



HPS Collaboration Meeting
06/04/2025

Progress on the Prompt A' Resonance Search

Emrys Peets
Stanford University
SLAC National Accelerator Laboratory

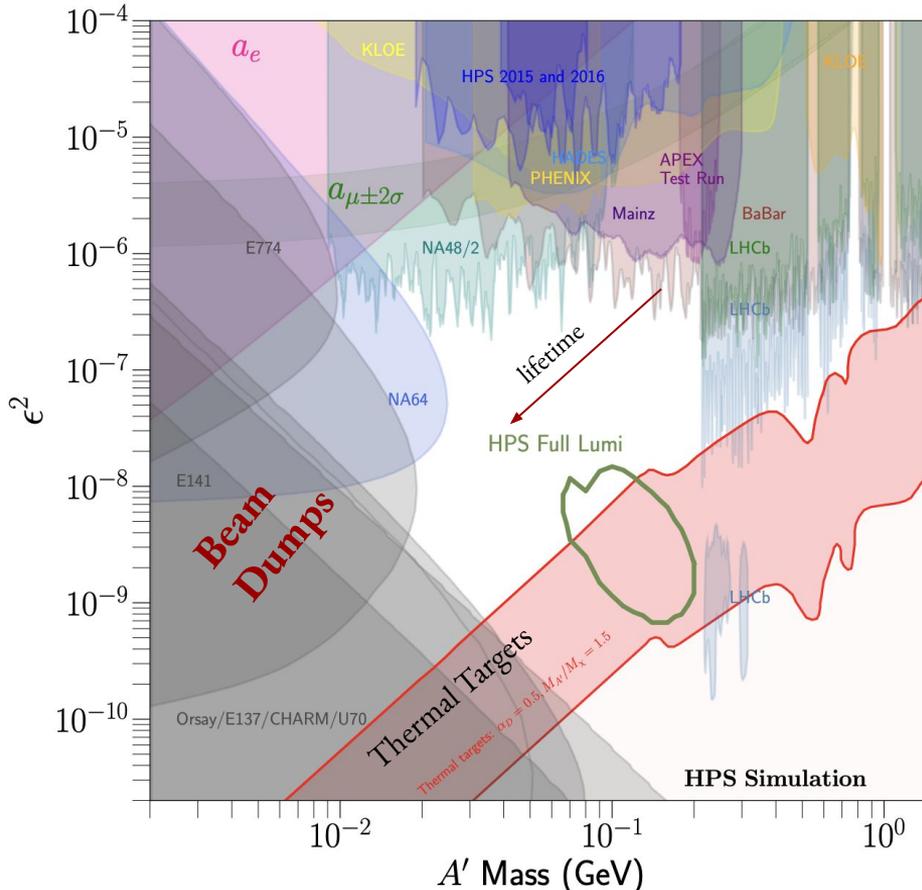


Stanford
University



NATIONAL
ACCELERATOR
LABORATORY

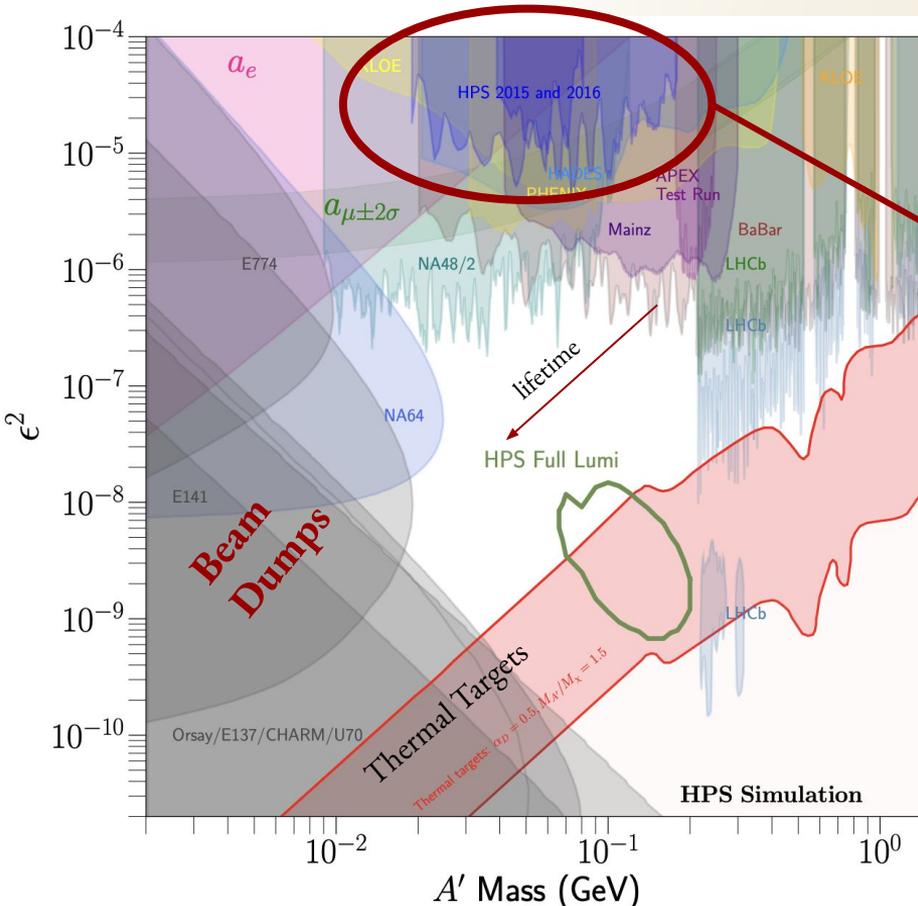
Physics Sensitivity of HPS



HPS has two primary search strategies for the A' depending on the lifetime / kinetic mixing, or coupling strength, (ϵ^2).

[HPS 2023 Publication](#)

Physics Sensitivity of HPS



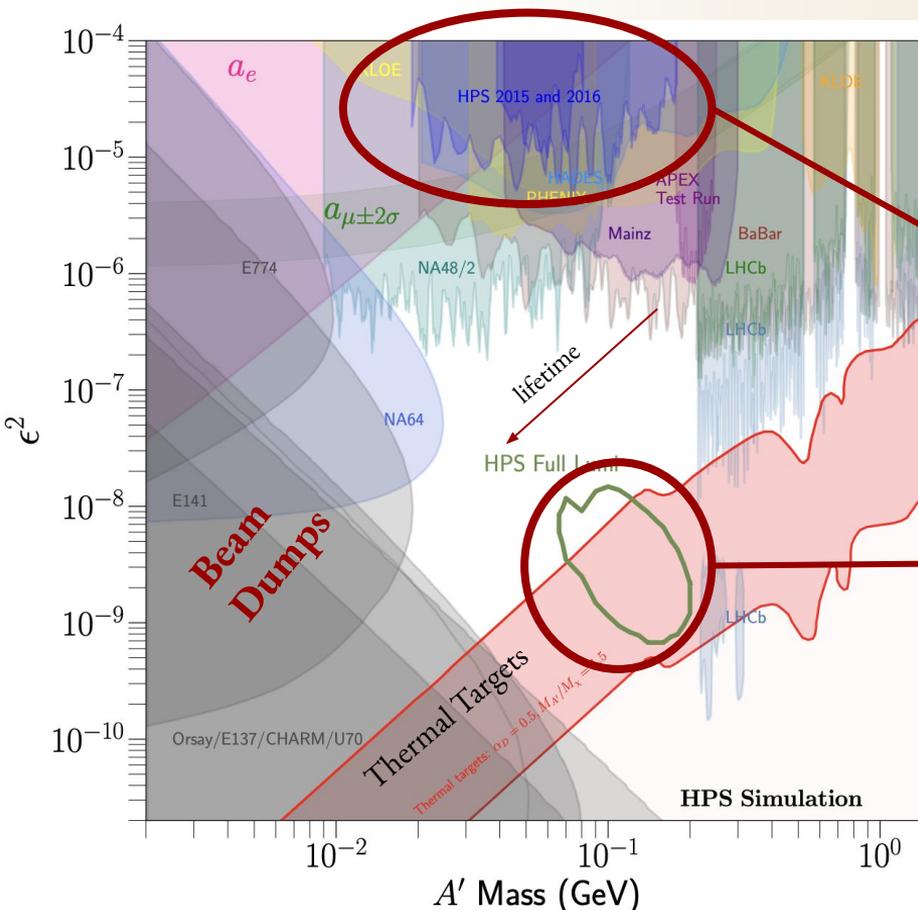
HPS has two primary search strategies for the A' depending on the lifetime / kinetic mixing, or coupling strength, (ϵ^2).

HPS Prompt Resonance Search Result

For higher coupling strengths (lower lifetime), A' 's are expected to decay extremely fast at the target and a signal is expected as a “bump” in the reconstructed e^+e^- invariant mass distribution (**IMD**).

[HPS 2023 Publication](#)

Physics Sensitivity of HPS



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HPS Displaced Vertex Search Reach Estimate

For lower coupling strengths, A' 's have a longer lifetime and the e^+e^- pairs are expected to be generated at characteristic distances away from the target.

[HPS 2023 Publication](#)

The Bumhunters' Chronicles

2023

Attempts at Fitting 2016 Distribution [*\[Spring JLAB Collab Meeting\]*](#)

Proof of Concept Fitting [*\[Fall SLAC Collab Meeting\]*](#)

2024

First Upper Limits* [*\[Spring JLAB Collab Meeting\]*](#)

Takumi Britt works on 2015 IMD [*\[Summer 2015 IMD Presentation\]*](#)

Aidan Hsu and Tom Eichlersmith develop gaussian process regression methodology

- Goal is to have validation of results and background characterization
- [*\[Aidan and TJ Winter SLAC Presentation\]*](#)

Full RooFit Implementation [*\[Fall SLAC Collab Meeting\]*](#)

2025

Blinded Analysis Technique Developed for control region
[this presentation]

TJ and Aidan work to finalize GP application to 100% 2015 and 10% 2016 *[Next Presentation]*

2025+

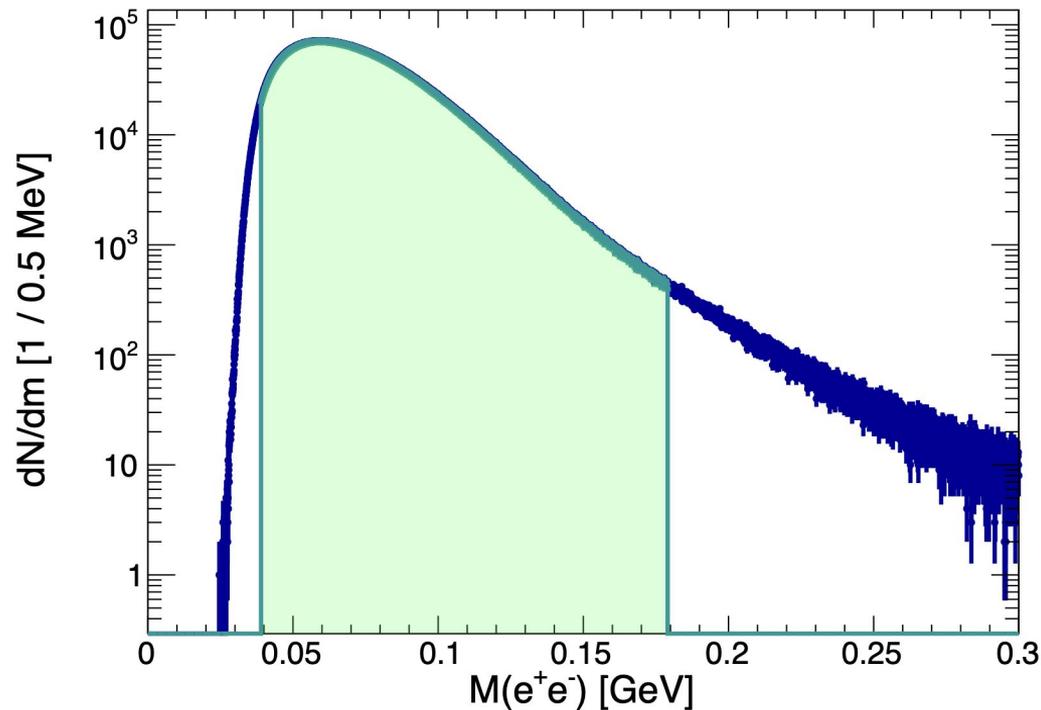
- Complete 2016 Control Region Study
- Recast 2015 and Validate with GPR
- Validate 100% 2016 with GPR
- 2021: 1% → 10% → 100%
 - Discover Heavy Photon
- 2019: 1% → 10% → 100%
 - Stockholm
- APEX: Data Exists, we should use it

HPS 2016 Reconstructed e^+e^- Invariant Mass Distribution

Data collected during 2016
engineering run

- total integrated luminosity of **10 pb⁻¹**.
- 67.2 mC or ~7 billion triggered events.

Raw data from the detector and MC
simulation are cleanly reconstructed to (e^+e^-)
pairs with shared vertices.

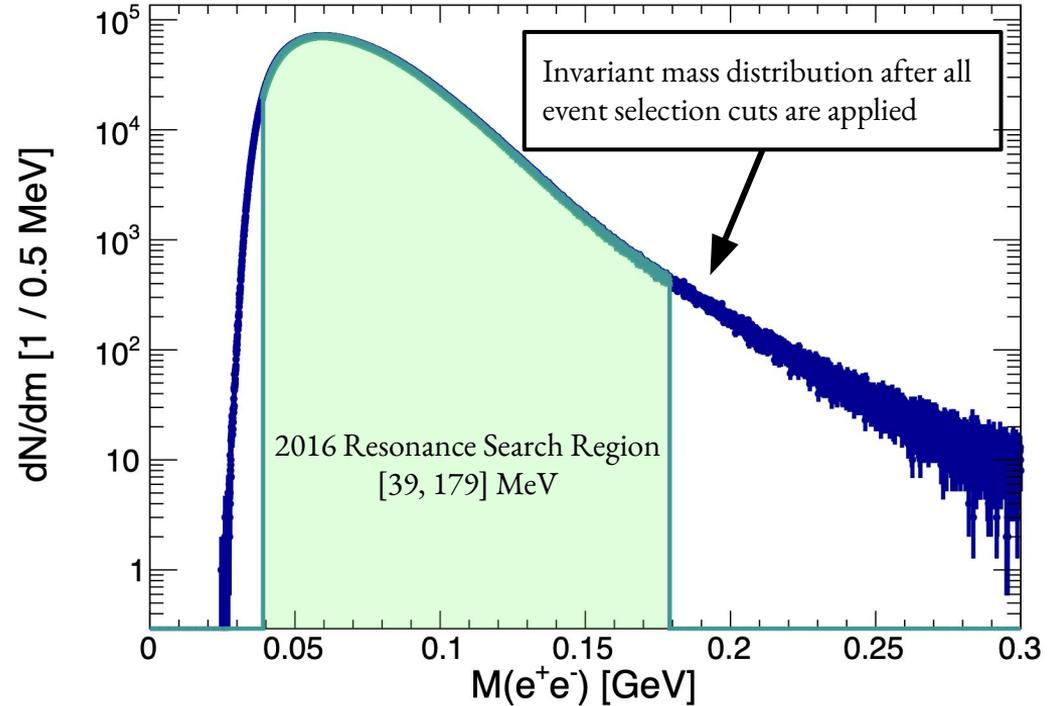


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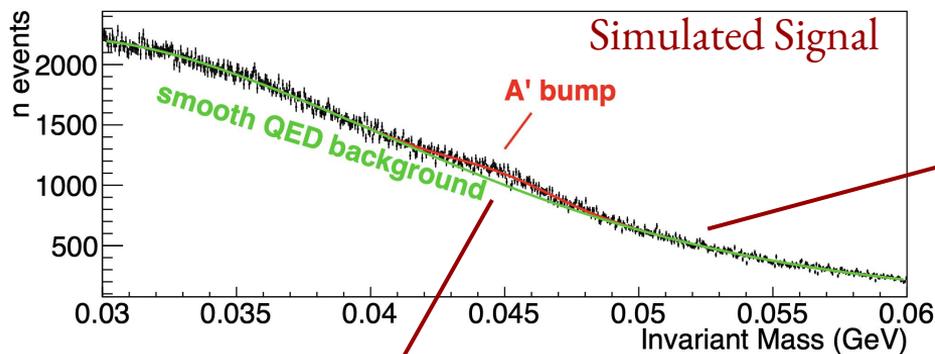
Raw data from the detector and MC simulation are cleanly reconstructed to (e^+e^-) pairs with shared vertices.



