Global QCD analysis of parton distribution functions

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Parton distributions in hadrons

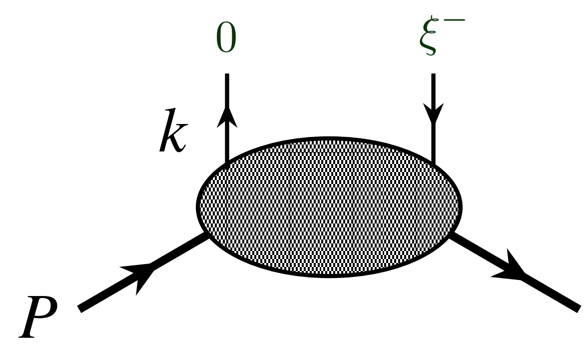
- Parton distribution functions (PDFs) are light-cone correlation functions

$$q(x) = \int_{-\infty}^{\infty} d\xi^- e^{-ixP^+\xi^-} \langle P \,|\, \overline{\psi}(\xi^-)\, \gamma^+ \mathcal{W}(\xi^-, 0)\, \psi(0) \,|\, P \rangle$$

→ parton light-cone momentum fraction $x = \dfrac{k^+}{P^+}$

→ Wilson line (gauge invariance)

$$\mathcal{W}(\xi^-, 0) = \exp\left\{ -\, ig \int_0^{\xi^-} d\eta^- \mathcal{A}^+(\eta^-) \right\}$$



- In $\mathcal{A}^+ = 0$ gauge, in fast-moving frame PDF has a probabilistic interpretation as a particle density

$$\int_{-1}^{1} dx\, q(x) \;=\; \langle P \,|\, \overline{\psi}(0)\, \gamma^+ \psi(0) \,|\, P \rangle \;\approx\; \langle P \,|\, \psi^\dagger(0)\, \psi(0) \,|\, P \rangle$$

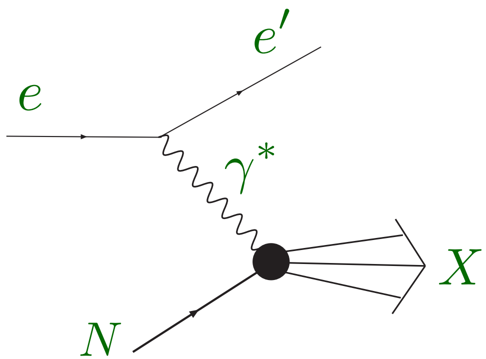
number density

$\overline{\psi}\, \gamma^0\, \psi \approx \overline{\psi}\, \gamma^z\, \psi$

number operator

Parton distributions in hadrons

- Most information on PDFs obtained from lepton-nucleon deep-inelastic scattering (DIS)



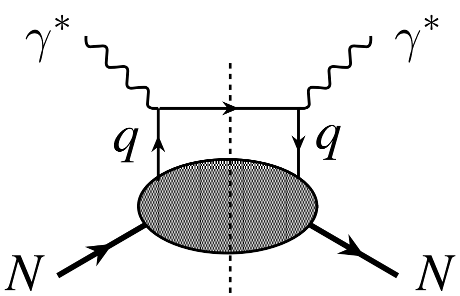
$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\frac{F_1}{M} + \frac{F_2}{\nu}\right)$$

$$x_B = \frac{Q^2}{2M\nu} \qquad \begin{aligned} Q^2 &= \vec{q}^{\,2} - \nu^2 \\ \nu &= E - E' \\ y &= \nu/E \end{aligned}$$

⟶ structure function given as convolution of Wilson coefficient and a PDF

$$F_2(x_B, Q^2) = x_B \sum_q e_q^2 \int_{x_B}^{1} \frac{dx}{x} C_q\left(\frac{x_B}{x}, \alpha_s\right) q(x, Q^2)$$

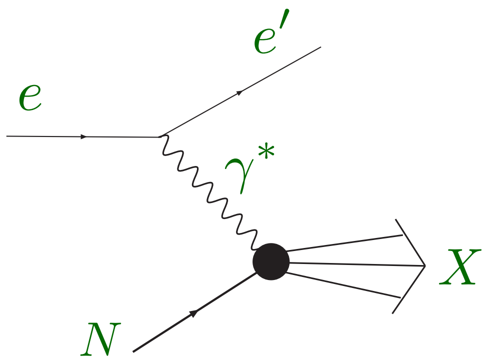
$$\longrightarrow x_B \sum_q e_q^2\, q(x_B, Q^2)$$



for leading order approximation $C_q \longrightarrow \delta\left(1 - \frac{x_B}{x}\right)$

Parton distributions in hadrons

- Most information on PDFs obtained from lepton-nucleon deep-inelastic scattering (DIS)



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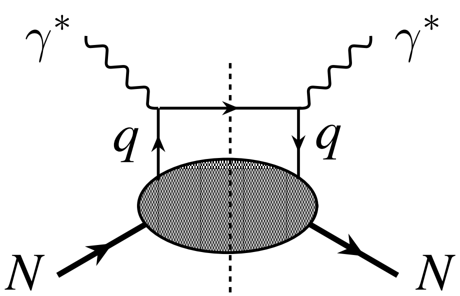
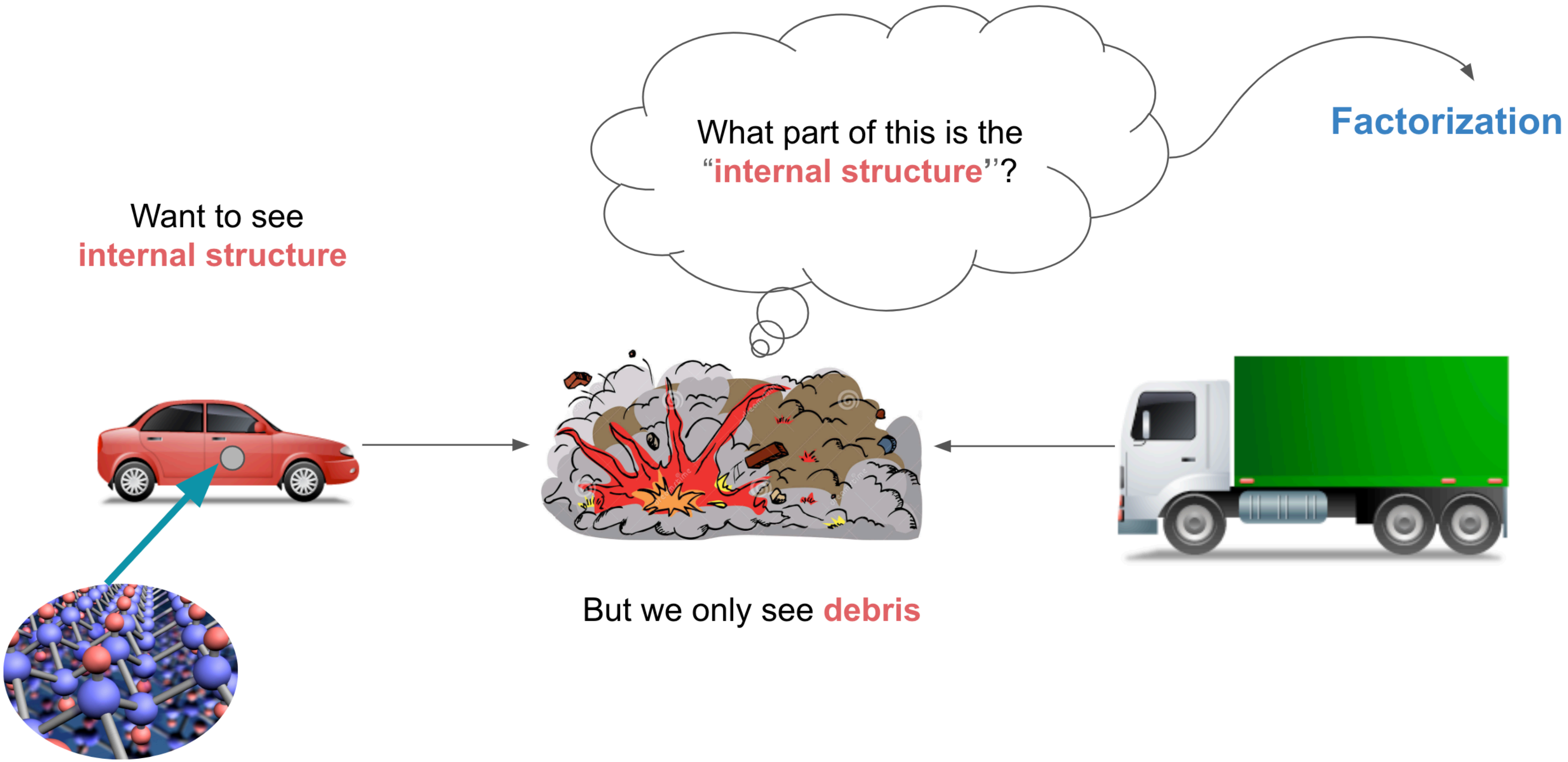


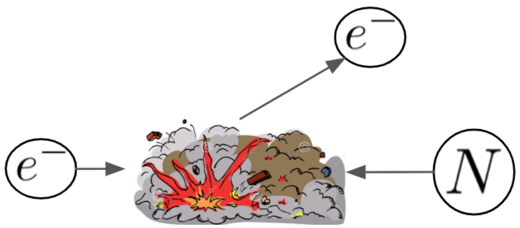
⟶ <u>note</u>: in general, parton fraction x (internal, unphysical) is <u>*not*</u> the same as Bjorken x_B (physical, specific to DIS)

Factorization in DIS

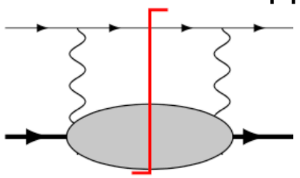
- how do we get hadron structure from experimental data?



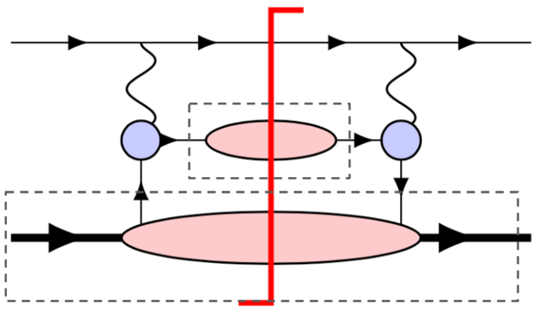
Factorization in DIS

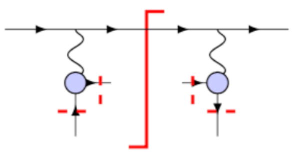


$$E' \frac{\mathrm{d}\sigma}{\mathrm{d}^3 l'}$$

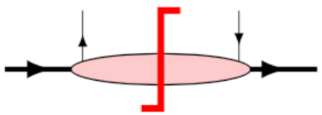


Approximations





Collision dependent factor

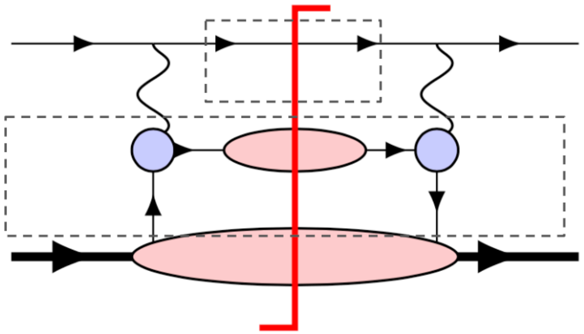


Internal structure

$$\sum_i \int_{x_B}^{1} \frac{\mathrm{d}\xi}{\xi} C_i(\xi) f_i\left(\frac{x_B}{\xi}\right) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

Error of approximations

Factorization in DIS



$$\frac{d^2\sigma}{dx_B\,dy} = \frac{2\pi\alpha^2}{x_B y Q^2}\left(\left(Y_+ + 2x_B^2 y^2 \frac{M^2}{Q^2}\right)\boxed{F_2} - y^2 \boxed{F_L} \mp Y_- x_B \boxed{F_3}\right)$$

$$F_2^p(x_B, Q^2) = \sum_q e_q^2 \int_{x_B}^1 \frac{d\xi}{\xi}\left[f_{q/p}(\xi,\mu^2) C_q \left(\frac{x_B}{\xi}, \frac{Q^2}{\mu^2}, \alpha_S(\mu^2)\right) + f_{g/p}(\xi,\mu^2) C_g \left(\frac{x_B}{\xi}, \frac{Q^2}{\mu^2}, \alpha_S(\mu^2)\right)\right]$$

