Nonlinear QCD at Small-x: Parton Model vs Dipole Model

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Abstract

The small-x regime of Quantum Chromodynamics (QCD) is a frontier in high-energy nuclear physics. In this highly non-linear domain, gluon densities grow rapidly and saturation effects emerge, rendering traditional perturbative frameworks, such as the Parton Model, inadequate. Instead, the Dipole Model, which is better suited for describing non-perturbative dynamics at small-x, gains relevance. One of the key research goals for the upcoming Electron-Ion Collider (EIC) is the experimental verification of these effects [1]. This research aims to leverage the well-established techniques used in the extraction of Parton Distribution Functions (PDFs) for the Dipole Model to support the experimental efforts. Preliminary results show that the dipole framework reproduces the parton model structure functions at medium-x, highlighting the overlap between the valid kinematic regions of the two descriptions.

Theoretical Background: QCD in the Saturation Regime

To understand the internal structure of a hadron (e.g., the proton), we need to fully comprehend the strong force that is governed by Quantum Chromodynamics (QCD). The interaction between quarks is mediated by color-charged gluons (g). However, hadrons are composed not only of valence quarks and gluons binding them together but also of a sea of quark-antiquark pairs, each carrying a momentum fraction of the hadron's total momentum.

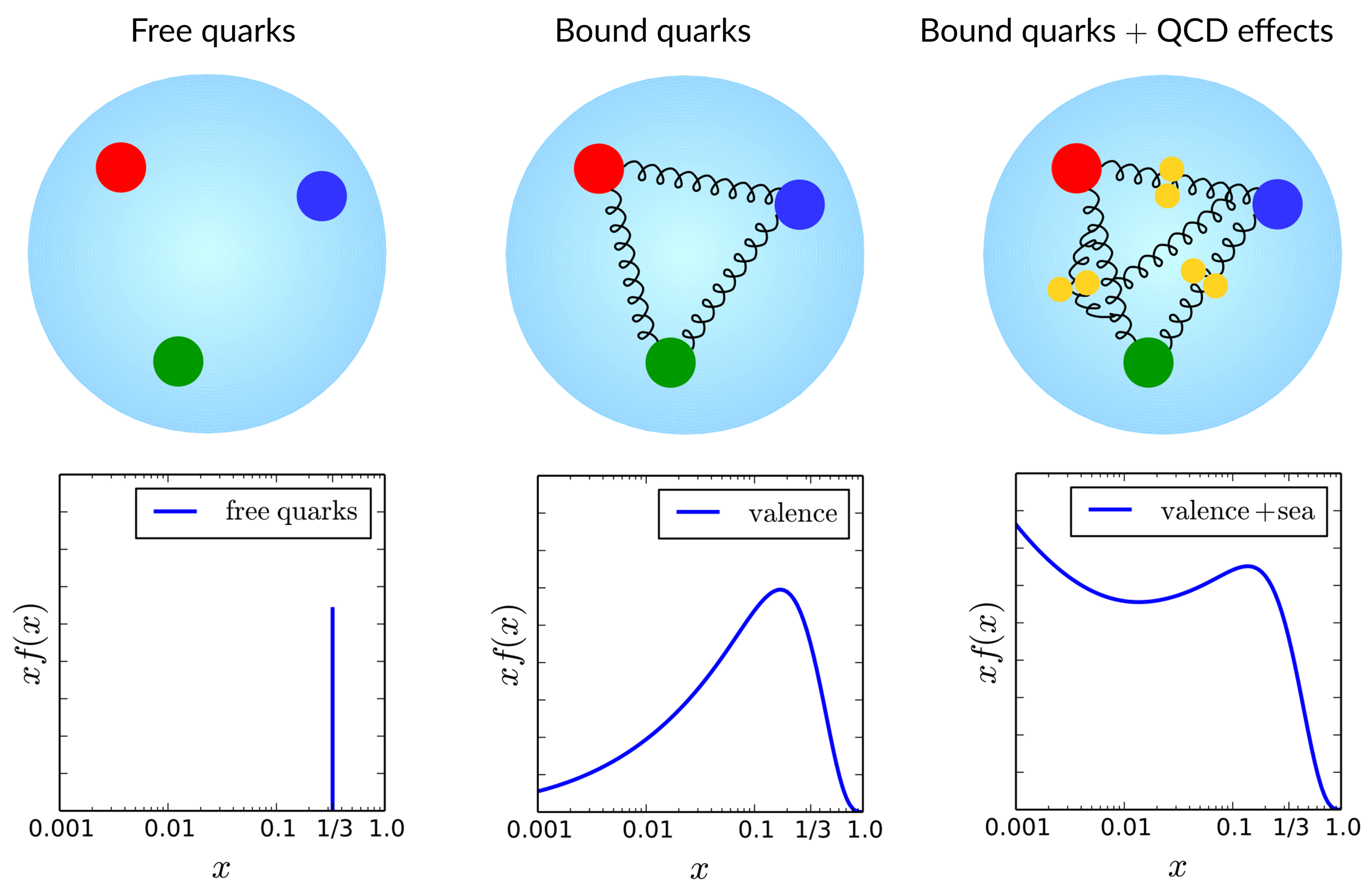


Figure 1. Illustration of the transition from the naive parton model to the inclusion of QCD dynamics.

In the parton model (illustrated above), the internal structure of hadrons is described by **Parton Distribution Functions** (PDFs) , which encode the momentum distribution of partons. However, standard PDFs do not account for gluon saturation at small-x.

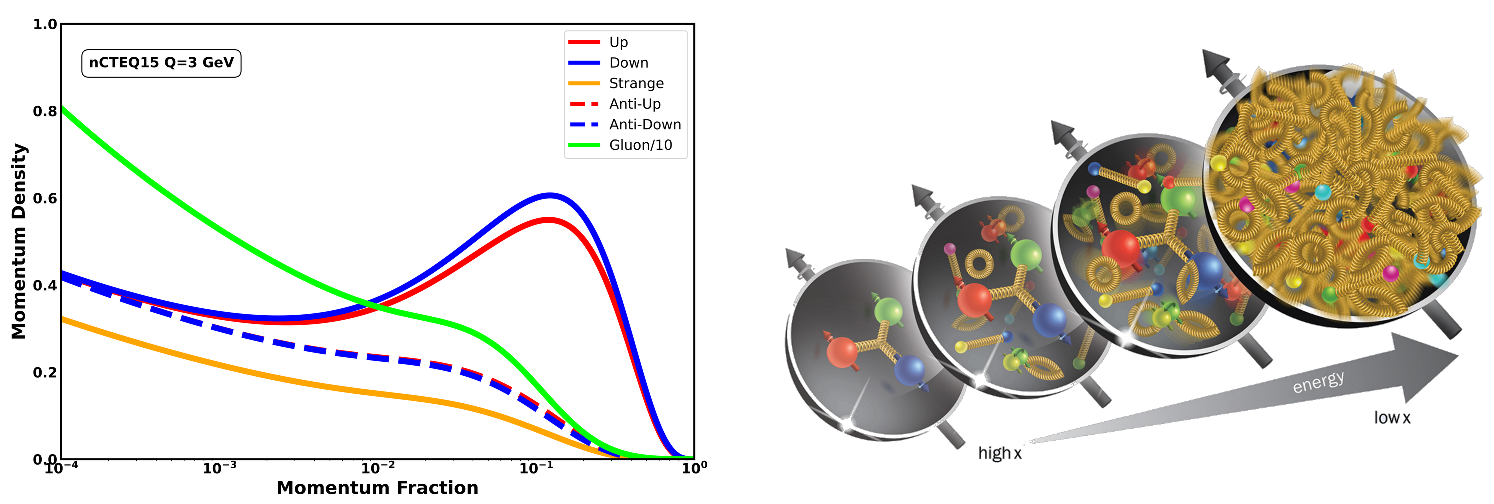


Figure 2. QCD at small-x: the structure of the hadron is dominated by gluons.

