Carnegie Mellon University

U.S. DEPARTMENT OF ENERGY

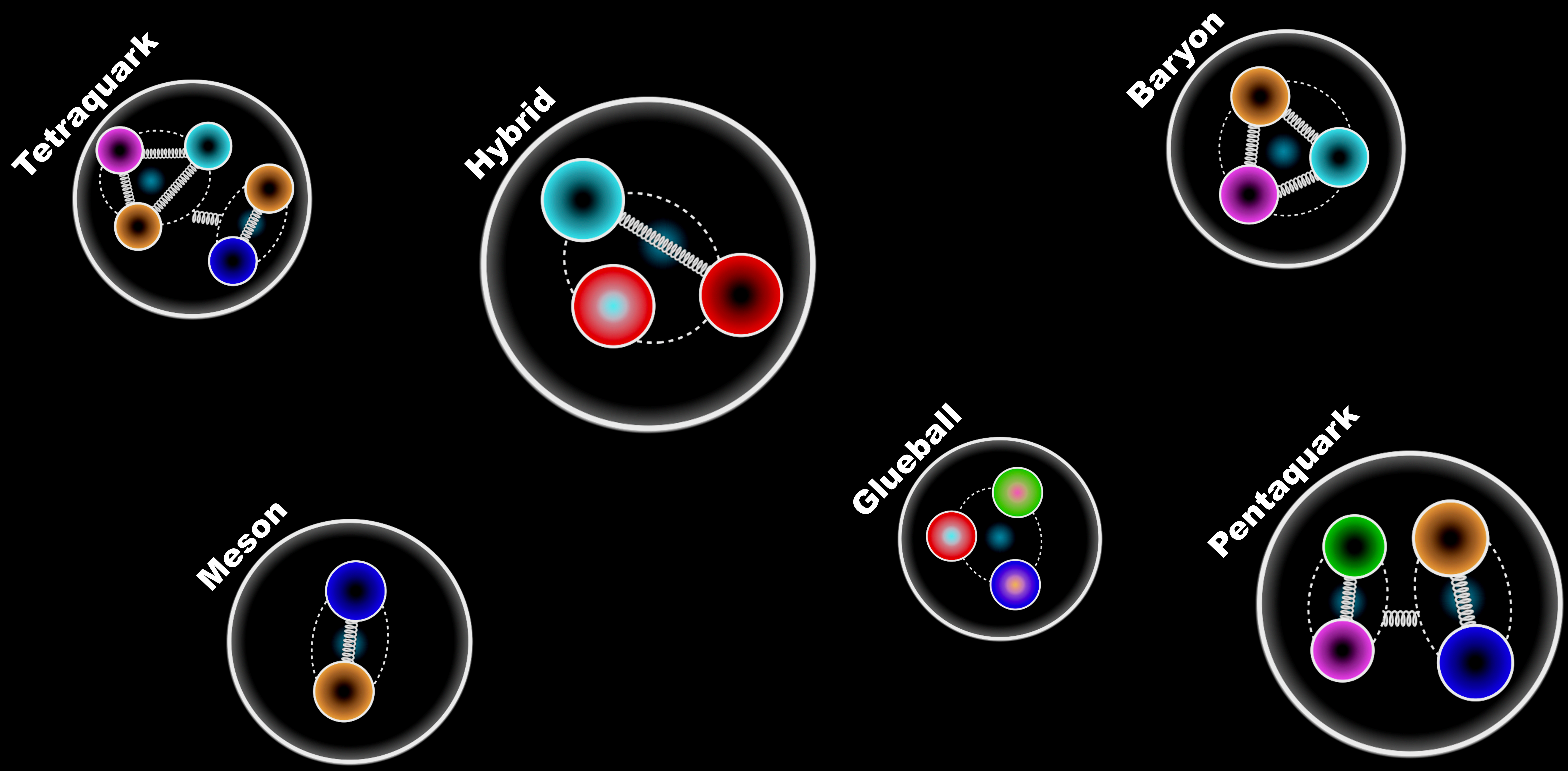
Office of Science

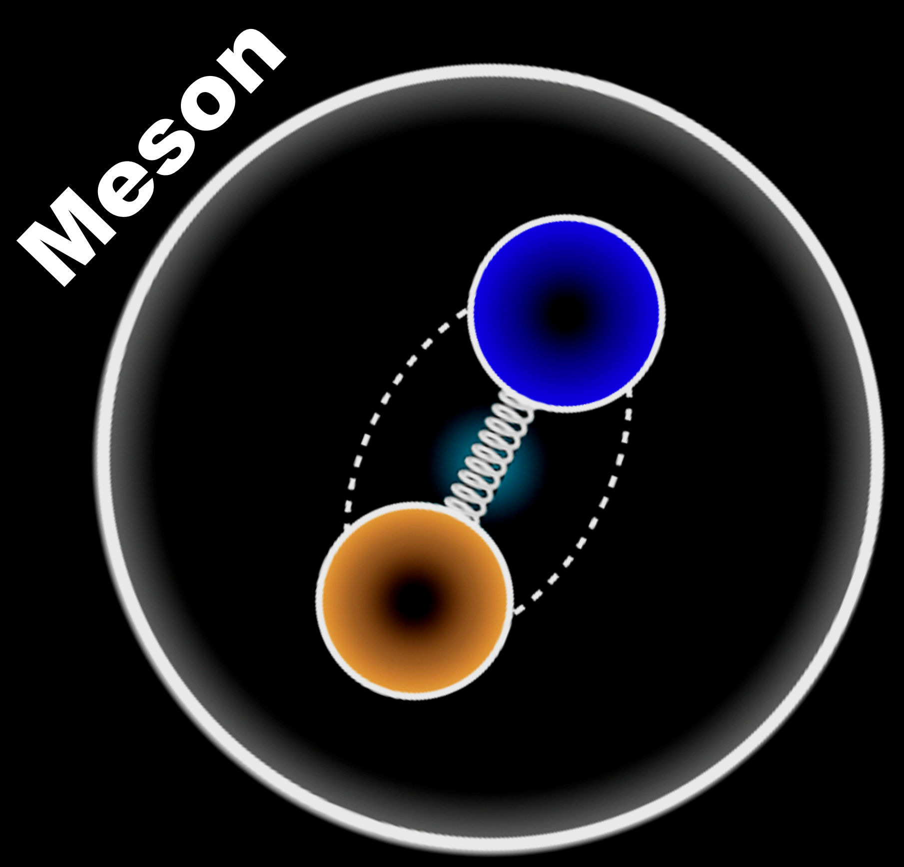
Exploring Photoproduced $\eta^{(\prime)}\pi^0$ Systems in the Search for Exotic Hadrons at GlueX

Thomas Jefferson National Lab

HUGS | 40 years!

Zachary Baldwin | June 13, 2025


Tetraquark
Hybrid
Baryon
Glueball
Pentaquark
Meson



Total angular momentum | $J = 0,1,2,...$

Parity | $P = (-1)^{L+1}$

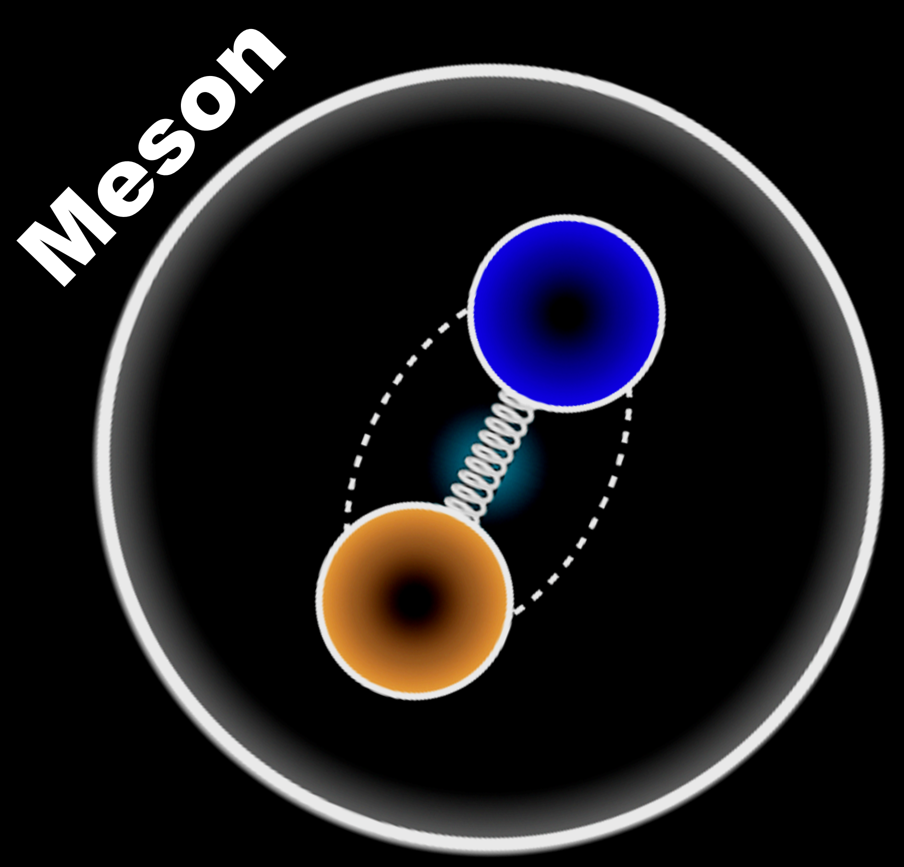
Charge Conjugation | $C = (-1)^{L+S}$

L is the relative orbital angular momentum of the q and $\bar{q}$

S is the total intrinsic spin of the $q\bar{q}$ pairs

Allowed J^{PC} quantum numbers

L	S	J^{PC}	L	S	J^{PC}	L	S	J^{PC}
0	0	0^{-+}	1	0	1^{+-}	2	0	2^{-+}
0	1	1^{--}	1	1	0^{++}	2	1	1^{--}
			1	1	1^{++}	2	1	2^{--}
			1	1	2^{++}	2	1	3^{--}



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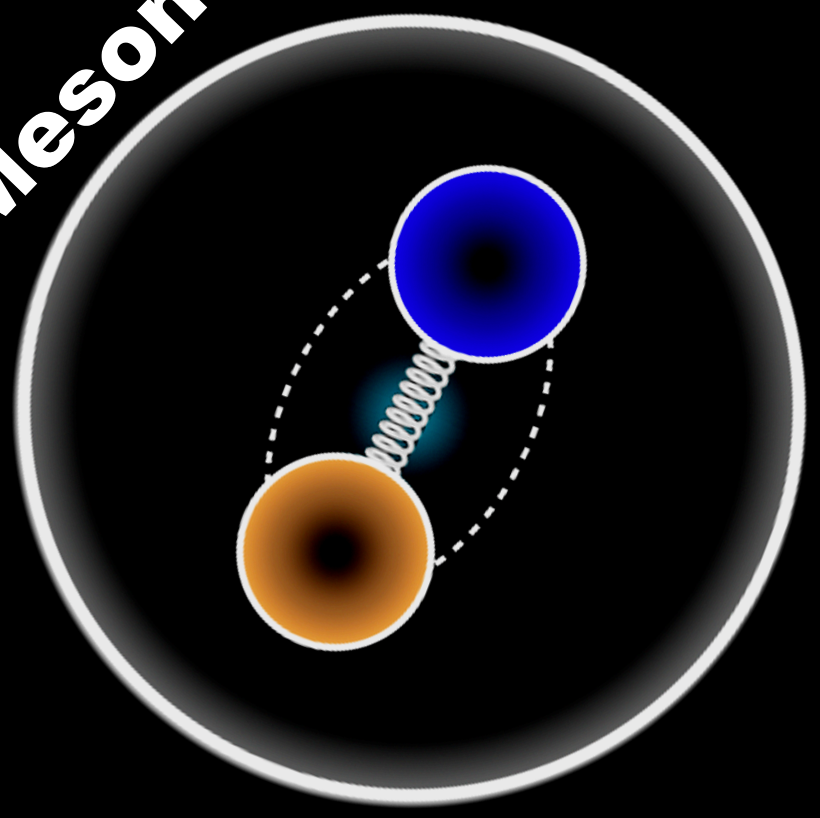
			1	1	1^{++}	2	1	2^{--}
			1	1	2^{++}	2	1	3^{--}

Forbidden J^{PC} quantum numbers

$0^{--},\ 0^{+-},\ 1^{-+},\ 2^{+-}$

Discovering forbidden quantum numbers would be immediate evidence of a non-$q\bar{q}$ state



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Do gluons play a larger role?

Lattice QCD predicts "gluonic excitations", mesons that are not in constituent quark model known as exotic mesons


