The Sherman function and its radiative corrections for elastic positron-nucleus scattering

Doris Jakubaßa-Amundsen
University of Munich (LMU)

JLab, March 2026

The Sherman function and its radiative corrections for elastic positron-nucleus scattering

Doris Jakubaßa-Amundsen
University of Munich (LMU)

JLab, March 2026

Contents:

1. Definition of the spin asymmetry for spin-zero nuclei
2. Motivation
3. Results for the ^{12}C nucleus
4. Results for the ^{208}Pb nucleus
5. Outlook for nuclei with spin (^{197}Au)

1. Definition of the beam-normal spin asymmetry

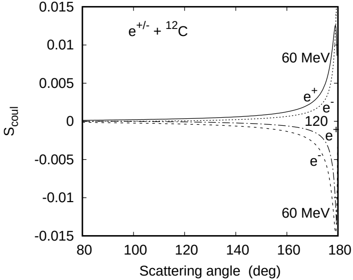
Polarized positron beam $\qquad e^+ \longrightarrow \uparrow \longrightarrow \bullet\, Z_T$ $\qquad \zeta$

Phase-shift analysis for $V_T(r) = -\int d\boldsymbol{r}'\varrho_0(r')\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}$

$$\frac{d\sigma}{d\Omega}(\zeta) = (|A|^2 + |B|^2)\,(1 + S\,\boldsymbol{n}\cdot\zeta), \quad \boldsymbol{n} \sim \boldsymbol{k}_i \times \boldsymbol{k}_f$$

Sherman function

$$S = \frac{d\sigma/d\Omega(\uparrow) - d\sigma/d\Omega(\downarrow)}{d\sigma/d\Omega(\uparrow) + d\sigma/d\Omega(\downarrow)} = \frac{2\,\mathrm{Re}\,\{AB^*\}}{|A|^2 + |B|^2}$$



$e^{+/-} +^{12} C$

60 and 120 MeV

2. Motivation for lepton-induced spin asymmetry

(below pion-production threshold, 135 MeV)

Probe phase-shift result between 15-120 MeV for heavier nuclei

Investigate two-photon exchange mechanisms (dispersion)

Understand the ^{208}Pb puzzle: why is dispersion so small?

Importance of QED effects for precision experiments

Positrons versus electrons: Coulomb distortion
Interference studies (diffraction)
Greater sensitivity to nuclear models

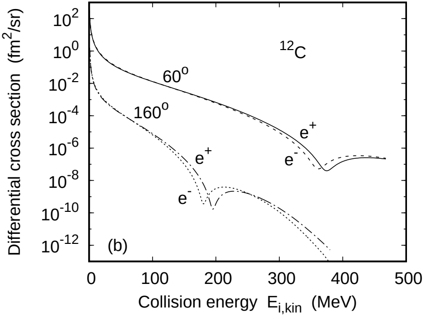
3. Results for ^{12}C

(a) Phase-shift analysis: $S(e^+) \approx -S(e^-)$

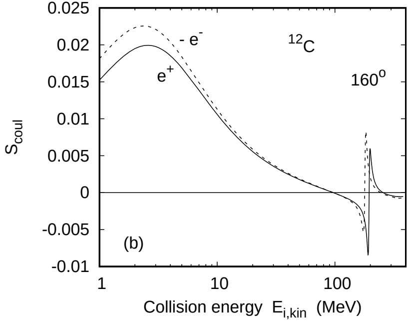
$$|S(e^+)| < |S(e^-)|$$

Energy distribution at fixed scattering angle:

Cross section $d\sigma/d\Omega$



Sherman function



Resonance structure from diffraction near 200 MeV at 160°

(b) QED corrections

Corrections via additional potentials :

Vacuum polarization: Uehling potential $V_{\mathrm{vac}}(r)$, depends on ϱ_0
Vertex + self-energy correction: $V_{vs}(r)$

Derive V_{vs} from first Born approximation for the vs process via inverse Fourier transform:

$$V_{vs}(r) \;=\; -\,\frac{2Z}{\pi}\int_0^\infty dq\,\frac{\sin(qr)}{qr}\,F_0^c(q)\,F_1^{vs}(q)$$

$$F_0^c \;=\; \text{ground-state charge form factor}$$

$$F_0^c(q) = -\,\frac{q^2}{4\pi Z}\int d\boldsymbol{r}\;e^{i\boldsymbol{q}\boldsymbol{r}}\;V_T(r)$$

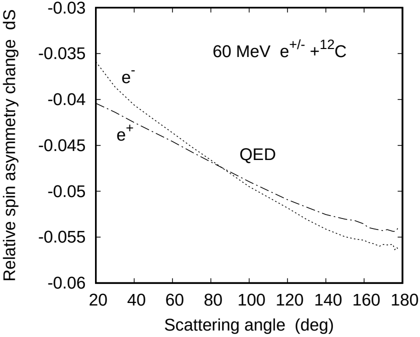
$$F_1^{vs} \;=\; \text{electric vs form factor} \qquad \text{(Tsai 1961)}$$

