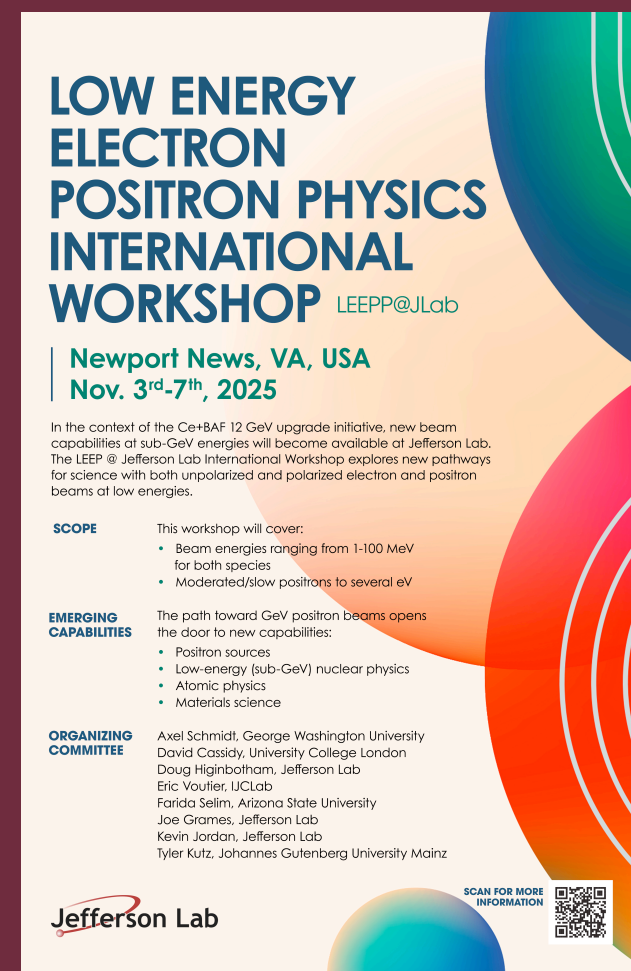


# Spin-Light Polarimetry for Low Energy Electron/Positron Beams

Dipangkar Dutta



Low Energy Electron Positron Physics Workshop at Jefferson Lab  
March 23-28, 2026

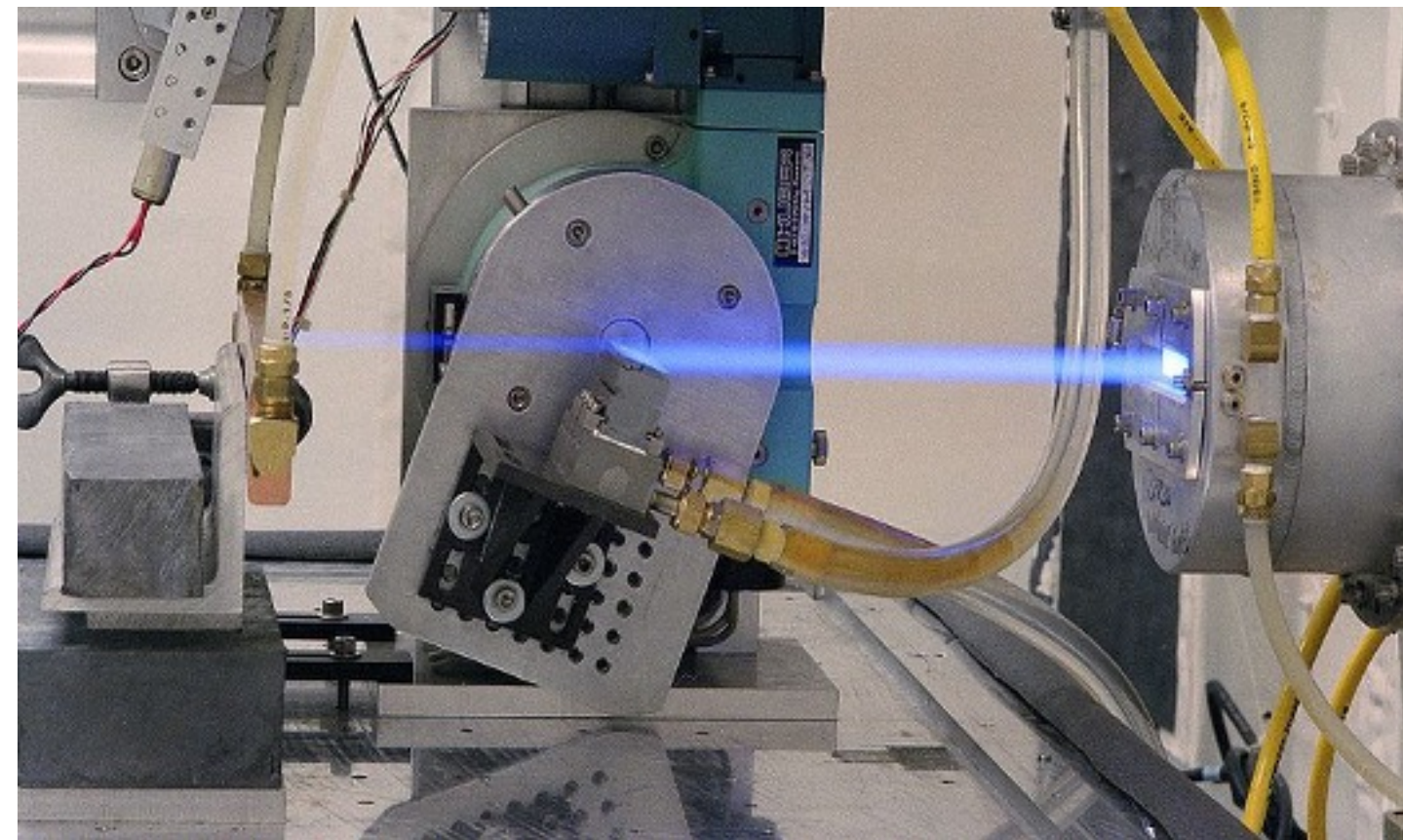
# Outline

- **Synchrotron Radiation & “Spin-Light”**
- **Conceptual Design of a Spin-Light Polarimeter**
  - Transverse polarimeter
  - Longitudinal polarimeter
- **Low energy challenges**
- **Summary**

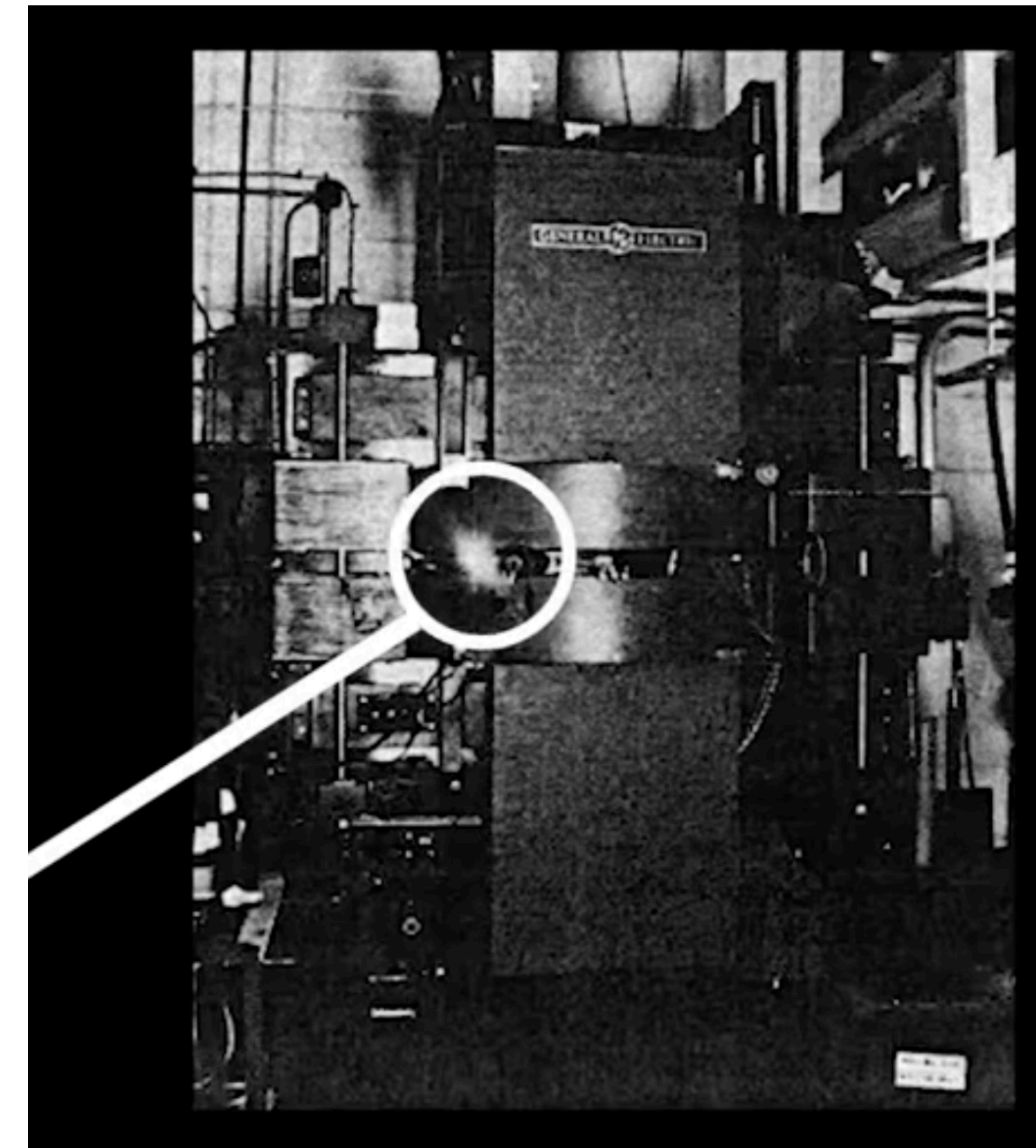
# Synchrotron radiation was discovered at the General Electric Research Labs in Upstate NY, in 1947.

Radiation from charged particle accelerated transverse to its velocity  
(consequence of finite velocity of light)

predicted by Ivanenko and Pomeranchuk(1944)



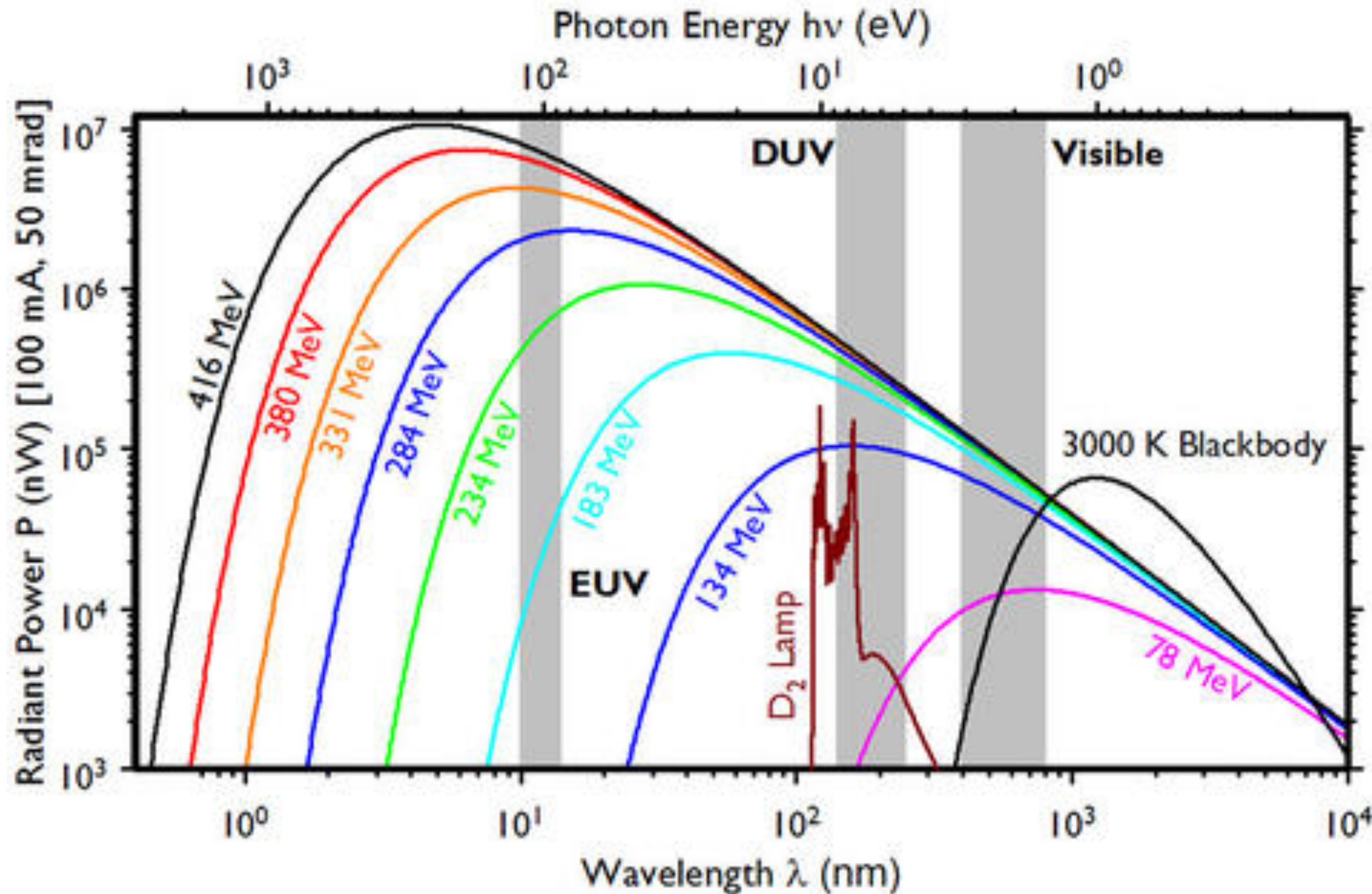
visible radiation from an electron beam of a 70 MeV synchrotron



