Secondary Beams at Jefferson Lab Workshop

Precision physics with low-energy muons

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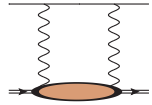
simplistic thoughts of a blissfully ignorant theorist

- theory for low-energy processes (with leptons) is typically limited by non-perturbative contributions
- can we turn this round and learn about non-perturbative physics ??
- replacing electrons by muons leads to complementarity information / advantages



- (1) overview of state-of-the-art for QED
- (2) three examples where $e \to \mu$ is theoretically interesting (not all are new)

QED status

- huge progress in past ~ 10 years (please do not use theory from the 70's any longer)
- fixed-order NNLO QED corrections (at low energies) available/planned for

$\ell \to \ell' \nu \bar{\nu}$ $\quad \ell \to \ell' \nu \bar{\nu} \gamma$ $\quad \ell \to \ell' \nu \bar{\nu} (e^+ e^-)$

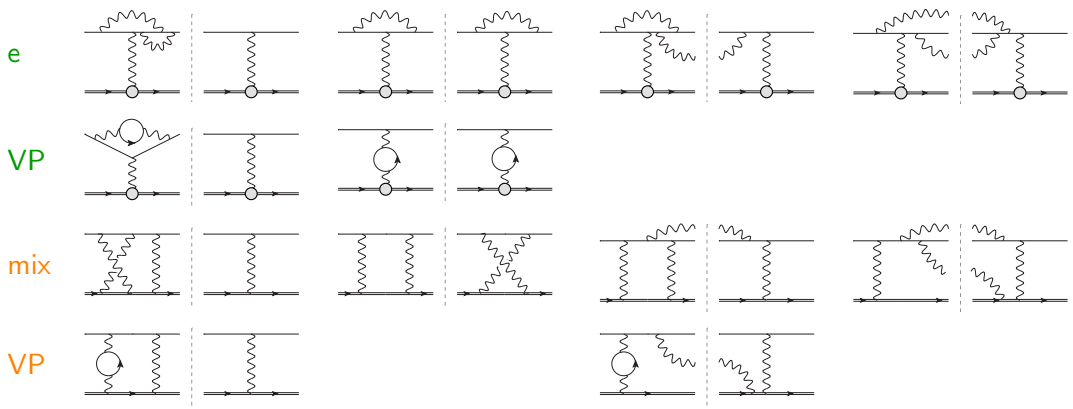
$e^\pm \mu \to e^\pm \mu$ $\quad e^\pm e^\pm \to \ell^\pm \ell^\pm$ $\quad \ell\, p \to \ell\, p$ $\quad \ell\, N \to \ell\, N$

$e^+ e^- \to \gamma^*$ $\quad e^+ e^- \to \gamma\, \gamma$ $\quad e^+ e^- \to \mu^+ \mu^- \gamma$

$e^+\, e^- \to \pi^+\, \pi^-$ $\quad e^+\, e^- \to \pi^+\, \pi^-\, \gamma$

- full NNLO (beware of liberal use of the letter N), fully differential, toying with N^{3}LO
- polarisation adds complication
- but largest uncertainty is often due to non-perturbative hadronic effects

example: $e^- N \to e^- N$ at NNLO, $N \in \{e, \mu, p, N\}$



also e → N corrections, but they are often tiny for "difficult" N

e.g. $\mu^\pm e^- \to \mu^\pm e^-$ at NNLO: many people involved, Monte Carlo and beyond !

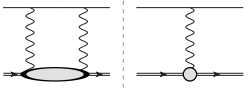
- many diagrams $\to$ automate generation of diagrams, algebra, reduction to master integrals $\Rightarrow$ amplitude $\sim$ 60 Mb

  [Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, Torres Bobadilla, Tramontano]
- MUonE two-loop integrals with $m_e = 0$ expressed in terms of generalised polylogs $\to$ develop Fortran tool for fast numerical evaluation [handyG]
- include effects of $m_e \neq 0$ in mixed NNLO approximately (massification)

  [Penin; Becher, Melnikov; Engel, Gnendiger, AS, Ulrich]
- delicate numerics for one-loop (up to pentagon) diagrams $\to$ use [OpenLoops]
- delicate numerics in phase space integration $\to$ use next-to-soft approach, extension of LBK theorem beyond NLO [Engel, AS, Ulrich, MCMULE]