Nonperturbative approach: solve Dirac equation

$$[-ic\boldsymbol{\alpha}\boldsymbol{\nabla} + V_T(r) + V_{\mathrm{vac}}(r) + V_{vs}(r)]\,\psi(\boldsymbol{r}) = E\,\psi(\boldsymbol{r})$$

Sherman function

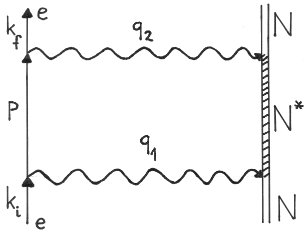
$$S_{QED} = \frac{2\,\mathrm{Re}\,\{A_{QED}B^*_{QED}\}}{|A_{QED}|^2 + |B_{QED}|^2}$$



Relative QED change:
$dS_{QED} = \Delta S_{QED}/S$

$\Delta S_{QED} = S_{QED} - S$
(QED modification)

(c) Dispersion: Transient nuclear excitation during scattering



Transition amplitude A_{fi}^{box}
from Feynman box diagram

(second-order Born)

Considered excited states: $E_x < 30$ MeV, $L \leq 3$

Most important: 1^- states at 23.5 MeV and 17.7 MeV

Form factors: Quasiparticle phonon model QPM (V.Ponomarev)

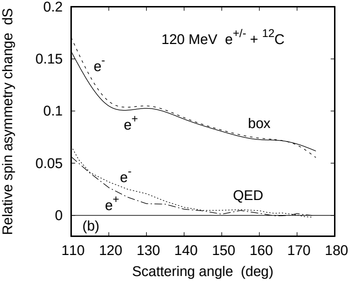
Scattering cross section including dispersion:

$$\frac{d\sigma_{\rm box}}{d\Omega}(\zeta) = \frac{d\sigma_{\rm coul}}{d\Omega}(\zeta) + 2 \text{ Re } \{ f^*_{\rm coul}(\zeta)\, A^{\rm box}_{fi}(\zeta)\}$$

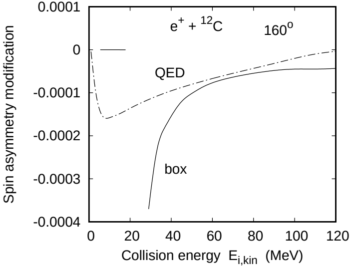
$f_{\rm coul}$ = amplitude for potential scattering

$$S_{\rm box} = \frac{d\sigma_{\rm box}/d\Omega(\uparrow) - d\sigma_{\rm box}/d\Omega(\downarrow)}{d\sigma_{\rm box}/d\Omega(\uparrow) + d\sigma_{\rm box}/d\Omega(\downarrow)} \approx S_{\rm coul} + \sum_{\omega_L, L} (S_{\rm box}(\omega_L, L) - S_{\rm coul})$$

$dS_{\rm box/QED}$ 120 MeV



$S_{\rm box/QED} - S_{\rm coul}$ 160°

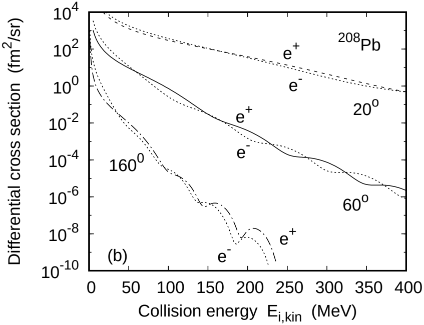


4. Results for ^{208}Pb

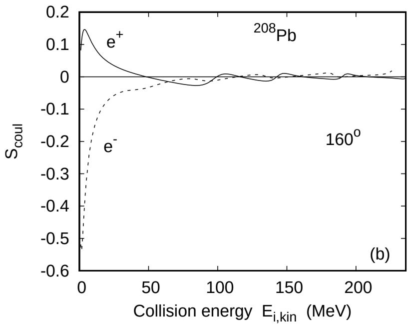
Different spin asymmetry for electrons and positrons

