lattice QCD and the hadron spectrum

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contents

how should we approach a
quantum field theory
so that it is possible to compute even when **non-perturbative** ?

(**not** a truncated power series in small α_S)

once again, step back to one-dim quantum mechanics …

path integrals in quantum mechanics

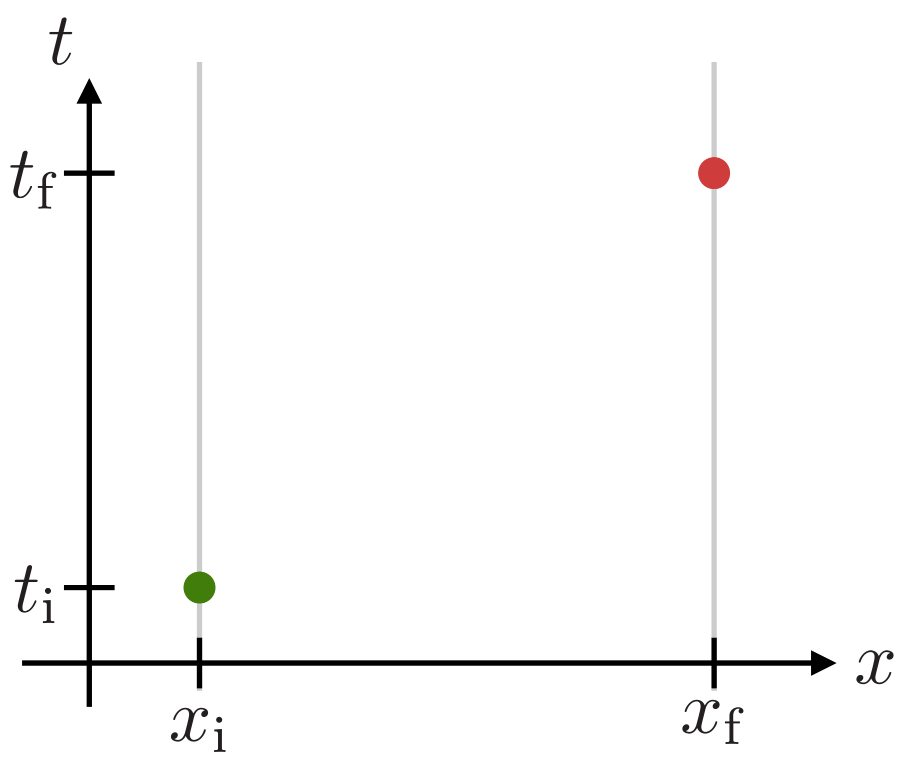
e.g. a free particle moving between a

definite initial position (x_i, t_i)

and a

definite final position (x_f, t_f)

space-time diagram



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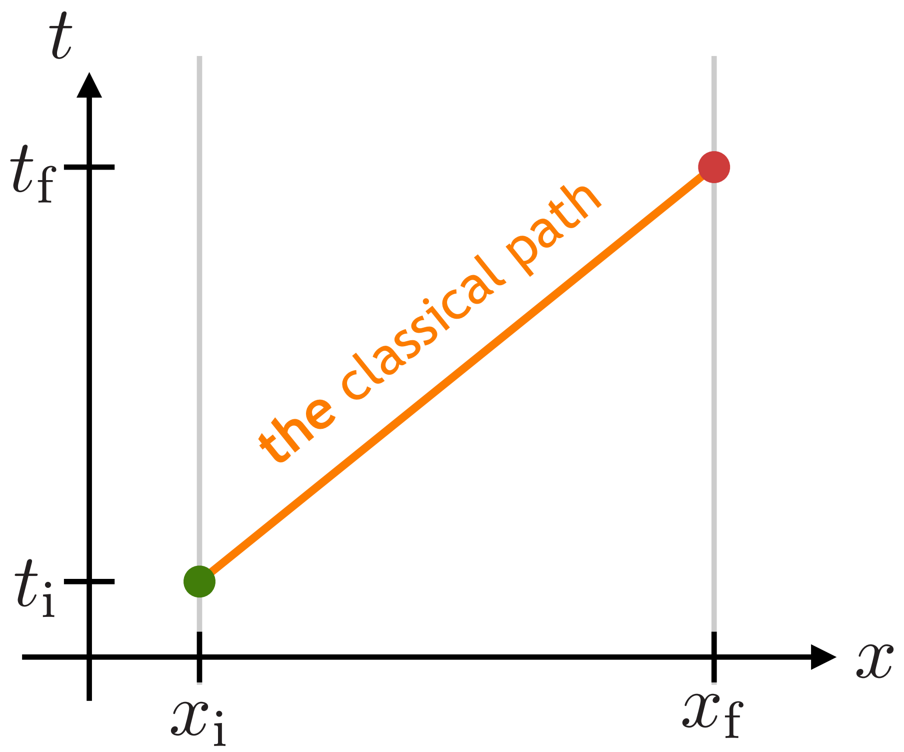

the **unique** classical path is the path of **minimum action**

the *action* $$S\big[x(t)\big] = \int_{t_\mathrm{i}}^{t_\mathrm{f}} dt\, L(x, \dot{x})$$

$$L_\mathrm{free} = \tfrac{1}{2} m \dot{x}^2$$

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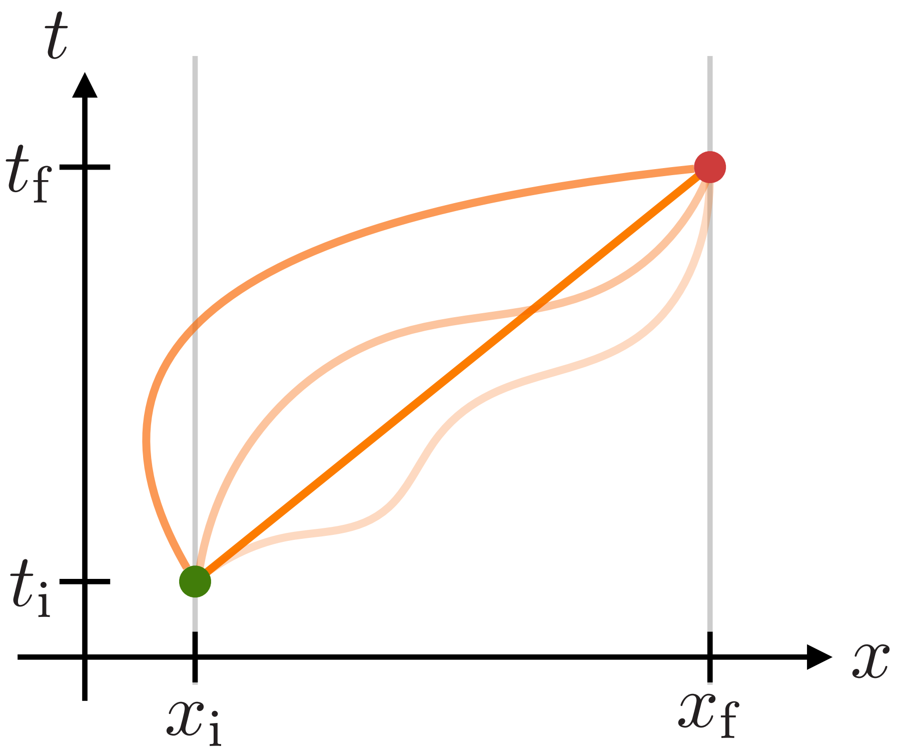