Quark contributions

Gluon contributions

at lowest order $C_g = 0$

Mellin convolutions

Definition of a convolution of two functions

$$\sigma(z) = \int_z^1 \frac{d\xi}{\xi} h(\xi) f\left(\frac{z}{\xi}\right)$$

$$\boxed{\Sigma(N) = \int_0^1 dz\ z^{N-1}\sigma(z)}$$

$$\boxed{\Sigma(N) = H(N)\ F(N)}$$

$$H(N) = \int_0^1 dy\ y^{N-1} h(y)$$

$$F(N) = \int_0^1 dy\ y^{N-1} f(y)$$

Mellin transform makes a convolution an ordinary product

DIS in Mellin space

$$F_i^p(x_B\,, Q^2) = \sum_q e_q^2 \int_{x_B}^1 \frac{d\xi}{\xi}\, f_{q/p}(\xi, \mu^2) C_{q,i}\left(\frac{x_B}{\xi}, \frac{Q^2}{\mu^2}, \alpha_S(\mu^2)\right) + (q \to g)$$

$$F_i^p(N, Q^2) = \sum_q e_q^2\, f_{q/p}(N, \mu^2) C_{q,i}\left(N, \frac{Q^2}{\mu^2}, \alpha_S(\mu^2)\right) + (q \to g)$$

$$C_j(N) = C_j^{[0]}(N) + \frac{\alpha_S}{4\pi} C_j^{[1]}(N) + O(\alpha_S^2)$$

Mellin transforms

Mellin transform of f(x)

$$F(N) = \int_0^1 dx x^{N-1} f(x)$$

Can be done numerically

$$f(x) = \frac{1}{2\pi i} \int_c dN x^{-N} F(N)$$

Inverse Mellin transform

Complex contour integration

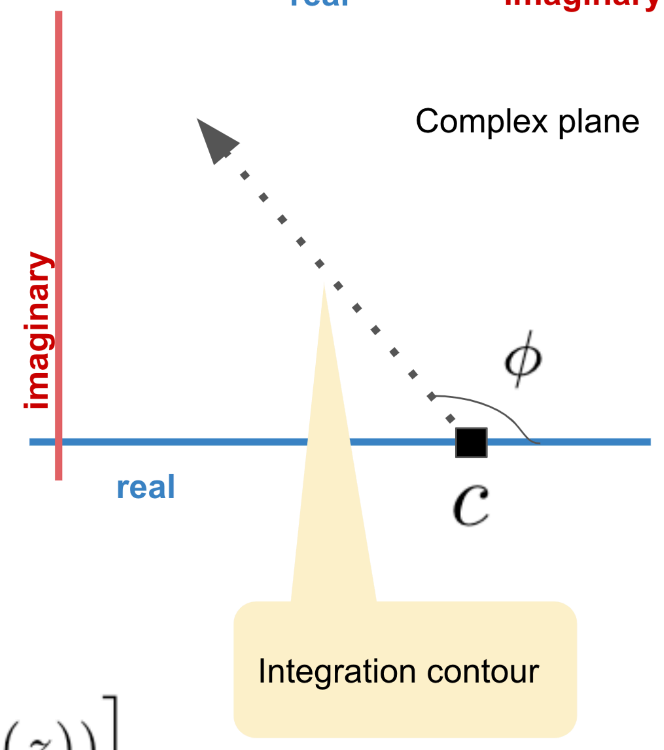
Numerical implementation

$$f(x) = \frac{1}{2\pi i} \int_c dN x^{-N} F(N)$$

$$N(z) = c + z e^{i\phi}$$

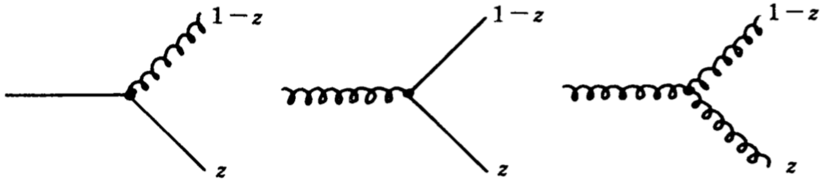
$$f(x) = \frac{1}{\pi} \int_0^\infty dz \mathrm{Im} \left[e^{i\phi} x^{-N(z)} F(N(z)) \right]$$

$$N(z) = \underbrace{[c + z\cos(\phi)]}_{\text{real}} + i \underbrace{[z\sin(\phi)]}_{\text{imaginary}}$$



Evolution equations

- Scale evolution equations arise from interactions between quarks & gluons



(a) (b) (c)

→ Definition of PDFs in field theory requires renormalization

PDFs will depend on renormalization scale and its RGEs are the famous DGLAP equations

UV singularity when the field separation is zero

$$f_i(\xi) \stackrel{!}{=} \int \frac{\mathrm{d}w^-}{4\pi} e^{-i\xi p^+ w^-} \langle N|\bar{\psi}_i(0, w^-, \mathbf{0}_\mathrm{T})\gamma^+\psi_i(0)|N\rangle$$

Renormalization

$$f = Z_F \otimes f_\mathrm{bare}$$

$$f(\xi) \to f(\xi, \mu)$$

Dokshitzer–**G**ribov–**L**ipatov–**A**ltarelli–**P**arisi

$$\boxed{\frac{\mathrm{d} f_i(\xi, \mu^2)}{\mathrm{d} \ln \mu^2} = \sum_j \int_\xi^1 \frac{\mathrm{d}y}{y} P_{ij}(\xi, \mu^2)\, f_j\left(\frac{y}{\xi}, \mu^2\right)}$$

aka **DGLAP**

parton $i \to j$ splitting function

Evolution equations

- DGLAP in Mellin space

Splitting kernels

$$\frac{\partial}{\partial \ln \mu^2} f_{j/H}(\xi, \mu) = \sum_{j'} \int_\xi^1 \frac{dz}{z} P_{jj'}(z, g) f_{j'/H}(\xi/z, \mu)$$

System of integro-differential equations

$$\frac{\partial}{\partial \ln \mu^2} F_{j/H}(N, \mu) = \sum_{j'} P_{jj'}(N, \mu)\ F_{j'/H}(N, \mu)$$

Ordinary system of differential equations

Can be solved Analytically!

Flavor composition

$$\frac{\partial}{\partial \ln \mu^2} F_{j/H}(N,\mu) = \sum_{j'} P_{jj'}(N,\mu)\; F_{j'/H}(N,\mu)$$

11 equations for 5 active quark flavors + glue

$$F_{j/H}(N) = F_j$$

$$F_{q^\pm} = F_q \pm F_{\bar{q}}$$

$$\begin{aligned}
F_{\pm 3} &= F_{u^\pm} - F_{d^\pm}, \\
F_{\pm 8} &= F_{u^\pm} + F_{d^\pm} - 2F_{s^\pm}, \\
F_{\pm 15} &= F_{u^\pm} + F_{d^\pm} + F_{s^\pm} - 3F_{c^\pm}, \\
F_{\pm 24} &= F_{u^\pm} + F_{d^\pm} + F_{s^\pm} + F_{c^\pm} - 4F_{b^\pm}, \\
F_{\pm} &= F_{u^\pm} + F_{d^\pm} + F_{s^\pm} + F_{c^\pm} + F_{b^\pm}
\end{aligned}$$

$$F_{j/H}(N) \Longrightarrow \boxed{F_{\pm j}} \quad \boxed{F_{\pm}} \quad \boxed{F_g}$$

Just linear transformations

11 = 8 + 2 + 1

Evolution equations

Flavor singlet and non-singlet evolution

$$\frac{\partial}{\partial \ln \mu^2} F_{j/H}(N,\mu) = \sum_{j'} P_{jj'}(N,\mu)\; F_{j'/H}(N,\mu)$$

Non singlet combinations decouples from glue

$$\boxed{\frac{\partial F_{\pm j}}{\partial \ln \mu^2} = P^{\pm}_{\mathrm{NS}} F_{\pm j}} \qquad \boxed{\frac{\partial F_{-}}{\partial \ln \mu^2} = P^{-}_{\mathrm{NS}} F_{-}}$$

⇒ **Non-singlet** evolution

$$\boxed{\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} F_{+} \\ F_{g} \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \begin{pmatrix} F_{+} \\ F_{g} \end{pmatrix}}$$

⇒ **Singlet** evolution

Solving the non-singlet evolution equations

$$\boxed{\frac{\partial F_{\pm j}}{\partial \ln \mu^2} = P_{\rm NS}^{\pm} F_{\pm j}} \qquad P_{ij}(a_S) = \sum_{m=0}^{\infty} a_S^{m+1}(\mu) P_{ij}^{(m)}$$

Splitting functions depend only on alphaS

$$\Downarrow$$

$$\frac{d a_S}{d \ln \mu^2} = \beta(a_s) \qquad a_s = \frac{\alpha_S}{2\pi}$$

$$\Downarrow$$

$$\boxed{\frac{\partial F_{\pm j}}{\partial a_S} = -\frac{1}{\beta_0 a_S} P_{\rm NS}^{\pm(0)} F_{\pm j}} \quad \Longrightarrow \quad \boxed{F_{\pm j}(a_S) = \left(\frac{a_S}{a_0}\right)^{-P_{\rm NS}^{\pm(0)}/\beta_0} F_{\pm j}(a_0)}$$

alphaS at the final scale

alphaS at the input scale

Solving the singlet evolution equations

Eigenvalue decomposition

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} F_+ \\ F_g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix}\begin{pmatrix} F_+ \\ F_g \end{pmatrix}$$

$$\Downarrow$$

$$\frac{\partial}{\partial a_S}\begin{pmatrix} F_+ \\ F_g \end{pmatrix} = \frac{-1}{\beta_0 a_S}\begin{pmatrix} P_{qq}^{(0)} & P_{qg}^{(0)} \\ P_{gq}^{(0)} & P_{gg}^{(0)} \end{pmatrix}\begin{pmatrix} F_+ \\ F_g \end{pmatrix}$$

$$\boldsymbol{R}_0 = \frac{1}{\beta_0}\begin{pmatrix} P_{qq}^{(0)} & P_{qg}^{(0)} \\ P_{gq}^{(0)} & P_{gg}^{(0)} \end{pmatrix} = r_-\boldsymbol{e}_- + r_+\boldsymbol{e}_+$$

$$\boldsymbol{e}_\pm = \frac{1}{r_\pm - r_\mp}\left[\boldsymbol{R}_0 - r_\mp \boldsymbol{I}\right]$$

$$r_\pm = \frac{1}{2\beta_0}\left[P_{qq}^{(0)} + P_{gg}^{(0)} \pm \sqrt{\left(P_{qq}^{(0)} - P_{gg}^{(0)}\right)^2 + 4P_{qg}^{(0)}P_{gq}^{(0)}}\right]$$

$$\boxed{\begin{pmatrix} F_+(a_S) \\ F_g(a_S) \end{pmatrix} = \left[\boldsymbol{e}_-\left(\frac{a_S}{a_0}\right)^{-r_-} + \boldsymbol{e}_+\left(\frac{a_S}{a_0}\right)^{-r_+}\right]\begin{pmatrix} F_+(a_0) \\ F_g(a_0) \end{pmatrix}}$$

Evolution equations

Flavor decomposition

$$\boxed{F_{\pm j}} \quad \boxed{F_{\pm}} \quad \boxed{F_g}$$

$$F_{b^\pm} = (F_{-} - F_{\pm 24})/5$$

$$F_{c^\pm} = F_{b^\pm} + (F_{\pm 24} - F_{\pm 15})/4$$

$$F_{s^\pm} = F_{c^\pm} + (F_{\pm 15} - F_{\pm 8})/3$$

$$F_{d^\pm} = F_{s^\pm} + (F_{\pm 8} - F_{\pm 3})/2$$

$$F_{u^\pm} = F_{s^\pm} + (F_{\pm 8} + F_{\pm 3})/2$$

$$F_q = \frac{1}{2}(F_{q^+} + F_{q^+})$$

$$F_{\bar{q}} = \frac{1}{2}(F_{q^+} - F_{q^-})$$

Evolution equations

Evolution flow

μ^2

$\boxed{F_{\pm j}}$ $\boxed{F_\pm}$ $\boxed{F_g}$ $\Rightarrow$

$$F_{b^\pm} = (F_- - F_{\pm 24})/5$$
$$F_{c^\pm} = F_{b^\pm} + (F_{\pm 24} - F_{\pm 15})/4$$
$$F_{s^\pm} = F_{c^\pm} + (F_{\pm 15} - F_{\pm 8})/3$$
$$F_{d^\pm} = F_{s^\pm} + (F_{\pm 8} - F_{\pm 3})/2$$
$$F_{u^\pm} = F_{s^\pm} + (F_{\pm 8} + F_{\pm 3})/2$$

$\Rightarrow$ $F_{j/H}(N)$

Evolved PDFs

$$F_{\pm j}(a_S) = \left(\frac{a_S}{a_0}\right)^{-P_{\rm NS}^{\pm(0)}/\beta_0} F_{\pm j}(a_0)$$

$$\begin{pmatrix} F_+(a_S) \\ F_g(a_S) \end{pmatrix} = \left[\boldsymbol{e}_- \left(\frac{a_S}{a_0}\right)^{-r_-} + \boldsymbol{e}_+ \left(\frac{a_S}{a_0}\right)^{-r_+} \right] \begin{pmatrix} F_+(a_0) \\ F_g(a_0) \end{pmatrix}$$

μ_0^2

Input scale PDFs

$\boxed{F_{\pm j}}$ $\boxed{F_\pm}$ $\boxed{F_g}$ $\Leftarrow$

$$F_{\pm 3} = F_{u^\pm} - F_{d^\pm},$$
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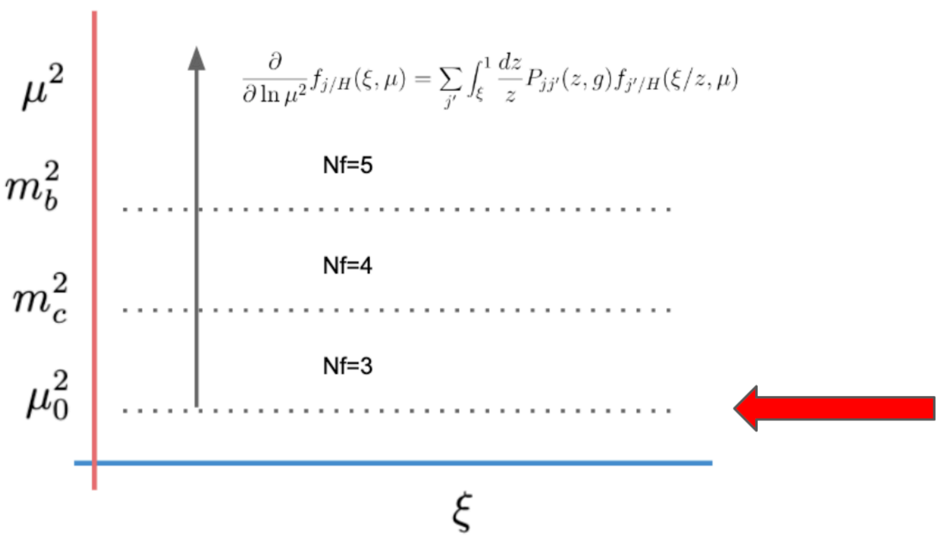
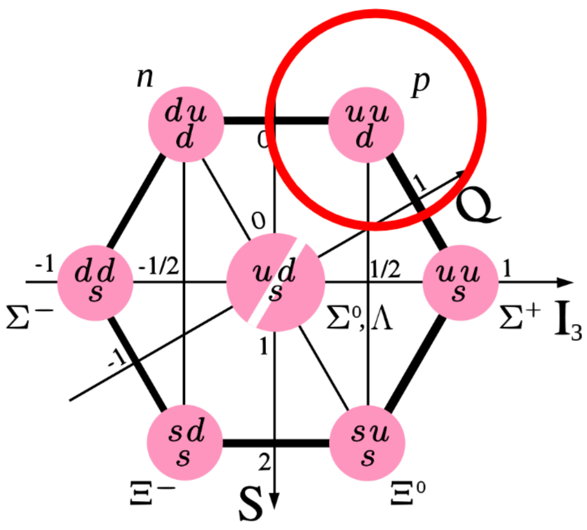
$$F_\pm = F_{u^\pm} + F_{d^\pm} + F_{s^\pm} + F_{c^\pm} + F_{b^\pm}$$

