Probing charge form factors with low energy leptons and hadronic beams

Paul Guèye
Professor
March 22, 2026

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Outline

Why e^-/e^+ to probe nuclei?

Beyond the Born Approximation

Nuclear charged radii database

Nuclear distributions: matter, charged, neutral

Sub-GeV electron/positron beams for nuclear radii

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Probing nuclei with electrons & positrons (Quasi-elastic scattering case)

Plane Wave Born Approximation (PWBA)

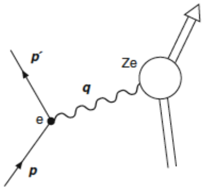
Distorted Wave Born Approximation (DWBA)

- Effective Momentum Approximation (EMA)

$$\left.\frac{d^3\sigma}{d\Omega_{e'}\,dE_{e'}}\right|^{\text{PWBA}} = \sigma_{\text{Mott}}\left\{\left(\frac{Q^2}{\mathbf{q}^2}\right)^2 R_L(|\mathbf{q}|,\omega) + \left(\frac{Q^2}{2\mathbf{q}^2}+\tan^2\frac{\theta}{2}\right)R_T(|\mathbf{q}|,\omega)\right\} = \sigma_{\text{Mott}}\times S_{\text{total}}(|\mathbf{q}|,\omega,\theta)$$

$$\mathbf{q}_{\text{eff}}=\mathbf{q}\left(1\boxed{-\frac{V_C}{E_e}}\right)-\omega\frac{V_C}{E_e}\hat{\mathbf{k}}_{e'}$$

$$\left.\frac{d^3\sigma}{d\Omega_{e'}\,dE_{e'}}\right|^{\text{EMA}} = \sigma_{\text{Mott}}\left\{\left(\frac{Q^2_{\text{eff}}}{\mathbf{q}^2_{\text{eff}}}\right)^2 R_L(|\mathbf{q}_{\text{eff}}|,\omega) + \left(\frac{Q^2_{\text{eff}}}{2\mathbf{q}^2_{\text{eff}}}+\tan^2\frac{\theta}{2}\right)R_T(|\mathbf{q}_{\text{eff}}|,\omega)\right\} = \sigma_{\text{Mott}}\times S_{\text{total}}(|\mathbf{q}_{\text{eff}}|,\omega,\theta),$$

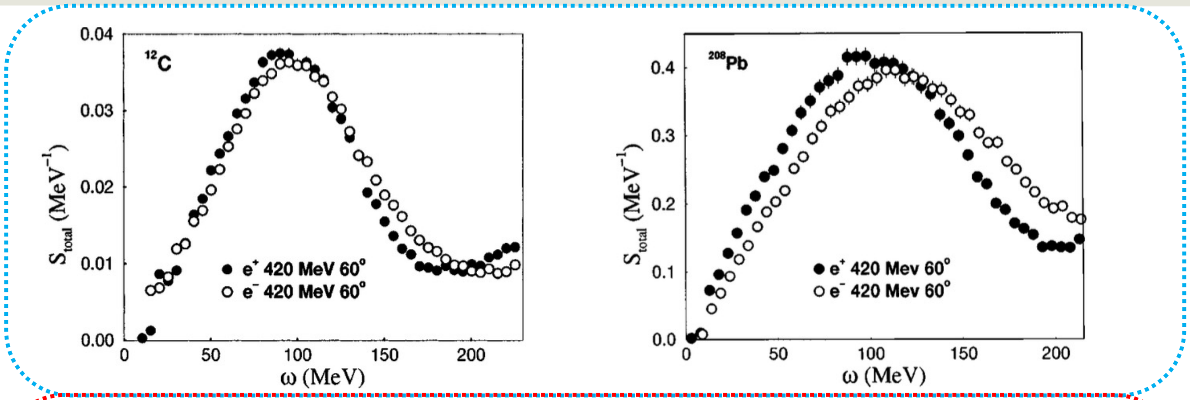


$$\left(\frac{d\sigma}{d\Omega}\right)_A = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left|F(Q^2)\right|^2$$

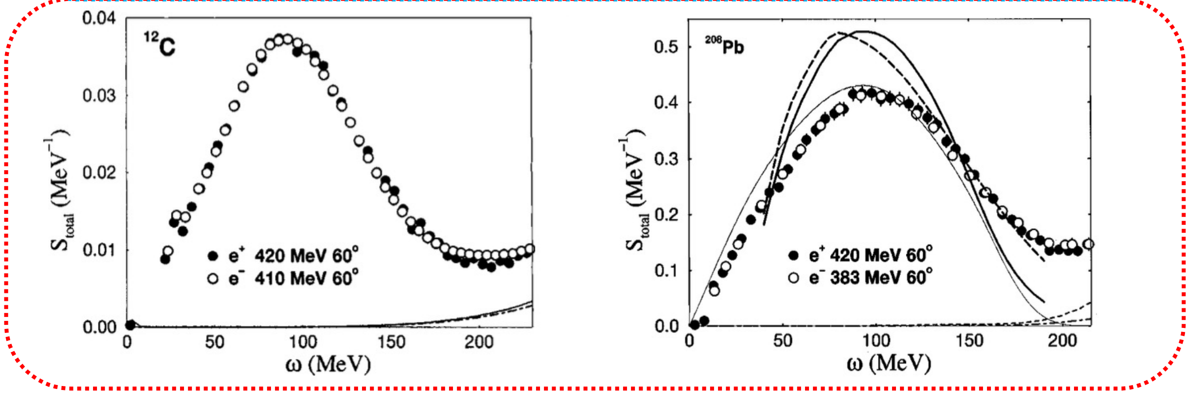
Coulomb field
electron: **p** increases and **p'** decreases
positron: **p** decreases and **p'** increases

Coulomb effects on xsec
electron: shift to higher energies + decrease S_{total}
positron: shift to lower energies + increase S_{total}

Coulomb Corrections, Ctnd.