TPE from e.g. MUSE, $p = 210\,\mathrm{MeV}$

find



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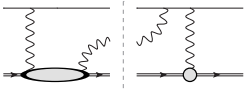
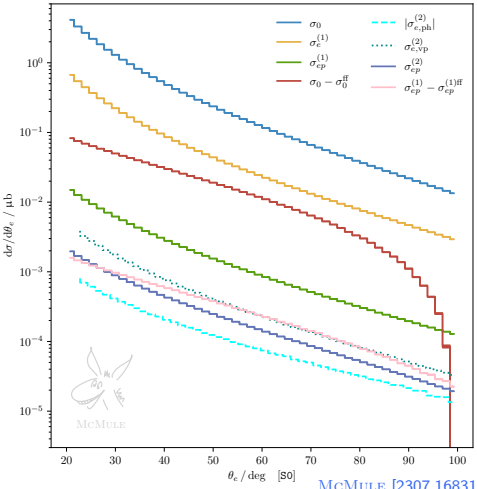


in slide 4
(NNLO QED is sufficient)

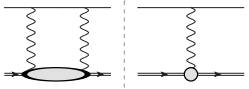
ep : no cut on (forward) γ



McMule [2307.16831]

TPE from e.g. MUSE, $p = 210\,\mathrm{MeV}$

find



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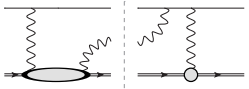
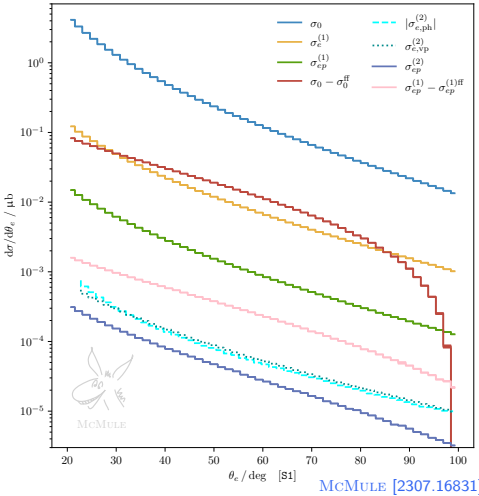


in slide 4
(NNLO QED is sufficient)

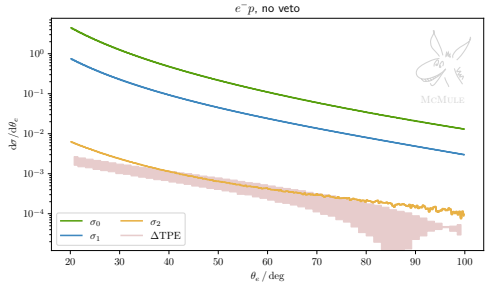
$e\,p$: with cut on (forward) γ



McMule [2307.16831]

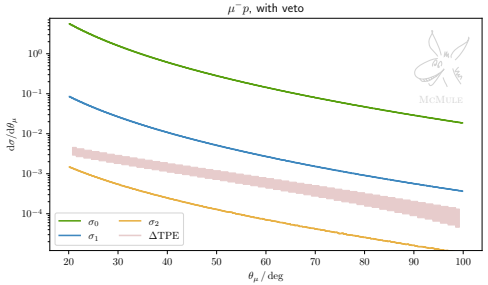
TPE from e.g. MUSE, $p = 210\,\mathrm{MeV}$

$e\,p$: no cuts on γ



QED corrections large

$\mu\,p$: no/cut on forward γ

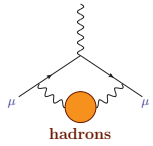


QED corrections small

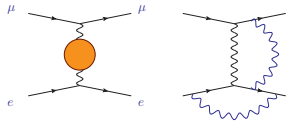
avoiding initial state collinear emission and $e \to \mu$ helps