Event selection methodology / figure described in full in [2016 Physics Result](#) as published in PRD.

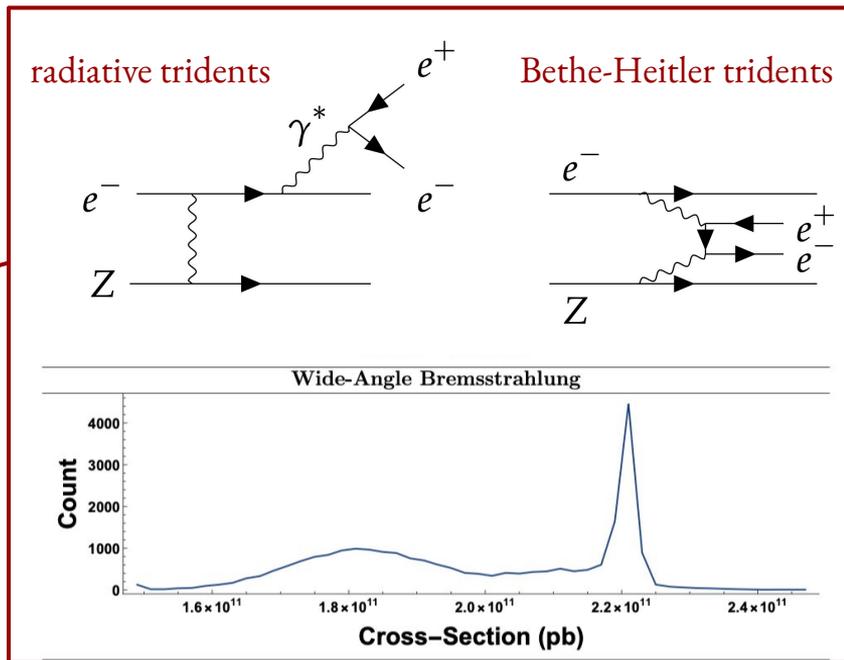
Prompt A' Signal Model and Backgrounds

If A' exists within the acceptance of HPS, it will present itself as a **gaussian excess above background** in the IMD.



natural width of A' \ll detector resolution
observed signal width = experimental mass resolution

Primary Backgrounds

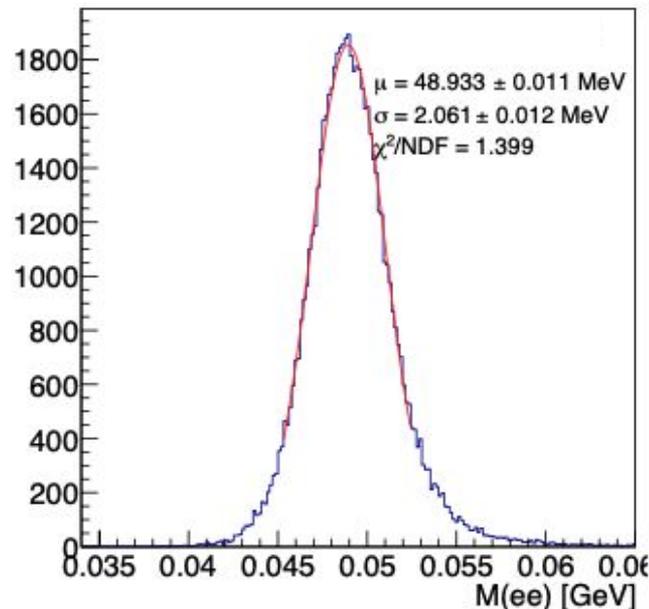
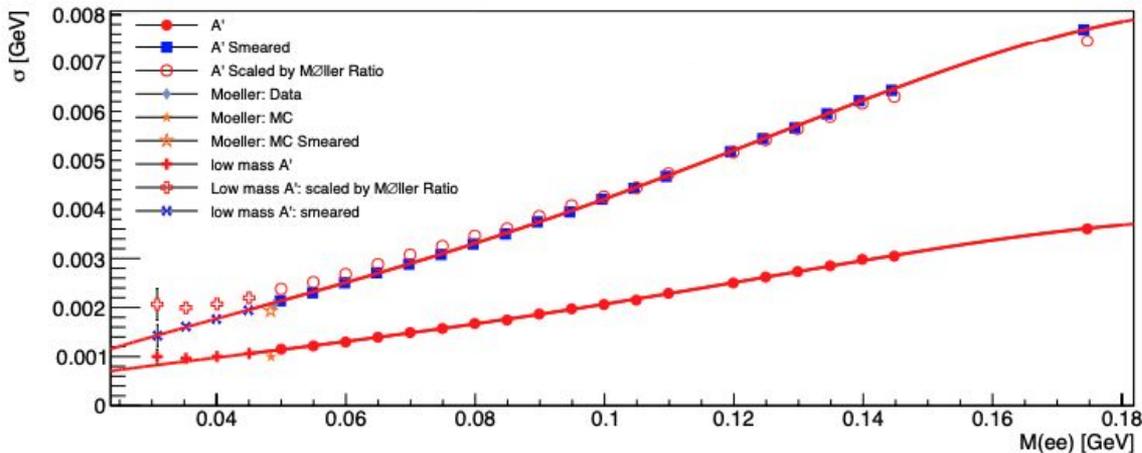


Prompt A' Mass Resolution in HPS

If A' exists within the acceptance of HPS, it will present itself as a **gaussian excess above background** in the IMD.

Prompt A' Mass Resolution determined by comparing Møller scattering in MC and data.

Møller Scattering 2016 Data



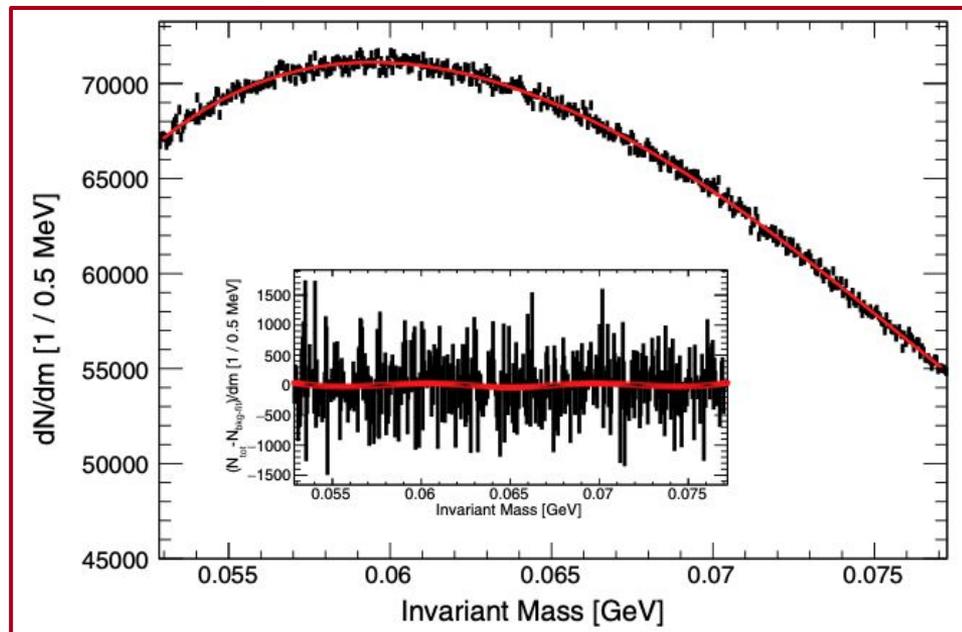
natural width of A' \ll detector resolution
observed signal width = experimental mass resolution

Primary Analysis Technique of 2023 PRD Publication

Published analysis used a sliding background model

- **centered around each mass hypothesis**
- fit window width determined by mass resolution
- shape use 3rd or 5th order Legendre polynomials

Background Model at 65 MeV



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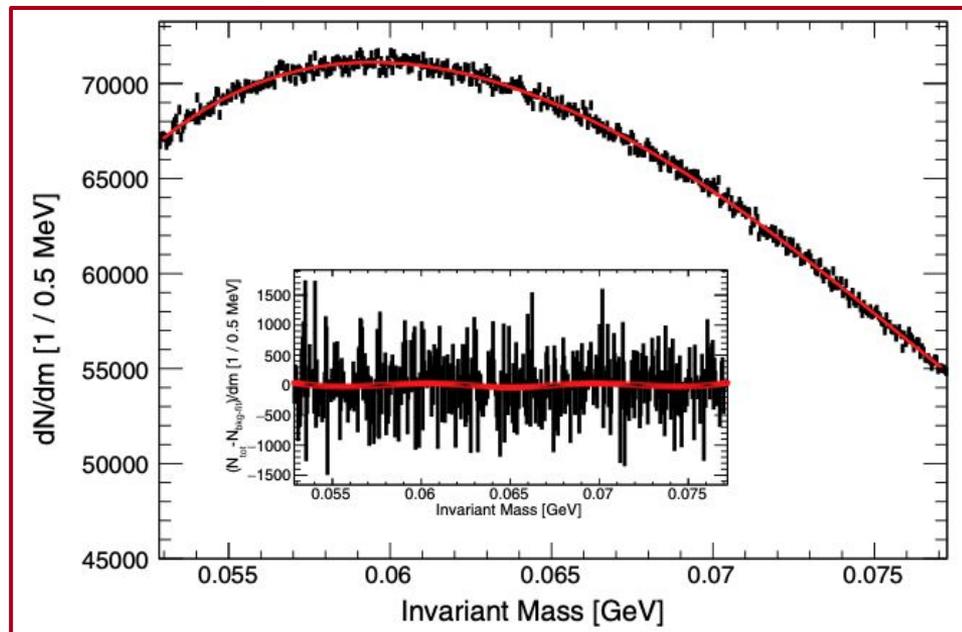
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$$10^L N(m_{e^+e^-} | \vec{t})$$

Background Model PDF Contribution

Flexibility of background model chosen to minimize signal yield bias comes at **cost to signal sensitivity**.

Background Model at 65 MeV



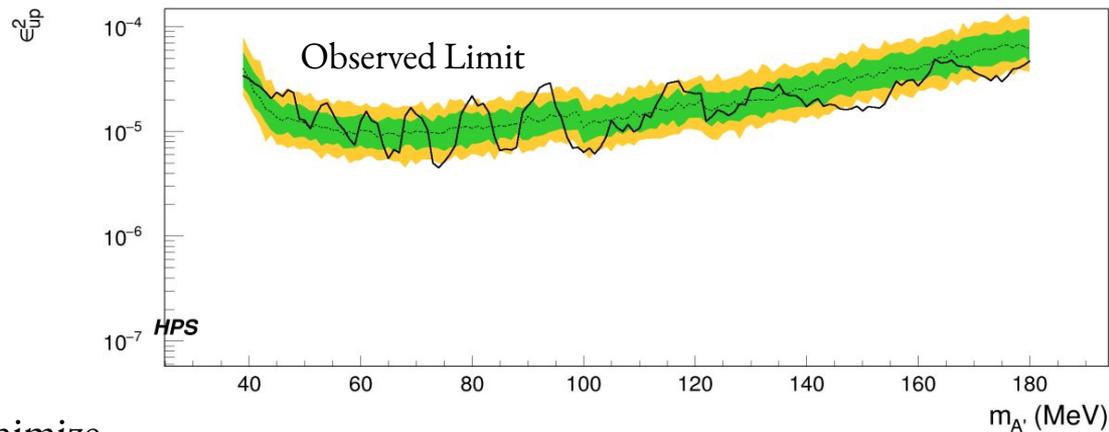
Changing the Background Model

$$10^{L_N(m_{e^+e^-} | \vec{t})}$$

Background Model PDF Contribution

Flexibility of background model chosen to minimize signal yield bias comes at **cost to signal sensitivity**.

ϵ^2 Upper Limit Published Result



Motivation

Based on the statistical uncertainty only limit, there is roughly an order-of-magnitude improvement in sensitivity possible for the background model.

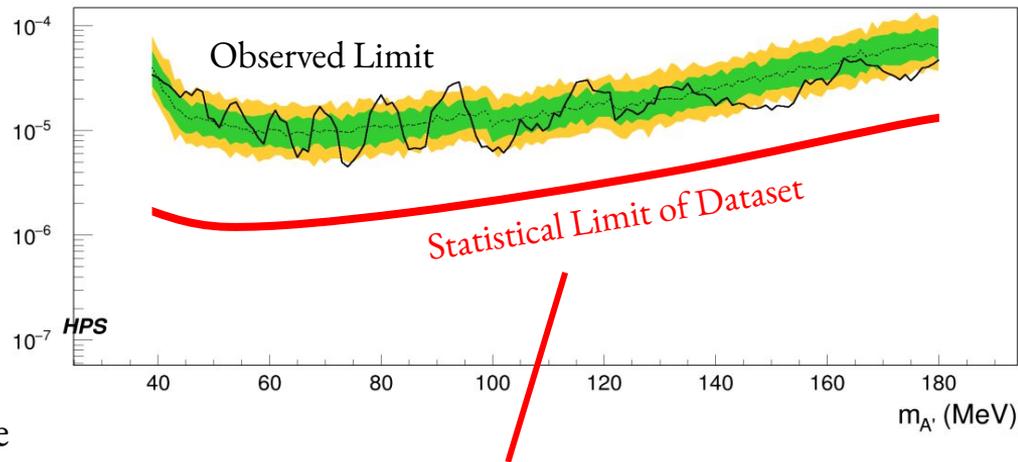
$$10^{L_N(m_{e^+e^-} | \vec{t})}$$

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Flexibility of background model chosen to minimize signal yield bias comes at **cost to signal sensitivity**.