$\Leftarrow$ $F_{j/H}(N)$

ξ

Modeling PDFs

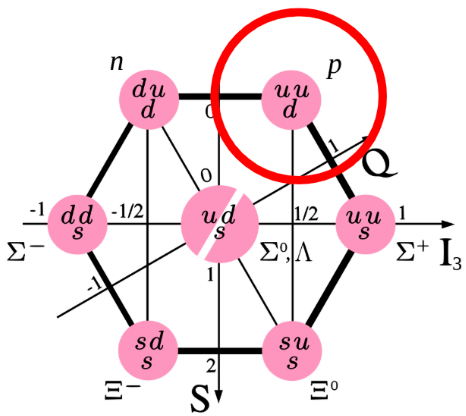




Modeling **input** scale PDFs

Modeling PDFs

Sum rules for proton PDFs

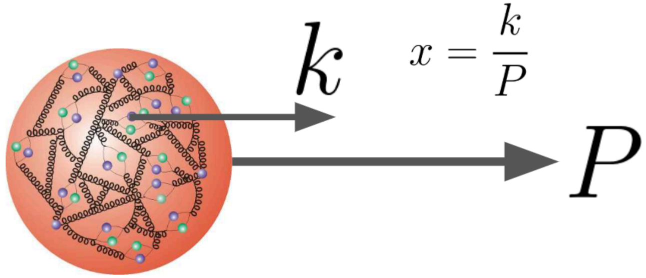


Valence number sum rules

$$\int_0^1 dx\, [f_{u/p}(x) - f_{\bar{u}/p}(x)] = 2$$

$$\int_0^1 dx\, [f_{d/p}(x) - f_{\bar{d}/p}(x)] = 1$$

$$\int_0^1 dx\, [f_{s/p}(x) - f_{\bar{s}/p}(x)] = 0$$



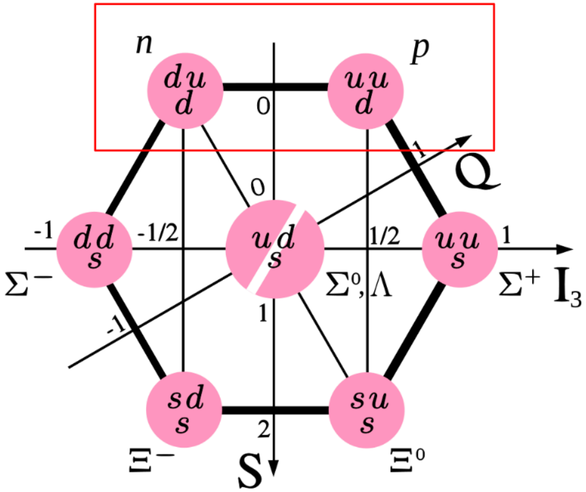
$$x = \frac{k}{P}$$

Momentum sum rule

$$\int_0^1 dx\; x\, [f_g + f_{u^+} + f_{d^+} + f_{s^+} + f_{c^+} + f_{b^+}] = 1$$

Modeling PDFs

- **neutron** PDFs ?



Isospin symmetry

$$f_{u/n} = f_{d/p}$$

$$f_{d/n} = f_{u/p}$$

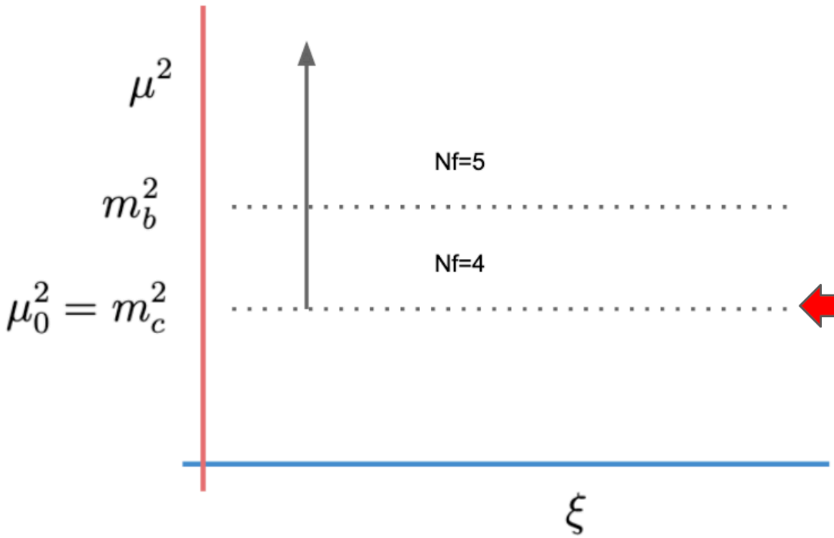
$$f_{\bar{u}/n} = f_{\bar{d}/p}$$

$$f_{\bar{d}/n} = f_{\bar{u}/p}$$

$$m_p \simeq m_n$$

Modeling PDFs

PDF parametrization



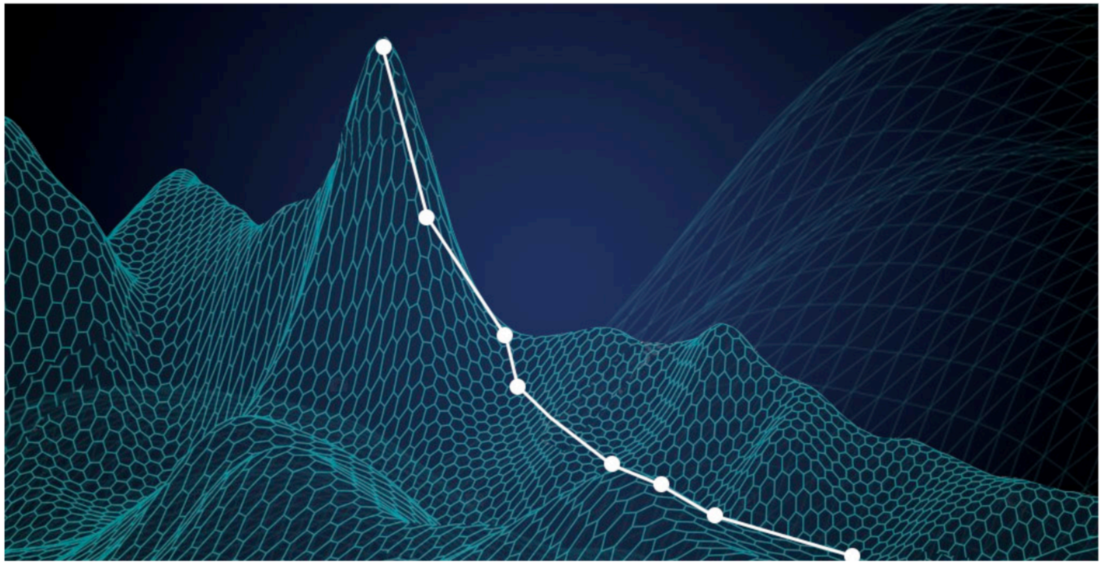
$$u(\xi, \mu_0^2) = u_v(\xi, \mu_0^2) + 2\bar{u}(\xi, \mu_0^2)$$
$$d(\xi, \mu_0^2) = d_v(\xi, \mu_0^2) + 2\bar{d}(\xi, \mu_0^2)$$
$$\bar{u}(\xi, \mu_0^2) = S_1(\xi, \mu_0^2) + \bar{u}_0(\xi, \mu_0^2)$$
$$\bar{d}(\xi, \mu_0^2) = S_1(\xi, \mu_0^2) + \bar{d}_0(\xi, \mu_0^2)$$
$$s(\xi, \mu_0^2) = S_2(\xi, \mu_0^2) + s_0(\xi, \mu_0^2)$$
$$\bar{s}(\xi, \mu_0^2) = S_2(\xi, \mu_0^2) + \bar{s}_0(\xi, \mu_0^2)$$

Generic template function

$$T(\xi; \boldsymbol{a}) = \mathcal{M} \frac{\xi^{\alpha} (1-\xi)^{\beta} \left(1 + \gamma\sqrt{\xi} + \delta\xi\right)}{\int_0^1 \mathrm{d}\xi\, \xi^{\alpha+1} (1-\xi)^{\beta} \left(1 + \gamma\sqrt{\xi} + \delta\xi\right)}$$

$$u_v,\ d_v,\ \bar{u}_0,\ \bar{d}_0,\ s_0,\ \bar{s}_0,\ S_1,\ S_2$$

- Determining PDF parameters from data



The **Loss** function

- Anatomy of **Chi2** function

Experimental data point

Point-by-point correlated systematic uncertainties

Theory with pdf parameters **a**

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{\underline{d_{i,e}} - \sum_k r_e^k \underline{\beta_{i,e}^k} - \underline{T_{i,e}}(\boldsymbol{a})/N_e}{\underline{\alpha_{i,e}}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\underline{\delta N_e}}\right)^2$$

Uncorrelated uncertainties added in quadrature

Overall normalization uncertainty

Anatomy of Chi2 function

Nuisance fitting parameter

Penalties for the Nuisance parameters

$$\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - \sum_k \underline{r_e^k}\beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/\underline{N_e}}{\alpha_{i,e}}\right)^2 + \sum_k \left(\underline{r_e^k}\right)^2 + \left(\frac{1-\underline{N_e}}{\delta N_e}\right)^2$$

$$\frac{\partial\chi^2}{\partial r_e^k} = 0 \quad \Longrightarrow \quad \boxed{\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}}\right)^2 - \sum_{k,k'} B_{k,e}A^{-1}_{kk',e}B_{k',e}}$$

$$B_e^k = \sum_i \frac{\beta_{i,e}^k\left(d_{i,e} - T_{i,e}/N_e\right)}{\alpha_i^2} \qquad A_{kk',e} = \delta_{kk'} + \sum_i \frac{\beta_{i,e}^k\beta_{i,e}^{k'}}{\alpha_i^2}$$

Anatomy of Chi2 function

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{\boxed{d_{i,e}} - \boxed{\sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\boxed{\delta N_e}} \right)^2$$

prior distribution

$$\rho(\boldsymbol{a}|\mathrm{data}) \sim \mathcal{L}(\boldsymbol{a}, \mathrm{data})\pi(\boldsymbol{a})$$

likelihood function

$$\mathrm{E}[\mathcal{O}] = \int d^n a \; \rho(\boldsymbol{a}|\mathrm{data}) \; \mathcal{O}(\boldsymbol{a})$$ expectation value

$$\mathrm{V}[\mathcal{O}] = \int d^n a \; \rho(\boldsymbol{a}|\mathrm{data}) \; \left[\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}]\right]^2$$ variance

Fitting PDFs

Loss function

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\delta N_e}\right)^2$$

⇧

Observables

$$F_i^p(x_{\mathrm{bj}},Q^2) = \sum_q e_q^2 \int_{x_{\mathrm{bj}}}^1 \frac{d\xi}{\xi}\, f_{q/p}(\xi,\mu^2) C_{q,i}\left(\frac{x_{\mathrm{bj}}}{\xi}, \frac{Q^2}{\mu^2}, \alpha_S(\mu^2)\right) + (q\to g)$$

⇧

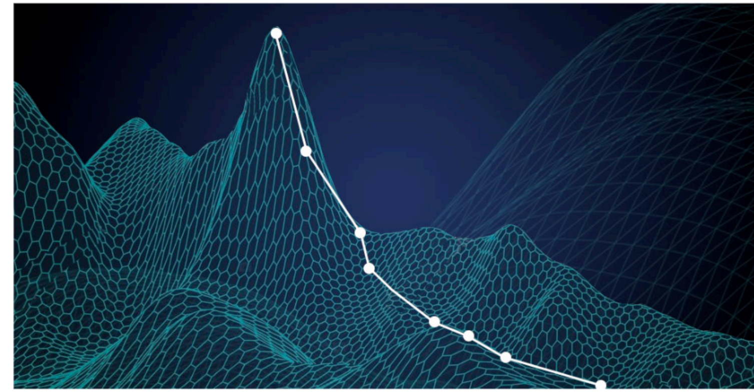
Reset boundary conditions

$$\frac{\partial}{\partial \ln\mu^2} f_{j/H}(\xi,\mu) = \sum_{j'} \int_\xi^1 \frac{dz}{z} P_{jj'}(z,g) f_{j'/H}(\xi/z,\mu)$$