quantum mechanical amplitude

$$\langle x_\mathrm{f} | e^{-i\hat{H}(t_\mathrm{f}-t_\mathrm{i})} | x_\mathrm{i} \rangle$$

$$= \int \mathcal{D}x \, e^{-iS[x(t)]}$$

path integrals in quantum mechanics

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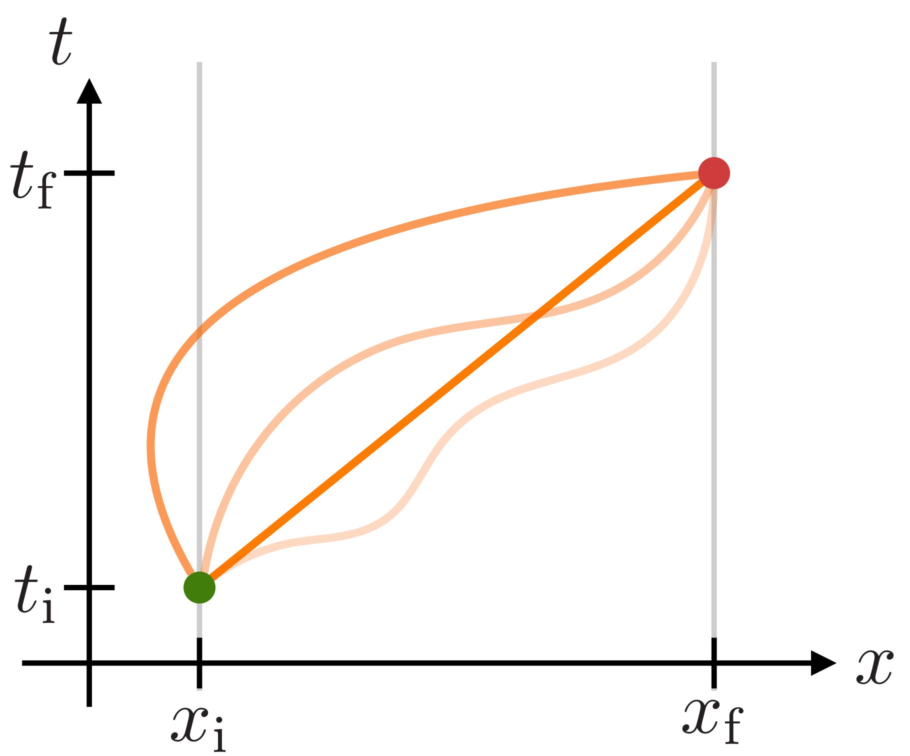

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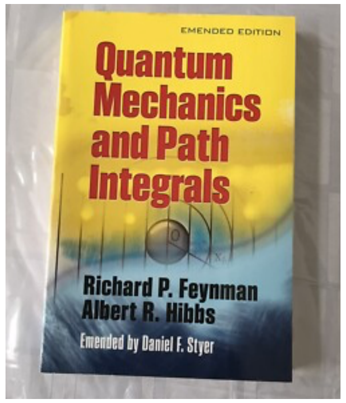
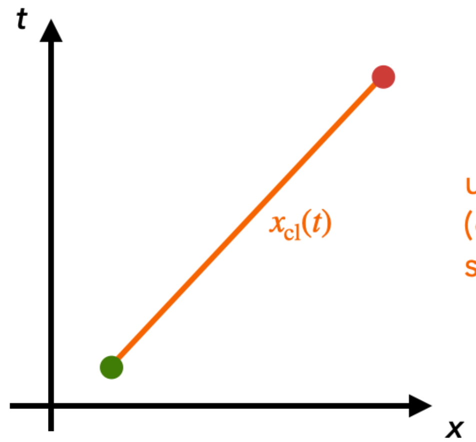
$$\langle x_\mathrm{f} | e^{-i\hat{H}(t_\mathrm{f}-t_\mathrm{i})} | x_\mathrm{i} \rangle = \int \mathcal{D}x \, e^{-iS[x(t)]}$$

"sum" over all paths — weighted by a phase set by the action

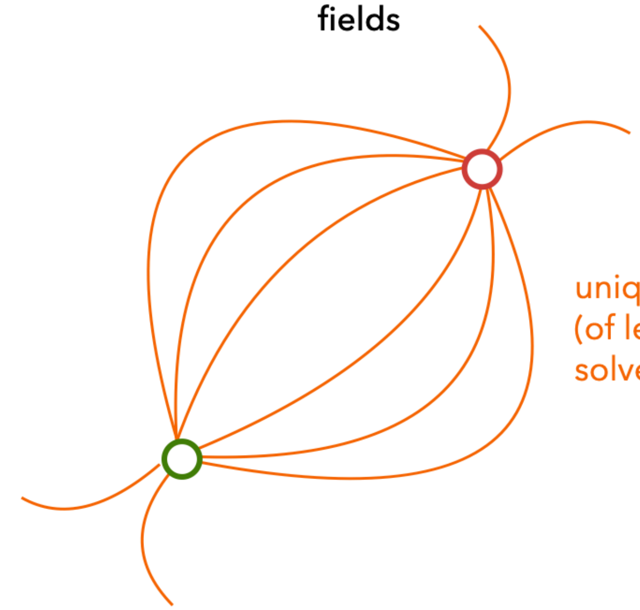
and conventional quantum mechanics follows ...



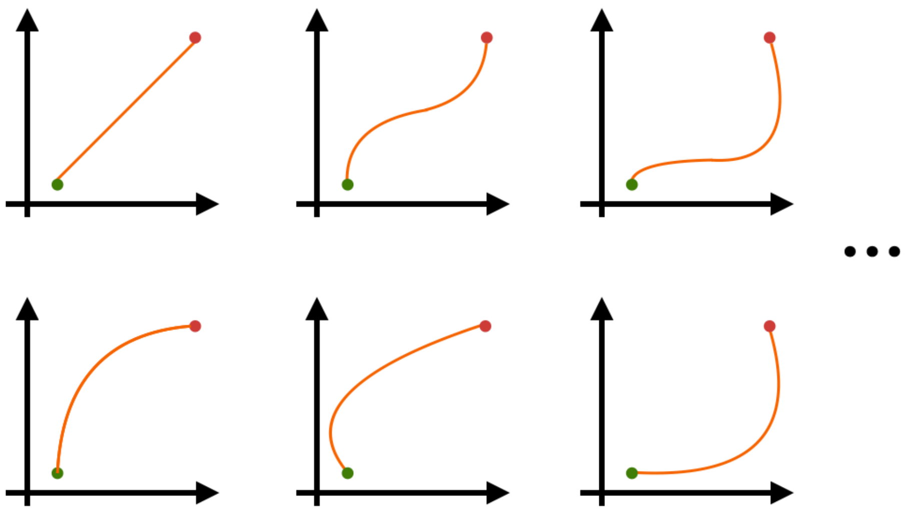
	mechanics	fields
classical	unique path (of least action) solves Newton's eqns	unique field configuration (of least action) solves Maxwell's eqns (e.g.)
quantum	weighted average over all possible paths	



unique path
(of least action)
solves Newton's eqns



unique field configuration
(of least action)
solves Maxwell's eqns (e.g.)