ϵ^2

ϵ^2 Upper Limit Published Result



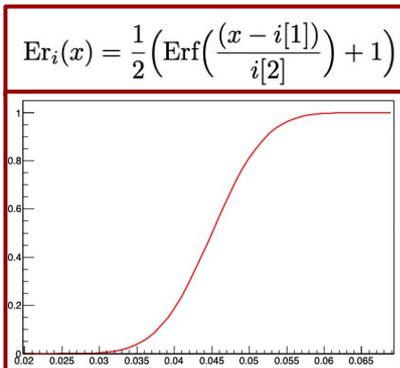
Optimistic \sqrt{N} limit on potential signal sensitivity in a sliding two-sigma mass window.

Looking for a Global Background Model

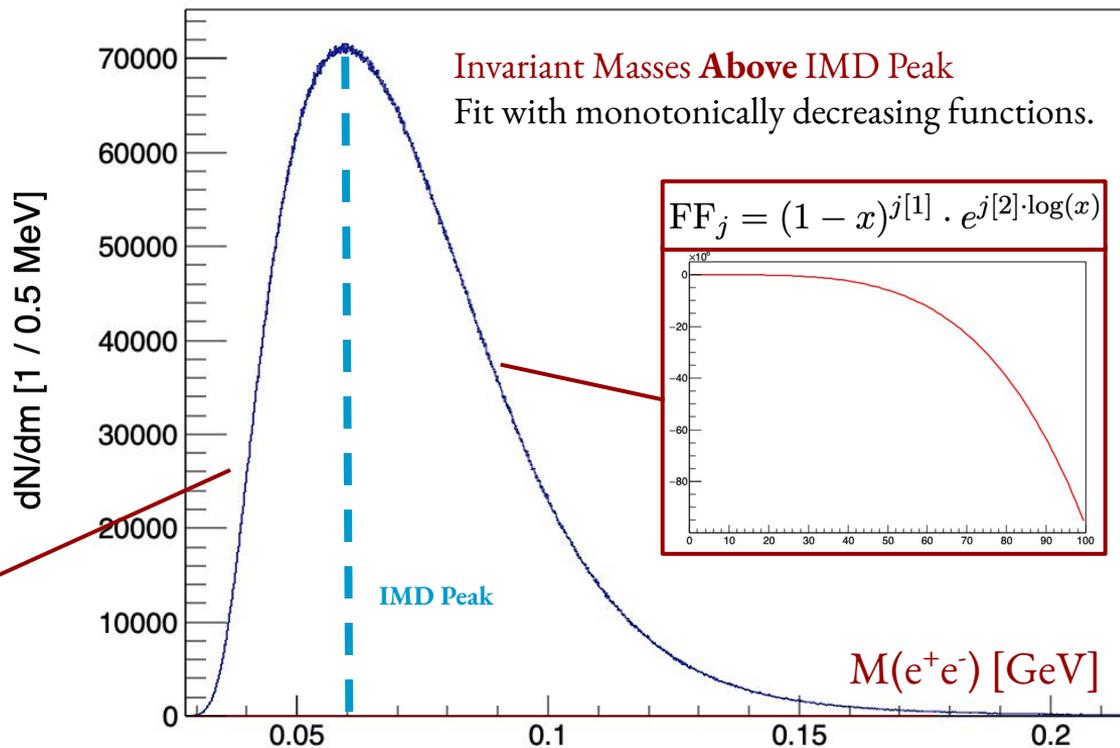
The general strategy for finding functions to fit the IMD is by modeling the broader scale features of the distribution.

The shape of the IMD is complicated by the complex geometric acceptance of the SVT and high statistics of background.

Invariant Masses Below IMD Peak
Fit with monotonically increasing functions.



2016 Invariant Mass Distribution



Looking for a Global Background Model

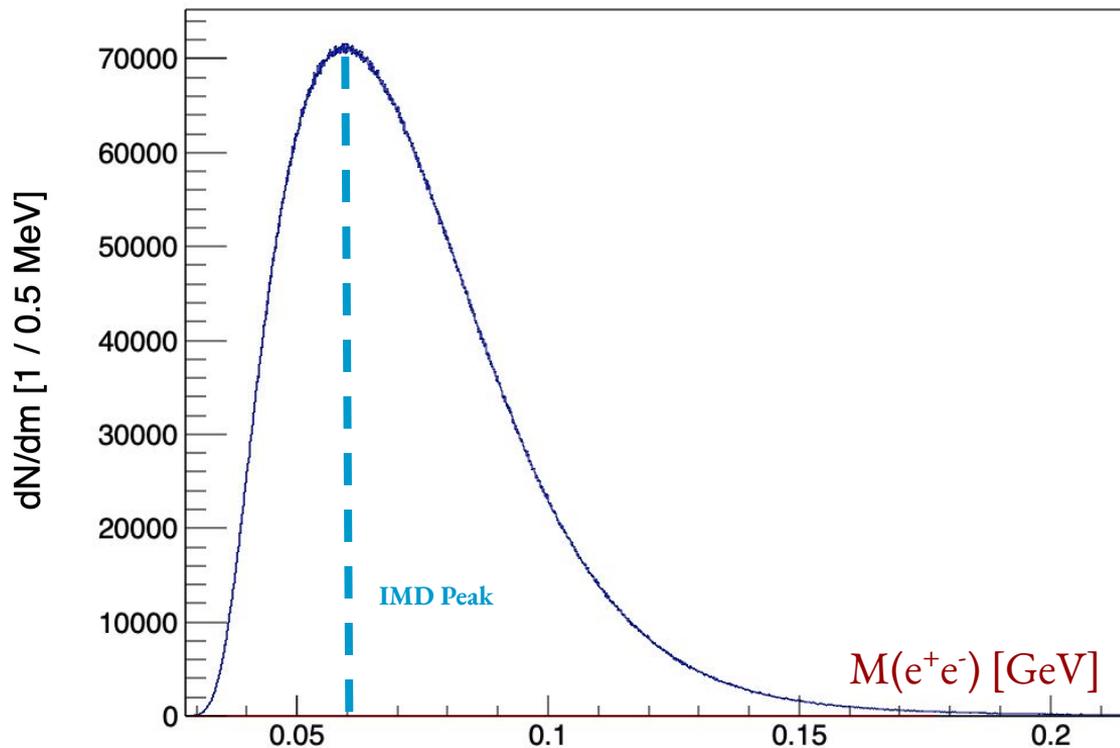
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Candidate Background Model Functional Form

$$\mathcal{F}_{\text{bkg}} = \sum_i \left(E r_i \cdot F F_i \right)$$

2016 Invariant Mass Distribution



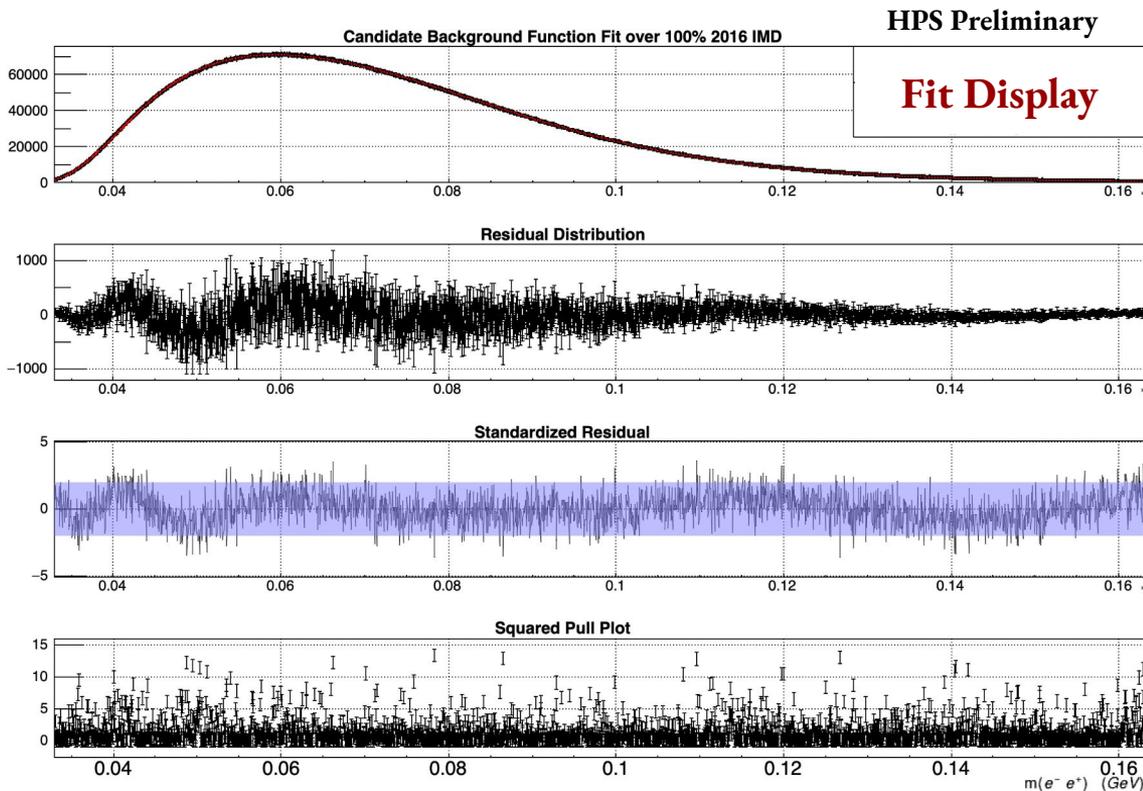
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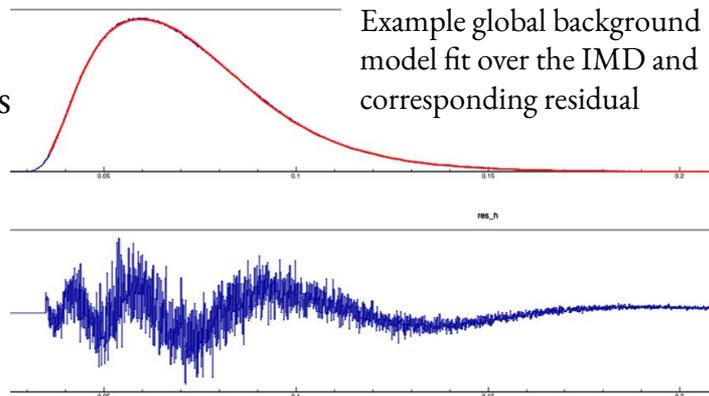
Candidate Background Model Functional Form

$$\mathcal{F}_{\text{bkg}} = \sum_i \left(\text{Er}_i \cdot \text{FF}_i \right)$$

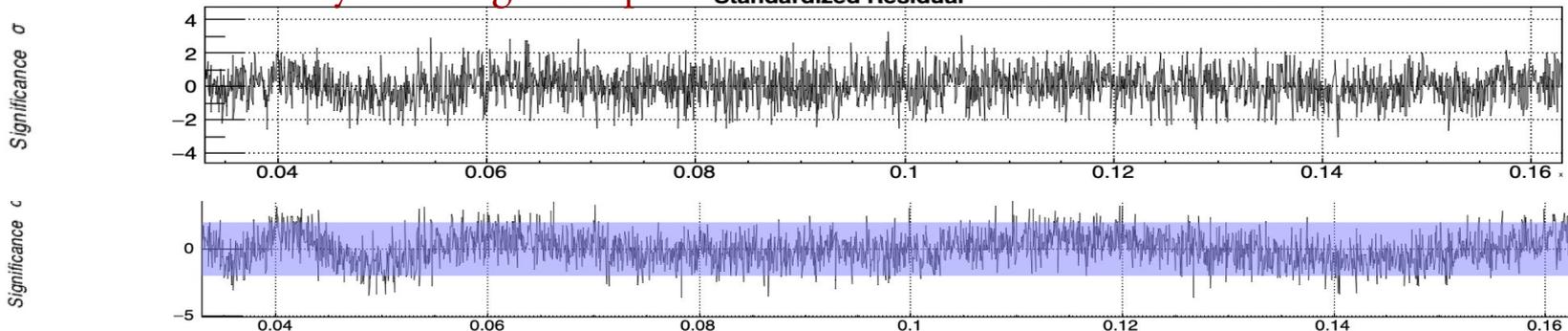


Complication of Global Fitting: The Wiggles

Challenges to fitting 100% are the existence of global scale fluctuations



early blinding attempt



10%

100%

Development of Global Background Model Analysis

131 functions of varying complexity tested on 2016 dataset.

- Lowest chi2 fits chosen to run through analysis chain.
- Candidate functions tested with different parameters floating and fixed to develop blinding procedure

$$\mathcal{F}_{\text{bkg}} = \sum_i \left(\text{Er}_i \cdot \text{FF}_i \right)$$

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$$\mathcal{F}_{\text{bkg}} = \sum_i \left(\text{Er}_i \cdot \text{FF}_i \right)$$

Asymptotic Likelihood Test and Iterative CLs Scan Performed

$$\text{CL}_s(\mu) = \frac{p_\mu}{1 - p_b} \quad \text{CL}_s(N_{\text{sig}}^{\text{up}}) = 0.05.$$

The ϵ^2 upper limit is found with the signal yield upper limit, radiative fraction of events, and estimated background of events.

$$\epsilon^2 = \frac{2\alpha N_{\text{sig}}^{\text{up}}}{3\pi m_{A'} f_{\text{rad}} \frac{dN_{\text{bkg}}}{dm}}$$

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Floating Shape

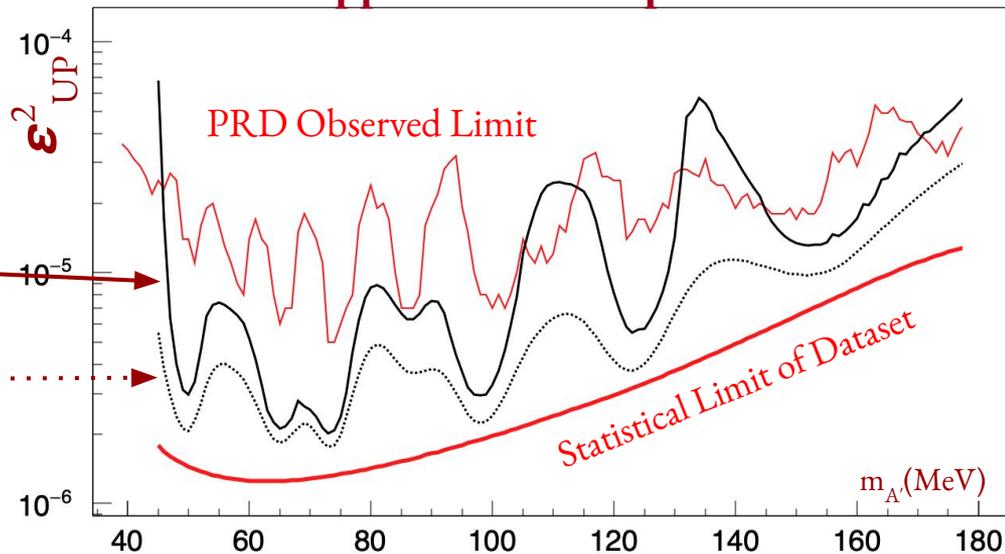
Fixed Shape

Asymptotic Likelihood Test and Iterative CLs Scan Performed

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ϵ^2 Upper Limit Comparison*



*unblinded

$$\epsilon^2 = \frac{2\alpha N_{\text{sig}}^{\text{up}}}{3\pi m_{A'} f_{\text{rad}} \frac{dN_{\text{bkg}}}{dm}}$$

Blinded Analysis Flow

Functional Form Tests

Filter by χ^2 probability threshold.