The dynamics of gluons inside hadrons behave very differently in the non-linear regime (low-x) compared to the linear regime (high-x). The gluon density increases rapidly at small-x until gluon recombination counteracts gluon splitting, thereby limiting further growth and leading to a state known as gluon saturation [2]. Understanding this phenomenon requires probing hadrons at small-x, where nonlinear QCD effects become significant.

Determination and Evolution of PDFs

PDFs can be determined from, e.g., deep inelastic scattering (DIS), and are essential for QCD predictions and calculating hadronic cross-sections in collider experiments [3]. Knowledge on PDFs is accessed through experimental data of hard-scattering experiments on nucleons at distinct values of Q^2 and x. `CTEQ`, `NNPDF`, and `MSHT` are some of the leading PDF determination groups. The PDFs are fit to the experimental data via chosen functional forms and optimizing the set of free parameters that minimize a *goodness-of-fit* χ^2 function. Once PDFs are determined at an initial Q_0 scale, the evolution through energy (Q^2) is determined using DGLAP equations.

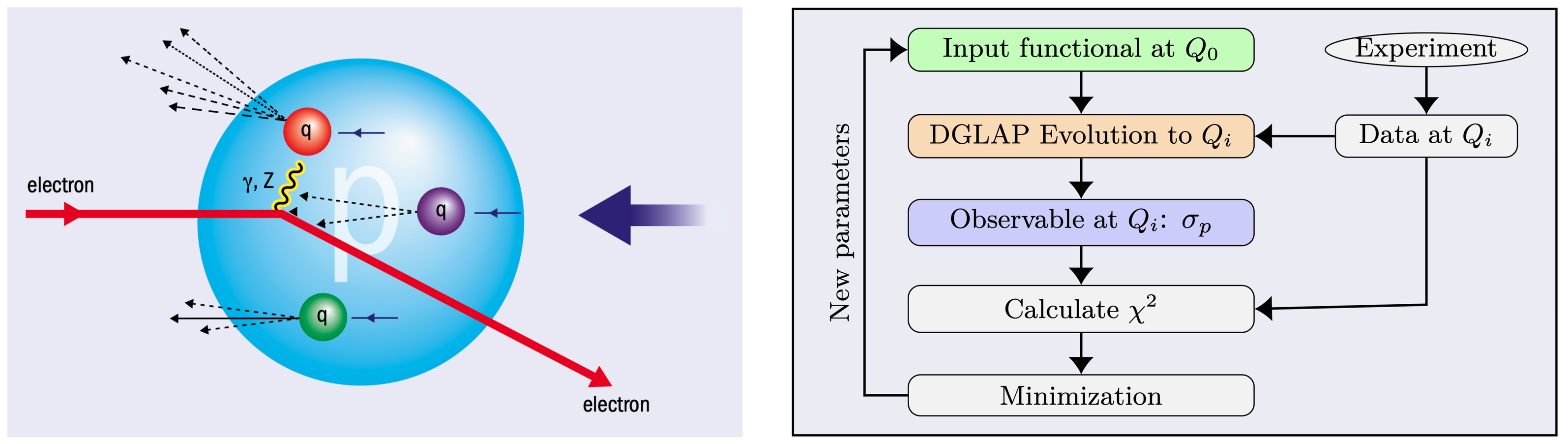


Figure 3. Schematic of DIS in the parton model (left) and the PDF fitting workflow with DGLAP evolution (right).

Parton – Dipole Correspondence

This project establishes a connection between the Parton Model framework, which includes DGLAP evolution, and the Dipole Framework, incorporating BK evolution [4].