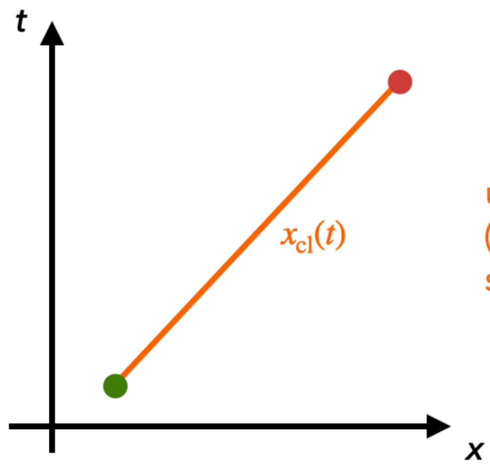


weighted average
over all possible paths

mechanics

fields

classical



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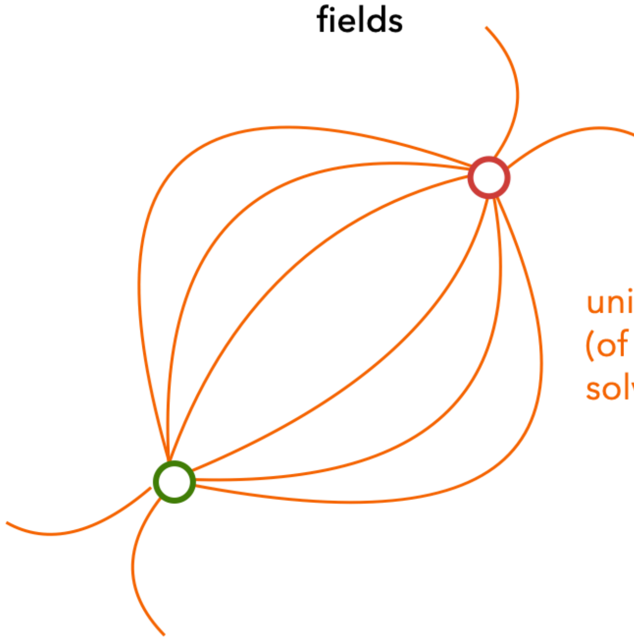

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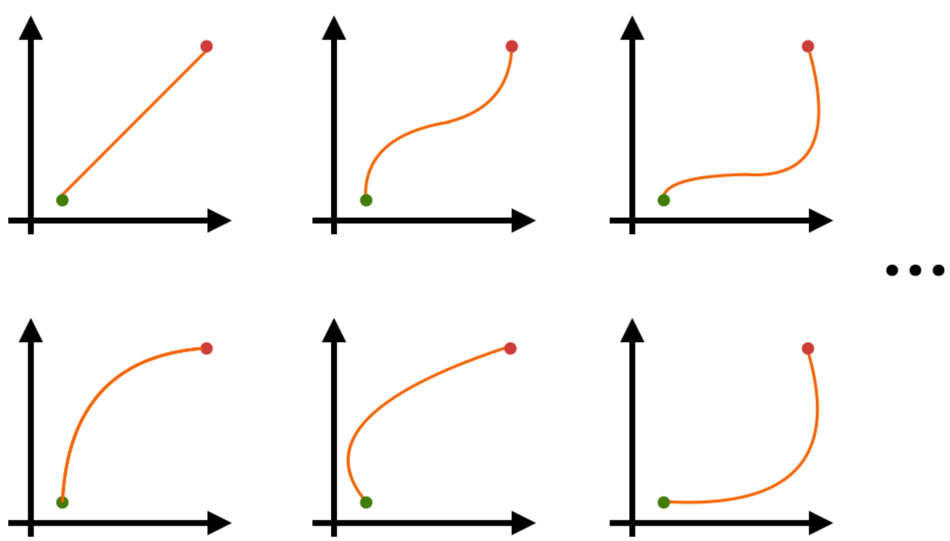
quantum



weighted average
over all possible paths

weighted average
over all possible
field configurations

consider a real scalar field theory

$$\varphi(\mathbf{x}, t)$$

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \varphi \, \partial^\mu \varphi - \tfrac{1}{2} m^2 \varphi^2 + V\left[\varphi\right]$$

?

path integrals in quantum field theory

consider a real scalar field theory

$$\varphi(\mathbf{x}, t)$$

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \tfrac{1}{2}m^2\varphi^2 + V\big[\varphi\big]$$

can define a path integral

$$Z = \int \mathcal{D}\varphi(x)\, e^{-iS[\varphi(x)]}$$

where the action is $S\big[\varphi(x)\big] = \int d^4x\, \mathcal{L}\big[\varphi(x)\big]$

"sum" over all
field configurations

?

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"sum" over all
field configurations

correlation functions can be expressed similarly

e.g. relationship between the field value at one space-time point
and the value at another space-time point

$$\big\langle 0\big|\hat{\varphi}(y)\,\hat{\varphi}(z)\big|0\big\rangle = \frac{1}{Z}\int \mathcal{D}\varphi(x)\;\varphi(y)\,\varphi(z)\;e^{-iS[\varphi(x)]}$$

the spin vectors in a ferromagnet is a nice classical example

?