10% Background Fit

Use 10% parameterization for initial seeds and covariance matrix for multivariate constraint.

100% Background Fit

Use 100% parameterization to generate MC Toy distributions.

100% BKG + SIG_Float

Signal normalization floats to best fit value.

100% BKG + SIG_Fix

Scan over range of fixed signal normalization values.

Using Data CLs
Observed Upper Limits

Using Toys CLs
Uncertainty Bands

Observed upper limits and uncertainties combine to form final result.

Importance of a Control Region

If extracting signal significance from fits, expect too significant result < 60 MeV

- Moving to “Control Region” – $[60, 180]$ MeV

Breakdown of in-progress work on Control Region

- 10% Function Selection — CHECK
- 100% BKG Only Fits (data) — CHECK
- 100% BKG + Sig Floating — CHECK
- Generate Upper limit
 - CLs SCAN on Data Complete
 - Observed Upper Limit on Signal Yield Computed, coupling to be complete soon
- Generate Toys — in progress
 - Generate Bands from toy CLs Scan

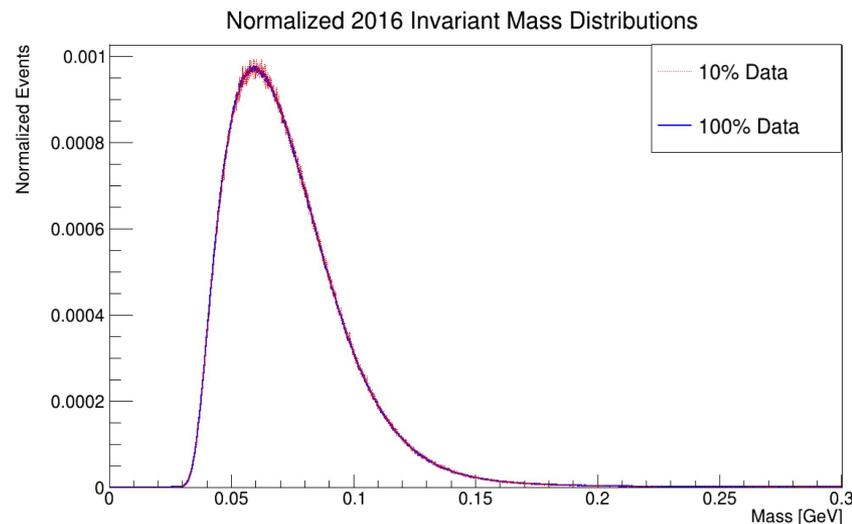
Blinding Procedure: Random 10% of 2016 Dataset

10% Background Only Hypothesis

- >100 functions created and tested
- Store parameters and covariance matrices for each function.
- Top χ^2 probabilities selects candidate functions.

Candidate Background Model Functional Form

$$\mathcal{F}_{\text{bkg}} = \sum_i \left(\text{Er}_i \cdot \text{FF}_i \right)$$



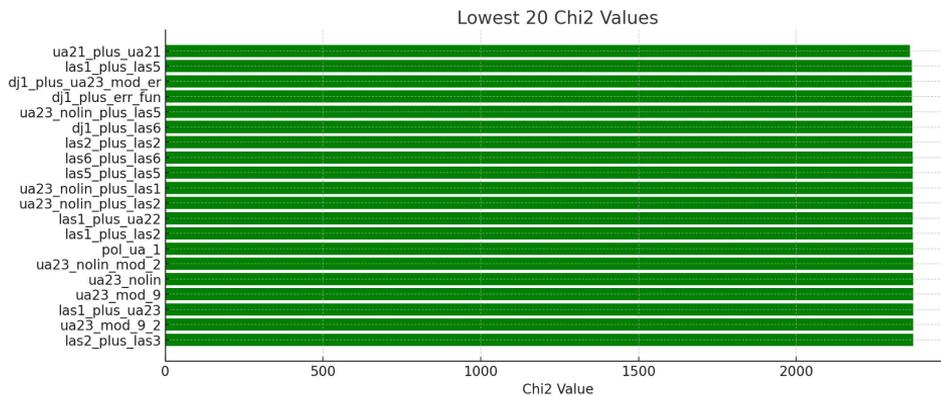
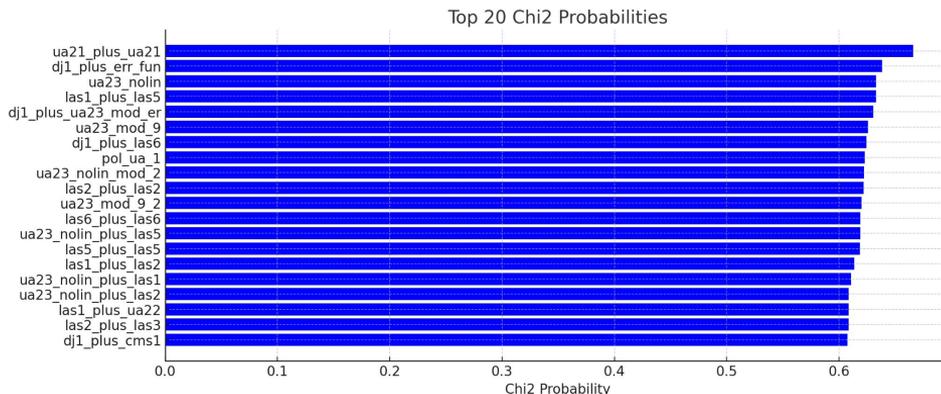
10% Function Selection in Control Region

Fit on range: [60, 180] MeV

Notable functions differ from previous results of best fit.

Note: 20 iteration limit on dynamic parameter seed selection.

Can improve and likely will once full analysis framework completed.



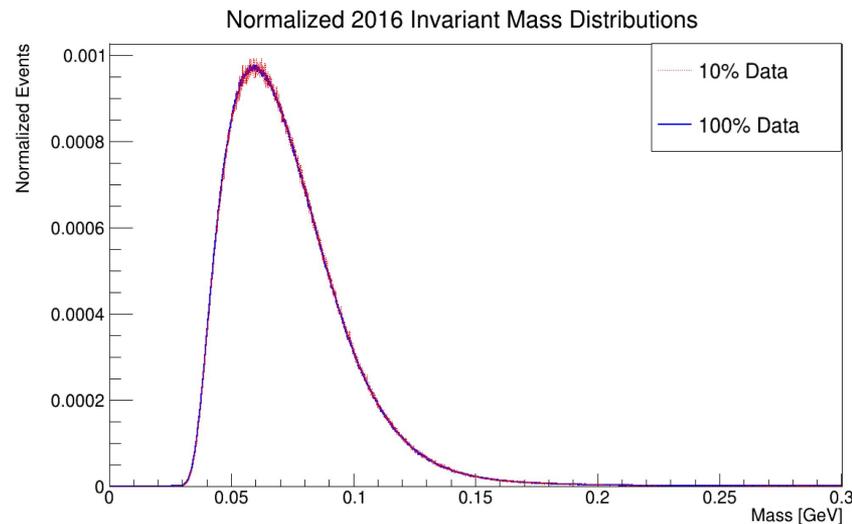
Blinding Procedure: 100% of 2016 Dataset

100% Background Only Hypothesis

- Use 10% Fit Parameters as initial seeds
- Constrained by multivariate gaussian using 10% covariance matrix.

Candidate Background Model Functional Form

$$\mathcal{F}_{\text{bkg}} = \sum_i \left(\text{Er}_i \cdot \text{FF}_i \right)$$



Blinding Procedure: 100% of 2016 Dataset

100% Background Only Hypothesis

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Candidate Background Model Functional Form

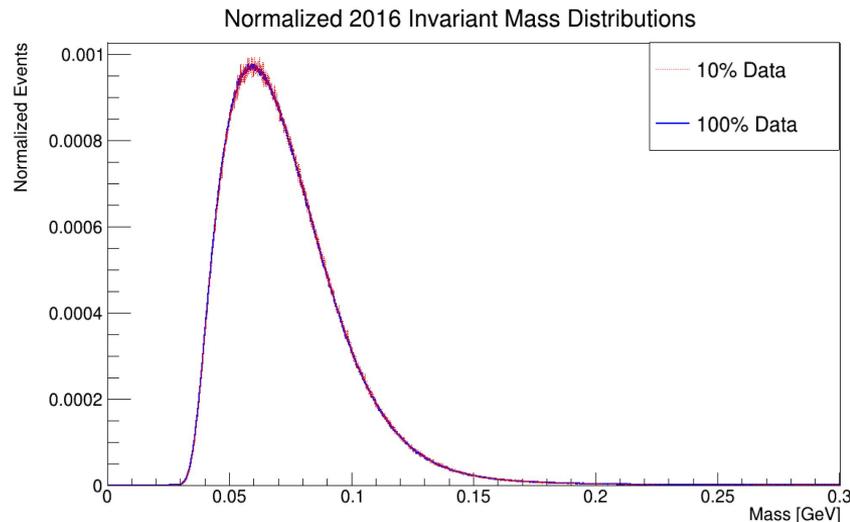
$$\mathcal{F}_{\text{bkg}} = \sum_i \left(\text{Er}_i \cdot \text{FF}_i \right)$$

100% Background + Signal Model (Signal Yield Floating)

- Float Background shape within constraint, Normalization floats
- Finds most probable signal value, error on Signal Yield stored

100% Background + Signal Model (Signal Yield Fixed)

- Iterative fixed signal strength CLs scan to find 95% upper limit on signal yield and corresponding coupling.

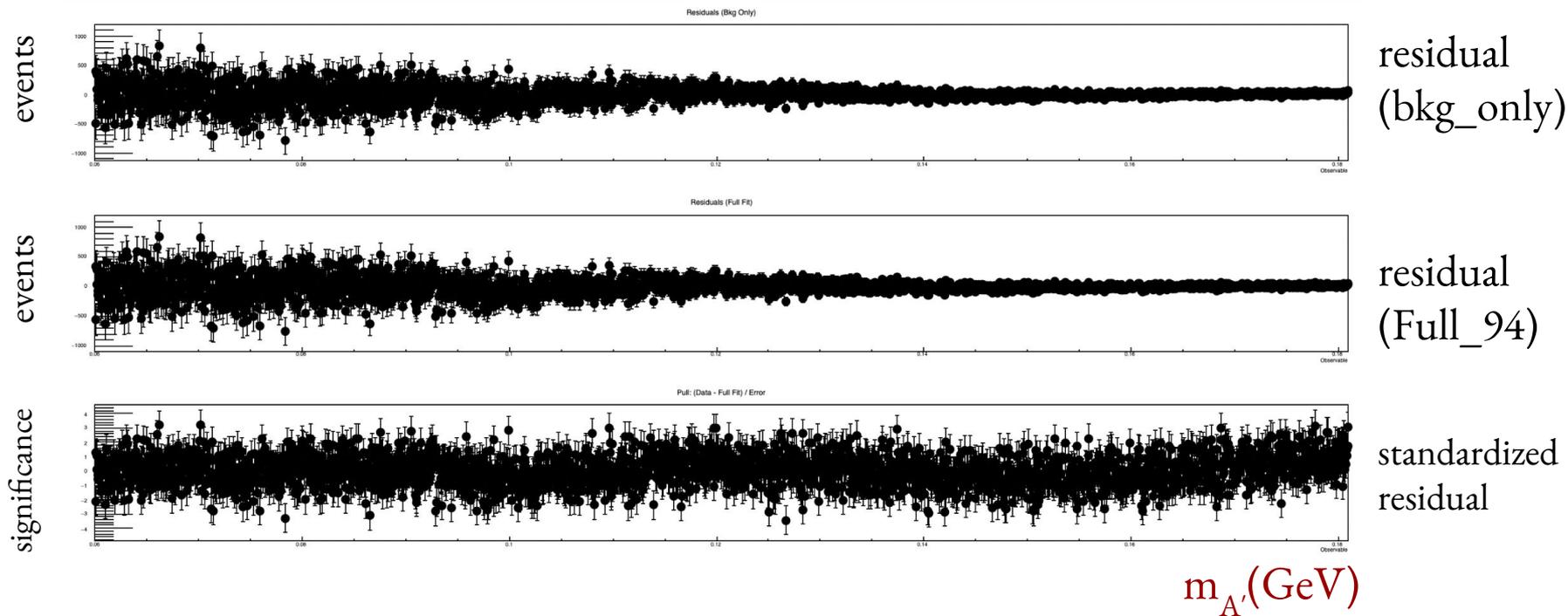


BKG+SIG Model Functional Form

$$\mathcal{F}_{\text{Full}} = C_{\text{bkg}} \mathcal{F}_{\text{bkg}} + C_{\text{sig}} \mathcal{F}_{\text{sig}}$$

$$C_{\text{bkg}} = \frac{N_{\text{bkg}}}{N_{\text{bkg}} + N_{\text{sig}}} \quad C_{\text{sig}} = \frac{N_{\text{sig}}}{N_{\text{bkg}} + N_{\text{sig}}}$$

New Display Tools



Current CLs Algorithm to Find Upper Limit

Old Algorithm

```
// if((CLs <= 0.051 && CLs > 0.049))
// {
//     std::cout << "[ BumpHunter ]: Upper limit: " << mu95up << std::endl;
//     std::cout << "[ BumpHunter ]: CLs: " << CLs << std::endl;
//
//     result->setUpperLimit(mu95up);
//     result->setUpperLimitPValue(CLs);
//
//     break;
// }
// else if(CLs <= 1e-10) { mu95up = mu95up*0.1; }
// else if(CLs <= 1e-8) { mu95up = mu95up*0.5; }
// else if(CLs <= 1e-4) { mu95up = mu95up*0.8; }
// else if(CLs <= 0.01) { mu95up = mu95up*0.9; }
// else if(CLs <= 0.04) { mu95up = mu95up*0.99; }
// else if(CLs <= 0.049) { mu95up = mu95up*0.999; }
// else if(CLs <= 0.1) { mu95up = mu95up*1.01; }
// else { mu95up = mu95up*1.1; }
```

Issues:

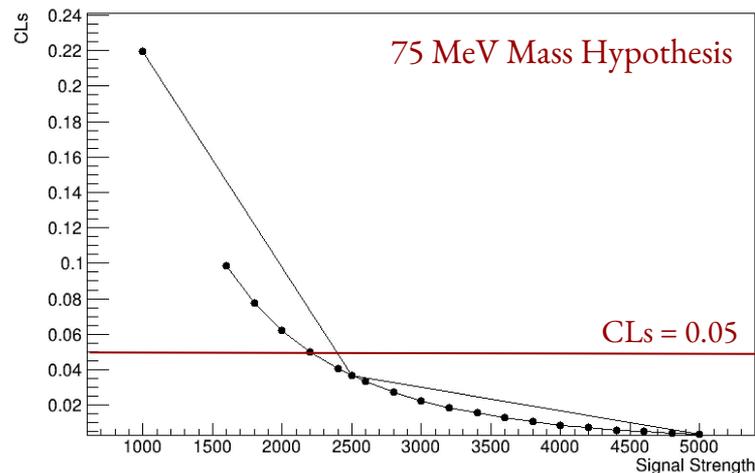
- runaway signal yield
- timing out
- not exactly 0.05

New Algorithm and Approach

Fixed Signal Strength Interpolation Scan

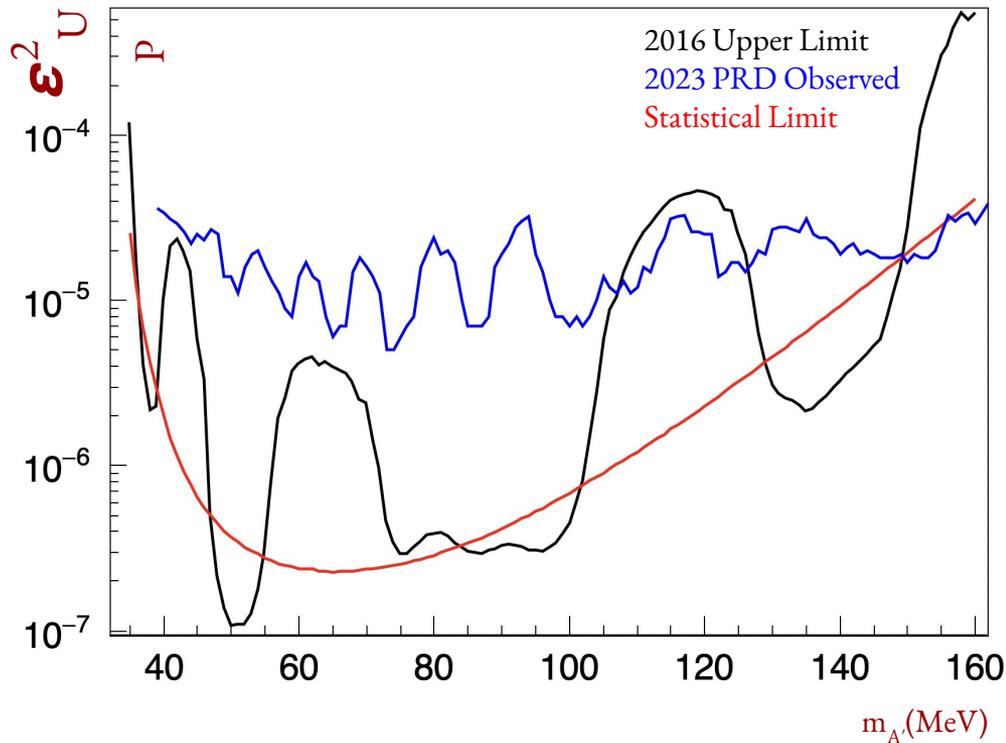
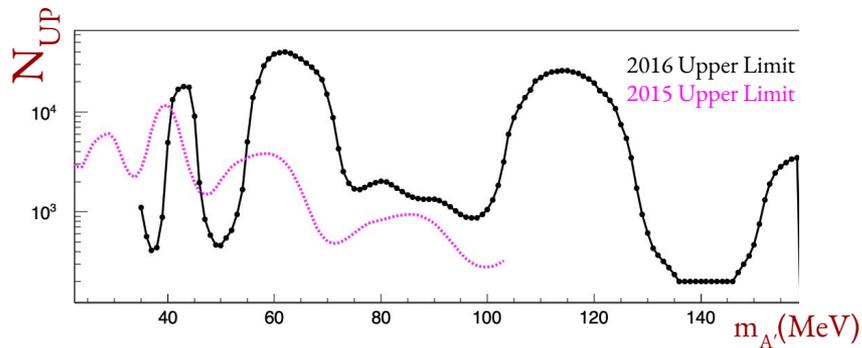
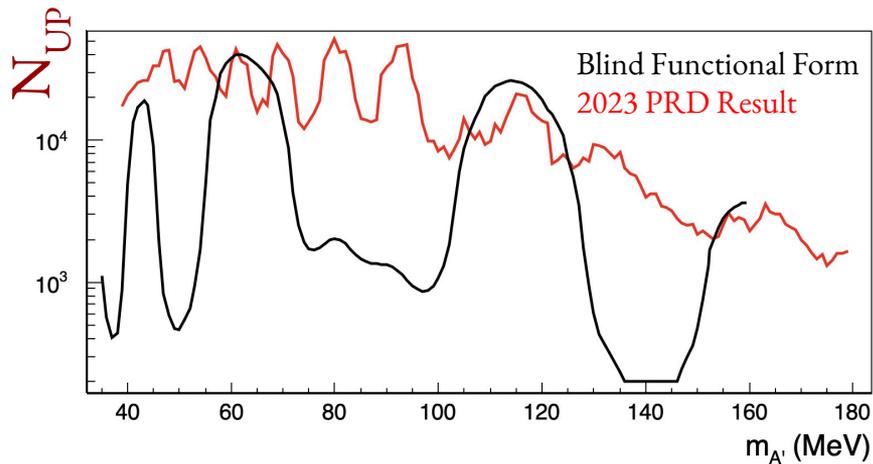
- Course Scan ~1500 Event Intervals until ~65000 Events
- If CLs drops below 0.03, start fine scan in reverse
 - stop fine scan at CLs of 0.1
- Stores CLs / Signal Strength pair for **Cubic Spline Interpolation**

CLs vs Signal Strength



Observed Upper Limits with Fixed Background Model

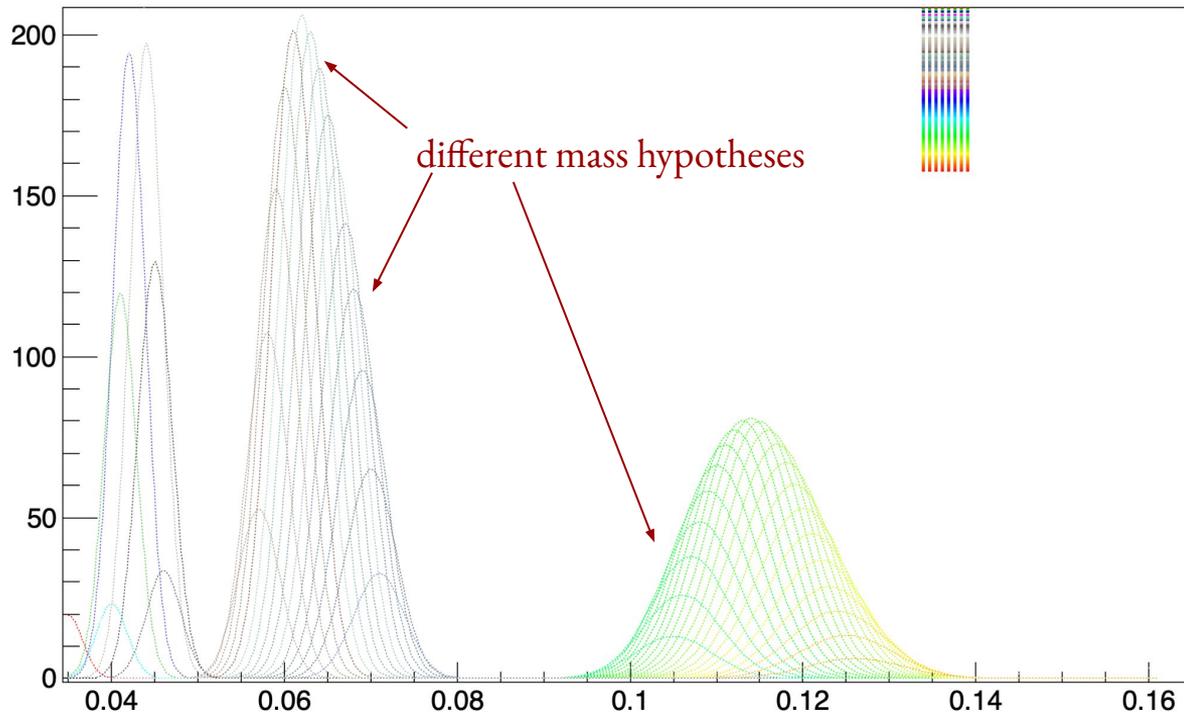
Observed Upper Limits with fixed background model



Scanning Entire Range for Signal Yield (non-control region)

Signal Yield Scan Compilation (bkg model fixed)

signal yield events
(N per 50 KeV)

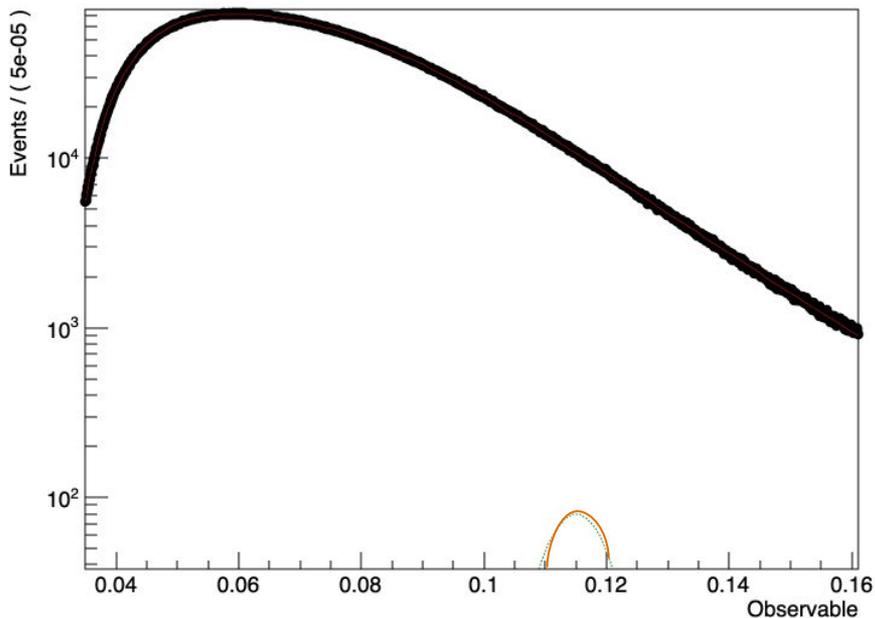


$m_{A'}(\text{GeV})$

Individual Projections of Signal Yield: Non Zero Projected Yield

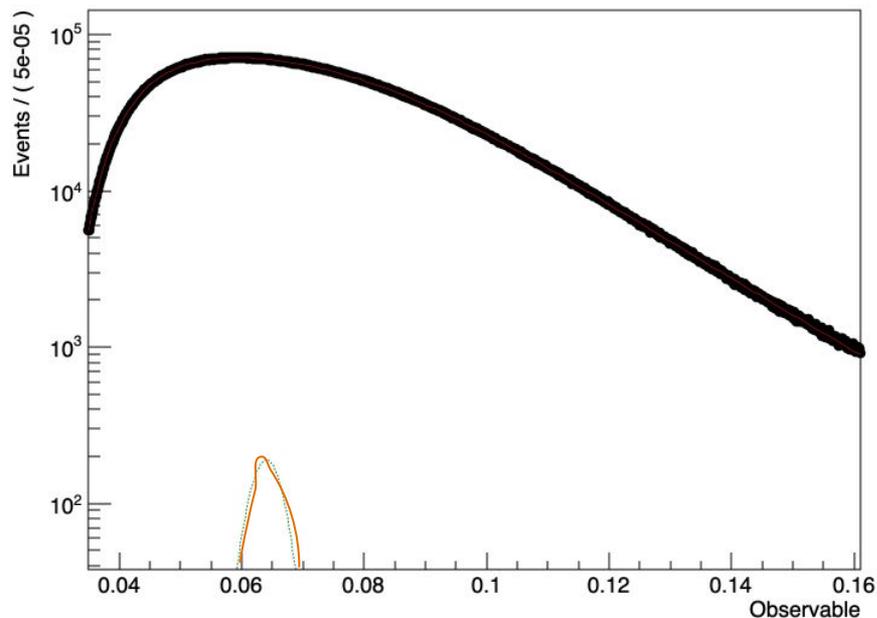
115 MeV Signal Hypothesis

Signal Component (Full - Bkg)

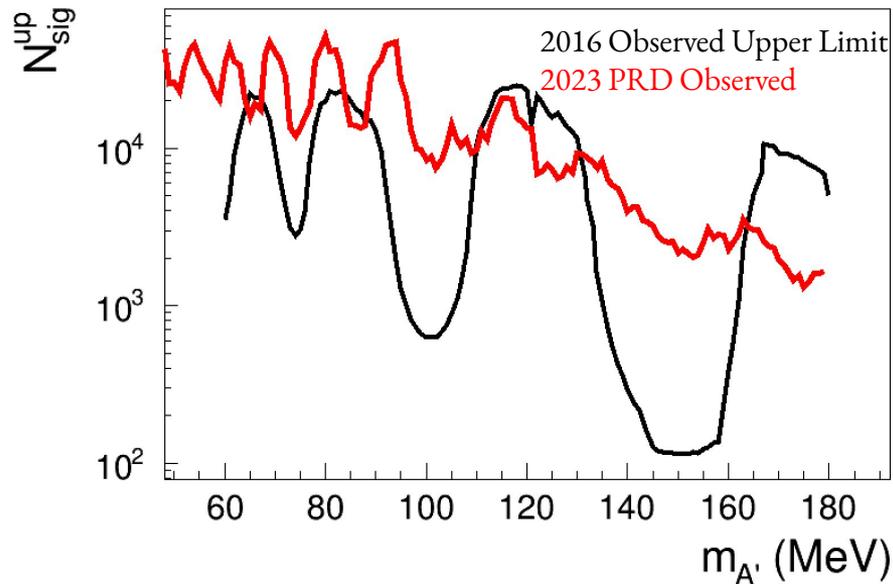
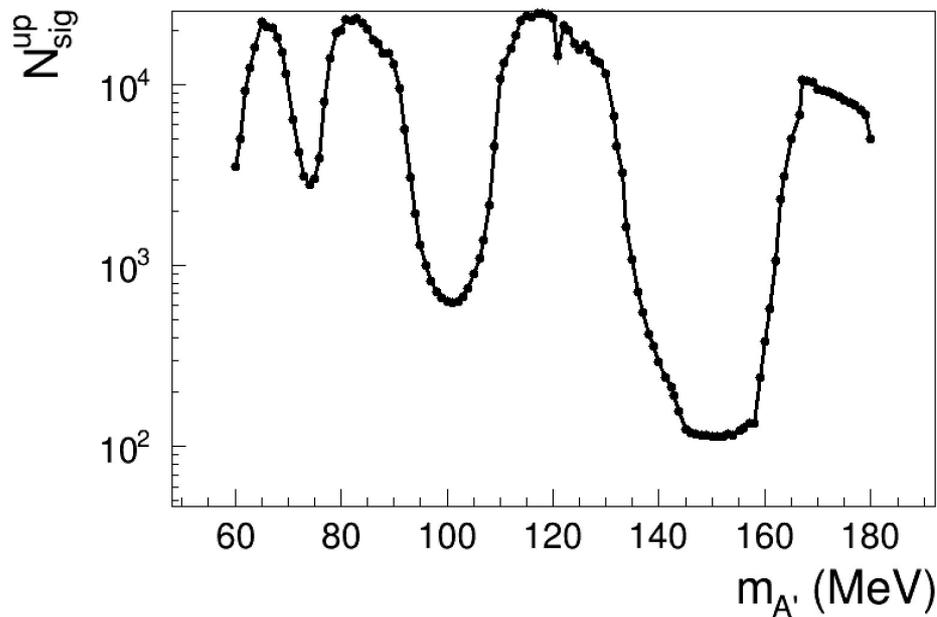


64 MeV Signal Hypothesis

Signal Component (Full - Bkg)



Observed Upper Limit on Signal Yield – Multivariate Constraint



Note: bug in upper limit on ϵ^2 code atm, will fix when working through limit bands on ϵ^2

Scan for Signal Yield in Control Region [60, 180 MeV]

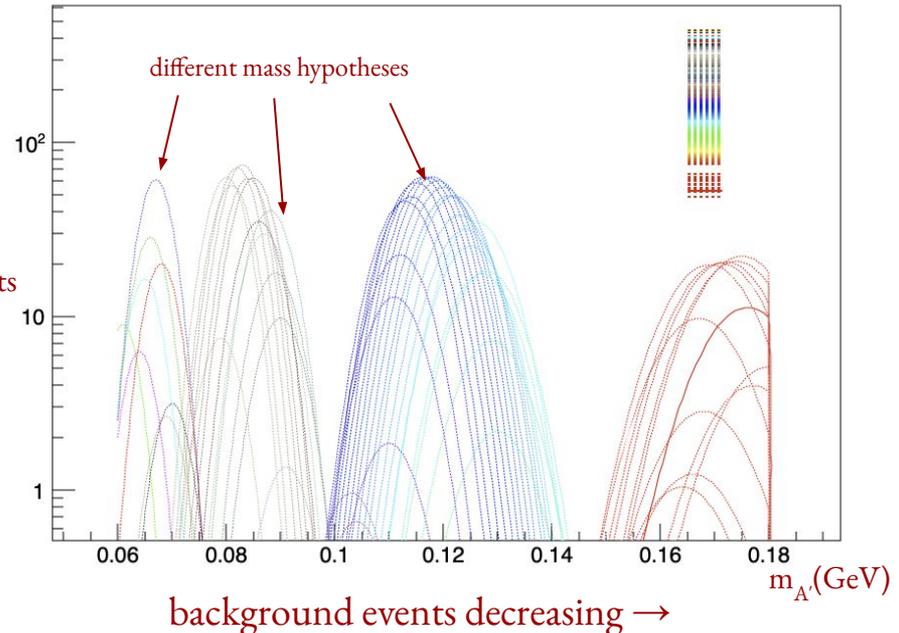
Differences from previous model fully fixed shape):

- “bumps” are of lower significance
- “bump” at 120 MeV has half observed yield of floating similar to fixed
- new “bump” at 80 MeV

Note: bugs found re: roofit memory issues and full range (< 60) hasn't been fit

Signal Yield Scan Compilation (blk model float w/constraints)

signal yield events
(N per 50 KeV)

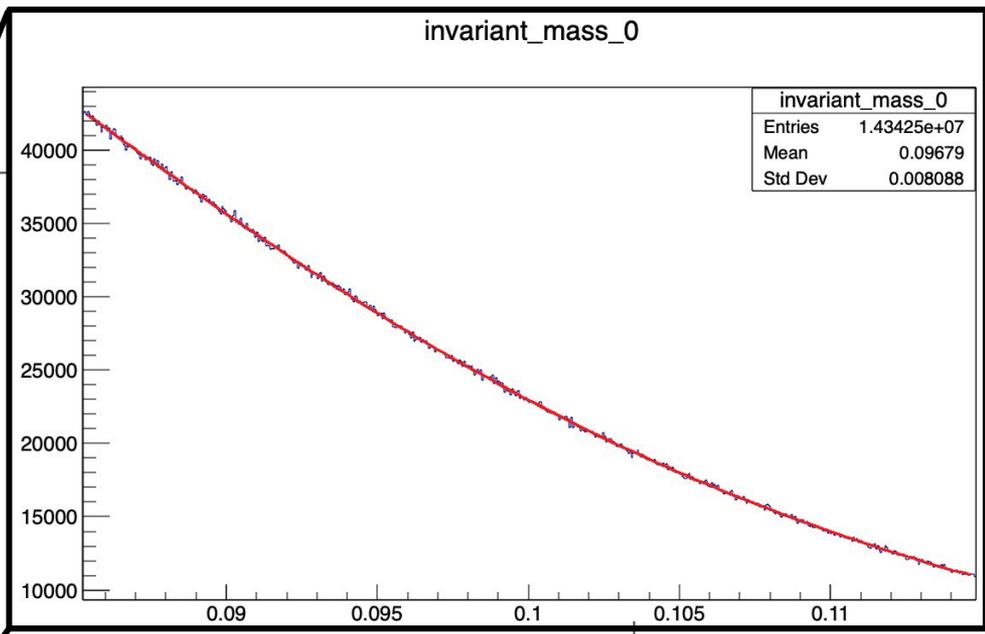
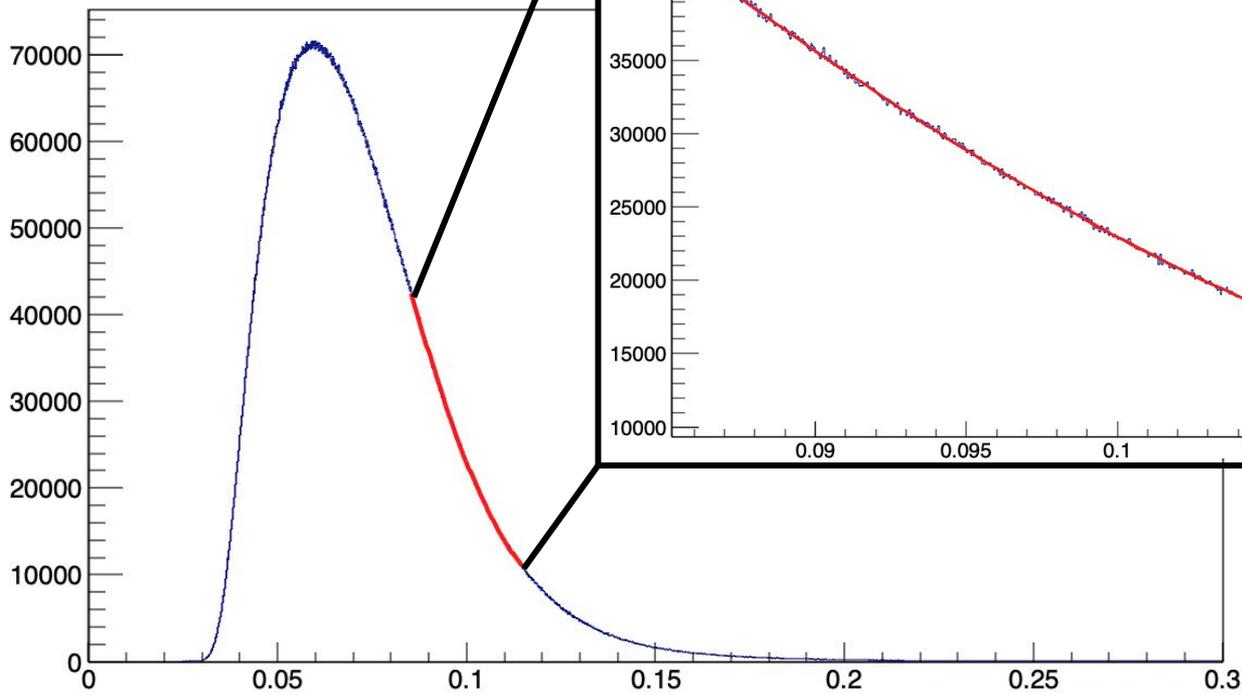




Visual Representation of Toy Experiment

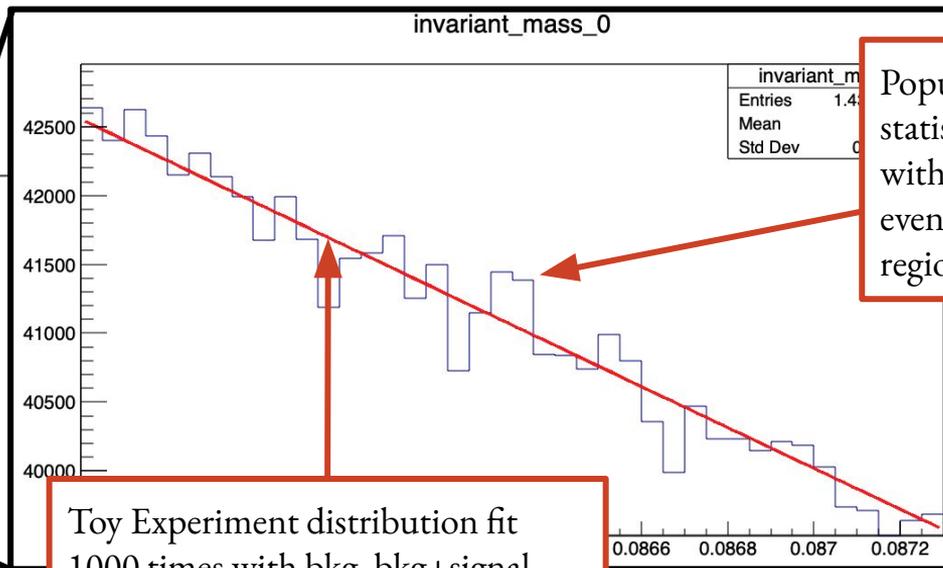
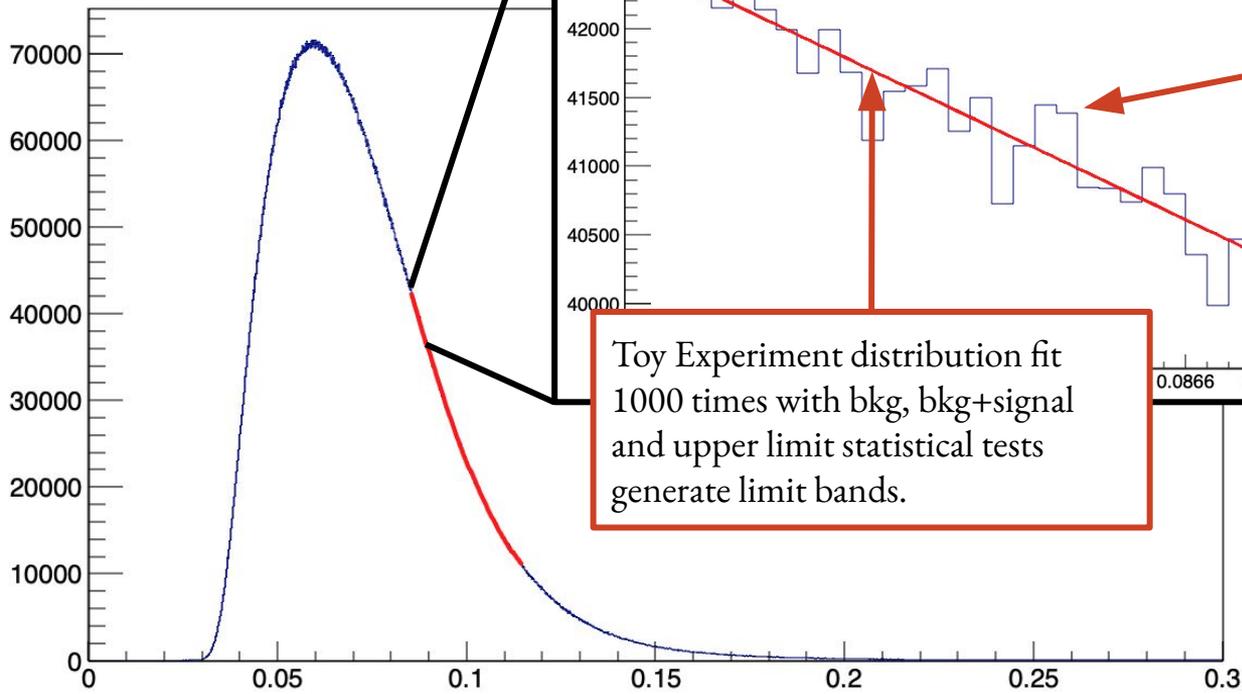
bhResSys_m100w7p3.root

- h_Minv_General_Final_1;1
- fit_toys;1
- invariant_mass_0;1
- invariant_mass_1;1



Visual Representation of Toy Experiment

File: bhResSys_m100w7p3.root
h_Minv_General_Final_1;1
fit_toys;1
invariant_mass_0;1
invariant_mass_1;1



Toy Experiment distribution fit 1000 times with bkg, bkg+signal and upper limit statistical tests generate limit bands.

Populated toy experiment statistics distributed randomly with total events equaling total events from data in the same region

Generated toys last week!