JLAB-THY-11-1314
TCD-MATH 11-02

Isoscalar meson spectroscopy from lattice QCD

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Michael J. Peardon,[3] David G. Richards,[1] and Christopher E. Thomas[1]
(for the Hadron Spectrum Collaboration)
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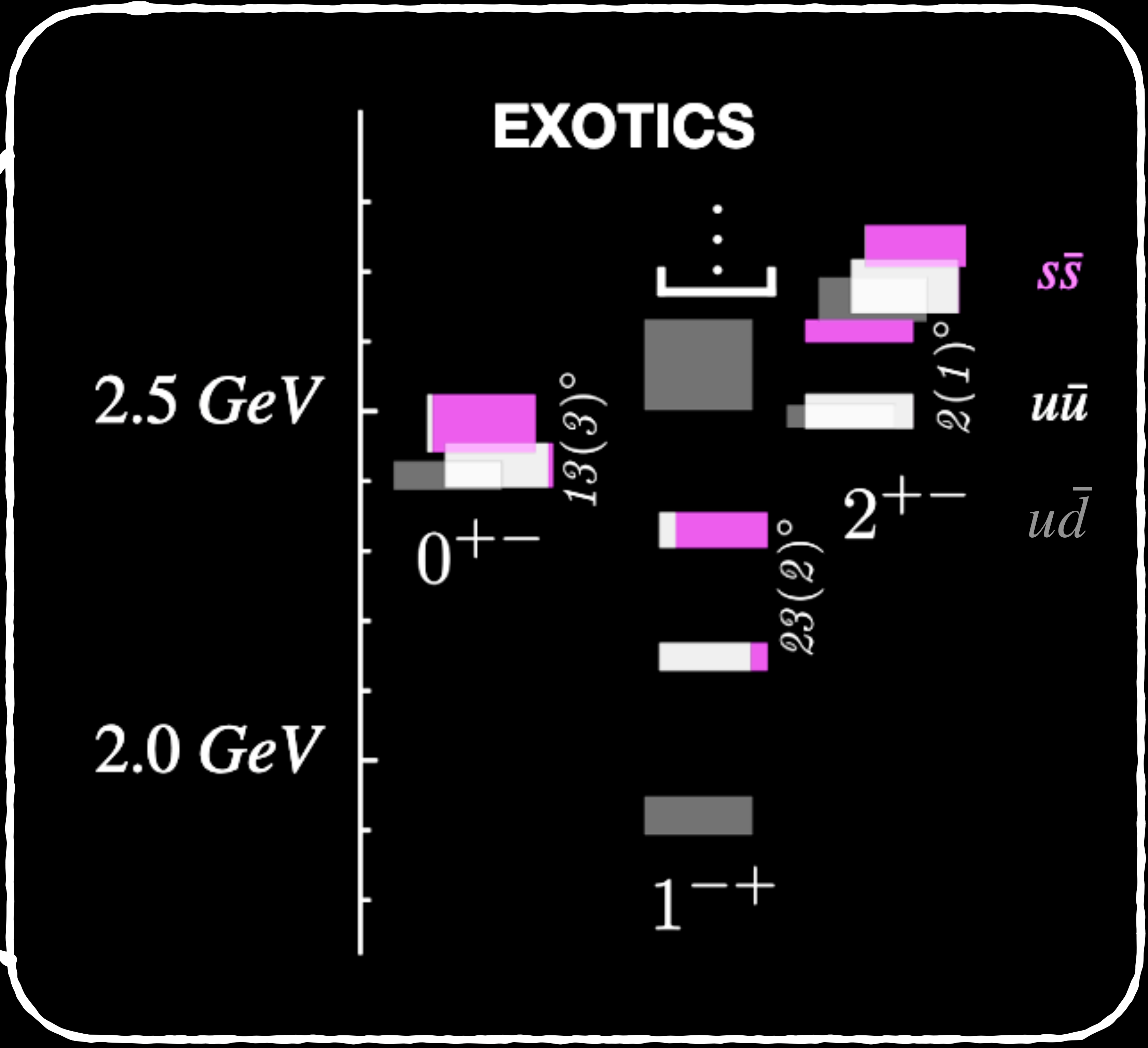
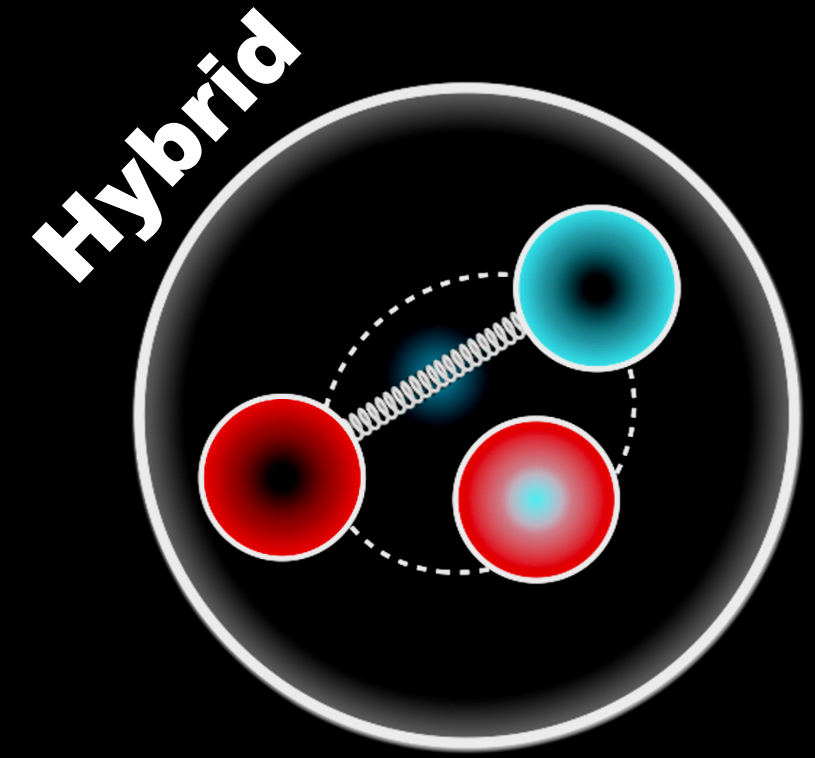
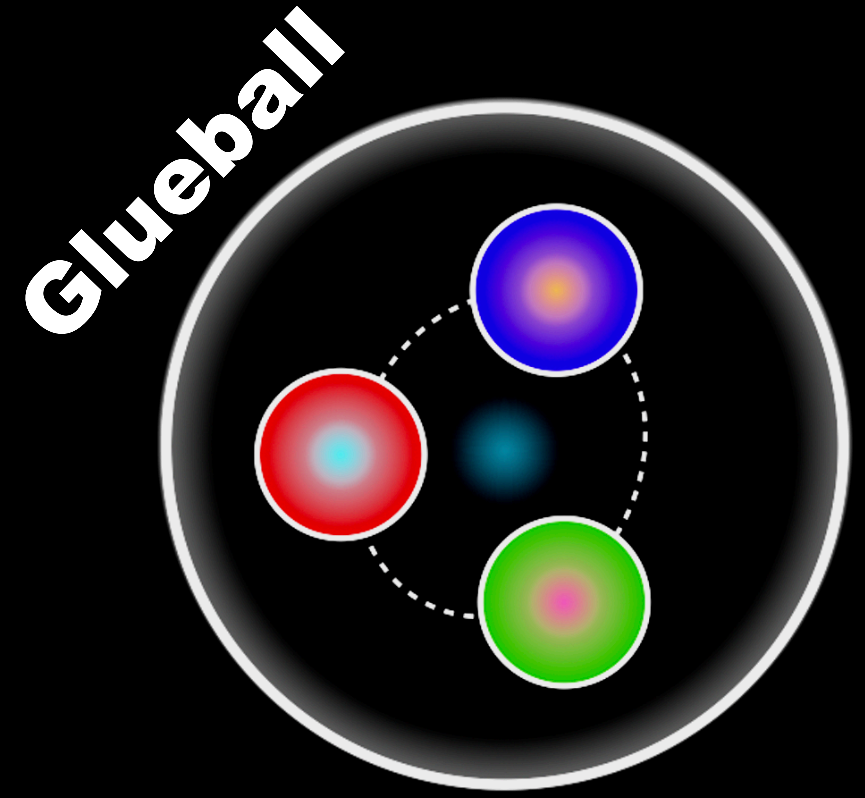
[2] Department of Physics, Old Dominion University, Norfolk, VA 23529, USA
[3] School of Mathematics, Trinity College, Dublin 2, Ireland

We extract to high statistical precision an excited spectrum of single-particle isoscalar mesons using lattice QCD, including states of high spin and, for the first time, light exotic J^{PC} isoscalars. The use of a novel quark field construction has enabled us to overcome the long-standing challenge of efficiently including quark-annihilation contributions. Hidden-flavor mixing angles are extracted and while most states are found to be close to ideally flavor mixed, there are examples of large mixing in the pseudoscalar and axial sectors in line with experiment. The exotic J^{PC} isoscalar states appear at a mass scale comparable to the exotic isovector states.

J. Dudek et al. [Hadron Spectrum Collab], Phys. Rev. D 83, 111502 (2011)







Lattice QCD predicts "gluonic excitations", mesons that are not in constituent quark model known as exotic mesons

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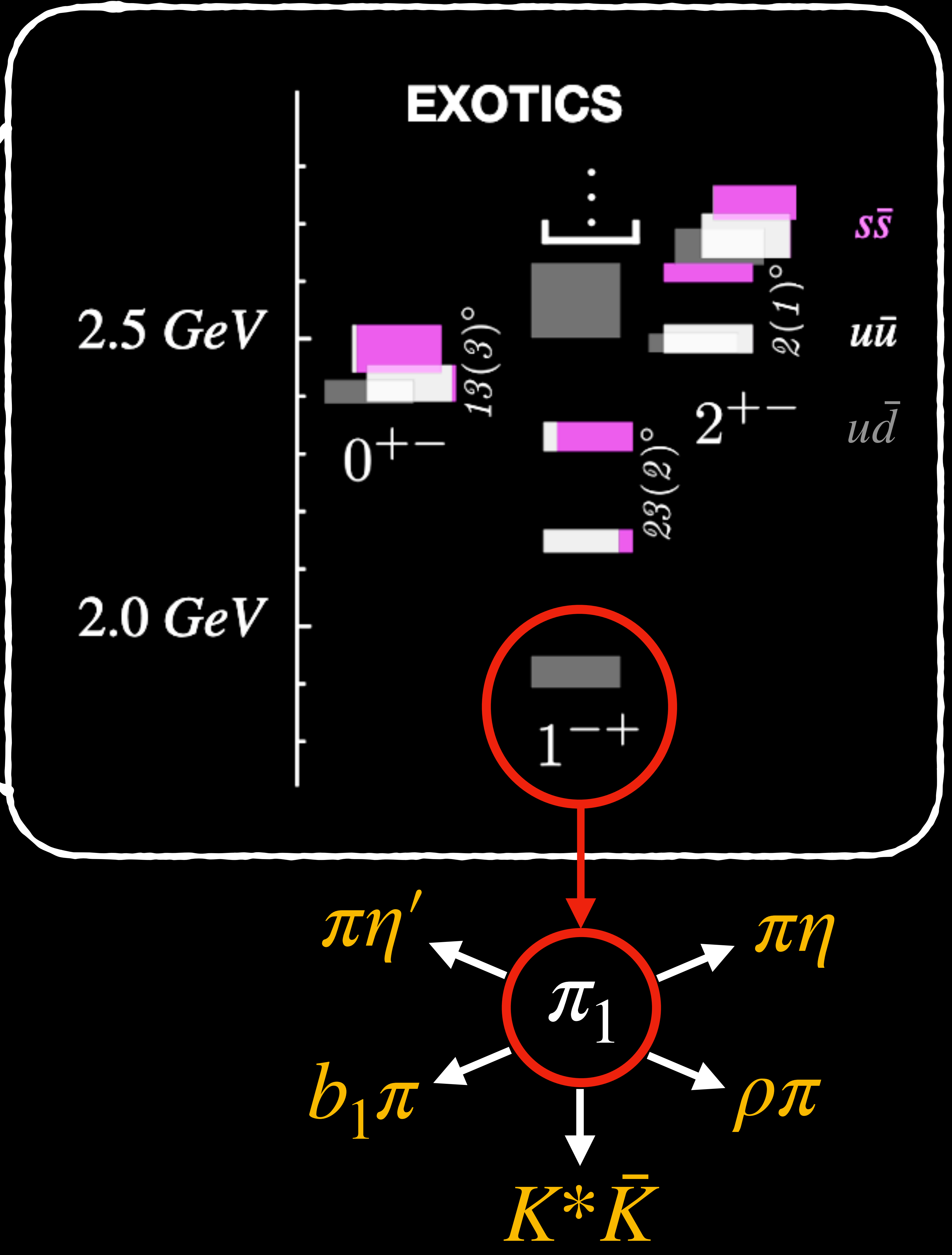

