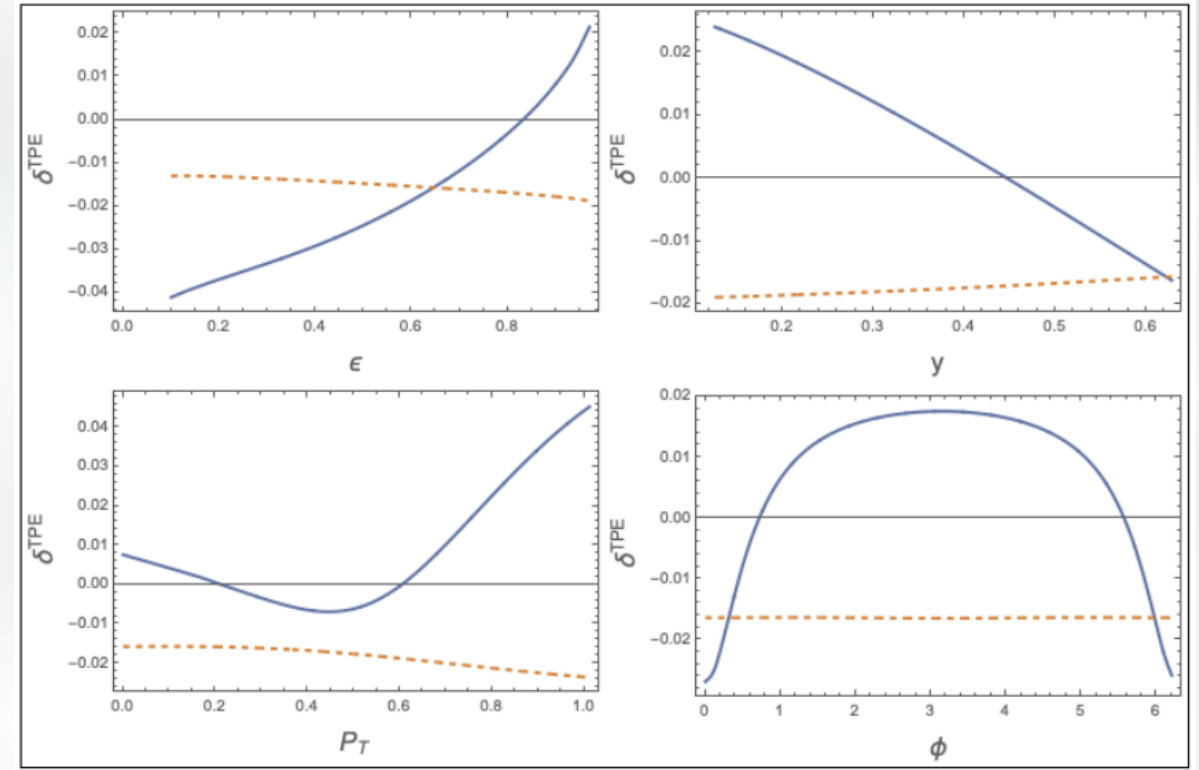
x	Q^2	E_0	ϵ	E_e'	θ_e'
0.25	3.3	8.6	0.32	1.6	28.7°
		10.7	0.59	3.7	16.7°
0.31	3.1	6.5	0.3	1.1	37.7°
		8.6	0.63	3.3	19.2°
		10.7	0.78	5.4	13.4°
0.1	1	6.5	0.34	1.2	20.9°
		8.6	0.66	3.3	10.8°
		10.7	0.8	5.4	7.6°
0.225	1.9	5.5	0.33	1	34.2°
		6.5	0.55	2	22°
		8.6	0.77	4.1	13.3°
		10.7	0.87	6.2	9.7°
0.3	1.9	4.4	0.41	1	37.9°
		5.5	0.66	2.1	23.3°
		6.5	0.78	3.1	17.6°
		8.6	0.89	5.2	11.8°
0.3	3	6.5	0.33	1.2	36.6°
		8.6	0.66	3.3	18.8°
		10.7	0.8	5.4	13.1°
0.4	3	5.5	0.48	1.5	35.1°
		6.5	0.66	2.5	24.8°
		8.6	0.83	4.6	15.8°
		10.7	0.9	6.7	11.7°
0.5	3	4.4	0.47	1.2	44.2°
		5.5	0.7	2.3	28.2°
		6.5	0.8	3.3	21.5°
		8.6	0.9	5.4	14.6°
		10.7	0.94	7.5	11.1°
0.5	5	6.5	0.31	1.2	47.8°
		8.6	0.65	3.3	24.3°
		10.7	0.8	5.4	17°
0.6	5	5.5	0.32	1.1	55.2°
		6.5	0.55	2.1	35.6°
		8.6	0.78	4.2	21.6°
		10.7	0.87	6.3	15.7°

Table 4: Kinematics for DIS TPE studies. The first two x , Q^2 entries are taken simultaneously with the SIDIS TPE data. The rest are recorded separately with $x = 0.5$ and above in the HMS and below $x = 0.5$ in the SHMS.

SIDIS TPE Projections



$E=10.6$ GeV, $x=0.31$, $Q^2=2.5$ GeV², and $z=0.5$
 From S. Lee [2024 PWG presentation](#)



$E=10.6$ GeV, $x=0.31$, $Q^2=2.5$ GeV², and $z=0.7$
 From S. Lee and A. Afanasev [arXiv:2504.17123](#)

DIS TPE Projections – Using Elastic Hard TPE Calculations

DIS TPE

The magnitude of TPE effects in both figures are calculated using a prescription a prescription from Ref. [43],

$$\delta_{2\gamma} = -0.069 (1 - \varepsilon) \ln (0.394 \text{GeV}^{-2} Q^2 + 1) . \quad (16)$$

This equation is based on a fit to world elastic data and assumes the full form factor discrepancy is attributable to “hard” TPE. The fit also assumes that there is a Q^2 dependent piece that incorporates quark degrees of freedom for high Q^2 scattering. We have opted to use this fit in the absence of *any* available calculations for TPE in DIS.

[43] Andrei Afanasev et al. Radiative corrections: from medium to high energy experiments. *Eur. Phys. J. A*, 60(4):91, 2024.