Synchrotron radiation = magnetic bremsstrahlung = electronic light

# Angular and spectral distribution first worked out in classical theory

Ivanenko and Pomeranchuk(1946), Schwinger (1947)



$$P = \frac{2 e^2 \gamma^4 c}{3 R^2} \propto E^4$$

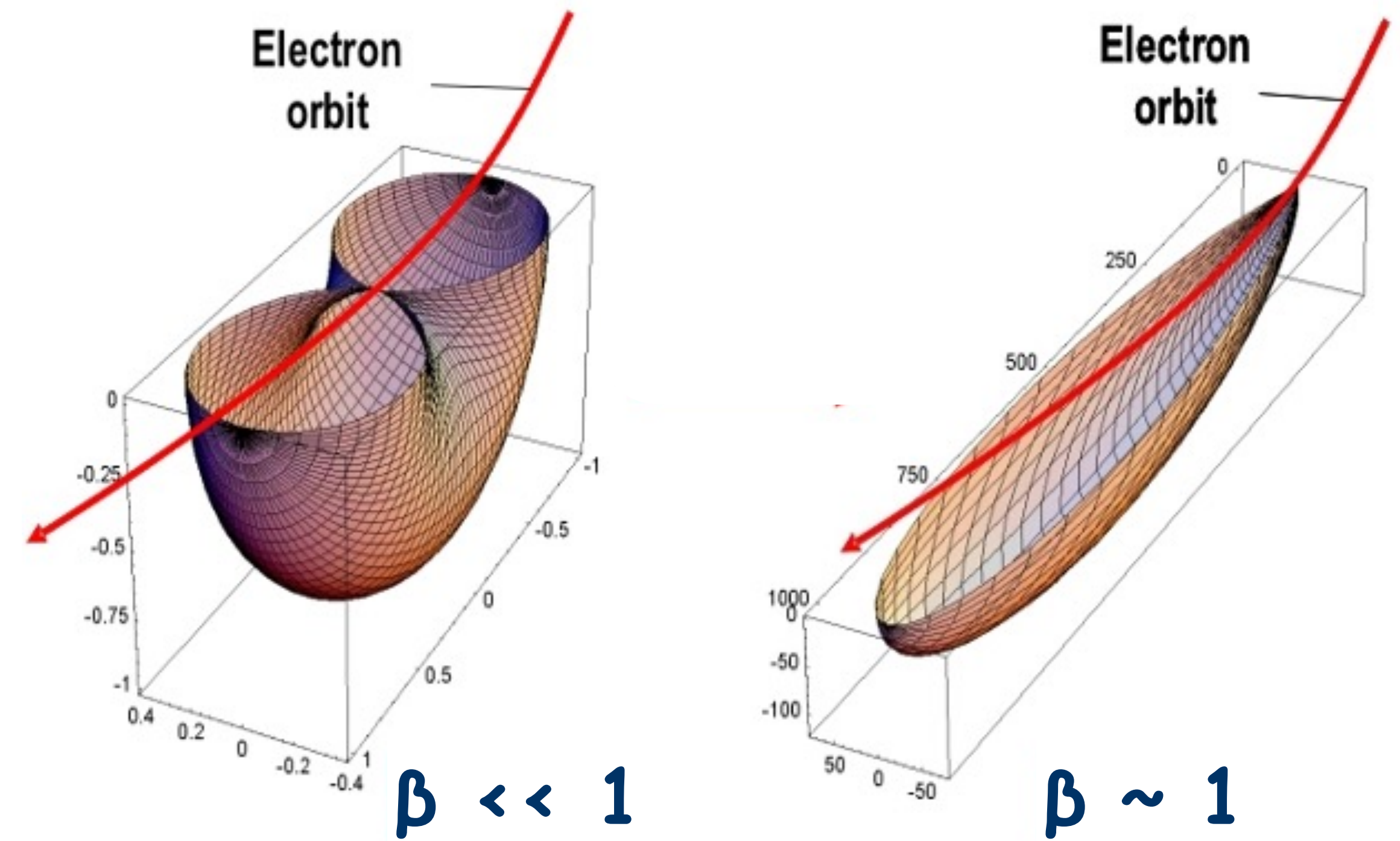
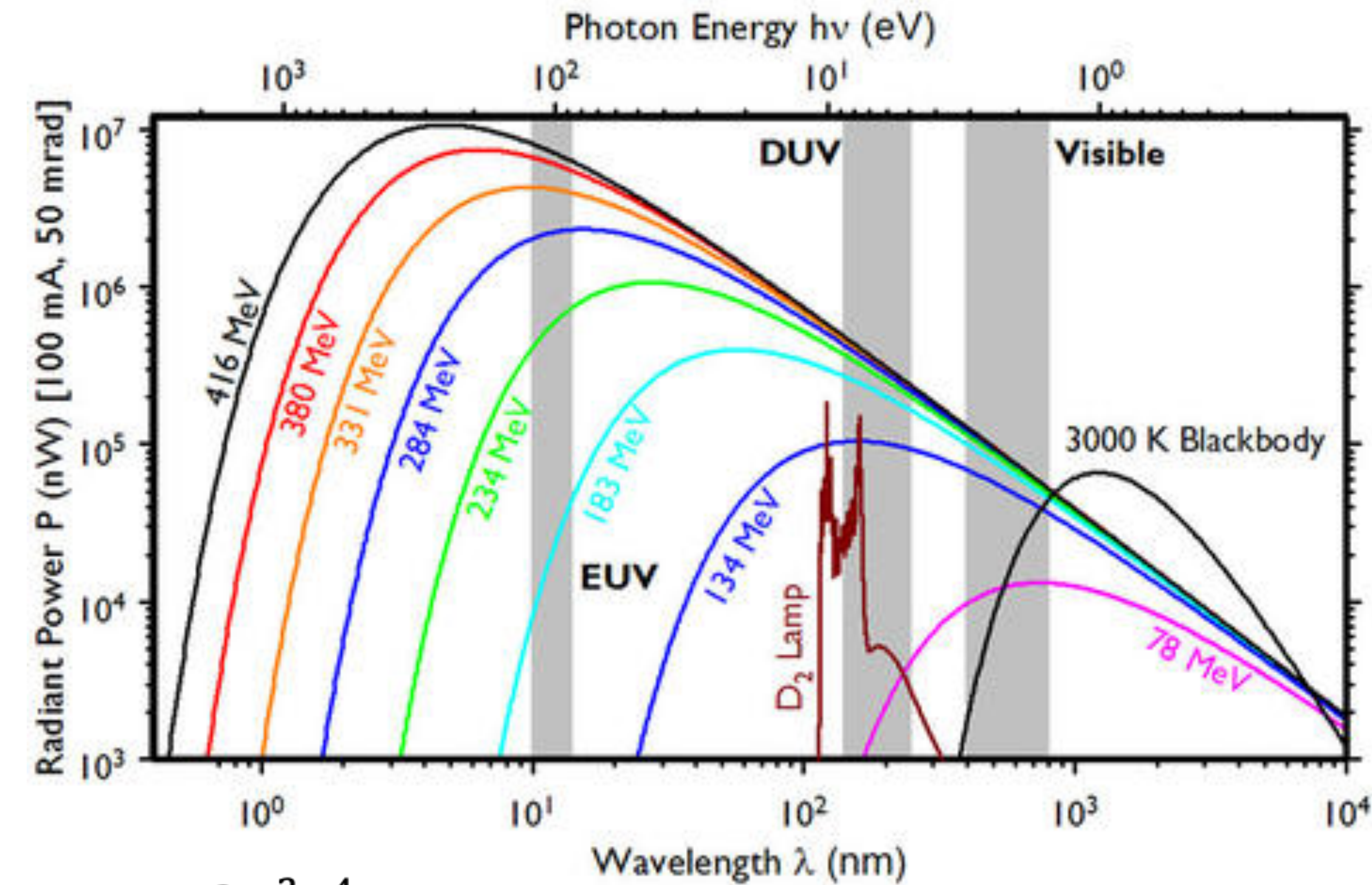
$$dP/d\Omega = \frac{e^2 \gamma^4 c (1 - \beta \cos\theta)^2 - (1 - \beta^2) \sin^2\theta \cos^2\phi}{4\pi R^2 (1 - \beta \cos\theta)^5}$$

$\theta, \phi$  relative to direction of motion

spectral width  $\Delta\omega \propto \gamma^3$

i.e. continuous

# Angular and spectral distribution first worked out in classical theory



For  $\gamma \gg 1$   $\theta \sim 1/\gamma$

$$P = \frac{2e^2\gamma^4c}{3R^2} \propto E^4$$

$$\frac{dP}{d\Omega} = \frac{e^2\gamma^4c}{4\pi R^2} \frac{(1 - \beta\cos\theta)^2 - (1 - \beta^2)\sin^2\theta\cos^2\phi}{(1 - \beta\cos\theta)^5}$$

for 0.1 GeV electrons  $\theta \sim 10$  mrad (spot size at 1 m  $\sim 1$  cm)

for 1 GeV electrons  $\theta \sim 1$  mrad (spot size at 10 m  $\sim 1$  cm)

for 11 GeV electrons  $\theta \sim 90$   $\mu$ rad (spot size at 10m  $\sim 1$ mm)

# Quantum corrections to SR introduce spin dependence

**Exact QED calculations by A.A. Sokolov and I. M. Ternov (1960s)**

**The classical theory (continuous SR spectrum) is valid for  $E_{\text{electron}} \ll E_{\text{critical}}$**

**$E_{\text{critical}}$  = when a single SR photon carries all of the electron's energy.**

**$E_{\text{crit}} \sim 10^6 \text{ GeV}$  and  $B_{\text{crit}} \sim 4 \times 10^9 \text{ T}$**

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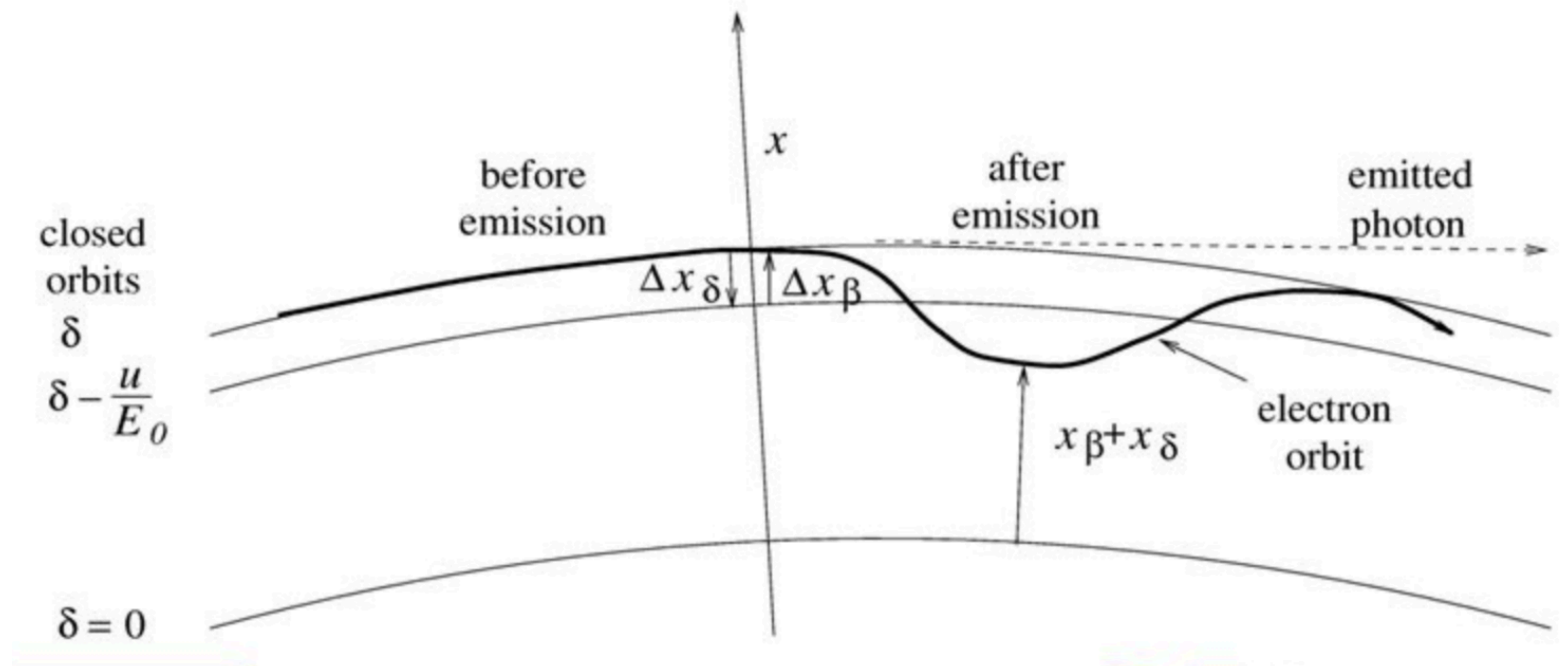
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**and QED corrections give electrons spin dependence in the radiated power**