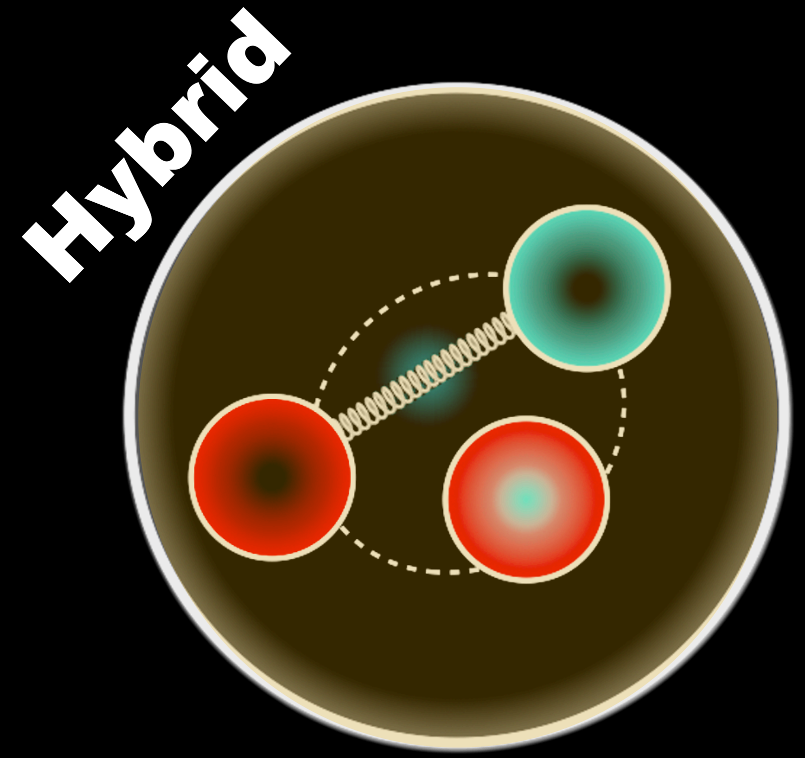

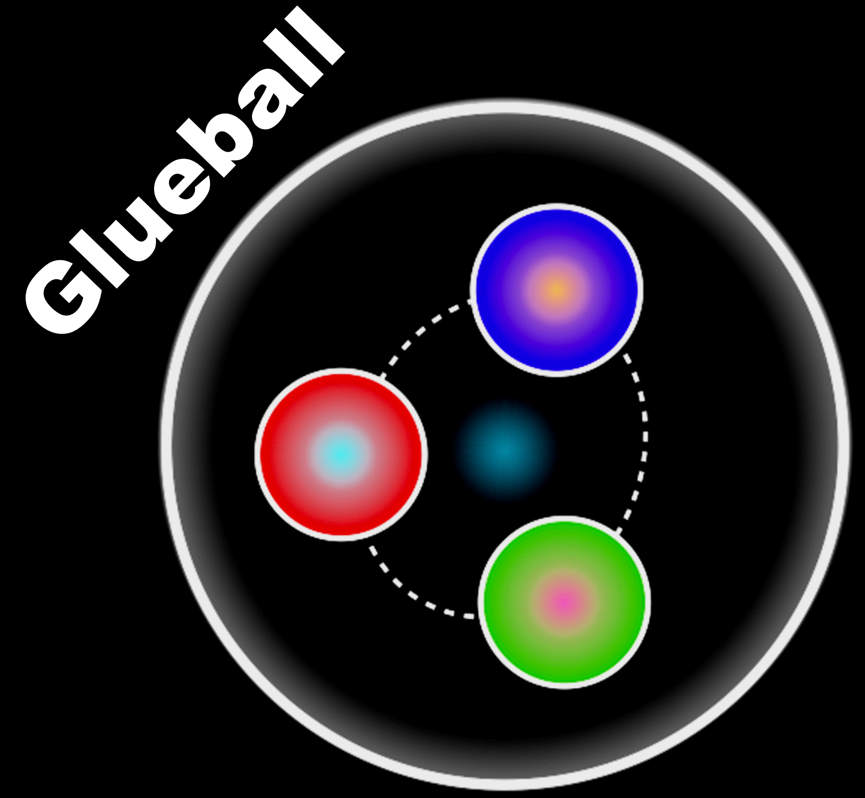

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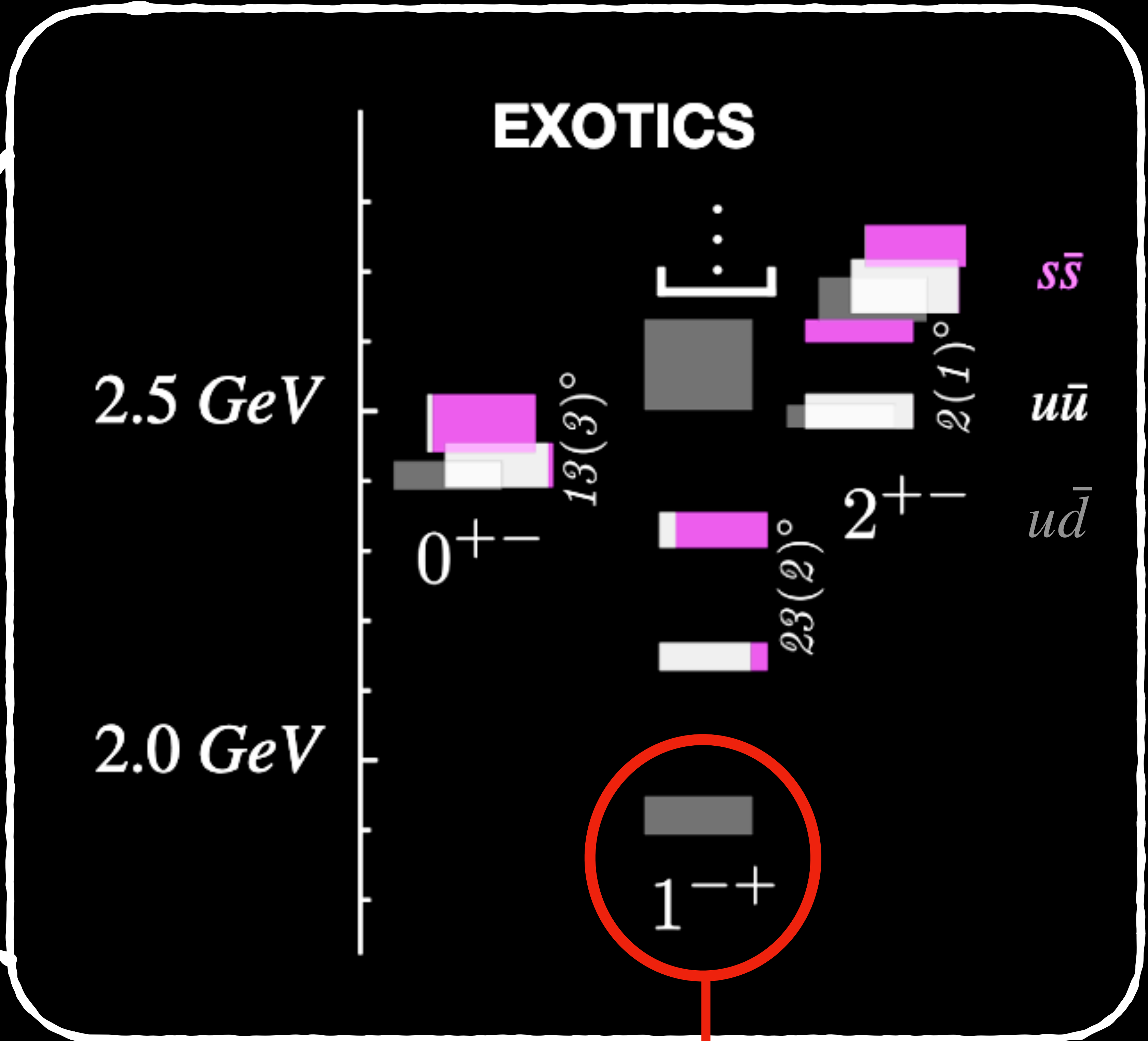
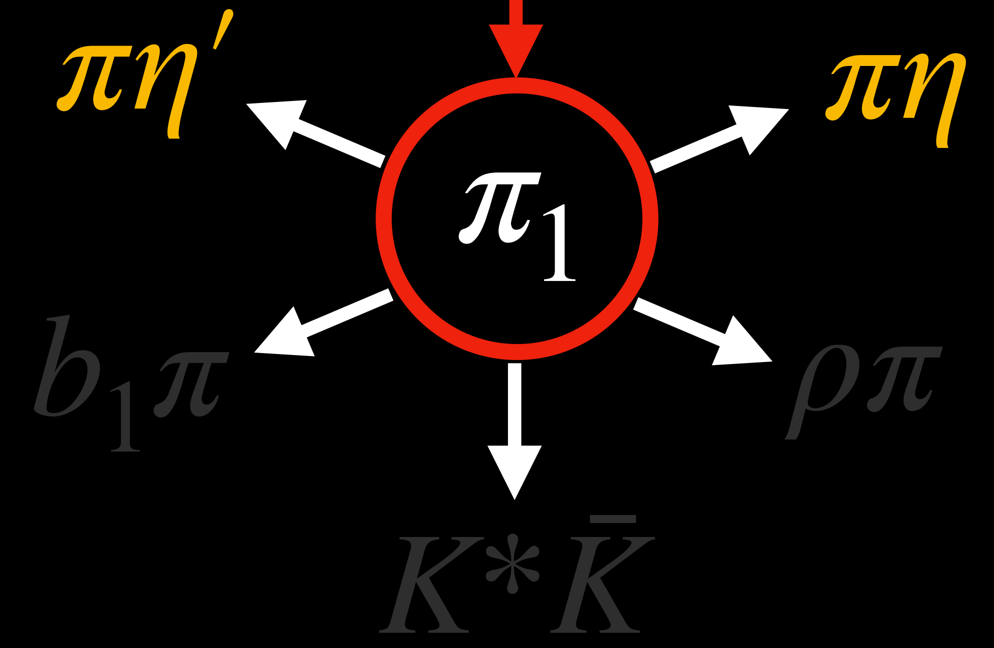
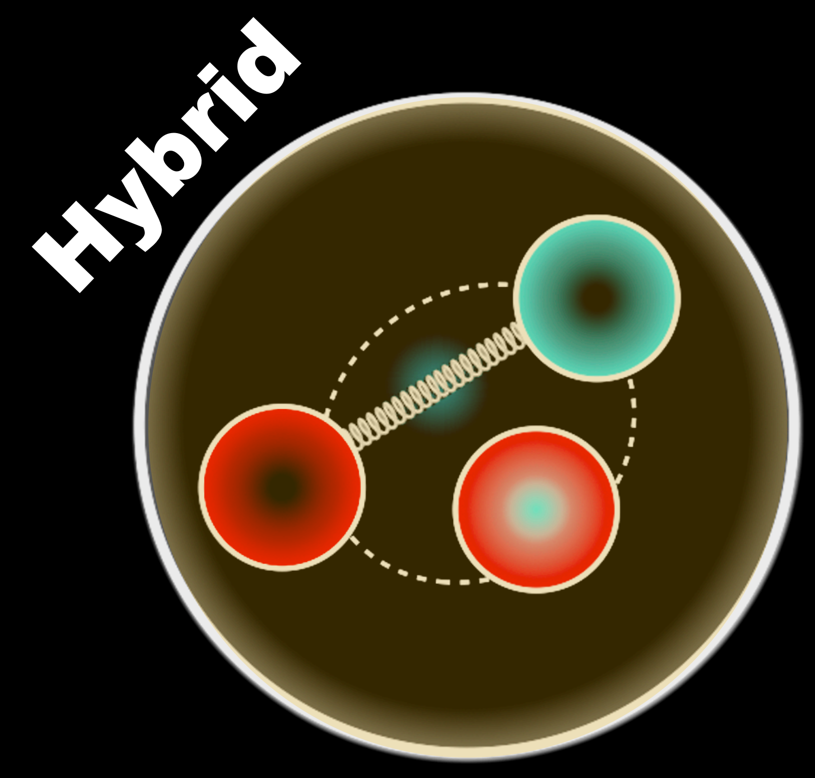
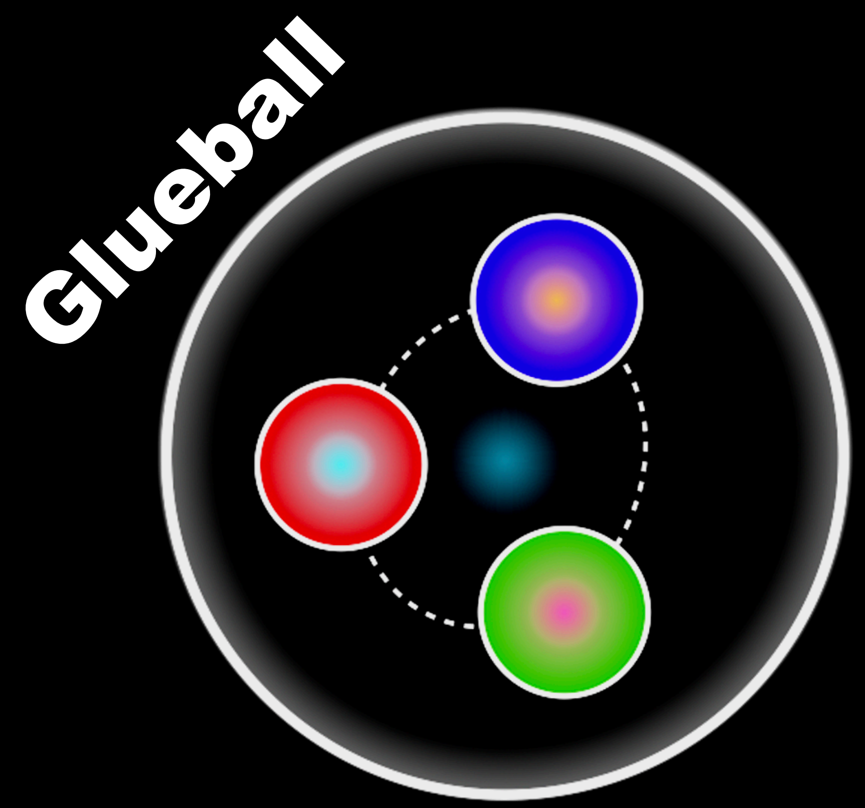


Focus

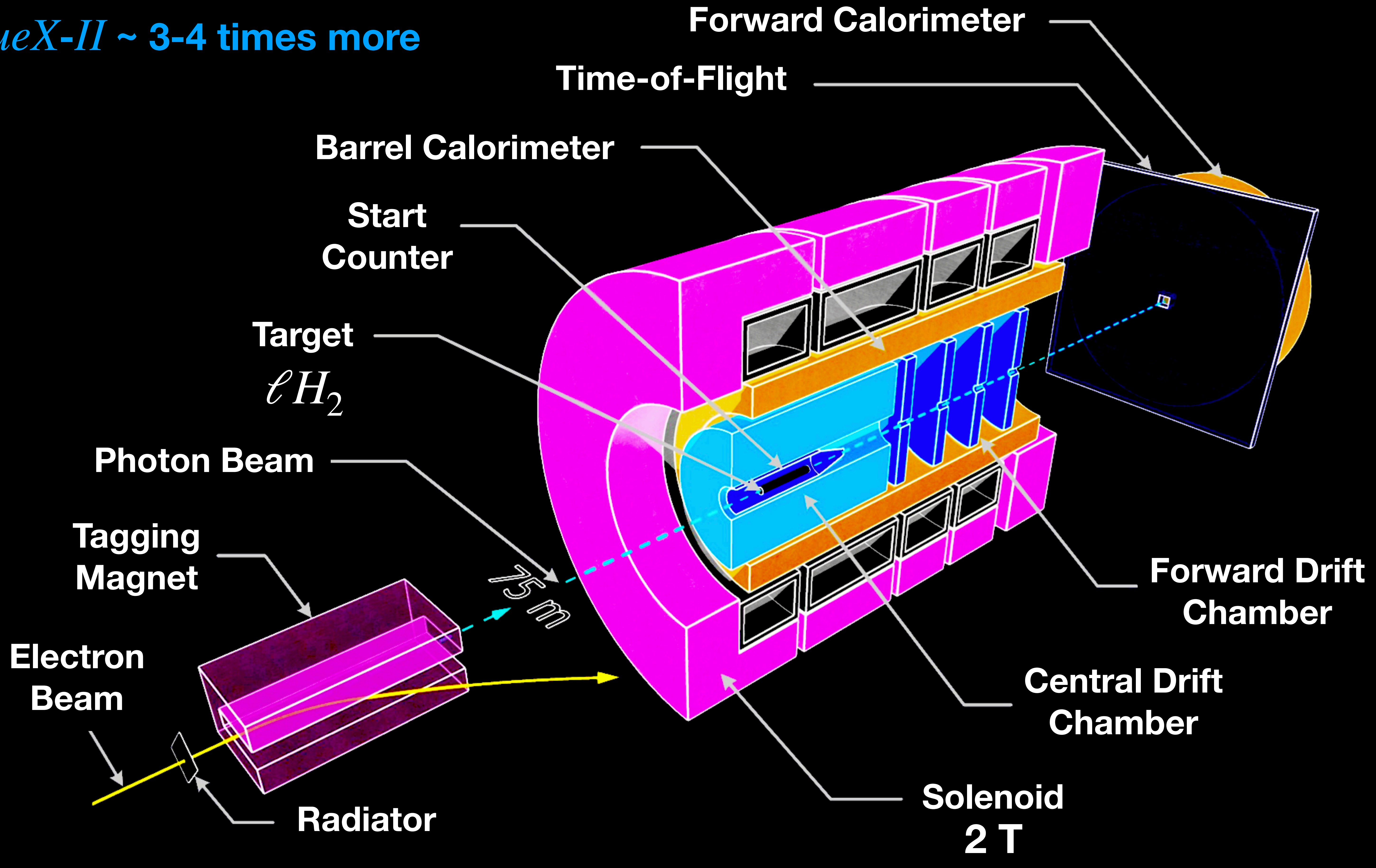




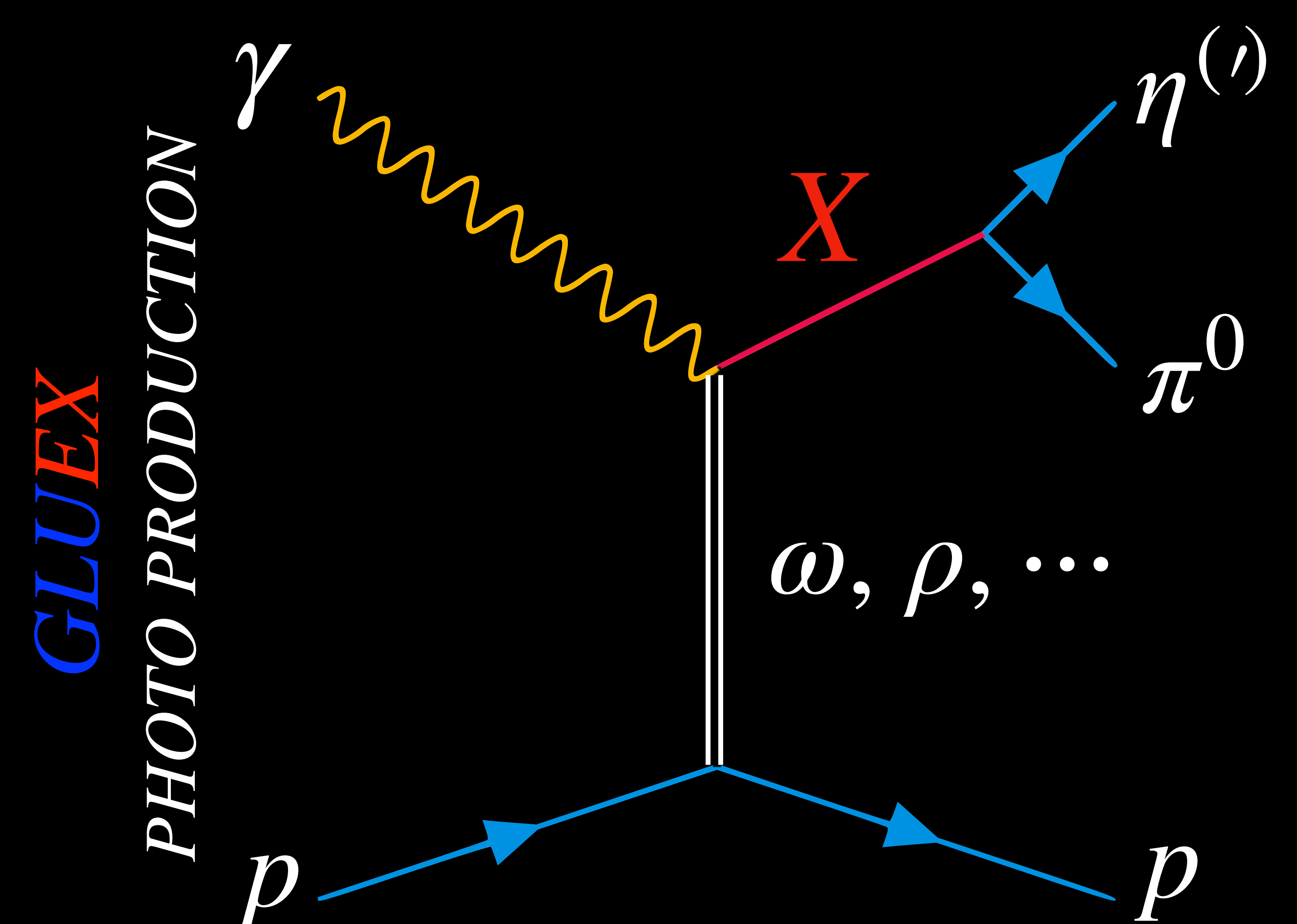


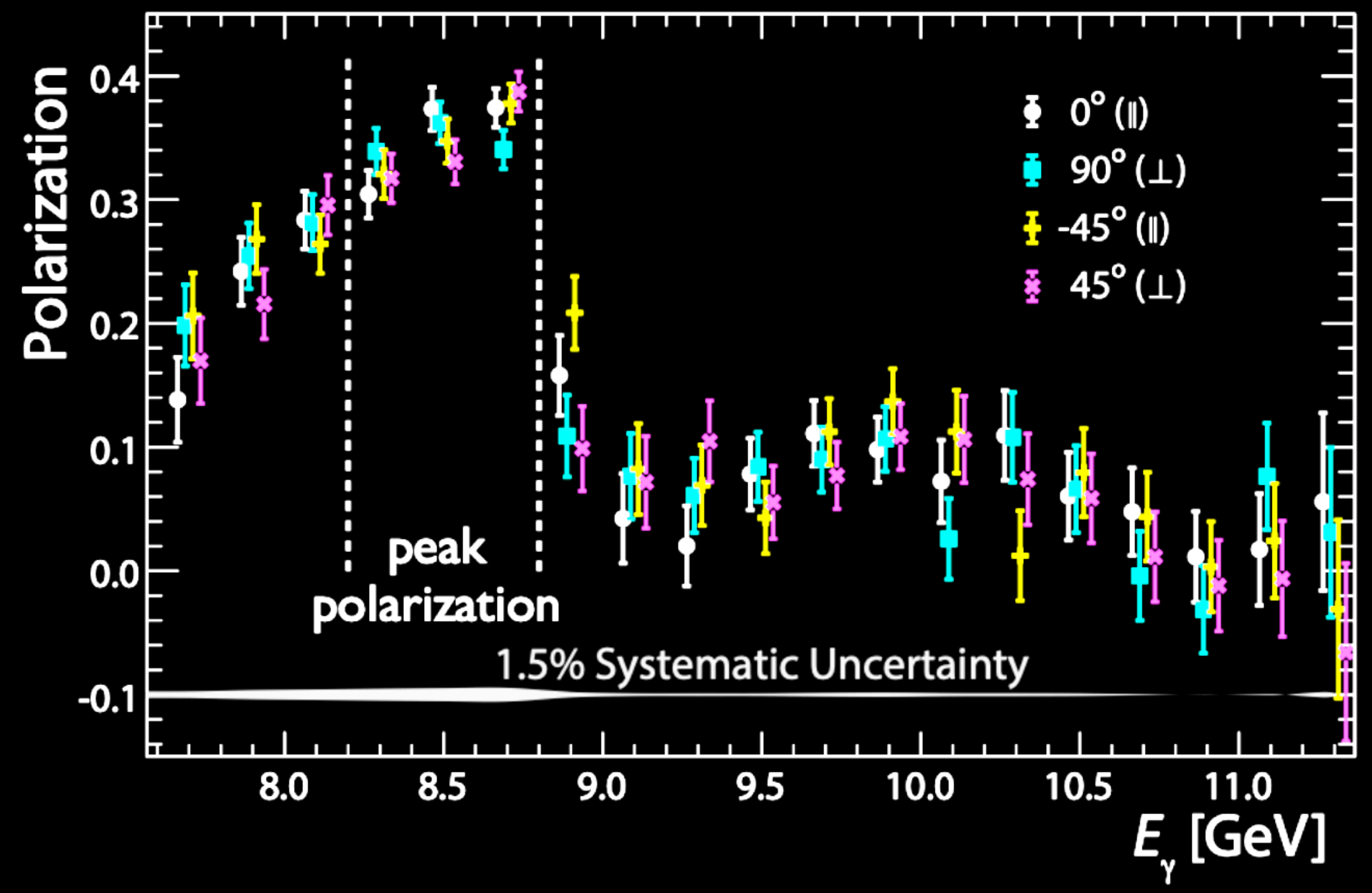
L	S	P	D	F	…
J^{PC}	0^{++}	1^{-+}	2^{++}	3^{-+}	…

- designed to reconstruct final state particles from $\gamma p \rightarrow pM$
- 4 polarization orientations
- *GlueX-I* collected $\int L = 125\ pb^{-1}$ in *coherent peak*
- *GlueX-II* ~ 3-4 times more

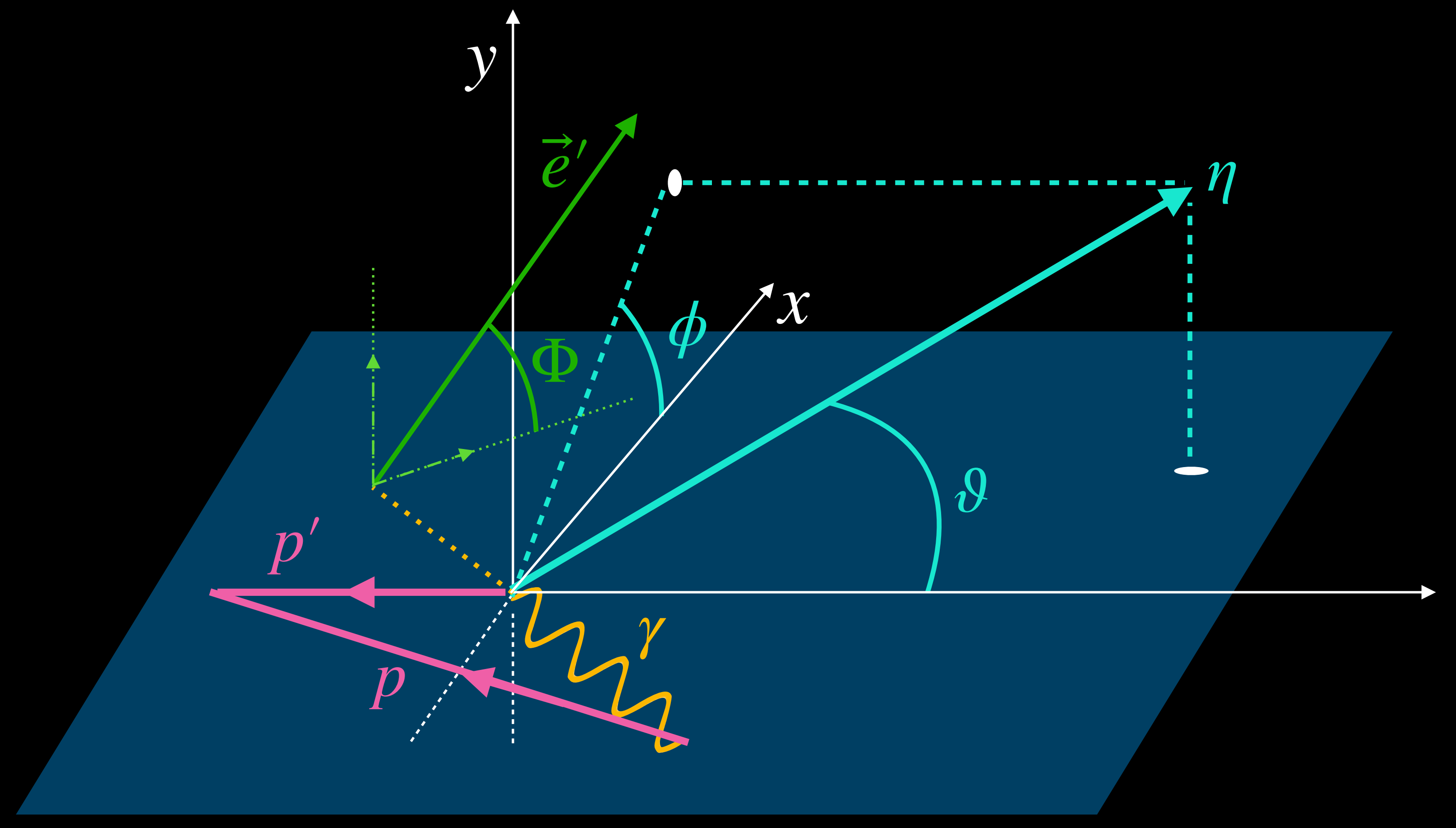


S. Adhikari et al., NIM A 987, 164807 (2021)





- **Describes all two-psuedoscalar systems (i.e. all $\eta^{(\prime)}\pi$)**
- **New basis** $\rightarrow Z_l^m(\Omega, \Phi) = Y_l^m(\Omega)e^{-i\Phi}$
- **Described by 3 angles:** $\cos\vartheta_{\eta^{(\prime)}}$, $\phi_{\eta^{(\prime)}}$ in the resonance frame; $\Phi \rightarrow$ btw the polarization and production plane



V. Mathieu et al. [JPAC], PRD 100, 054017 (2019)

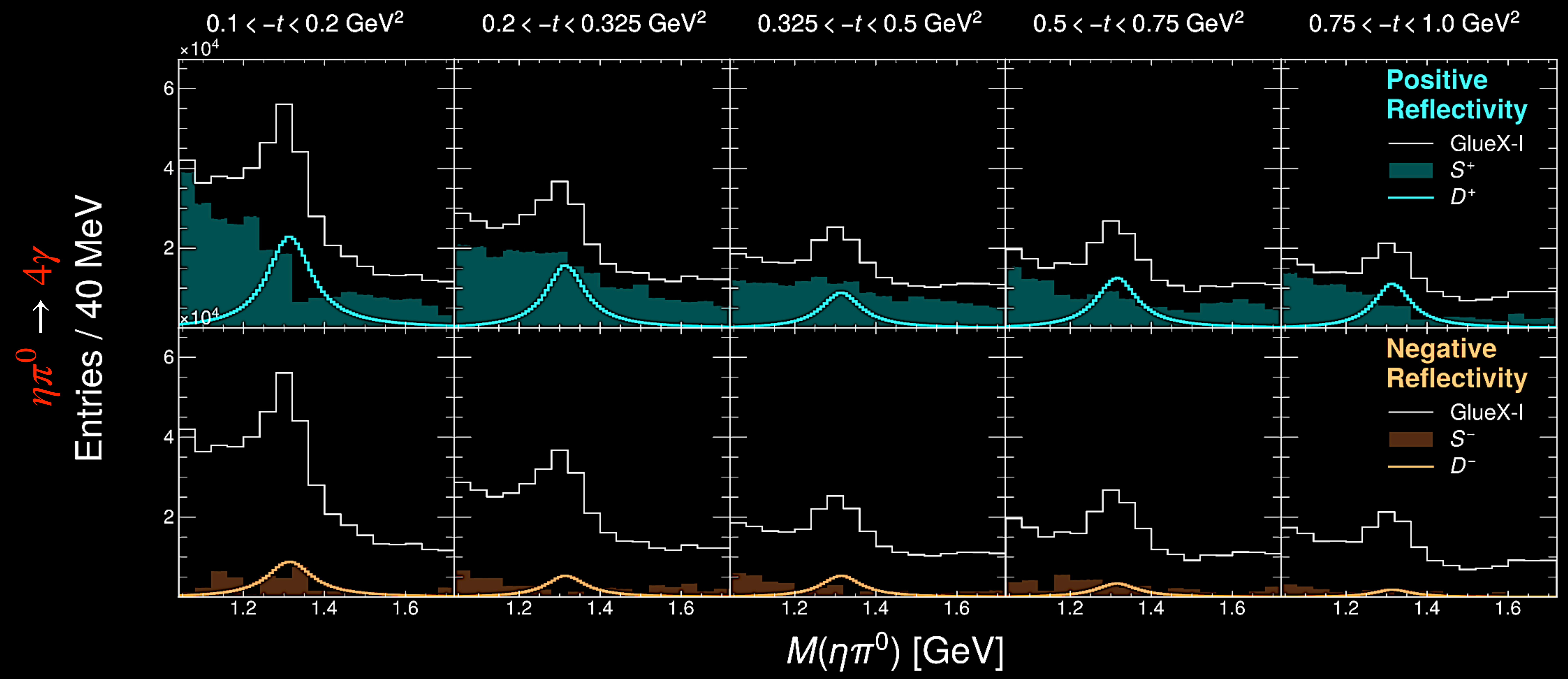
- **Reflectivity corresponds to exchange being natural ($+1$) and unnatural (-1) parity**
- **4x more amplitudes than hadro-production**

$$\Rightarrow \mathcal{I}(\Omega,\Phi) = 2\kappa\sum_k \Big\{(1-P_\gamma)\Big|\sum_{l,m}[l]_m^{(-)}\,\mathcal{Re}[Z_l^m(\Omega,\Phi)]\Big|^2 + (1-P_\gamma)\Big|\sum_{l,m}[l]_m^{(+)}\,\mathcal{Im}[Z_l^m(\Omega,\Phi)]\Big|^2 +$$

$$(1+P_\gamma)\Big|\sum_{l,m}[l]_m^{(+)}\,\mathcal{Re}[Z_l^m(\Omega,\Phi)]\Big|^2 + (1+P_\gamma)\Big|\sum_{l,m}[l]_m^{(-)}\,\mathcal{Im}[Z_l^m(\Omega,\Phi)]\Big|^2\Big\}$$

Fit $[l]_{m;k}^{(\pm)}$ to the data
W/ $m = {}^-l, \ldots, {}^+l$

- **Assume $a_2(1320)$ and $a_2(1700)$ are text book Breit-Wigner resonances**
  - share only 1 common phase parameter for each in the D_{waves}
- **S_{wave} contributions more complicated**
  - define *mass independent* piecewise parameterization
- **Individual fit results across $-t$**
  - coherent sums of (+) and (-) reflectivities



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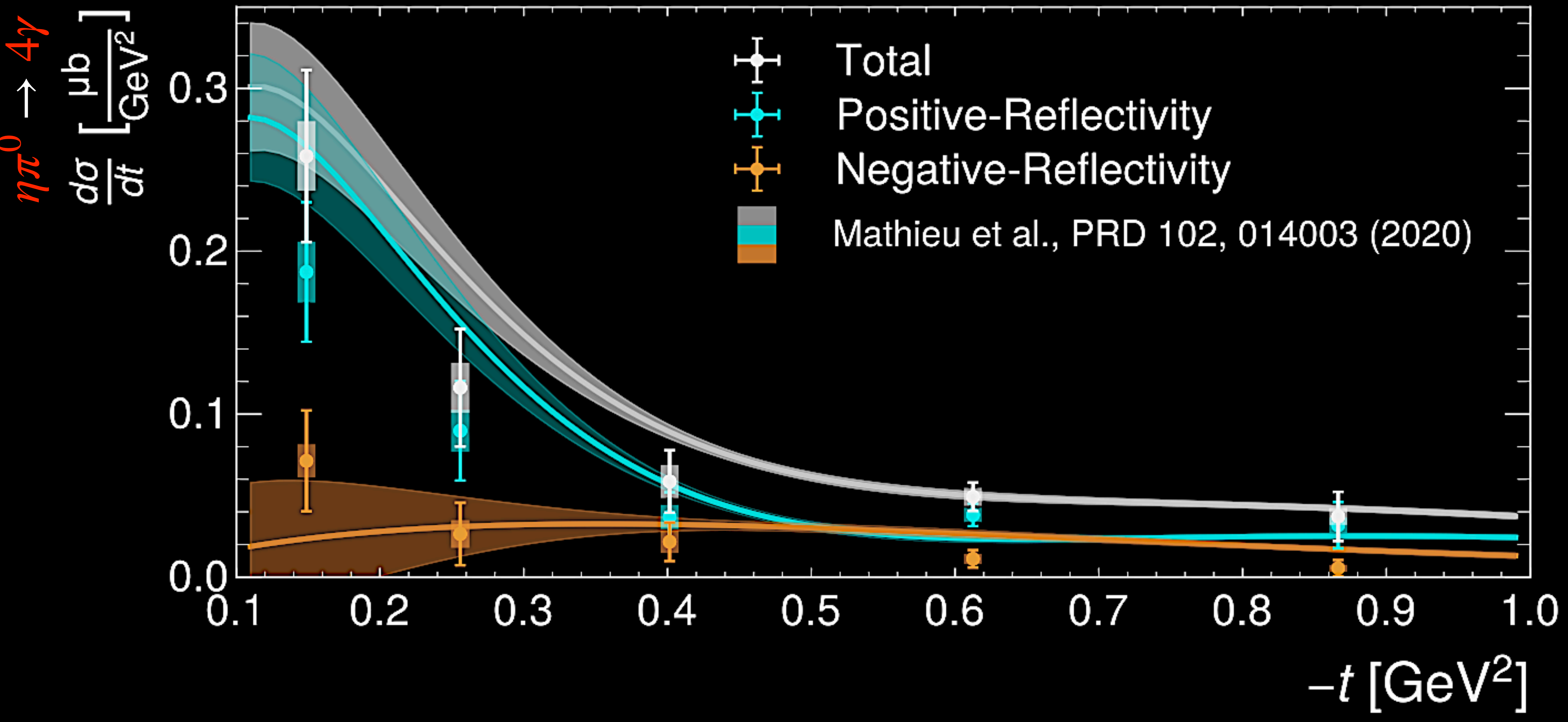
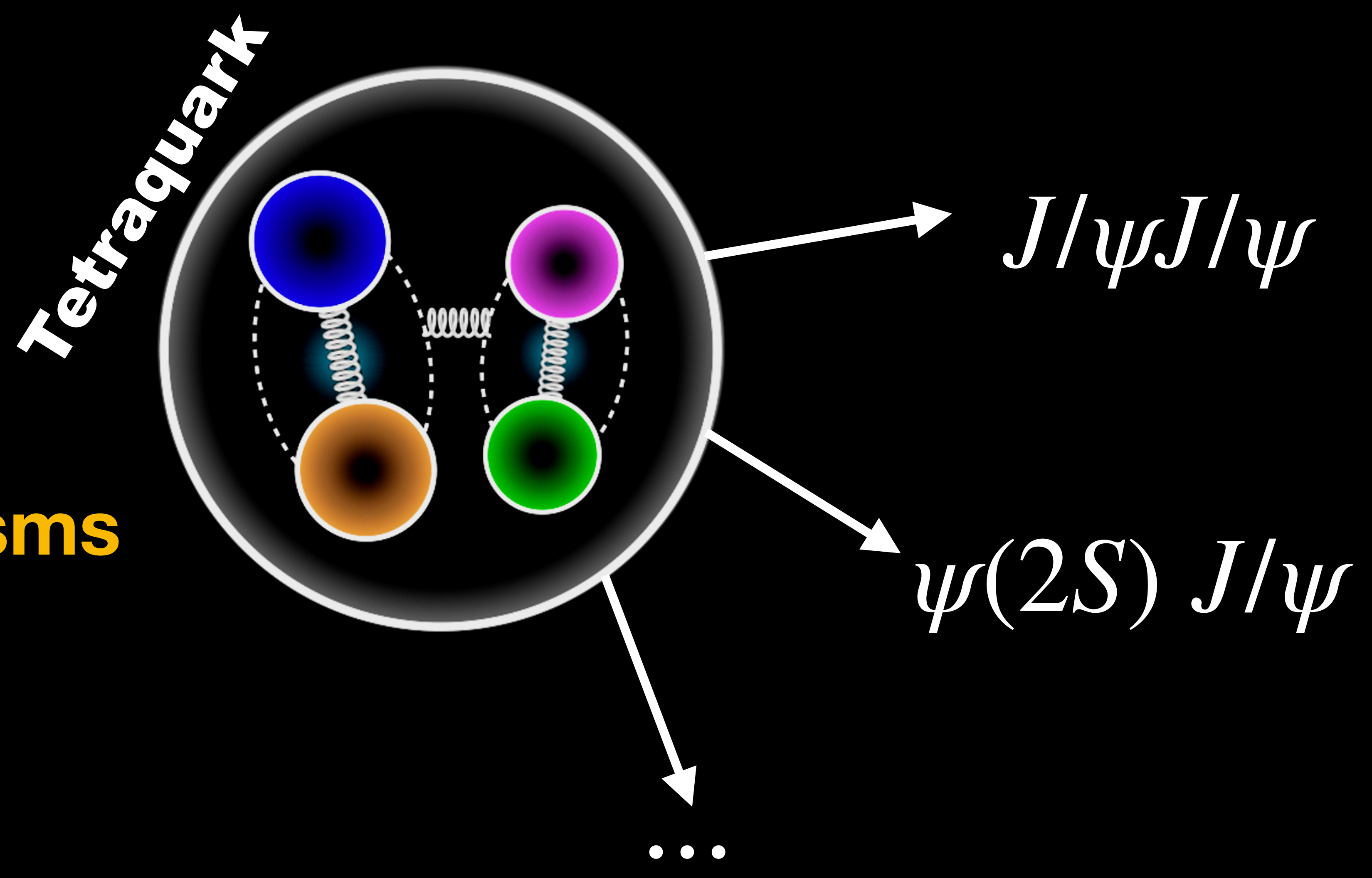
  - share only 1 common phase parameter for each in the D_{waves}
- **S_{wave} contributions more complicated**
  - define *mass independent* piecewise parameterization
- **Decent agreement between JPAC theoretical predictions**
  - systematics finalized

Just Published!

First Measurement of $a_2^0(1320)$ Polarized Photoproduction Cross Section



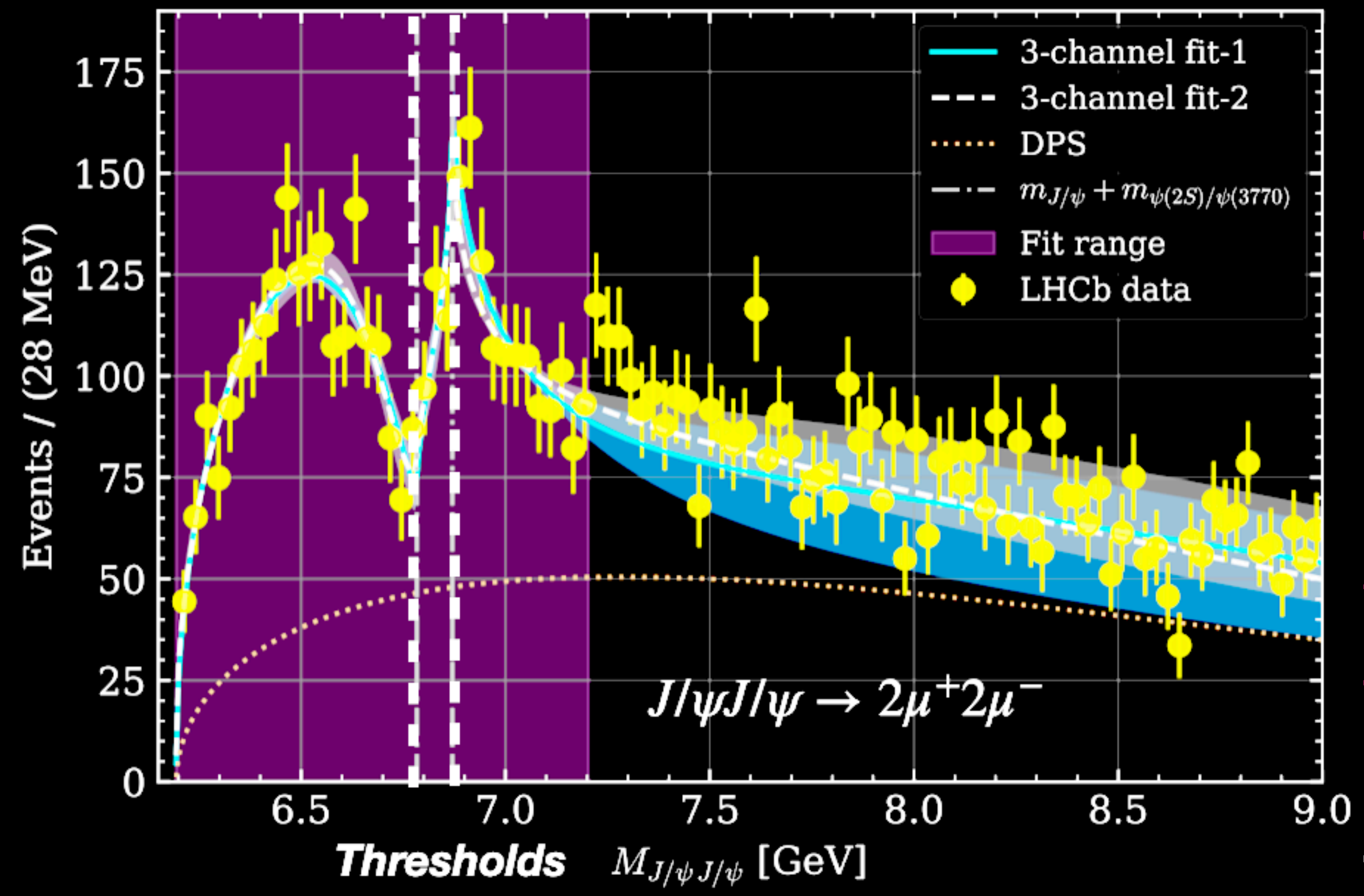
- **Nature of strong interactions:**
  - governed by non-perturbative QCD
  - allows resonance formation and decay across multiple channels



- **Essential resonance parameters vary across production/decay mechanisms**
  - the structure and classification of hadrons emerge from their full decay pattern and interference structure

- **Motivation from experimental reality ⇒**
  - each final state provides only partial access to the underlying pole structure
  - several exciting physics opportunities occur near threshold

Theory Prediction

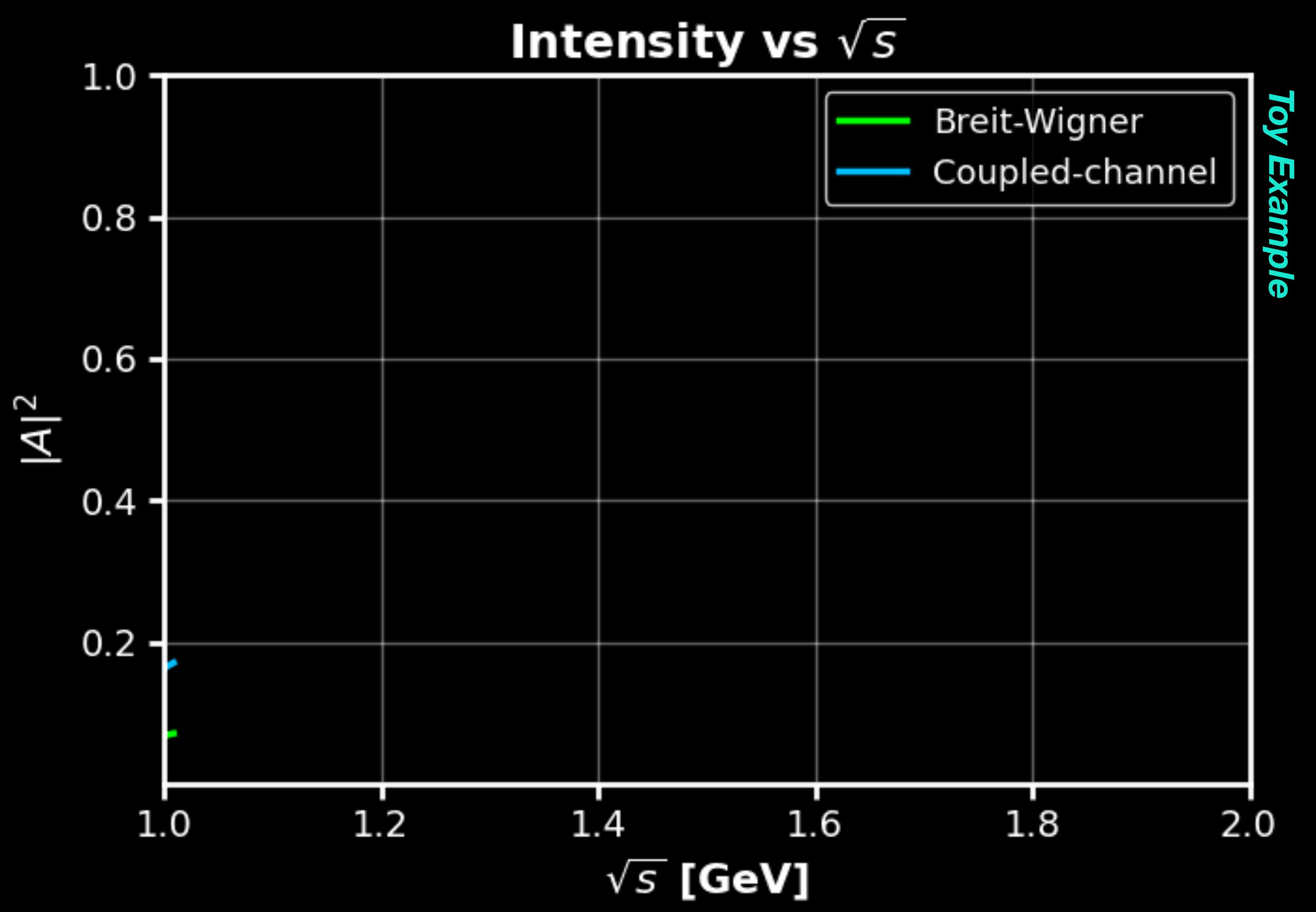


Thresholds

Xiang-Kun Dong et al., Phys. Rev. Lett. 126 (2021)

Overall, single channel *analyses* *cannot* *fully disentangle complex interference or threshold behavior*

- Overlapping resonances interfere differently depending on model
  - **BW**: underestimates interference
  - **CC**: enhances the first peak through self-consistent interference across poles

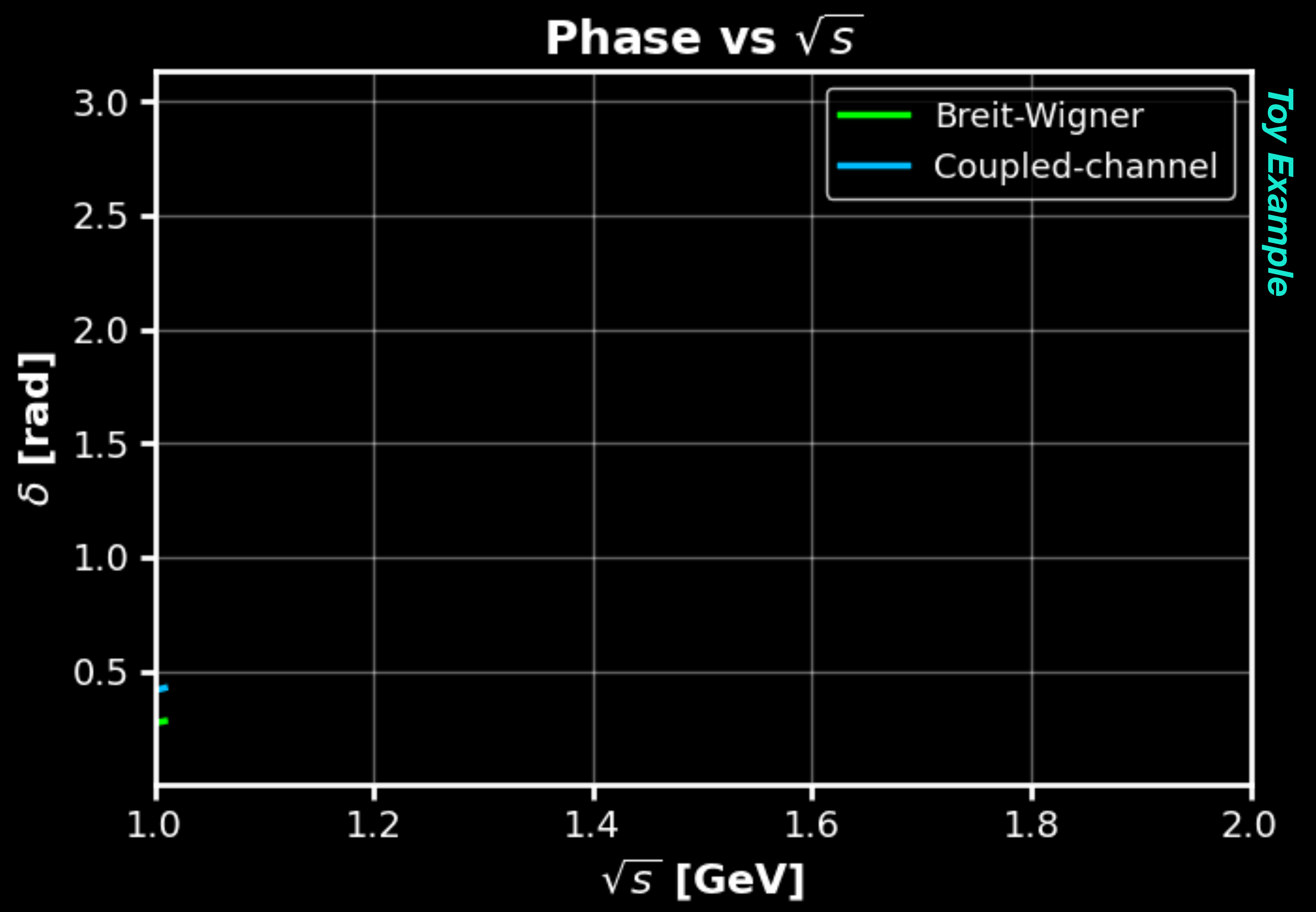


PHASE MOTION

ARGAND LOOP

INTENSITY EVOLUTION

- **Overlapping resonances interfere differently depending on model**
  - **BW**: no threshold structure and ignores threshold effects
  - **CC**: shows a non-analytic phase jump showcasing multiple channel feedback