$e\,p$ with cut looks similar to $\mu\,p$ (with/out) cut

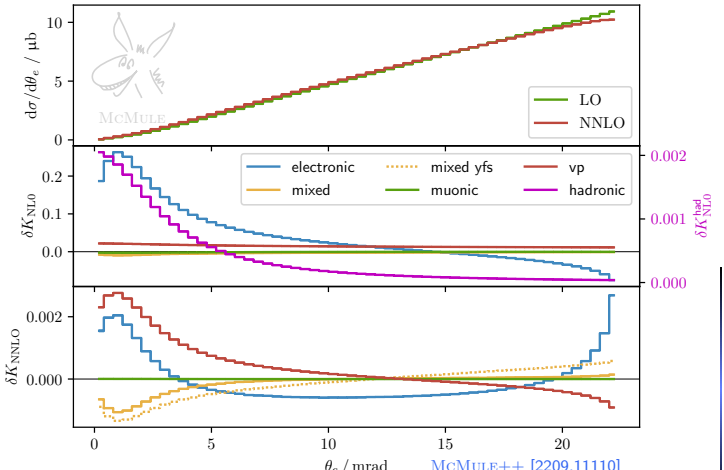
$\Pi_{\gamma\gamma}(q^2)$ from MUonE



from



measure a 0.1% effect to 1%
(need to go beyond NNLO)

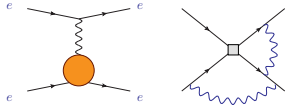


$\Pi_{\gamma Z}(q^2)$ for MOLLER

PV 'kills' QED, EW pops up

hadrons

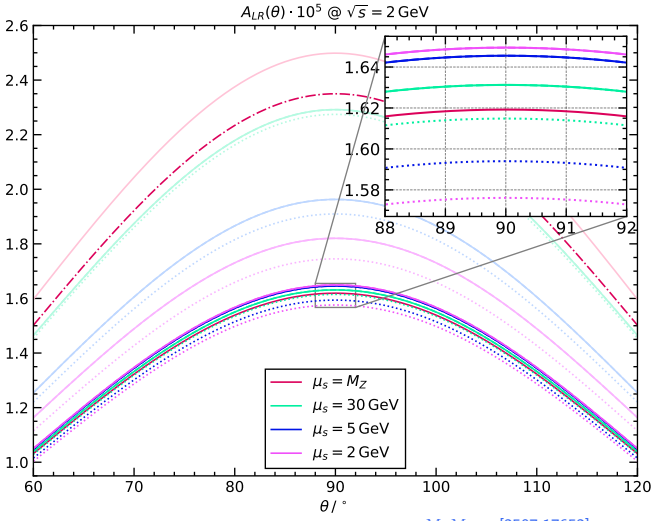
from



measure a large effect to ?%

RGE improved (L)EFT

non-perturbative matching at $\sim 2\,\text{GeV}$



McMule [2507.17652]

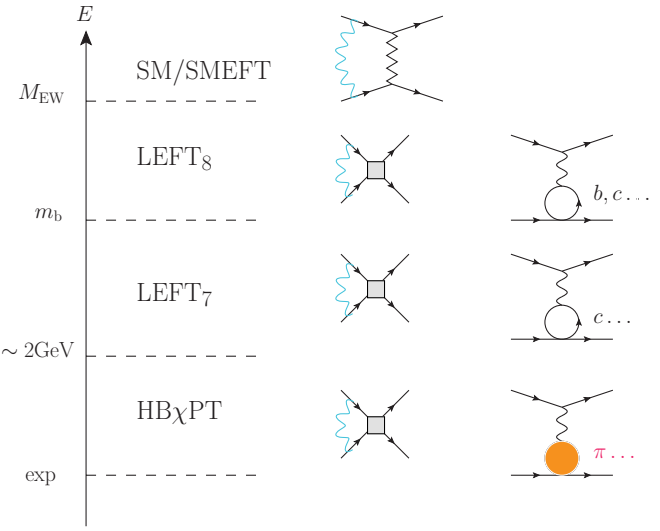
use $\mu^\pm X \to \mu^\pm X$ with $X \in \{e, p, {}^{12}C, \ldots\}$

extract information on EW / non-perturbative effects ???

facilitate non-pertubative matching LEFT$\to$ EFT$_{@\mu_{\text{exp}}}$

maybe Lattice will also help at some point

learn from $(g-2)_\mu$ history



the end

since you encouraged crazy ideas ...

if you wanted to study

- TPE effects in $\mu^{\pm} N \to \mu^{\pm} N$ for $N \neq p$ but also deuteron or ^{12}C or ...
- weak / non-perturbative effects in polarised μ scattering on N
- weak / non-perturbative effects in (un/polarised) μ on (unpolarised) e
- produce and use data to fix / constrain non-perturbative effects

QED effects are (will be) known to sufficient precision to be disentangled

but they have to be taken into account !!