⇧

parametrization

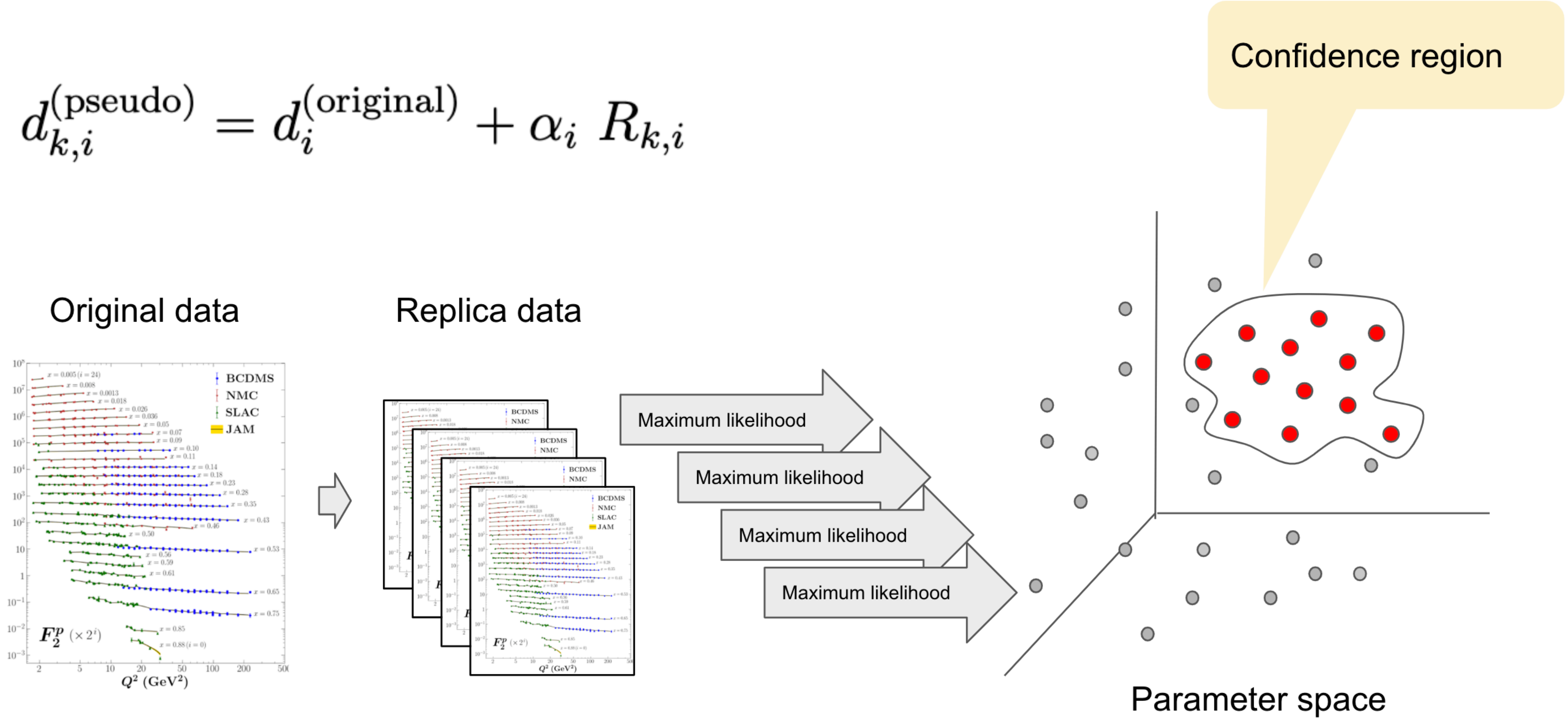
$$T(\xi;\boldsymbol{a}) = \mathcal{M} \frac{\xi^\alpha (1-\xi)^\beta \left(1+\gamma\sqrt{\xi}+\delta\xi\right)}{\int_0^1 \mathrm{d}\xi\, \xi^{\alpha+1}(1-\xi)^\beta\left(1+\gamma\sqrt{\xi}+\delta\xi\right)}$$



Maximum Likelihood

Data resampling

$$d_{k,i}^{(\mathrm{pseudo})} = d_i^{(\mathrm{original})} + \alpha_i \; R_{k,i}$$



Fitting PDFs

- Repeat fits ~ 1,000 times

$$\mathrm{E}[\mathcal{O}] = \int d^n a \ \rho(\boldsymbol{a}|\mathrm{data}) \ \mathcal{O}(\boldsymbol{a})$$

$$\mathrm{V}[\mathcal{O}] = \int d^n a \ \rho(\boldsymbol{a}|\mathrm{data}) \ \left[\mathcal{O}(\boldsymbol{a}) - \mathrm{E}[\mathcal{O}]\right]^2$$

⟶ mean, variance from MC replicas

Build an MC ensemble ($$$)

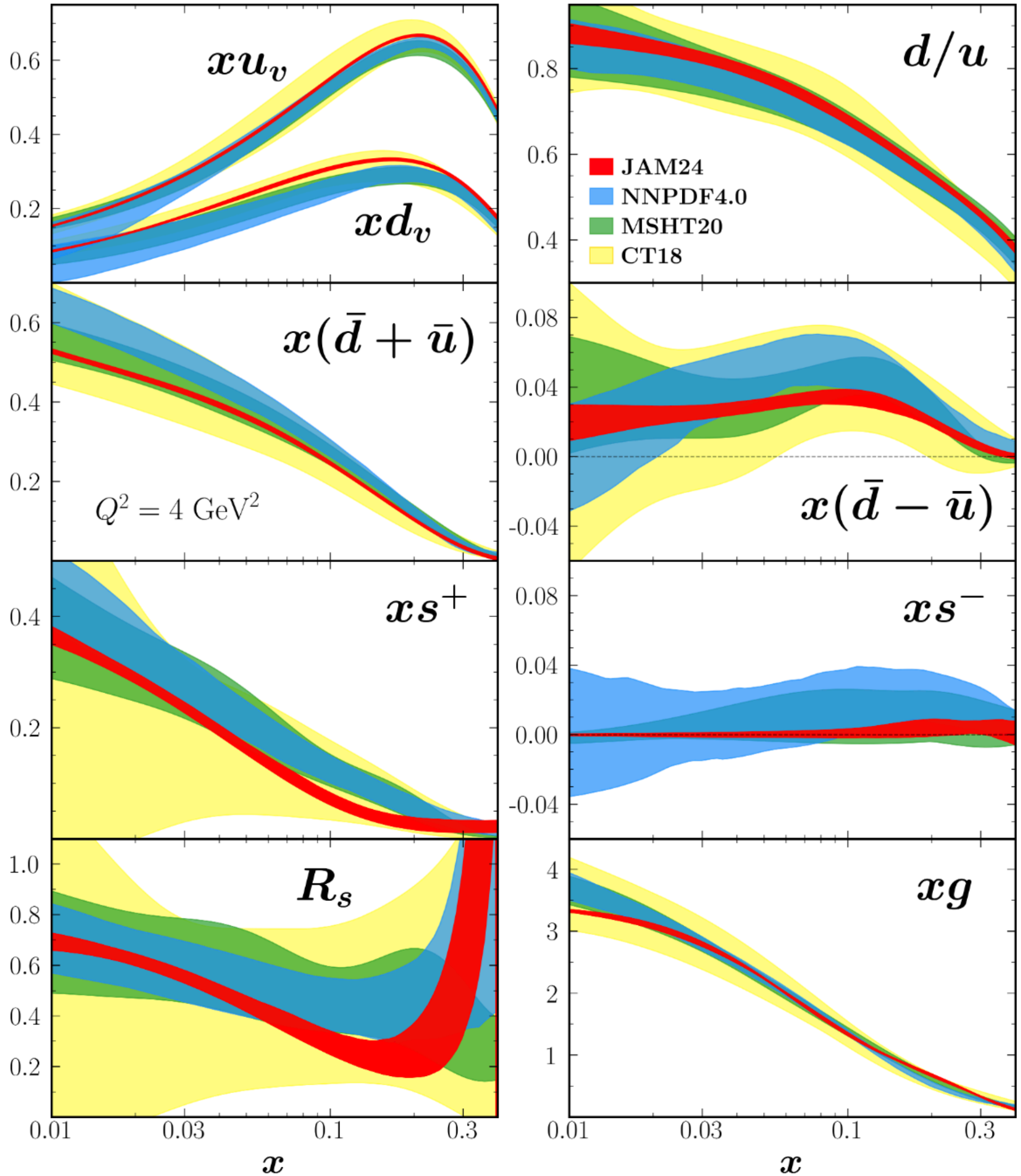
$$\mathrm{E}[\mathcal{O}] \simeq \frac{1}{N}\sum_k \mathcal{O}(\boldsymbol{a}_k)$$

$$\mathrm{V}[\mathcal{O}] =\simeq \frac{1}{N}\sum_k \left[\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}]\right]^2$$

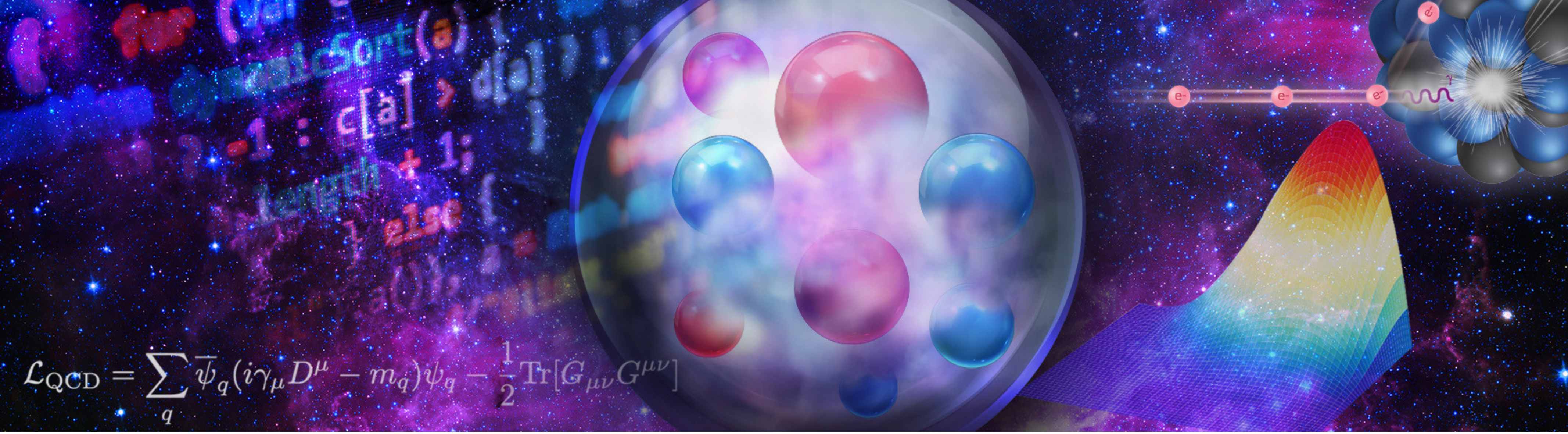
Many algorithms

- MCMC
- HMC
- Data resampling
- ...

Output PDFs



Anderson, WM, Sato
arXiv:2501.00665



- Jefferson Lab Angular Momentum (JAM) collaboration — an enterprise involving <u>theorists</u>, <u>experimentalists</u>, and <u>computer scientists</u> using QCD to study internal structure of hadrons

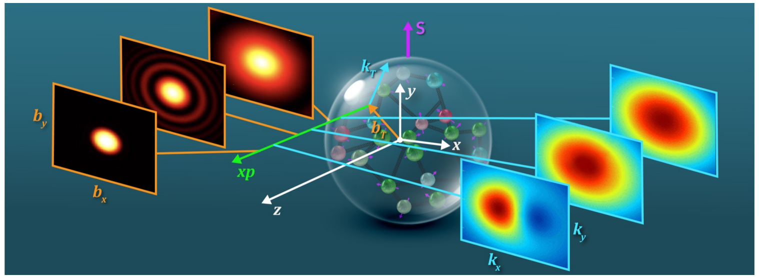
→ analyze data using modern Monte Carlo techniques & uncertainty quantification to <u>simultaneously</u> extract various quantum correlation functions

— parton distribution functions (PDFs)
— fragmentation functions
— transverse momentum dependent (TMD) distributions
— generalized parton distributions (GPDs)

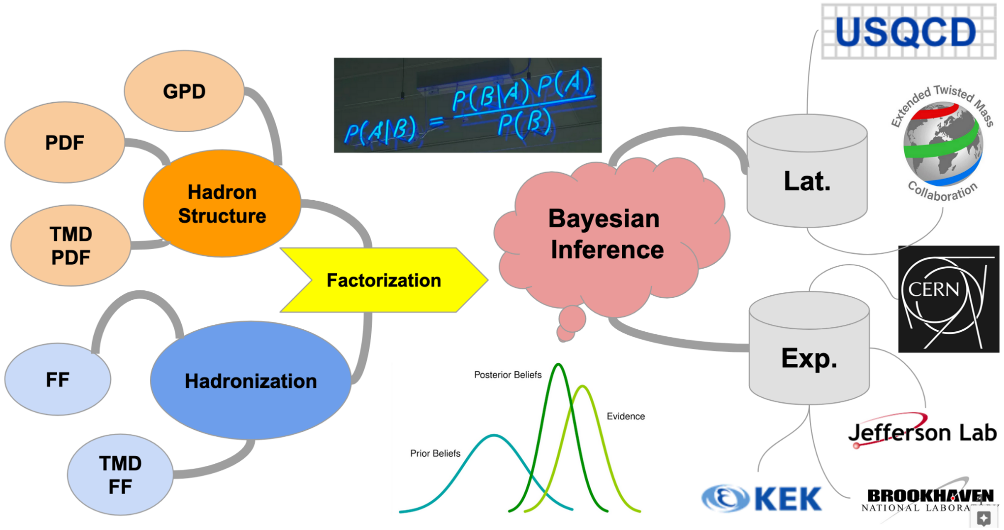
http://www.jlab.org/jam

→ inclusion of lattice data and ML algorithms to potentially expand reach and efficacy of JAM analyses and understanding of hadron structure in QCD