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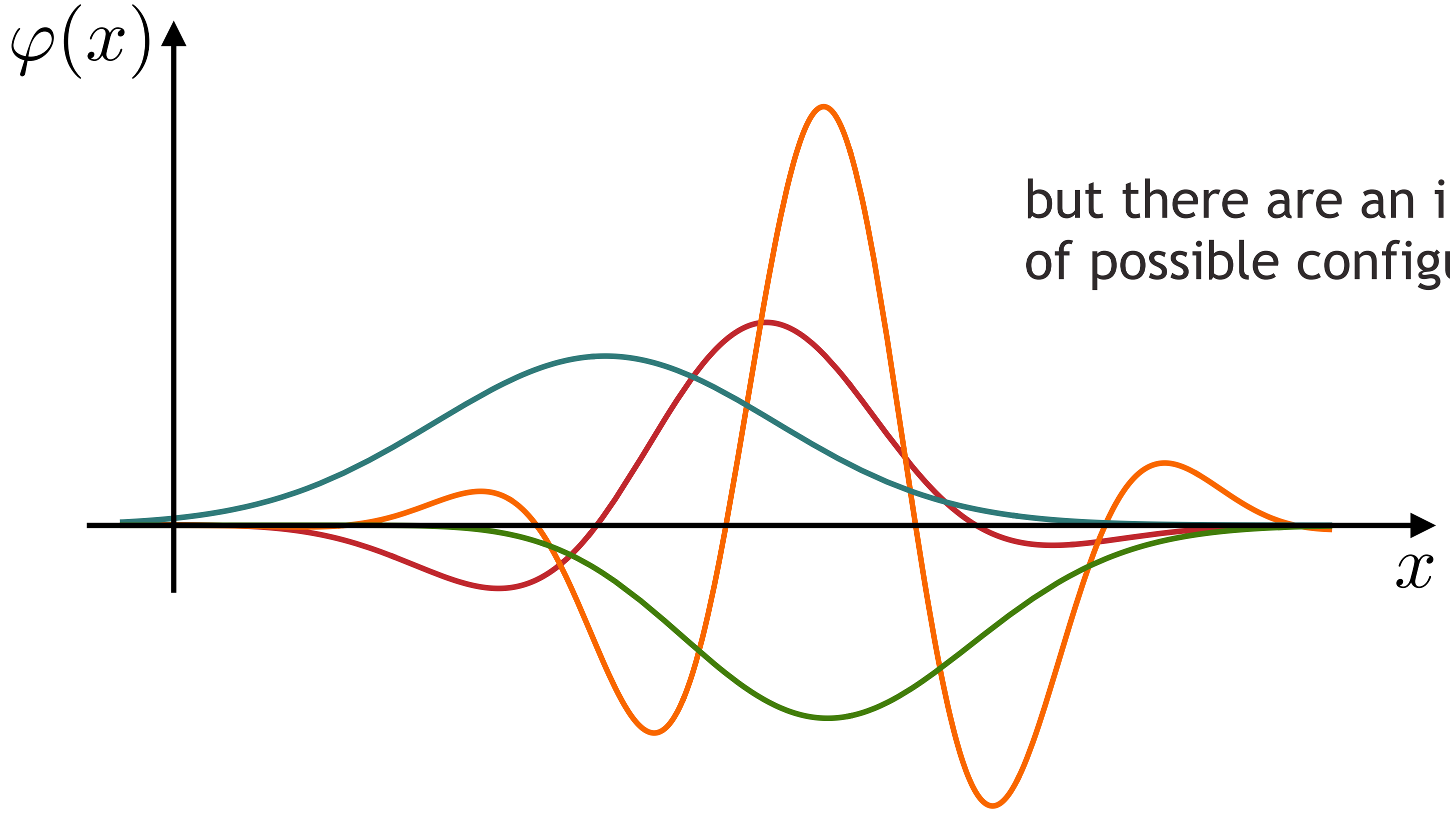
the spin vectors in a ferromagnet is a nice classical example

but practically how does one 'do' the integral $\int\!\mathcal{D}\varphi(x)$?

go to one dimension for simplicity of illustration

scalar field configurations



but there are an infinite number of possible configurations ...

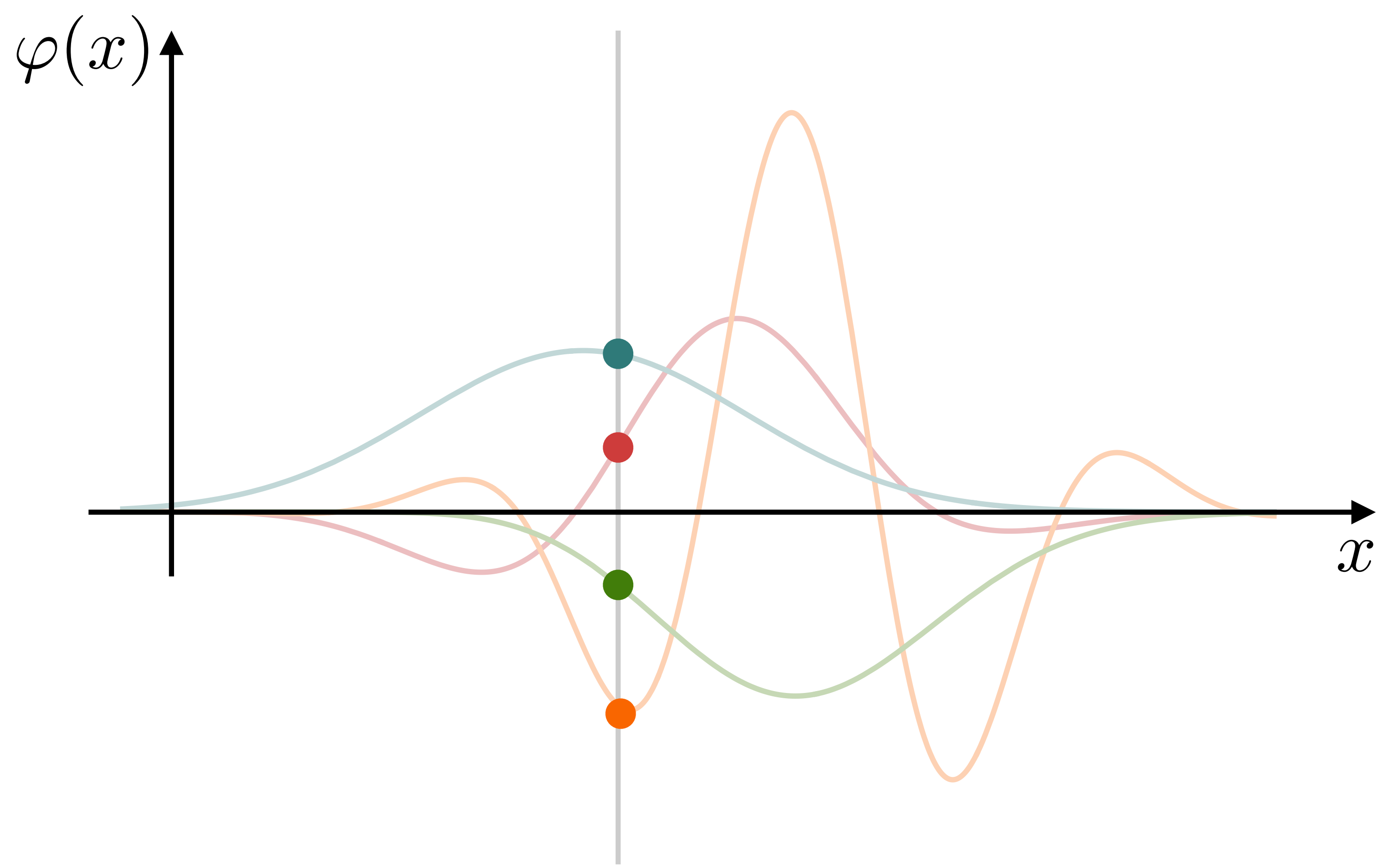
lattice quantum field theory

discretize the space

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x = \int d\varphi_1 \int d\varphi_2 \int d\varphi_3 \cdots$$