- non-trivial with RooFit
- hpstr/BumpHunter integration
- used wrong parameterization...
- should scale up extremely soon (next week)

```
Generating 2 Toys
  Signal Injection      :: 0
  Signal Shape         :: Gaussian
  Background Multiplier :: 1
[Init] bins: 2421, window: [0.06, 0.181], mass hypo: 0.095
[INFO] Initializing RooFit PDF normalization...
Generating Toy 0
[Debug] bins_ = 2421, window_start_ = 0.06, window_end_ = 0.181
[#1] INFO:NumericIntegration --
RooRealIntegral::init(las3pluslas6_toy_model_Int[x]) using
numeric integrator RooIntegrator1D to calculate Int(x)
[#1] INFO:NumericIntegration --
RooRealIntegral::init(las3pluslas6_toy_model_Int[x]) using
numeric integrator RooIntegrator1D to calculate Int(x)
[Debug] bins_ = 2421, window_start_ = 0.06, window_end_ = 0.181
[#1] INFO:NumericIntegration --
RooRealIntegral::init(las3pluslas6_toy_model_Int[x]) using
numeric integrator RooIntegrator1D to calculate Int(x)
[#1] INFO:NumericIntegration --
RooRealIntegral::init(las3pluslas6_toy_model_Int[x]) using
numeric integrator RooIntegrator1D to calculate Int(x)
Fitting Toy 0
```

Conclusions and Moving Forward

Successfully developed competitive blinded analysis technique for setting upper limits.

Once limit bands are generated

- Full Proof of Concept Analysis Chain Complete
- Shift to top performing control region function
- Move on

Getting caught up to speed on preselection studies / 2021 IMD production for bump hunting scaffolding.

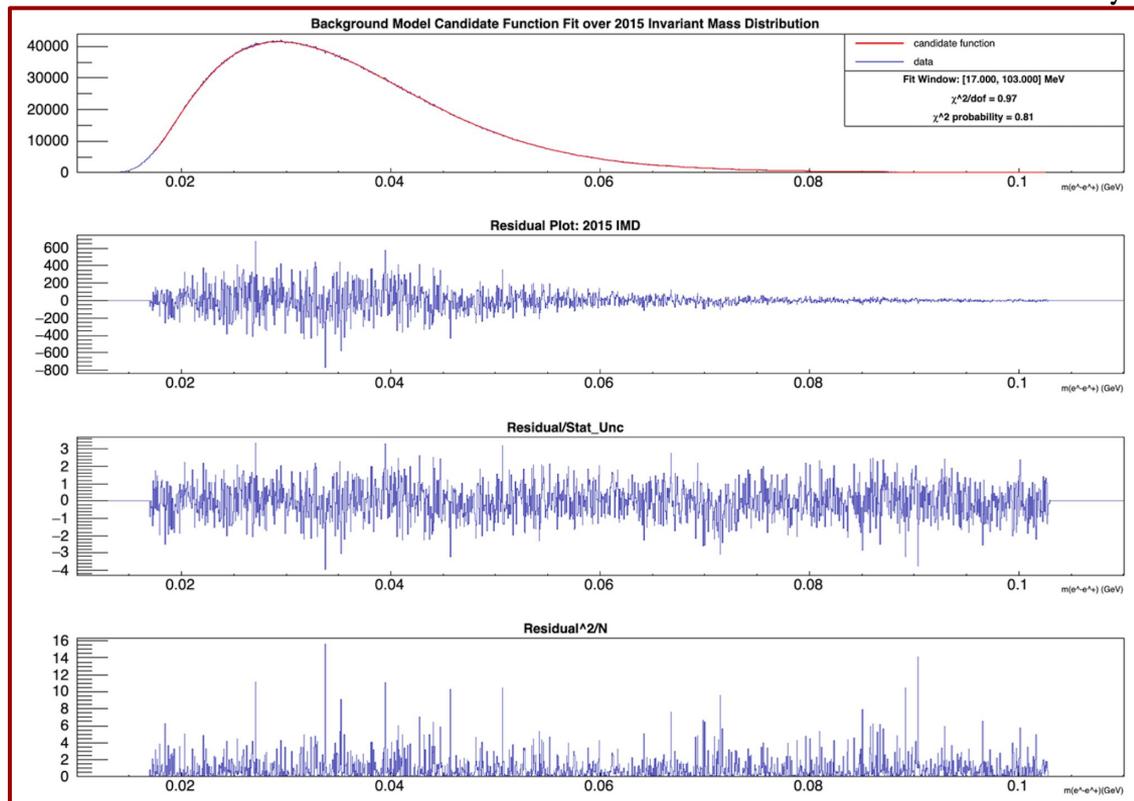
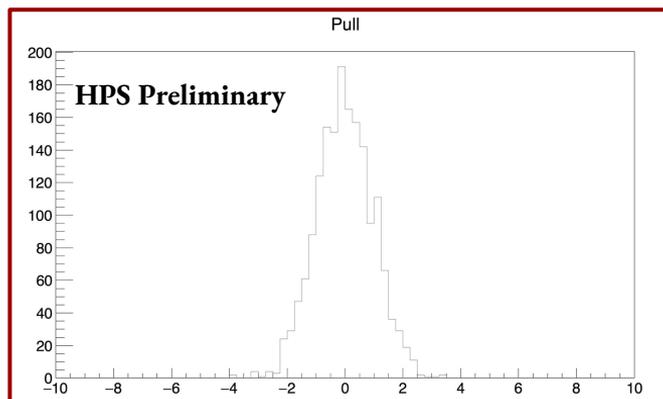
TJ and Aidan will work with me this summer. Go team bump hunters!

- I. Functional Form Global Fit of 2015 Dataset
 - A. Corresponding 2015 Upper Limits
- II. Additional Use Case: APEX
- III. Detailed Upper Limit Calculation
- IV. Gaussian Process Bonus Slides
- V. Look Elsewhere Effect
- VI. Freeze out Thermal Relics

Functional Form Global Background Fit of 2015

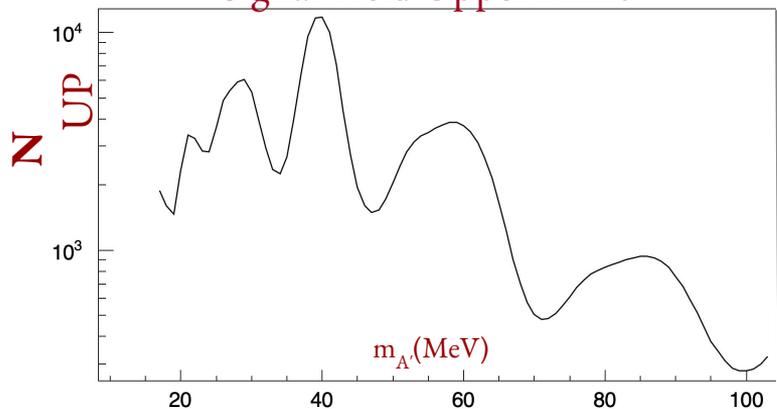
From TJ Britt's Summer Project

- parameters stored for use in bkg+signal model
- **chi2 probability = 0.81**
- **chi2/dof = 0.97**

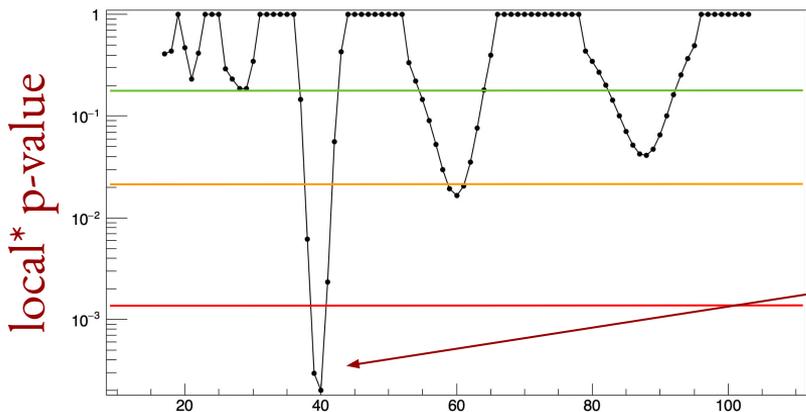
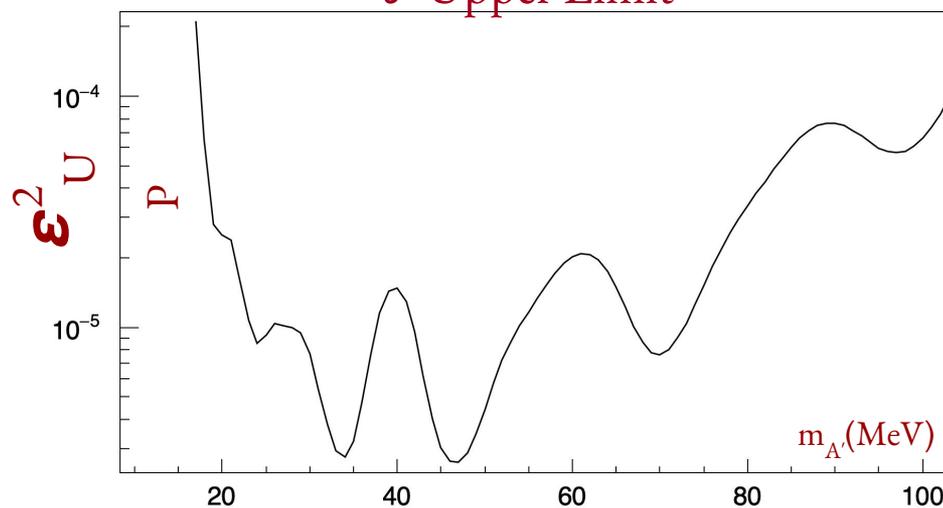


Corresponding 2015 Upper limits and pvalues

Signal Yield Upper Limit



ϵ^2 Upper Limit



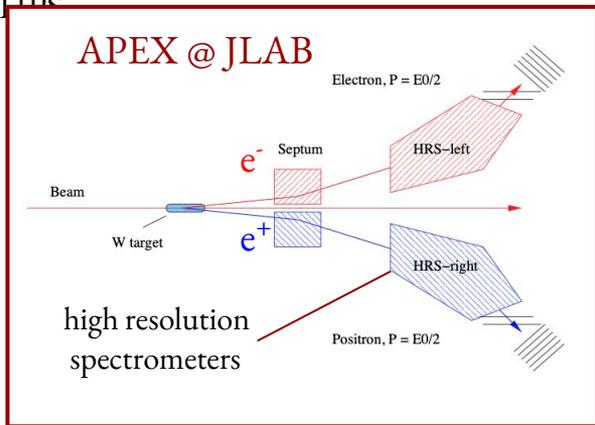
local 3.54 σ

*Global pvalues to be calculated when bumphunter rewritten.

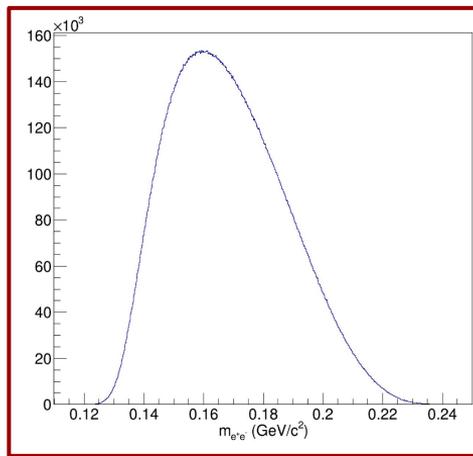
Additional Use Case: APEX

APEX, a JLAB fixed target experiment, has nearly identical resonance search methodology to

INDC

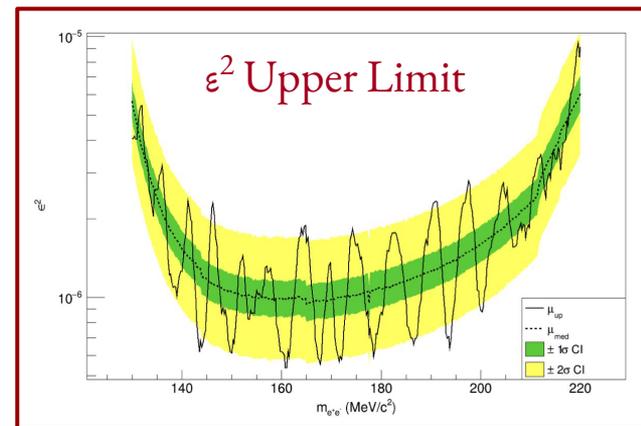


APEX Blinded 10% e^+e^- IMD



Only 10% of 2019 dataset has been analyzed.

Systematic similarities to HPS results.



[APEX: A Search for Dark Photons in Hall A](#)

[Williamson, John Thomas Austin \(2022\) APEX \(A' Experiment\): The search for a dark photon at Jefferson Lab. PhD thesis.](#)

APEX is an opportunity to leverage HPS analysis techniques, improve physics sensitivity in **well motivated parameter space**, and publish a result.

Determining Upper Limits for each Mass Hypothesis

Method: Asymptotic Likelihood Test

Constructing a Test Statistic

Negative Signal Case Positive Signal Case

$$\lambda_1 \equiv \frac{\mathcal{L}(\mu_0)}{\mathcal{L}(0)}$$

$$\lambda_2 \equiv \frac{\mathcal{L}(\mu_0)}{\mathcal{L}(\hat{\mu})}$$

$$\tilde{q}_\mu = \begin{cases} -2 \ln \lambda(\mu) & \hat{\mu} \leq \mu \\ +2 \ln \lambda(\mu) & \hat{\mu} > \mu \end{cases}$$

Background Model Probability

$$p_b = \begin{cases} \Phi(-\sqrt{-\tilde{q}_\mu} - \mu/\sigma) & \tilde{q}_\mu < 0 \\ \Phi(\sqrt{\tilde{q}_\mu} - \mu/\sigma) & 0 \leq \tilde{q}_\mu \leq \mu^2/\sigma^2 \\ \Phi\left(\frac{\tilde{q}_\mu - \mu^2/\sigma^2}{2\mu/\sigma}\right) & \mu^2/\sigma^2 < \tilde{q}_\mu \end{cases}$$

Signal Model Probability

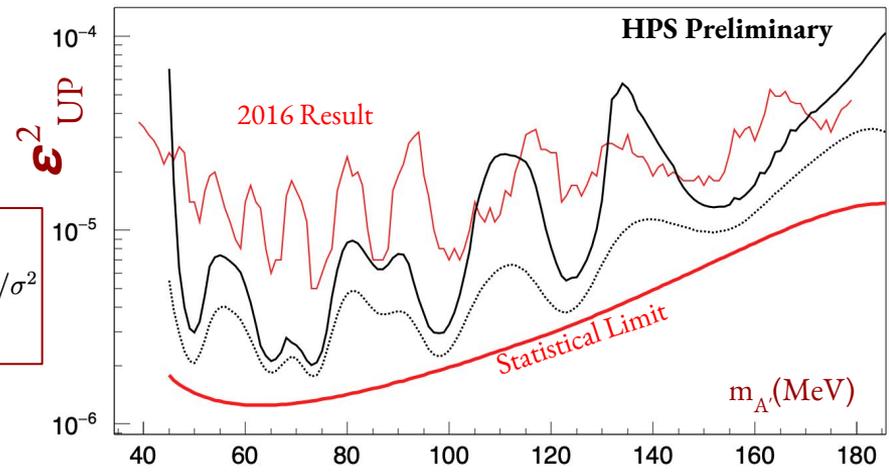
$$p_\mu = \begin{cases} 1 - \Phi(-\sqrt{-\tilde{q}_\mu}) & \tilde{q}_\mu < 0 \\ 1 - \Phi(\sqrt{\tilde{q}_\mu}) & 0 \leq \tilde{q}_\mu \leq \mu^2/\sigma^2 \\ 1 - \Phi\left(\frac{\tilde{q}_\mu + \mu^2/\sigma^2}{2\mu/\sigma}\right) & \mu^2/\sigma^2 < \tilde{q}_\mu \end{cases}$$

Iteratively done to find maximum fixed signal yield necessary to hit target confidence level threshold (CLs scan on next slide).

$$CL_s(\mu) = \frac{p_\mu}{1 - p_b} \quad CL_s(N_{sig}^{up}) = 0.05.$$

The ϵ^2 upper limit is found with the signal yield upper limit, radiative fraction of events, and estimated background of events.

