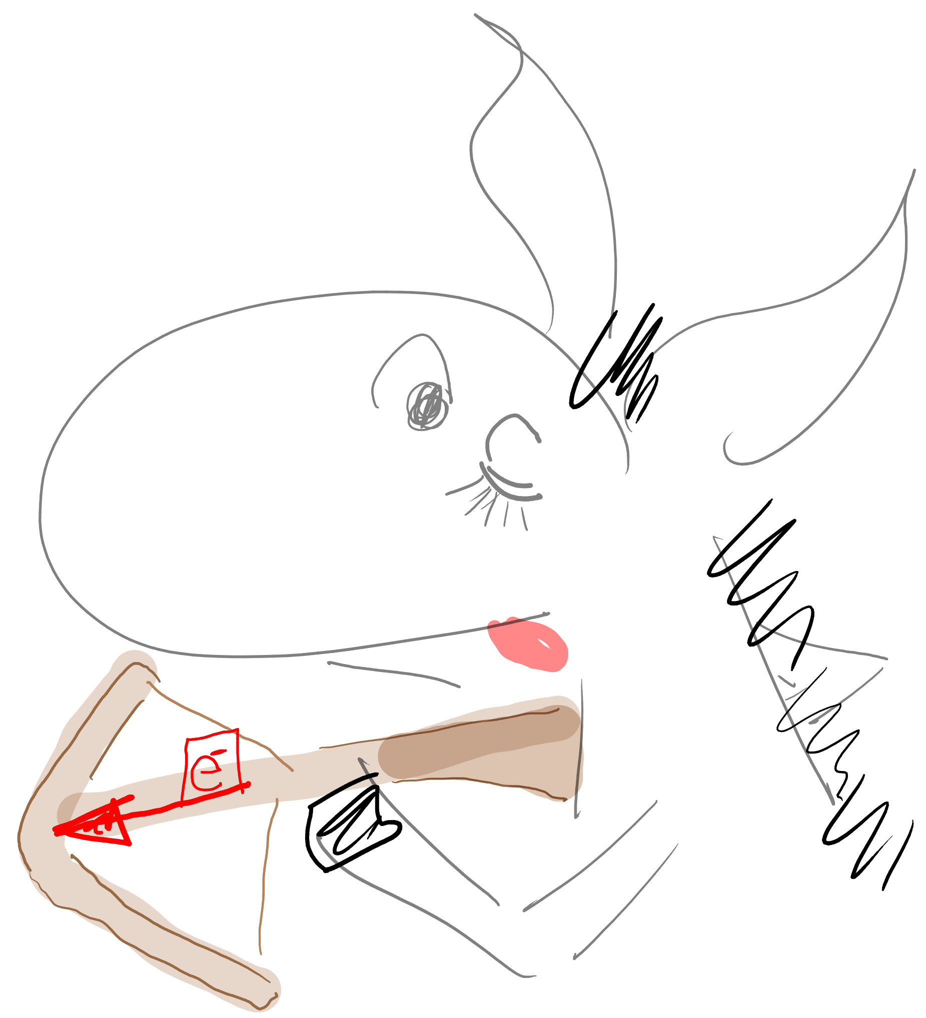

Radiative corrections for lepton-proton scattering with McMule

Matteo Ronchi

Johannes Gutenberg University, Mainz

LEEPP Conference@JLab

23^{rd}-27^{th} March 2026

What is McMule?

McMule

Monte Carlo for MUons and other LEptons
https://mule-tools.gitlab.io
P. Banerjee, G. Billis, A. Coutinho, Y. Fang, S. Gündogdu, F. Hagelstein,
S. Kollatzsch, T. Oruç, D. Radic, M. Rocco, M. Ronchi, N. Schalch,
V. Sharkovska, A. Signer, Y. Ulrich, J. Wilson

- ❑ Framework for fully-differential higher-order QED calculations

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- Framework for fully-differential higher-order QED calculations

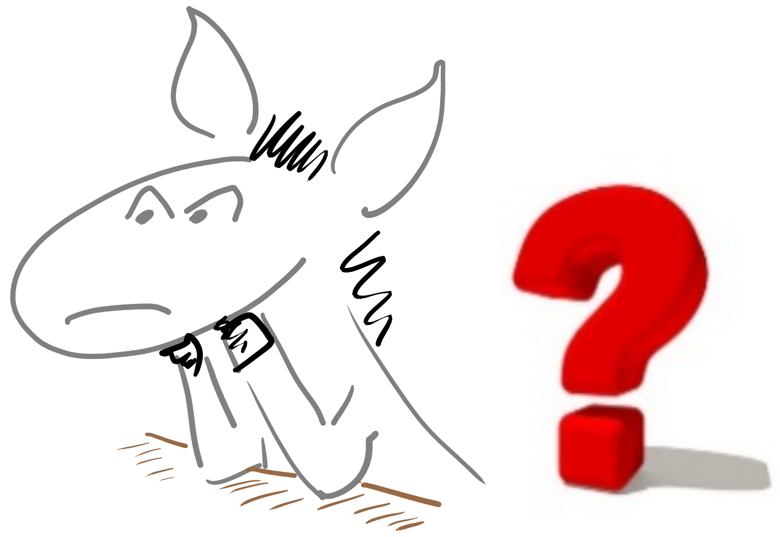
- Advanced detectors (new accelerators) → increased experimental precision (new kinematics)
- Advanced experimental setups → e.g., recoil proton detection, ISR, …
- Dark-sector searches → crucial to control background

Experiments are improving

↓

theory needs to follow!

Why should we improve radiative corrections?



- The effects are small, natural expansion parameter: $\frac{\alpha}{\pi} \approx 2 \times 10^{-3}$

- The effects are small, **but enhancement exists:** $\frac{\alpha}{\pi} \log\left(\frac{m}{E}\right) \approx 0.05$

Not only lepton-proton scattering ...

process	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO+
$\ell p \to \ell p$	P2, Muse, PRad, QWeak, MAGIX	proton radius and weak charge	NNLO
$eN \to eN$	PRad, ULQ2, MAGIX	background	+
$e^-e^- \to e^-e^-$	PRad2, MOLLER, ...	normalisation, $\sin^2\theta_W$ at low Q^2	NNLO
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \ell\ell$	VEPP, BES, Daphne, Belle	R-ratio, τ properties	NNLO±
$ee \to \gamma\gamma$	Daphne, any e^+e^- collider	dark searches, luminosity measurement	NNLO−
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO−
$\mu \to \nu\bar{\nu}e$	MEG, DUNE	ALP searches, beam-line profiling	NNLO+
$\mu \to \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \to \nu\bar{\nu}eee$	MEG, Mu3e	background	NLO
$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	+

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[***] $e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
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$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	+
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	+

[***] [P. Banerjee et al. (2021) https://arxiv.org/abs/2106.07469]

process	experiment	on	order
$e\mu \to e\mu$	MUonE		NNLO+
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$\mu \to \nu\bar{\nu}e\gamma$	MEG, Mu3e, I		NLO
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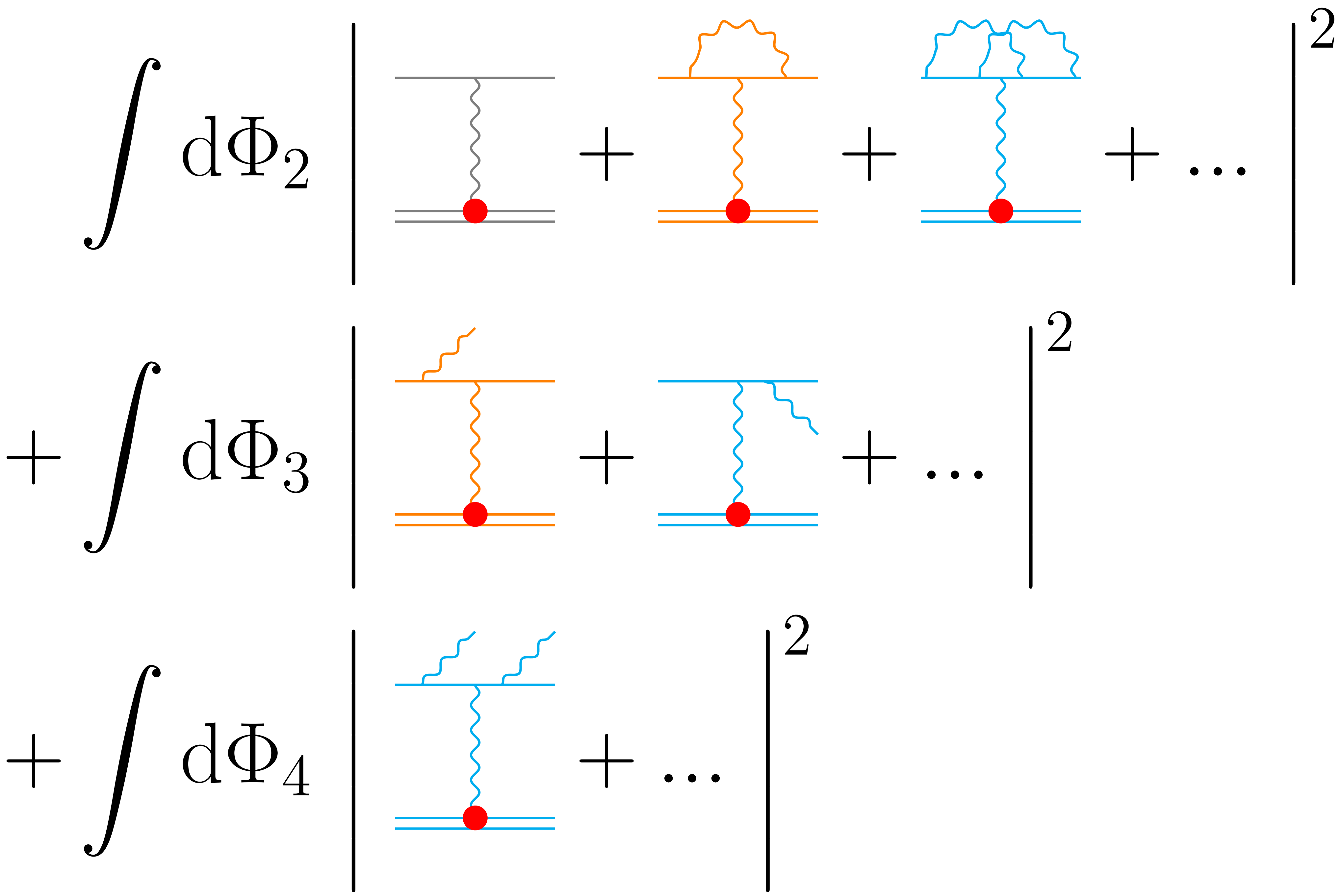
$ee \to \pi\pi$	VEPP, BES, I		+
$ee \to \ell\ell\gamma$	VEPP, BES, I		+

Goal:
world domination

How does McMule work?

McMule (Monte Carlo for muons and other leptons) Collaboration



- **input:** matrix elements evaluated by McMule coll. or others
- **McMule:** integrator (generator is work in progress)
- **output**: differential physical cross sections for any physical quantity at fixed order through phase-space Monte Carlo integration

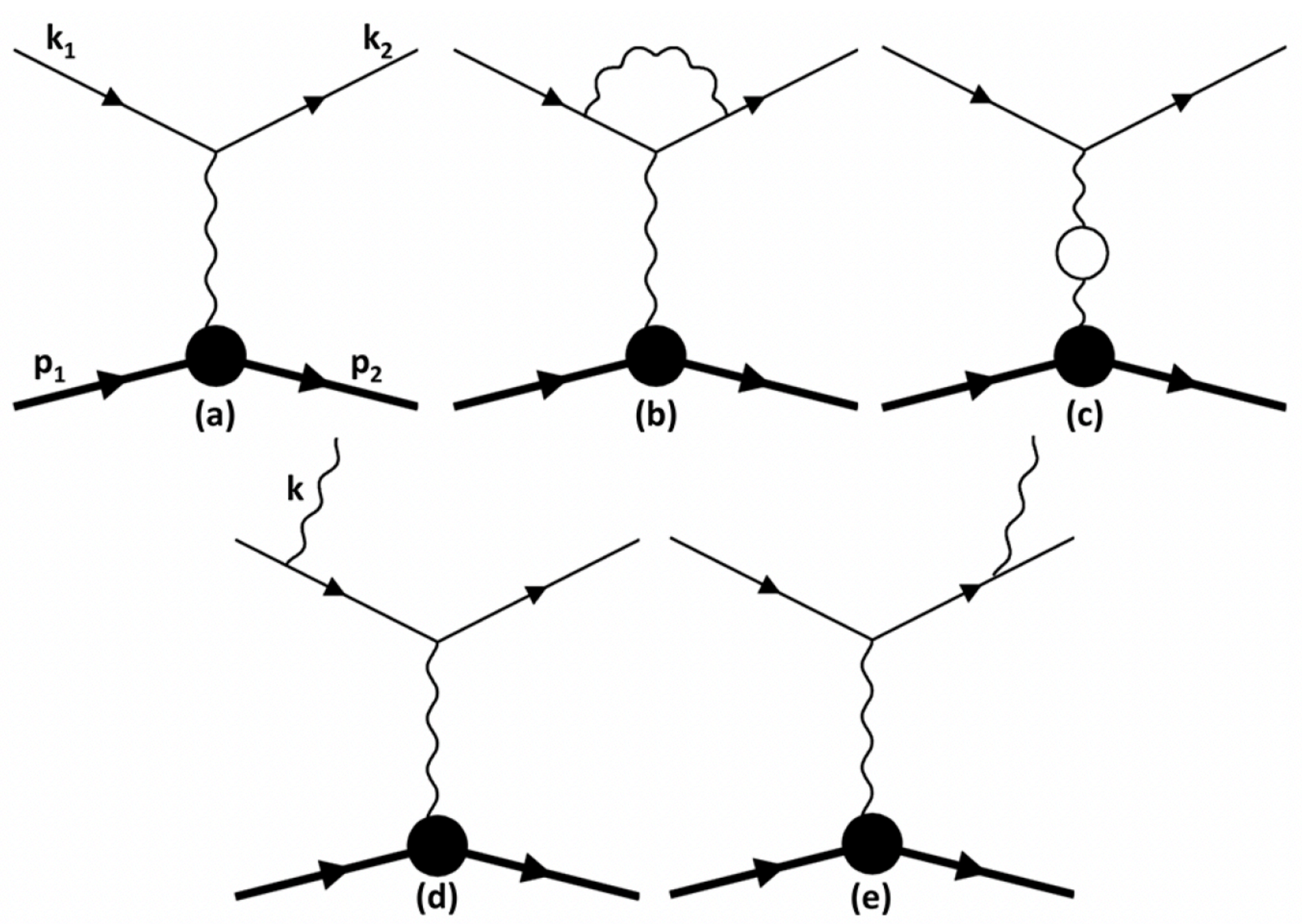
Which radiative corrections have been considered so far?

A small proton charge radius from an electron–proton scattering experiment
[W. Xiong et al. (2019) Nature, 575, 147–150 (2019)]

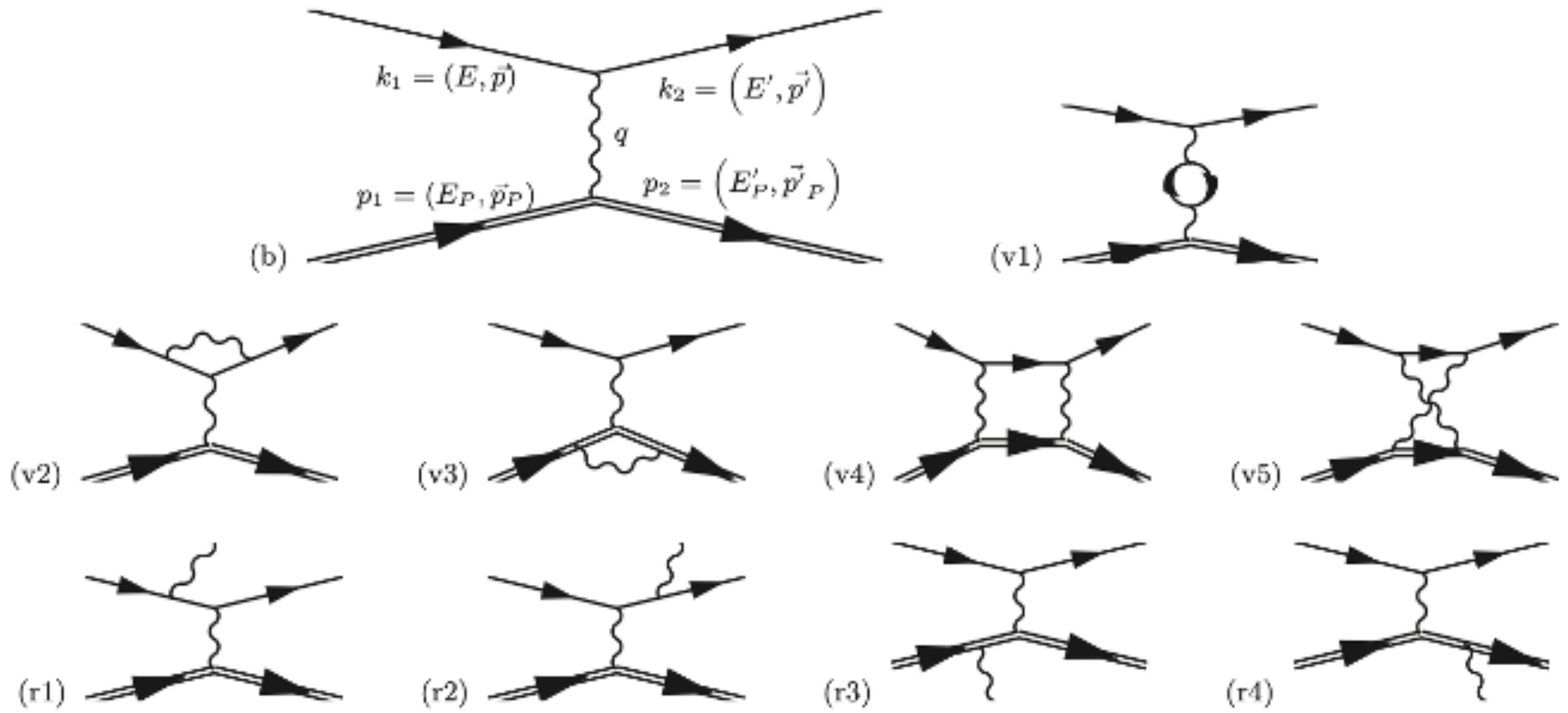
Most extensive data set for proton electromagnetic form factors (A1 Collaboration)
[J.C. Bernauer et al. (2014) https://arxiv.org/abs/1307.6227]

(*) (1969) https://inspirehep.net/literature/52657
(**) (2000) https://arxiv.org/abs/nucl-th/0002058
(***) (2000) https://arxiv.org/abs/hep-ph/0001100

NLO corrections considered



[I. Akushevich et al. (2015) Eur.Phys.J.A 51 (2015), 1]



[Mo&Tsai (*), Maximon&Tjon (**), Vanderhaeghen et al. (***)]

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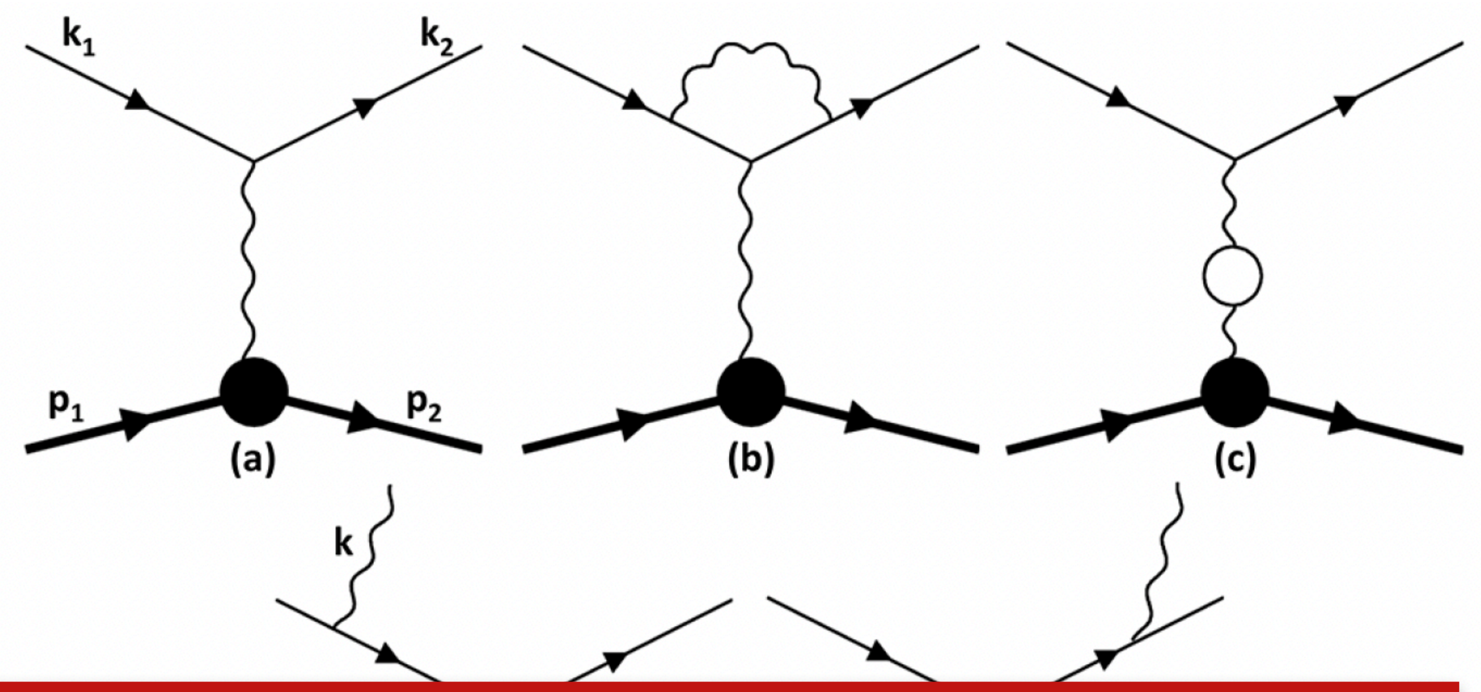
NLO corrections considered