an integral over all the values the field can take at x_2

scalar field configurations



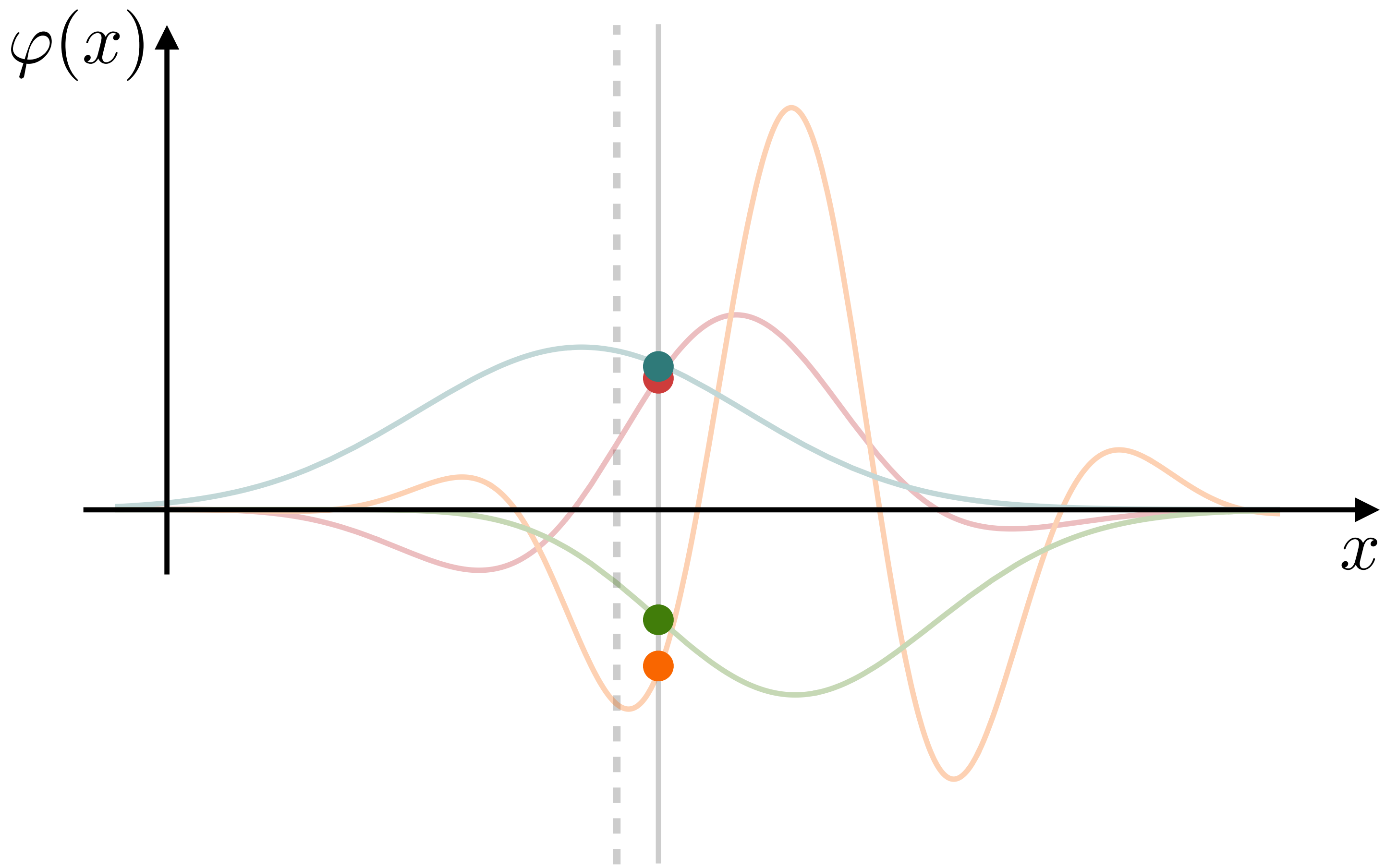
(i'm handwaving the rigorous math here)

discretize the space

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x = \int d\varphi_1 \int d\varphi_2 \int d\varphi_3 \cdots$$

an integral over all the values the field can take at x_3

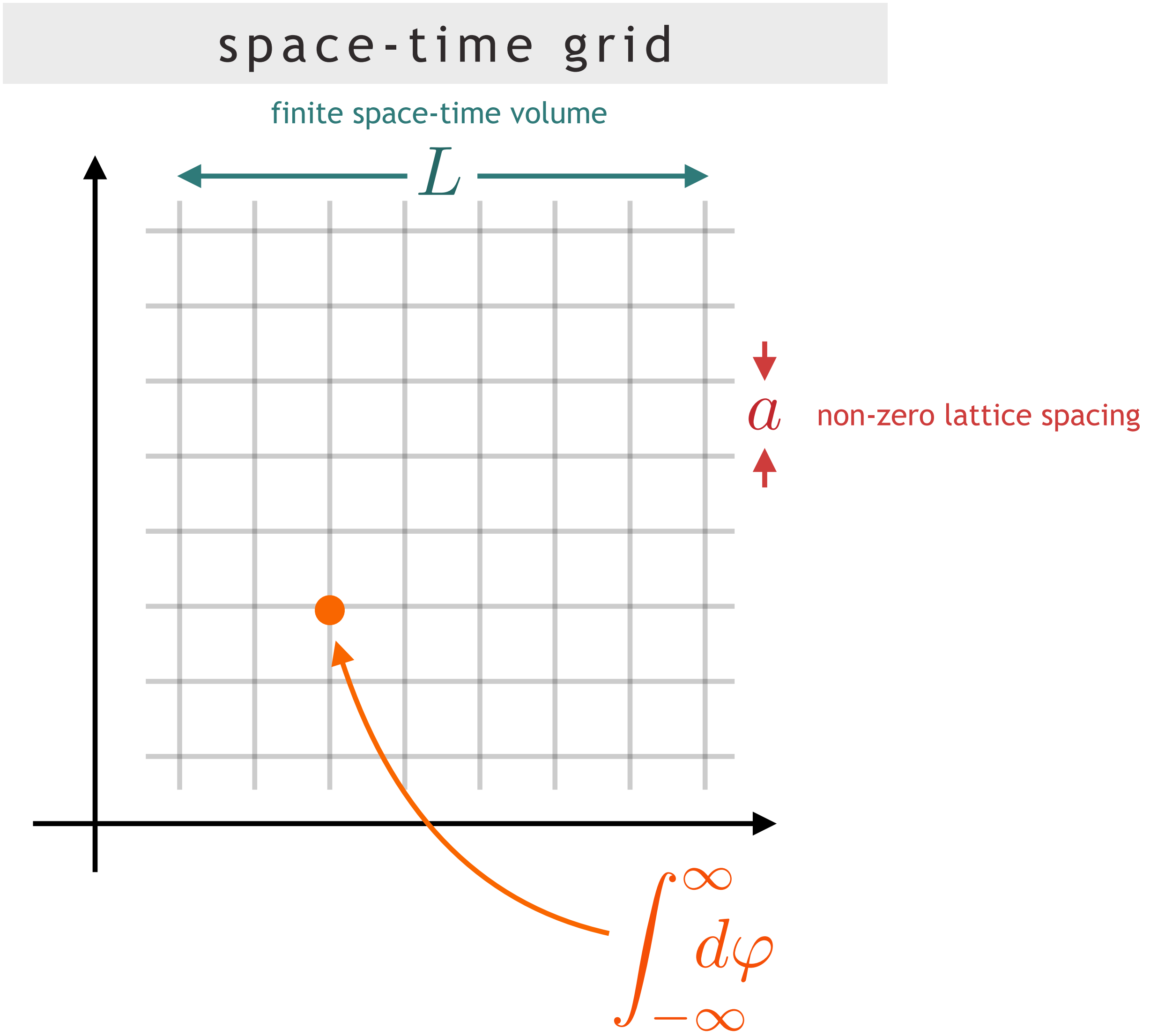
scalar field configurations



lattice quantum field theory

approach generally is to use a (hyper)cubic grid

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x$$



hiding it here,
but boundary conditions
are important

even with the grid, still not practical: $Z = \int \mathcal{D}\varphi(x)\, e^{-iS[\varphi(x)]}$

a **phase** is not ideal for averaging

make a variable transform $t \to -it$ then $-iS = -i\int d^3x\, dt\, \mathcal{L} \to -\int d^3x\, dt\, \mathcal{L}_{\mathrm{E}} = -S_{\mathrm{E}}$

euclidean path integral

$$Z_{\mathrm{E}} = \int \mathcal{D}\varphi(x)\, e^{-S_{\mathrm{E}}[\varphi(x)]}$$

a bounded real number
⇝ a probability ?