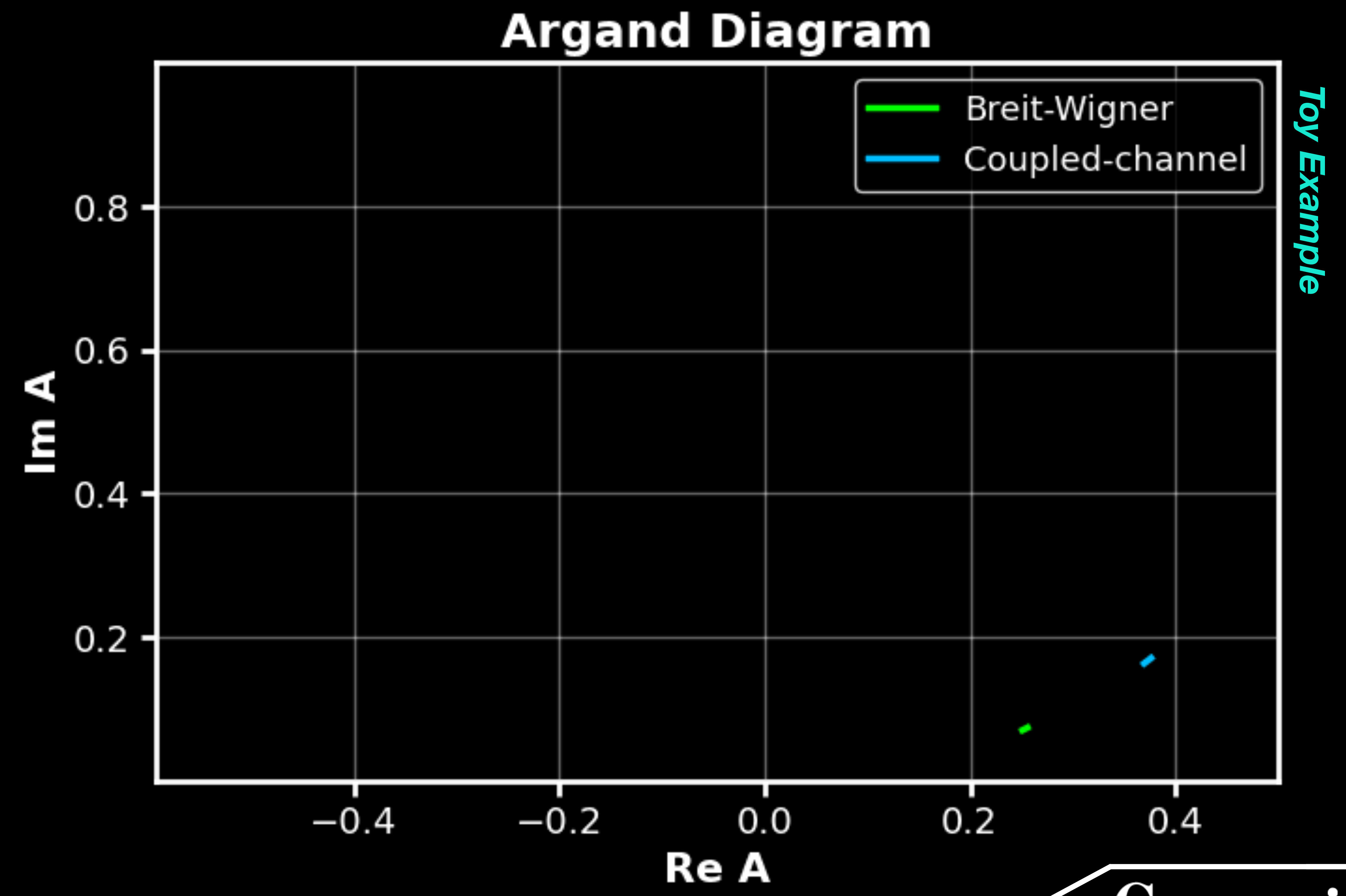


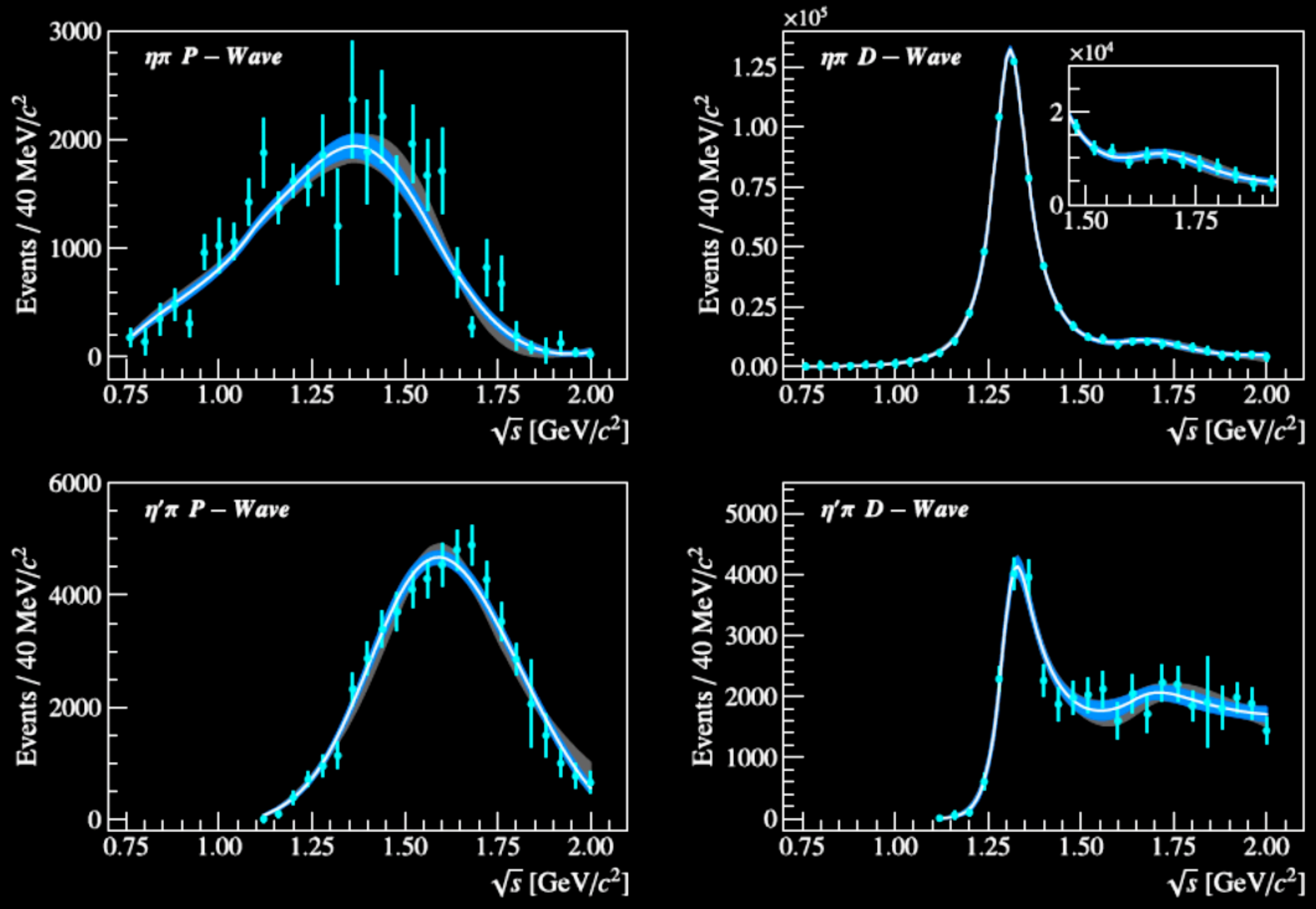
ARGAND LOOP

INTENSITY EVOLUTION

PHASE MOTION

- Overlapping resonances interfere differently depending on model
  - **BW**: erratic, non-unitary motion
  - **CC**: clean, unitary counter-clockwise trajectory





Kopf, B., Albrecht, M., Eur. Phys. J. C 81, 1056 (2021)

- Follow up to the 2020 Crystal Barrel analysis and JPAC results, but with extended framework

  - incorporates additional $\pi^- p \to \eta^{(\prime)}\pi$ and $\pi\pi \to X$ including COMPASS results

  - refines the treatment of the π_1 wave especially to its coupling to $\eta\pi$ and $\eta'\pi$

  - allows single pole for the π_1 to describe both $\eta\pi$ and $\eta'\pi$ (doesn't rule out 2 poles)

Since parameterization is universal, can use these to fit photoproduced $\eta\pi$ *and* $\eta'\pi$!

$\eta\pi^0 \rightarrow 4\gamma\pi^+\pi$

GlueX Phase1 Data

$\eta'\pi^0 \rightarrow 4\gamma\pi^+\pi$

GlueX Phase1 Data

Mass-independent piecewise defined S_{wave} needed to account for no a_0 K-Matrix parameters in $\eta'\pi^0$

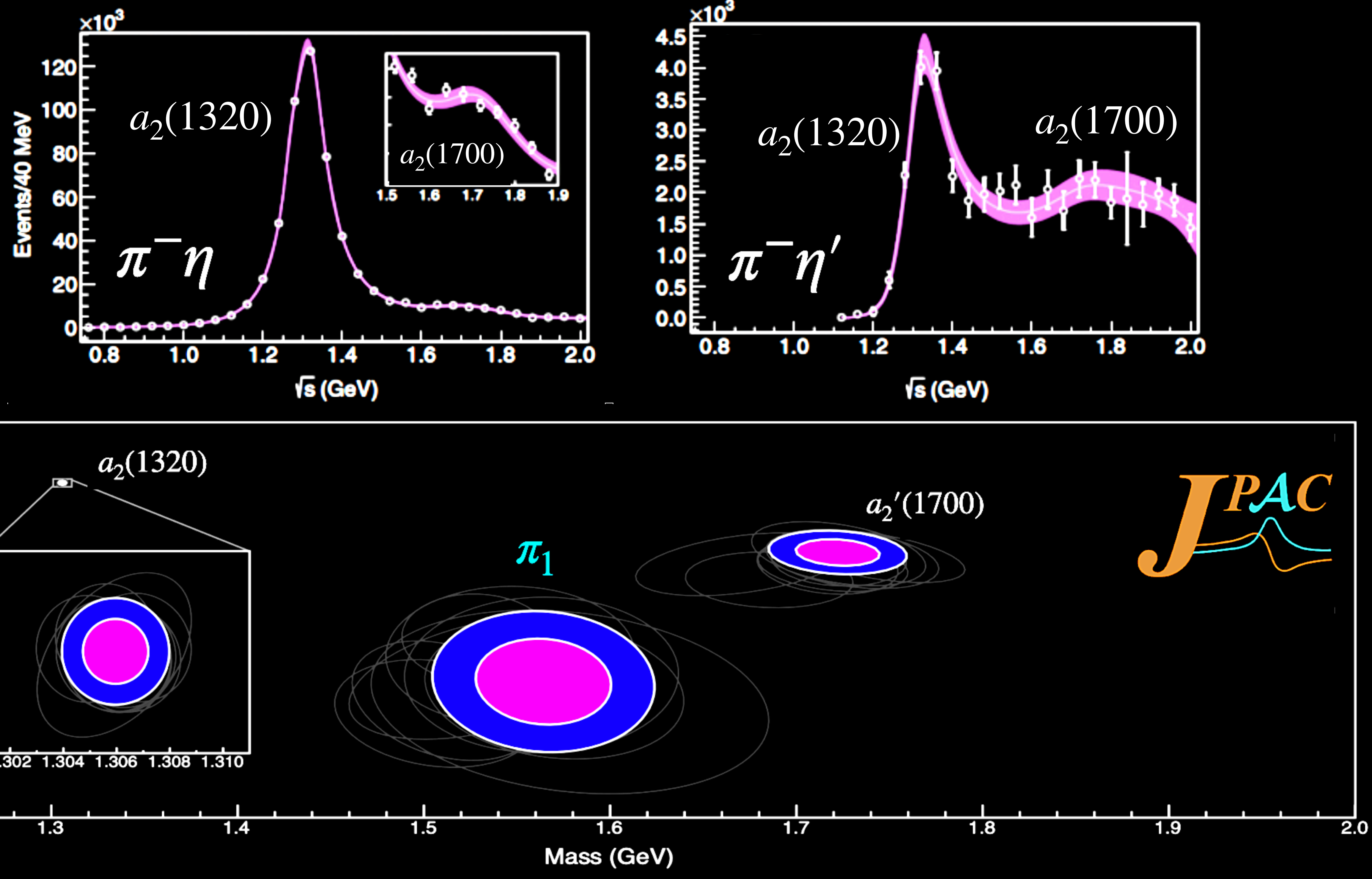
$\eta\pi^0 \rightarrow 4\gamma\pi^+\pi$

$\eta'\pi^0 \rightarrow 4\gamma\pi^+\pi$

GlueX Phase1 Data

- **JPAC analysis utilizing COMPASS data**
  - coupled channel fit to both $\eta^{(\prime)}\pi$ systems
  - describes dominate a_2 resonances and the π_1

- **Model line shapes using the analytic, unitary N/D formalism**
  - extraction of poles more rigorous than K-Matrix or BW approaches



A. Rodas et al.
[Joint Physics Analysis Center], PRL 122, 042002 (2019)

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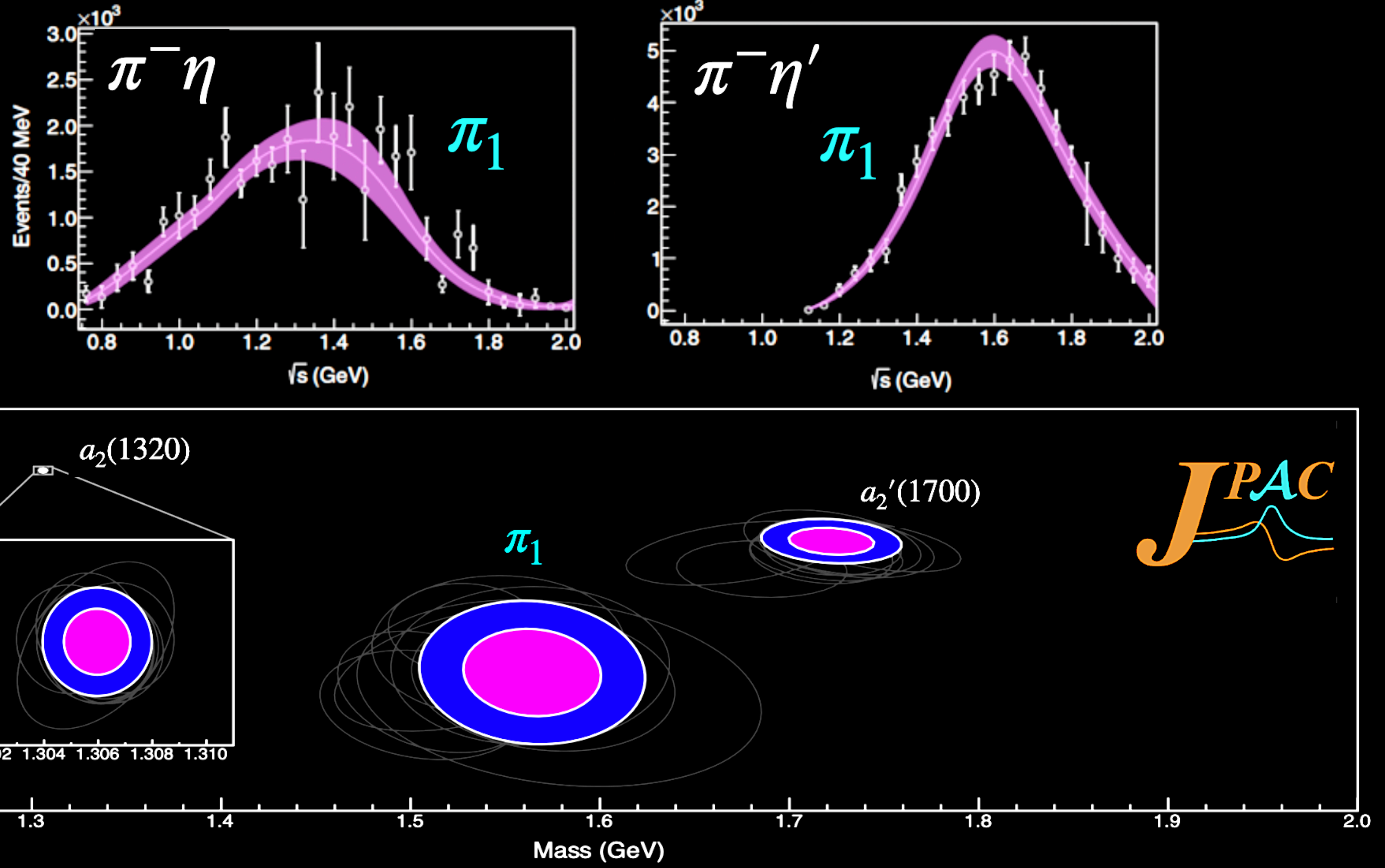


