Mapping the transverse spin sum rule in position space

Based on: C. Lorce, A. Mukherjee, R. Singh and H.Y. Won [arXiv:2505.20468]

Ravi Singh

Department of Physics, Indian Institute of Technology Bombay, India

HUGS | 40 years!

Introduction

Proton is a spin 1/2 particle.

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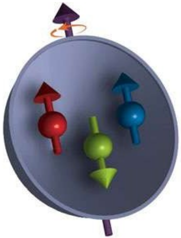

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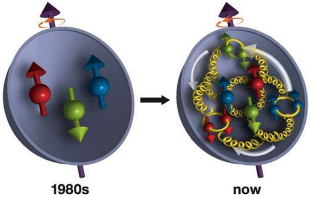

Non-relativistic $\longrightarrow$ relativistic

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Even less known: **spatial distribution of transverse component of angular momentum**.

Challenge with the transverse AM

$$\vec{J} = \vec{L} + \vec{S} = \vec{R} \times \vec{P} + \vec{S}$$

$[J^\perp, K^z] \neq 0$ $\Longrightarrow$ **frame-dependent effects**

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Let $\frac{1}{2}\vec{s}$ be a rest frame spin-vector

Longitudinal spin sum rule

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Transverse spin sum rule

$$\langle J_a^T \rangle = \frac{1}{2} s_T$$ B.L.G Bakker, et al., PRD 70, 114001 (2004)

$$\langle J_a^T \rangle = \gamma \frac{1}{2} s_T$$ X.Ji, F.Yuan, PLB 810, 135786 (2020)

$$\langle J_a^T \rangle = \gamma^{-1} \frac{1}{2} s_T$$ C.Lorce, EPJC 81 (5) (2021)413.

where $\gamma = p^0/M$

Why this contradiction - Answer lies in the Pivot

Center of

+ energy
× spin
• mass

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$\times$ spin

$\bullet$ mass

$$R^{\mu}_{E} = \frac{1}{P^{0}} \int d^{3}r \; r^{\mu} T^{00}(r)$$

$$R^{\mu}_{M} = \text{Center of energy in the rest frame of system}$$

$$R^{\mu}_{c} = \frac{P^{0} R^{\mu}_{E} + M R^{\mu}_{M}}{P^{0} + M}$$

Why this contradiction - Answer lies in the Pivot

Center of

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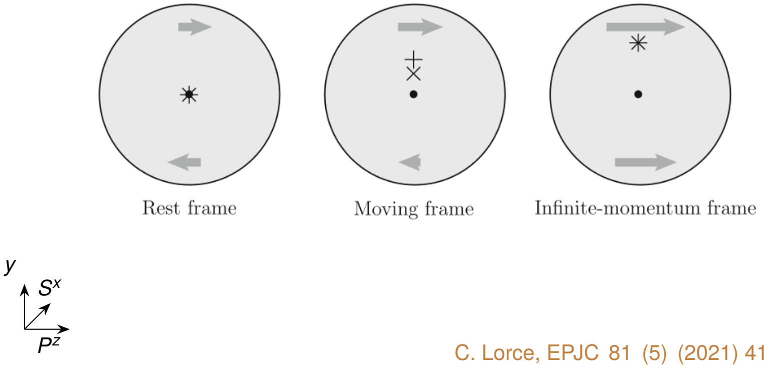

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Densities in position space (Spatial distribution)

$$\langle L^i \rangle(x) = \epsilon^{ijk} x^j \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} e^{i\Delta . x} \langle T^{0k} \rangle$$ **(Phase space formalism)**

where $\langle T^{\mu\nu} \rangle = \dfrac{\langle p', \boldsymbol{s} \,|T^{\mu\nu}(0)|\, p, \boldsymbol{s} \rangle}{2P^0}$, $\Delta = p' - p$ and $P = \dfrac{p' + p}{2}$.