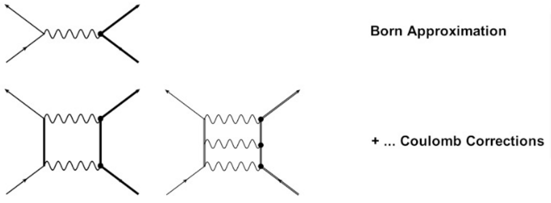
before correction



after correction

Z-dependent (V_c)

Scaling: $1+\delta_{>1\gamma}$



See W. Melnitchouk talk

P. Guèye, PRC, **60**, 044308 (1999)

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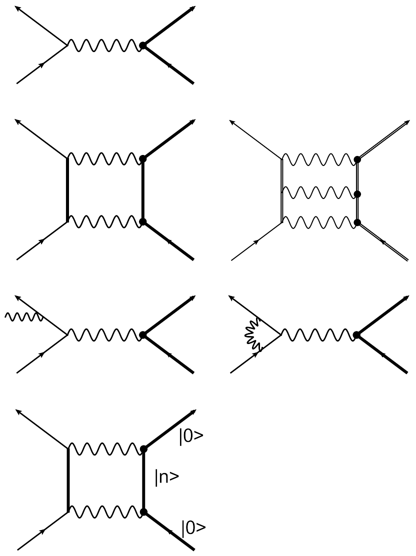
Beyond the (1γ-exchange) Born Approximation

J. Friar and M. Rosen, Annals Phys., 87:289–326, 1974.



✓ Born Approximation

✓ + ... Coulomb Corrections

✓ + ... Radiative Corrections

✗ + ... Dispersive Corrections

partial-wave analysis

static analysis

dynamic corrections

$$\sum |n\rangle\langle n| = 1$$

$$\mathcal{M}_{elast+disp} = (\alpha Z)F(q^2) + (\alpha Z)^2 G(q^2)$$

$$\begin{aligned} |\mathcal{M}_{elast+disp}|^2 &= (\alpha Z)^2\left[F(q^2)\right]^2 \\ &+ \boxed{2(\alpha Z)^3\left[F(q^2)\mathcal{R}e\{G(q^2)\}\right]} \\ &+ \boxed{(\alpha Z)^4\left[|\mathcal{R}e\{G(q^2)\}|^2 + |\mathcal{I}m\{G(q^2)\}|^2\right]} \end{aligned}$$

$$\mathcal{M}_{disp} = \sum_{n\neq 0}\int \frac{d^3\vec{p}}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}} \frac{\langle 0|\boxed{\rho(\vec{q}_2)}|n\rangle\langle n|\boxed{\rho(\vec{q}_1)}|0\rangle}{p^2 - p_n^2 - i\varepsilon}a(p_n)$$

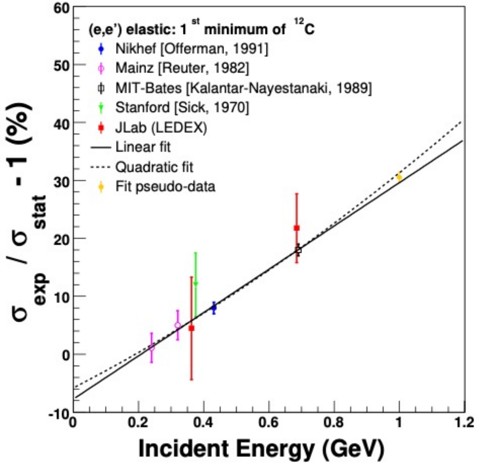
Dispersive Effects

Minima of diffraction : $F(q^2) \rightarrow 0$; $\mathcal{R}e\{G(q^2)\} + \mathcal{I}m\{G(q^2)\}$

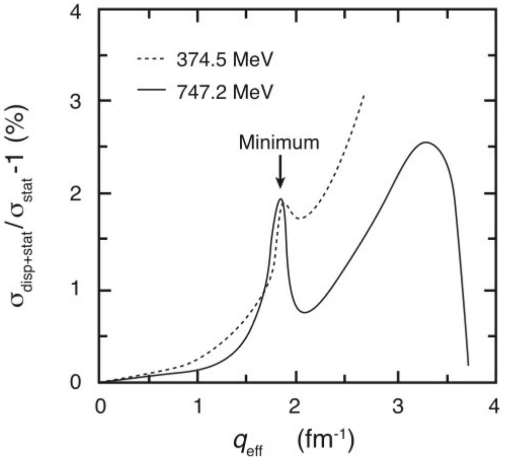
Comparison e^--e^+ : even terms canceled; $\mathcal{R}e\{G(q^2)\}$