ϵ^2 Upper Limit Proof of Concept*



$$\epsilon^2 = \frac{2\alpha N_{sig}^{up}}{3\pi m_{A'} f_{rad} \frac{dN_{bkg}}{dm}}$$

*unblinded

- Functional Form Switch
 - 1 of the three functions I have integrated fully into BumpHunter is a top 5 10% 2016 function based on recent study. Will create a switch to allow easier switch between each of the functional forms and create pertinent upper limits for easy comparison

Understanding Gaussian Process Regression

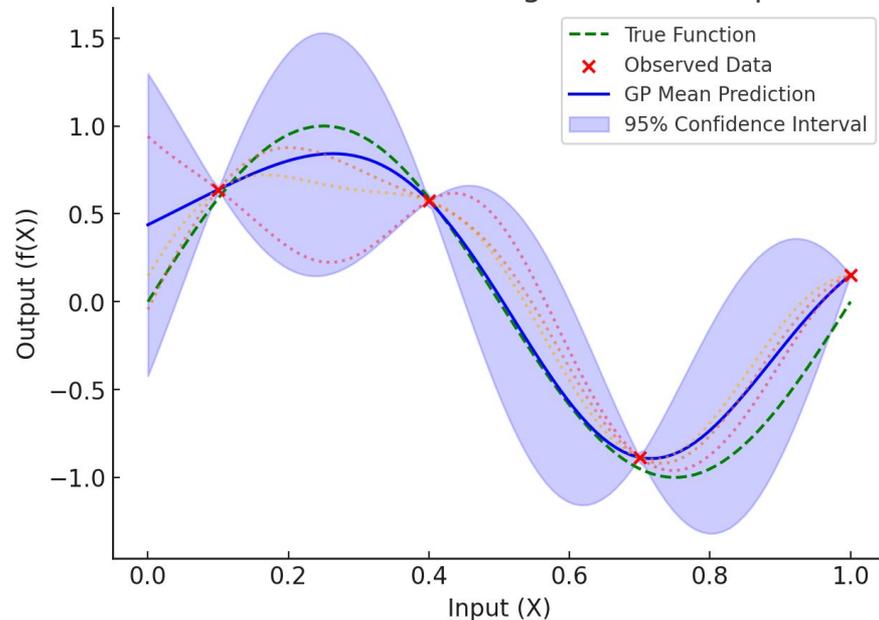
Collaborative effort with Tom Eichlersmith (Minnesota, PhD), Aidan Hsu (Stanford Undergraduate), Takumi Britt (High School).

What is Gaussian Process Regression (GPR)?

- A **flexible, non-parametric Bayesian approach** that models distributions over functions.
- Unlike traditional regression, **GPR does not assume a fixed set of parameters**—it learns a distribution of possible functions.
- **Built-in uncertainty quantification** makes it ideal for noisy and complex datasets.

The kernel function (covariance function) governs how data points interact and influence one another.

Gaussian Process Regression Example



The choice of kernel shapes the model's **smoothness, flexibility, and generalization ability**, making it crucial for capturing underlying data patterns.

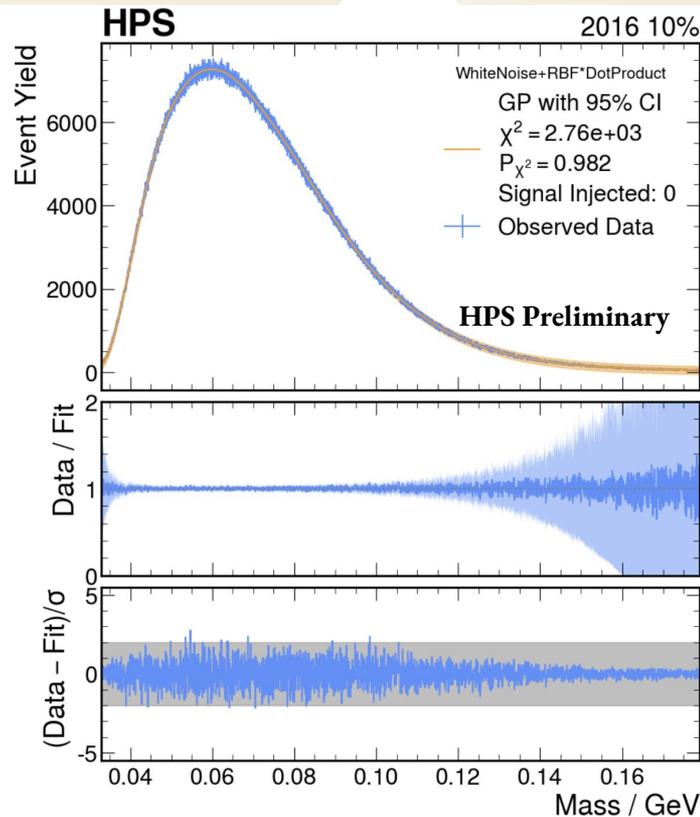
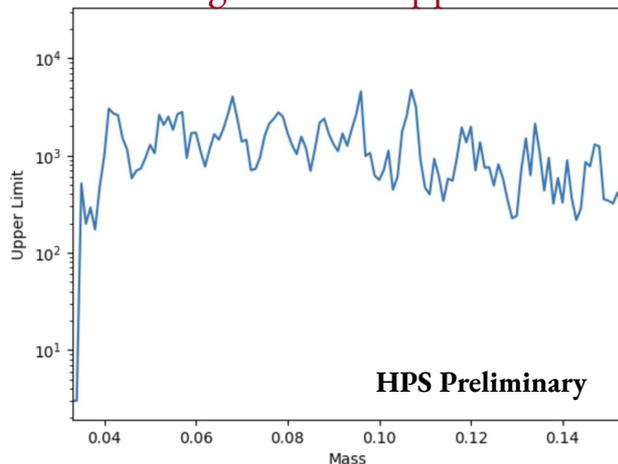
Application of GPR to HPS Datasets

The GP model provides a strong fit to the datasets with well-defined uncertainty estimates.

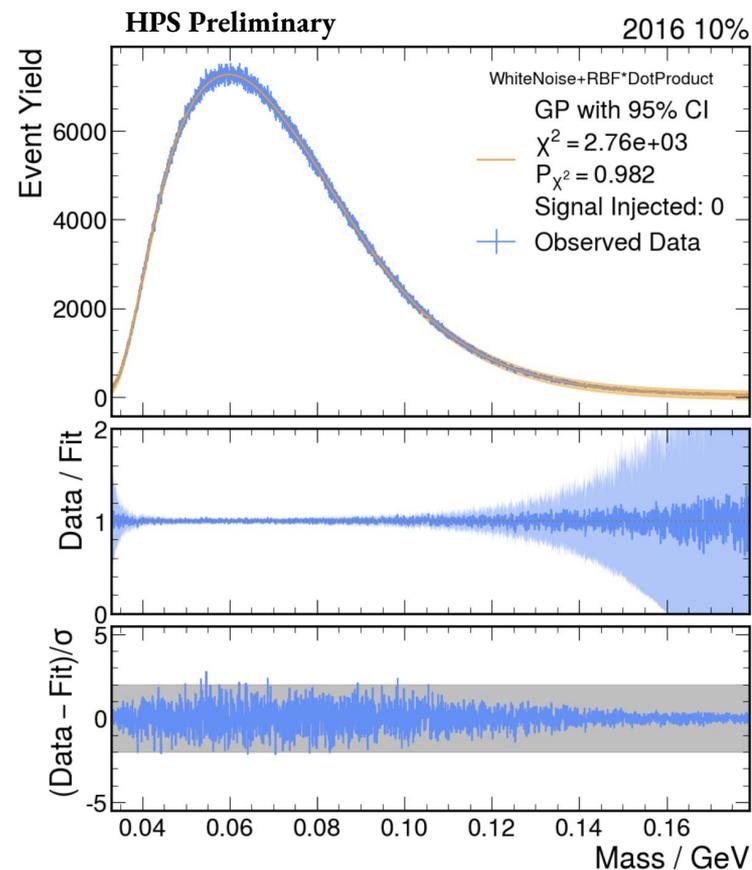
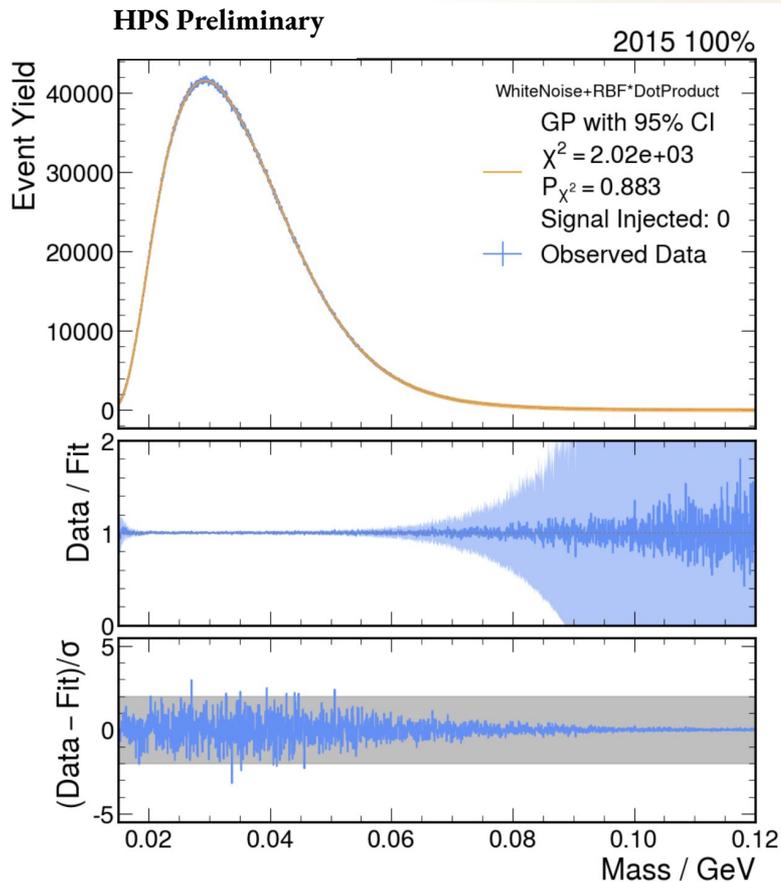
Preliminary Upper Limits determined to be competitive with functional form fitting.

10% Signal Yield Upper Limit

Kernel Choices:
WhiteNoise - models broad noise
RBF Kernel - models local correlations



Application of GPR to HPS Datasets



Local p-values and the Look Elsewhere Effect

Each mass hypothesis has a representative background fit as determined by the 2016 fit selection.

- corresponding χ^2 probabilities are “local” to the fit window
- global pvalues must be determined and take into account statistical fluctuations expected when searching **multiple independent regions**

The Look-Elsewhere Effect defines global p-values as being proportional to the number of independent regions:

$$p_{\text{global}} = p_{\text{local}} * N_{\text{regions}}$$

where

$$N_{\text{regions}} = W / \sigma_{\text{ave}}$$

total search
window size

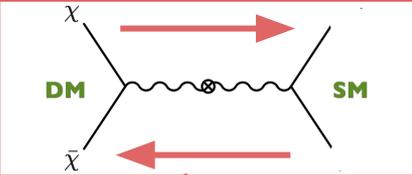
average mass
resolution

In 2016, $N_{\text{regions}} \sim 32$

- implying a sufficiently independent search region on average every ~ 4.4 MeV

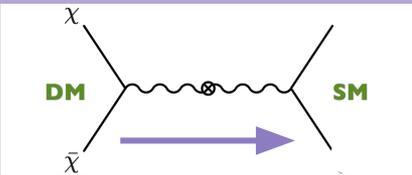
Freeze Out Thermal Relic Dark Matter Models

Early Universe: Thermal Equilibrium
Production = Annihilation



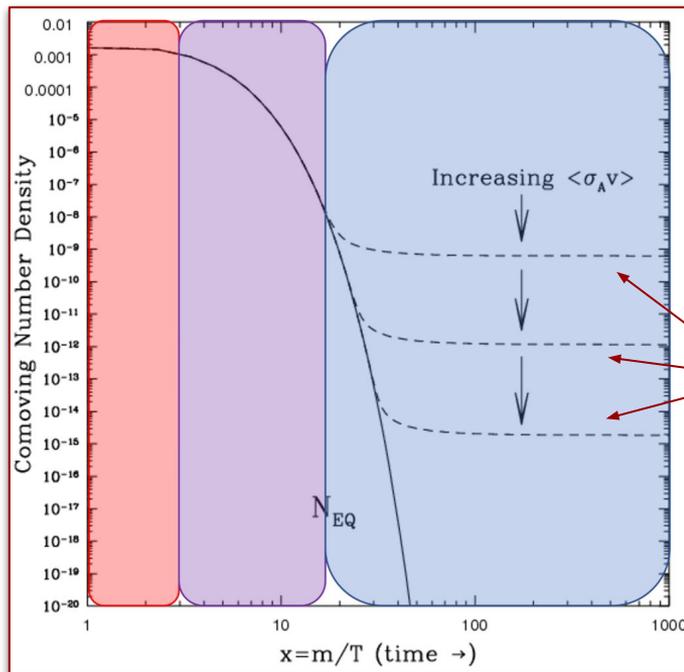
ended as universe cooled

Annihilation
Production < Annihilation

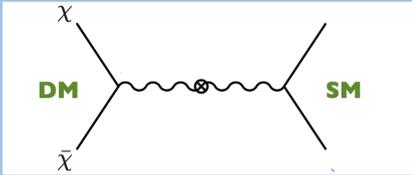


ended as universe expanded

Thermal Relic Density: Ω_χ



Now: Freeze-Out
Relic Density Set by $\langle\sigma_A v\rangle$



Observed DM abundance can correspond to different number densities depending on the characteristic mass and annihilation cross section.

Model Dependent

$$\Omega_\chi \propto \frac{1}{\langle\sigma v\rangle}$$

$$\sigma v \propto \epsilon^2 \alpha_D \frac{m_\chi^2}{m_{A'}^4}$$