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$$i \frac{\partial}{\partial \boldsymbol{\Delta}} e^{i\Delta.x} = i \frac{\partial}{\partial \boldsymbol{\Delta}} \left(e^{i \frac{\boldsymbol{P}.\boldsymbol{\Delta}}{P^0} x^0} e^{-i\boldsymbol{\Delta}.\boldsymbol{x}} \right) = \left(\boldsymbol{x} - \frac{\boldsymbol{P}}{P^0} x^0 \right) e^{i\Delta.x}$$ **(On-shell condition $P.\Delta = 0$)**

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$$\langle L^i \rangle(\boldsymbol{x}) = \epsilon^{ijk} \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} \left[i \frac{\partial}{\partial \boldsymbol{\Delta}} e^{i\Delta.x} \right] \langle T^{0k} \rangle + \epsilon^{ijk} \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} x^0 \frac{\boldsymbol{P}}{P^0} e^{i\Delta.x} \langle T^{0k} \rangle$$

$$= \epsilon^{ijk} \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} e^{i\Delta.x} \left[-i \frac{\partial \langle T^{0k} \rangle}{\partial \boldsymbol{\Delta}} + \underbrace{\frac{x^0}{2} \frac{\boldsymbol{P}}{P^0} \langle T^{0k} \rangle}_{\text{Time dependent}} \right] \longrightarrow \text{Lack probabilistic interpretation}$$

Densities in position space (Spatial distribution)

$$\langle L^i\rangle(x) = \epsilon^{ijk}x^j\int\frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}e^{i\Delta.x}\langle T^{0k}\rangle$$ **(Phase space formalism)**

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$$= \epsilon^{ijk}\int\frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}e^{i\Delta.x}\left[-i\frac{\partial\langle T^{0k}\rangle}{\partial\boldsymbol{\Delta}} + \underbrace{\frac{x^0}{2}\frac{\boldsymbol{P}}{P^0}\langle T^{0k}\rangle}_{\text{Time dependent}}\right] \longrightarrow \text{Lack probabilistic interpretation}$$

In Breit frame i.e. $\boldsymbol{P} = 0$,

$$\langle L^i\rangle(\boldsymbol{x}) = -i\epsilon^{ijk}\int\frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}e^{-i\boldsymbol{\Delta}.\boldsymbol{x}}\frac{\partial\left\langle T^{0k}\right\rangle}{\partial\Delta^j}$$

Elastic frame

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Momentum transfer is confined to the transverse plane

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Access to only 2D distribution but benefit of interpolation between Briet frame and infinite momentum frame

Elastic frame

$$\langle L^i \rangle(\boldsymbol{x}) = \epsilon^{ijk} \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} e^{i\Delta . x} \left[-i \frac{\partial \langle T^{0k} \rangle}{\partial \boldsymbol{\Delta}} + \frac{x^0}{2} \frac{\boldsymbol{P}}{P^0} \langle T^{0k} \rangle \right]$$

Integrating over z-coordinate

Spatial distribution of OAM in elastic frame

$$\langle L^{\perp} \rangle(\boldsymbol{b}_{\perp}, P^z) = \epsilon^{\perp jk} \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_{\perp} . \boldsymbol{b}_{\perp}} \left[-i \frac{\partial \langle T^{0k} \rangle}{\partial \Delta^j} \right]_{\Delta^z = 0}$$

where $\perp = 1, 2$

C. Lorce, *et al.*, PLB 776 (2018).

Spin-0 energy momentum tensor

$$\mathcal{M}[\hat{T}_a^{\mu\nu}] = 2P^\mu P^\nu A_a + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{2} D_a + 2M^2 g^{\mu\nu} \bar{C}_a,$$

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Spin-1/2 Energy momentum tensor

$$\mathcal{M}_{s's}[\hat{T}_a^{\mu\nu}] = \bar{u}' \left[\frac{P^\mu P^\nu}{M} A_a + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4M} D_a + M g^{\mu\nu} \bar{C}_a \right.$$
$$\left. + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} J_a - \frac{iP^{[\mu}\sigma^{\nu]\rho}\Delta_\rho}{2M} S_a \right] u,$$

where $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$, $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$ and $\mathcal{M}_{s's}[\hat{O}] = \langle p', s' | \hat{O}(0) | p, s \rangle$

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Generalized intrinsic spin tensor

$$\mathcal{M}_{s's}[\hat{S}_q^{\mu\alpha\beta}] = \frac{1}{2} \epsilon^{\mu\alpha\beta\lambda} \bar{u}' \left[\gamma_\lambda \gamma_5 G_A^q + \frac{\Delta_\lambda \gamma_5}{2M} G_P^q \right] u.$$

AM spatial distributions for spin-0 target

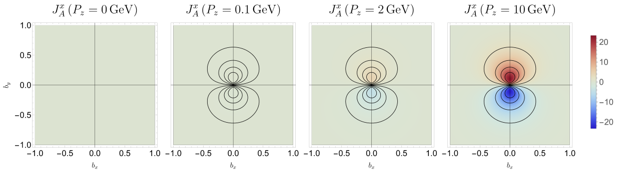
Spin-0

$$J^i_{\perp,a}\left(b_\perp, P^z\right) = \int dr^z \, L^i_{\perp,a}\left(r, P^z\right)$$

$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp\cdot b_\perp} i\epsilon^{ij}_\perp \sqrt{\tau}\, \underline{X^j_1} \left[4MP^z \frac{d}{dt} A_a + \frac{MP^z}{2\left(P^0\right)^2} D_a\right]_{t=-\Delta^2_\perp}$$

where $X^i_1 = \Delta^i_\perp / \left|\Delta_\perp\right|$ denotes that the contribution is **dipole** type

AM spatial distributions for spin-0 target



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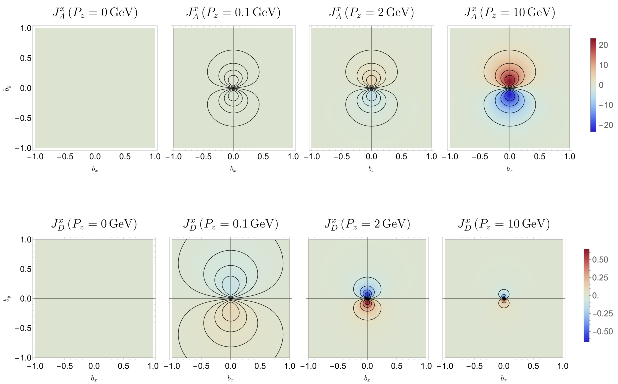

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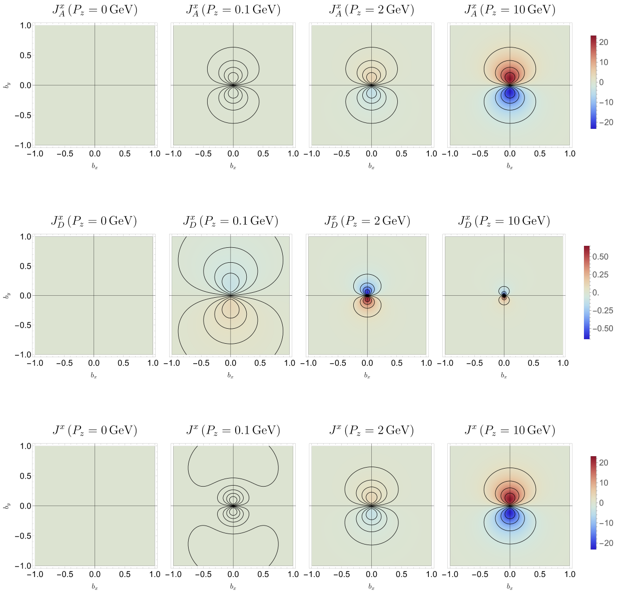


Spin-1/2

$$L^i_{\perp,a}(b_\perp, P^z; s', s) = \int dr^z\, L^i_{\perp,a}(r, P^z; s', s)$$

$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp\cdot b_\perp} \Big[\delta_{s's}\, i\epsilon^{ij}_\perp \sqrt{\tau}\, \underline{X^j_1} \tilde{L}^U_{1,a}(t)$$

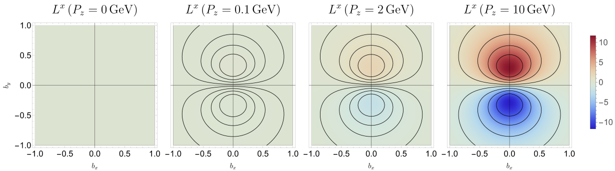
$$+ (\sigma_\perp)^i_{s's}\, \underline{X_0} \tilde{L}^T_{0,a}(t) + (\sigma_\perp)^j_{s's}\, \tau\, \underline{X^{ij}_2} \tilde{L}^T_{2,a}(t)\Big]_{t=-\Delta^2_\perp}$$

where $X_0 = 1$ denotes **monopole**,

$X^i_1 = \Delta^i_\perp / |\Delta_\perp|$ denote **dipole**,

$X^{ij}_2 = \Delta^i_\perp \Delta^j_\perp / |\Delta_\perp|^2 - \delta^{ij}_\perp / 2$ denotes **quadrupole**

Spin-1/2 unpolarized target



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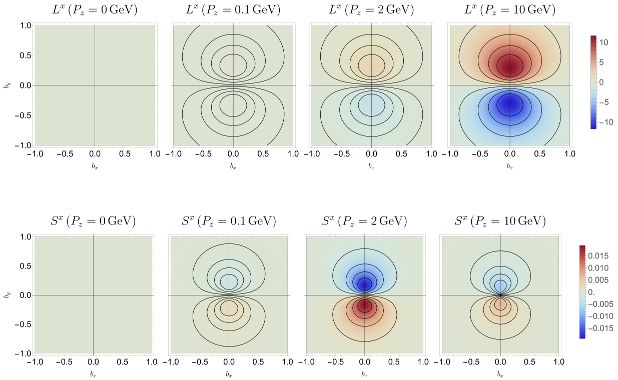

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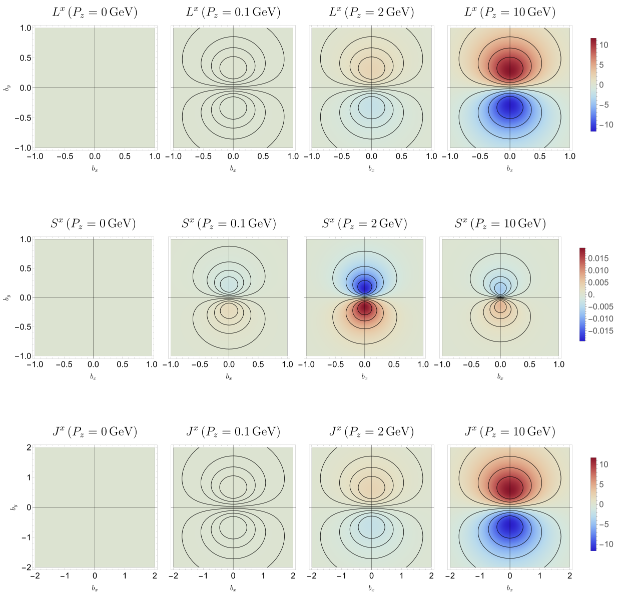
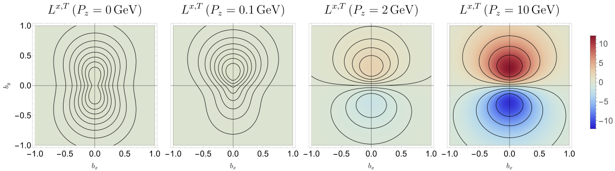


Spin-1/2 transversely polarized target



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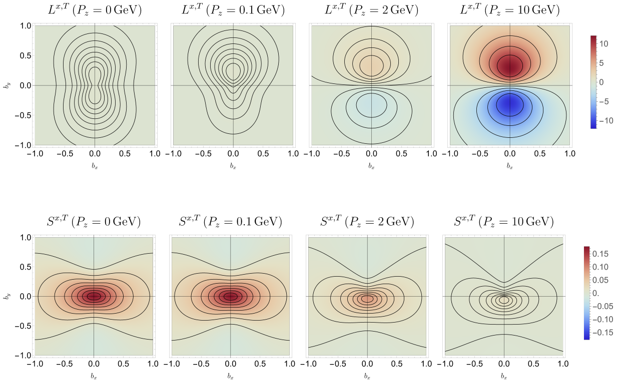

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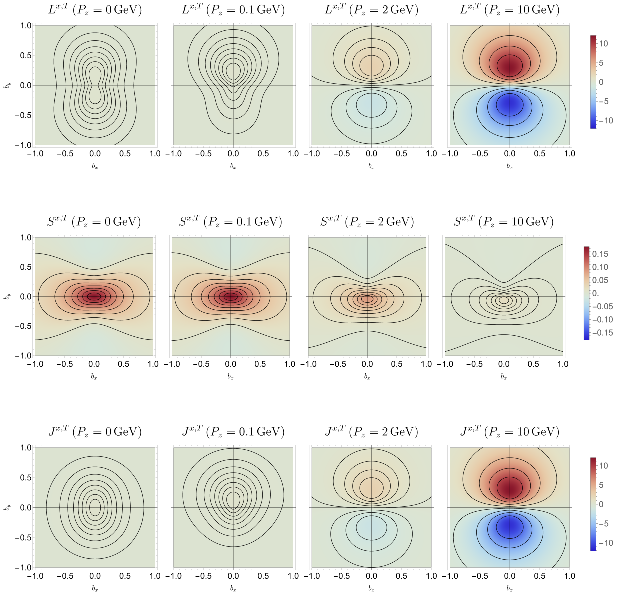


Conclusion

- Spatial distributions of angular momentum provide richer picture of hadron spin.
- Phase space formalism can be used to study them in elastic (or any) frame.
- Even spin-0 targets have a non-trivial transverse angular momentum distribution.
- Spin-1/2 targets have very complicated transverse angular momentum distribution consisting of monopole, dipole and quadrupole contributions.
- Transverse spin sum rule is verified.

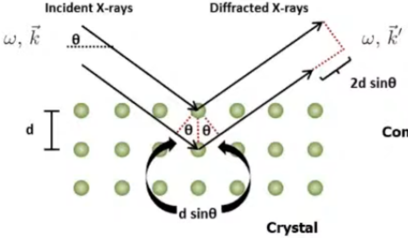
Future Goals:

- Study in detail the effect of pivot on the distributions.
- Study the distributions in light front frame and infinite momentum frame and connect all the pictures.
- Create a complete and coherent map of transverse AM distributions in position space.

Thank you

Backup Slides

Example: X-ray diffraction

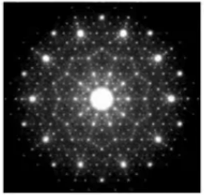


Constructive interference (Bragg's law)

$$2d \sin\theta = n\lambda$$

$$\lambda \sim d$$

Diffraction pattern



$\propto |A_{\text{scatt}}|^2$

Scattered amplitude

$$A_{\text{scatt}} \propto F(\vec{q}) = \int \mathrm{d}^3r\, e^{i\vec{q}\cdot\vec{r}}\, \rho(\vec{r}) \qquad \vec{q} = \vec{k} - \vec{k}'$$

Form factor — **Scatterer distribution**

Credit: Prof. Cedric Lorce

Quantum phase space formalism

$$\langle O \rangle = \int \frac{d^3p}{(2\pi)^3} d^3\mathcal{R}\, \rho_\psi(\vec{\mathcal{R}}, \vec{p}) \langle O \rangle_{\vec{\mathcal{R}}, \vec{p}}$$

$$\rho_\psi(\vec{\mathcal{R}}, \vec{p}) = \int d^3z\, e^{-i\vec{p}.\vec{z}} \psi^* \left(\vec{R} - \frac{\vec{z}}{2} \right) \psi(\vec{R} + \frac{\vec{z}}{2})$$

Phase space density

$$\langle O \rangle_{\vec{\mathcal{R}}, \vec{p}} = \int \frac{d^3\Delta}{(2\pi)^3} e^{i\vec{\Delta}.\vec{\mathcal{R}}} \frac{\langle p + \frac{\Delta}{2} \,|O|\, p - \frac{\Delta}{2} \rangle}{\sqrt{4(p^0)^2 - (\Delta^0)^2}}$$

System localized in phase space

$$\langle \int d^3r\, r^i\, O(r) \rangle_{\vec{\mathcal{R}}, \vec{p}} = \mathcal{R}^i \frac{\langle p | O(0) | p \rangle}{2p^0} + \frac{1}{2p^0} \left[-i \frac{\partial}{\partial \Delta^i} \left\langle p + \frac{\Delta}{2} \,|O|\, p - \frac{\Delta}{2} \right\rangle \right]_{\vec{\Delta} = \vec{0}}$$

OAM multipole amplitudes

$$\tilde{L}^U_{1,a} = \frac{d}{dt}\left[\frac{4MP^z\left[P^0+M(1+\tau)\right]}{P^0+M}A_a\right] + \frac{MP^z\left[P^0+M(1+\tau)\right]}{2\left(P^0\right)^2\left(P^0+M\right)}D_a - \frac{MP^z}{2P^0\left(P^0+M\right)}L_a + \frac{d}{dt}\left[tP^z\left(\frac{L_a}{P^0+M}+\frac{J_a+S_a}{P^0}\right)\right],$$

$$\tilde{L}^T_{0,a} = -\frac{d}{dt}\left[\frac{(P^z)^2}{2M\left(P^0+M\right)}tA_a\right] - \frac{(P^z)^2}{16M\left(P^0\right)^2\left(P^0+M\right)}tD_a + \frac{M\left[P^0+M(1+\tau)\right]}{2P^0\left(P^0+M\right)}L_a + \frac{1}{2}\frac{d}{dt}\left[tL_a\right] + \frac{d}{dt}\left[\frac{t\left(P^z\right)^2}{2M}\left(\frac{L_a}{P^0+M}+\frac{J_a+S_a}{P^0}\right)\right],$$

$$\tilde{L}^T_{2,a} = -\frac{d}{dt}\left[\frac{4M\left(P^z\right)^2}{P^0+M}A_a\right] - \frac{M\left(P^z\right)^2}{2\left(P^0\right)^2\left(P^0+M\right)}D_a + 4M^2\frac{d}{dt}L_a + \frac{d}{dt}\left[4M\left(P^z\right)^2\left(\frac{L_a}{P^0+M}+\frac{J_a+S_a}{P^0}\right)\right].$$

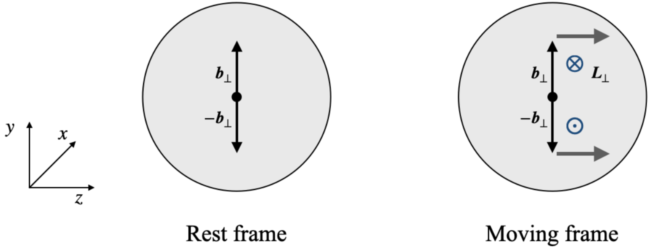
Intrinsic spin multipole amplitudes

$$\tilde{S}^{U}_{1,a} = -\frac{MP^z}{P^0\left(P^0+M\right)} S_a,$$

$$\tilde{S}^{T}_{0,a} = \frac{\left(P^0+M\right)^2-(P^z)^2}{2P^0\left(P^0+M\right)} S_a + \frac{t}{16MP^0} G^a_P,$$

$$\tilde{S}^{T}_{2,a} = -\frac{M^2}{P^0\left(P^0+M\right)} S_a - \frac{M}{2P^0} G^a_P$$