$\eta\pi^0 \to 4\gamma\pi^+\pi$

GlueX Phase1 Data

$\eta'\pi^0 \to 4\gamma\pi^+\pi$

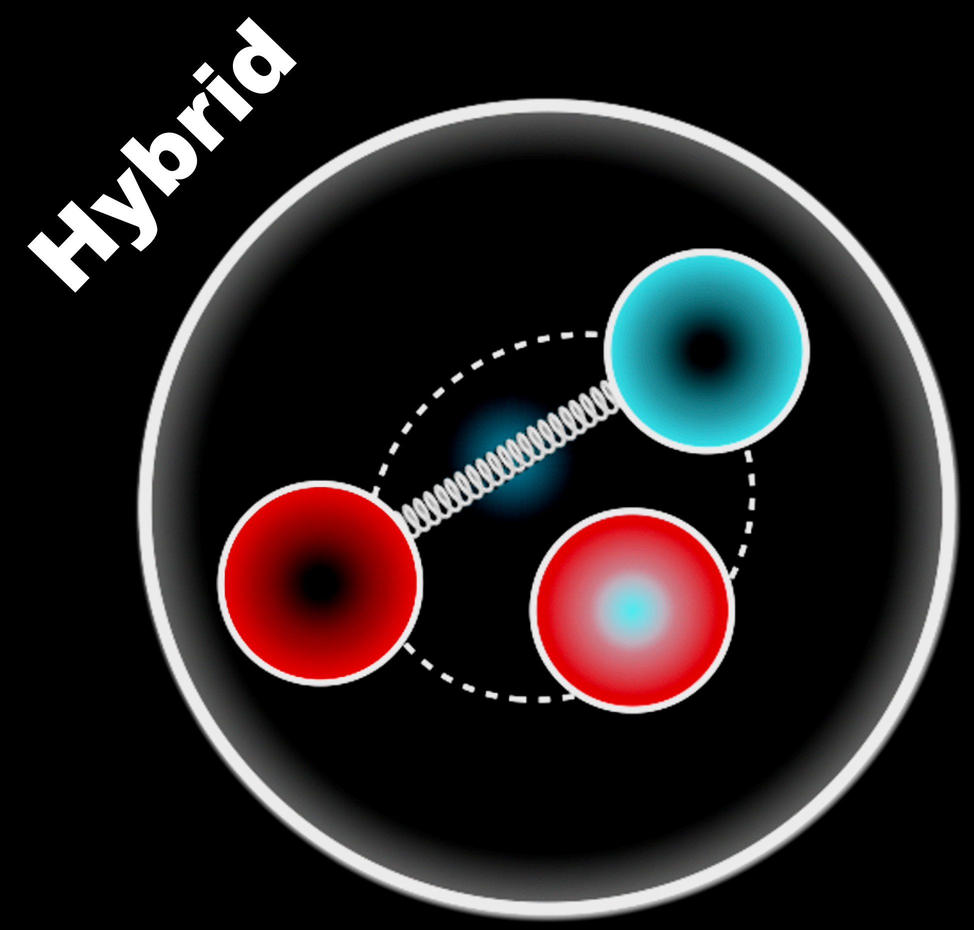
GlueX Phase1 Data

Piecewise defined S_{wave} needed to account for both channels since no S_0 configurations

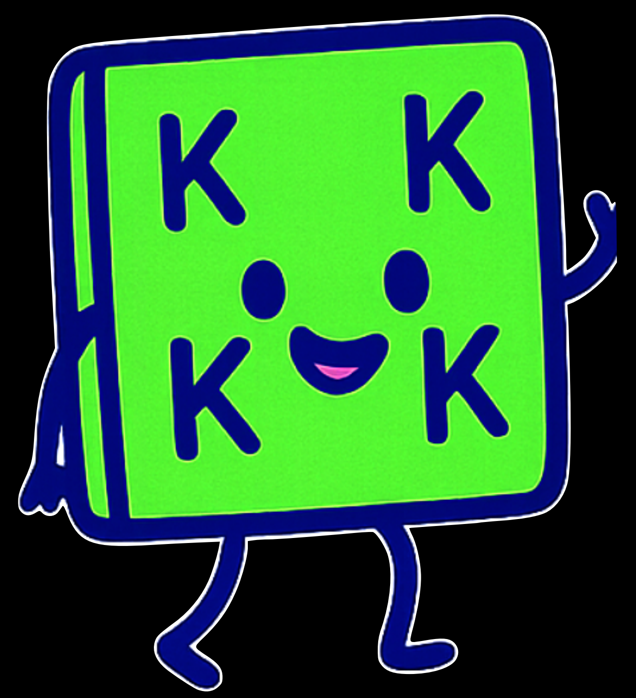
— working with JPAC to develop more sophisticated model

- **GlueX has collected large quantity of photoproduced data**
  - recent results extracted a_2 cross section which will be used as a reference signal
  - can analyze production mechanisms using polarization info
  - strong effort to look for *exotic hybrid* π_1 meson in $\eta^{(\prime)}\pi$ systems using several different analysis methods *
  - coupled-channel framework produce stable and reasonable fits
    — potential exotic signal?

(with significantly larger amounts of π_1 observed in $\eta'\pi^0$ than in $\eta\pi^0$
working on projecting π_1 upper limit simulations onto these results)



- **Next immediate steps:**
  - Further analyze both neutral and charged $\eta^{(\prime)}\pi$
    — extract a_2^- cross section, etc.
  - find the pole positions, and make sure they are correct (or fits are meaningless!)

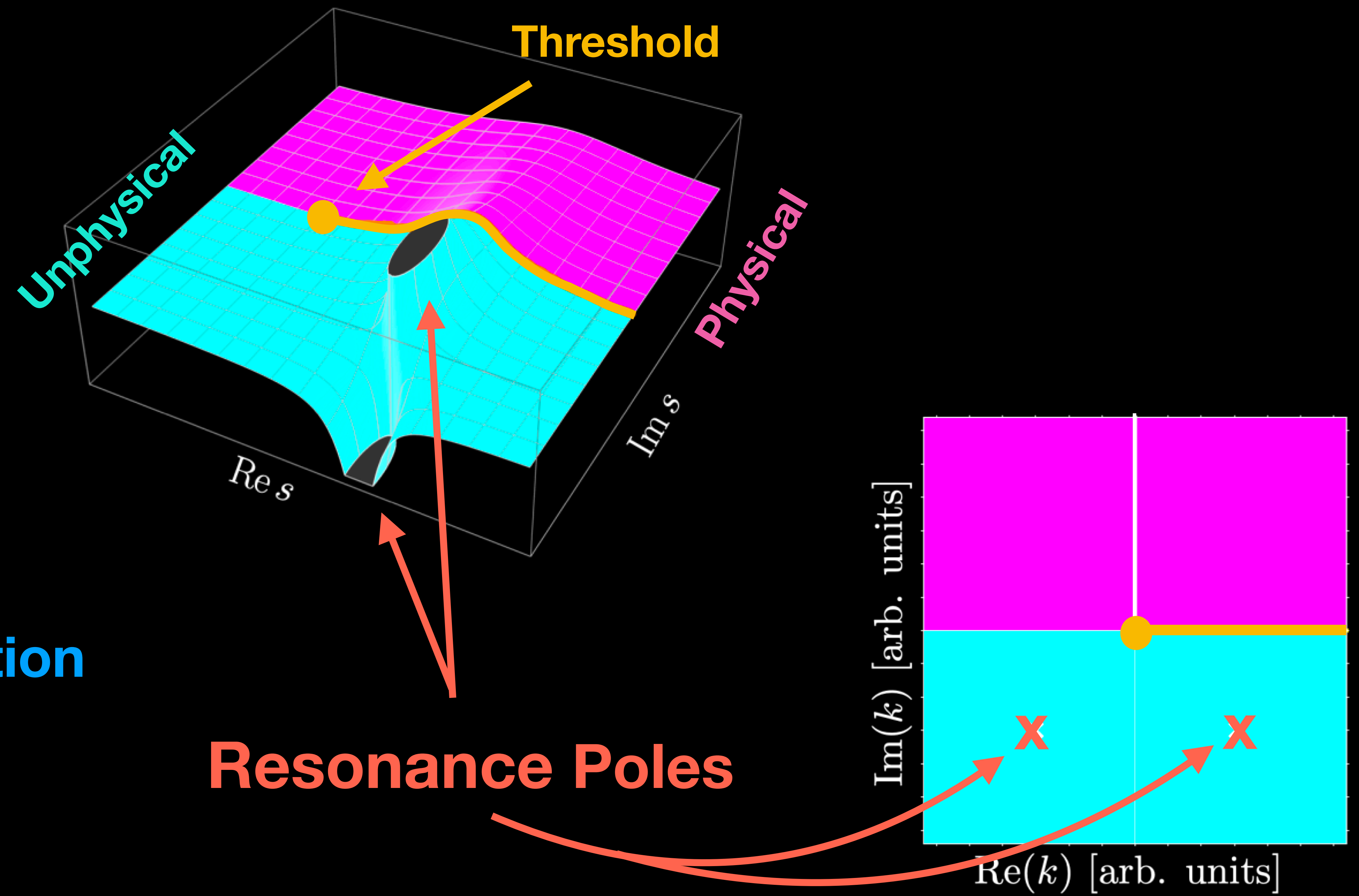


EXCITING TIMES FOR EXOTICS!

For complete derivations/examples with further information on coupled-channel approach see
International K-Matrix Day Indico
Küßner

BACKUP SLIDES

- In QFT → resonances correspond to poles of the S-matrix in the complex energy plane
  - these poles lie off the real energy axis, reflecting the unstable nature of resonances
  - not directly visible, but they influence observables (i.e. cross sections)
  - resonance position and width → encoded in complex pole location



- Breit-Wigner functions are the historical standard for describing resonances
  - *algebraically simple*
  - *work well for isolated, narrow resonances*

  useful first approximation!

But…

- fails to conserve total probability in multi-channel scattering
- do not account for:
  1) coupled-channel effects
  2) overlapping resonances
  3) nearby thresholds

To understand what a true resonance is,
we must ask where its pole position lies … not where its bump appears!

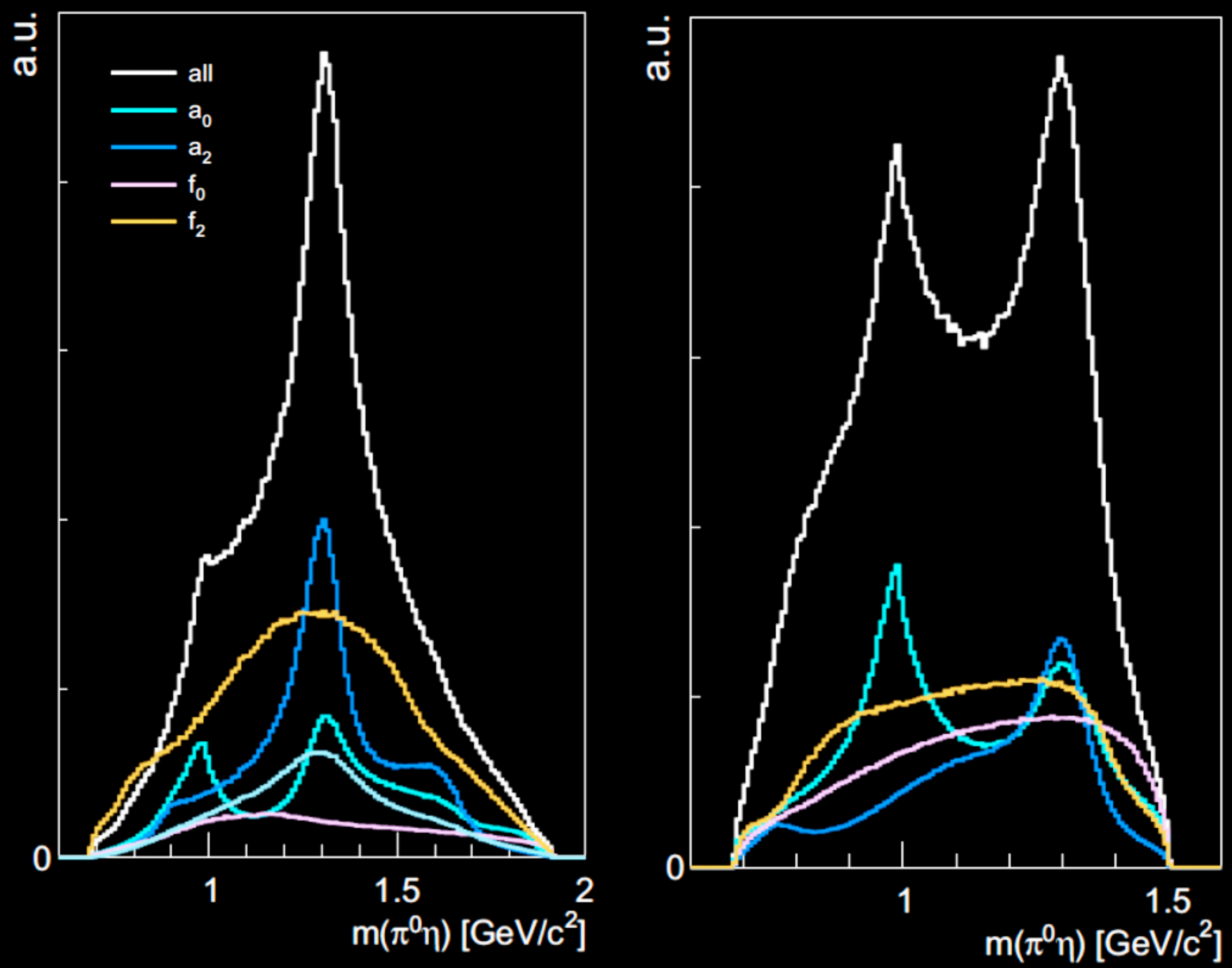
- **Recent comprehensive coupled-channel partial wave analysis of $p\bar{p} \rightarrow \pi^0\pi^0\eta$, $\pi^0\eta\eta$, and $K^+K^-\pi^0$**

  - measured at Crystal Barrel

  - analyzed alongside $\pi\pi$ scattering data using the KMatrix formalism and Chew-Mandelstam functions

  - identified significant contributions from: f_0, f_2, a_0, a_2, and ρ

  also prominent π_1 wave (~20% wave contribution)



Albrecht, M., et al. [Crystal Barrel Collab.] Eur. Phys. J. C 80, 453 (2020)

- **Formalism following the approach in** *Kopf, B et al* **gives the scattering amplitude as:**

$$F_i^p(s) = \sum_j \left[\left(I + K(s)C(s) \right)^{-1} \right]_{ij} P_j^p(s)$$

- **Formalism following the approach in *Kopf, B et al* gives the scattering amplitude as:**

P-vector encoding production dynamics

$$P_j^p(s) = \sum_\alpha \left(\frac{\beta_\alpha^p g_{\alpha j}^{\text{bare}}}{m_\alpha^2 - s} + \sum_k c_{kj} \cdot s^k \right) \cdot B^\ell(q_j, q_{\alpha j})$$

$$F_i^p(s) = \sum_j \left[\left(I + K(s) C(s) \right)^{-1} \right]_{ij} P_j^p(s)$$

K-Matrix

$$K_{ij}(s) = \sum_\alpha B^\ell(q_i, q_{\alpha i}) \left(\frac{g_{\alpha i}^{\text{bare}} g_{\alpha j}^{\text{bare}}}{m_\alpha^2 - s} + \tilde{c}_{ij} \right) B^\ell(q_j, q_{\alpha j})$$

$$C_{ii}(s) = \frac{\rho_i(s)}{\pi} \log \left(\frac{\xi_i(s) + \rho_i(s)}{\xi_i(s) - \rho_i(s)} \right) - \frac{\xi_i(s)}{\pi} \frac{m_{2i} - m_{1i}}{m_{1i} + m_{2i}} \log \left(\frac{m_{2i}}{m_{1i}} \right)$$

Chew-Mandelstam

- In order to use the previous results on photoproduced data - need to fix the following parameters ⋯

$$K_{ij}(s) = \sum_{\alpha} B^{\ell}(q_i, q_{\alpha i}) \left(\frac{g_{\alpha i}^{\text{bare}} g_{\alpha j}^{\text{bare}}}{m_{\alpha}^2 - s} + \tilde{c}_{ij} \right) B^{\ell}(q_j, q_{\alpha j})$$

$$P_j^p(s) = \sum_{\alpha} \left(\frac{\beta_{\alpha}^p g_{\alpha j}^{\text{bare}}}{m_{\alpha}^2 - s} + \sum_k c_{kj} \cdot s^k \right) \cdot B^{\ell}(q_j, q_{\alpha j})$$

$\alpha \rightarrow$ Resonances

$B^{\ell} \rightarrow$ Blatt-Weisskopf Barrier Factors

$g \rightarrow$ Final State Couplings

$c \rightarrow$ Backgrounds in KMatrix

⋯ and the only parameters floated in the fit are the complex photocouplings and the complex background amplitudes to the photoproduced data ⋯

$\beta_{\alpha}^p \rightarrow$ Photocouplings $\qquad$ $c_{kj} \rightarrow$ Constant Background

⋯ in each reflectivity ($+/-$)