$$P = P^{\text{clas}} \times \frac{9\sqrt{3}}{16\pi} \sum_{s'} \int_0^\infty \frac{y dy}{(1 + \xi y)^4} I_{ss'}^2(x) F(y), \quad y = \frac{\omega}{\omega_c}; \quad \xi = \frac{3}{2} \frac{B}{B_{\text{crit}}} \gamma;$$

**$s$  = radial quantum #  
 $I$  = Laguerre func.  
 $y \propto 1/\gamma^3$   
 $x = f(y, \xi)$**

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$\propto$  electron spin  $j, j'$   
+ modified Bessel fns.

$s = \text{radial quantum \#}$   
 $I = \text{Laguerre func.}$   
 $y \propto 1/\gamma^3$   
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 $y \propto 1/\gamma^3$   
 $x = f(y, \xi)$

For  $\xi \ll 1$  and electron spin  $j, j' = \pm 1$

$$P = P^{\text{clas}} \left[ \left( 1 - \frac{55\sqrt{3}}{24} \xi + \frac{64}{3} \xi^2 \right) - \left( \frac{1 + jj'}{2} \right) \left( j\xi + \frac{5}{9} \xi^2 + \frac{245\sqrt{3}}{48} j\xi^2 \right) + \left( \frac{1 - jj'}{2} \right) \left( \frac{4}{3} \xi^2 + \frac{315\sqrt{3}}{432} j\xi^2 \right) + \dots \right] \left( \int_{\text{mod. Bessel}} dy \right)$$

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lowest order spin dependent term
lowest order spin flip dependent term

# Spin dependent SR - “Spin Light” was demonstrated in the 1980s

$$P^{pol} - P^{unpol} = P^{spin} = -j\xi P^{clas} \int_0^\infty \frac{9\sqrt{3}}{8\pi} y^2 K_{1/3}(y) dy. \quad \begin{array}{l} K_{1/3} \text{ modified} \\ \text{Bessel function} \end{array}$$

To the first order in  $\xi$  the difference in SR intensity between polarized and unpolarized electrons is  $\delta = \xi j \sim 10^{-4}$  for 100  $\mu\text{A}$ , 5 GeV electrons, 2T field, 0.9 degree of polarization

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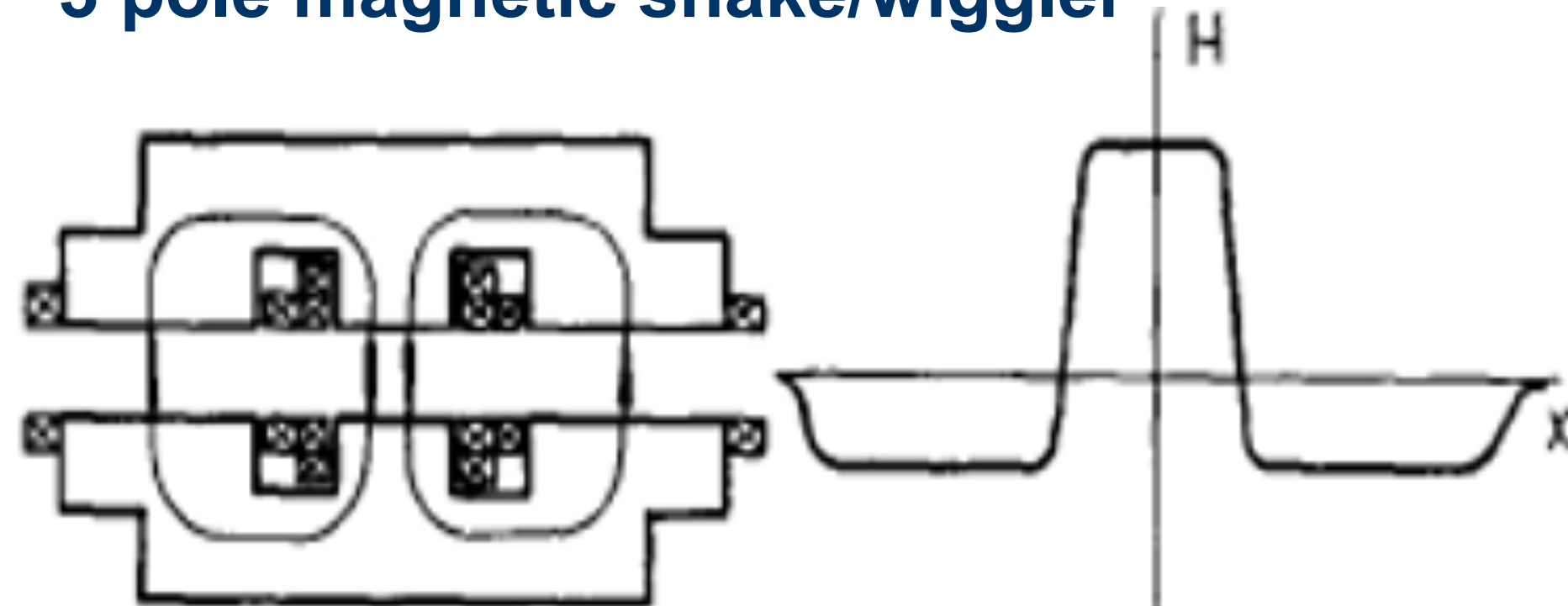
$K_{1/3}$  modified Bessel function

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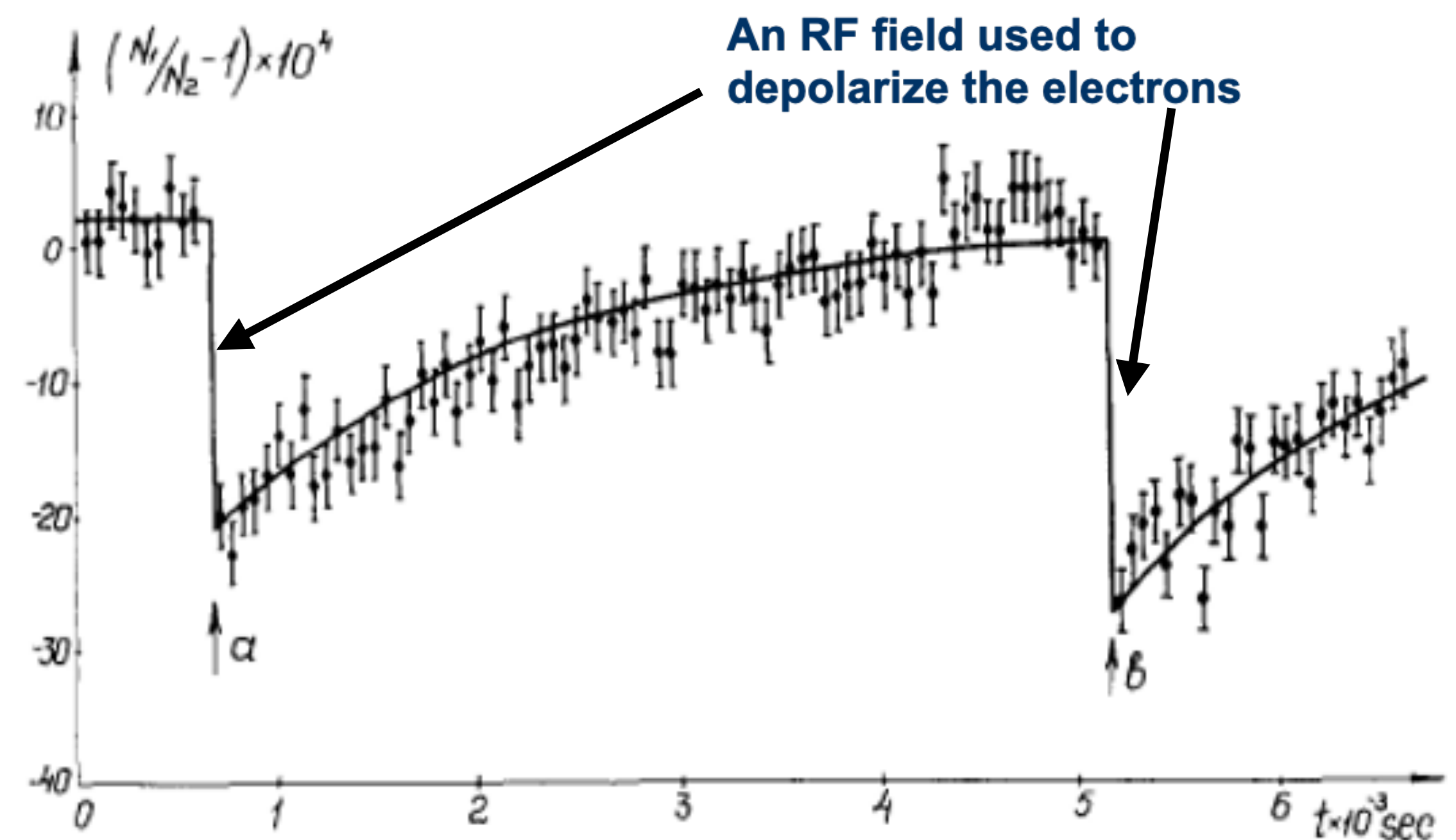
Verified experimentally at the VEPP-4 storage ring in Novosibirsk