Parton Model:

$$\sigma_p = \begin{cases} f(x,Q) \otimes \hat{\sigma}_p \\ \text{non-perturbative part} \otimes \text{perturbative part} \end{cases}$$

- $f(x,Q)$: PDF
- $\hat{\sigma}_p$: hard scattering cross-section

Dipole Model:

$$\sigma_d = \begin{cases} |\Psi(\vec{x}_\perp, z)|^2 \otimes \hat{\sigma}_d \\ \text{perturbative part} \otimes \text{non-perturbative part} \end{cases}$$

- $\Psi(\vec{x}_\perp, z)$: wavefunction for photon → quark-antiquark dipole.
- $\hat{\sigma}_d$: dipole–nuclear interaction cross-section.

It is hard to measure $\hat{\sigma}_d$ directly but can be estimated using QCD models (e.g., MV$^\gamma$). This work presents one of the first systematic comparisons of `rcBK` and `bkEval` evolution frameworks with HERA data, using the emerging `SURGE-DipoleFit` workflow.

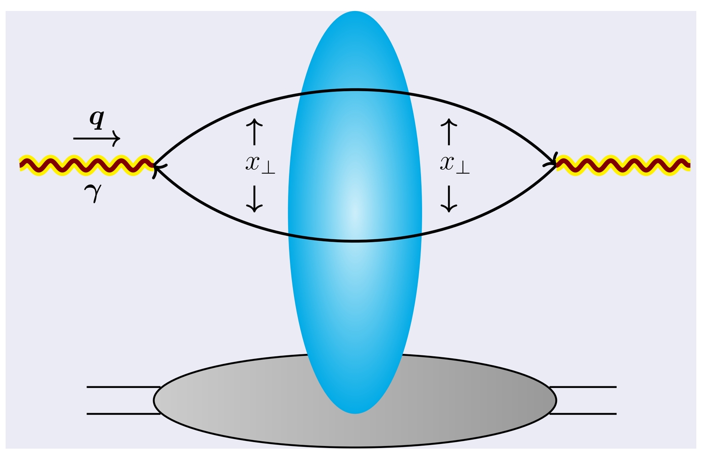


Figure 4. Schematic of DIS in the dipole model.

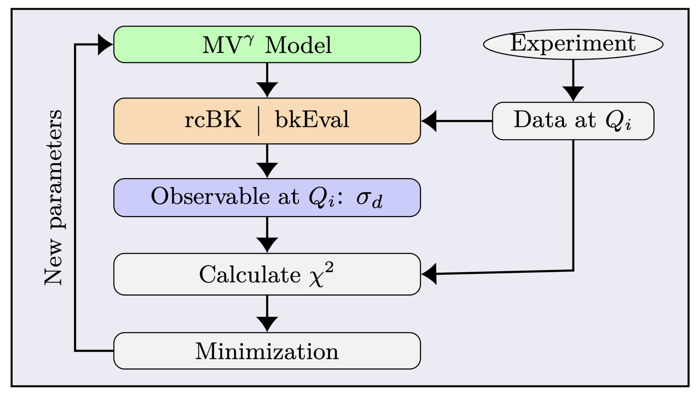


Figure 5. Dipole fitting workflow (BK evolution).

