Incoherent Deeply Virtual Compton Scattering on the Deuteron

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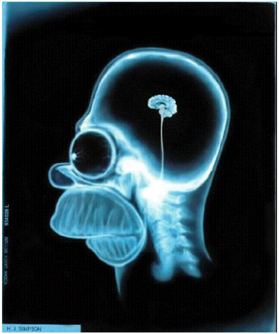
Analogy

Let's start with an analogy ...

If I want to study the structure of a person how could I do it?

- 2D picture of Bones $\rightarrow$ X-Ray
- 3D picture of Bones $\rightarrow$ CT scan
- 2D picture of Brain and stuff $\rightarrow$ MRI
- 2D picture of Baby in Uterus $\rightarrow$ UltraSound
- Other stuff, I am not a doctor ...



We have something similar with Hadrons ...

Information from Distribution Functions

Probability distributions **(Leading Order!)**

- Parton Distribution Functions (PDFs) $\rightarrow$ **1-D Momenta distribution**
- Electromagnetic Form Factors $\rightarrow$ **2-D Spatial Distribution**
- Generalized Parton Distribution Functions (GPDs) $\rightarrow$ **2-D spatial distribution + 1-D Momenta Distribution**
- Transverse Momentum Dependent Parton Distribution Functions (TMDs)$\rightarrow$ **3-D momenta distribution**
- Too lazy to write more ...

Note: GPDs and EFFs do not give this information directly you need to look at Impact Parameter Distributions of these to get this information ...

GPDs

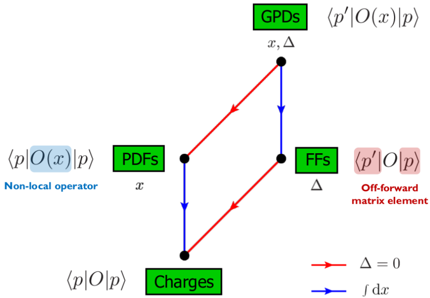


Figure: Relation between GPDs, FFs, and PDFs (took figure from Lorce slides)

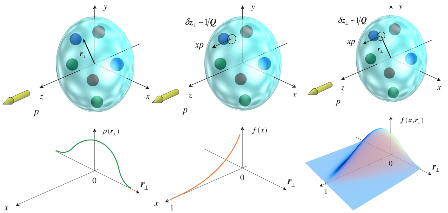


Figure: Shows relation between the Impact Parameter Distribution (IPDs) of GPDs relation to PDFs and IPDs of FFs

Generalized Parton Distribution functions (GPDs)

GPDs importance for hadron structure [1]

- Can be used to get 2D transverse spatial distribution of partons
- 1D longitudinal momentum distributions of partons
- Information of orbital angular momentum distribution
- Can **maybe** give information of pressure distributions (See Adam Freese Talk)
- Used for Mass decomposition

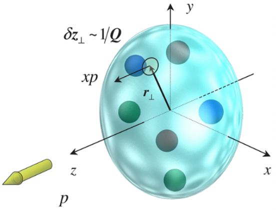
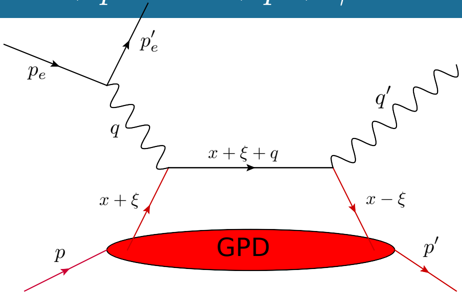


Figure: Illustration to show how IPD gotten from GPD gives 3D picture of proton [2]

Processes that probe GPDs → **DVCS**, DVMP, DDVCS, TCS,etc.