*Belomestnykh et al., NIM 227, 173 (1984)*

3 pole magnetic snake/wiggler



2T field



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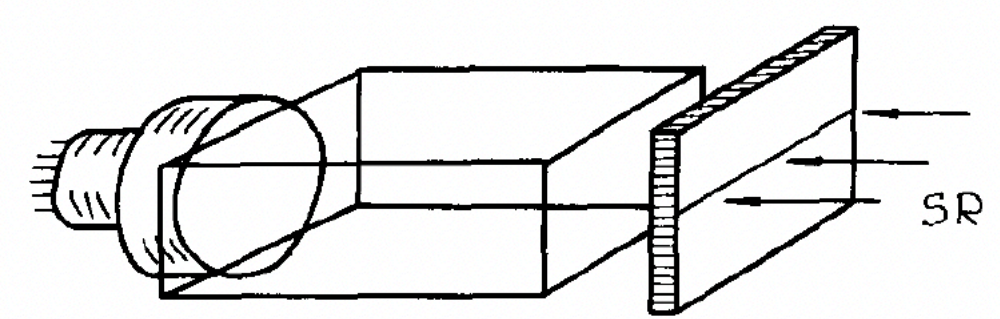
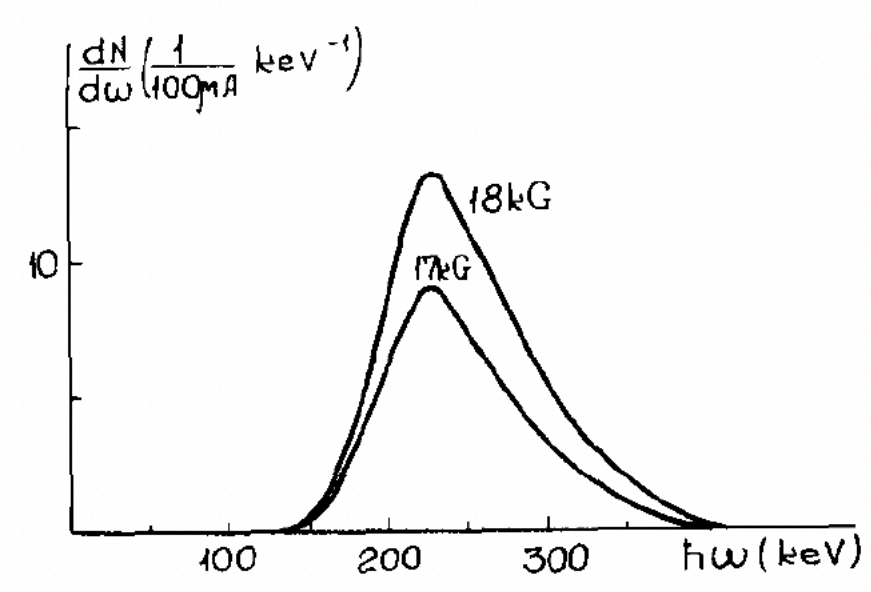
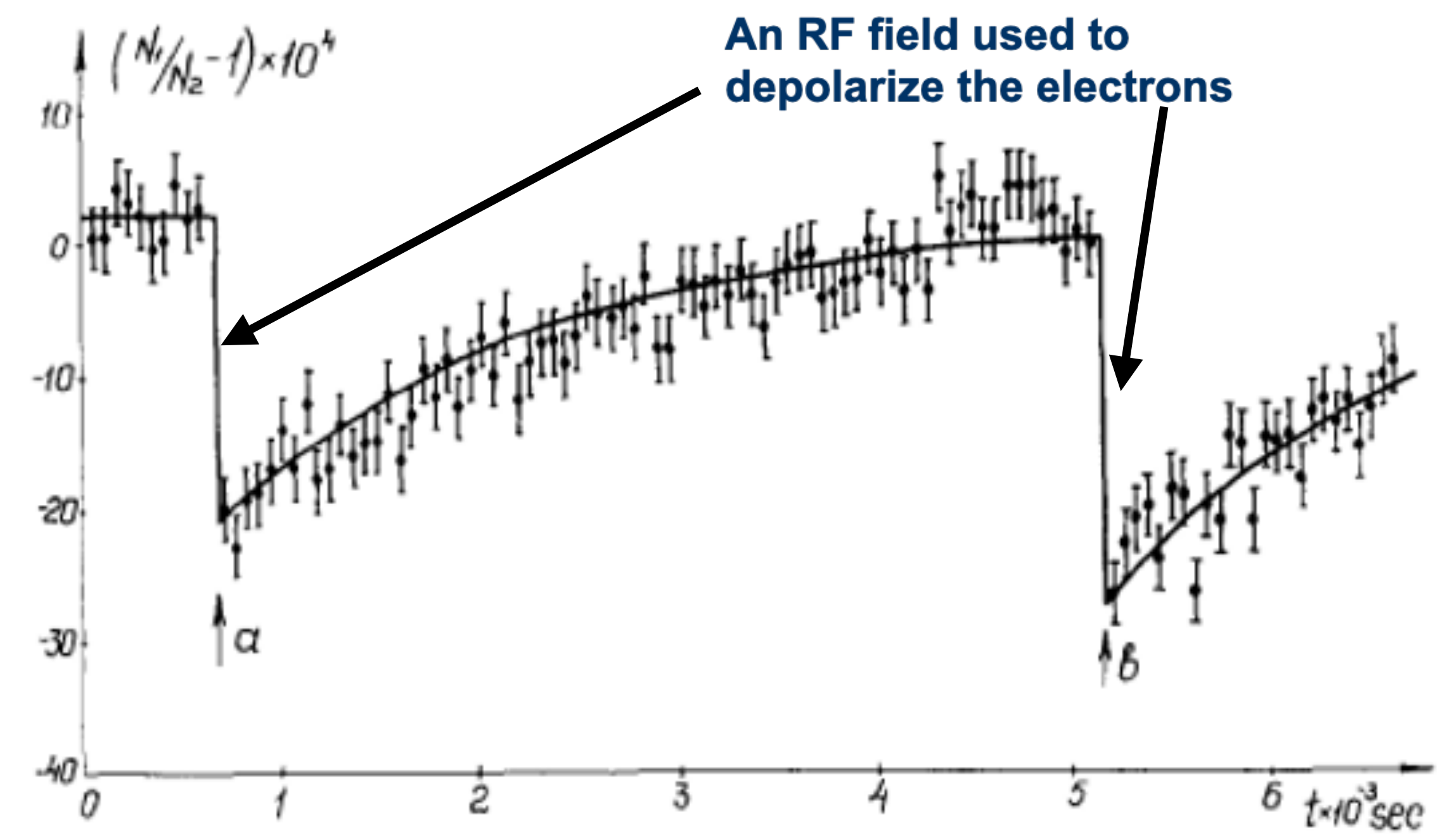


Fig. 2. Layout of the detector.

4mm Pb converter in front of scintillator



Spectrum of highest energy SR photons



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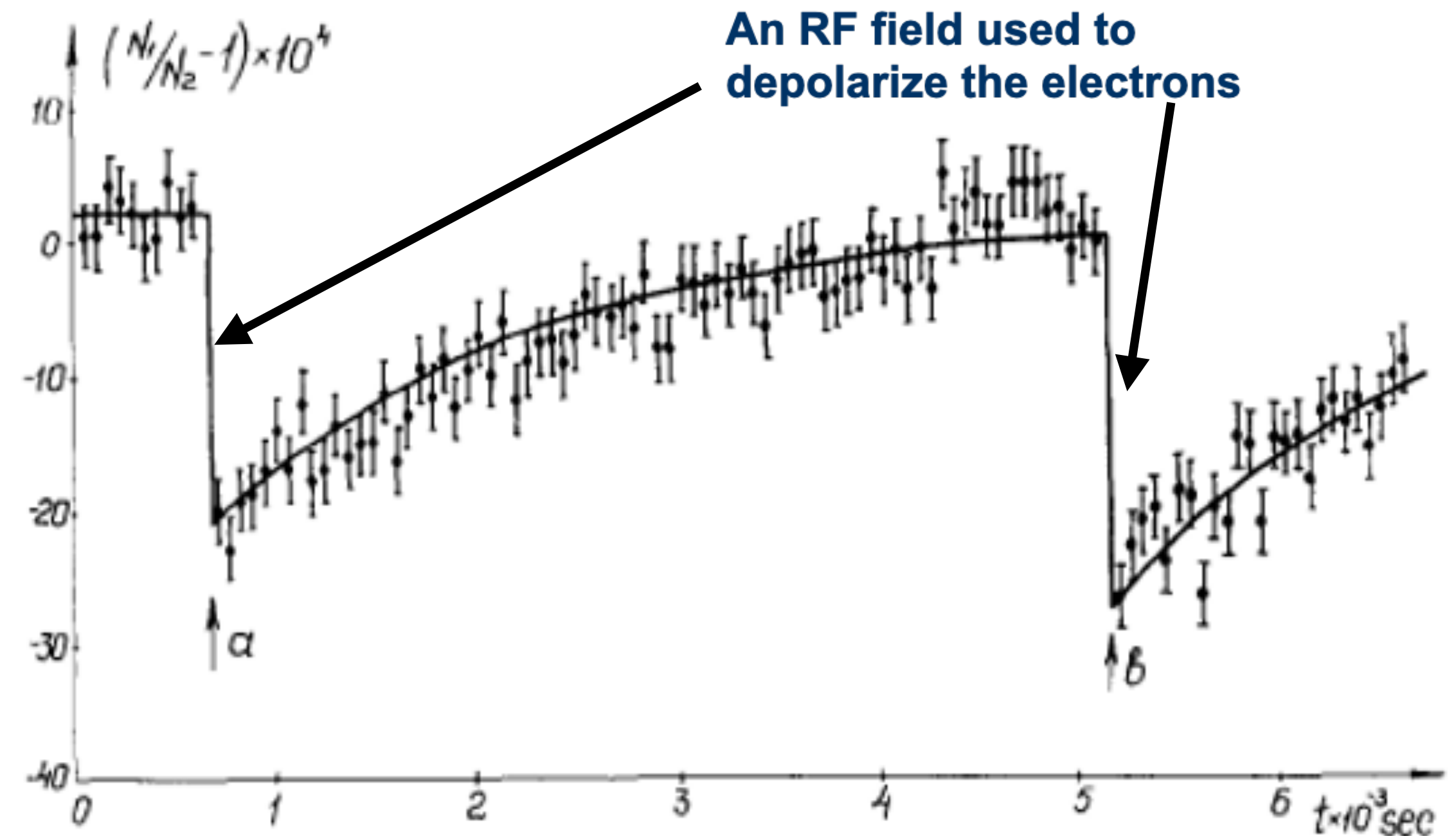
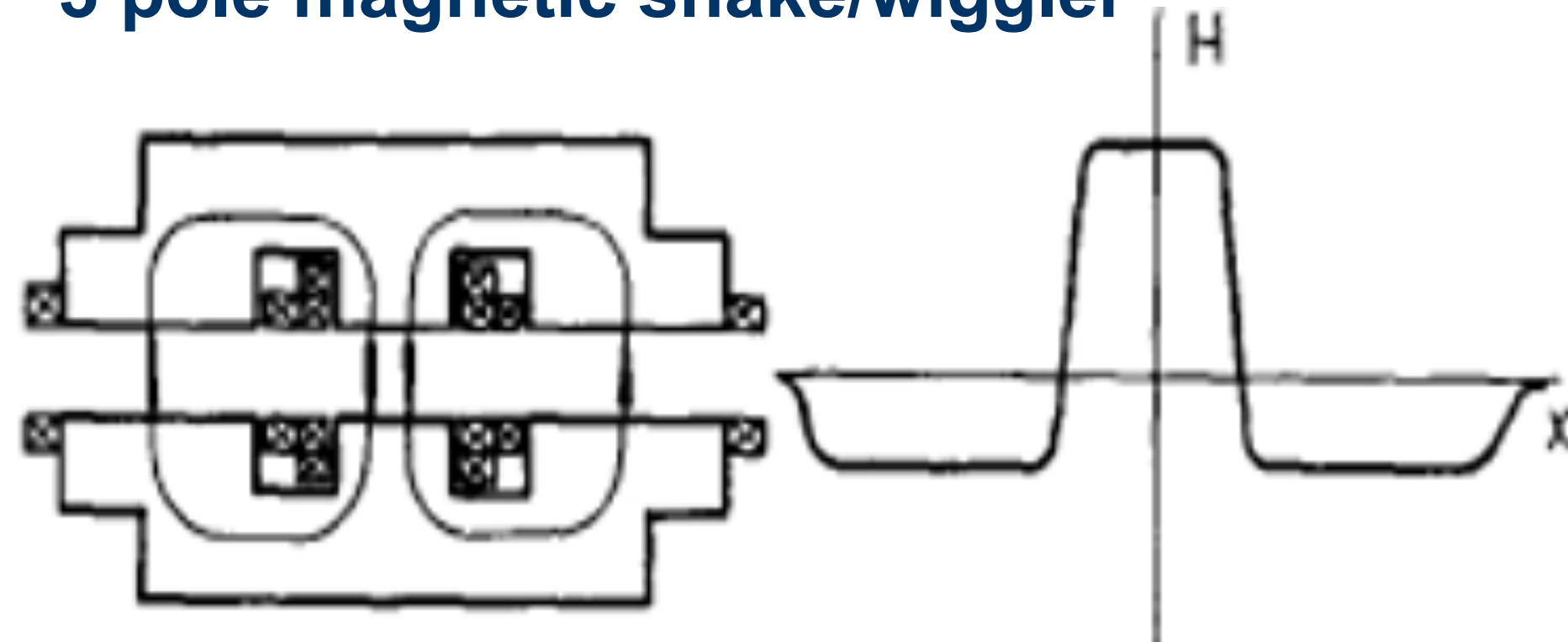
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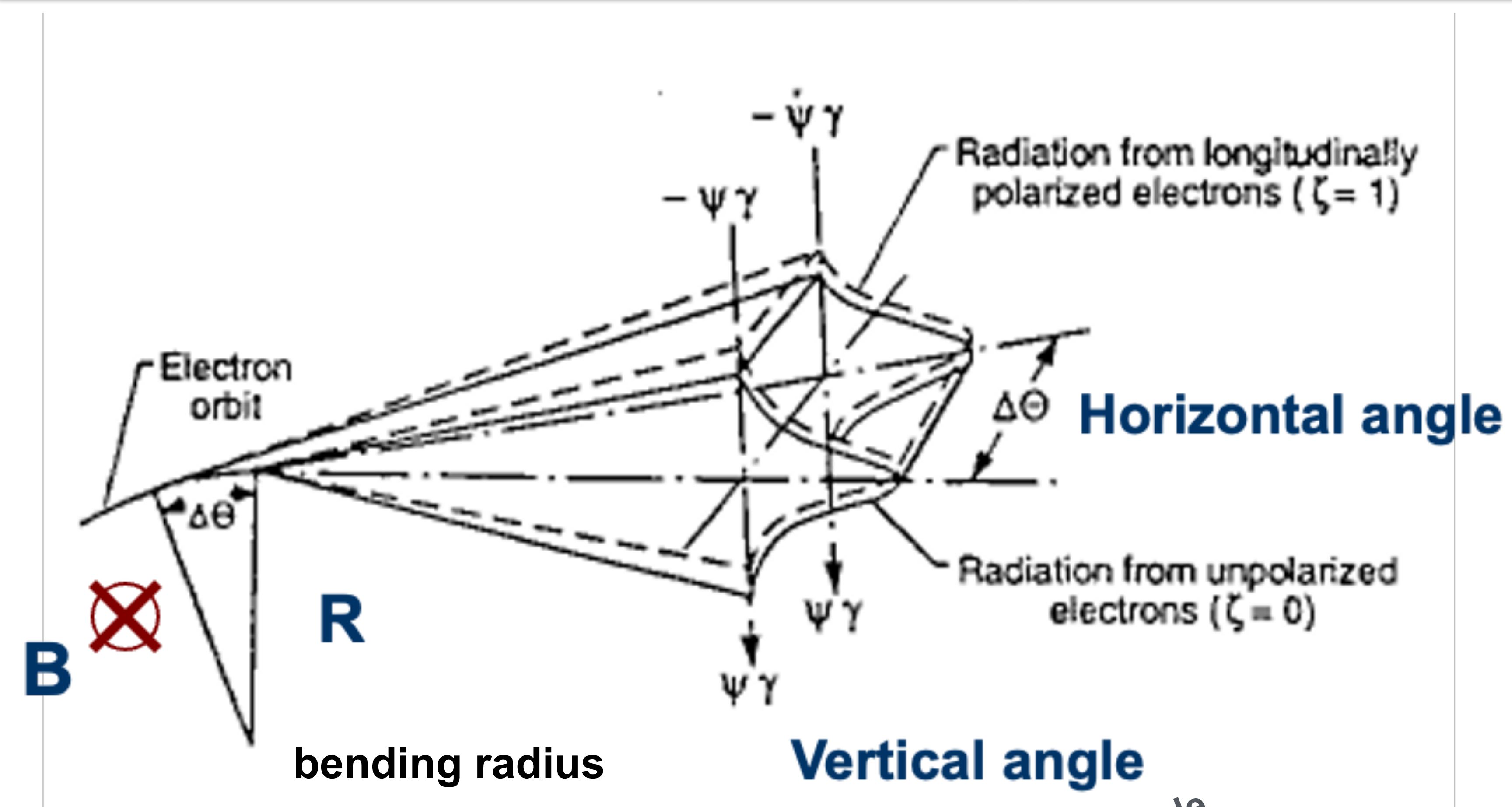
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3 pole magnetic snake/wiggler