euclidean path integral

$$Z_{\mathrm{E}} = \int \mathcal{D}\varphi(x)\, e^{-S_{\mathrm{E}}[\varphi(x)]}$$

probability for a field configuration $\varphi(x)$

⇒ **importance sampled Monte Carlo generation of field configurations**

obtain an **ensemble** of configurations $\{\varphi_x\}_{i=1\ldots N}$

[value of the field at each point on the grid]

computing observables

for some observable (vacuum matrix element)

$$\langle 0 | \, O[\hat{\varphi}] \, | 0 \rangle = \int \mathcal{D}\varphi \, O[\varphi] \, e^{-S_{\mathrm{E}}[\varphi]}$$

can now be estimated as an **average over the ensemble**

$$\langle 0 | \, O[\hat{\varphi}] \, | 0 \rangle \approx \overline{O} = \frac{1}{N} \sum_{i=1}^{N} O[\varphi^{(i)}]$$

plus get an **uncertainty estimate** from the variance

$$\sigma(O) = \sqrt{\tfrac{1}{N(N-1)} \sum_{i=1}^{N} \left(O[\varphi^{(i)}] - \overline{O} \right)^2}$$

ensemble mean and error

$$\langle 0 | \, O[\hat{\varphi}] \, | 0 \rangle \approx \overline{O} \pm \sigma(O)$$

what about the theory we require: **quantum chromo dynamics** ?

gauge field theory of quarks (fermions) and gluons (vector gauge fields) with SU(3) 'color' symmetry

qcd lagrangian

$$\mathcal{L} = \bar{\psi}\big(i\gamma^\mu D_\mu - m\big)\psi - \tfrac{1}{2}\mathrm{tr}\big(F_{\mu\nu}F^{\mu\nu}\big)$$

gauge covariant derivative

$$D_\mu = \partial_\mu + igA_\mu$$

field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig\big[A_\mu, A_\nu\big]$$

qcd fields

color index $i = 1\ldots3$

quark field $\psi^i_\alpha(x)$

Dirac spin index $\alpha = 1\ldots4$

traceless matrix in color

gluon field $A^{ij}_\mu(x)$

Lorentz vector

$$= \sum_{a=1\ldots8} A^a_\mu(x)\, t^a_{ij}$$

expansion in SU(3) generators

$$t^a = \tfrac{1}{2}\lambda^a$$

$$\big[t^a, t^b\big] = if^{abc}t^c$$

$$\mathrm{tr}\big(t^a t^b\big) = \tfrac{1}{2}\delta^{ab}$$

quantum chromodynamics

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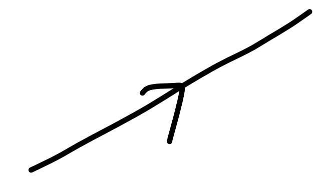
qcd ingredients

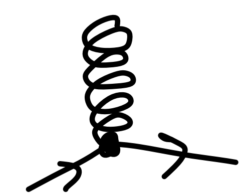
relativistic fermions

$$\bar{\psi}\big(i\gamma^\mu\partial_\mu - m\big)\psi$$



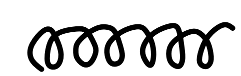
color vector current

$$g\ \big(\bar{\psi}\gamma^\mu t^a\psi\big)\ A^a_\mu$$



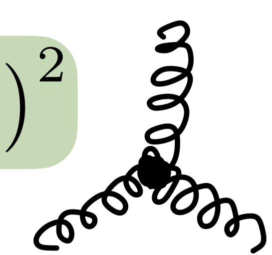
massless gluons

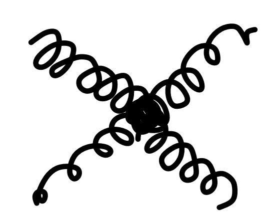
$$\big(\partial_\mu A_\nu - \partial_\mu A_\nu\big)^2$$



gluon self-interactions

$$g[A,A]\partial A,\ g^2\big([A,A]\big)^2$$





transforming to Euclidean space-time
required for Monte-Carlo sampling

$$(x_{\mathrm{M}})^0 = -i(x_{\mathrm{E}})_4 \qquad (x_{\mathrm{M}})^i = (x_{\mathrm{E}})_i$$

$$(\gamma_{\mathrm{M}})^0 = (\gamma_{\mathrm{E}})_4 \qquad (\gamma_{\mathrm{M}})^i = i(\gamma_{\mathrm{E}})_i$$

$$(A_{\mathrm{M}})^0 = i(A_{\mathrm{E}})_4 \qquad (A_{\mathrm{M}})^i = -(A_{\mathrm{E}})_i$$

euclidean qcd action

(entirely in Euclidean variables)

$$S_{\mathrm{E}} = \int d^4x \left[\overline{\psi}\left(\gamma_\mu D_\mu + m\right)\psi + \tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right]$$

we'd like to discretize on a hypercubic grid

quark fields take values
on the sites of the lattice

$$\psi^i_\alpha(x_\mu = a n_\mu)$$

with one wrinkle
still to be dealt with

but what shall we do with the **gluon fields** … ?

parallel transporters and gauge invariance

in the continuum theory,
consider a quark field pair separated by some distance

the combination $\overline{\psi}^{\,j}(y)\,\delta_{ji}\,\psi^{i}(x)$ is not **gauge-invariant**

we can perform **different** local gauge transformations at locations ***x*** and ***y***

q$\bar{\text{q}}$ pair

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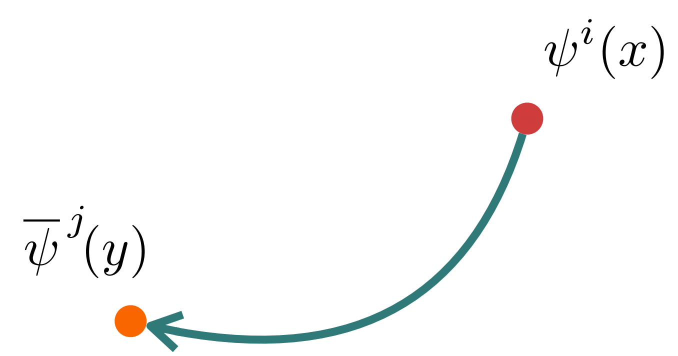
q$\overline{\text{q}}$ pair

a **gauge-invariant** combination is $\overline{\psi}^j(y) \left[e^{ig\int_x^y dz_\mu A^\mu(z)} \right]_{ji} \psi^i(x)$