- 3D tomography in terms of quantum correlation functions (QCFs)
  - ⟶ hadron structure (PDFs, TMD PDFs, GPDs)
  - ⟶ hadronization (FFs, TMD FFs)



- "Holistic" approach to global QCD analysis
  - ⟶ factorization theorems
  - ⟶ Monte Carlo analysis
  - ⟶ simultaneous determination of various QCFs
  - ⟶ Bayesian inference
  - ⟶ use of lattice QCD data to supplement global analysis (with caution)

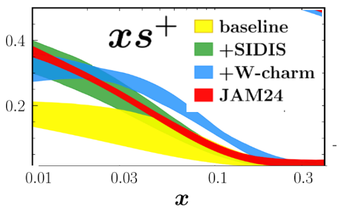


JAM analysis groups

Unpolarized PDFs (and fragmentation functions)

Strangeness in the proton from W+charm production and SI
T. Anderson, W. Melnitchouk, N. Sato
arXiv:2503.21006 [hep-ph]

Global QCD analysis and dark photons
N. T. Hunt-Smith, W. Melnitchouk, N. Sato, A. W. Thomas, X. (
JHEP **09**, 096 (2023), *arXiv:2302.11126 [hep-ph]*



Bayesian Monte Carlo extraction of the sea asymmetry with SeaQuest and STAR data
C. Cocuzza, W. Melntichouk, A. Metz, N. Sato
Phys. Rev. D **104**, 074031 (2021), *arXiv:2109.00677 [hep-ph]*

Simultaneous Monte Carlo analysis of parton densities and fragmentation functions
E. Moffat, W. Melnitchouk, T. C. Rogers, N. Sato
Phys. Rev. D **104**, 016015 (2021), *arXiv:2101.04664 [hep-ph]*

Constraints on the U(1)$_{B-L}$ model from global QCD analysis
X. Wang, N. Hunt-Smith, W. Melnitchouk, N. Sato, A.W. Thomas
Phys. Rev. D 111, 015019 (2025), *arXiv:2410.01205 [hep-ph]*

Isovector EMC effect from global QCD analysis with MARATHON data
C. Cocuzza, C. E. Keppel, H. Liu, W. Melnitchouk, A. Metz, N. Sato, A. W. Thomas
Phys. Rev. Lett. **127**, 242001 (2021), *arXiv:2104.06946 [hep-ph]*

Strange quark suppression from a simultaneous Monte Carlo analysis of parton distributions and
N. Sato, C. Andres, J.J. Ethier, W. Melnitchouk
Phys. Rev. D **101**, 074020 (2020), *arXiv:1905.03788 [hep-ph]*

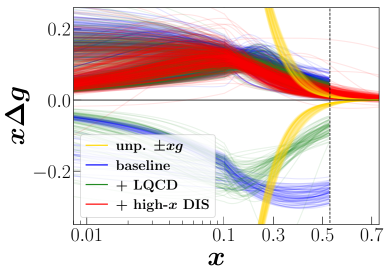
First Monte Carlo analysis of fragmentation functions from e^+e^- annihilation
N. Sato, J. J. Ethier, M. Hirai, S. Kumano, W. Melnitchouk
Phys. Rev. D **94**, 114004 (2016), *arXiv:1609.00899 [hep-ph]*

Helicity PDFs

New data-driven constraints on the sign of gluon polarizati
N. T. Hunt-Smith, C. Cocuzza, W. Melnitchouk, N. Sato, A. \
Phys. Rev. Lett. 133, 161901 (2024), *arXiv:2403.08117 [hep-p*

Accessing gluon polarization with high-PT hadrons in SIDIS
R. M. Whitehill, Y. Zhou, N. Sato, W. Melnitchouk
Phys. Rev. D **107**, 034033 (2023), *arXiv:2210.12295 [hep-ph]*

How well do we know the gluon polarization in the proton?
Y. Zhou, N. Sato, W. Melnitchouk
Phys. Rev. D **105**, 074022 (2022), *arXiv:2201.02075 [hep-ph]*



Gluon helicity from global analysis of experimental data and lattice QCD Ioffe time distributions
J. Karpie, R.M. Whitehill, W. Melnitchouk, C. Monahan, K. Orginos, J.-W. Qiu, D.G. Richards, N. Sato,
Phys. Rev. D 109, 036031 (2024), *arXiv:2310.18179 [hep-ph]*

Polarized antimatter in the proton from global QCD analysis
C. Cocuzza, W. Melnitchouk, A. Metz, N. Sato
Phys. Rev. D **106**, L031502 (2022), *arXiv:2202.03372 [hep-ph]*

First simultaneous extraction of spin-dependent parton distributions and fragmentation functions
J. J. Ethier, N. Sato, W. Melnitchouk
Phys. Rev. Lett. **119**, 132001 (2017), *arXiv:1705.05889 [hep-ph]*

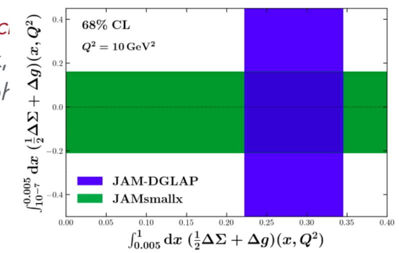
Iterative Monte Carlo analysis of spin-dependent parton distributions
N. Sato, W. Melnitchouk, S. E. Kuhn, J. J. Ethier, A. Accardi
Phys. Rev. D **93**, 074005 (2016), *arXiv:1601.07782 [hep-ph]*

Small-x PDFs

First study of polarized proton-proton scattering with small-x helicity evolution
D. Adamiak, N. Baldonado, Y. Kovchegov, M. Li, W. Melnitchouk, D. Pitonyak, N. Sato,
arXiv:2503.21006 [hep-ph]

Global analysis of polarized DIS and SIDIS data with improved small-x helicity evolution
D. Adamiak, N. Baldonado, Y. V. Kovchegov, W. Melnitchouk, D. Pitonyak, N. Sato, M. D. Sievert, A. Tarasov, Y. Tawabutr
Phys. Rev. D **108**, 114007 (2023), *arXiv:2308.07461 [hep-ph]*

First analysis of world polarized DIS data with small-x helic
D. Adamiak, Y. V. Kovchegov, W. Melnitchouk, D. Pitonyak,
Phys. Rev. D **104**, L031501 (2021), *arXiv:2102.06159 [hep-ph*

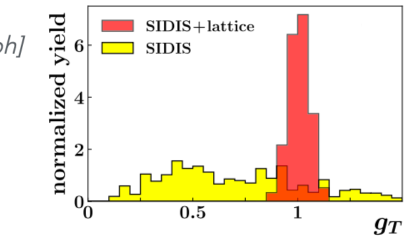


Transversity PDFs

First simultaneous global QCD analysis of dihadron fragmentation functions and transve
C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl
Phys. Rev. D **109**, 034024 (2024), *arXiv:2308.14857 [hep-ph]*

Transversity distributions and tensor charges of the nucleon: Extraction from dihadron production and their universal nature
C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl
Phys. Rev. Lett. **132**, 091901 (2024), *arXiv:2306.12998 [hep-ph]*

First Monte Carlo global analysis of nucleon transversity with lattice QCD constraints
H.-W. Lin, W. Melnitchouk, A. Prokudin, N. Sato, H. Shows
Phys. Rev. Lett. **120**, 152502 (2018), *arXiv:1710.09858 [hep-ph]*



Pion distributions (collinear and TMD)

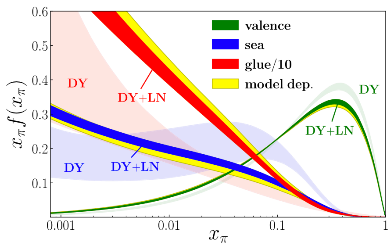
Tomography of pions and protons via transverse momentum dependent distributions
P. C. Barry, L. Gamberg, W. Melnitchouk, E. Moffat, D. Pitonyak, A. Prokudin, N. Sato
Phys. Rev. D **108**, L091504 (2023), *arXiv:2302.01192 [hep-ph]*

Towards the three-dimensional parton structure of the pion: Integrating transverse momentum dat
N. Y. Cao, P. C. Barry, N. Sato, W. Melnitchouk
Phys. Rev. D **103**, 114014 (2021), *arXiv:2103.02159 [hep-ph]*

Complementarity of experimental and lattice QCD data on pion parton distributions
P. C. Barry, C. Egerer, J. Karpie, W. Melnitchouk, C. Monahan, K. Orginos, Jian-Wei Qiu, D. Richards, N. Sato, R. S. Sufian, S. Zafeiropoulos
Phys. Rev. D **105**, 114051 (2022), *arXiv:2204.00543 [hep-ph]*

Global QCD analysis of pion parton distributions with threshold resummation
P. C. Barry, C.-R. Ji, N. Sato, W. Melnitchouk
Phys. Rev. Lett. **127**, 232001 (2021), *arXiv:2108.05822 [hep-ph]*

First Monte Carlo global QCD analysis of pion parton distribut
P. C. Barry, N. Sato, W. Melnitchouk, C.-R. Ji
Phys. Rev. Lett. **121**, 152001 (2018), *arXiv:1804.01965 [hep-ph]*
* highlighted in Phys. Rev. "50 years of QCD"



TMD PDFs

Updated QCD global analysis of single transverse-spin asymmetries: Extracting H˜, and the role of the Soffer bound and lattice QCD
L. Gamberg, M. Malda, J. A. Miller, D. Pitonyak, A. Prokudin, N. Sato
Phys. Rev. D **106**, 034014 (2022), *arXiv:2205.00999 [hep-ph]*

New tool for kinematic regime estimation in semi-inclusive deep-inelastic scattering
M. Boglione, M. Diefenthaler, S. Dolan, L. Gamberg, W. Melnitchouk, D. Pitonyak, A. Prokud
JHEP **04** (2022) 084, *arXiv:2201.12197 [hep-ph]*

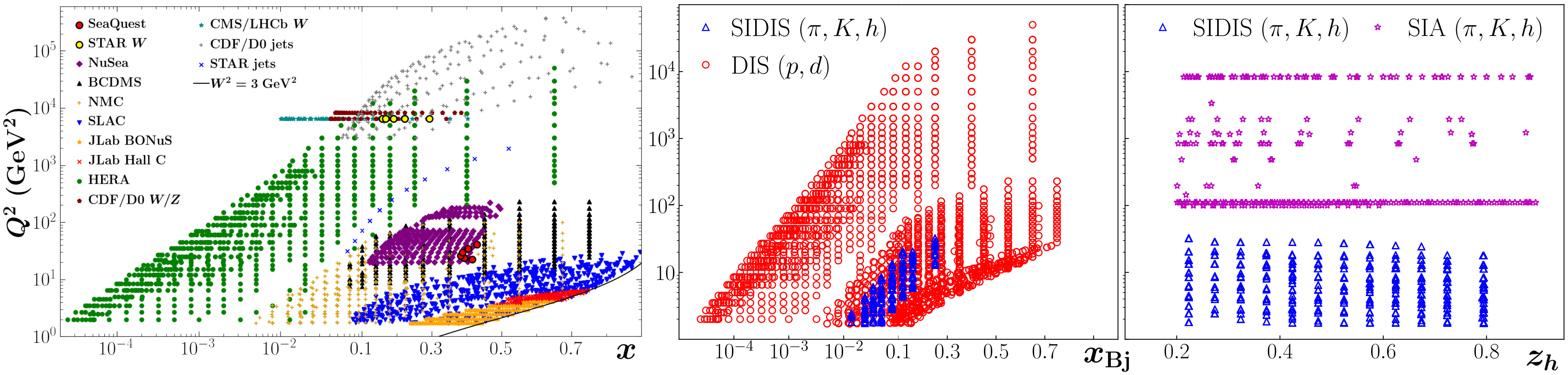
Origin of single transverse-spin asymmetries in high-energy collisions
J. Cammarota, L. Gamberg, Z.-B. Kang, J.A. Miller, D. Pitonyak, A. Prokudin, T.C. Rogers, N. Sato
Phys. Rev. D **102**, 054002 (2020), *arXiv:2002.08384 [hep-ph]*

GPDs

Kernel methods for evolution of GPDs
A. Freese, D. Adamiak, I. Cloët, W. Melnitchouk, J.-W. Qiu, N. Sato, M. Zaccheddu
Comput. Phys. Commun. 311, 109552 (2025), *arXiv:2412.13450 [hep-ph]*

Shedding light on shadow generalized parton distributions
E. Moffat, A. Freese, I. Cloët, T. Donohoe, L. Gamberg, W. Melnitchouk, A. Metz, A. Prokudin, N. Satc
Phys. Rev. D **108**, 036027 (2023), *arXiv:2303.12006 [hep-ph]*

Unpolarized proton PDFs

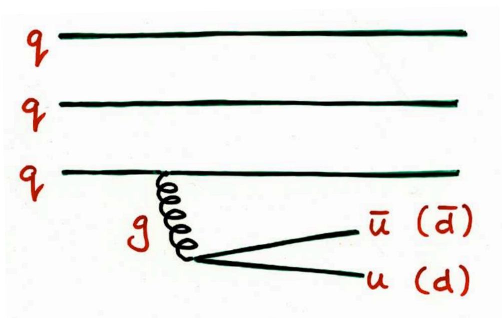


- DIS, Drell-Yan, W/Z/jet production $\longrightarrow$ PDFs $f(x)$
- SIA (single-inclusive e^+e^- annihilation $e^+e^- \to h\,X$) $\longrightarrow$ FFs $D^h(z)$
- SIDIS (semi-inclusive DIS $\ell N \to \ell' \, h \, X$) $\longrightarrow$ PDFs & FFs $f(x) \otimes D^h(z)$