- In order to use the previous results on photoproduced data - need to fix the following parameters ⋯

$$K_{ij}(s) = \sum_{\alpha} B^{\ell}(q_i, q_{\alpha i}) \left(\frac{g_{\alpha i}^{\text{bare}} g_{\alpha j}^{\text{bare}}}{m_{\alpha}^2 - s} + \tilde{c}_{ij} \right) B^{\ell}(q_j, q_{\alpha j})$$

$$P_j^p(s) = \sum_{\alpha} \left(\frac{\beta_{\alpha}^p g_{\alpha j}^{\text{bare}}}{m_{\alpha}^2 - s} + \sum_k c_{kj} \cdot s^k \right) \cdot B^{\ell}(q_j, q_{\alpha j})$$

α → Resonances

B^{ℓ} → Blatt-Weisskopf Barrier Factors

g → Final State Couplings

c → Backgrounds in KMatrix

first terms in each amplitude (the resonance poles) account for the well-defined, resonant structures

BUT these alone cannot describe the full structure of real data

- phenomenologically encode smooth, slowly-varying background that absorbs non-resonant effects

(currently a constant complex value but can increase to higher order)

- **Formalism following the approach in** *Rodas, A et al* **gives the production amplitude as:**

Angular Momentum Barrier Factors

Production Term (smooth, real functions)

$$A_i^J(s) = q^{J-1} p_i^J \sum_k n_k^J(s) \left[D^J(s)^{-1} \right]_{ki}$$

KMatrix

$$K_{ki}^J(s) = \sum_R \frac{g_k^{J,R} g_i^{J,R}}{m_R^2 - s} + c_{ki}^J + d_{ki}^J s$$

$$D_{ki}^J(s) = \left[K^J(s)^{-1} \right]_{ki} - \frac{s}{\pi} \int_{s_k}^{\infty} ds' \frac{\rho N_{ki}^J(s')}{s'(s' - s - i\epsilon)}$$

Analytic Denominator Matrix

- Similar to before, in order to use the previous results on photoproduced data - need to fix the following parameters⋯

$$n_k^J(s) = \sum_{i=0}^{3} a_i^{(J,k)}\, T_i\left(\frac{s}{s+1}\right)$$

$$K_{ki}^J(s) = \sum_R \frac{g_k^{J,R} g_i^{J,R}}{m_R^2 - s} + c_{ki}^J + d_{ki}^J s$$

$a \rightarrow$ Chebyshev Coefficients

$g \rightarrow$ Final State Couplings

$c, d \rightarrow$ Backgrounds in K-Matrix

⋯ and the only parameters floated in the fit are the complex photocouplings ⋯

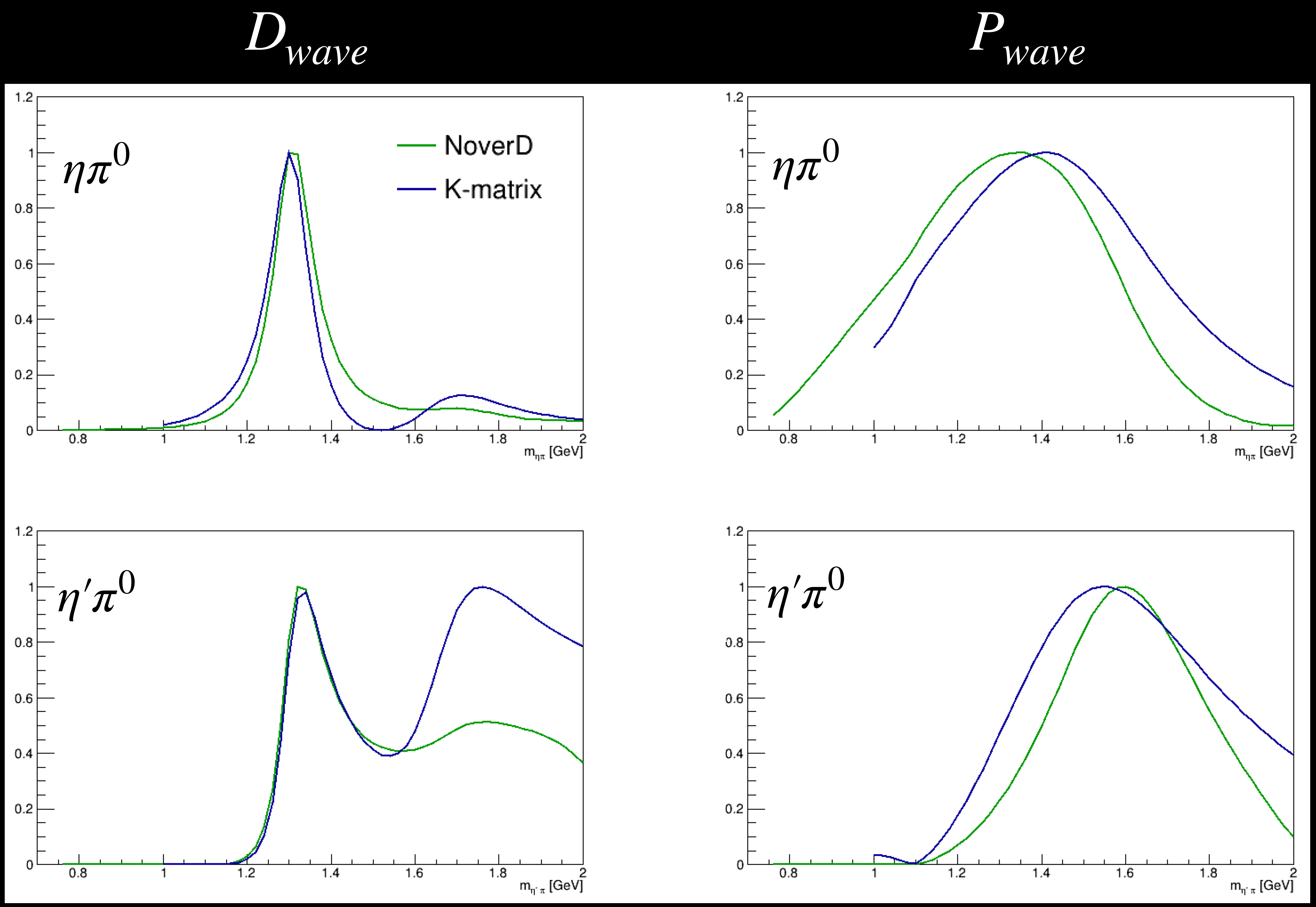
$$A_i^J(s) = \beta^P \cdot q^{J-1}(s) \cdot p_i^J(s) \cdot \sum_{k=1}\left(\sum_{n=0}^{3} a_n^{(J,k)}\, T_n\left(\frac{s}{s+1}\right)\right) \cdot \left[D^J(s)^{-1}\right]_{ki}$$

⋯ in each reflectivity ($+/-$)

D_{wave} S_{wave} P_{wave}

GlueX Phase1 Data

- **Both N/D and K-matrix approaches offer fairly consistent descriptions!**
  - deviations observed predominantly in $\eta'\pi^0$ channel
  - the P-wave exhibits notable sensitivity to model assumptions



- **Calculating the line shapes directly from code highlights the differences in amplitude tails and peak structures**

  - improved line shape agreement near resonance peaks without fits

  - K-matrix and N/D parameterizations handle analytic constraints differently

K-matrix $\rightarrow$ more sensitive to background and threshold effects

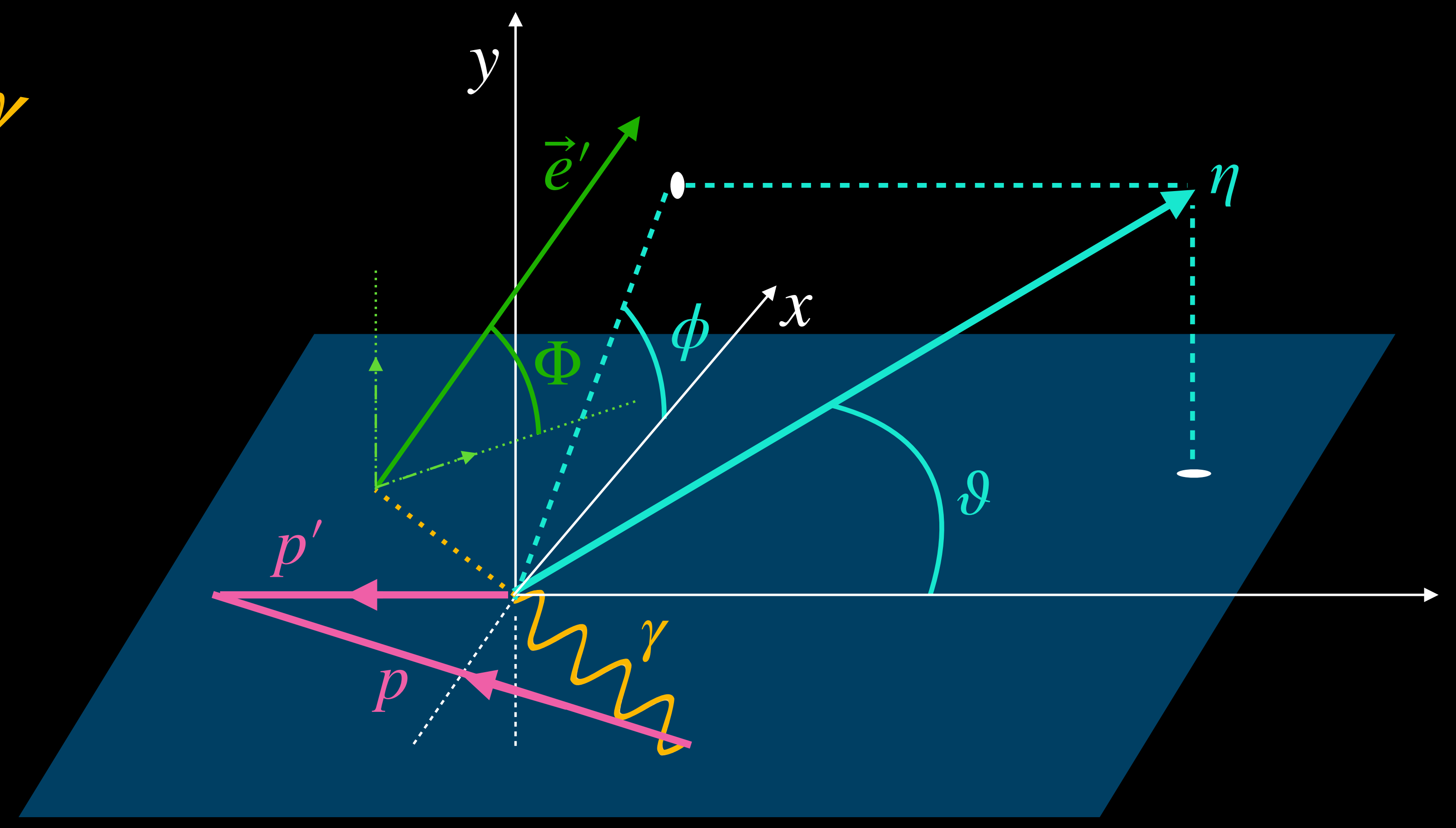
N/D $\rightarrow$ better preserves resonance dominated line shapes

$$\mathcal{I}(\Omega, \Phi) = \sum_{\alpha=0}^{3} \mathcal{I}_\alpha(\Omega) P_\gamma^\alpha(\Phi)$$

Linearly polarized γ

$$P_\gamma^\alpha(\Phi) = (1, \; -P_\gamma \cos(2\Phi), \; -P_\gamma \sin(2\Phi), \; 0)$$

$$\mathcal{I}(\Omega, \Phi) = \mathcal{I}_0(\Omega) - \mathcal{I}_1(\Omega) P_\gamma \cos(2\Phi) - \mathcal{I}_2(\Omega) P_\gamma \sin(2\Phi)$$



V. Mathieu et al. [JPAC], PRD 100, 054017 (2019)

- K-matrix parameters for $\cdots$

pole name α	m_α^{bare}	$g_{\alpha\,\pi\eta}^{\text{bare}}$	$g_{\alpha\,K\bar{K}}^{\text{bare}}$
$a_0(980)$	0.95395	0.43215	−0.28825
$a_0(1450)$	1.26767	0.19000	0.43372

$\} \; a_0 \to \eta\pi^0,\, K\bar{K}$

pole name α	m_α^{bare}	$g_{\alpha\,\pi\eta}^{\text{bare}}$	$g_{\alpha\,K\bar{K}}^{\text{bare}}$	$g_{\alpha\,\pi\eta'}^{\text{bare}}$
$a_2(1320)$	1.30080	0.30073	0.21426	−0.09162
$a_2(1700)$	1.75351	0.68567	0.12543	0.00184
background terms				
$\tilde{c}_{00}$	−0.40184			
$\tilde{c}_{01}$	0.00033	$\tilde{c}_{11}$	−0.21416	
$\tilde{c}_{02}$	−0.08707	$\tilde{c}_{12}$		$\tilde{c}_{22}$
				−0.17435

$\} \; a_2 \to \eta\pi^0,\, K\bar{K},\, \eta'\pi^0$

pole name α	m_α^{bare}	$g_{\alpha\,\pi\eta}^{\text{bare}}$	$g_{\alpha\,\pi\eta'}^{\text{bare}}$
$\pi_1(1600)$	1.38552	0.80564	1.04695
background terms			
$\tilde{c}_{00}$	1.0500		
$\tilde{c}_{01}$	0.15163	$\tilde{c}_{11}$	−0.24611

$\} \; \pi_1 \to \eta\pi^0,\, \eta'\pi^0$

- N/D **parameters for** $\cdots$

Coefficient	$\eta\pi$	$\eta'\pi$
a_0^P	408.75	−47.05
	356 ± 334	−43 ± 39
a_1^P	−632.57	65.84
	−547 ± 534	59 ± 63
a_2^P	281.48	−20.96
	240 ± 255	−17 ± 30
a_3^P	−57.98	1.20
	−47 ± 63	0 ± 8
a_0^D	−247.80	230.92
	−247 ± 28	233 ± 79
a_1^D	413.91	−290.66
	415 ± 39	−290 ± 125
a_2^D	−190.94	176.88
	−192 ± 39	177 ± 83
a_3^D	59.25	−3.82
	61 ± 29	−1 ± 62

K-matrix background	
$c^P_{\eta\pi,\eta\pi}$	−15.43
	−14.77 ± 7.22
$c^P_{\eta\pi,\eta'\pi}$	−67.22
	−65.28 ± 13.91
$c^P_{\eta'\pi,\eta'\pi}$	−190.73
	−184.19 ± 38.21
$d^P_{\eta\pi,\eta\pi}$	1.82
	1.93 ± 2.24
$d^P_{\eta\pi,\eta'\pi}$	7.64
	7.59 ± 5.09
$d^P_{\eta'\pi,\eta'\pi}$	63.85
	60.54 ± 18.59
$c^D_{\eta\pi,\eta\pi}$	−2402.56
	−2385.05 ± 273.87
$c^D_{\eta\pi,\eta'\pi}$	462.60
	469.55 ± 55.87
$c^D_{\eta'\pi,\eta'\pi}$	−86.60
	−92.25 ± 28.11
$d^D_{\eta\pi,\eta\pi}$	−614.58
	−608.35 ± 49.32
$d^D_{\eta\pi,\eta'\pi}$	164.72
	166.85 ± 17.46
$d^D_{\eta'\pi,\eta'\pi}$	−42.19
	−44.45 ± 11.59

Resonating terms	
$g^P_{\eta\pi}$	−0.68
	−0.55 ± 0.38
$g^P_{\eta'\pi}$	−13.12
	−13.12 ± 0.95
$m^2_{P,1}$	3.52
	3.52 ± 0.08
$g^D_{\eta\pi,1}$	5.63
	5.64 ± 0.34
$g^D_{\eta'\pi,1}$	−3.77
	−3.78 ± 0.10
$m^2_{D,1}$	1.86
	1.86 ± 0.02
$g^D_{\eta\pi,2}$	147.79
	147.17 ± 9.88
$g^D_{\eta'\pi,2}$	−33.39
	−34.07 ± 3.41
$m^2_{D,2}$	8.06
	8.06 ± 0.30

- Enforced unitarity of S when T-matrix is related to the real K-matrix

$$T = (I + iKC)^{-1}K$$

generalized phase space factor

$\rho = -\,Im\;C$

$$T(s) = \begin{pmatrix} T_{\eta\pi\to\eta\pi} & T_{\eta\pi\to\eta'\pi} \\ T_{\eta'\pi\to\eta\pi} & T_{\eta'\pi\to\eta'\pi} \end{pmatrix}$$

- off-diagonal entries like $T_{\eta\pi\to\eta'\pi}$ encode channel mixing
- shared resonance poles manifest in both channels due to unitarity and analytic structure