A small proton charge radius from an electron–proton scattering experiment
[W. Xiong et al. (2019) Nature, 575, 147–150 (2019)]

Old theoretical framework stopped at NLO QED corrections

[I. Akushevich et al. (2015) Eur.Phys.J.A 51 (2015), 1]

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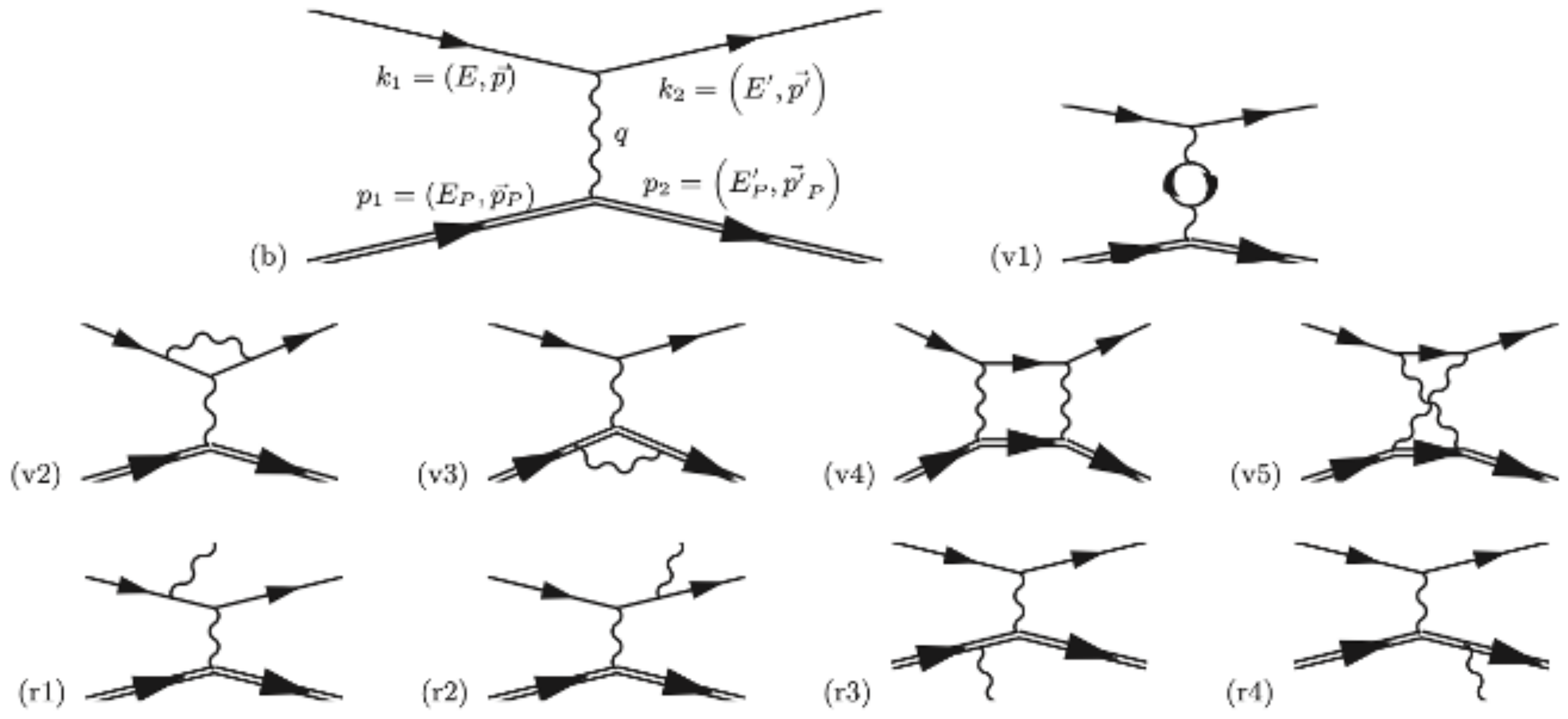

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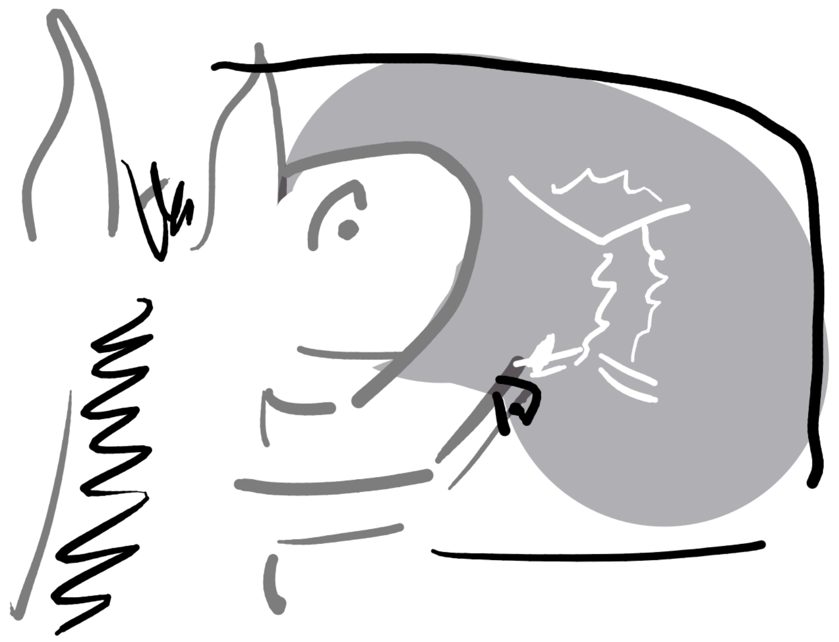
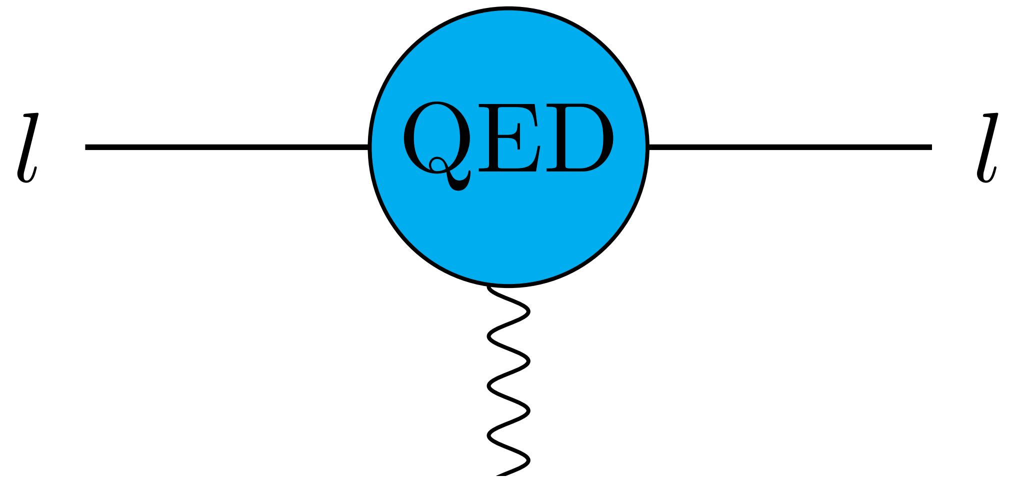
[Mo&Tsai (*), Maximon&Tjon (**), Vanderhaeghen et al. (***)]

Which are the benefits of McMule?



1. NNLO QED corrections
   [McMule collaboration (2023), MUSE] $lp \to lp$
   https://arxiv.org/abs/2307.16831
   [McMule collaboration (2021), Møller] $e^-e^- \to e^-e^-$
   https://arxiv.org/abs/2107.12311

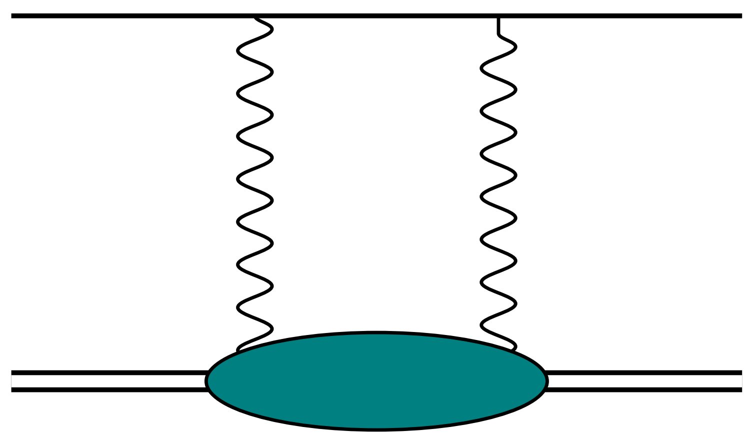


2. Dimensional regularization for IR divergencies

3. No limitation on emitted photon energies

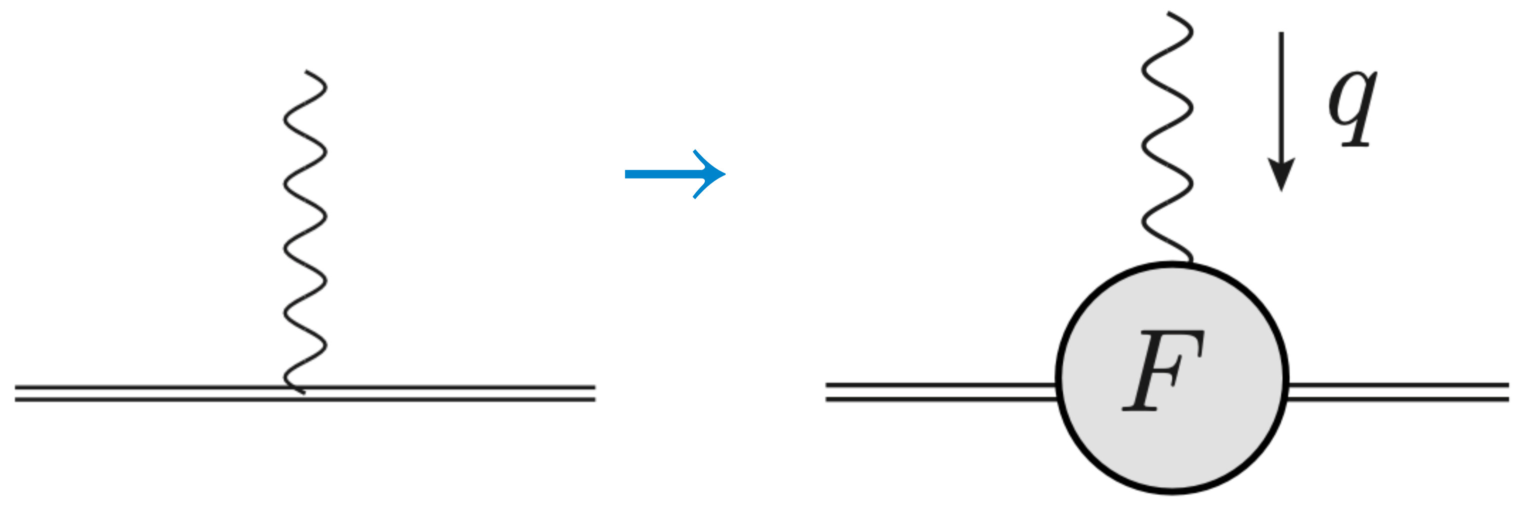
4. Lepton mass included

5. Hard (inelastic) TPE can be included



From leptons to hadrons

- Natural to extend to lp scattering by including proton form factors



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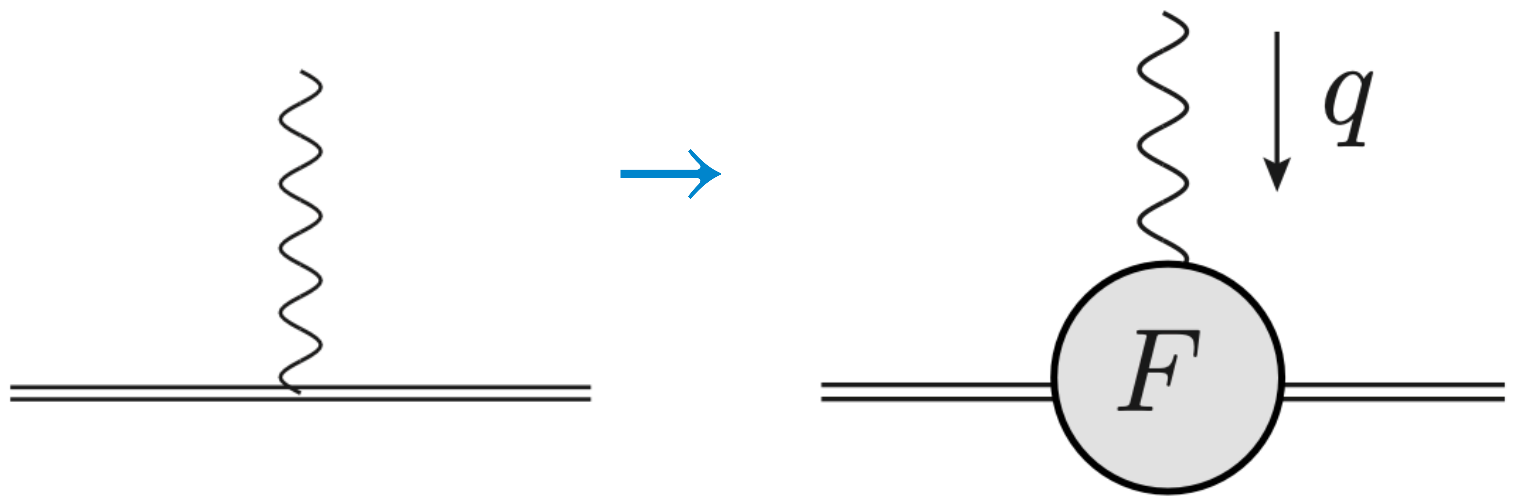
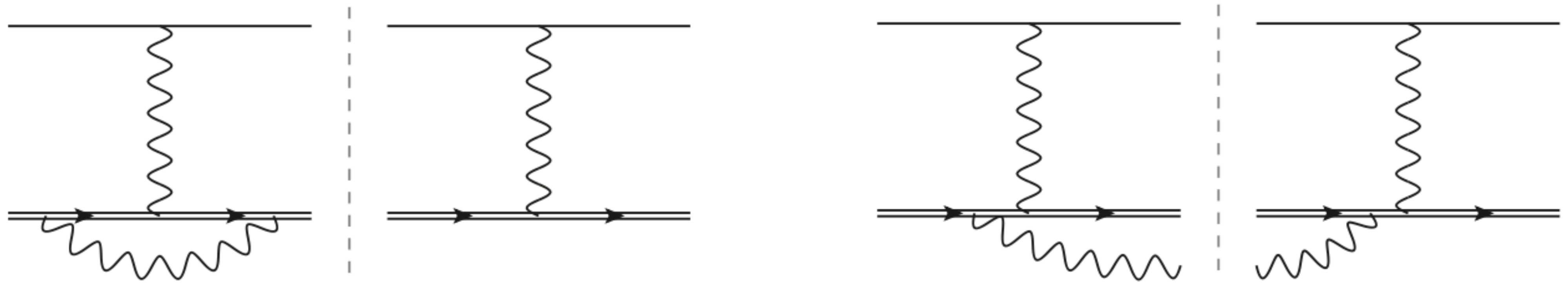
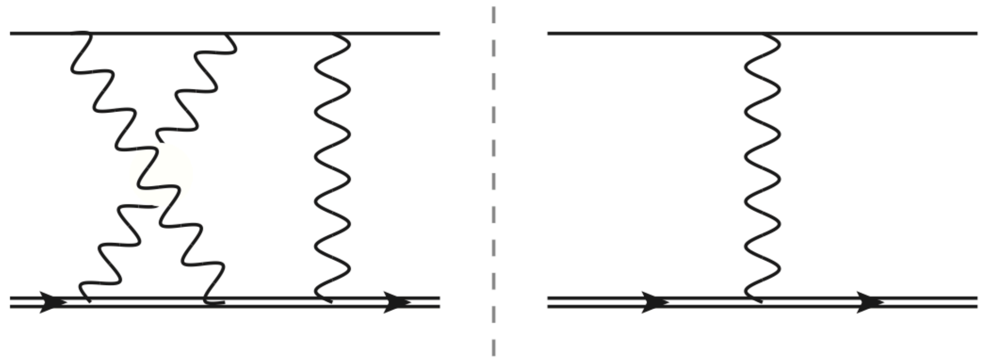


  ➢ LO+NLO: proton structure included except pure proton-line corrections →



  ➢ NNLO: proton point-like except electronic corrections →



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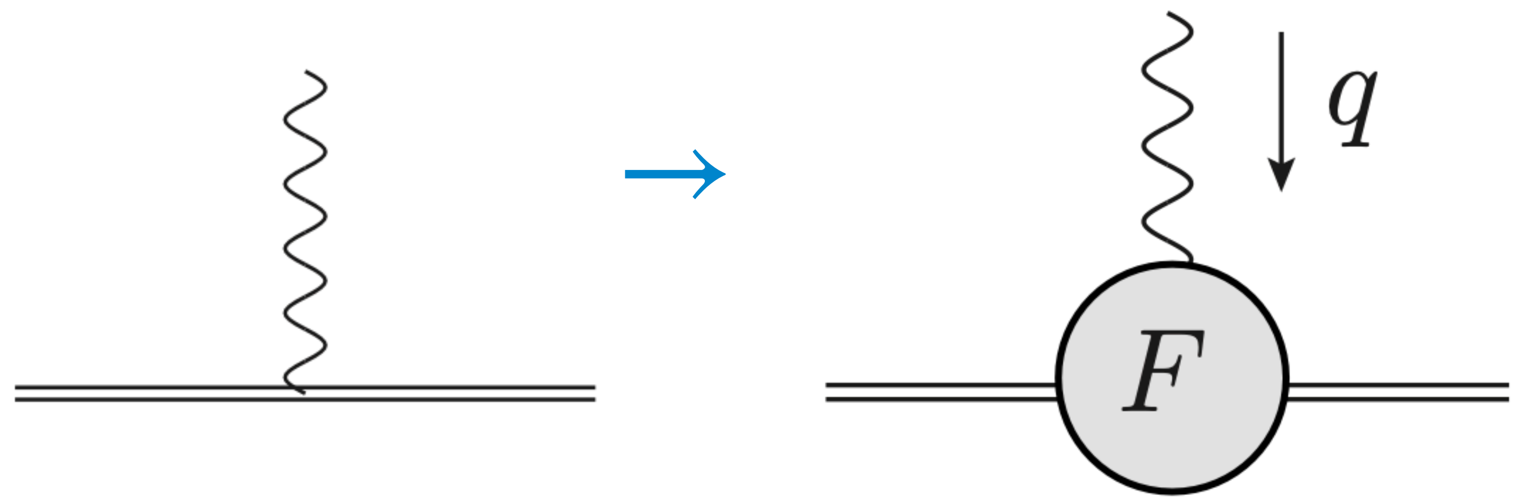

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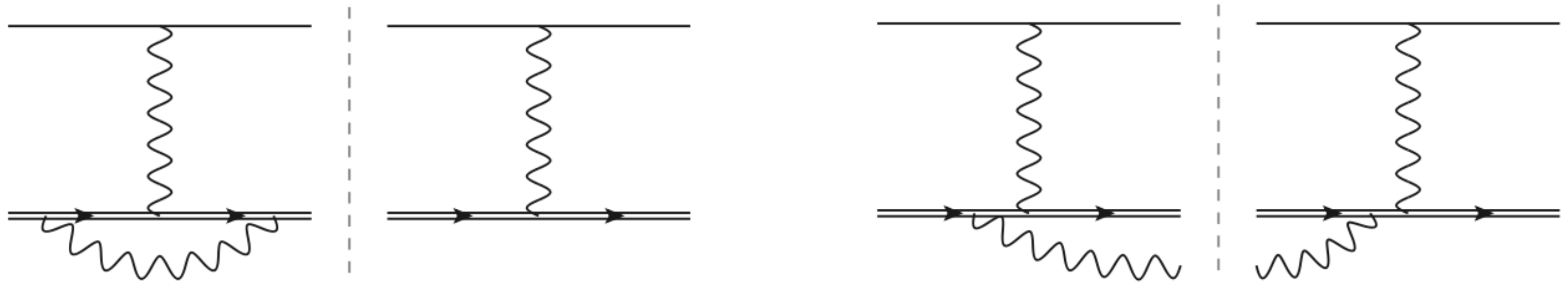