Interpretation of spin-0 AM distribution



Transverse spin sum rule

The transverse OAM and intrinsic spin distributions are normalized as

$$L^i_\perp(P^z; s', s) := \int d^2b_\perp\ L^i_\perp\ (b_\perp, P^z; s', s)$$

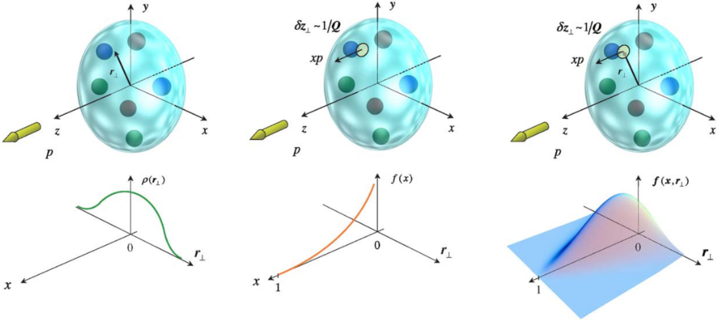
$$= (\sigma_\perp)^i_{s's} \left[-\frac{(P^z)^2}{2M\,(E_P + M)} A(0) + \frac{E_P}{M} J(0) - \frac{M}{E_P} S(0) \right], \quad (1)$$

$$S^i_\perp(P^z; s', s) := \int d^2b_\perp\ S^i_\perp\ (b_\perp, P^z; s', s)$$

$$= (\sigma_\perp)^i_{s's}\, \frac{M}{E_P} S(0), \quad (2)$$

with $E_P = P^0|_{t=0} = \sqrt{(P^z)^2 + M^2}$. Combining Eqs. (1) and (2), we obtain the transverse spin sum rule for a spin-1/2 target

$$J^i_\perp(P^z; s', s) = L^i_\perp(P^z; s', s) + S^i_\perp(P^z; s', s) = \frac{(\sigma_\perp)^i_{s's}}{2}, \quad (3)$$



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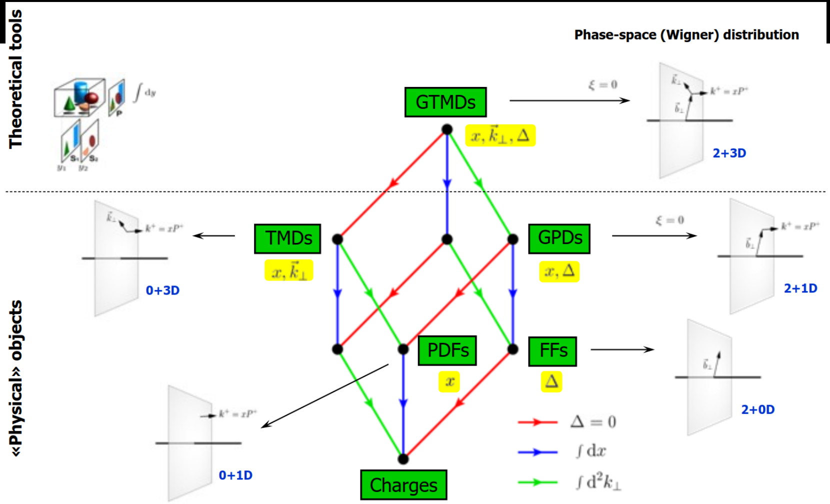

Figure: Map of all the parton distributions

Cedric Lorce, Barbara Pasquini, Vanderhaegen (2011)