a **'Wilson line'** transports the color

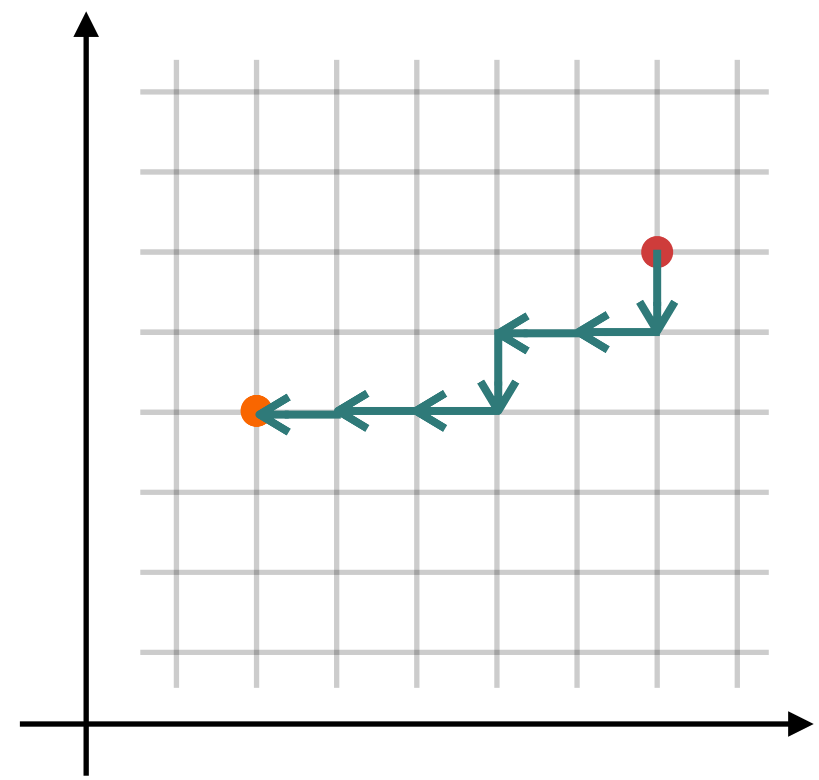
q$\overline{\text{q}}$ pair with Wilson line



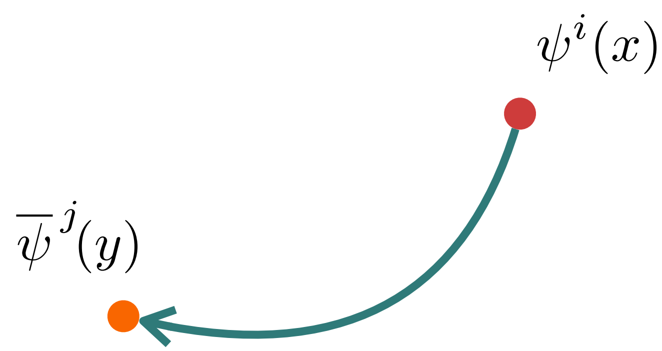
gauge ‘links’

on a lattice, make hops to neighboring sites

space-time grid

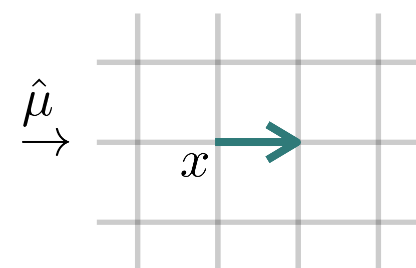


$q\bar{q}$ pair with Wilson line



$$\overline{\psi}^j(y)\left[e^{ig\int_x^y dz_\mu A^\mu(z)}\right]_{ji}\psi^i(x)$$

shortest path between neighboring sites = a **‘link’**



$$\left[e^{igaA^\mu(x)}\right]_{ji}$$

$$U_\mu(x) = e^{igaA^\mu(x)}$$ SU(3) matrix on each **link** of the lattice

lattice QCD action

can construct a gauge-invariant **finite-difference** – approximation to a derivative ?

$$\overline{\psi}(x)\,\gamma_\mu\,U_\mu(x)\,\psi(x+\hat{\mu}a) - \overline{\psi}(x)\,\gamma_\mu\,U^\dagger_\mu(x-\hat{\mu}a)\,\psi(x-\hat{\mu}a)$$

c.f. $\frac{1}{2a}\big(f(x+a)-f(x-a)\big) \xrightarrow{a\to 0} \frac{df}{dx} + O(a^2)$

$$\xrightarrow{a\to 0} 2a\,\overline{\psi}\,\gamma_\mu\big(\partial_\mu + igA_\mu\big)\,\psi + \dots$$

and using constructions like these we can build **discretized actions**

e.g. $\int d^4x\,\overline{\psi}\big(\gamma_\mu D_\mu + m\big)\psi \quad \rightsquigarrow \quad \overline{\psi}^{\,i\alpha}_x M^{i\alpha,j\beta}_{x,y}[U]\,\psi^{j\beta}_y$

Dirac matrix

matrix in
color, spin, spacetime

N.B. **large** matrix, but **sparse**

e.g. for a $24^3\times128$ lattice, ~ 21M×21M

most of the elements are zero

(100 Pb !!!)

what about the quarks ?

a gauge-field ‘configuration’ is simple — it’s an SU(3) matrix on each link

but what about a quark-field configuration?

fermion fields operators **anticommute**

e.g. equal-time anticommutator $\left\{\hat{\psi}_\alpha(\vec{x},t), \hat{\psi}^\dagger_\beta(\vec{y},t)\right\} = \delta_{\alpha\beta}\,\delta(\vec{x}-\vec{y})$

but under the path integral, operators become numbers,
and ordinary numbers don’t anticommute … ?

⇒ **Grassmann variables**

well-defined mathematics of anti-commuting numbers

ordinary (complex) numbers	Grassmann (complex) numbers
$x\,y = y\,x$	$\theta_i\,\theta_j = -\,\theta_j\,\theta_i$
$x^2 \neq 0 \,\text{unless}\, x = 0$	$(\theta_i)^2 = 0$

$$\theta_i\,x = x\,\theta_i$$

Taylor series terminate,

$$f(\theta) = a + b\theta$$

integration is simple

$$\int d\theta = 0 \qquad \int \theta\,d\theta = 1$$

a gauge-field ‘configuration’ is simple — it’s an SU(3) matrix on each link

but what about a quark-field configuration? **fermion fields anticommute ⇒ Grassmann variables**

actually we can do the quark field integration exactly in the path integral:

$$\int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \, e^{-S_{\mathrm{E}}[\psi,\overline{\psi},U]} = \int \mathcal{D}U \, e^{-S^{\mathrm{g}}_{\mathrm{E}}[U]} \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \, e^{-\overline{\psi} M[U] \psi}$$

$$= \det M[U]$$

follows from integration properties of Grassmann variables

$$= \int \mathcal{D}U \, \det M[U] \, e^{-S^{\mathrm{g}}_{\mathrm{E}}[U]}$$

interpret as the probability for configuration $U_\mu(x)$

a (gauge-variant) correlation function

$$\langle 0 | \hat{\psi}_x^{i\alpha} \, \hat{\overline{\psi}}_y^{j\beta} | 0 \rangle = \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \, \psi_x^{i\alpha} \, \overline{\psi}_y^{j\beta} \, e^{-S_{\mathrm{E}}[\psi, \overline{\psi}, U]}$$

correlation between
quark at *x*, color *i*, spin *α*
and
quark at *y*, color *j*, spin *β*