Dispersive Corrections

P. Guèye et al., EPA, 56(5), 126 (2020)



J. Friar and M. Rosen, Annals Phys., 87:289–326, 1974

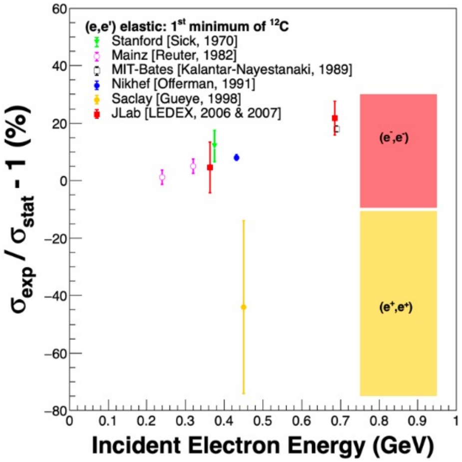


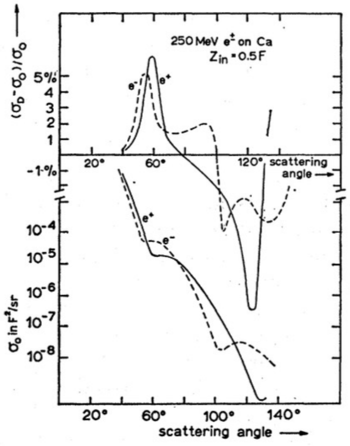
Energy dependence

- (A,Z) evolution?
- Unpolarized vs. polarized beam?
- …

Lepton charge

- Accurate prediction needed!





P. Guèye, Phys. Rev. **C**, 57:2107–2110 (1998)
G. H. Rawitscher, Phys. Rev., 151:846–852 (1966)

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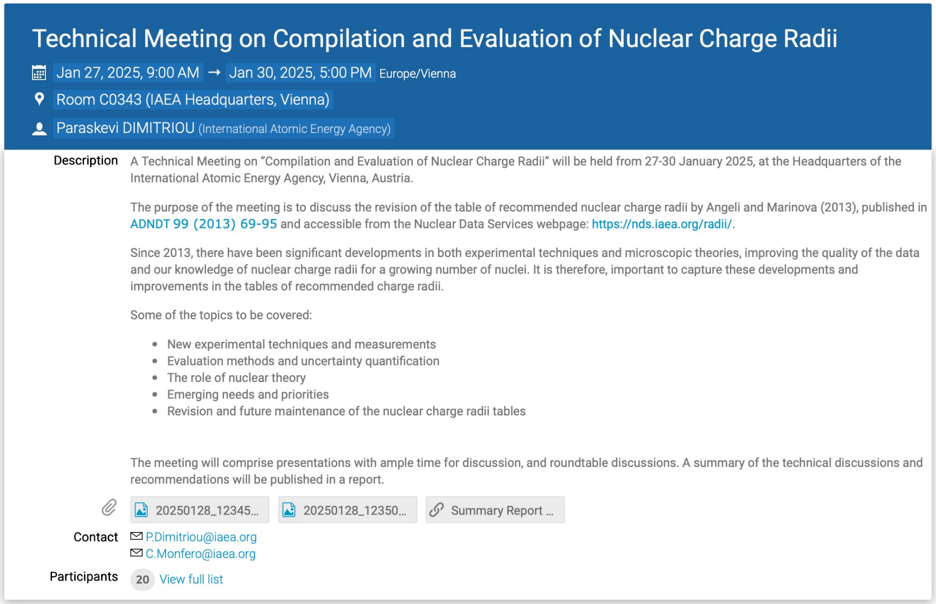
IAEA Technical Meeting [1]

More accurate compilation (rms nuclear charge radii)

- Latest: Angeli, I. & Marinova, K. P
  - Atom. Data Nucl. Data Table, **99**, 69 (2013)

Challenges

- Large uncertainties: mainly theory
- Need guidance for experiments
- No dispersive effects in past compilations!



Technical Meeting on Compilation and Evaluation of Nuclear Charge Radii

Jan 27, 2025, 9:00 AM → Jan 30, 2025, 5:00 PM Europe/Vienna

Room C0343 (IAEA Headquarters, Vienna)

Paraskevi DIMITRIOU (International Atomic Energy Agency)

Description A Technical Meeting on "Compilation and Evaluation of Nuclear Charge Radii" will be held from 27-30 January 2025, at the Headquarters of the International Atomic Energy Agency, Vienna, Austria.

The purpose of the meeting is to discuss the revision of the table of recommended nuclear charge radii by Angeli and Marinova (2013), published in ADNDT 99 (2013) 69-95 and accessible from the Nuclear Data Services webpage: https://nds.iaea.org/radii/.

Since 2013, there have been significant developments in both experimental techniques and microscopic theories, improving the quality of the data and our knowledge of nuclear charge radii for a growing number of nuclei. It is therefore, important to capture these developments and improvements in the tables of recommended charge radii.

Some of the topics to be covered:

- New experimental techniques and measurements
- Evaluation methods and uncertainty quantification
- The role of nuclear theory
- Emerging needs and priorities
- Revision and future maintenance of the nuclear charge radii tables

The meeting will comprise presentations with ample time for discussion, and roundtable discussions. A summary of the technical discussions and recommendations will be published in a report.