Low energy: $|S(e^+)| \ll |S(e^-)|$

$d\sigma_{\rm coul}/d\Omega$



$S_{\rm coul}$

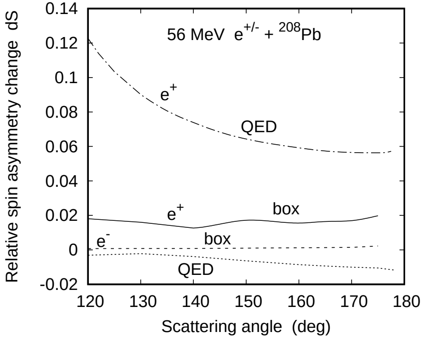


Considered states for dispersion: $E_x < 30$ MeV, $L \leq 3$

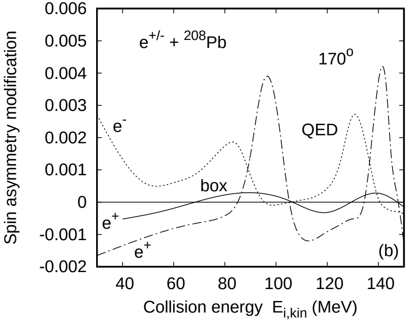
Low energy ($\lesssim 60$ MeV): 1^- states dominant
High energy (> 70 MeV: 2^+ and 3^- states dominant

Form factors from the Skyrme interaction (X.Roca-Maza)

$S_{\mathrm{box/QED}}/S_{\mathrm{coul}} - 1$



$S_{\mathrm{box/QED}} - S_{\mathrm{coul}}$



Summary for the beam-normal spin asymmetry

^{12}C: $S(e^+) \approx -S(e^-)$

$$dS_{\mathrm{box/QED}}(e^+) \approx dS_{\mathrm{box/QED}}(e^-)$$

QED effects: a few percent above 20 MeV

Dispersion: often large, mostly induced by dipole states
Subject to inaccurate nuclear models (mean-field theories)

^{208}Pb: $|S(e^+)| \ll |S(e^-)|$ below diffraction structures
(Onset of diffraction structures near 50 MeV)

QED effects: about $5-10\%$
$|dS_{\mathrm{QED}}(e^+)| > |dS_{\mathrm{QED}}(e^-)|$ above 30 MeV

Dispersion: much smaller than QED effects
(mutual cancellation of the L=2,3 contributions;
dipole only dominant at lower energies (below 65 MeV))

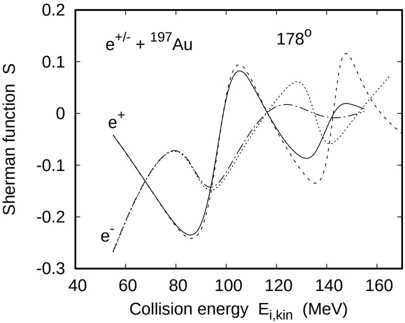
Outlook for nuclei with spin

Nonzero spin $J \Rightarrow$ Contribution of higher multipoles ($L \leq 2J$)

$\varrho_L(r) = \int d\Omega\, \varrho(\boldsymbol{r}) Y^*_{L0}(\Omega)$, $\quad \varrho =$ nuclear charge density
$J_{LL}(r) = \int d\Omega\, \boldsymbol{J}(\boldsymbol{r})\, \boldsymbol{Y}^{0*}_{JJ}(\Omega)$, $\quad \boldsymbol{J} =$ nuclear current density

$$\text{DWBA:} \qquad \frac{d\sigma}{d\Omega}(\zeta) = \frac{k_f}{k_i}\, \frac{1}{f_{\mathrm{rec}}}\, \frac{1}{2J+1} \sum_{\sigma_f}$$

$$\times \sum_{M_f M_i} \left| f_{\mathrm{coul}}\, \delta_{M_f,M_i} + A^{\mathrm{ch}}_{fi}(M_f, M_i) + A^{\mathrm{mag}}_{fi}(M_f, M_i) \right|^2$$

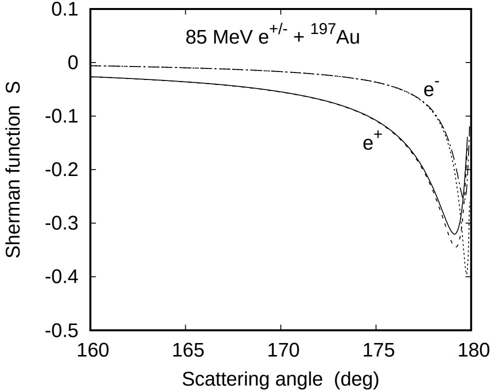


$e^+, e^- + {}^{197}$Au
Spin $= 3/2$

Large $e^+ - e^-$ asymmetry

Strong damping of S
at large E_i, θ

Angular distribution at 85 MeV



Change by magnetic scattering and QED:

$$\leq 176^\circ : \begin{cases} \leq\ 1\% & (e^+) \\ 10 - 20\% & (e^-) \end{cases}$$

179° : $5.7\%(e^+)$, $17\%(e^-)$

Cross section at 176°:

6.18e-6 $\mathrm{fm}^2/\mathrm{sr}$ (e^+)
3.54e-6 $\mathrm{fm}^2/\mathrm{sr}$ (e^-)

Thank you!

QED corrections for the Sherman function

(with and without magnetic contribution)