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$$= \int \mathcal{D}U \, e^{-S_\mathrm{E}^\mathrm{g}[U]} \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \, \psi_x^{i\alpha} \overline{\psi}_y^{j\beta} \, e^{-\overline{\psi} M[U] \psi}$$

$$= \int \mathcal{D}U \left[M^{-1}[U] \right]_{x,y}^{i\alpha, j\beta} \det M[U] \, e^{-S_\mathrm{E}^\mathrm{g}[U]}$$

probability

c.f. Wick's theorem
(look in your favorite QFT textbook)

a (gauge-variant) correlation function

$$\langle 0 | \hat{\psi}^{i\alpha}_x \, \hat{\overline{\psi}}^{j\beta}_y | 0 \rangle = \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \, \psi^{i\alpha}_x \, \overline{\psi}^{j\beta}_y \, e^{-S_{\mathrm{E}}[\psi, \overline{\psi}, U]}$$

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$$= \int \mathcal{D}U \left[M^{-1}[U] \right]^{i\alpha, j\beta}_{x,y} \det M[U] \, e^{-S^{\mathrm{g}}_{\mathrm{E}}[U]}$$

probability

c.f. Wick's theorem
(look in your favorite QFT textbook)

$$= \sum_{\{U\}} \left[M^{-1}[U] \right]^{i\alpha, j\beta}_{x,y}$$

compute
'quark propagator'
on each
configuration

a (gauge-variant) correlation function

$$\langle 0 | \hat{\psi}_x^{i\alpha} \, \hat{\overline{\psi}}_y^{j\beta} | 0 \rangle = \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \, \psi_x^{i\alpha} \, \overline{\psi}_y^{j\beta} \, e^{-S_{\mathrm{E}}[\psi, \overline{\psi}, U]}$$

correlation between
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$$= \int \mathcal{D}U \, e^{-S_{\mathrm{E}}^{\mathrm{g}}[U]} \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \, \psi_x^{i\alpha} \overline{\psi}_y^{j\beta} \, e^{-\overline{\psi} M[U] \psi}$$

$$= \int \mathcal{D}U \, \Big[M^{-1}[U] \Big]_{x,y}^{i\alpha, j\beta} \det M[U] \, e^{-S_{\mathrm{E}}^{\mathrm{g}}[U]}$$

probability

c.f. Wick's theorem
(look in your favorite QFT textbook)

$$= \sum_{\{U\}} \Big[M^{-1}[U] \Big]_{x,y}^{i\alpha, j\beta}$$

compute
'quark propagator'
on each
configuration

actually, don't do this
because it will average to zero

a meson correlation function

$$\langle 0 | \sum_{\vec{x}} \left(\overline{\psi}\gamma_5\psi\right)_{\vec{x},t} \left(\overline{\psi}\gamma_5\psi\right)_{\vec{0},0} | 0 \rangle$$

$\overline{\psi}\gamma_5\psi$ pseudoscalar quantum numbers

$\sum_{\vec{x}} f(\vec{x}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} f(\vec{x})\Big|_{\vec{p}=\vec{0}}$ projection into zero momentum

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$$= -\sum_{\{U\}} \sum_{\vec{x}} \mathrm{tr}\Big(\big[M^{-1}[U]\big]_{\vec{0}0,\vec{x}t} \gamma_5 \big[M^{-1}[U]\big]_{\vec{x}t,\vec{0}0} \gamma_5 \Big)$$

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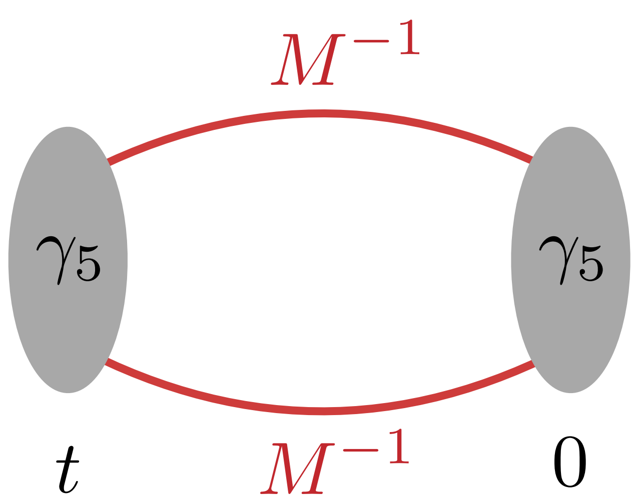

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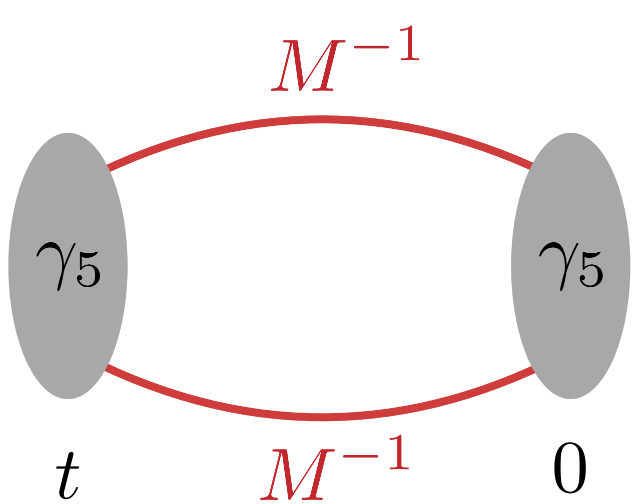

point – all propagator

$$\big[M[U]\big]_{\vec{y}t',\vec{x}t}\, \chi_{\vec{x}t} = \delta_{\vec{y},\vec{0}}\, \delta_{t',0}$$

sparse matrix — point source

$$\chi_{\vec{x}t} = \big[M^{-1}[U]\big]_{\vec{x}t,\vec{0}0}$$

point-all propagator

solving a sparse linear system: **A** · *x* = *b*

e.g. for a
$24^3\times128$ lattice,
~ 21M×12
(few Gb)

the lattice QCD workflow

select a discretization

'tune' the parameters

i.e. to choose a lattice spacing you want, and quark masses that you want ...

generate 100s / 1000s of gauge-field configurations

serious parallel supercomputing

whole industry of numerical algorithm development to make this step as efficient as possible

compute quark propagators

serious computing

GPUs very useful

'contract' into correlation functions

capacity computing

'bookkeeping' / memory management

•
•
•

PHYSICS ?

lattice systematics (much simplified)

finite lattice spacing

acts as a UV cutoff $\Lambda \sim \frac{1}{a}$ required, regularization of a renormalizable theory

appears as a scale $\hat{m} = am$

discretization errors $X(a) = X(0) + a\,\delta X_1 + \ldots$

extrapolate $a \to 0$

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discretization choice

all should agree in the *a*→0 limit

impact at finite *a* depends on observable

choose a discretization appropriate to your quantities

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finite lattice volume

need $L \gg \frac{1}{m_\pi}$

impacts multi-hadron systems
in an interesting way

carefully understand QFT in a finite volume

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quark mass choice

many calculations done with $m_{u,d} > m_{u,d}^{\text{phys}}$

use quark mass as a tool to understand QCD