The spin-flip term contributes only as  $\sim \xi^2$  (Sokolov-Ternov effect), responsible for the transverse self polarization of electron beams in storage rings (first observed at Orsay). J. Le Duff, P. C. Marin, J. L. Manson, and M. Sommev, Orsay - Rapport Technique, 4-73 (1973).

# Longitudinal Spin Light radiates different number of photons above and below the orbital plane



$P_y$  (long) = odd function of vertical angle

Integrated over all vertical angles the total SR power is spin **independent**

# of photons radiated above and below the orbital plane are not equal

$$\Delta N_\gamma(j) = \frac{3}{\pi^2} \frac{1}{137} \frac{I_e}{e} j \xi \gamma \Delta\theta \int_{y_1}^{y_2} y^2 dy \int_0^{\alpha = \text{vertical angle}} \alpha (1 + \alpha^2)^{3/2} K_{1/3}(z) K_{2/3}(z) d\alpha$$

$K_{2/3}, K_{1/3}$  modified Bessel function

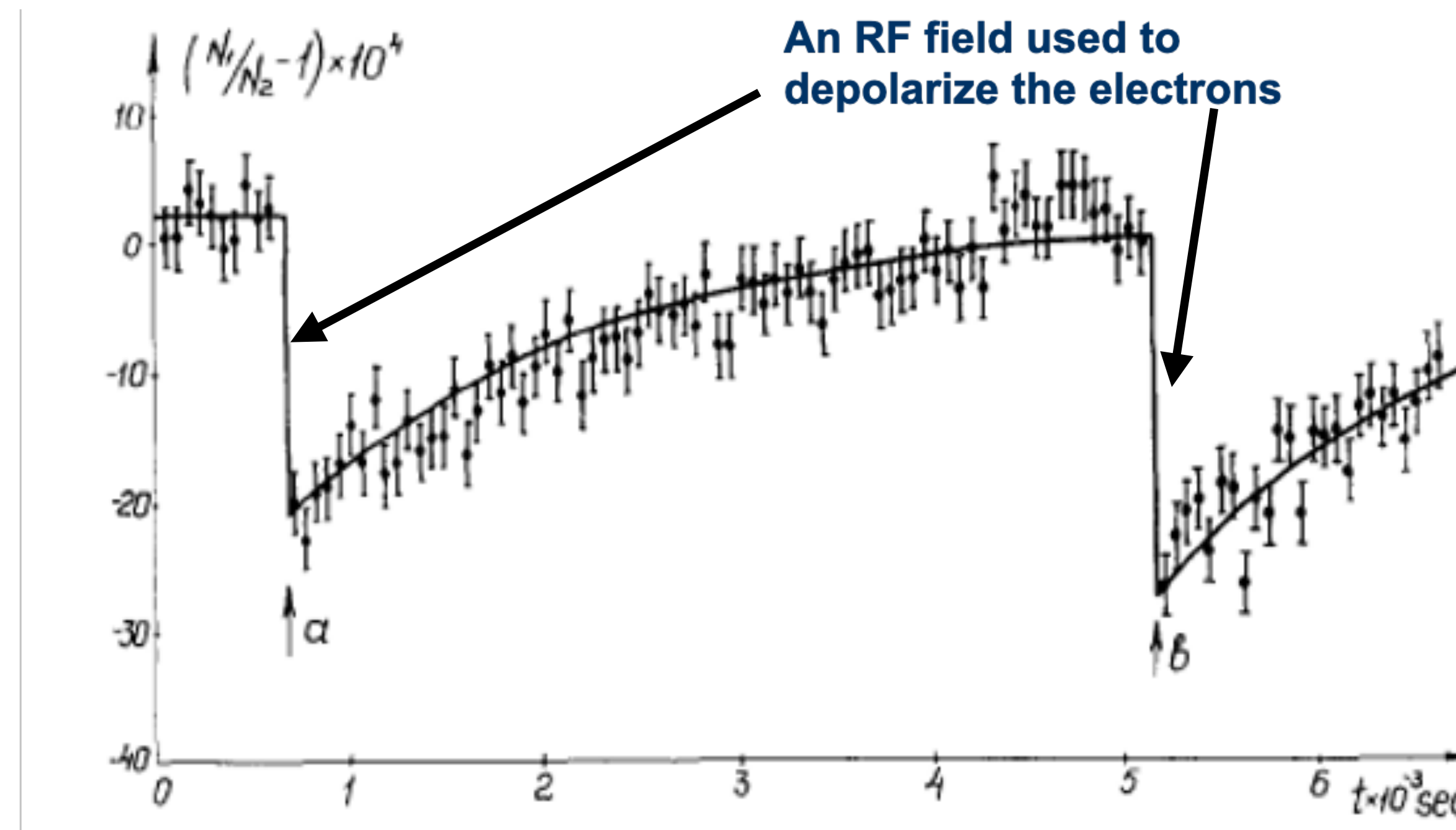
# Transverse Spin Light has an absolute spin dependent asymmetry

$P_\gamma$  (trans) = even function of vertical angle

Integrated over all vertical angles the total SR power is spin dependent

$$\Delta N_\gamma(j) = \frac{3}{\pi^2} \frac{1}{137} \frac{I_e}{e} j \xi \gamma \Delta\theta \int_{y_1}^{y_2} y^2 dy \int_{-\alpha}^{\alpha} (1 + \alpha^2)^{3/2} K_{1/3}(z) K_{2/3}(z) d\alpha$$

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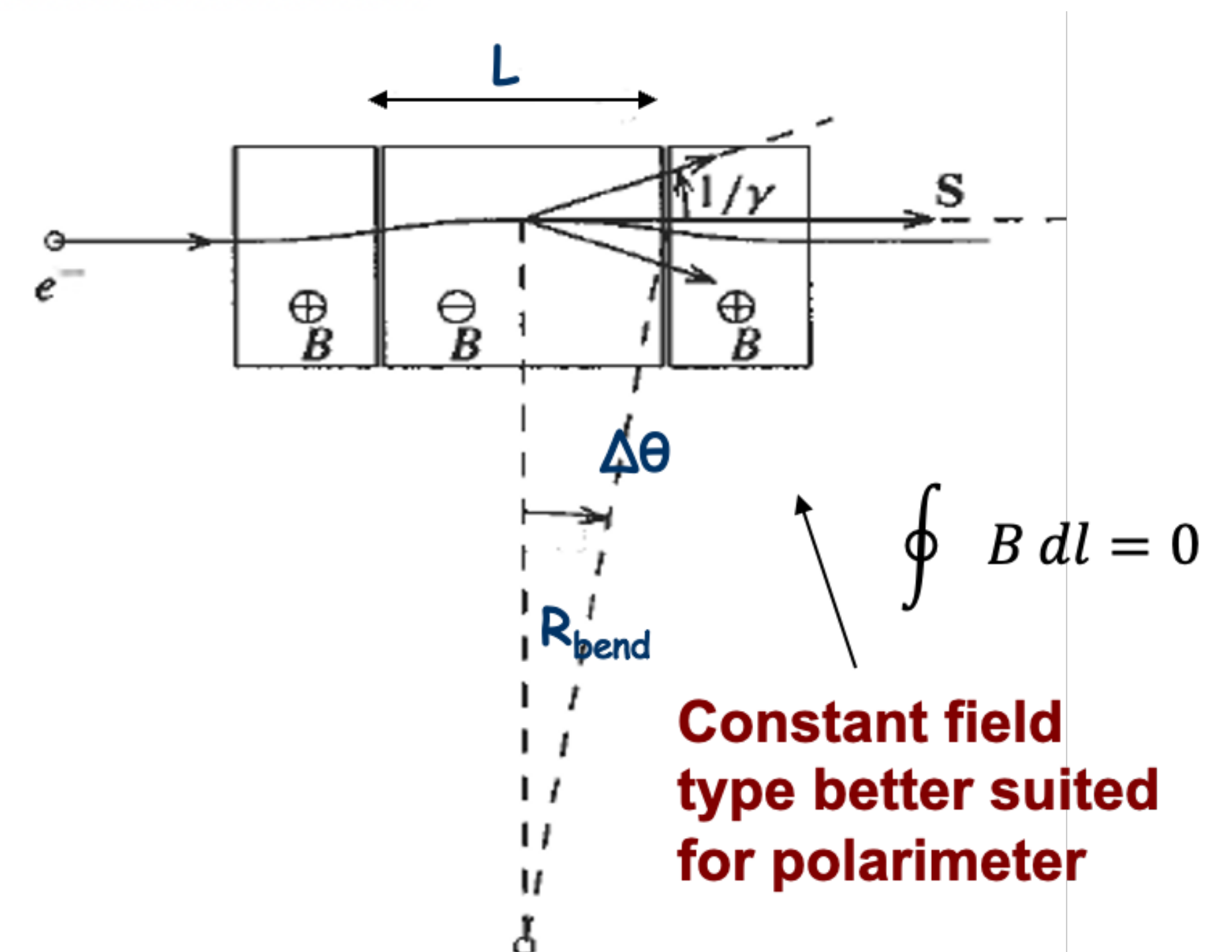
$K_{2/3}, K_{1/3}$  modified Bessel function