Contact P.Dimitriou@iaea.org
C.Monfero@iaea.org

Participants 20 View full list

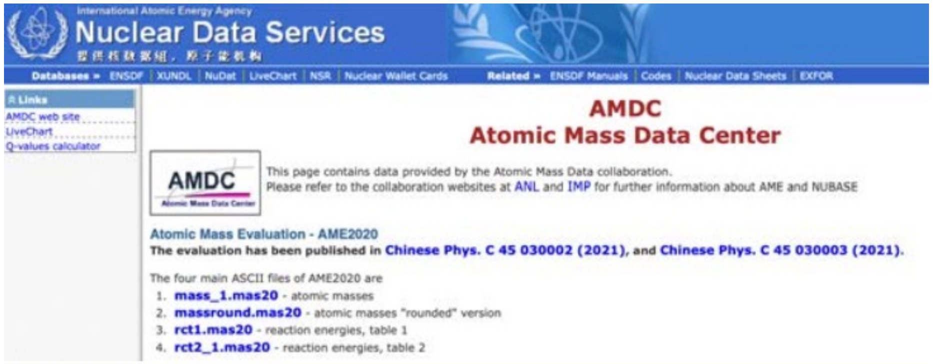
IAEA Technical Meeting [2]

Best practices to extract precise rms from nuclei?

- Stable nuclei
    - electron scattering (deVries, 1987!), muon or laser spectroscopy
- Unstable nuclei
    - laser spectroscopy (need ref. nuclei), electrons (SCRIT @ RIKEN and ULQ2 @ Tohoku Univ.)
- Standardize
    - e.g., Barrett moments from μ-spectroscopy (accuracy: ≤0.1%)

How to get there?

- Need both experimental and theoretical work
    - Across the nuclear chart
- Regular charge nuclear radii compilation
    - “à la” AMDC



AMDC
Atomic Mass Data Center

This page contains data provided by the Atomic Mass Data collaboration.
Please refer to the collaboration websites at ANL and IMP for further information about AME and NUBASE

Atomic Mass Evaluation - AME2020

The evaluation has been published in Chinese Phys. C 45 030002 (2021), and Chinese Phys. C 45 030003 (2021).

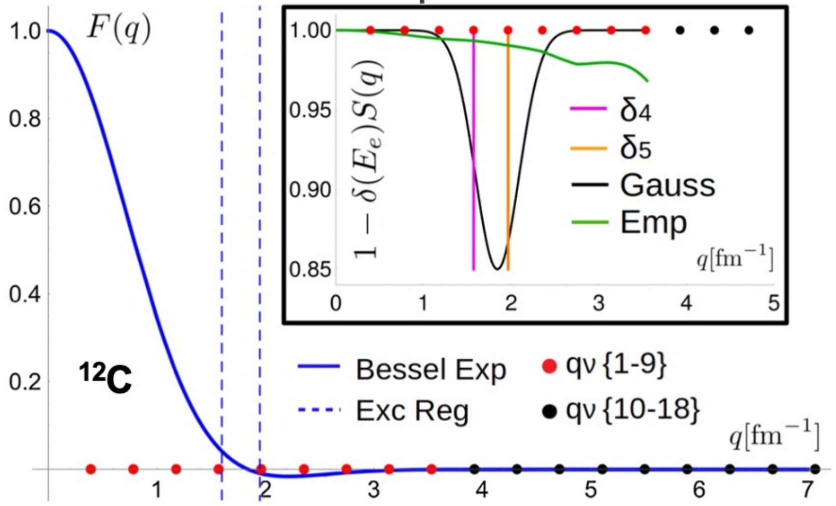
The four main ASCII files of AME2020 are

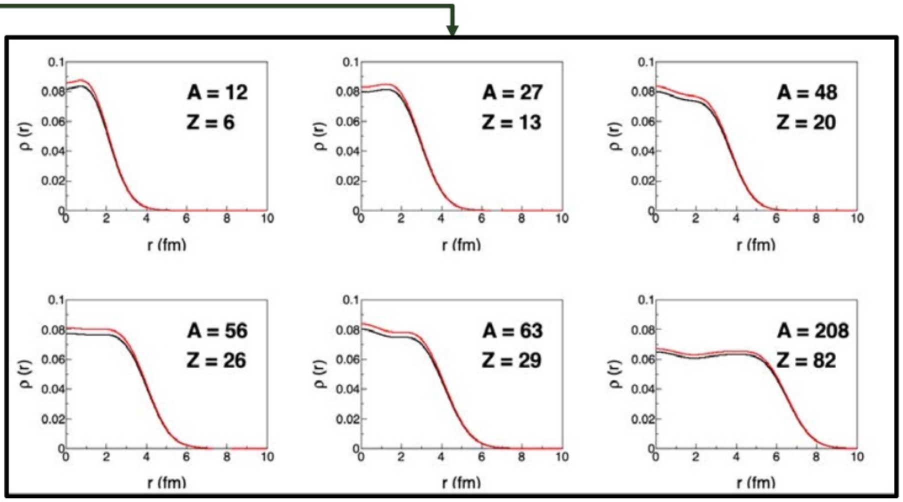
1. **mass_1.mas20** - atomic masses
2. **massround.mas20** - atomic masses "rounded" version
3. **rct1.mas20** - reaction energies, table 1
4. **rct2_1.mas20** - reaction energies, table 2