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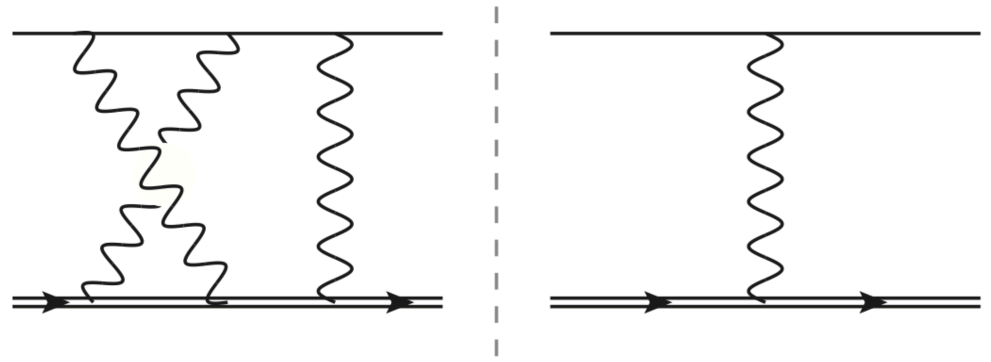
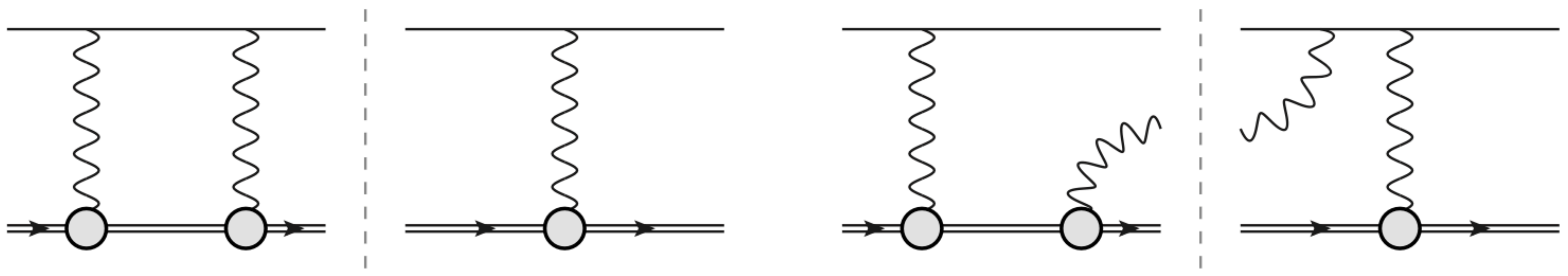
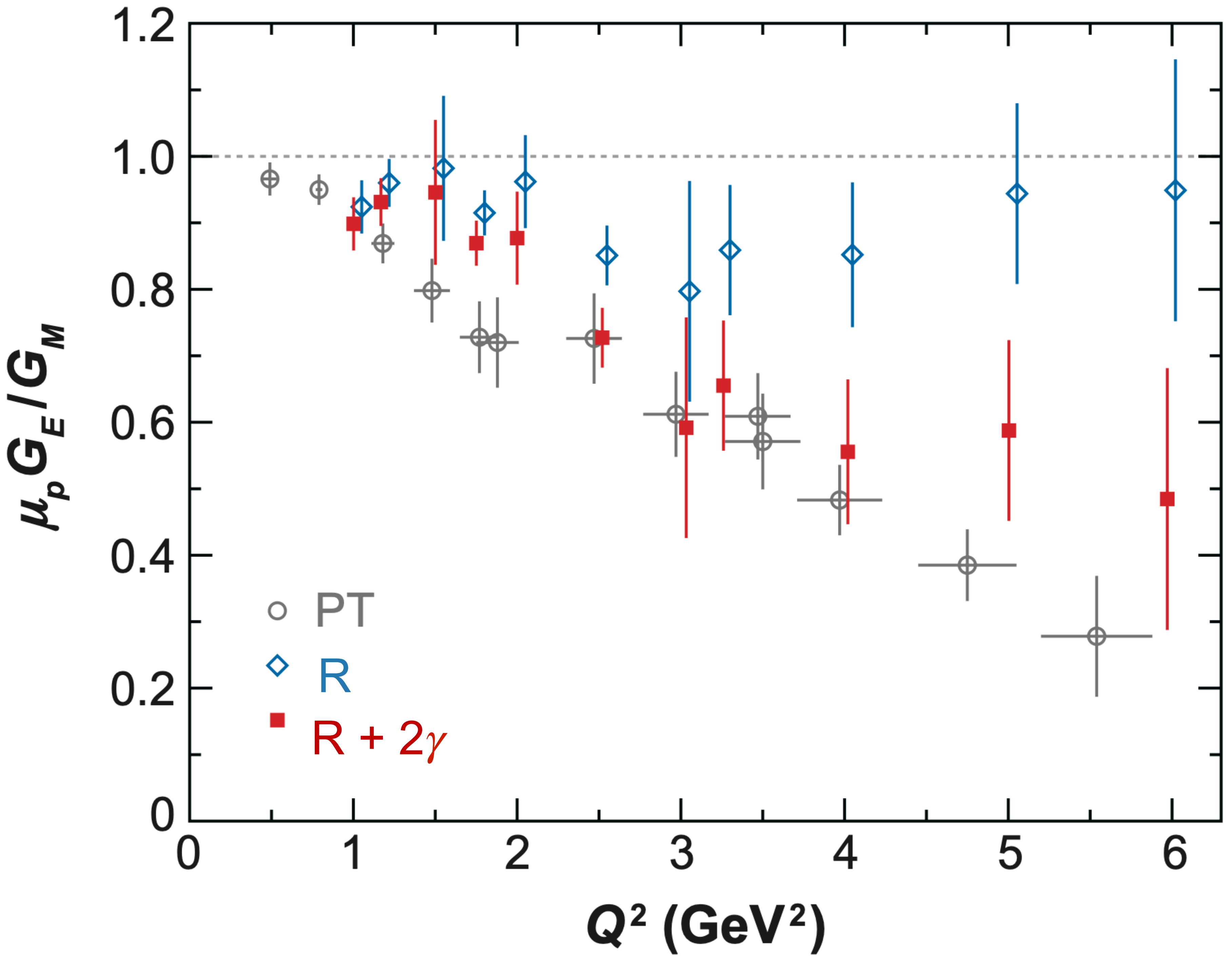


- McMule can connect QED current for the lepton side "in principle" to any current provided for the target side

Importance of two-photon-exchange TPE

Presently in McMule only simple Born TPE model with dipole FF





[C E. Carlson, M. Vanderhaeghen (2007)] https://arxiv.org/abs/hep-ph/0701272

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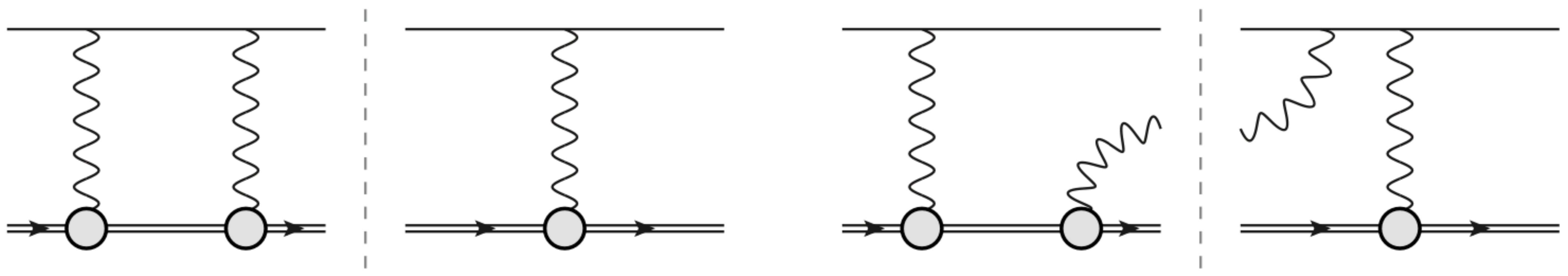

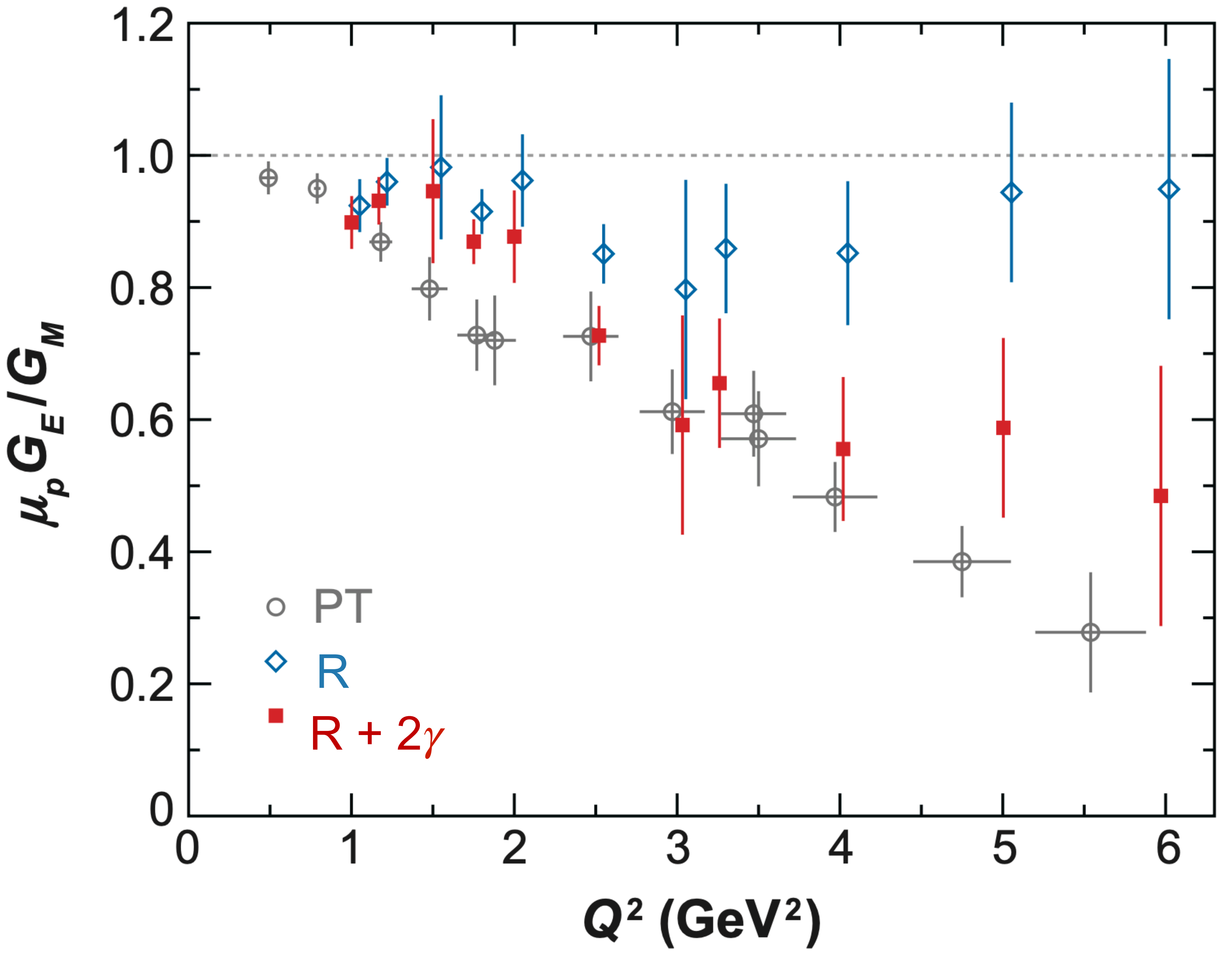

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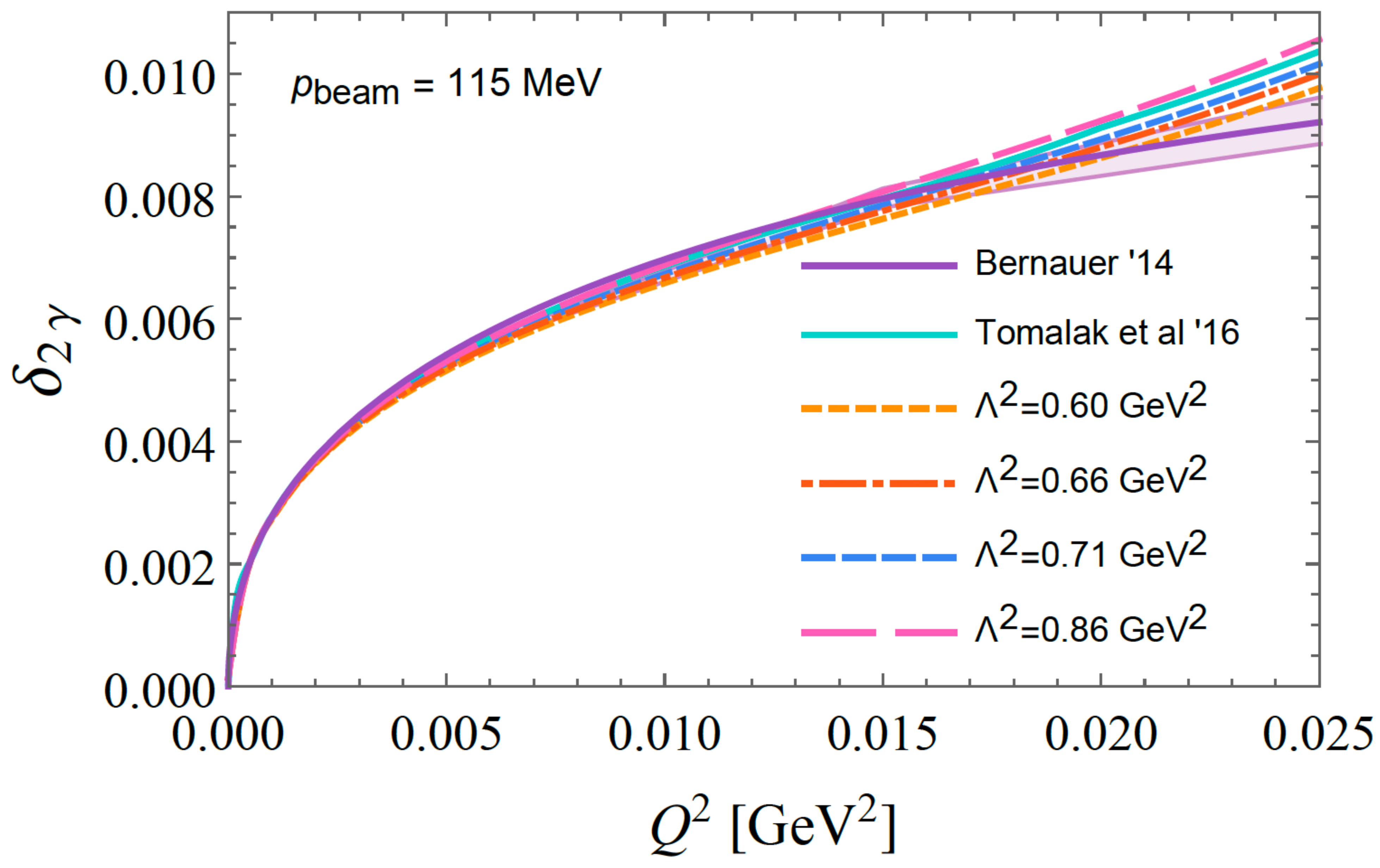
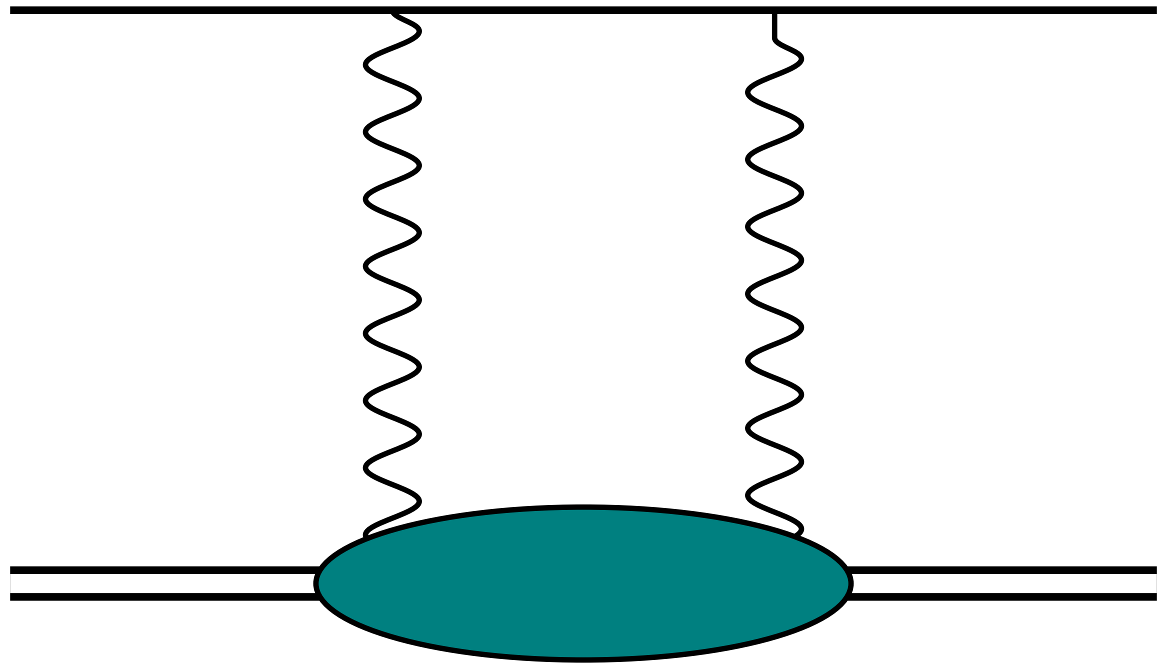
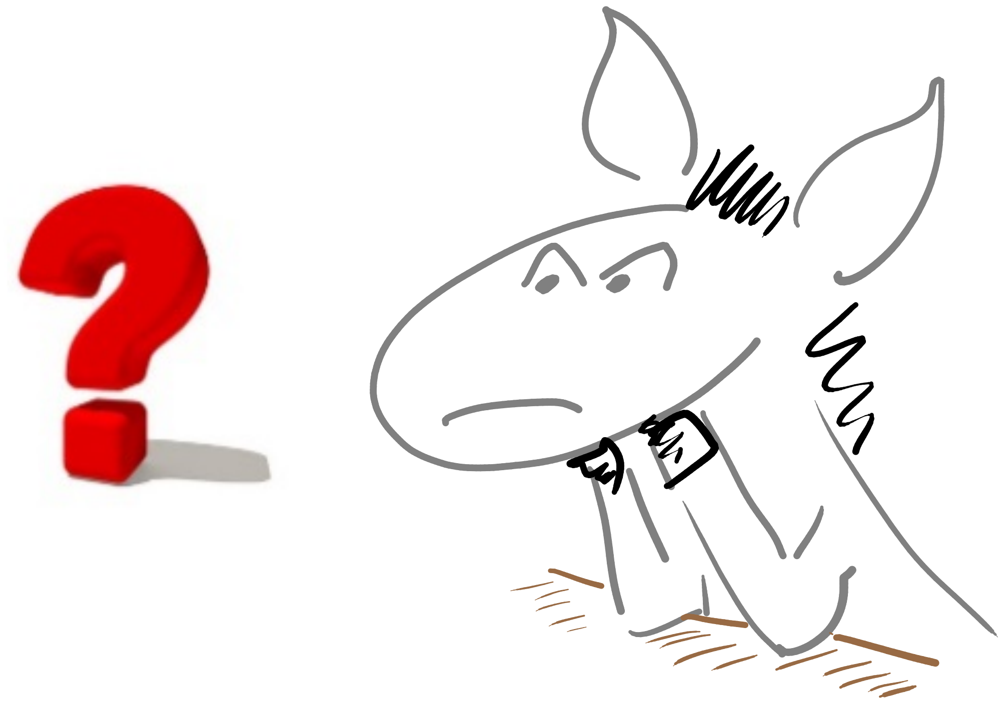


[McMule collaboration (2023), MUSE] $lp \to lp$ https://arxiv.org/abs/2307.16831

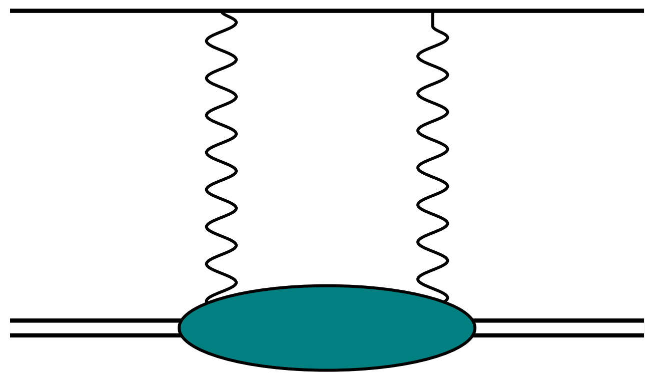


What is new

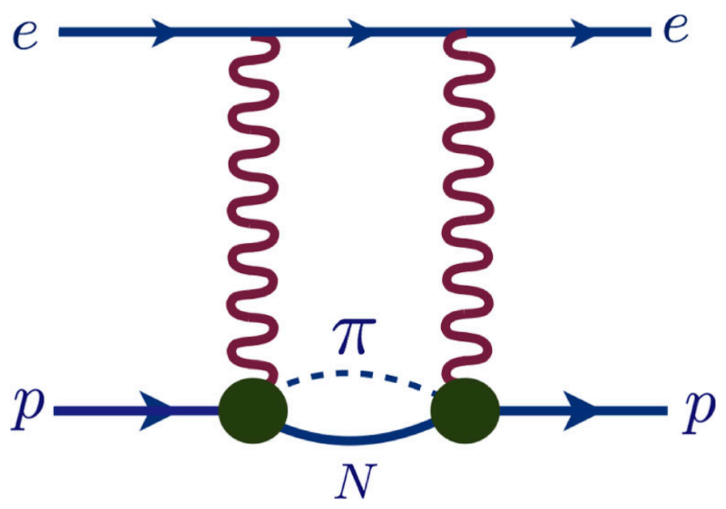


What is new?