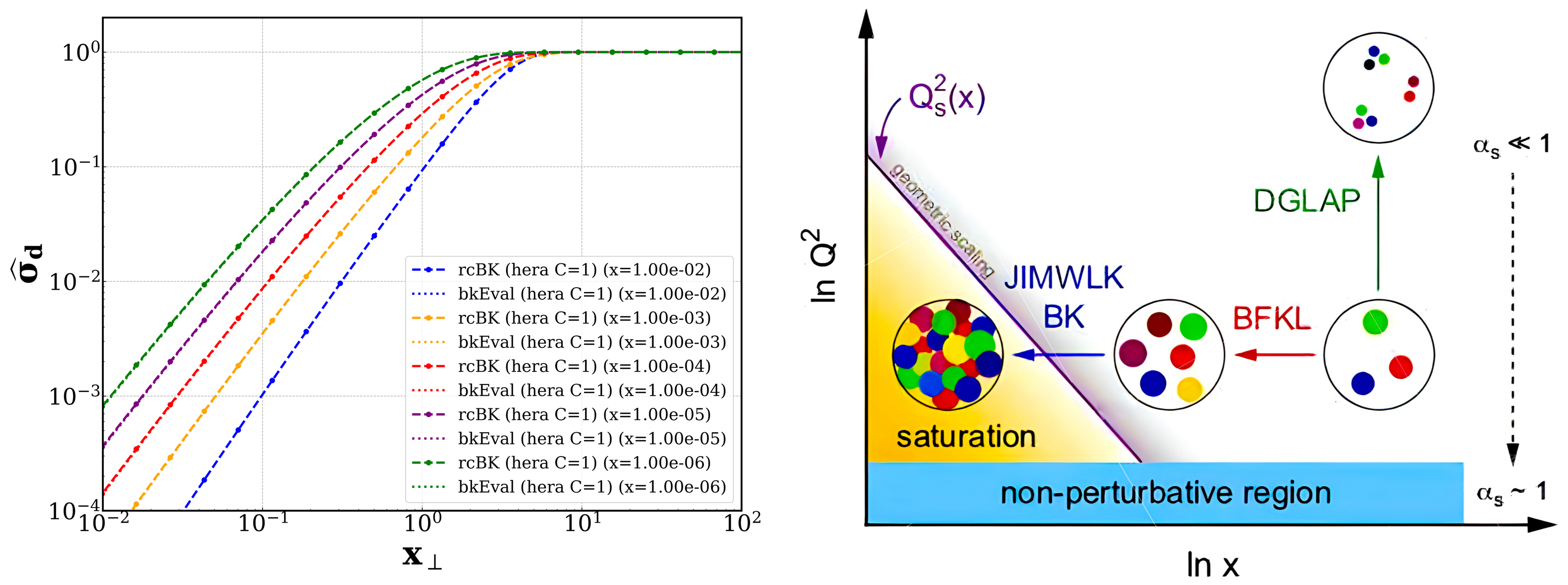
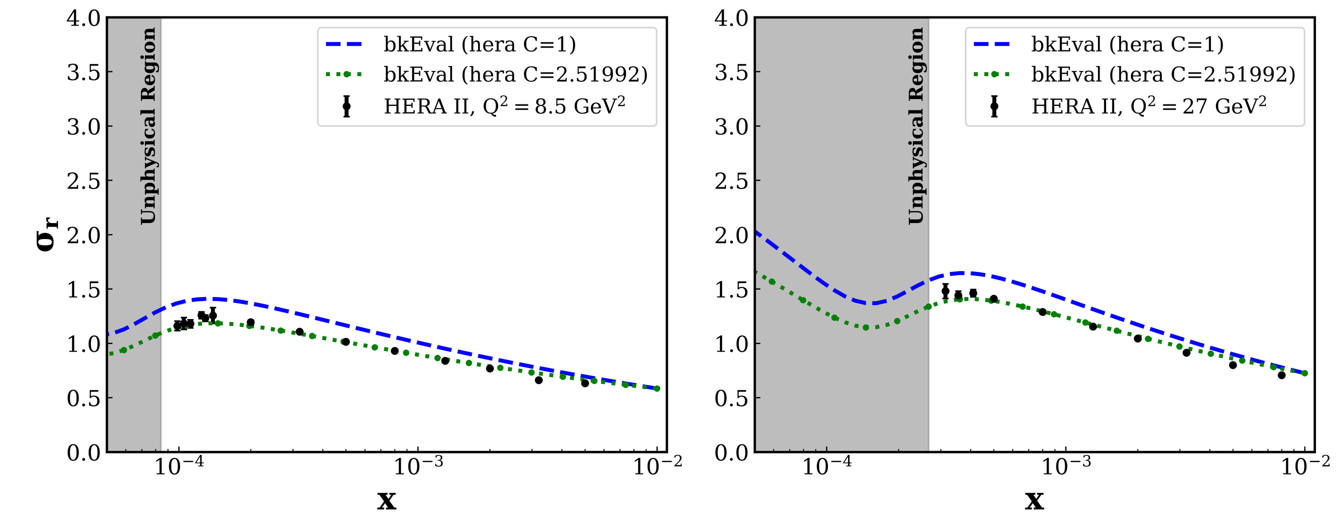