Dispersive Effects and Charge Distributions

P. Guèye et al., EPJ **A56**(5) 2020

F(q) = 1 - 0.00833q





	^{12}C	^{27}Al	^{48}Ca	^{56}Fe	^{63}Cu	^{208}Pb
$\rho_{disp}(r)/\rho(r)$ [%]	4.91	4.11	4.58	4.65	4.65	3.49

Jlab/PAC53 proposal: not recommended
Plan to re-submit (e^-: qualitative information; $e^\pm$: absolute)

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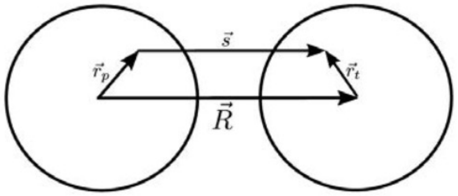
Matter Radii [1]

Double-folding model

- Optical potential to describe two colliding ions obtained by
  - » Averaging an appropriate N-N interaction over the matter distributions
  - » Similar approach as treatment of the Coulomb interaction
- Two components: real (Coulomb) and imaginary (nuclear)
- Use two-Fermi parameter distribution

G. Satchler, W. Love, Phys. Rep. **55**(3) (1979) 183–254
L. Chamon et al., Comp. Phys. Com. 267 (2021) 108061

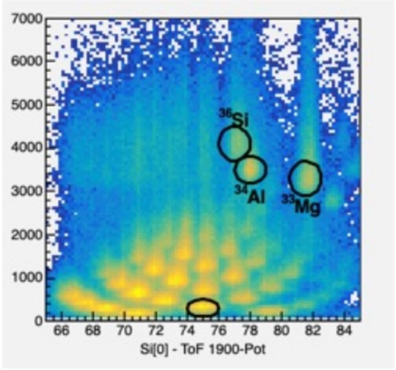


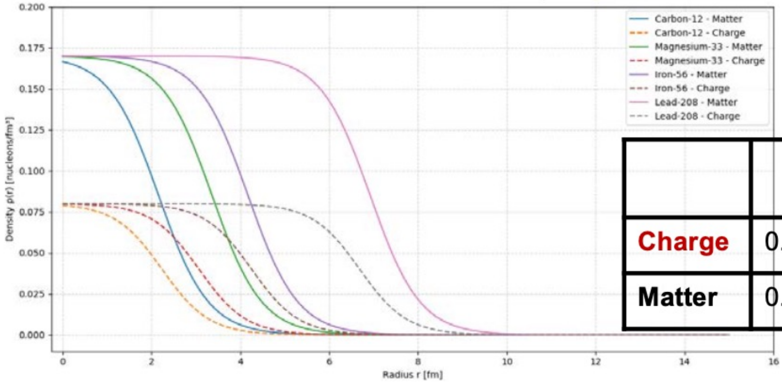
$$\rho(r) = \frac{\rho_0}{1 + e^{\left(\frac{r - R_0}{a}\right)}}$$

$$R_0 = 1.76 Z^{1/3} - 0.96 \text{ fm } a = 0.53 \text{ fm}$$

Application to ^{9}Be(^{A}X, ^{A}X') at 88.8 MeV/u (^{33}Mg)

PHY-2012040
PHY-2310078



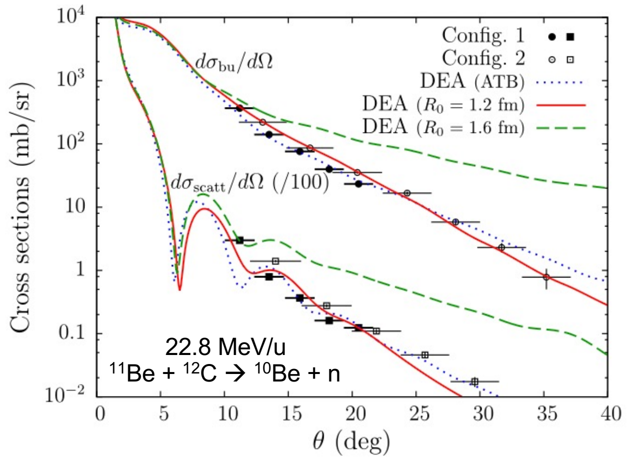


	ρ_0	R_0 (fm)	a (fm)
Charge	0.08 protons/fm^3	1.76*Z*1/3 − 0.96 fm	0.53
Matter	0.17 nucleons/fm^3	1.31*A*1/3 − 0.84 fm	0.56

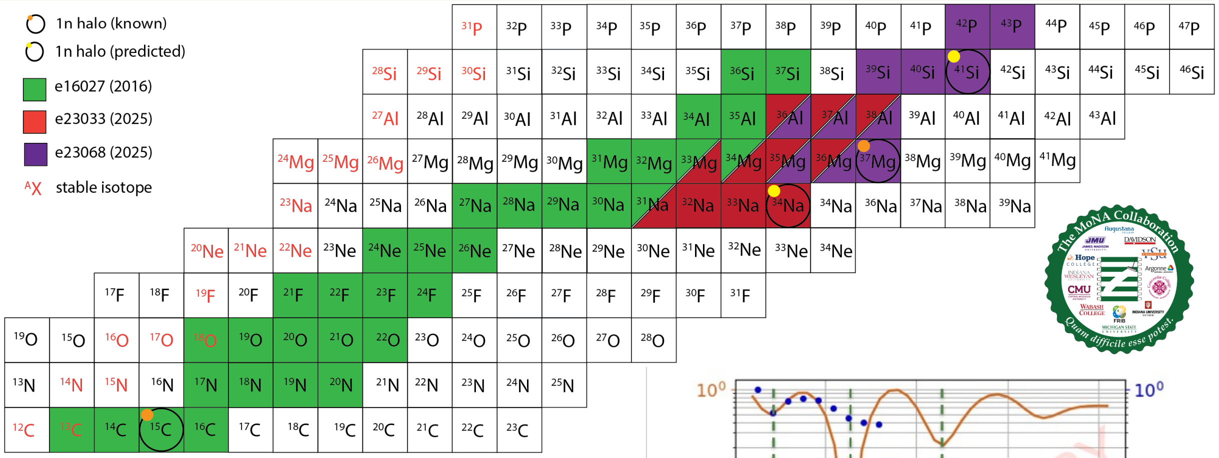
Makaila Parks
(Spelman, Undergrad student)

Matter Radii [2]

$$R = \frac{\sigma^{{}^{A}_{Z}X}_{elas}(\theta)}{\sigma^{{}^{A-1}_{Z}X}_{elas}(\theta)}$$



S. Ota et al., PRL, **134**, 212501 (2025)
P. Capel, R. Johnson, and F. Nunes, PLB 705, 112 (2011).
P. Capel, R. C. Johnson, and F. M. Nunes, PRC 88, 044602 (2013).
P. Capel, R. Johnson, and F. Nunes, EPA 56, 300 (2020).



										^{31}P	^{32}P	^{33}P	^{34}P	^{35}P	^{36}P	^{37}P	^{38}P	^{39}P	^{40}P	^{41}P	^{42}P	^{43}P	^{44}P	^{45}P	^{46}P	^{47}P
								^{28}Si	^{29}Si	^{30}Si	^{31}Si	^{32}Si	^{33}Si	^{34}Si	^{35}Si	^{36}Si	^{37}Si	^{38}Si	^{39}Si	^{40}Si	^{41}Si	^{42}Si	^{43}Si	^{44}Si	^{45}Si	^{46}Si
								^{27}Al	^{28}Al	^{29}Al	^{30}Al	^{31}Al	^{32}Al	^{33}Al	^{34}Al	^{35}Al	^{36}Al	^{37}Al	^{38}Al	^{39}Al	^{40}Al	^{41}Al	^{42}Al	^{43}Al		
						^{24}Mg	^{25}Mg	^{26}Mg	^{27}Mg	^{28}Mg	^{29}Mg	^{30}Mg	^{31}Mg	^{32}Mg	^{33}Mg	^{34}Mg	^{35}Mg	^{36}Mg	^{37}Mg	^{38}Mg	^{39}Mg	^{40}Mg	^{41}Mg			
						^{23}Na	^{24}Na	^{25}Na	^{26}Na	^{27}Na	^{28}Na	^{29}Na	^{30}Na	^{31}Na	^{32}Na	^{33}Na	^{34}Na	^{34}Na	^{36}Na	^{37}Na	^{38}Na	^{39}Na				
				^{20}Ne	^{21}Ne	^{22}Ne	^{23}Ne	^{24}Ne	^{25}Ne	^{26}Ne	^{27}Ne	^{28}Ne	^{29}Ne	^{30}Ne	^{31}Ne	^{32}Ne	^{33}Ne	^{34}Ne								
		^{17}F	^{18}F	^{19}F	^{20}F	^{21}F	^{22}F	^{23}F	^{24}F	^{25}F	^{26}F	^{27}F	^{28}F	^{29}F	^{30}F	^{31}F										
^{19}O	^{15}O	^{16}O	^{17}O	^{18}O	^{19}O	^{20}O	^{21}O	^{22}O	^{23}O	^{24}O	^{25}O	^{26}O	^{27}O	^{28}O												
^{13}N	^{14}N	^{15}N	^{16}N	^{17}N	^{18}N	^{19}N	^{20}N	^{21}N	^{22}N	^{23}N	^{24}N	^{25}N														
^{12}C	^{13}C	^{14}C	^{15}C	^{16}C	^{17}C	^{18}C	^{19}C	^{20}C	^{21}C	^{22}C	^{23}C															

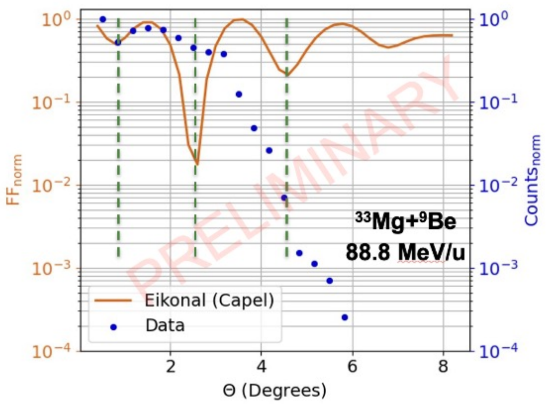
Ratio method

- Access to S_n, r_n

Minima of diffraction

- Access to matter radii
- New data (January 2026!)