Nucleon DVCS $e^- + p \rightarrow e^- + p + \gamma$



$$A_{DVCS} = \left[-\frac{i|e|^3}{2q^2} \sum_f [\bar{u}(p'_e)\gamma^\rho u(p_e)] \epsilon^*_\mu(q') g_{\rho\nu} \right] \quad (1)$$

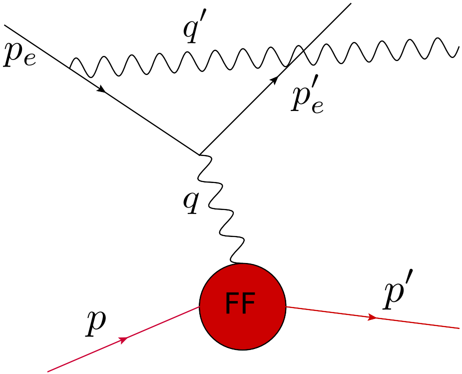
$$\times\, e_q^2 \left\{ g_\perp^{\mu\nu} \left[\mathscr{H}^q(\xi,t)\bar{u}(p'_N)\gamma^+ u(p_N) + \mathscr{E}^q(\xi,t)\bar{u}(p'_N)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p_N) \right] \right. \quad (2)$$

$$\left. +\, i\epsilon^{\mu\nu+-} \left[\tilde{\mathscr{H}}^q(\xi,t)\bar{u}(p'_N)\gamma^+\gamma_5 u(p_N) + \tilde{\mathscr{E}}^q(\xi,t)\bar{u}(p'_N)\frac{\gamma_5\Delta^+}{2m}u(p_N) \right] \right\} \quad (3)$$

$$\mathscr{H}^q(\xi,t) = \int dx \left[\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon} \right] H^q(x,\xi,t) \quad (4)$$

Bethe-Heitler Nucleon $e^- + p \to e^- + p + \gamma$

We need to look at all terms that contribute to reaction $e^- + p \to e^- + p + \gamma$



$$A_{BH} = \left[-\frac{i|e|^3}{q^2} \bar{u}(p'_e) \not{\epsilon}^*(q') \frac{\not{p}_e - \not{q}'}{(p_e - q')^2} \gamma_\mu u(p_e) \right] \times \left[F_1(t)\bar{u}(p'_N)\gamma^\mu u(p_N) + F_2(t)\bar{u}(p'_N)\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} u(p_N) \right] \tag{5}$$

Proton Structure

Complications with this process $e^- + p \to e^- + p + \gamma$

$$\sigma \propto |A|^2 = |A_{DVCS}|^2 + |A_{BH}|^2 + A^*_{BH}A_{DVCS} + A^*_{DVCS}A_{BH} \tag{6}$$

$$A_{DVCS} \propto \text{CFFs} \tag{7}$$

So $e^- + p \to e^- + p + \gamma$ does not even give you direct access to GPDs, but it gives CFFs.

For extraction of GPDs, need more data than just proton ...

Neutron Structure Importance

Use the Neutron! Zach holds key to flavor separation of Distribution functions **(Image is not to scale)**

Introduce other equations using isospin symmetry!

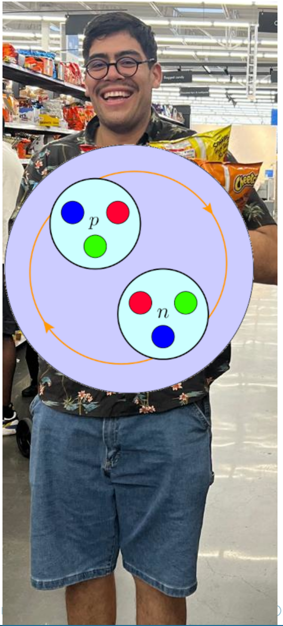
$$H_d^n(x,\xi,t) = H_u^p(x,\xi,t), \quad H_u^n(x,\xi,t) = H_d^p(x,\xi,t)$$
$$H_{\bar{d}}^n(x,\xi,t) = H_{\bar{u}}^p(x,\xi,t), \quad H_{\bar{u}}^n(x,\xi,t) = H_{\bar{d}}^p(x,\xi,t)$$

Slight problem, no free Neutron target ...

Axel holds key to Neutron Structure which is the **Deuteron!** (Image is not to scale)

- Simplest Nucleus with one proton and one neutron
- Spin-1 particle
- Nucleons in Deuteron can be described with non-relativistic wavefunction

Slight problem with getting Neutron Structure from the Deuteron ...



EMC Effect

Nucleon parton distributions are changed while bound in nucleus ...

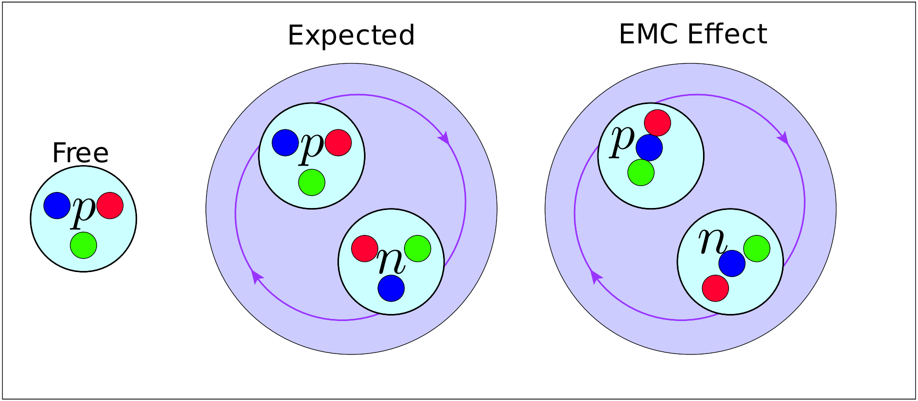
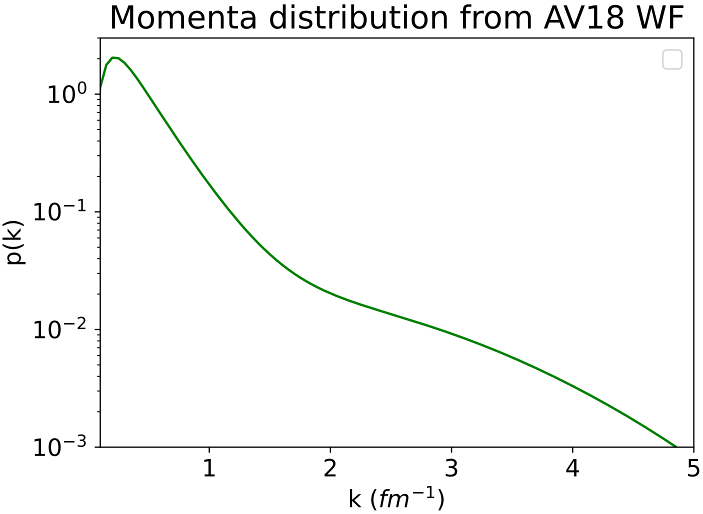


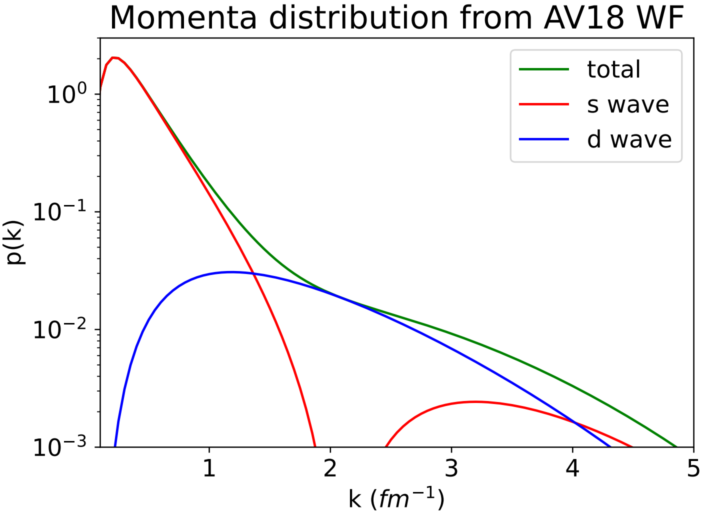
Figure: Dramatized Cartoon Illustration of EMC Effect

Need momentum distribution of nucleon so we introduce a Deuteron Wavefunction



Deuteron Wavefunction

Need momentum distribution of nucleon so we introduce a Deuteron Wavefunction



LFWF for Deuteron

Proton GPDs have/are being extensively studied. Neutron GPDs need to be looked at for quark flavor seperation of GPDs. How do we access Neutron GPDs?

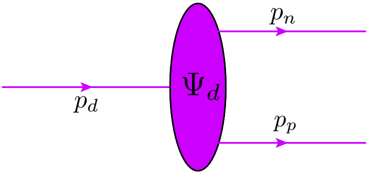


Figure: LFWF lowest Fock States for Deuteron

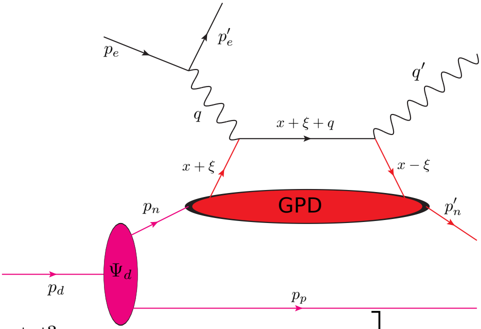
$$\Psi_d = \langle pn|d\rangle = \bar{u}_{LF}(p_n)\Gamma_\alpha v_{LF}(p_p)\epsilon^\alpha_{pn}(p_{pn}) \tag{8}$$

Where

$$\Gamma^\alpha = \gamma^\alpha G_1 + (p_p - p_n)^\alpha G_2 \tag{9}$$

Parametrization of bilinears exactly same as parametrizing form factors. To get values of G_1 and G_2 it is possible to relate these to S and D-Wave contributions in nonrelativistic wavefunctions. [3]

Incoherent DVCS PWIA $e^- + D \to e^- + p + n + \gamma$



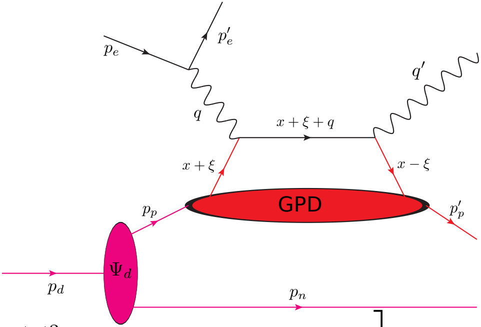
$$A_{nDVCS} = \left[-\frac{i|e|^3}{2q^2} \sum_f [\bar{u}(p'_e)\gamma^\rho u(p_e)] \epsilon^*_\mu(q') g_{\rho\nu} \right] \tag{10}$$

$$\times\, e_q^2 \left\{ g_\perp^{\mu\nu} \left[\mathscr{H}_n^q(\xi,t)\bar{u}(p'_n)\gamma^+ u(p_n) + \mathscr{E}_n^q(\xi,t)\bar{u}(p'_n)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p_n) \right] \right. \tag{11}$$

$$\left. + \, i\epsilon^{\mu\nu+-} \left[\tilde{\mathscr{H}}_n^q(\xi,t)\bar{u}(p'_n)\gamma^+\gamma_5 u(p_n) + \tilde{\mathscr{E}}_n^q(\xi,t)\bar{u}(p'_n)\frac{\gamma_5\Delta^+}{2m}u(p_n) \right] \right\}$$

$$\times\, \bar{u}_{LF}(p_n)\Gamma_\alpha v_{LF}(p_p)\epsilon^\alpha_{pn}(p_{pn})$$

Incoherent DVCS PWIA



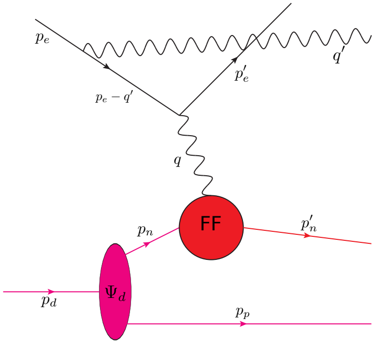
$$A_{pDVCS} = \left[-\frac{i|e|^3}{2q^2} \sum_f [\bar{u}(p'_e)\gamma^\rho u(p_e)]\epsilon^*_\mu(q')g_{\rho\nu} \right] \tag{12}$$

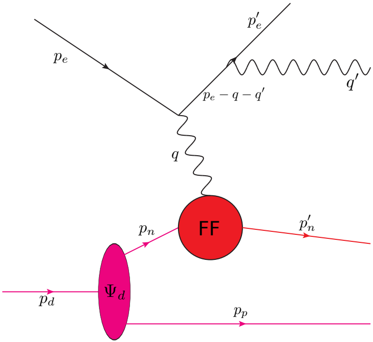
$$\times\, e_q^2 \left\{ g_\perp^{\mu\nu} \left[\mathscr{H}_p^q(\xi,t)\bar{u}(p'_p)\gamma^+ u(p_p) + \mathscr{E}_p^q(\xi,t)\bar{u}(p'_p)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p_p) \right] \right. \tag{13}$$

$$\left. + i\epsilon^{\mu\nu+-} \left[\tilde{\mathscr{H}}_p^q(\xi,t)\bar{u}(p'_p)\gamma^+\gamma_5 u(p_p) + \tilde{\mathscr{E}}_p^q(\xi,t)\bar{u}(p'_N)\frac{\gamma_5\Delta^+}{2m}u(p_N) \right] \right\}$$

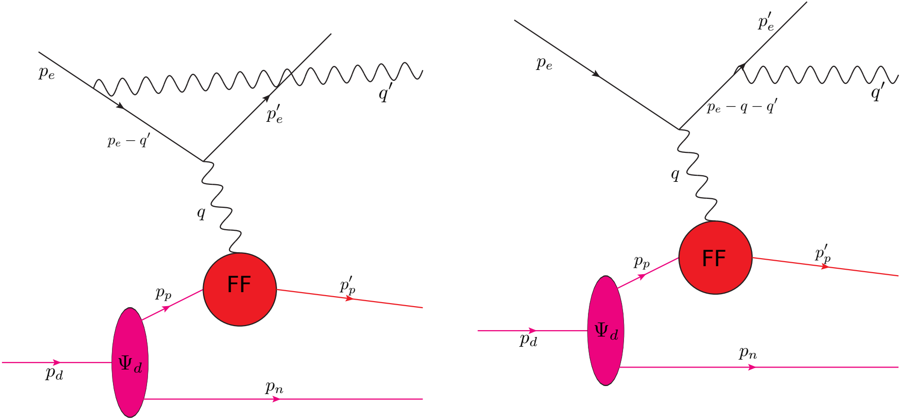
$$\times\, \bar{u}_{LF}(p_n)\Gamma_\alpha v_{LF}(p_p)\epsilon^\alpha_{pn}(p_{pn})$$

Incoherent BH $e^- + D \to e^- + p + n + \gamma$





Neutron Tagging Bethe-Heitler Amplitude PWIA



Final State Interactions

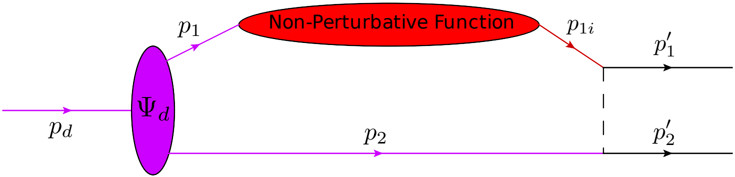


Figure: Dash Line corresponds to interaction between nucleons

$$\mathscr{M}_{\lambda'_1\lambda'_2;\lambda_1\lambda_2} = (\bar{u}_{\lambda'_1}(p'_1))_a(\bar{u}_{\lambda'_2}(p'_2))_b M_{ab;cd}(u_{\lambda_1}(p_1))_c(u_{\lambda_2}(p_2))_d \quad (14)$$

$$M_{ab;cd} = F_S(s,t)\delta_{ac}\delta_{bd} + F_V(s,t)\gamma_{ac}\cdot\gamma_{bd} + F_T(s,t)\sigma^{\mu\nu}_{ac}(\sigma_\mu\nu)_b d \quad (15)$$

$$+ F_P(s,t)\gamma^5_{ac}\gamma^5_{bd} + F_A(s,t)(\gamma^5\gamma)_{ac}\cdot(\gamma^5\gamma)_{bd} \quad (16)$$

$\mathscr{M}_{\lambda'_1\lambda'_2;\lambda_1\lambda_2}$ is a helicity amplitude that is accessible via SAID parametrizations of nucleon nucleon scattering data

Final State Interactions

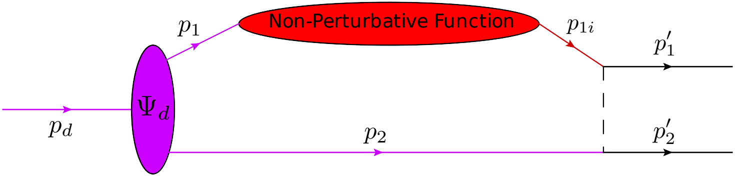


Figure: Dash Line corresponds to interaction between nucleons

$$A_{FSI,1} = \int \frac{d^3 p_2}{2E_2} N_N [\bar{u}(p_e') O_{L2} u(p_e)][\bar{u}(p_1')\bar{u}(p_2') M u(p_2) u(p_{1i})] \tag{17}$$

$$\times [\bar{u}(p_{1i}) O_1 u(p_1)][\bar{u}(p_1)\Gamma_\alpha v(p_2)]\epsilon^\alpha \tag{18}$$

Inputs for Model

- Proton GPDs
- Neutron GPDs
- Proton Form Factors
- Neutron Form Factors
- Deuteron Wavefunction
- Final State Interactions parametrizations

$$A_{N,PWIA} \propto (\text{Non-Perturbative Function})(\text{Deuteron WF}) \tag{19}$$

$$A_{N,FSIs} \propto (\text{Non-Perturbative Function})(\text{Deuteron WF}) \times (\text{Amplitude for nucleon-nucleon scattering})$$

$$\sigma \propto |A|^2 \tag{20}$$

Steps for unpolarized cross section

Deuteron LFWF lowest Fock Space State (Proton and Neutron)

↓

Amplitudes of $e^- + D \to e^- + p + n + \gamma$ in PWIA with LFWF

↓

Take into account Final State Interactions

↓

Model unpolarized Cross section and compare to experiment

↓

Look into kinematics where Neutron tagging is suppressed and vice versa

Thanks

References

A. Belitsky, A. Radyushkin, Physics Reports **418**(1–6), 1–387 (2005).
DOI 10.1016/j.physrep.2005.06.002.
URL http://dx.doi.org/10.1016/j.physrep.2005.06.002

D. Müller, Nuclear Physics A **755**, 71 (2005).
DOI https://doi.org/10.1016/j.nuclphysa.2005.03.151.
URL https://www.sciencedirect.com/science/article/pii/S0375947405003933.
Proceedings of the 10th International Conference on the Structure of Baryons

W. Cosyn, C. Weiss, Physical Review C **102**(6) (2020).
DOI 10.1103/physrevc.102.065204.
URL http://dx.doi.org/10.1103/PhysRevC.102.065204

J.W. Qiu, Z. Yu, Physical Review D **107**(1) (2023).
DOI 10.1103/physrevd.107.014007.
URL http://dx.doi.org/10.1103/PhysRevD.107.014007

S. Wallon.
Hard exclusive processes (2013).
URL https://arxiv.org/abs/1302.2888

Back Up Slides

Hadron Structure

- Parton Distribution Functions (PDFs) **Do not work on these, Yorgo does this stuff**
- Electromagnetic Form Factors (EFFs) **Do not work on these, Andrew works on this stuff**
- **Generalized Parton Distribution Functions (GPDs)**
- Transverse Momentum Dependent Parton Distribution Functions (TMDs). **Do not work on these, Matteo does this stuff**
- Too lazy to write more ...

Reactions that probe Distribution Functions

- Parton Distribution Functions (PDFs) $\rightarrow$ **Deep Inelastic Scattering (DIS), Drell-Yan (DY), ...**
- Electromagnetic Form Factors $\rightarrow$ **Elastic Scattering and Bethe-Heitler interaction, ...**
- Generalized Parton Distribution Functions (GPDs) $\rightarrow$ **Deeply Virtual Compton Scattering (DVCS), Deeply Virtual Meson Production (DVMP),...**
- Transverse Momentum Dependent Parton Distribution Functions (TMDs)$\rightarrow$ **Semi-Inclusive DIS (SIDIS), DY**
- Too lazy to write more ...

Mellin Moments of GPDs

Moments of GPDs [1]

$$\int dx\, x^n \int \frac{dz^-}{2\pi} e^{ixP^+z^-}\,\bar{q}(0)\gamma^+ q(z^-) = \frac{1}{(P^+)^{n+1}}\bar{q}(0)\gamma^+(\frac{i}{2}D^+)^n q(0) \tag{21}$$

With $n = 0$ which is first moment we have

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-}\,\bar{q}(0)\gamma^+ q(z^-) = \frac{1}{P^+}\bar{q}(0)\gamma^+ q(0) \tag{22}$$

Remember though these are matrix elements for Electromagnetic Form Factors

$$\langle p'|\bar{q}(0)\gamma^+ q(0)|p\rangle = F_1^q(t)\bar{u}(p')\gamma^+ u(p) + F_2^q(t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p) \tag{23}$$

Therefore we have

$$\int H_q(x,\xi,t)dx = F_1^q(t) \tag{24}$$

$$\int E_q(x,\xi,t)dx = F_2^q(t) \tag{25}$$

Second moment and GFFs

$$\int dx\; x \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \bar{q}(0)\gamma^+ q(z^-) = \frac{1}{(P^+)^2} \bar{q}(0)\gamma^+ (\frac{i}{2} D^+) q(0) \qquad (26)$$

Which the operator $i\gamma^\mu D^\nu = T^{\mu\nu}$ is the Energy Momentum tensor. Parametrize this matrix element with gravitational form factors.

$$\langle p'|\bar{q}(0)T^{\mu\nu}q(0)|p\rangle = \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_a(t) + M g^{\mu\nu} \bar{C}_a(t) \qquad (27)$$

$$+ \frac{P^{\{\mu} i\sigma^{\nu\}\lambda}\Delta_\lambda}{2M} J_a(t) - \frac{P^{[\mu} i\sigma^{\nu]\lambda}\Delta_\lambda}{2M} S_a(t) \qquad (28)$$

Therefore

$$\int H_q(x,\xi,t) x dx = A_q(t) + 4\xi^2 C_q(t) \qquad (29)$$

$$\int \frac{1}{2}[H_q(x,\xi,t) + E_q(x,\xi,t)] x dx = J_q(t) \qquad (30)$$

Coordinate Systems

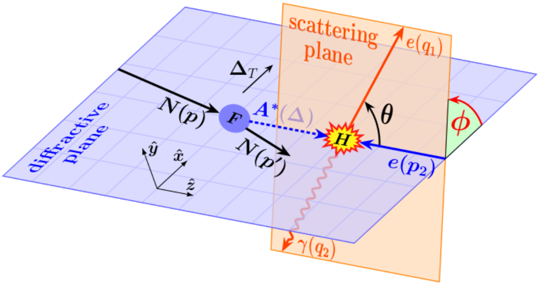


Figure: SDHEP for Nucleon

We look at SDHEP frame where separated into Soft and Hard parts of interaction. More information in Qiu and Yu [4]

- Hard part we have 3 independent 4 momenta Δ, k, k'
- Soft part we have 3 independent 4 momenta Δ, p_d, p_p

Nucleon DVCS

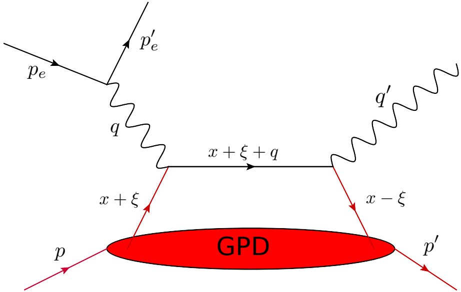


Figure: One of the DVCS diagrams for Nucleon

The Amplitude for this diagram can be written as

$$A = -i \sum_f \int \frac{d^4k}{(2\pi)^4} \, H_{ab} S_{ab} \tag{31}$$

H is the perturbative part of process (hard) and S is nonperturbative (soft) and ab are explicit spinor indices. Also note that $k = (x+\xi)p$