$\longrightarrow$ PDFs and fragmentation functions fitted simultaneously

Sea quark asymmetries

- From perturbative QCD expect symmetric $q\bar{q}$ sea generated by gluon radiation into $q\bar{q}$ pairs (if quark masses are the same)

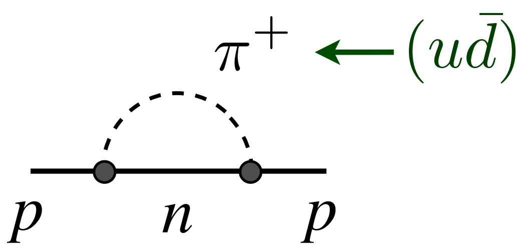


$\longrightarrow$ since u and d quarks nearly degenerate, expect flavour-symmetric light-quark sea

$$\bar{d} \approx \bar{u}$$

Ross, Sachrajda (1979)

- Chiral symmetry of QCD (important at low energy) should have consequences for antiquark PDFs in nucleon (measured at high energy)

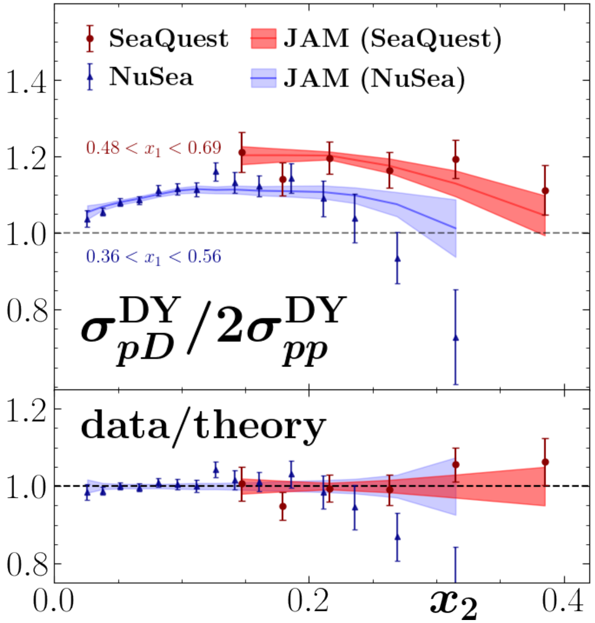


$\longrightarrow$ $\bar{d} > \bar{u}$

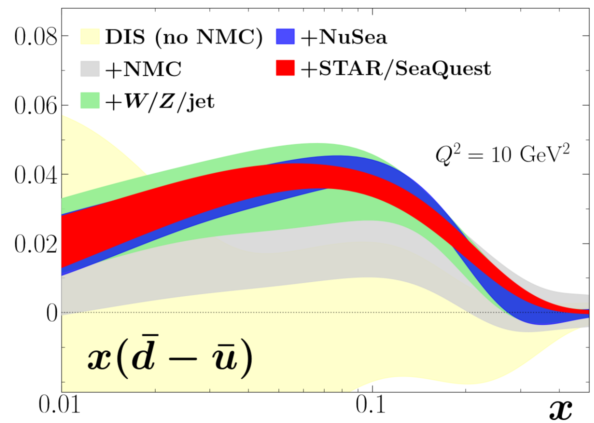
Thomas (1983)

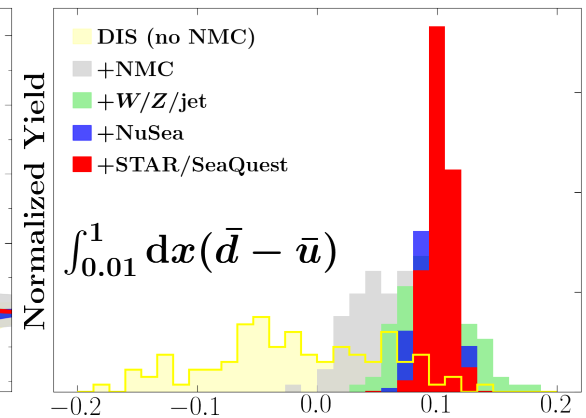
Sea quark asymmetries

Impact of SeaQuest & RHIC W-production data



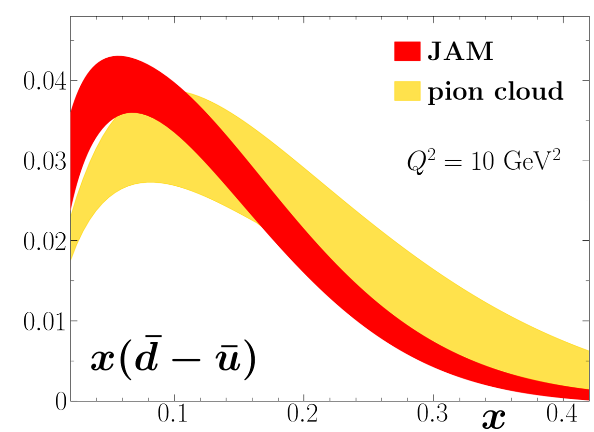
$$\frac{\sigma^{pd}}{\sigma^{pp}} \approx 1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)} \quad \text{for } x_1 \gg x_2$$





→ strong constraints on integrated $\bar{d}-\bar{u}$ in measured region

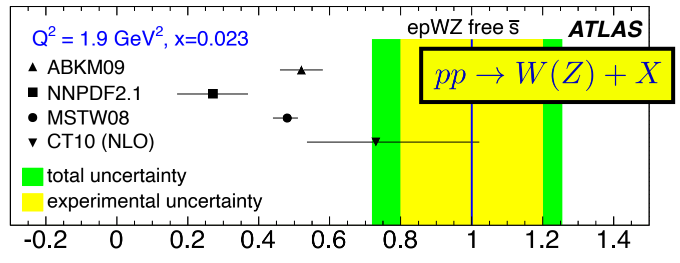
→ shape and magnitude consistent with pion cloud models of nucleon



Cocuzza, WM, Metz, Sato
*PRD **104**, 074031 (2021)*

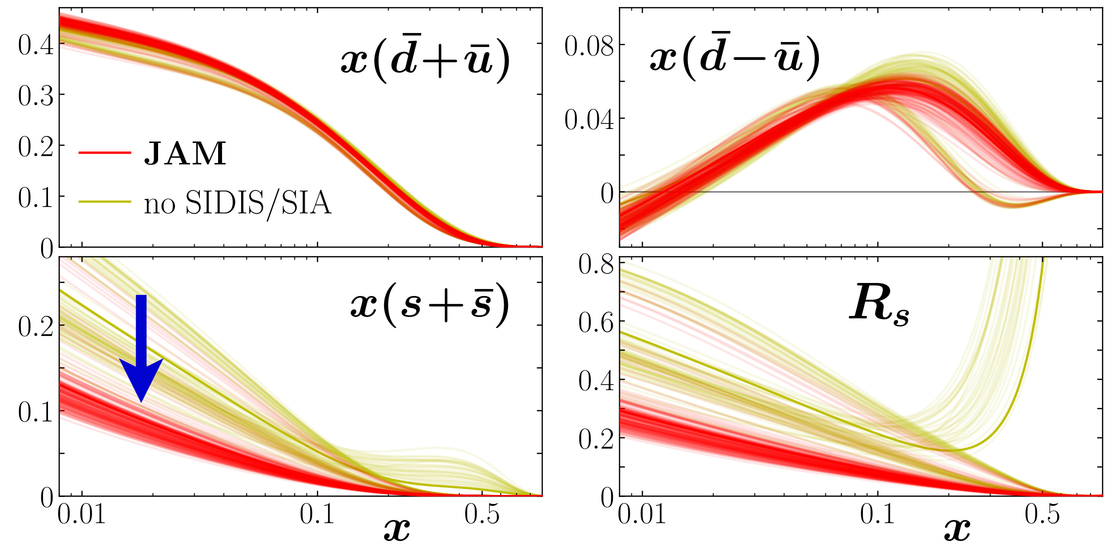
Strange quark sea

- Shape and magnitude of strange quark PDF is controversial
  - → historically, strange to nonstrange ratio $R_s = \dfrac{s+\bar{s}}{\bar{u}+\bar{d}} \sim 0.4$
  - → larger than expected strangeness extracted from ATLAS W/Z data



$r_s = (s+\bar{s})/2\bar{d}$
$= 1.00^{+0.25}_{-0.28}$

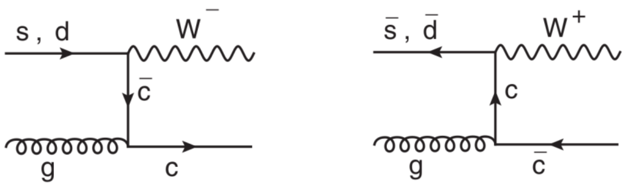
- Impact of SIDIS & SIA data on unpolarised PDFs
  - → SIA data at large z disfavor small $s \to K$ FF
  - → larger $s \to K$ FF requires smaller strange PDF
  - → <u>suppression of strange</u> PDF compared to ATLAS extraction

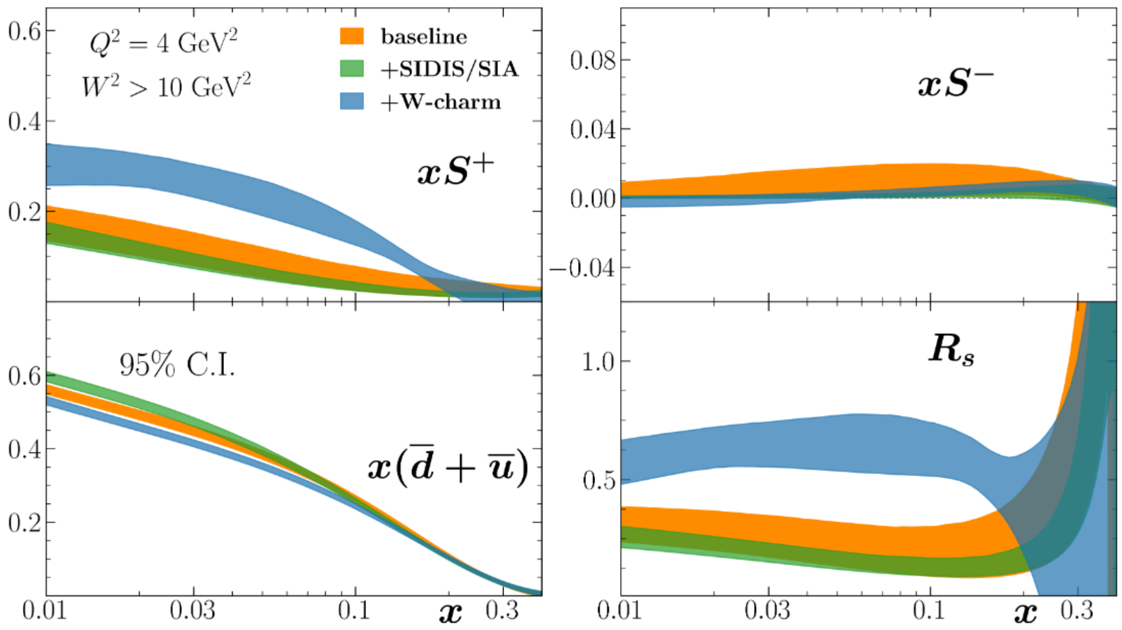


Sato, Andres, Ethier, WM
*PRD **101**, 074020 (2020)*

Strange quark sea

- $W{+}c$ production in pp collisions
  - ⟶ sensitive to s and $\bar{s}$ distributions
  - ⟶ ATLAS and CMS data indicate larger strange content

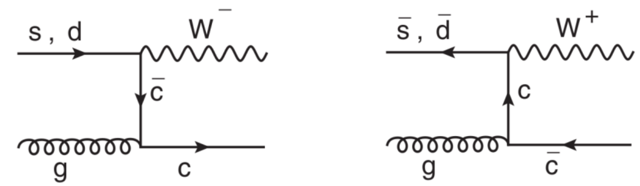




Anderson, WM, Sato, in preparation (2024)