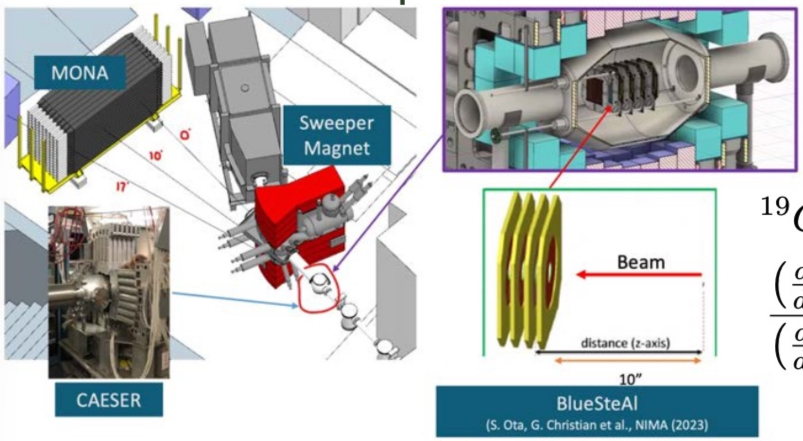
^{51,52}K, ^{52,53}Ca, ^{53,54,55}Sc, ^{55,56}Ti



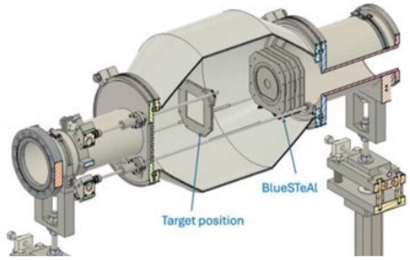
Ratio Method: $^{19}C \rightarrow {}^{18}C$ Experiment

FRIB PAC3 proposal

- PR25076: spokespersons are S. Ota (BNL), P. Capel (JHUM) & P. Guèye (FRIB)
- Establish a complete picture of the halo nucleus ^{19}C using a novel technique: the ratio method

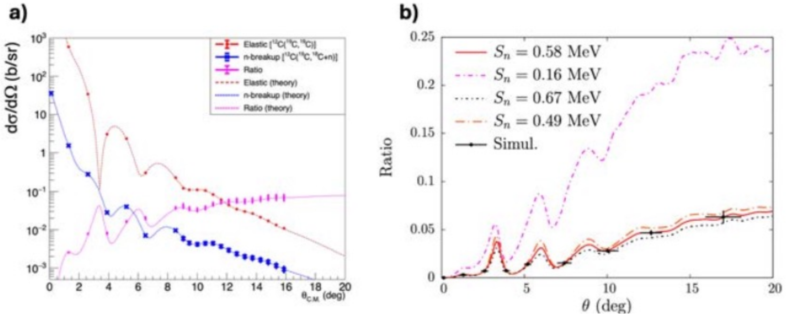


BlueSteAl
(S. Ota, G. Christian et al., NIMA (2023)



$$^{19}C \to {}^{18}C + n$$

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)_{^{19}C}}{\left(\frac{d\sigma}{d\Omega}\right)_{^{18}C}} \to S_n$$



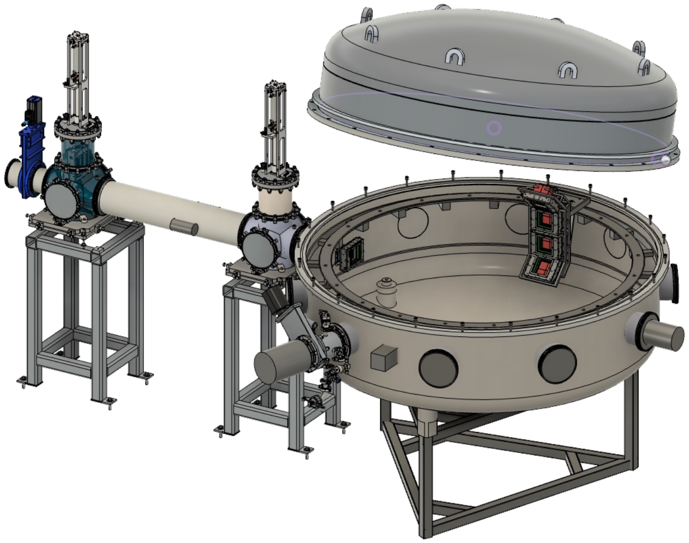
Western Michigan University [1]

DoE Proposal (submitted Fall 2025; waiting for review)

- WMU: Asghar Kayani, Zbigniew Chajecki; MSU: Paul Guèye; SJS: Nicholas Esker
- One focus: measurements of low-energy elastic scattering cross-sections from H to Cu beams
  - on stable and radioactive targets, including some residing in the lanthanide and actinide region
- Complement charge radii to infer neutron distributions!