Implement State-Of-The-Art TPE models, including inelastic TPE

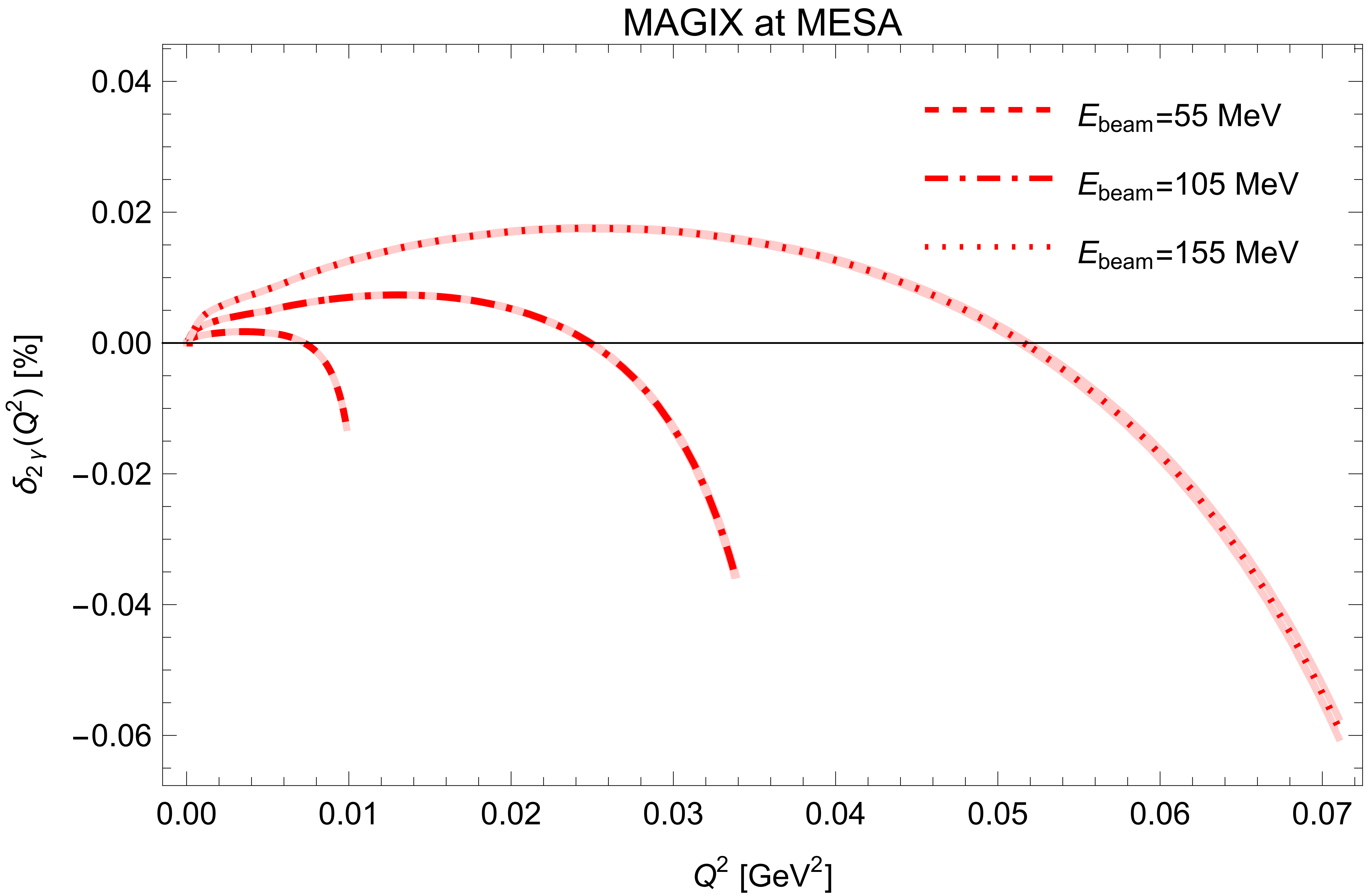


$\lesssim 1\ \mathrm{GeV}^2$ Q^2



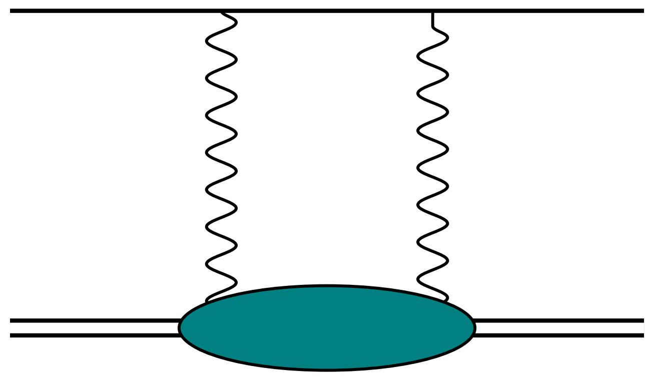
TPE with πN intermediate state

[O. Tomalak, B Pasquini, M. Vandergaeghen (2017)]
https://arxiv.org/abs/1708.03303v2



Outlook on inelastic TPE

Implement State-Of-The-Art TPE models, including inelastic TPE



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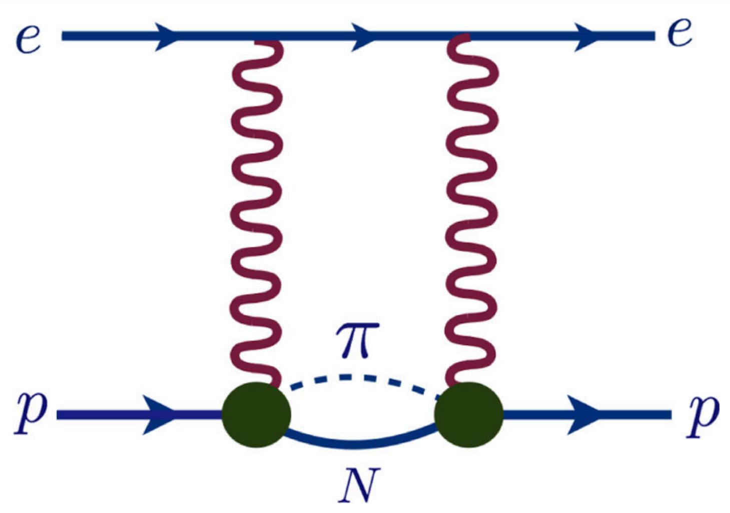
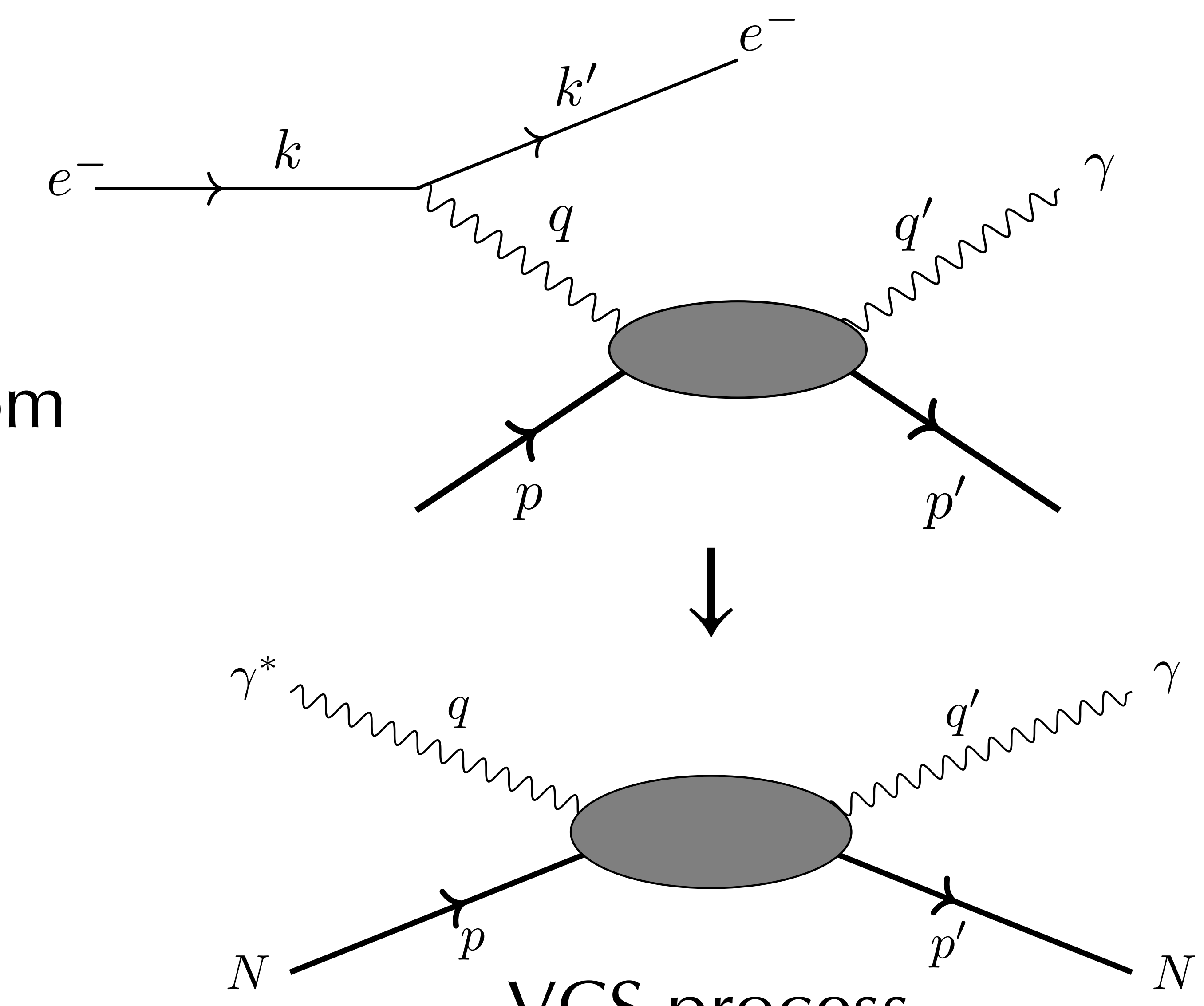


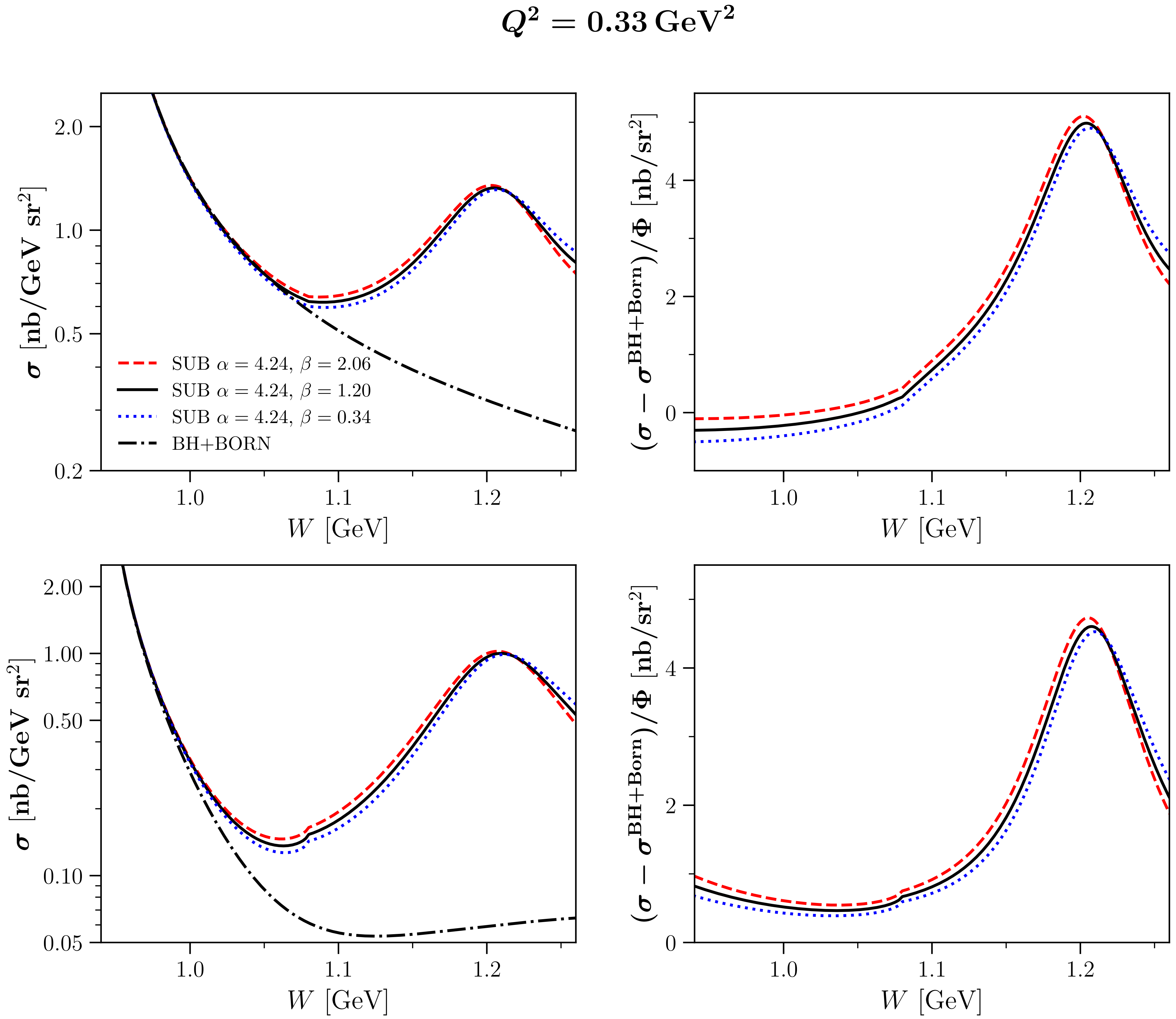
TPE with πN intermediate state



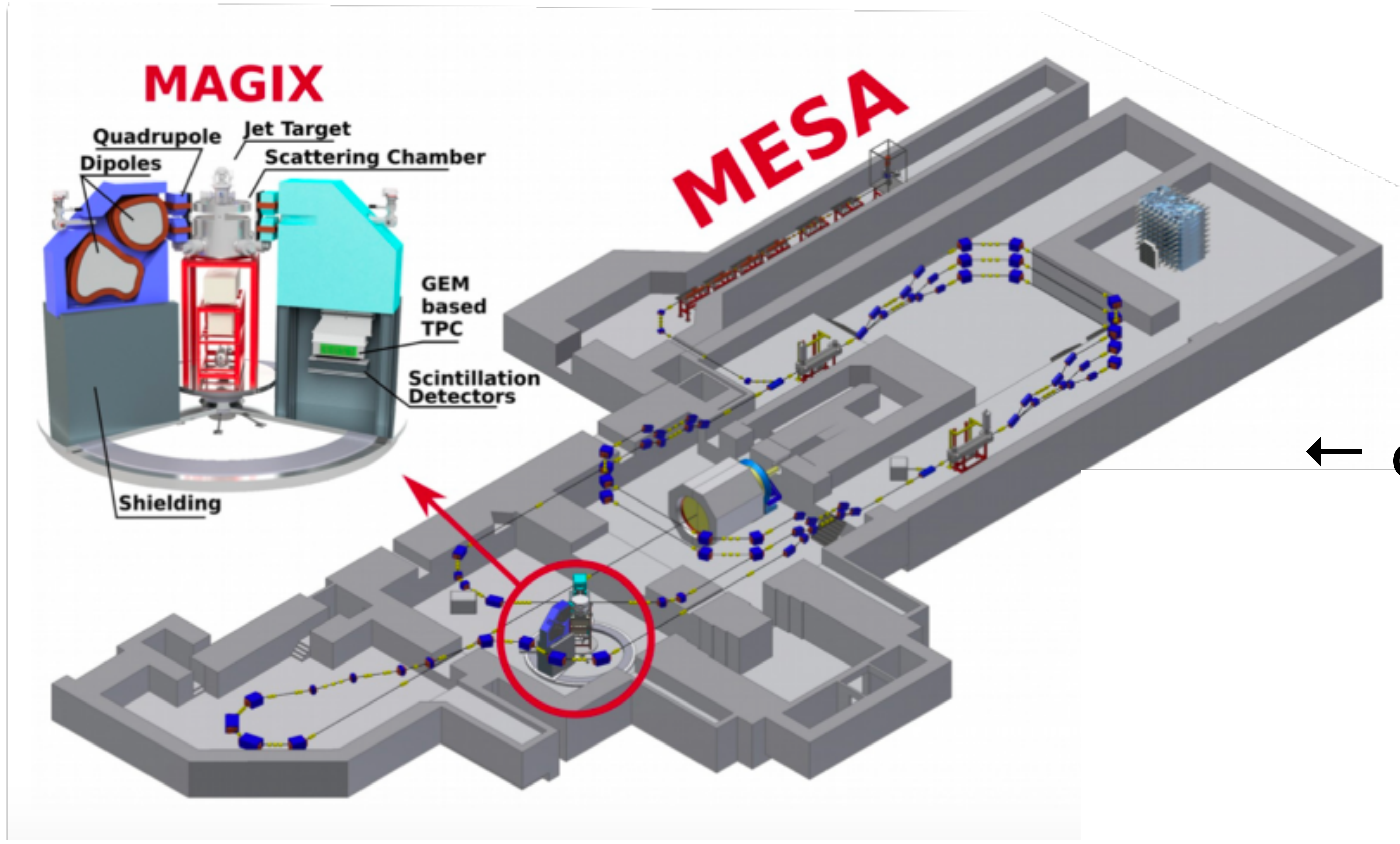
VCS process

Bremsstrahlung can be included from Virtual Compton Scattering (VCS) amplitudes

[I. Danikin, B. Pasquini, M. Ronchi, M. Vanderhaeghen, (2026)]
https://arxiv.org/abs/2603.09831

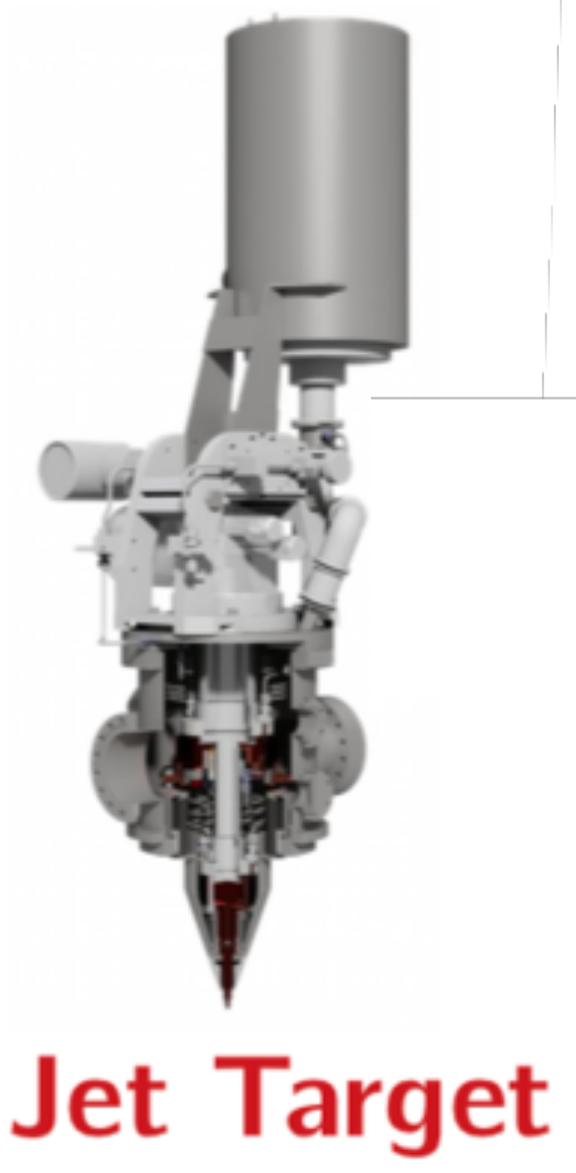


MAGIX@MESA electron scattering



← can provide:

- Low beam energy
- High beam current
- High resolution

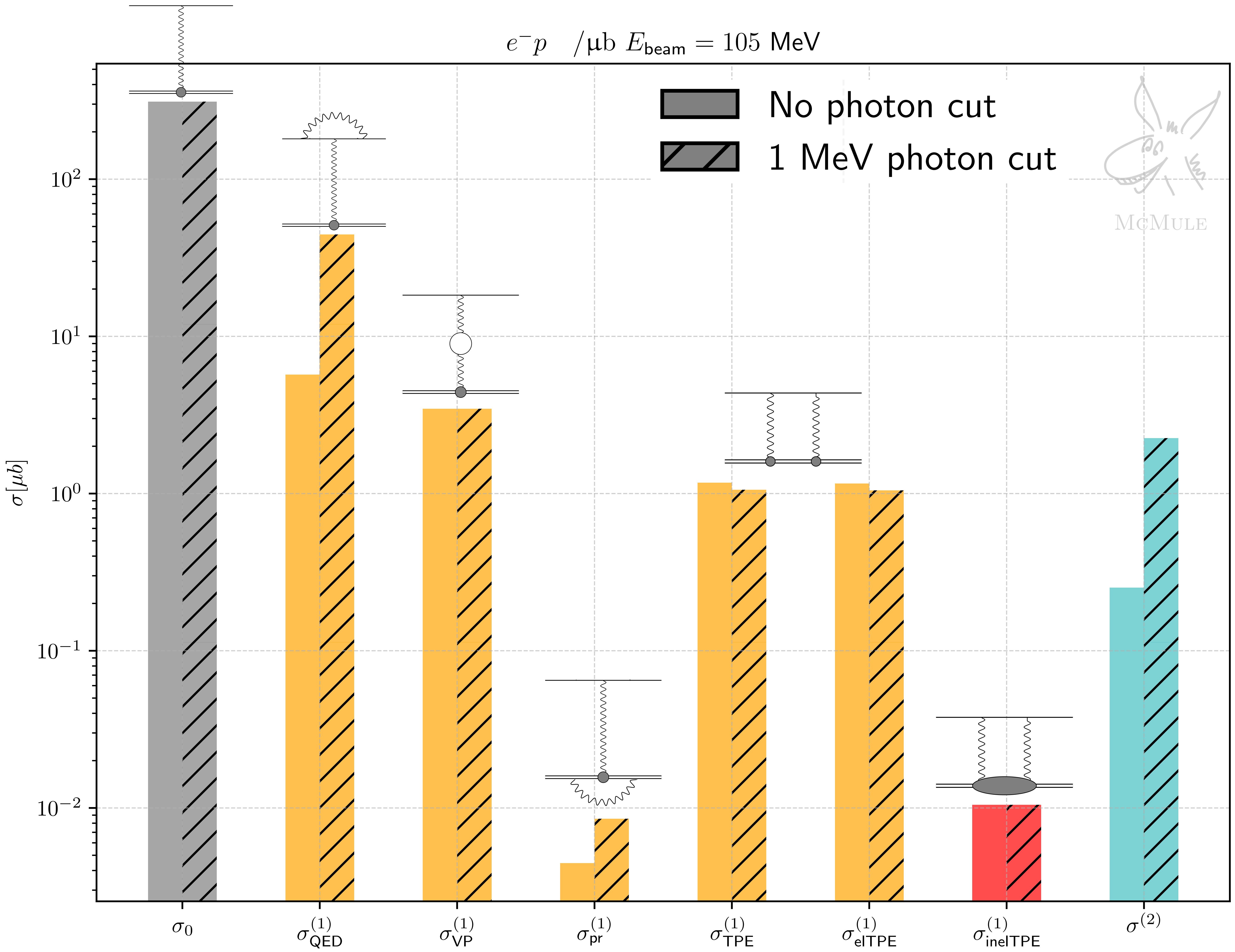


← can provide:

- Proton target
- Deuteron target
- Helium target
- …

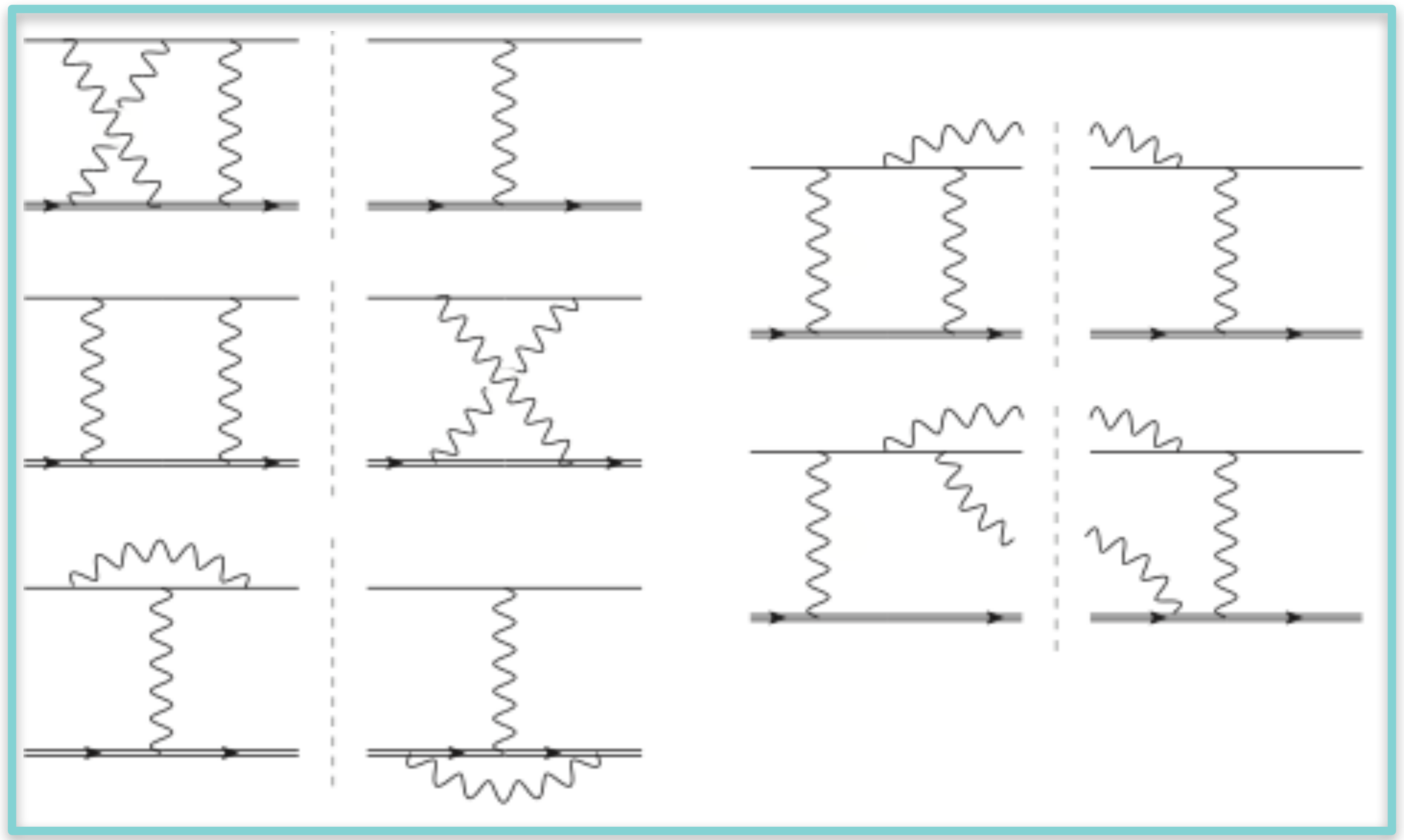
Hosted by the *Institute of Nuclear Physics* of the *Johannes Gutenberg University* of Mainz

Results for MAGIX

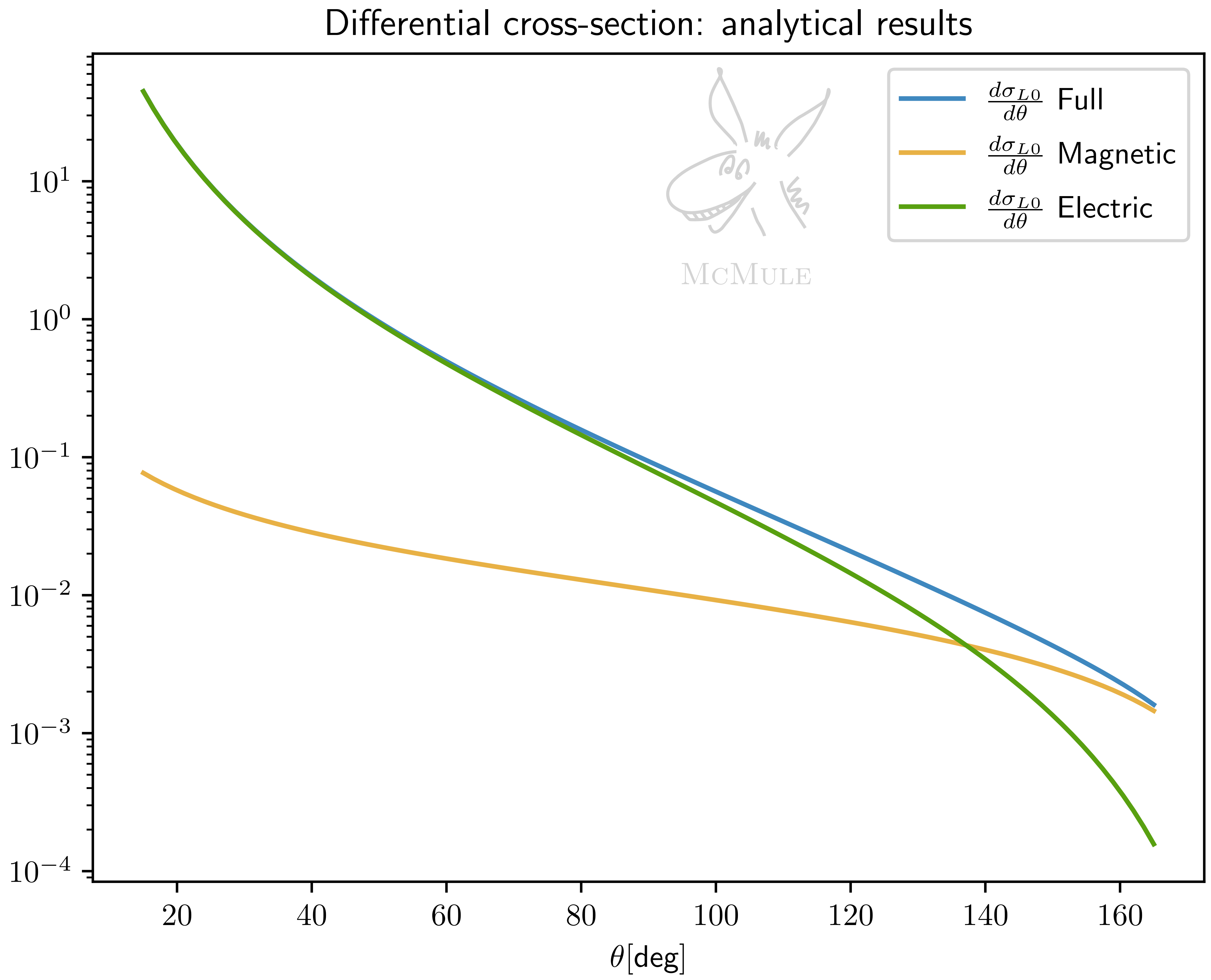


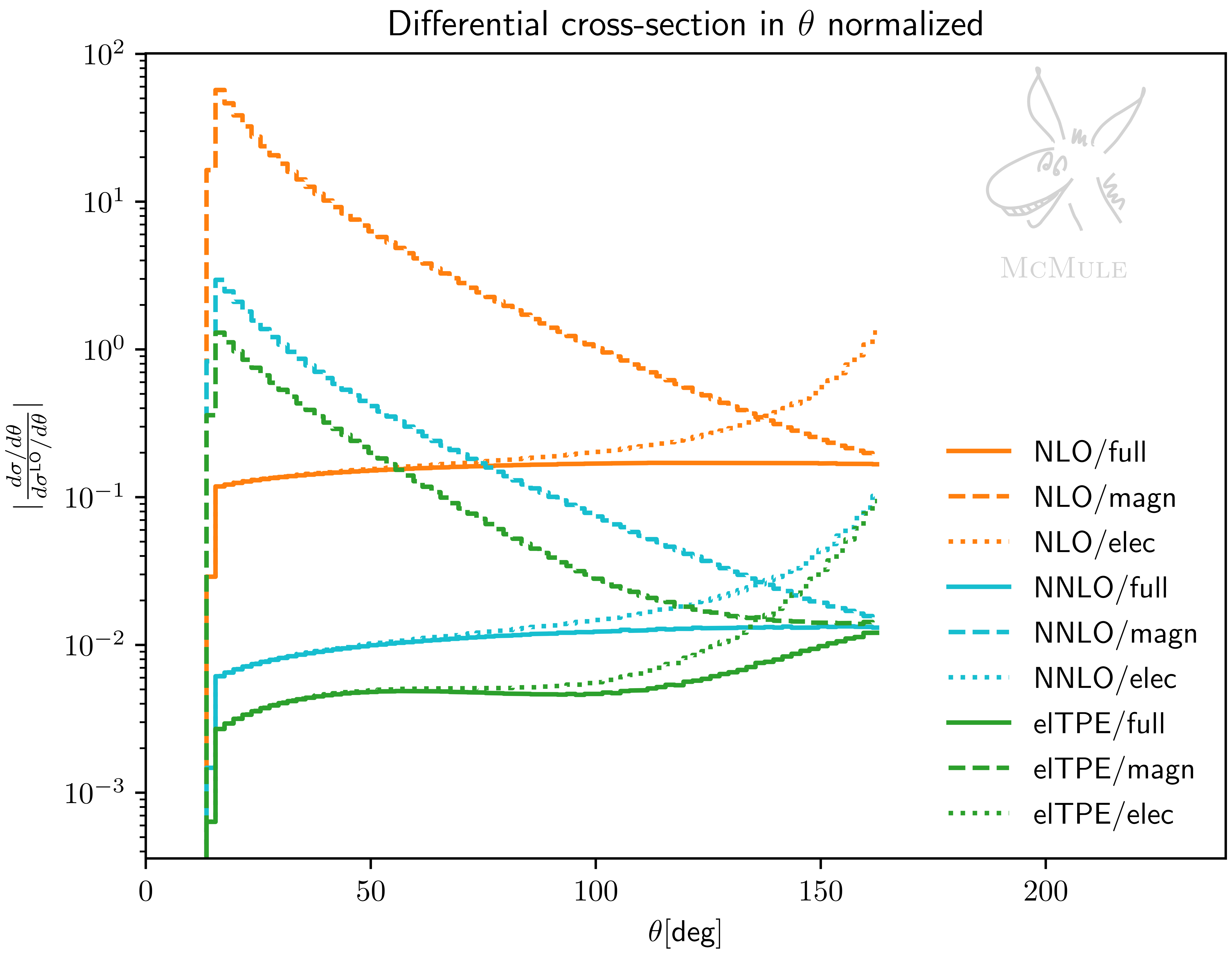
Parameters:

- $E_{\text{beam}} = 105\,\text{MeV}$
- $15^\circ \leq \theta_l \leq 165^\circ$



Results for MAGIX





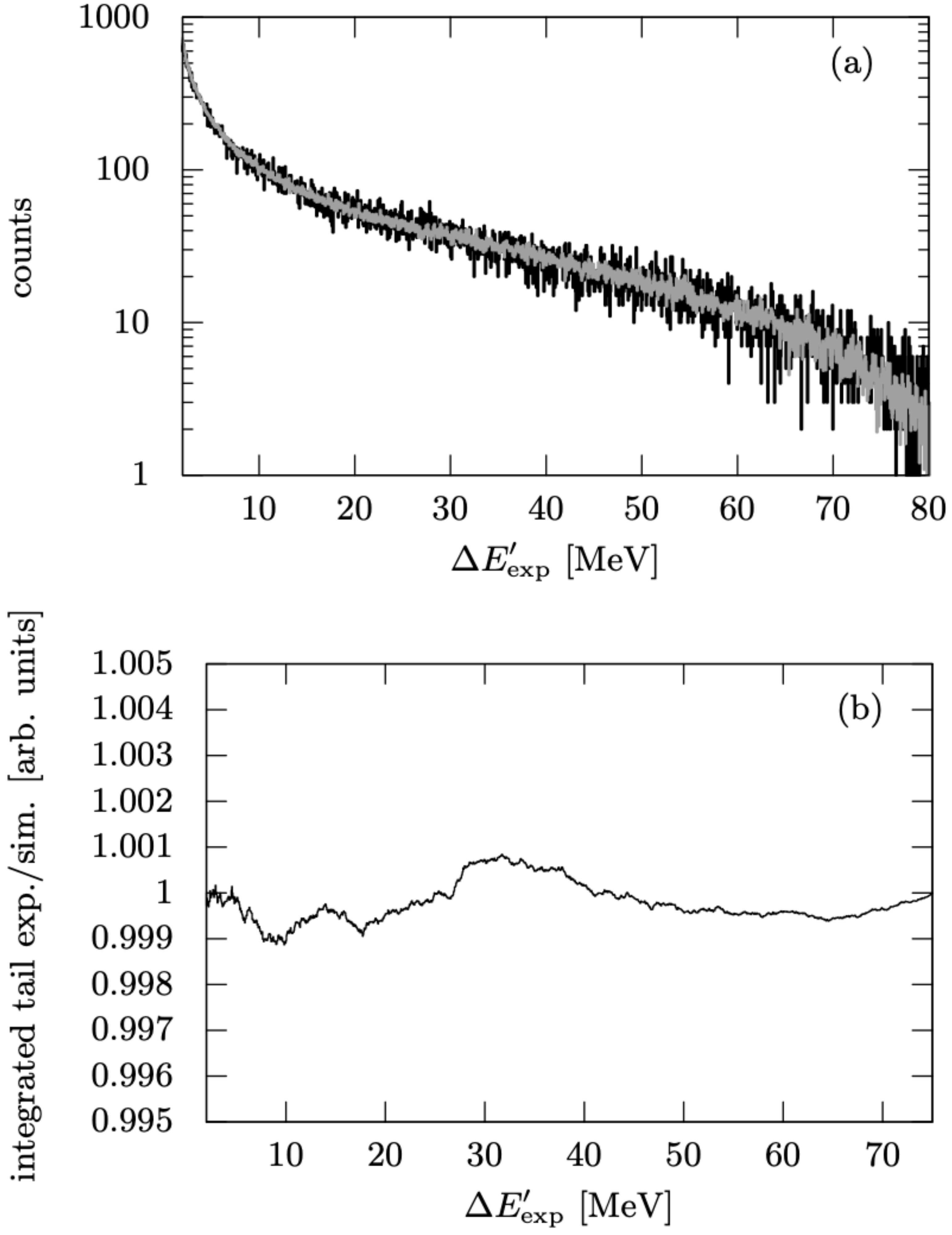
Sensitivity to NNLO: radiative tail

$$\frac{d\sigma^{exp}(E')}{d\sigma^{(0)}\left[1+\sum_{i=1}^{N}\delta^{(i)}(E')\right]}$$

E': scattered electron energy

$i = 1$: next-to-leading order (NLO)
$i = 2$: NNLO

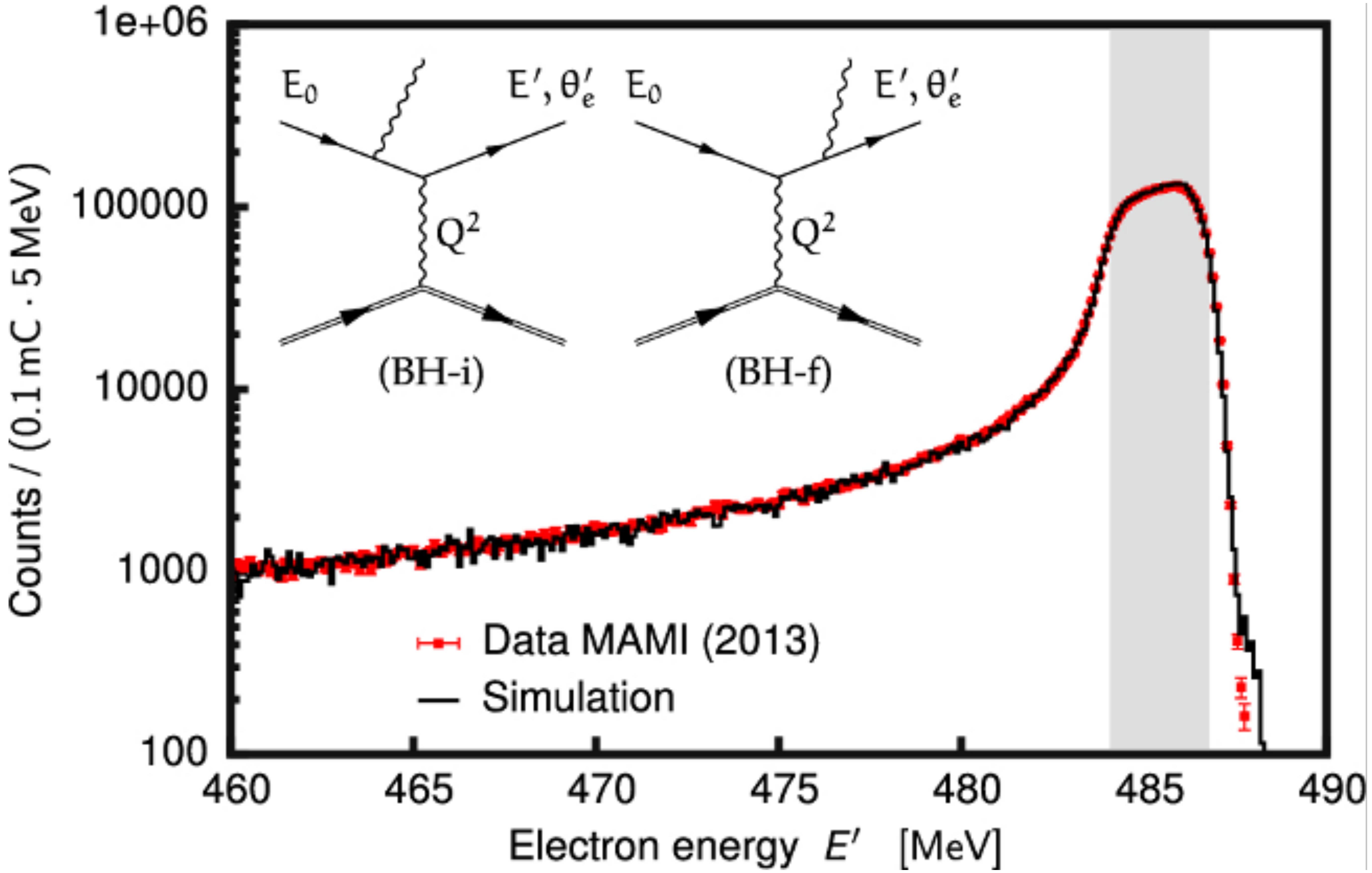
$\Delta E' = E_{elastic} - E'$



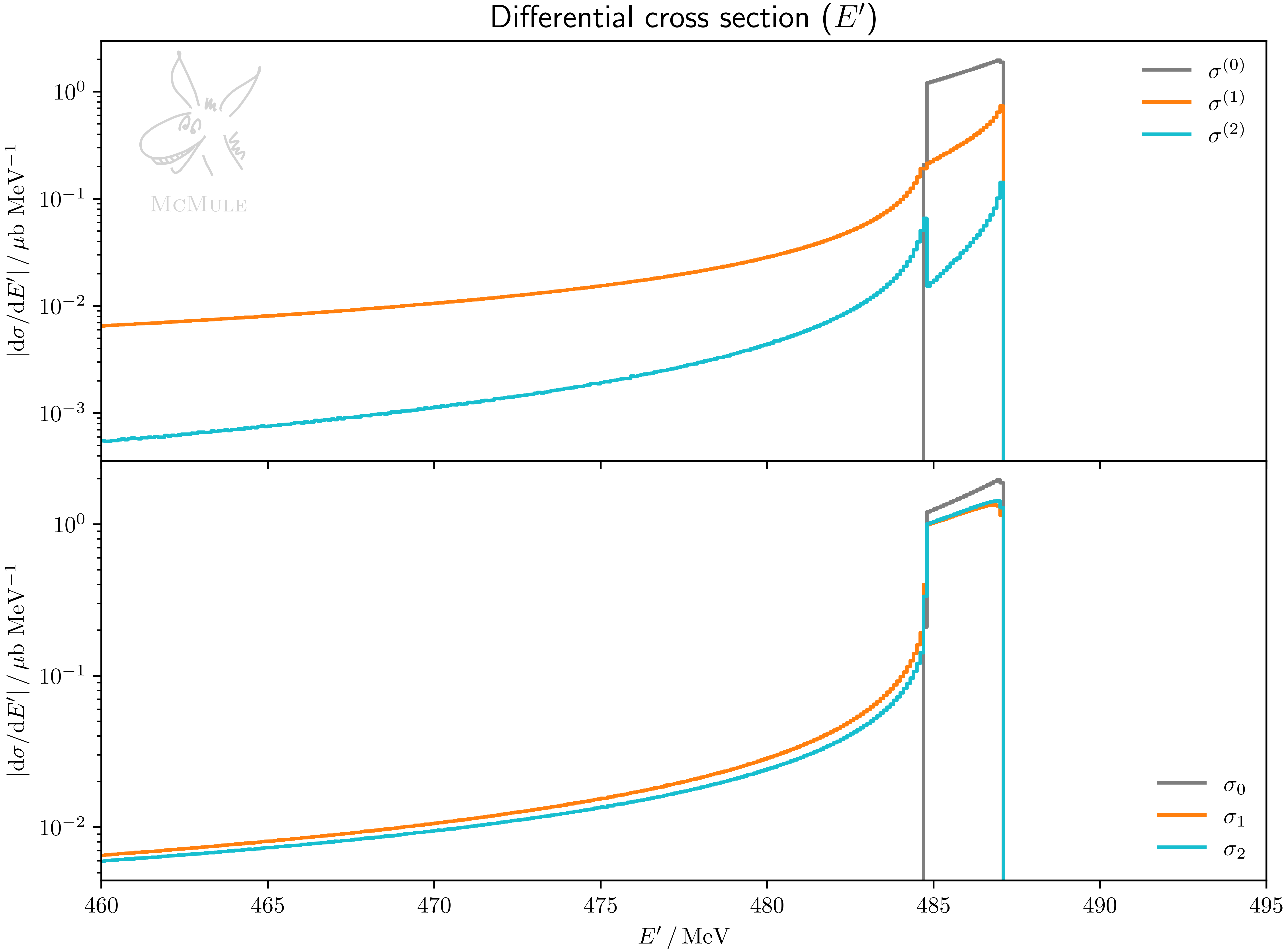
[J.C. Bernauer et al. (2014) https://arxiv.org/abs/1307.6227]

Initial-state radiation technique and radiative tail

Proton charge radius extraction from initial-state radiation (ISR) experiment at MAMI: $E_0 = 495\,\mathrm{MeV}$
[M. Mihovilovič et al. (2021) https://arxiv.org/abs/1905.11182]



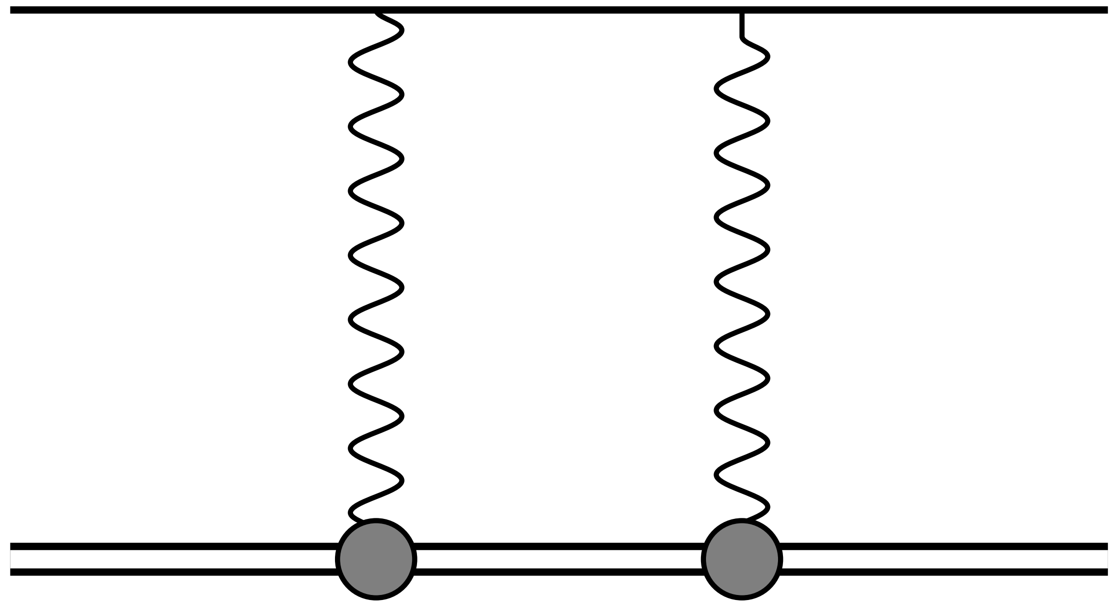
Sensitivity to NNLO: radiative tail



Parameters:

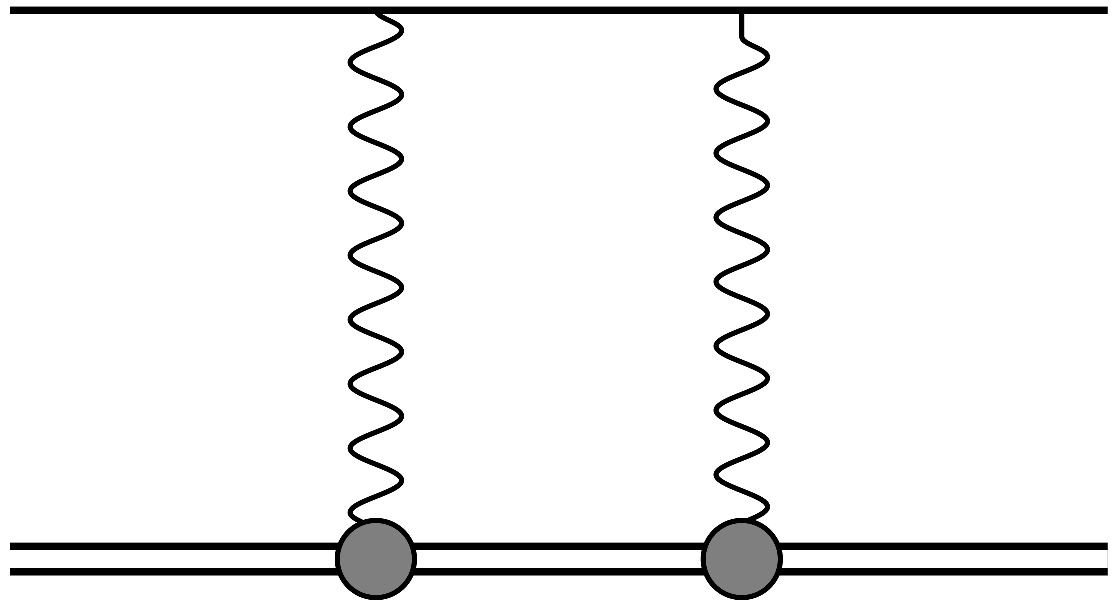
- $E_{\text{beam}} = 495\,\text{MeV}$
- $p_{\text{cut}} = 280\,\text{MeV}$
- $\theta_l = 15^\circ \pm 15\,\text{mrad}$

Iterative analysis of electron scattering data



- ❑ Aim: **self-consistent** extraction of FFs including elastic TPE

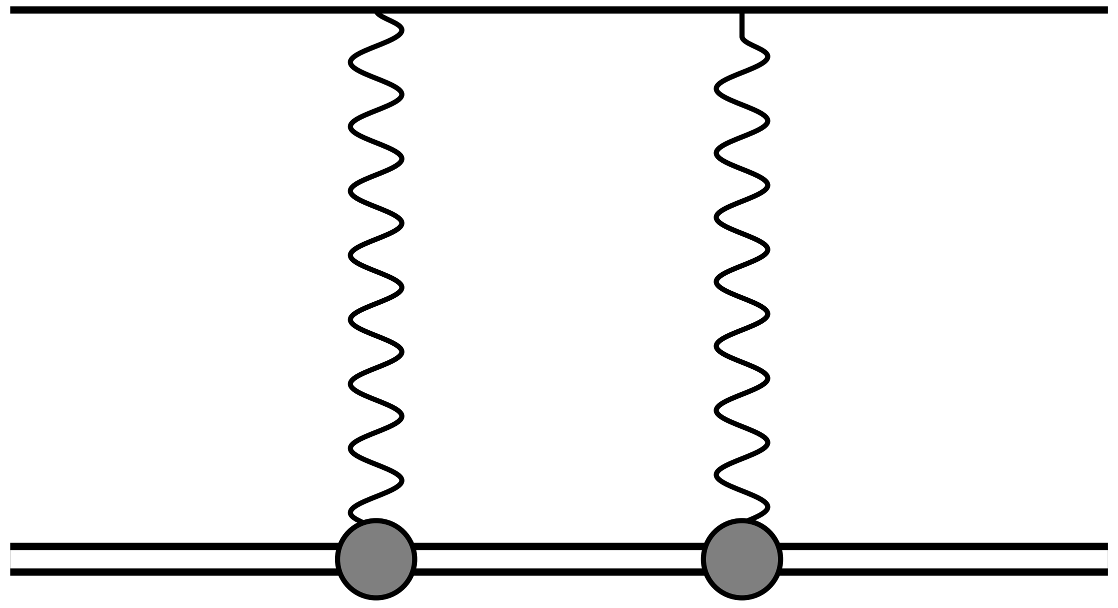
Iterative analysis of electron scattering data



- Aim: **self-consistent** extraction of FFs including elastic TPE

  Provide flexible analytical formulas of TPE (virtual TPE + bremsstrahlung) with general form factor $G_E(\vec{a}, Q^2), G_M(\vec{a}, Q^2)$
  G_E and G_M ansatz: sum of monopoles

Iterative analysis of electron scattering data



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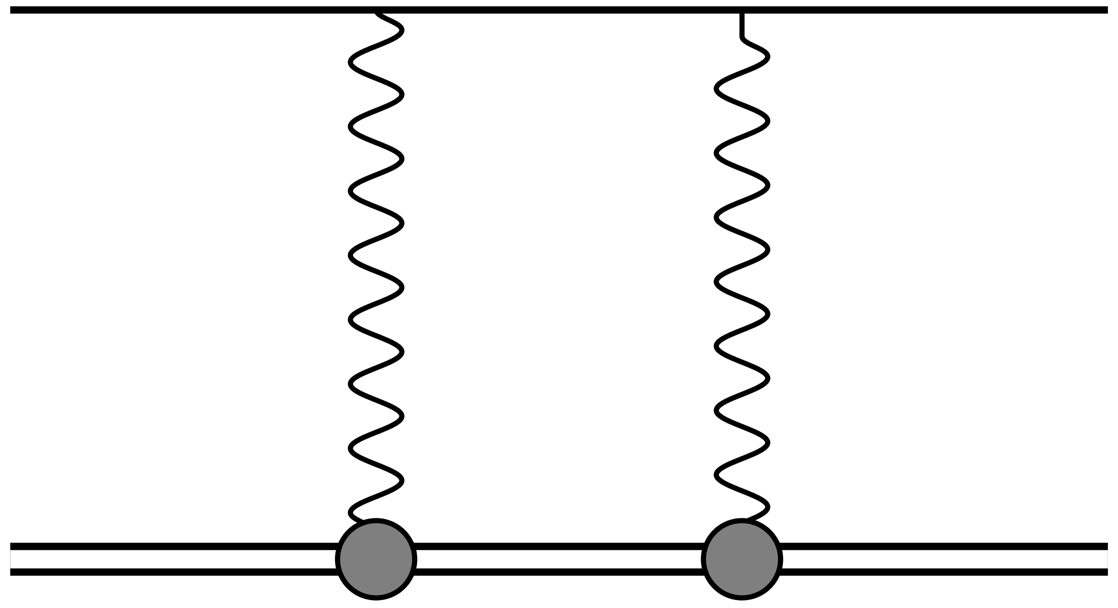
  G_E and G_M ansatz: sum of monopoles

Choose a "reasonable" set of parameters $\vec{a}^{(0)}$ and calculate the radiative corrections, including TPE $\rightarrow$ rad corr $(\vec{a}^{(0)})$

Iterative analysis of electron scattering data



- Aim: **self-consistent** extraction of FFs including elastic TPE

  Provide flexible analytical formulas of TPE (virtual TPE + bremsstrahlung) with general form factor $G_E(\vec{a}, Q^2), G_M(\vec{a}, Q^2)$
  G_E and G_M ansatz: sum of monopoles

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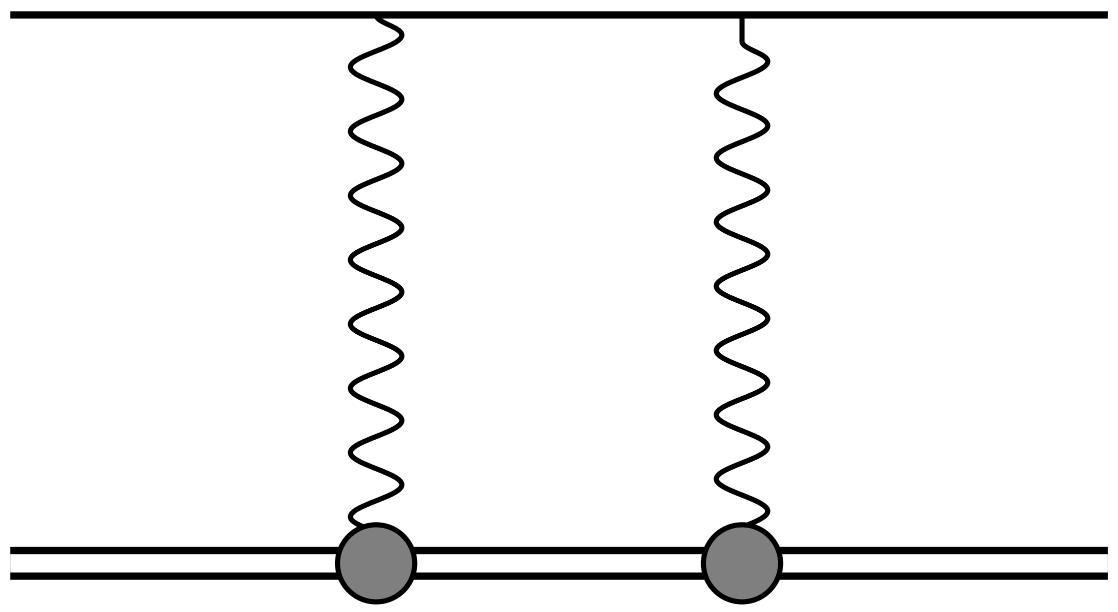
$\rightarrow$ … $\rightarrow$

Iterative analysis of electron scattering data



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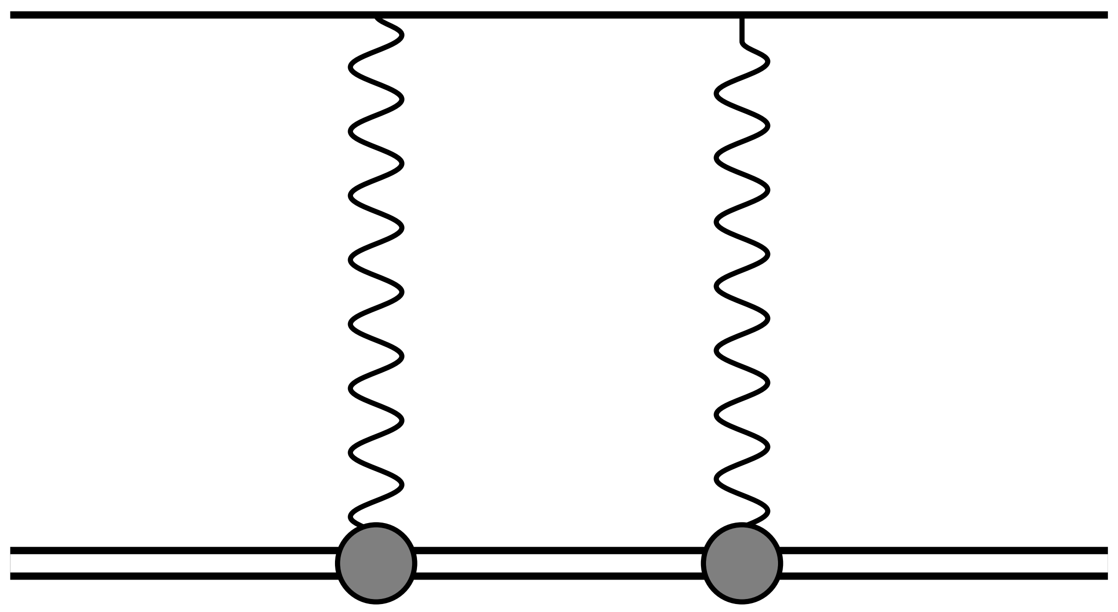
Fit $\vec{a}^{(1)}$ to data

data(1)=
raw data - rad corr $(\vec{a}^{(0)})$
Fit σ_0 to data(1) $\rightarrow \vec{a}^{(1)}$

$\rightarrow$ [ ] $\rightarrow$ … $\rightarrow$ [ ]

Iterative analysis of electron scattering data



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Fit $\vec{a}^{(1)}$ to data

data(1)=
raw data - rad corr $(\vec{a}^{(0)})$
Fit σ_0 to data(1) → $\vec{a}^{(1)}$

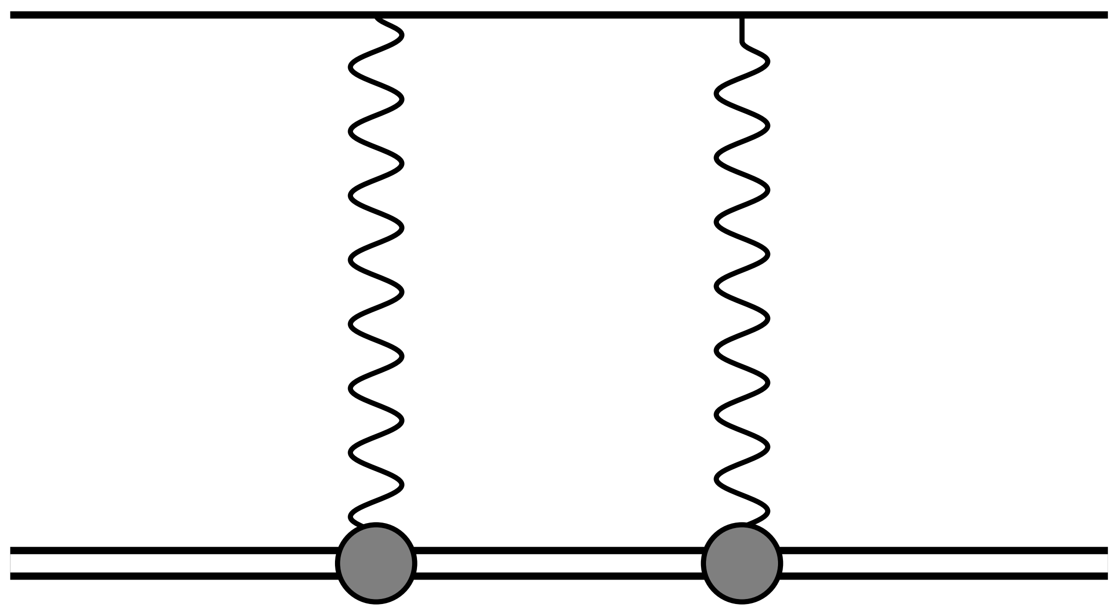
→

Fit $\vec{a}^{(2)}$ to data

data(2)=
raw data - rad corr $(\vec{a}^{(1)})$
Fit σ_0 to data(2) → $\vec{a}^{(2)}$

→ … →

Iterative analysis of electron scattering data



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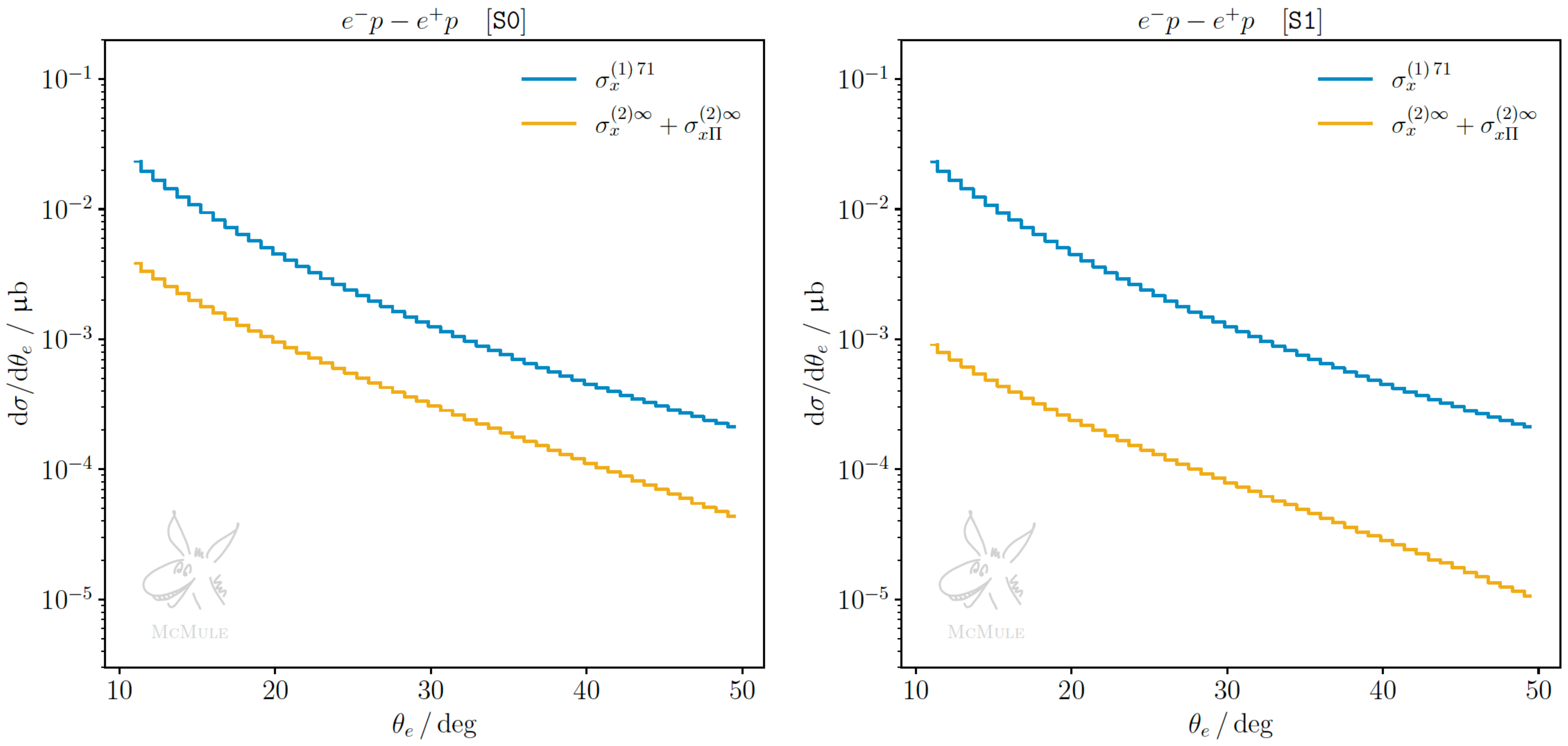
Choose a "reasonable" set of parameters $\vec{a}^{(0)}$ and calculate the radiative corrections, including TPE → rad corr ($\vec{a}^{(0)}$)

Fit $\vec{a}^{(1)}$ to data		Fit $\vec{a}^{(2)}$ to data				Fit $\vec{a}^{(i)}$ to data
data(1)= raw data - rad corr ($\vec{a}^{(0)}$) Fit σ_0 to data(1) → $\vec{a}^{(1)}$	→	data(2)= raw data - rad corr ($\vec{a}^{(1)}$) Fit σ_0 to data(2) → $\vec{a}^{(2)}$	→	…	→	data(i)= raw data - rad corr ($\vec{a}^{(i-1)}$) Fit σ_0 to data(i) → $\vec{a}^{(i)}$

Iteration until: FFs don't change anymore within (experimental) uncertainty

Also positrons as a probe



[S0]:

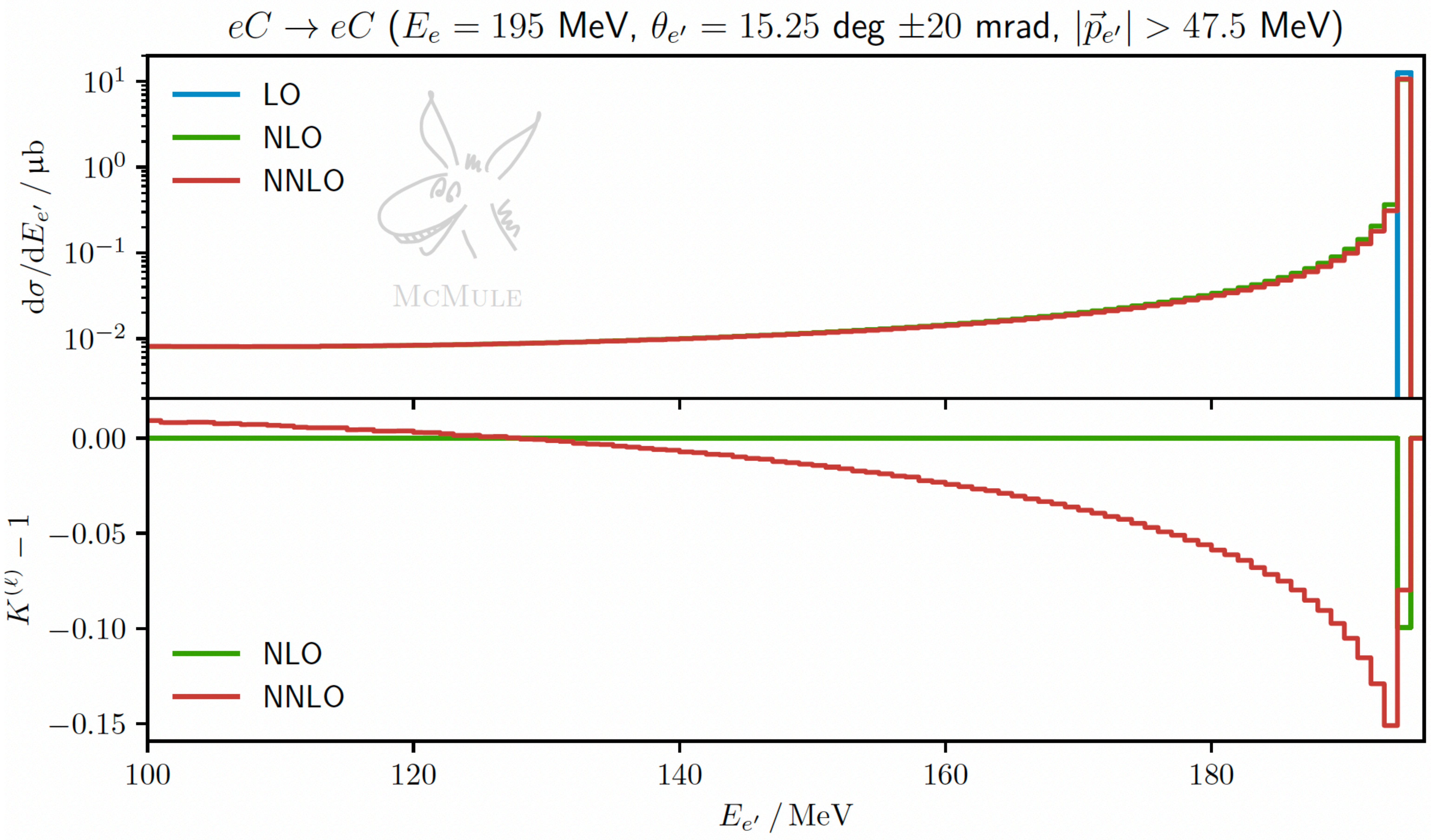
- $p_{\text{beam}} = 210\,\text{MeV}$
- $20^\circ \leq \theta_l \leq 100^\circ$

[S1]:

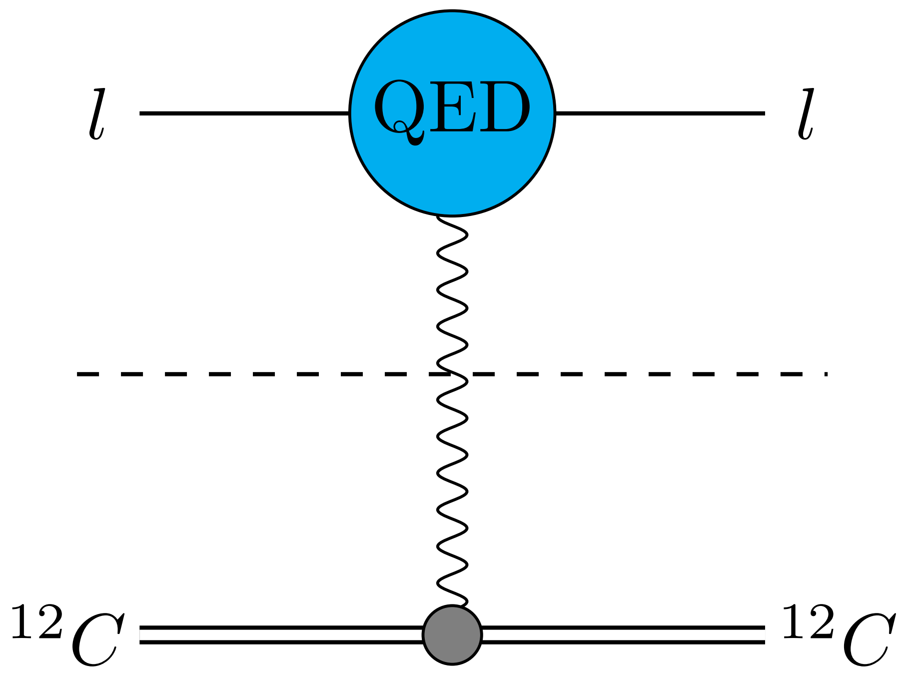
- $p_{\text{beam}} = 210\,\text{MeV}$
- $20^\circ \leq \theta_l \leq 100^\circ$
- $\Delta E_\gamma = 0.4\,p_{\text{beam}}$

[McMule collaboration (2023), MUSE] $lp \to lp$ https://arxiv.org/abs/2307.16831

Outlook for any target: we can do e.g. ^{12}C, ^{208}Pb

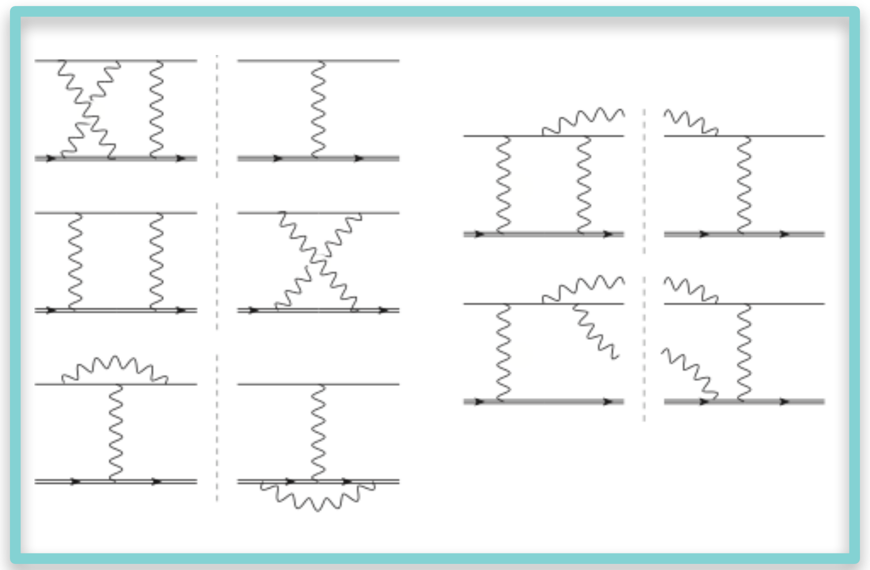


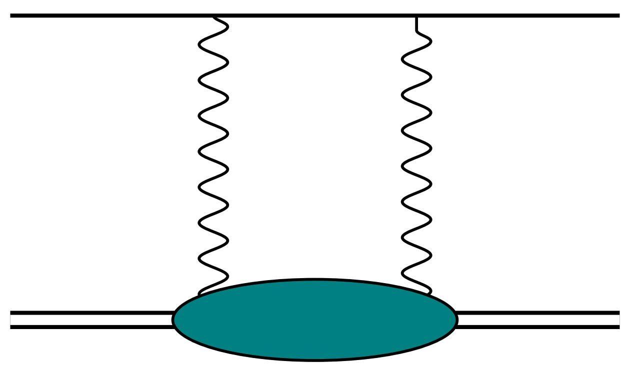
[M. Rocco's plot]



Outlook and conclusions

McMule is a framework for fully differential higher-order QED correction
Initially, it worked only with leptons → fully NNLO QED corrections with no approximation





- NNLO QED correction also for $lp \to lp$
- Inelastic TPE implemented

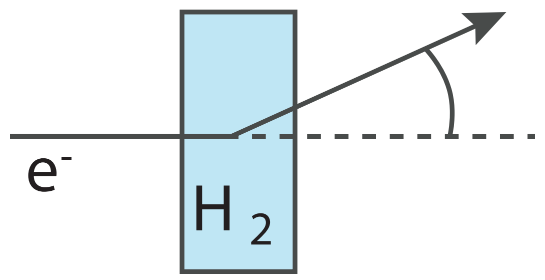
- Self consistent extraction including elastic TPE
- Polarized observables?

- Arbitrary spin extension implemented for "elastic" contributions → form factor input needed

from left to right: Franziska Hagelstein, Sara Gündogdu, Yizhou Fang, Franziska Rauscher, Antonio Coutinho, Georgios Billis, Marco Rocco, Sophie Kollatzsch, Adrian Signer, Yannick Ulrich, Matteo Ronchi, David Radic

Any questions?

Charge radii and form factors (FFs)



e^--p scattering

- Form factors (FF): Fourier transform of charge and magnetization distributions:

$$\rho_{\mathrm{E,M}}(r) = \int \frac{d\vec{q}}{(2\pi)^3} G_{\mathrm{E,M}}(\vec{q}^2) e^{-i\vec{q}\cdot\vec{r}}$$

- Charge radius:

$$< r^2 >_{\mathrm{E}} = \int d\vec{r}\, r^2 \rho_{\mathrm{E}}(\vec{r}) = -6 \left(\frac{\mathrm{d}}{\mathrm{d}Q^2} G_{\mathrm{E}}(Q^2) \right)_{|Q^2=0}$$

- Extraction of the charge radius from electron scattering requires extrapolation of FF data to zero momentum transfer

Integrate cross-section: MUSE kinematics

		$\sigma/\mu b$ [S0]				$\sigma/\mu b$ [S1]			
		$\Lambda\infty$	$\Lambda 60$	$\Lambda 71$	$\Lambda 86$	$\Lambda\infty$	$\Lambda 60$	$\Lambda 71$	$\Lambda 86$
σ_0		40.6564	38.5302	39.0482	39.5432	40.6564	38.5302	39.0482	39.5432
$\sigma_e^{(1)}$		6.3603	6.3721	6.3705	6.3687	0.9438	0.9735	0.9672	0.9610
$\sigma_x^{(1)}$	+	−0.1931	−0.1526	−0.1609	−0.1696	−0.1924	−0.1520	−0.1603	−0.1689
	−	0.1931	0.1526	0.1609	0.1696	0.1924	0.1520	0.1603	0.1689
$\sigma_p^{(1)}$		−0.0020				−0.0020			
$\sigma_\Pi^{(1)}$		0.5878	0.5554	0.5634	0.5711	0.5878	0.5554	0.5634	0.5711
$\sigma_e^{(2)}$		−0.0134		−0.0080		−0.0102		−0.0049	
$\sigma_x^{(2)}$	+	−0.0240				−0.0009			
	−	0.0279				0.0049			
$\sigma_p^{(2)}$		−0.0000				−0.0000			
$\sigma_{e\Pi}^{(2)}$		0.0540		0.0542		0.0094		0.0098	
$\sigma_{x\Pi}^{(2)}$	+	−0.0046				−0.0046			
	−	0.0046				0.0046			
$\sigma_{p\Pi}^{(2)}$		−0.0001				−0.0001			

[S0]:
- $p_{\text{beam}} = 210\,\text{MeV}$
- $20^\circ \leq \theta_l \leq 100^\circ$

[S1]:
- $p_{\text{beam}} = 210\,\text{MeV}$
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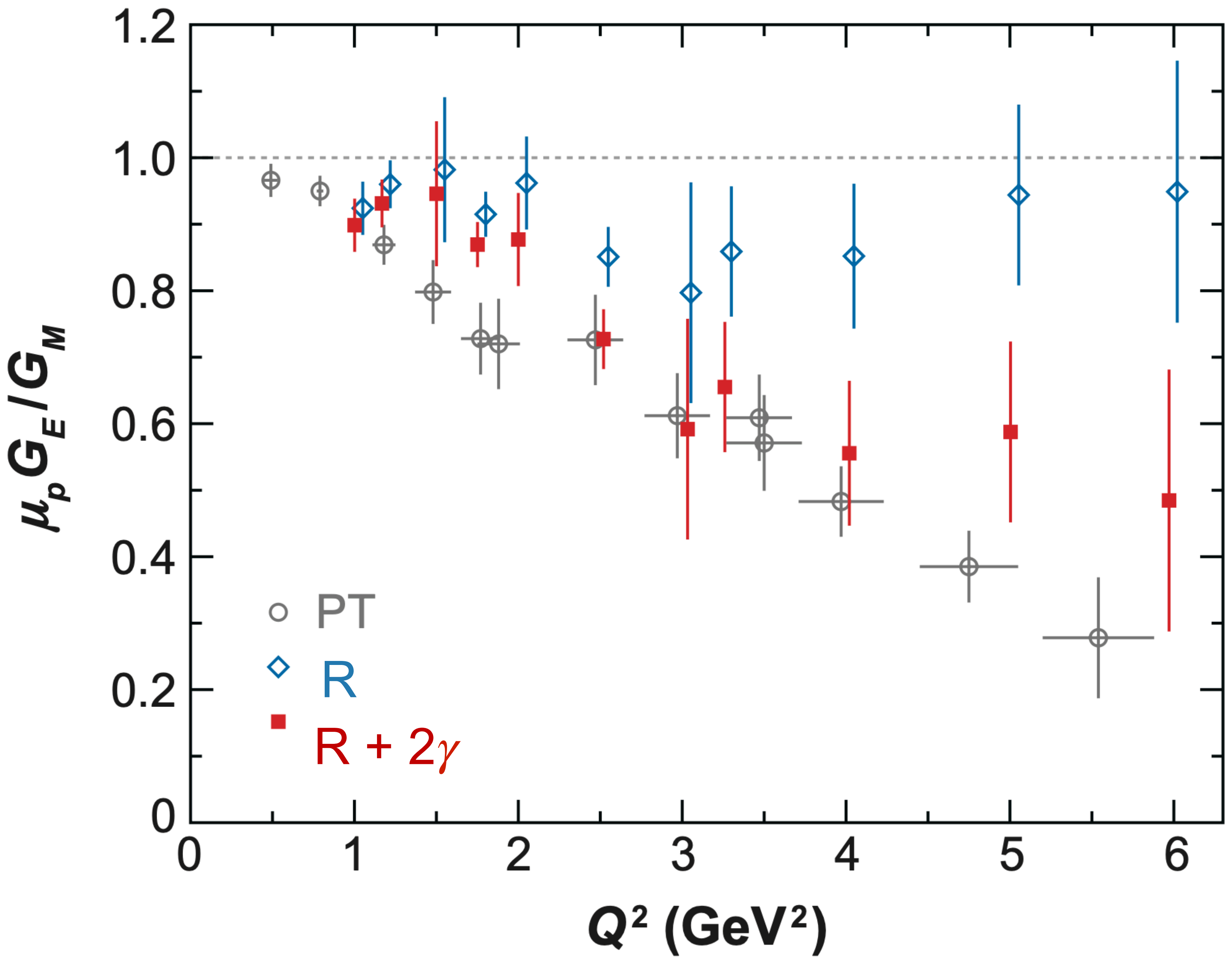
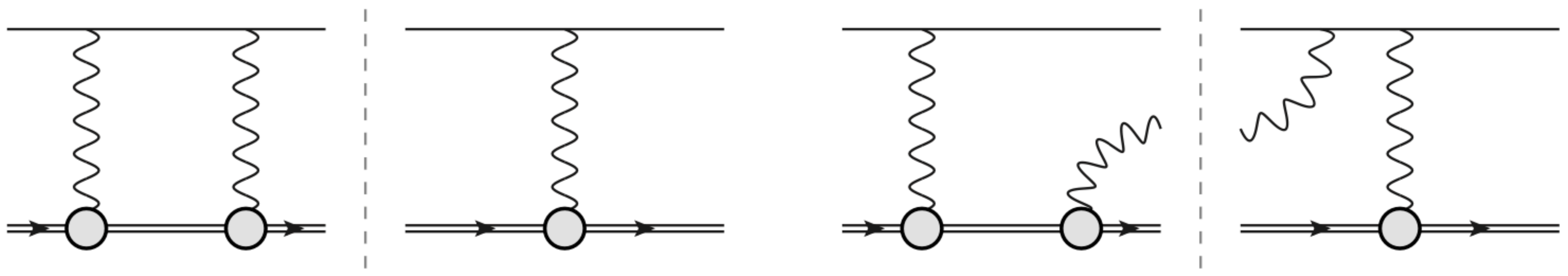
[McMule collaboration (2023), MUSE] $lp \to lp$ https://arxiv.org/abs/2307.16831

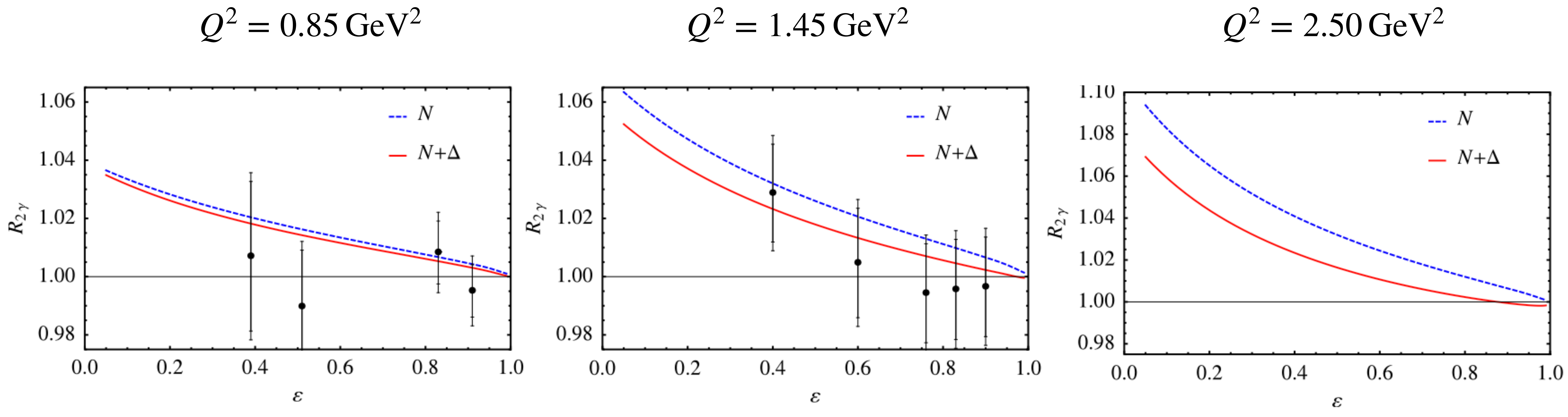
Importance of two-photon-exchange TPE

Presently in McMule only simple Born TPE model



[C E. Carlson, M. Vanderhaeghen (2007)] https://arxiv.org/abs/hep-ph/0701272





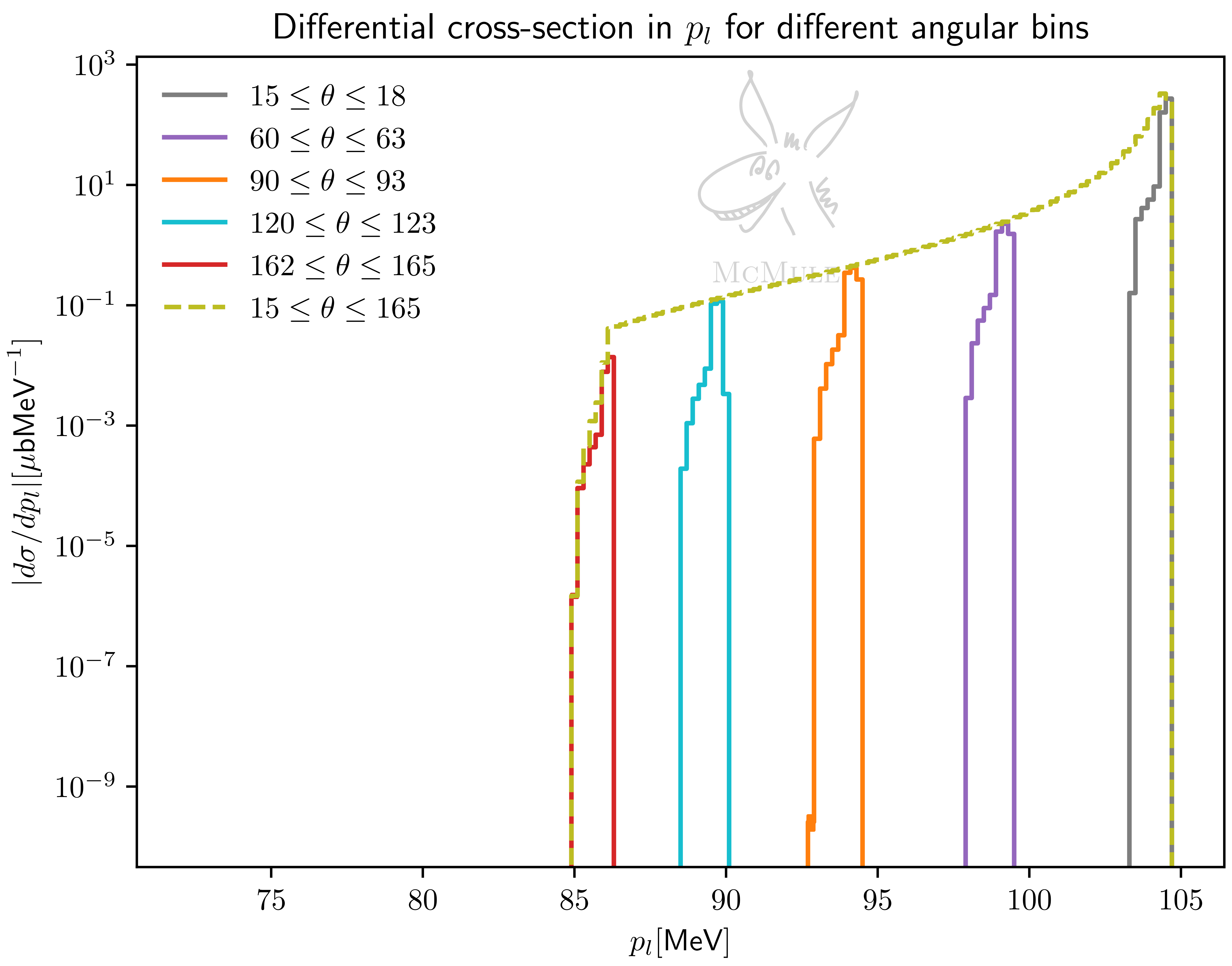
[A. Afanasev, P.G. Blunden, D. Hasell, B.A. Raue (2017)]
https://arxiv.org/abs/1703.03874

In principle fit the all corrections

Provide tpe (flexible model) in a way that could be used in an online analysis

In principle in some r.c. FF can be factored out and included in the fit

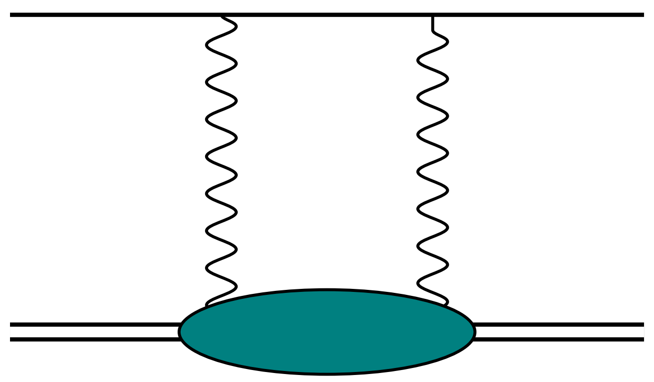
Result for MAGIX

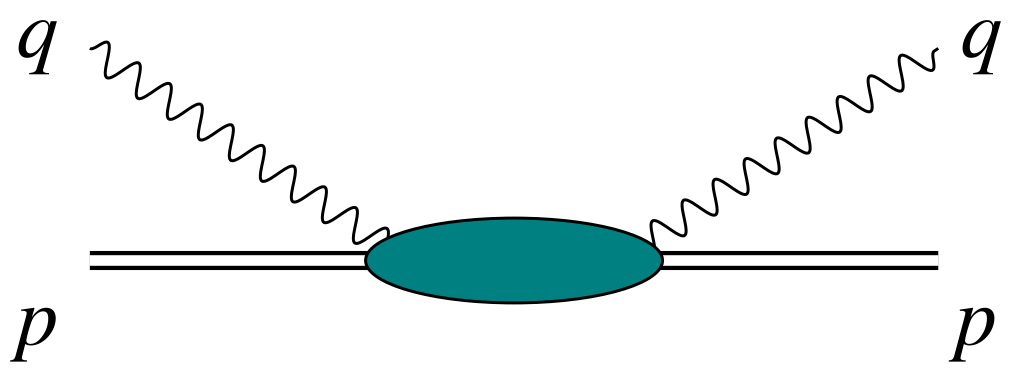


State-of-the-art TPE models

Implement State-of-the-art TPE models, including inelastic TPE

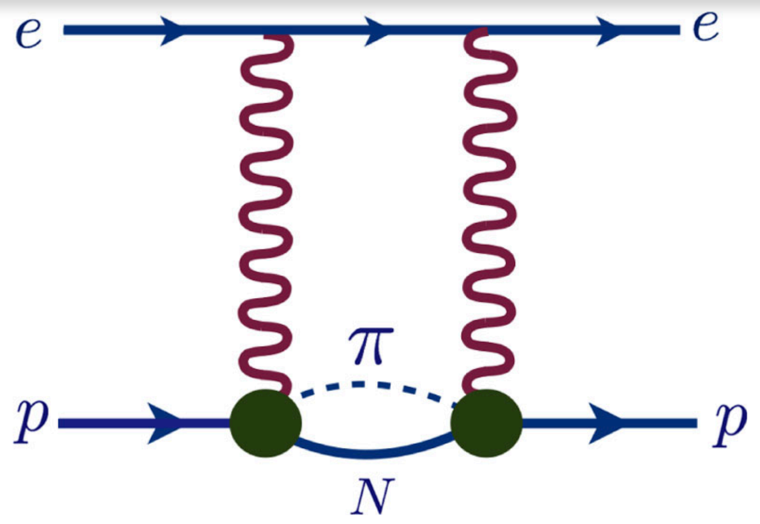
$\lesssim 0.1\ \mathrm{GeV}^2$ $\lesssim 1\ \mathrm{GeV}^2$ $\lesssim 5\ \mathrm{GeV}^2$ Q^2





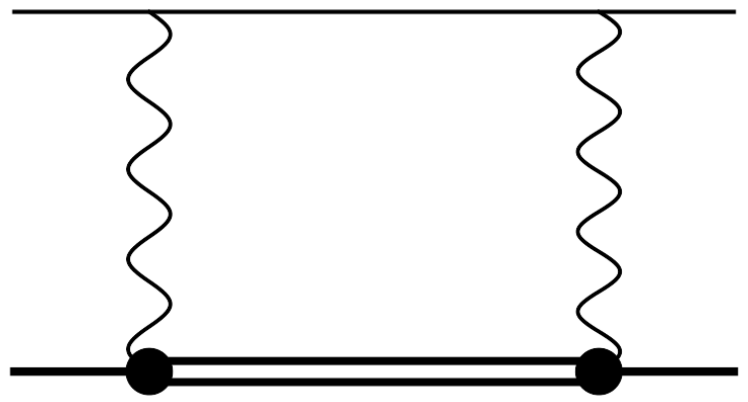
Near-forward approximation through VVCS

[O. Tomalak, M. Vanderhaeghen (2016)]
https://arxiv.org/abs/1508.03759



TPE with πN intermediate state

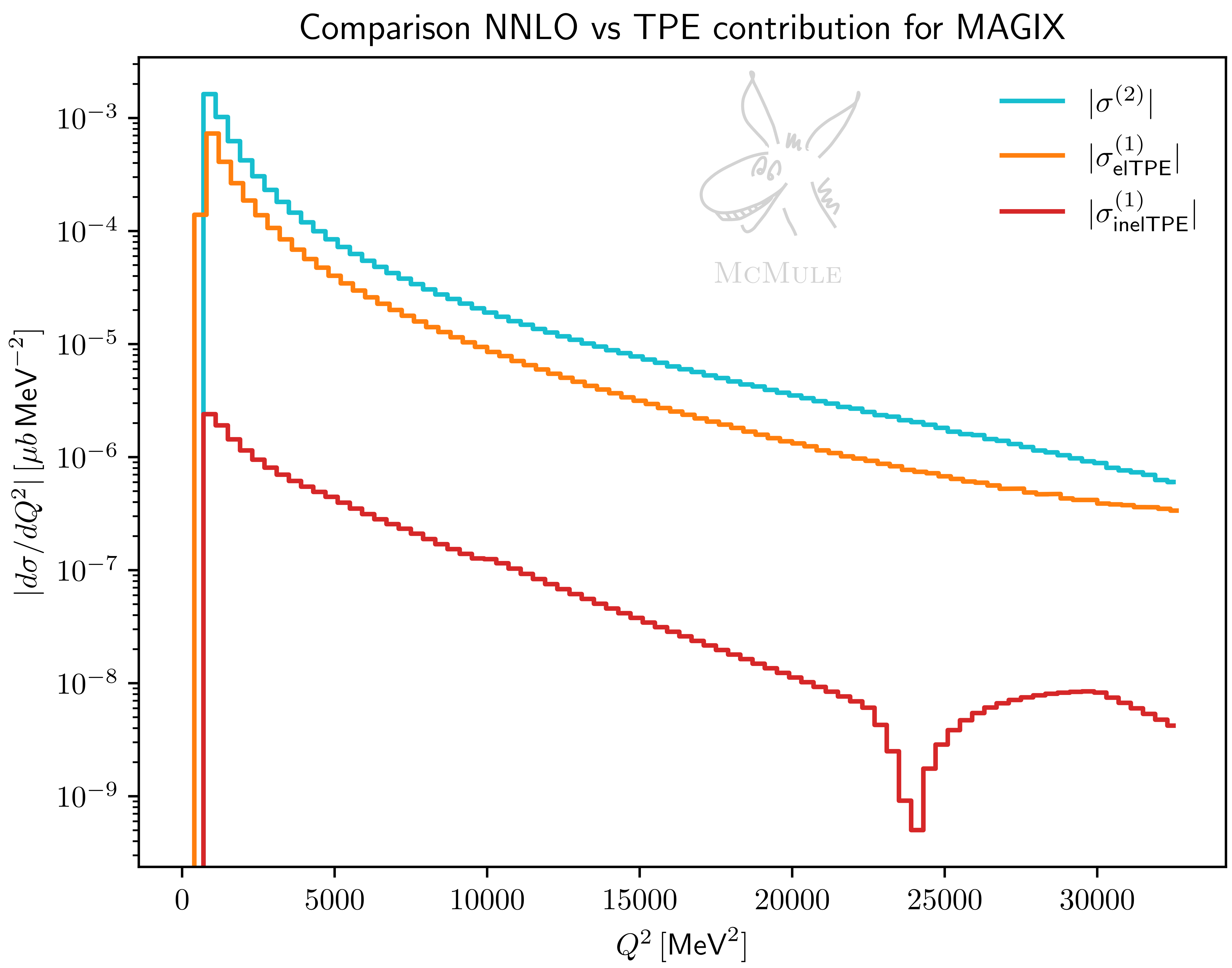
[O. Tomalak, B Pasquini, M. Vandergaeghen (2017)]
https://arxiv.org/abs/1708.03303v2



TPE from intermediate state resonances

[J. Ahmed, P.G. Blunden, W. Meltnitchouk (2020)]
https://arxiv.org/abs/2006.12543

NNLO vs TPE



Comparison NNLO vs TPE contribution for MAGIX
$|\sigma^{(2)}|$
$|\sigma^{(1)}_{\text{elTPE}}|$
$|\sigma^{(1)}_{\text{inelTPE}}|$
McMule
$|d\sigma/dQ^2|$ [μb MeV^{-2}]
Q^2 [MeV^2]
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0
5000
10000
15000
20000
25000
30000

process	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO$+$
$\ell p \to \ell p$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO
$eN \to eN$	PRad, ULQ2	background	$+$
$e^-e^- \to e^-e^-$	Prad 2	normalisation	NNLO
	MOLLER, ...	$\sin^2\theta_W$ at low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \ell\ell$	VEPP, BES, Daphne, ...	R-ratio	NNLO$\pm$
	Belle	τ properties	
$ee \to \gamma\gamma$	Daphne	dark searches	NNLO$-$
	any e^+e^- collider	luminosity measurement	
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO$-$
$\mu \to \nu\bar{\nu}e$	MEG	ALP searches	NNLO$+$
	DUNE	beam-line profiling	
$\mu \to \nu\bar{\nu}e\gamma$	MEG, Mu3e, Pioneer	background	NLO
$\mu \to \nu\bar{\nu}eee$	MEG, Mu3e	background	NLO
$ee \to \pi\pi$	VEPP, BES, Daphne, ...	R-ratio	$+$
$ee \to \ell\ell\gamma$	VEPP, BES, Daphne, ...	R-ratio	$+$