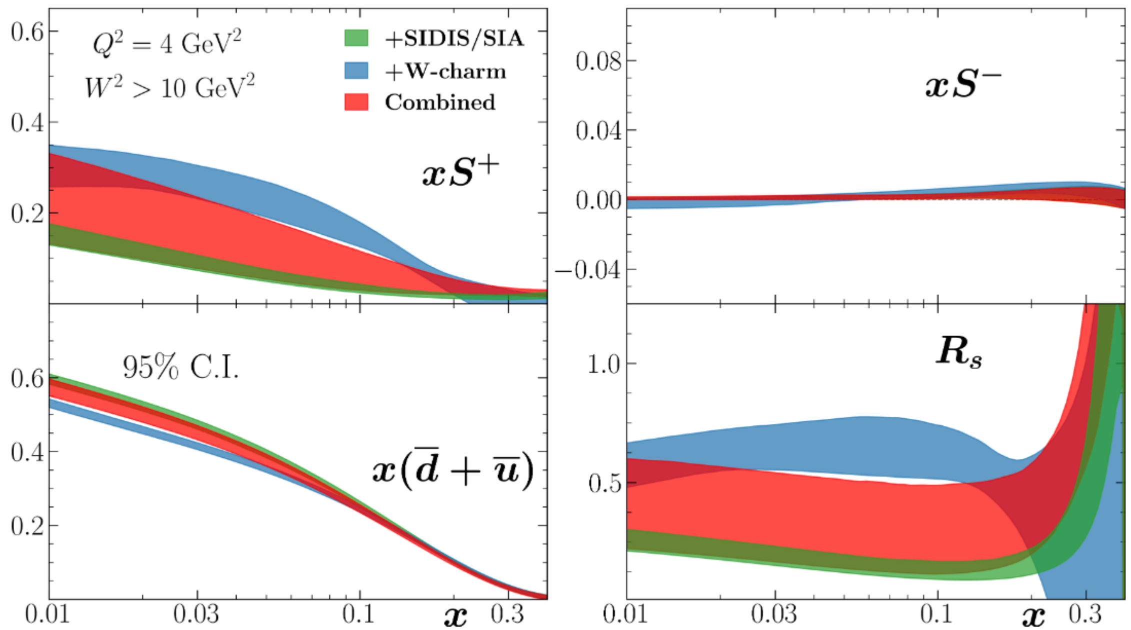
Strange quark sea

- $W{+}c$ production in pp collisions



→ sensitive to s and $\bar{s}$ distributions

→ ATLAS and CMS data indicate larger strange content



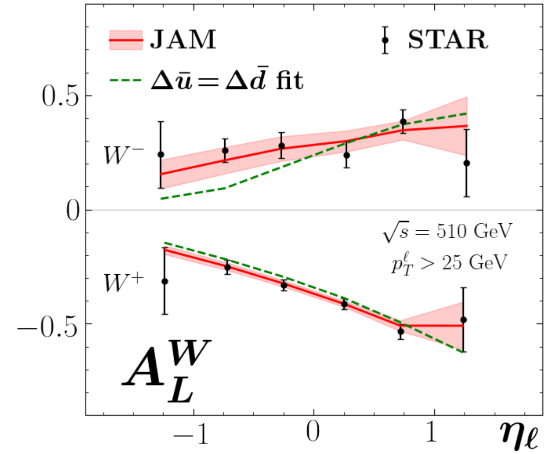
Anderson, WM, Sato, in preparation (2024)

→ combined analysis gives $R_s \approx 0.2 - 0.5$ for $0.01 \lesssim x \lesssim 0.25$ with larger overall uncertainty

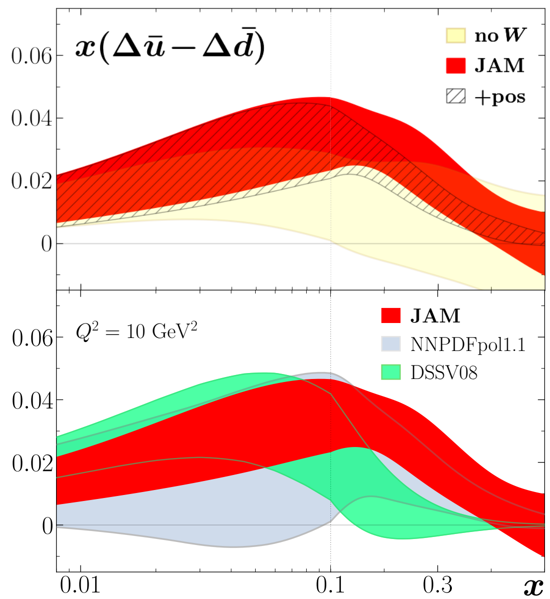
Polarized sea quarks

- Flavor asymmetries in spin PDFs expected from antisymmetrization

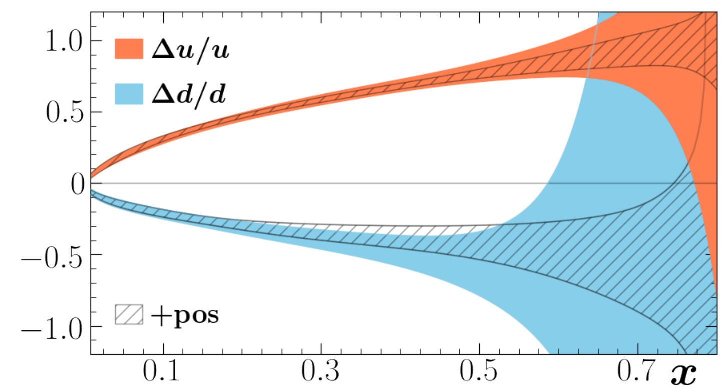
→ tested in W production in polarized pp collisions



$$A_L^{W^+} \sim \Delta\bar{d}(x_1)u(x_2) - \Delta u(x_1)\bar{d}(x_2)$$
$$A_L^{W^-} \sim \Delta\bar{u}(x_1)d(x_2) - \Delta d(x_1)\bar{u}(x_2)$$



Cocuzza, WM, Metz, Sato
PRD ***106****, L031502 (2022)*



→ excess of $\Delta\bar{u}$ over $\Delta\bar{d}$ at intermediate x

→ first consistent (simultaneous) extraction of ratios $\Delta q/q$

Where does the spin of the proton come from?

- Proton spin crisis (European Muon Collaboration, 1988) — total spin $\Delta\Sigma$ carried by quarks and antiquarks consistent with zero!

- Global experimental program in polarized ℓN & pp scattering
  $\longrightarrow$ more refined picture, in which $\Delta\Sigma \sim 0.3$, and evidence that spin carried by gluons ΔG is positive

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

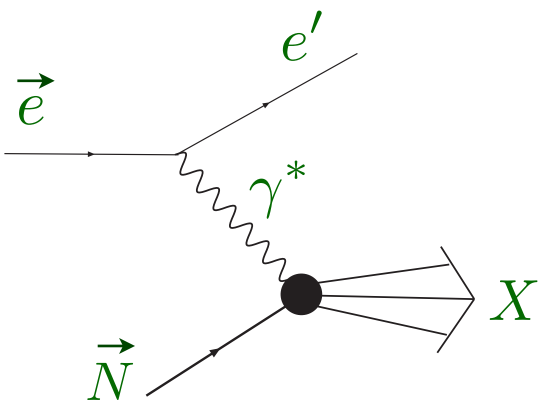
quark helicity ($\sim 30\%$)

gluon helicity (positive?)

quark & gluon orbital angular momentum (largely unknown)

- Quest to unravel spin decomposition continues!

Information on helicity-dependent PDFs obtained from polarized ℓN DIS



$$A_\parallel = \frac{\sigma^{\downarrow\Uparrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\downarrow\Uparrow} + \sigma^{\uparrow\Uparrow}} \xrightarrow[Q^2\to\infty]{} A_1 = \frac{1}{F_1}\left(g_1 - \frac{4M^2x^2}{Q^2} g_2\right)$$

$\longrightarrow$ polarized structure functions given by helicity parton distributions

$$g_1(x,Q^2) \overset{\text{LO}}{=} \frac{1}{2}\sum_q e_q^2\, \Delta q^+(x,Q^2)$$

$$\Delta q = q^{\uparrow\Uparrow} - q^{\downarrow\Uparrow}$$

$\longrightarrow$ total helicity carried by quarks & antiquarks

$$\Delta\Sigma(Q^2) = \sum_q \int_0^1 dx\, \Delta q^+(x,Q^2)$$

$\longrightarrow$ triplet axial charge

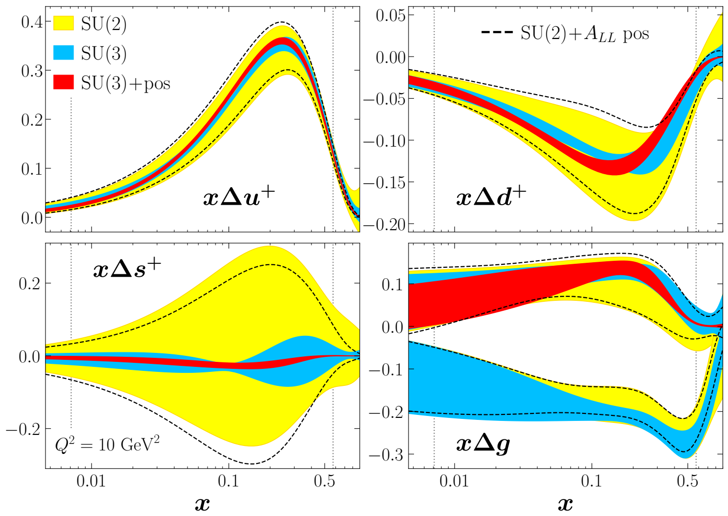
$$\int_0^1 dx\, \left[\Delta u^+ - \Delta d^+\right](x,Q^2) \;=\; g_A$$

octet axial charge

$$\int_0^1 dx\, \left[\Delta u^+ + \Delta d^+ - 2\Delta s^+\right](x,Q^2) \;=\; a_8$$

Polarized glue

- First simultaneous analysis including polarized *and* unpolarized <u>jets</u> in proton-proton and proton-antiproton collisions



SU(2):
$$\int_0^1 dx\,[\Delta u^+ - \Delta d^+](x, Q^2) = g_A$$
✓

SU(3):
$$\int_0^1 dx\,[\Delta u^+ + \Delta d^+ - 2\Delta s^+](x, Q^2) = a_8$$
?

PDF positivity:
$$|\Delta f_i(x, Q^2)| \leq f_i(x, Q^2)$$
✗

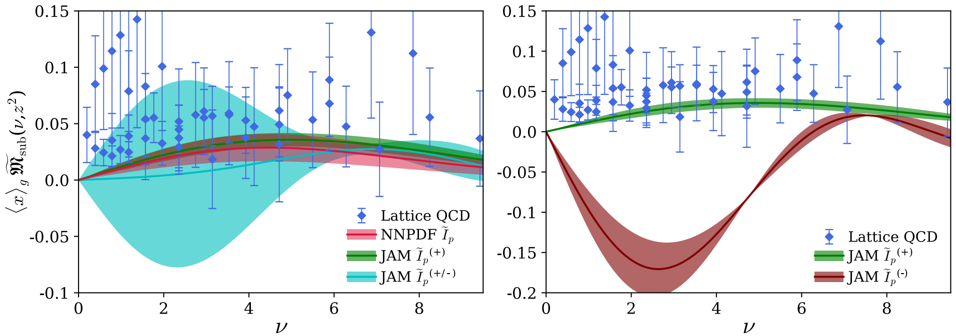
Zhou, Sato, WM, PRD ***105****, 074022 (2022)*

⟶ polarized strange and gluon spin PDFs depend strongly on theoretical assumptions, especially positivity of (unpolarized) PDFs

⟶ cannot rule out negative gluon polarization from experiment alone!

Polarized glue

- New lattice QCD calculations of Ioffe-time pseudo-distributions



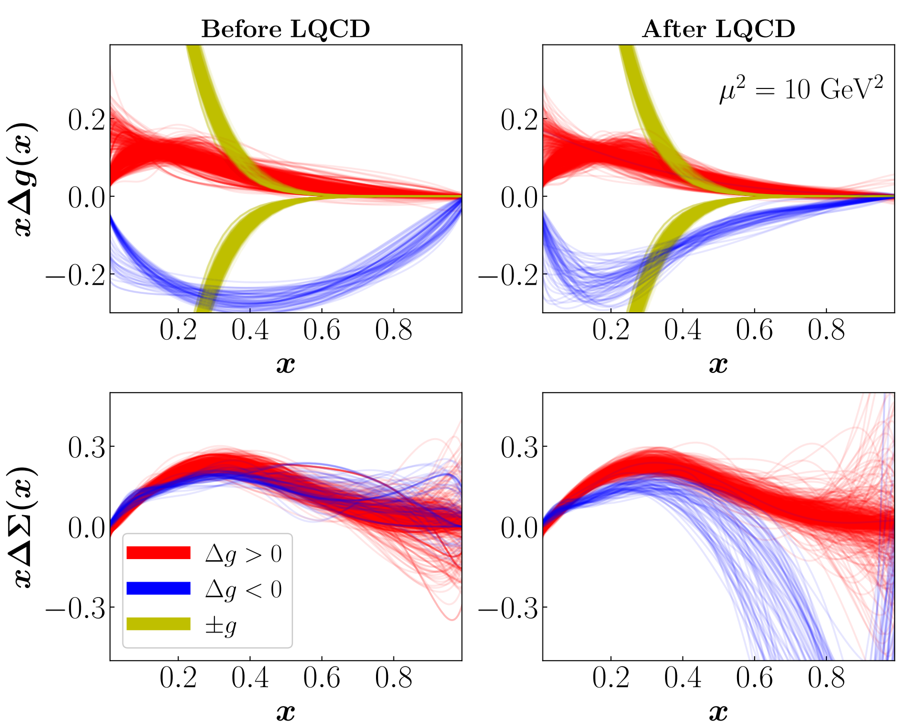
$\widetilde{\mathfrak{M}}(\nu, z^2)$ depends on

$$\widetilde{\mathcal{I}}_p(\nu) = \frac{i}{2} \int_{-1}^{1} \mathrm{d}x \, e^{-ix\nu} \, x \, \Delta g(x) \, .$$

Egerer et al. [HadStruc Collaboration]
*PRD **106**, 094511 (2022)*

⟶ favors positive gluon polarization?