**A Source of Spin light : a 3 pole wiggler**

$$R_{bend} = \frac{\gamma m_e c}{eB}$$

$$L = R_{bend} \Delta\theta$$

Horizontal angular acceptance  $\Delta\theta$  fixed to 10 mrad



# Transverse Spin Light polarimetry is a well demonstrated technique

$P_\gamma$  (trans) = even function of vertical angle

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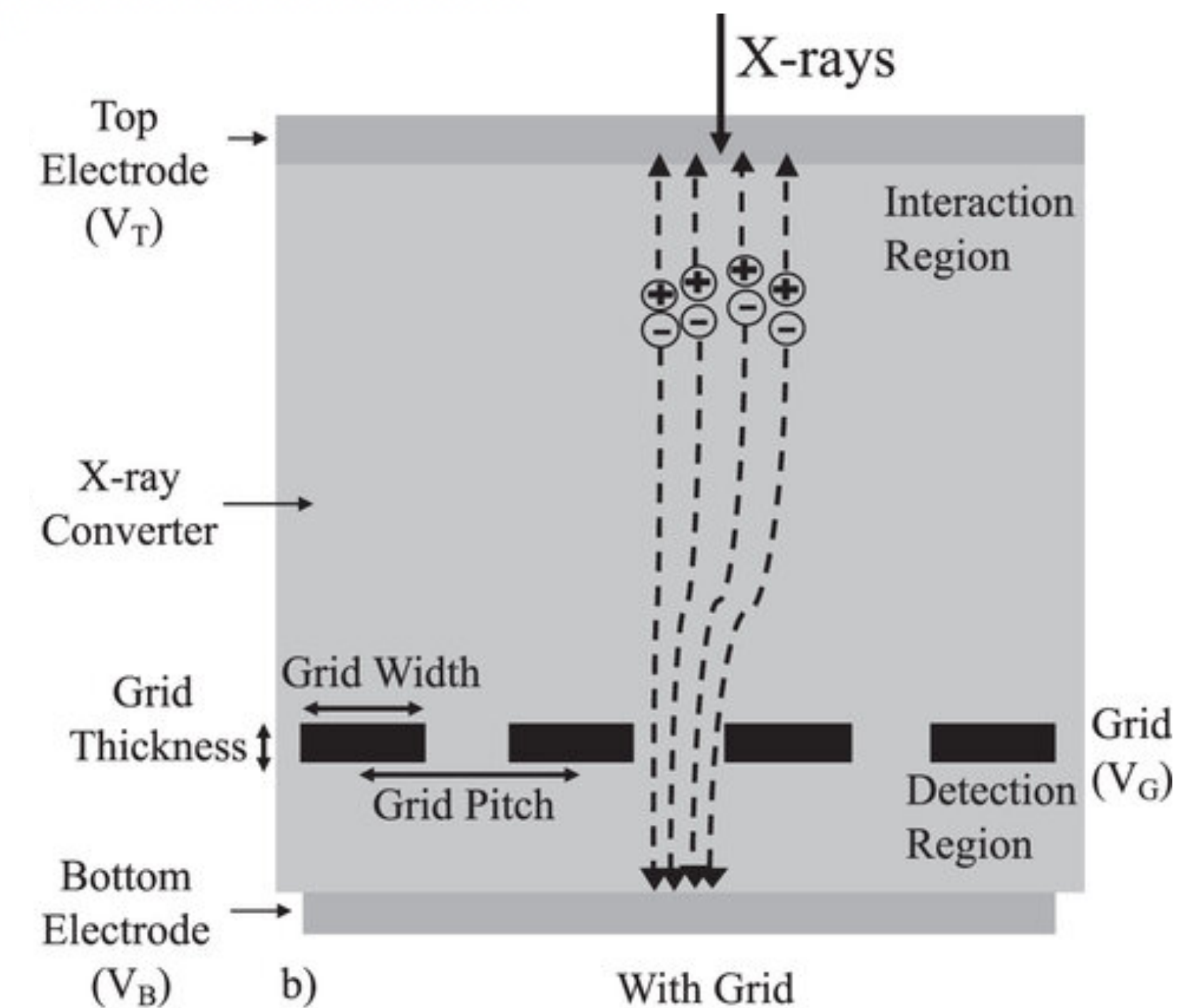
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$\alpha =$  vertical angle  
 detector frequency response

$K_{2/3}, K_{1/3}$  modified Bessel function

**A Spin light Detector : ion chamber or converter + scintillator**

- Can handle high rates
- Radiation hard
- Low dark current/noise
- Wide range of ICs commercially available



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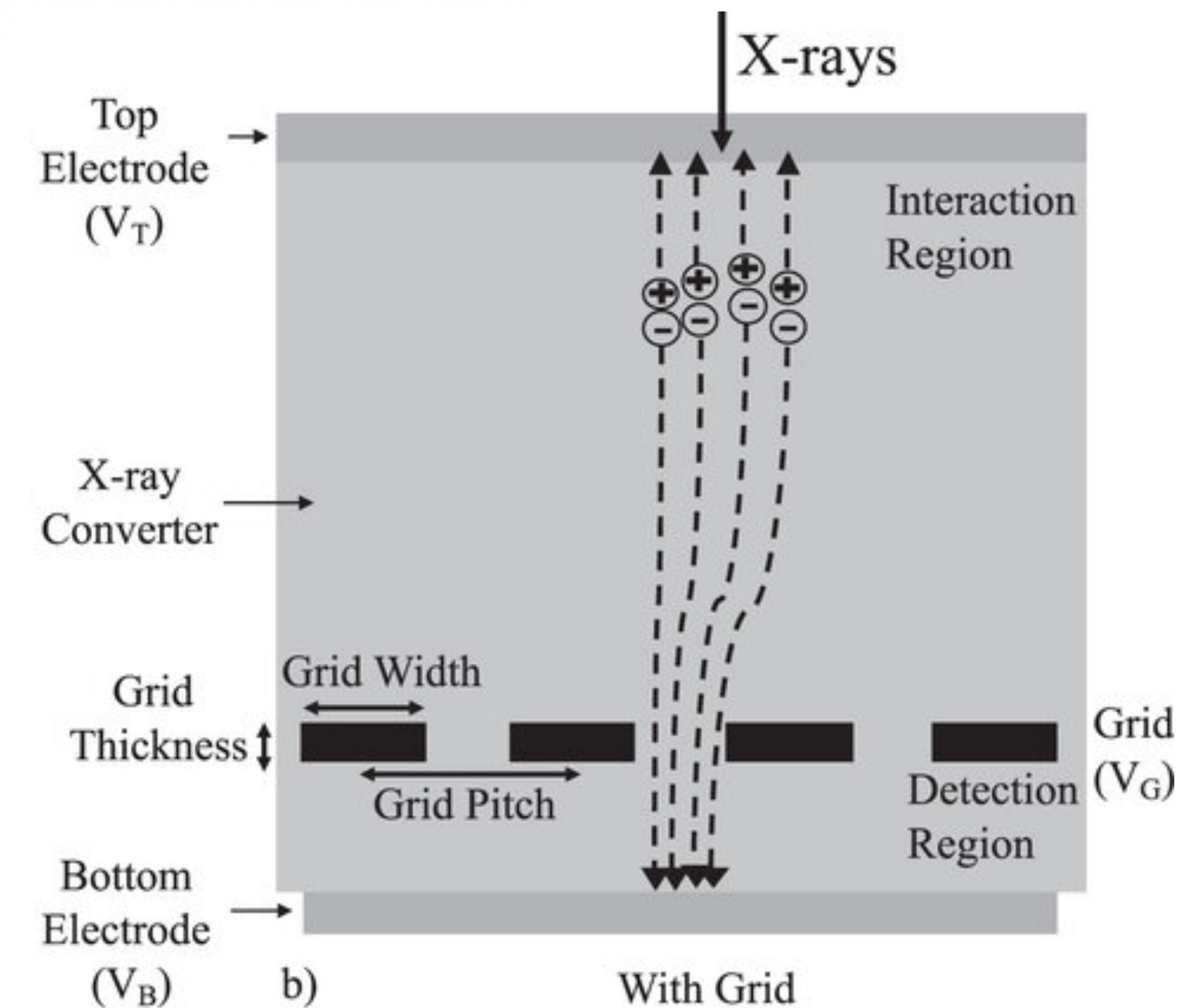
$\alpha = \text{vertical angle}$

detector frequency response

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**A Spin light Detector : ion chamber or converter + scintillator**

$$\text{Asym} = \frac{(N^{h+}_{SR} + \Delta N^{h+}_{spin} - N^{h-}_{SR} - \Delta N^{h-}_{spin})}{(N^{h+}_{SR} + \Delta N^{h+}_{spin} + N^{h-}_{SR} - \Delta N^{h-}_{spin})} \propto 2P_e \xi \gamma A_{det}$$



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$\alpha = \text{vertical angle}$   
 detector frequency response

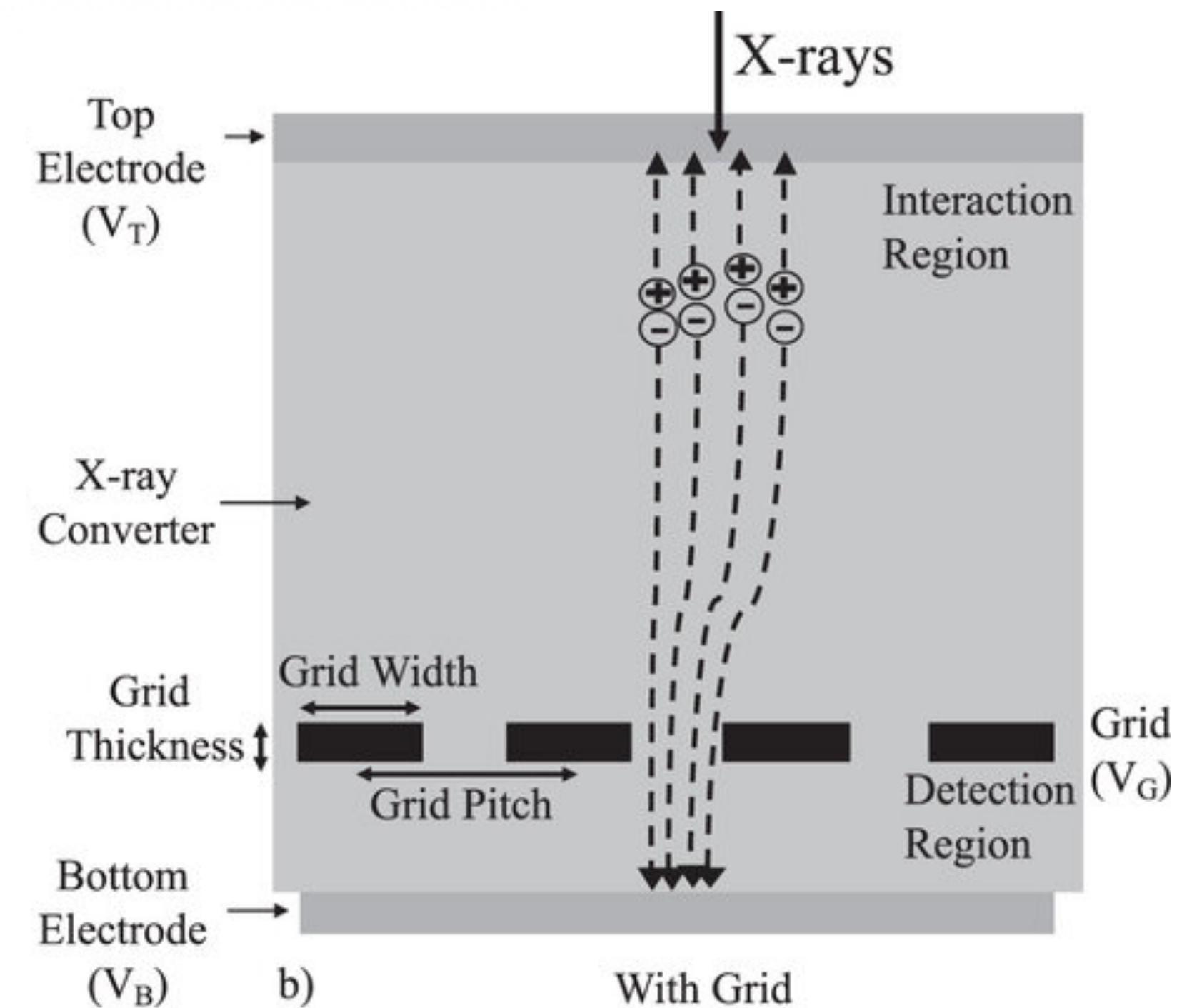
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$$\propto 2P_e \xi \gamma \textcircled{A_{det}}$$

acceptance function from simulation or calibration with RF depolarizer



# Transverse polarimetry is feasible for $\geq 0.5$ GeV beams

For  $E_{\text{beam}} = 0.5$  GeV

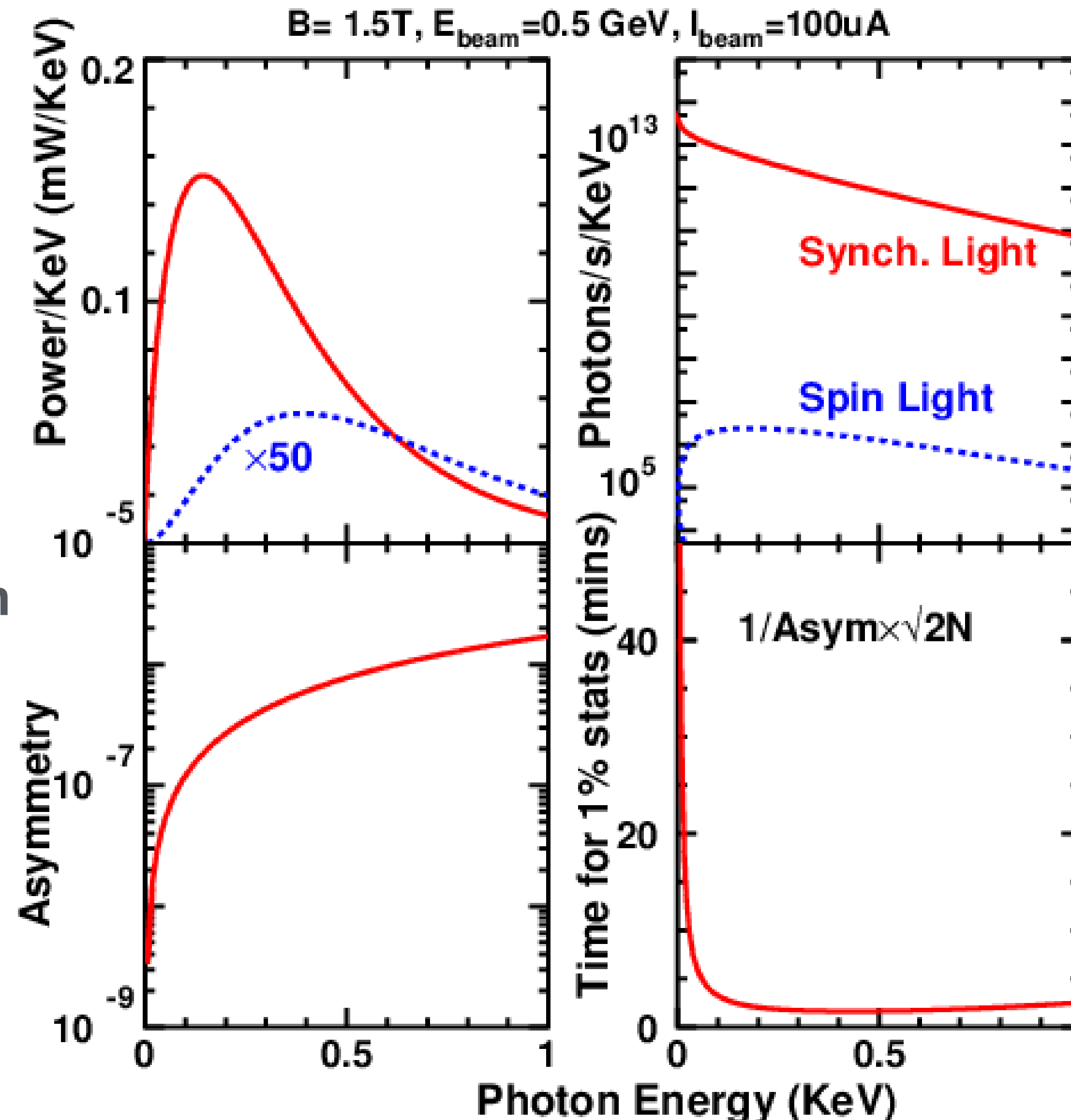
$B = 1.5$  T

Peak spin light power  
@ 0.35 KeV

Asym (0.35KeV)  $\sim 0.6$  ppm

For 100uA current

1% statistics in  
few minutes



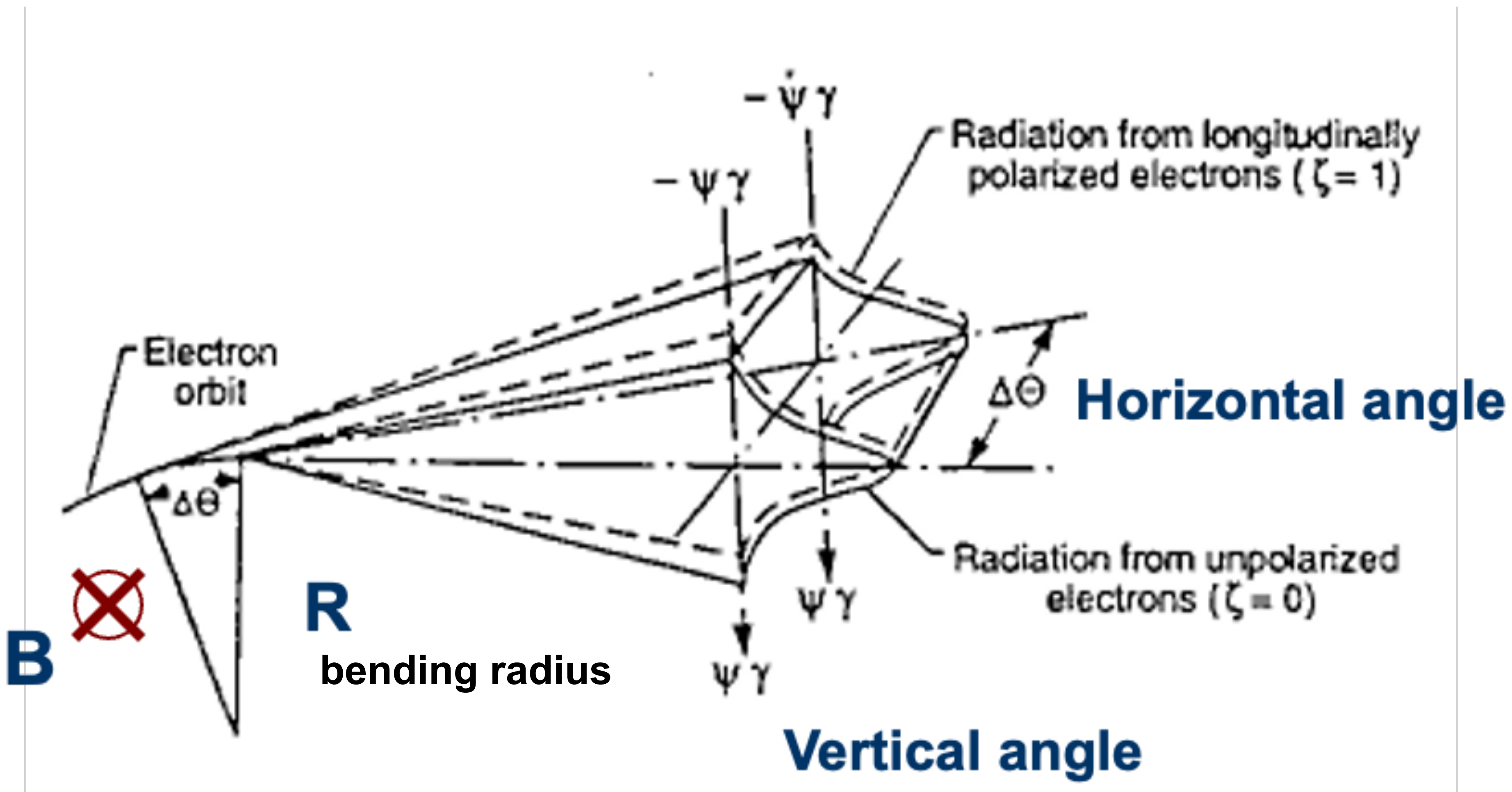
$$\text{Asym} \propto E_{\text{beam}}^2$$

$$\text{Asym} \propto B$$

Transverse polarimeters is  
easier for  $E_{\text{beam}} > 0.5$  GeV

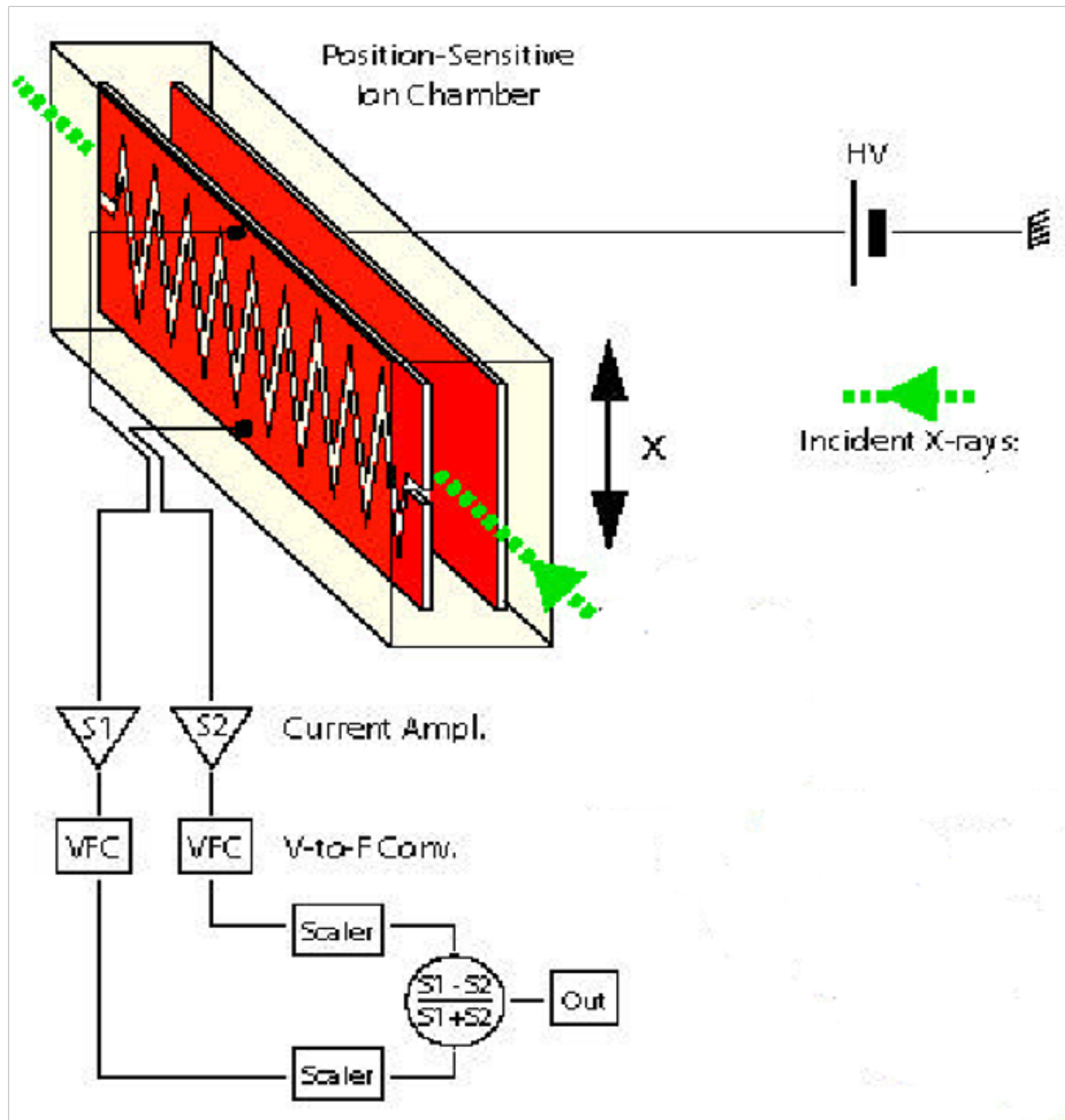
with higher field wiggler  
magnets it maybe feasible  
at  $\sim 0.2$  GeV

# Longitudinal Spin Light radiates different number of photons above and below the orbital plane



**Position dependent asymmetry**

# Longitudinal polarimetry is more complex but still feasible for few GeV beams (relative only)



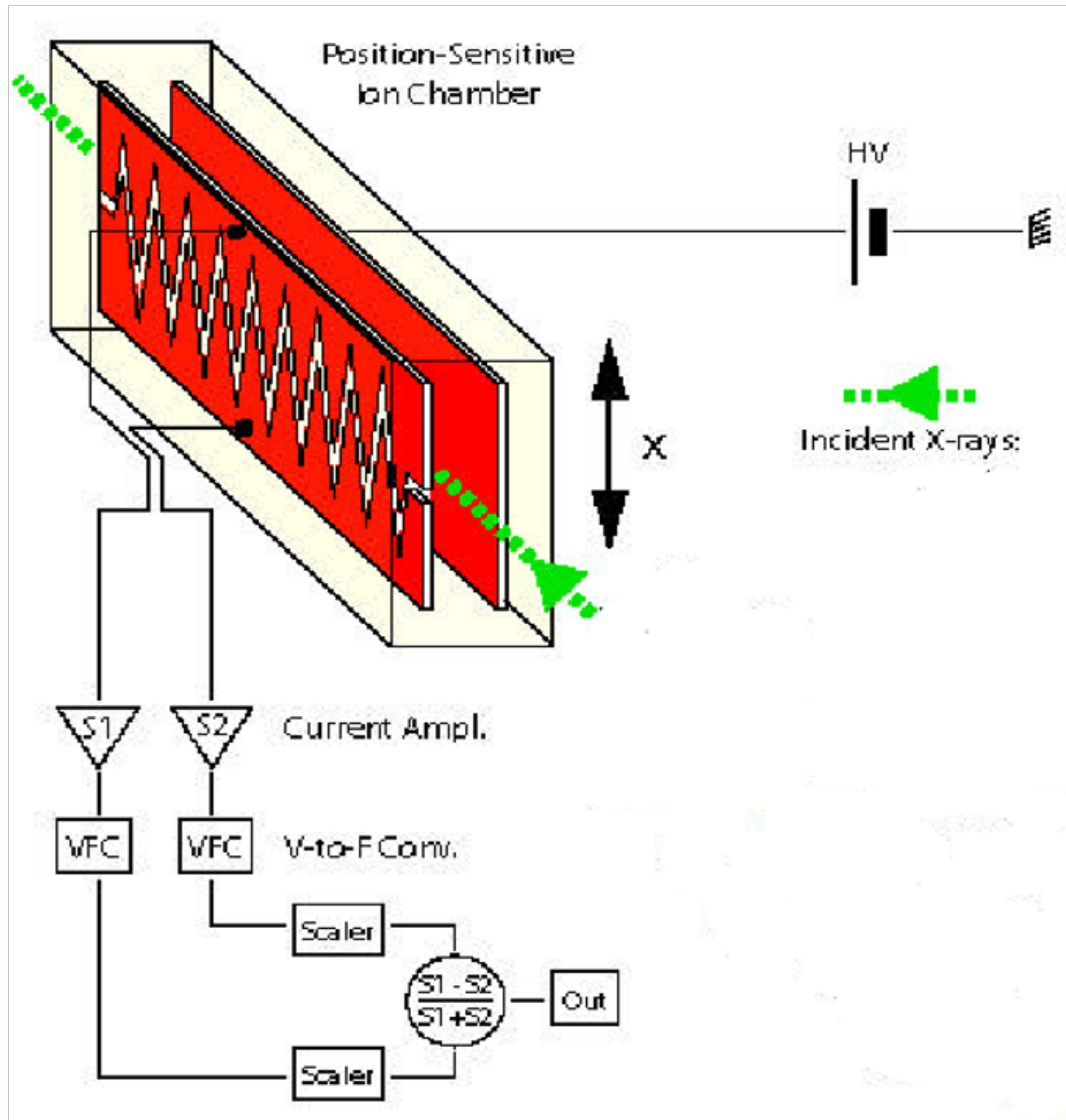
Source is same 3 pole wiggler

But detector must pick out a small position sensitive asymmetry

A transparent differential ionization chamber or converter+ diamond strip detector

K. Sato, J. of Synchrotron Rad., 8, 378 (2001)

# Longitudinal polarimetry is more complex but still feasible for few GeV beams (relative only)



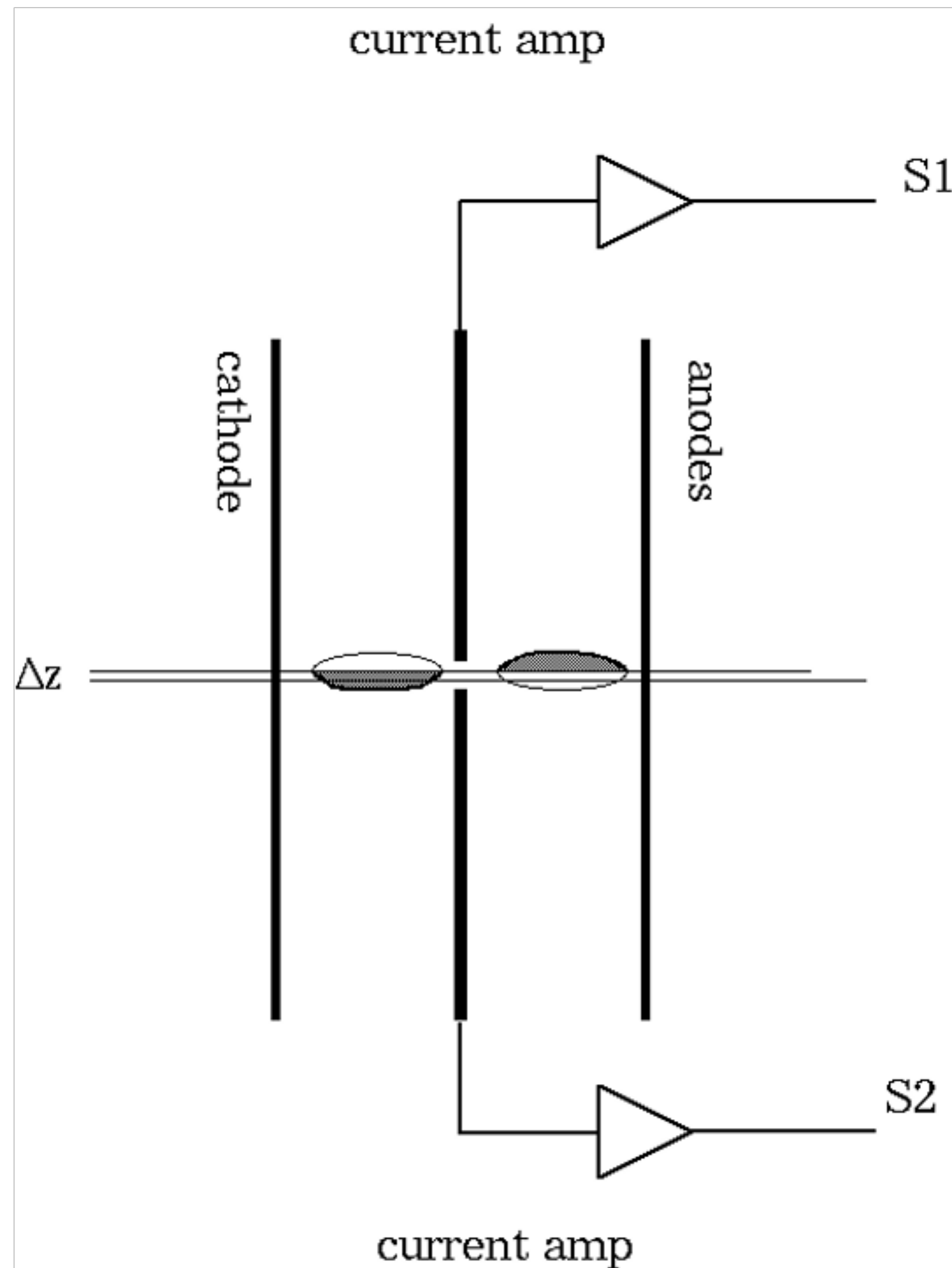
**A transparent differential ionization chamber**

**Split plane chamber design helps detect small position sensitive X-ray signal (5 $\mu$ m resolution)**

**Visible portion of SR can be used to align chamber**

K. Sato, J. of Synchrotron Rad., 8, 378 (2001)

# Longitudinal polarimetry is more complex but still feasible for few GeV beams (relative only)



$$S1 = (N^l_{SR} + \Delta N^l_{spin} + \Delta N^l_z) - (N^r_{SR} - \Delta N^r_{spin} + \Delta N^r_z)$$

Vertical beam motion cancels out

$$S2 = (N^l_{SR} - \Delta N^l_{spin} - \Delta N^l_z) - (N^r_{SR} + \Delta N^r_{spin} - \Delta N^r_z)$$

$$(S1 - S2) = 4\Delta N_{spin}$$

$$(S1 + S2) = 0$$

The sum of the two helicities can be used to measure efficiency and B-field related dilutions.

# Longitudinal polarimetry is challenging for 0.5 GeV beams, but maybe feasible for few GeV beams

For  $E_{\text{beam}} = 0.5 \text{ GeV}$

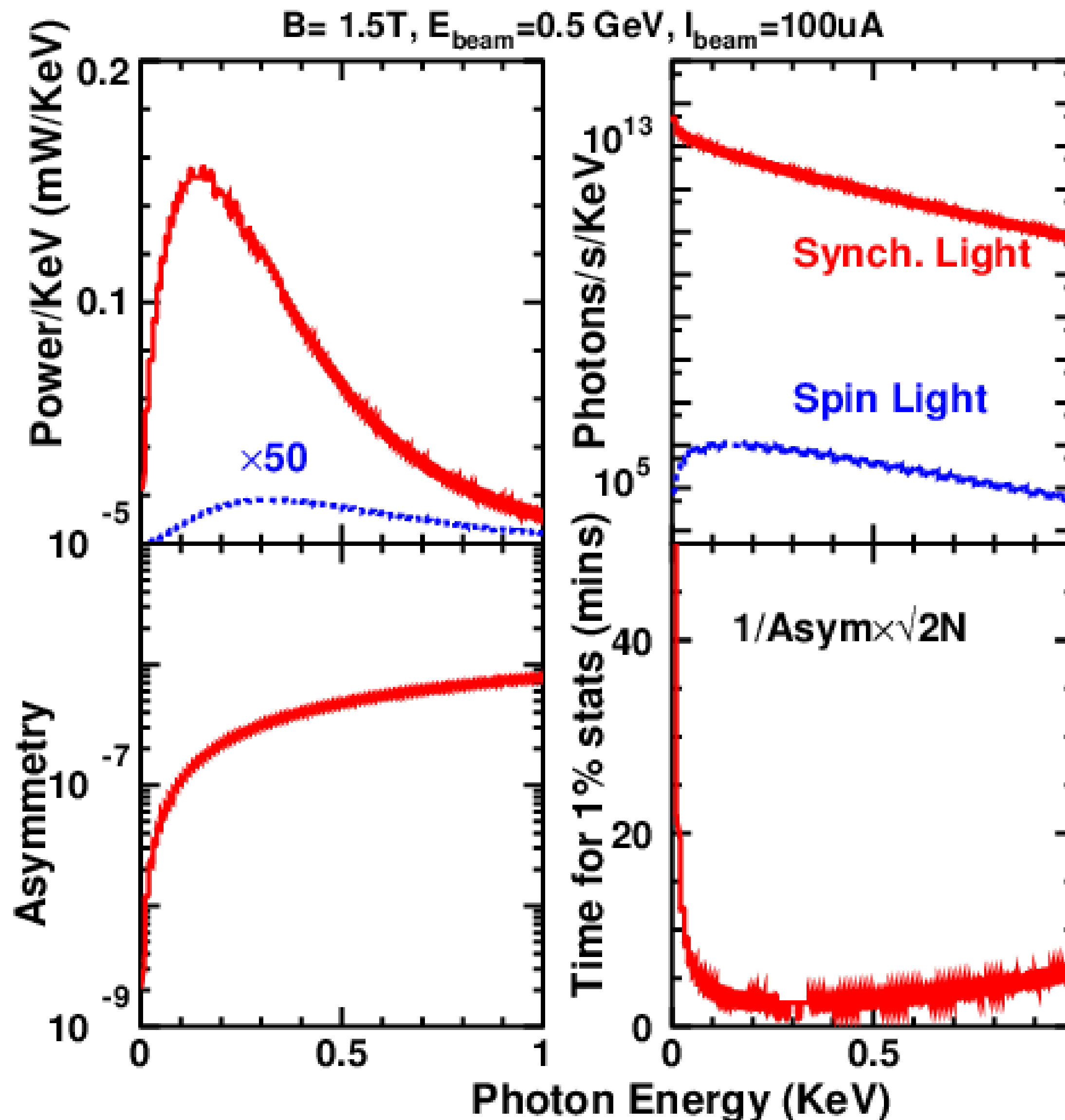
$B = 1.5 \text{ T}$

Peak spin light power  
@ 0.35 KeV

Asym (0.35KeV)  $\sim 0.3 \text{ ppm}$

For 100uA current

1% statistics in  
few minutes



$$\text{Asym} \propto E_{\text{beam}}^2$$

$$\text{Asym} \propto B$$

Longitudinal polarimeters  
marginally easier for  
 $E_{\text{beam}} > 0.5 \text{ GeV}$

or with higher field wiggler  
magnets

# Summary

**Spin light based polarimeter is a viable option for precision non-invasive polarimetry**

**It is based on a well demonstrated concept (for transversely polarized electrons), the necessary technology is readily available and widely used in light sources across the world.**

**It is most viable for beam energies  $\geq 0.5$  GeV as a absolute transverse polarimeter**

**Longitudinal polarimetry is more complicated but relative measurements are feasible**

**Spin light based polarimetry may be marginally viable for 0.2 - 0.5 GeV beams.**