Hard and Soft Parts

Hard part of diagram gives

$$H_{ab} = -i|e|^3 e_f^2 [\epsilon_\rho^*(q')\gamma_{bc}^\rho] \left[\frac{\not{k} + \not{q} + m}{(k+q)^2 - m^2} \gamma^\mu \right]_{ca} \frac{g_{\mu\nu}}{q^2} [\bar{u}_e(p_e')\gamma^\nu u_e(p_e)] \tag{32}$$

The soft part gives

$$S_{ab} = \int d^4 z \, e^{ikz} \langle p' | \bar{q}_b(0) W(0,z) q_a(z) | p \rangle \tag{33}$$

How is the soft part of this process related to GPDs?

Matrix Elements for Soft Part

Looking at the matrix elements that define soft part of diagram

$$S_{ab} = \int d^4z\, e^{ikz} \langle p'|\bar{q}_b(0)W(0,z)q_a(z)|p\rangle \tag{34}$$

We use Fierz Identity [5]

$$\bar{q}_b q_a = \frac{1}{4}\gamma^{\lambda}_{ab}\bar{q}\gamma_\lambda q + \frac{1}{4}(\gamma_5\gamma^\lambda)_{ab}\bar{q}\gamma_\lambda\gamma_5 q + \frac{1}{4}(I)_{ab}\bar{q}q + \frac{1}{4}(\gamma_5)_{ab}\bar{q}\gamma_5 q + \frac{1}{4}\sigma^{\alpha\beta}_{ab}\bar{q}\sigma_{\alpha\beta}q \tag{35}$$

For when we do trace later and knowing odd number of gamma matrix trace=0 we have the following left over elements

$$S_{ab} = \frac{1}{4}\int d^4z\, e^{iz\cdot k}[\gamma^{\lambda}_{ab}\langle p'|\bar{q}(0)\gamma_\lambda W(0,z)q(z)|p\rangle \tag{36}$$

$$+ (\gamma_5\gamma^\lambda)_{ab}\langle p'|\bar{q}(0)\gamma_\lambda\gamma_5 W(0,z)q(z)|p\rangle \tag{37}$$

$$+ \sigma^{\alpha\beta}_{ab}\langle p'|\bar{q}(0)\sigma_{\alpha\beta}W(0,z)q(z)|p\rangle] \tag{38}$$

Leading twist matrix elements

Considering mass of quarks=0 then Leading twist matrix elements are

$$S_{ab} = \frac{1}{4}\int d^4z\; e^{iz\cdot k}[\gamma^{-}_{ab}\langle p'|\bar{q}(0)\gamma^{+}q(z)|p\rangle \tag{39}$$

$$+ (\gamma_5\gamma^{-})_{ab}\langle p'|\bar{q}(0)\gamma^{+}\gamma_5 W(0,z)q(z)|p\rangle \tag{40}$$

Using the leading twist matrix elements, neglecting mass, taking the collinear limit $k^{-} = k_{\perp} = 0$ and using Lightcone gauge $A^{+} = 0$ so $W(0,z) = 1$

$$A \propto \int dx H(x,\xi)\left[\frac{1}{2}\int \frac{dz^{+}}{2\pi}e^{ixP^{+}z^{-}}\langle p'|\bar{q}(0)\gamma^{+}q(z)|p\rangle\right] + ... \tag{41}$$

Where we can get [1],[5]

$$\frac{1}{2}\int \frac{dz^{+}}{2\pi}e^{ixP^{+}z^{-}}\langle p'|\bar{q}(0)\gamma^{+}q(z)|p\rangle = \tag{42}$$

$$\frac{1}{2P^{+}}\left[H^{q}(x,\xi,t)\bar{u}(p')\gamma^{+}u(p) + E^{q}(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_{\alpha}}{2m}u(p)\right] \tag{43}$$