⟶ fit experimental + lattice data simultaneously



*Karpie, Whitlehill, WM et al., PRD **109**, 036031 (2024)*

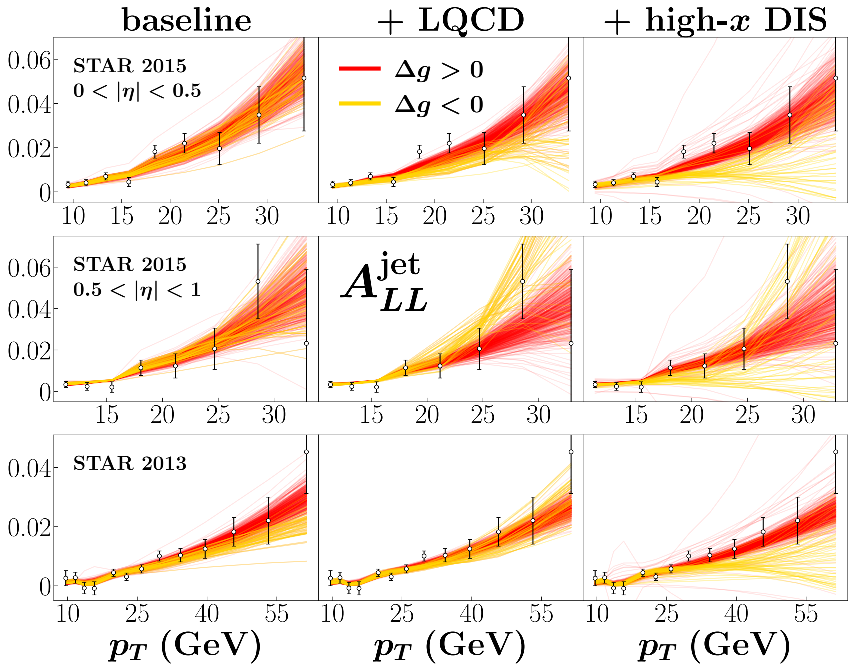
⟶ good description of data after inclusion of LQCD for both solutions for Δg

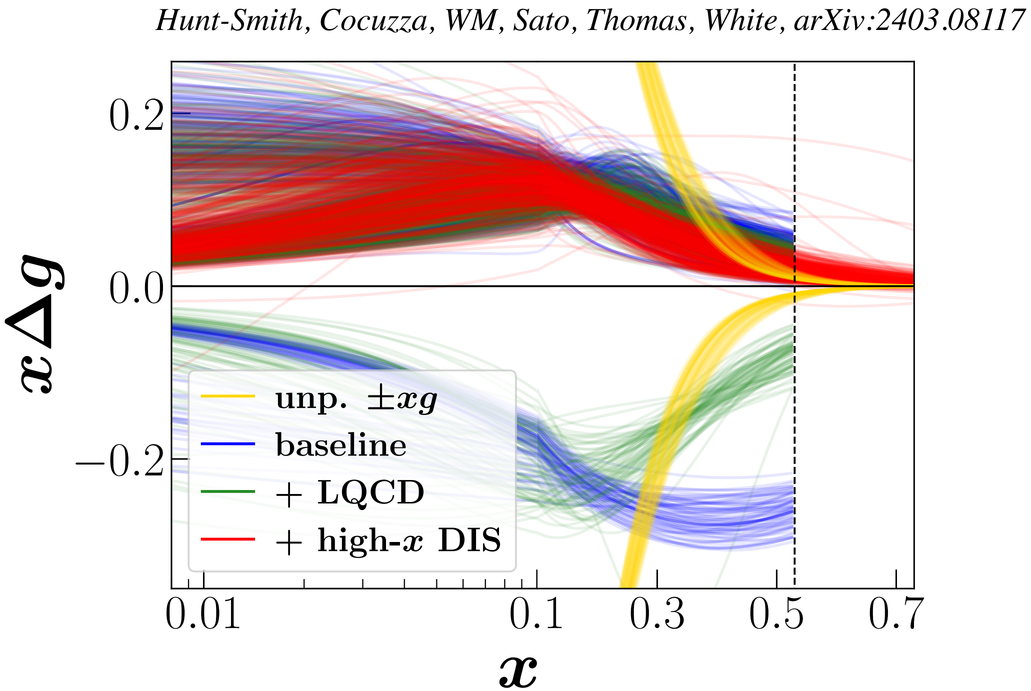
⟶ from χ^2 alone, LQCD cannot discriminate sign of Δg

but … negative Δg gives rise to negative $\Delta\Sigma$ at large x

Polarized glue

- Lower W^2 cut from $10\ \mathrm{GeV}^2$ to $4\ \mathrm{GeV}^2$ to include high-x region



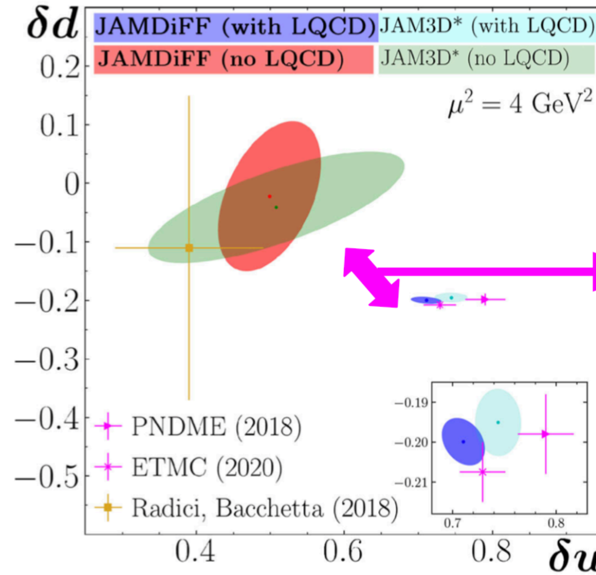


→ including high-x DIS data (CLAS, Hall A, SANE), LQCD strongly disfavors negative $\Delta\Sigma$ solutions at $x > 0.5$

→ in data-driven approach, $\Delta g < 0$ can be ruled out <u>only</u> with inclusion of polarized jet, lattice, and high-x DIS data!

Transversity

- Reconstruct transversity h_1 PDFs from
  - single spin asymmetries (SIDIS, pp) within TMD+CT3 framework
    $\longrightarrow$ "JAM3D"
  - dihadron production in SIDIS, pp and e^+e^- data
    $\longrightarrow$ "JAMDiFF"



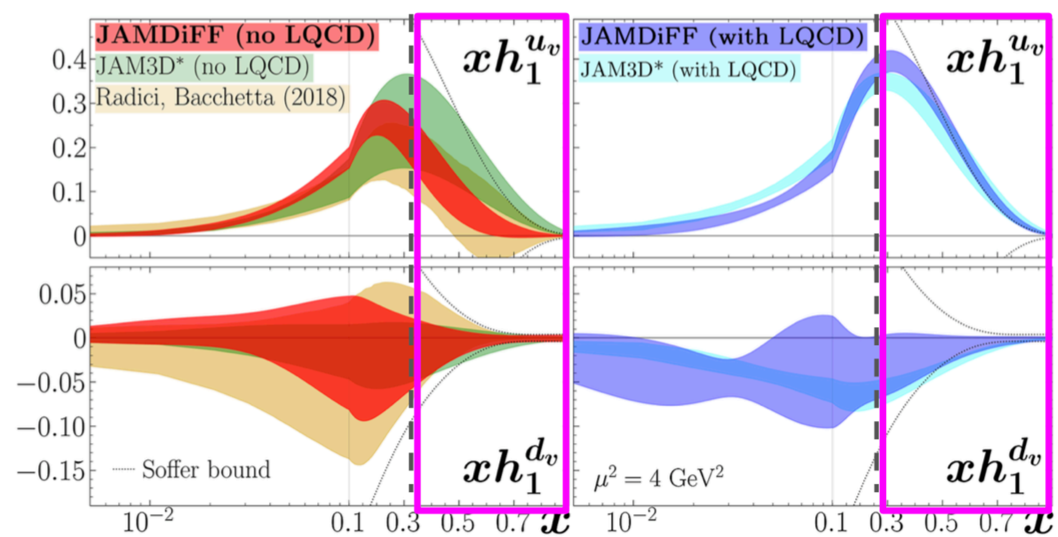
tension between LQCD & experiment removed by adjusting h_1 in (extrapolated) large-x region

- Tension between experiment and lattice QCD data?

  $\longrightarrow$ weak constraints on h_1 from experiment at $x > 0.3$

  $\longrightarrow$ LQCD moments suggest large contributions at high x

  $\longrightarrow$ more high-x data needed to test compatibility



Cocuzza, Metz, Pitonyak, Prokudin, Sato, Seidl
PRD ***109****, 034024 (2024)*
PRL ***132****, 091901 (2024)*

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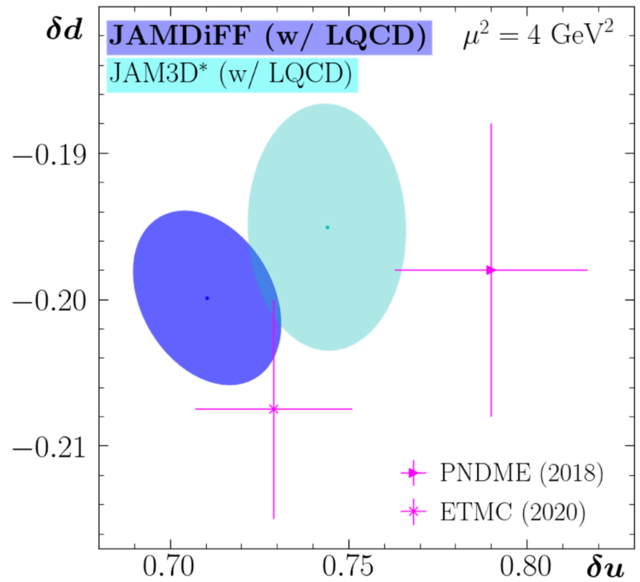

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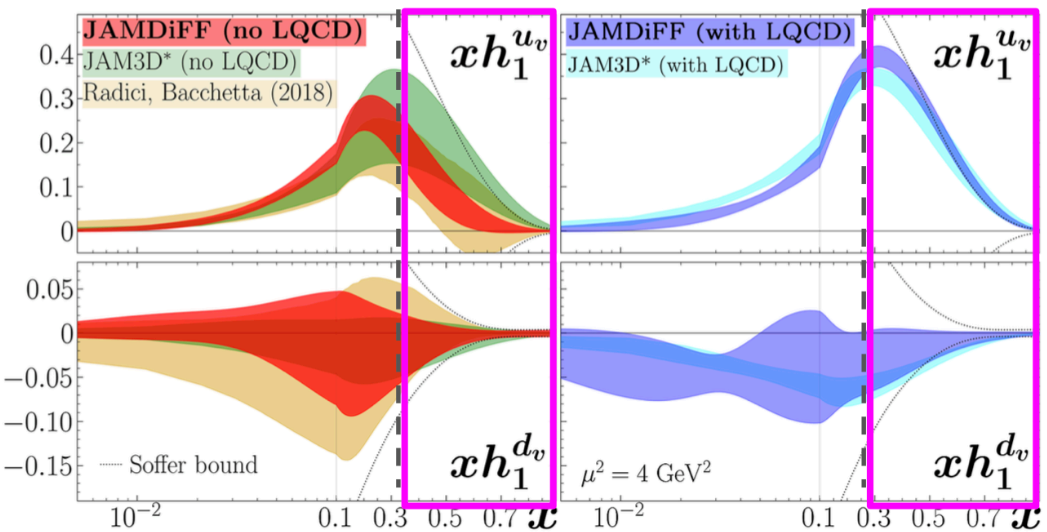

Cocuzza, Metz, Pitonyak, Prokudin, Sato, Seidl
*PRD **109**, 034024 (2024)*
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Generalized parton distributions

- JAM's GPD efforts moving towards ML-based analysis
  - ⟶ GPDs more complicated, traditional methods require more modeling
  - ⟶ capitalize on tools developed for <u>imaging in ML</u>
  - ⟶ model GPDs as <u>pixels</u> instead of assuming functional forms
    … address resolution with which images can be reconstructed

- JAM participating in multi-institutional projects
  - <u>Quark-Gluon Tomography (QGT) Topical Collaboration</u>
    — global analysis of GPDs from DVCS & DVMP data with LQCD
  - <u>LDRD</u>
    — SDHEPS to reconstruct x dependence: photoproduction in Hall D
  - <u>QuantOm</u>
    — integrated experiment and theory <u>event-level</u> analysis framework for hadron structure studies: fold detector effects with QCD reaction

⟶ ML-enabled framework requires differentiable programming libraries (PyTorch)

⟶ develop data analysis framework for simultaneous extractions of QCFs
— capitalize on JAM PDF/TMD machinery

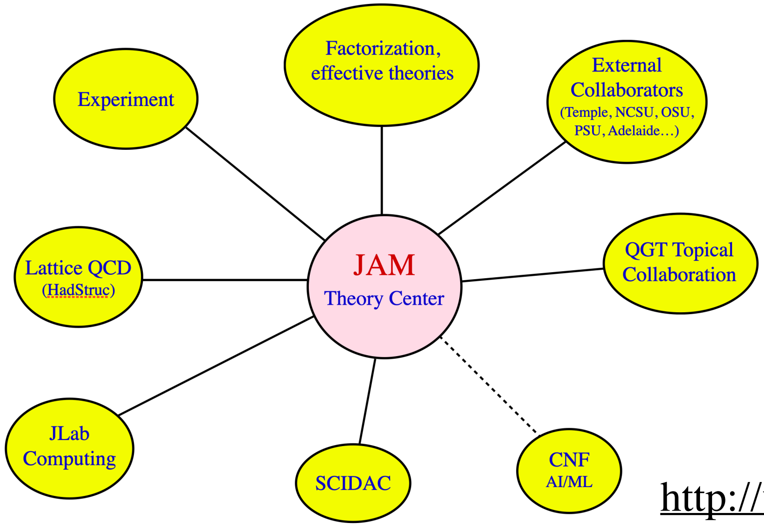
Outlook

- Progress made by JAM Collaboration towards simultaneous QCD analysis of all observables sensitive to collinear (spin-averaged and spin-dependent) PDFs and FFs, as well as TMD PDFs and TMD FFs

- Incorporation of lattice QCD data into global analysis (with caution)

- Increasing utilization of AI/ML tools to meet complexity challenge

- JAM machinery being leveraged in development of ML-based analysis framework for GPDs, TMDs, PDFs $\longrightarrow$ 3-D structure of hadrons



http://www.jlab.org/jam