Main systems

- 6MV Tandem
  - Charged states: H (via CH_2), ^{12}C, ^{27}Al, ^{48}Ca, ^{56}Fe, ^{197}Au, ^{208}Pb
- PING beamline
  - Physicists Inspiring the Next Generation (PING)
  - Participants: middle/High school + undergraduate students
  - Summer + year long programs
  - Undergrads mentoring pre-college students
  - Entire beamline designed by PING students!
    - Leverage Rutherford scattering kit from Leybold™

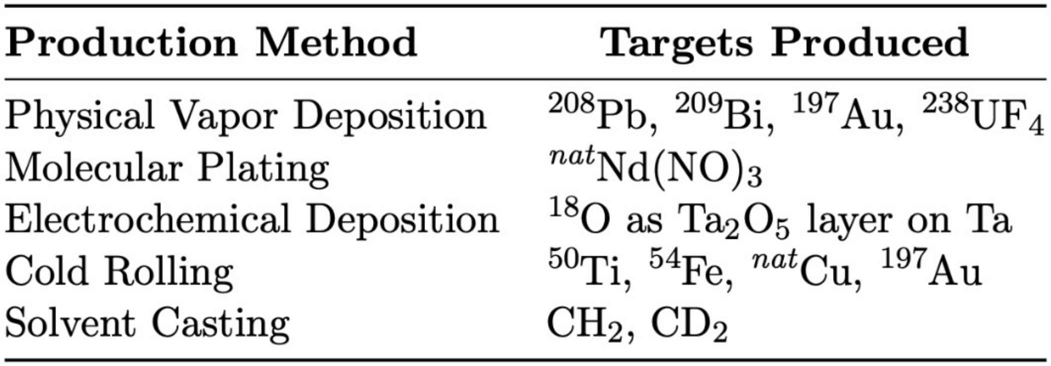


Western Michigan University [2]

Collaboration with San Jose University (N. Esker, V. Zakusilova [FRIB])

Possibility to use create radioactive targets

- Already produced



Production Method	**Targets Produced**
Physical Vapor Deposition	^{208}Pb, ^{209}Bi, ^{197}Au, $^{238}UF_4$
Molecular Plating	$^{nat}Nd(NO)_3$
Electrochemical Deposition	^{18}O as Ta_2O_5 layer on Ta
Cold Rolling	^{50}Ti, ^{54}Fe, ^{nat}Cu, ^{197}Au
Solvent Casting	CH_2, CD_2

- In progress (Lanthanide/Actinide region) – in discussion with JLab target group
  - ^{140}Nd, $^{139}Nd^{m}$, ^{168}Tm, and ^{171}Tm
  - Production (average): 10-100 mCi/week

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Electron/Positron (elastic) scattering: from Form Factors to Charge Distributions

Technique assumes spherical nuclei

$$\rho_{\text{matter}}^{\text{spherical}} = \rho_{\text{p}} + \rho_{\text{n}} \longrightarrow \rho_{\text{matter}}^{\text{deformed}} = \rho_{\text{matter}}^{\text{spherical}}(1+\delta)$$

$$\left(\frac{d\sigma(\theta_{e'})}{d\Omega}\right)_{exp} = \frac{N_d(\theta_{e'})}{N_{inc}}\frac{1}{N_{tar}}\frac{1}{\Delta\Omega}C_{corr}$$

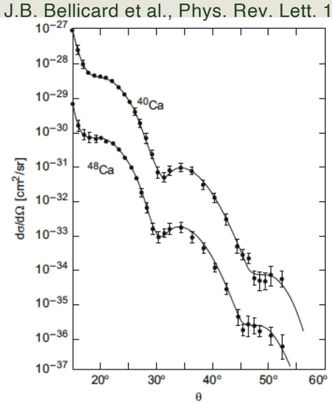
$$\left|F(q^2)\right|^2 = \frac{\left(\frac{d\sigma}{d\Omega}\right)_{exp}}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}}$$

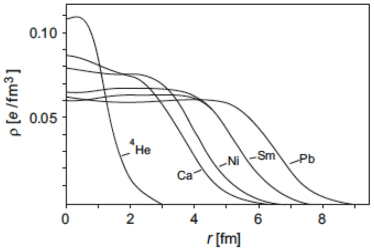
$$\rho(r) = \frac{1}{2\pi^2}\int_{q_{min}}^{q_{max}} F(q^2)j_0(qr)q^2dq$$

$$\rho_{\text{ch}}^{\text{FB}}(r) = \begin{cases} \sum_\nu a_\nu j_0\left(\frac{\nu\pi r}{R_{\text{cut}}}\right) & \text{for} \quad r \le R_{\text{cut}} \\ 0 & \text{for} \quad r > R_{\text{cut}} \end{cases}$$

D. Verney, *History of the concept of nuclear shape*, EPJA, **61**:82 (2025)

J.B. Bellicard et al., Phys. Rev. Lett. 19, 527 (1967)





OR

$$F(q^2) = 1 - \frac{1}{6}\frac{q^2 < r^2 >}{\hbar^2} + \cdots$$

$$\Longrightarrow < r^2 > = -6\hbar^2 \left.\frac{dF(q^2)}{dq^2}\right|_{q^2=0}$$

Should use both techniques in analysis!

Check for energy dependence

Compare: e^-A, e^+A, $\overrightarrow{e^-}$A, $\overrightarrow{e^+}$A (spin 0 & non-0)

Thank you!