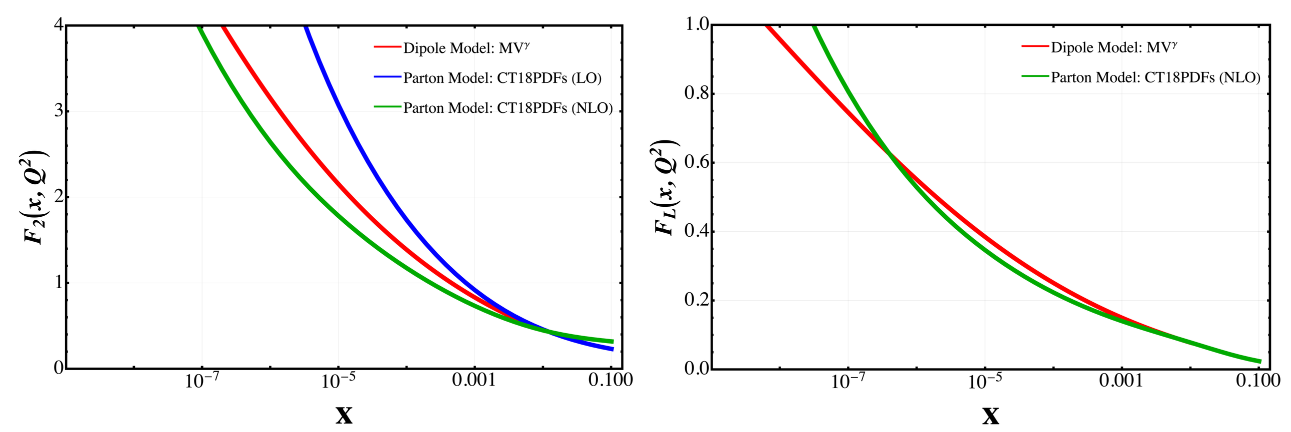
Figure 6. Comparison of rcBK and bkEval evolutions (left) and QCD evolution phase diagram (right).

Preliminary Results and Future Plans

- **Code Validation:** `rcBK` and `bkEval` exhibit excellent agreement across all x-values with matched internal settings.
- **Reduced Cross Section:** Using `bkEval`, we compute the reduced cross-section and compare it with HERA data at two sample C values [5]; quark masses are neglected.
- **Structure Functions:** Dipole model F_2 and F_L are compared with parton model predictions at leading order (LO) and next-to-leading order (NLO).



(a) Dipole model σ_r comparison with HERA data at different Q^2.



(b) Structure functions from Dipole and Parton models.

Figure 7. Preliminary results from Dipole and Parton model comparisons.

Future plans: (a) Incorporate quark mass effects to improve the accuracy of dipole cross-section predictions; (b) Employ machine learning techniques to optimize and automate parameter evolution across the (x, Q^2) grid, enhancing model adaptability; and (c) Develop an open and reproducible global χ^2 analysis workflow using `xFitter` code base to enable direct comparison with experimental structure function data.

References

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Acronyms:

EIC: Electron-Ion Collider.
BK: Balitsky-Kovchegov equation.
MV model: McLerran-Venugopalan model
SURGE: Saturated Glue Topical Collaboration.
DGLAP: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi.