lattice QCD and the hadron spectrum

Jozef Dudek

hadspec
hadron spectrum collaboration
hadspec.org

William & Mary
CHARTERED 1693

Jefferson Lab
Thomas Jefferson National Accelerator Facility

contents

meson spectroscopy *"illustrating the problem"*

resonances, scattering, elastic phase-shifts

lattice QCD *"introducing the tool"*

discrete spectrum, finite volume, computing the spectrum

elastic scattering *"solving the simplest problem"*

lattice QCD phase-shift results

coupled-channel scattering *"a more realistic situation"*

mapping the discrete spectrum to the ***t***-matrix

lattice QCD calculation results

the complex energy plane *"well-defined quantities"*

rigorously determining resonances

not-so-recent pedagogic review

REVIEWS OF MODERN PHYSICS, VOLUME 90, APRIL–JUNE 2018

Scattering processes and resonances from lattice QCD

Raúl A. Briceño*

Thomas Jefferson National Accelerator Facility,
12000 Jefferson Avenue, Newport News, Virginia 23606, USA
and Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA

Jozef J. Dudek†

Thomas Jefferson National Accelerator Facility,
12000 Jefferson Avenue, Newport News, Virginia 23606, USA
and Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA

Ross D. Young‡

Special Research Center for the Subatomic Structure of Matter (CSSM),
Department of Physics, University of Adelaide, Adelaide 5005, Australia

(published 18 April 2018)

very little referencing in these lectures
most references can be found in this review

contents

meson spectroscopy *"illustrating the problem"*

resonances, scattering, elastic phase-shifts

lattice QCD *"introducing the tool"*

discrete spectrum, finite volume, computing the spectrum

elastic scattering *"solving the simplest problem"*

lattice QCD phase-shift results

coupled-channel scattering *"a more realistic situation"*

mapping the discrete spectrum to the t-matrix

lattice QCD calculation results

the complex energy plane *"well-defined quantities"*

rigorously determining resonances

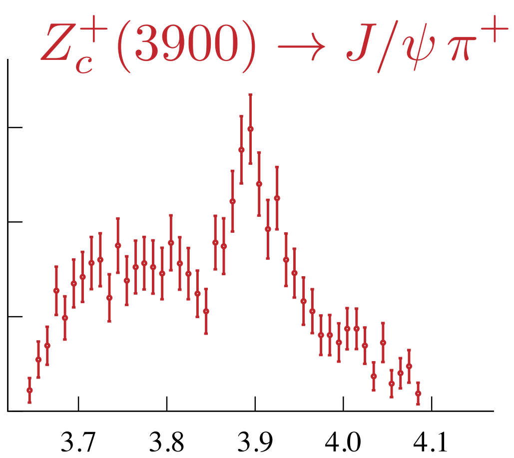
if I have time at the end …

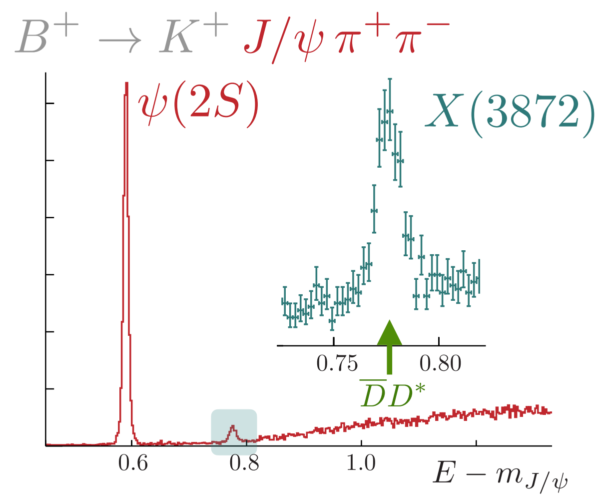
other applications of lattice QCD technology

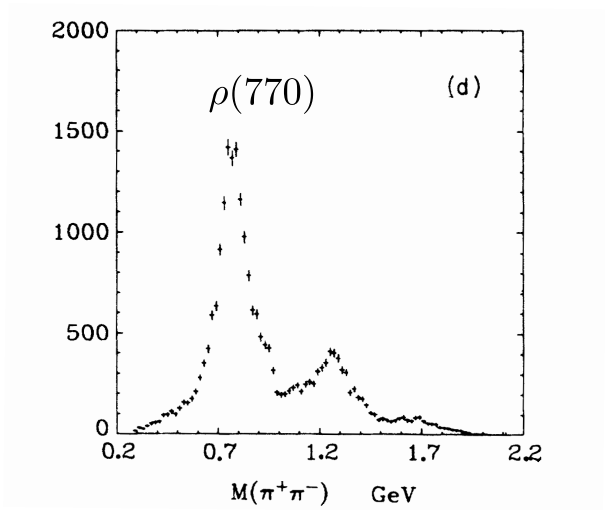
contents

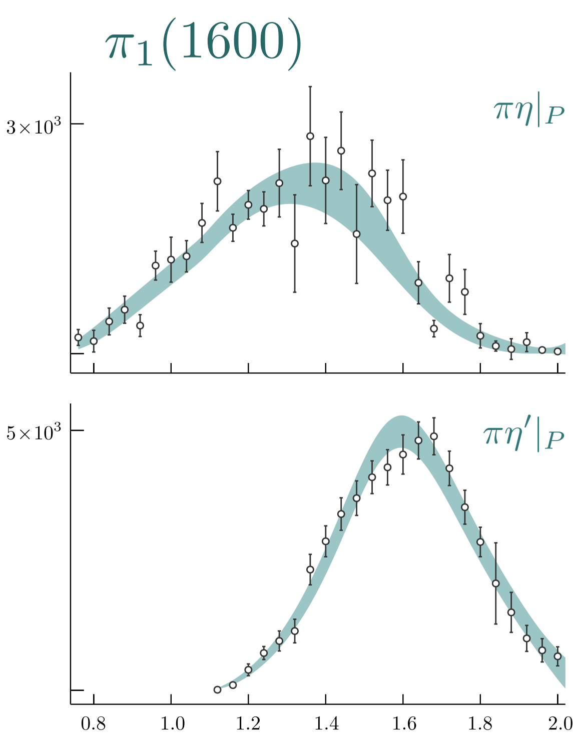
meson spectroscopy

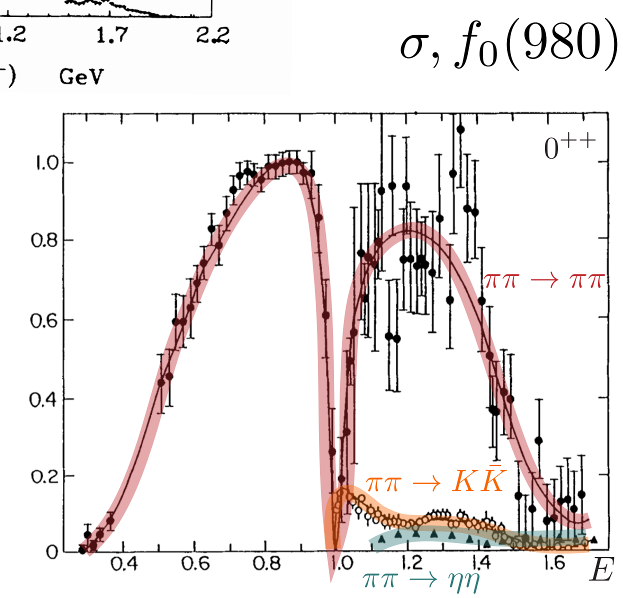
experimental signals











'theory' ?

$q\bar{q}$ mesons
glueballs
hybrids
tetraquarks
molecules
...

these are **'pictures'**

the theory is QCD

... how do we bridge the gap ?

I'll try to show you over the next few lectures

want to study excited hadrons as they really are — **rapidly decaying resonances**

$$\tau_\rho = \frac{\hbar}{\Gamma_\rho} \to 4 \times 10^{-24}\,\mathrm{s}$$

same strong dynamics that binds them also causes their decay
can’t treat decay as a perturbation

e.g. ρ resonance: $m_\rho \sim 800\,\mathrm{MeV}$ $\Gamma_\rho \sim 150\,\mathrm{MeV}$

compare hydrogen: $E_{2P} \sim \mathrm{eV}$ $\Gamma_{2P} = \frac{\hbar}{\tau_{2P}} \sim 10^{-7}\,\mathrm{eV}$

meson spectroscopy — ‘rigorously’

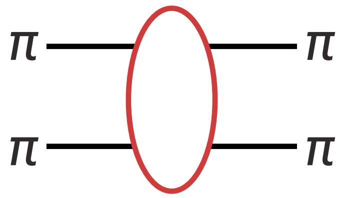
want to study excited hadrons as they really are — **rapidly decaying resonances**

same strong dynamics that binds them also causes their decay
can’t treat decay as a perturbation

we need to compute **scattering amplitudes** and see if they resonate

start with the simplest case: **elastic scattering** …

e.g.



amplitude

$t(E, \cos\theta)$

differential cross-section

$$\frac{d\sigma}{d\Omega} \propto \left|t(E, \cos\theta)\right|^2$$

pion, π, is a J^P=0^- hadron,

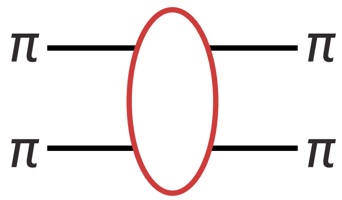
stable in pure QCD

mass near 140 MeV

isospin=1, π^+, π^-, π^0

long-lived enough to make charged beams

elastic partial-waves & unitarity



elastic scattering amplitude
can be expanded in partial-waves

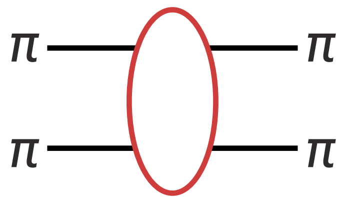
$$t(E,\cos\theta) = \sum_\ell (2\ell+1)\, t_\ell(E)\, P_\ell(\cos\theta)$$

partial-wave amplitude

resonances appear in a single partial-wave

ℓ	0	1	2	...
J^P	0^+	1^-	2^+	

elastic partial-waves & unitarity



elastic scattering amplitude
can be expanded in partial-waves

$$t(E, \cos\theta) = \sum_\ell (2\ell+1)\, t_\ell(E)\, P_\ell(\cos\theta)$$

partial-wave amplitude

resonances appear in a single partial-wave

ℓ	0	1	2	...
J^P	0^+	1^-	2^+	

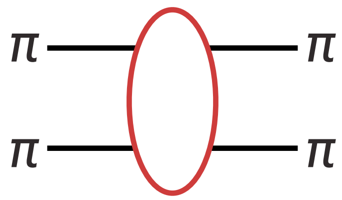
conservation of probability
a.k.a **elastic unitarity**

$$\mathrm{Im}\, t_\ell(E) = \rho(E) \big| t_\ell(E) \big|^2 \quad \text{or} \quad \mathrm{Im} \frac{1}{t_\ell(E)} = -\rho(E)$$

'phase-space' $\rho(E) = \dfrac{2k(E)}{E}$

c.m. momentum $k(E) = \frac{1}{2}\sqrt{E^2 - 4m^2}$

elastic partial-waves & unitarity



elastic scattering amplitude
can be expanded in partial-waves

$$t(E,\cos\theta) = \sum_\ell (2\ell+1)\, t_\ell(E)\, P_\ell(\cos\theta)$$

partial-wave amplitude

resonances appear in a single partial-wave

ℓ	0	1	2	…
J^P	0^+	1^-	2^+	

conservation of probability
a.k.a **elastic unitarity**

$$\mathrm{Im}\, t_\ell(E) = \rho(E)\big|t_\ell(E)\big|^2 \quad \text{or} \quad \mathrm{Im}\,\frac{1}{t_\ell(E)} = -\rho(E)$$

'phase-space' $\rho(E) = \dfrac{2k(E)}{E}$

c.m. momentum $k(E) = \frac{1}{2}\sqrt{E^2 - 4m^2}$

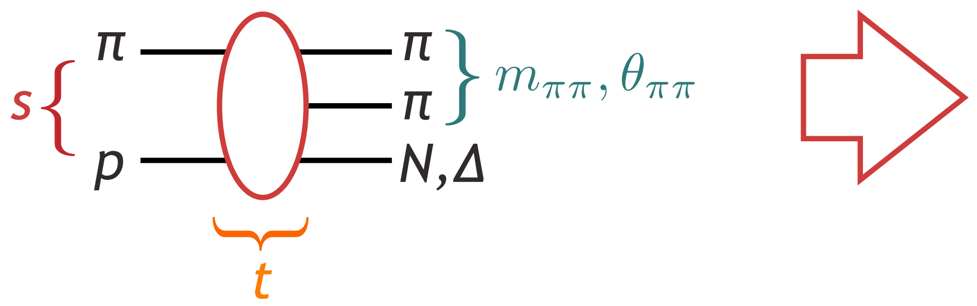
can parameterise elastic scattering
in terms of a single real parameter

$$t_\ell(E) = \frac{1}{\rho(E)}\, e^{i\delta_\ell(E)} \sin\delta_\ell(E)$$

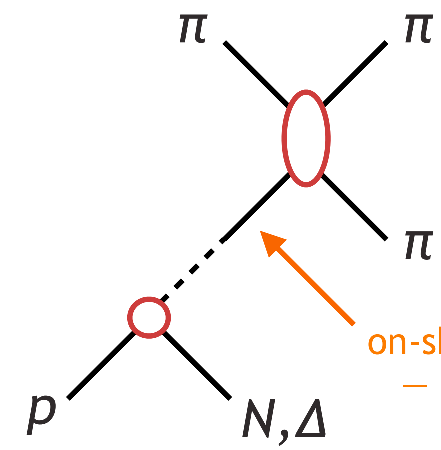
'phase-shift'

the “simplest” case: $\pi\pi$ elastic scattering

extract from **charged pion beams** on **nucleon targets**



physical scattering
has negative *t*



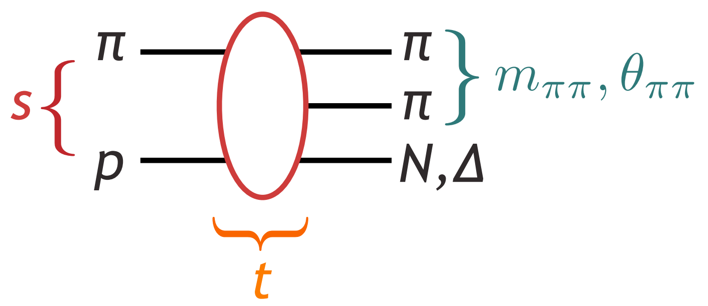
$\pi\pi$ partial-waves
— project $P_\ell(\cos\theta_{\pi\pi})$

on-shell π exchange
— extrapolate to $t = m_\pi^2$

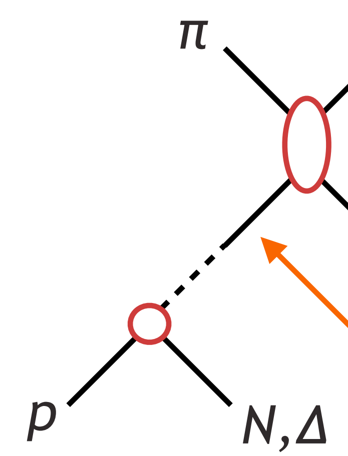
this bit is somewhat
model dependent

the “simplest” case: $\pi\pi$ elastic scattering

extract from **charged pion beams** on **nucleon targets**



physical scattering
has negative t

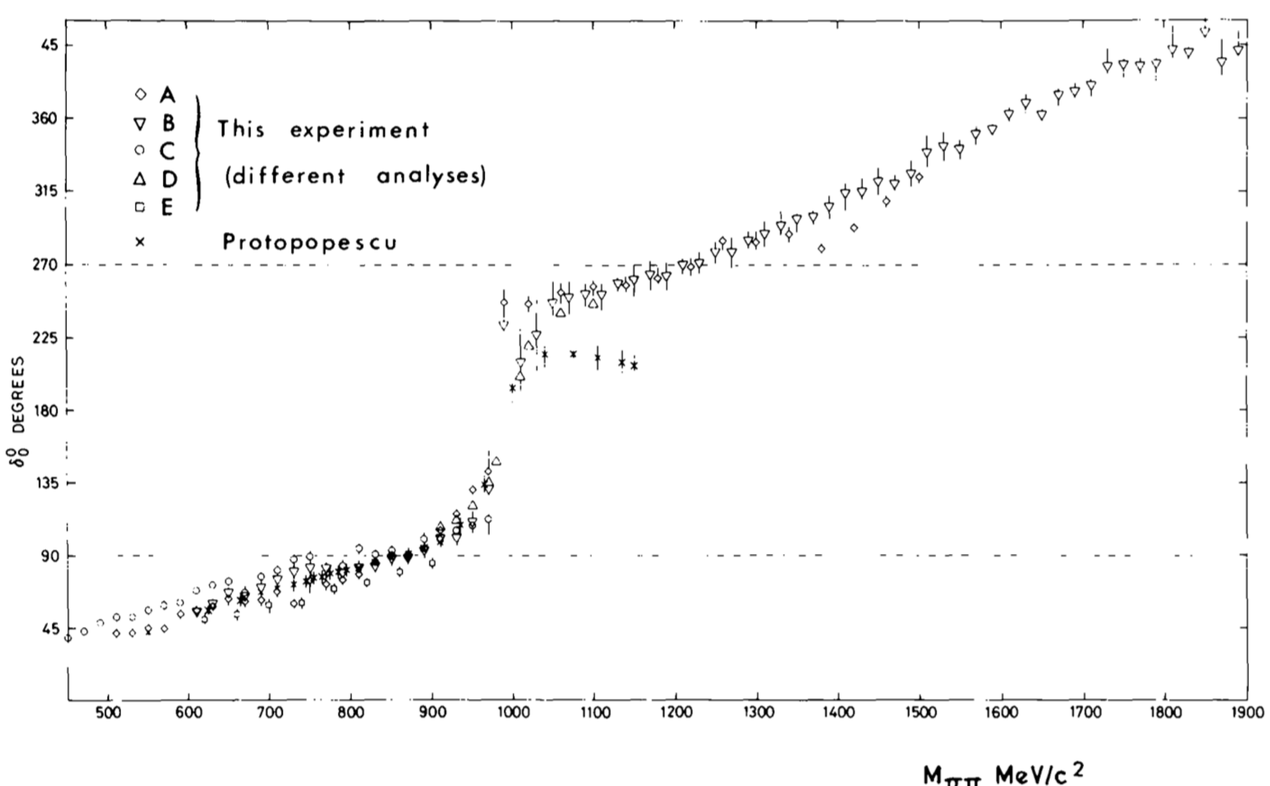


$\pi\pi$ partial-waves
— project $P_\ell(\cos\theta_{\pi\pi})$

on-shell π exchange
— extrapolate to $t = m_\pi^2$

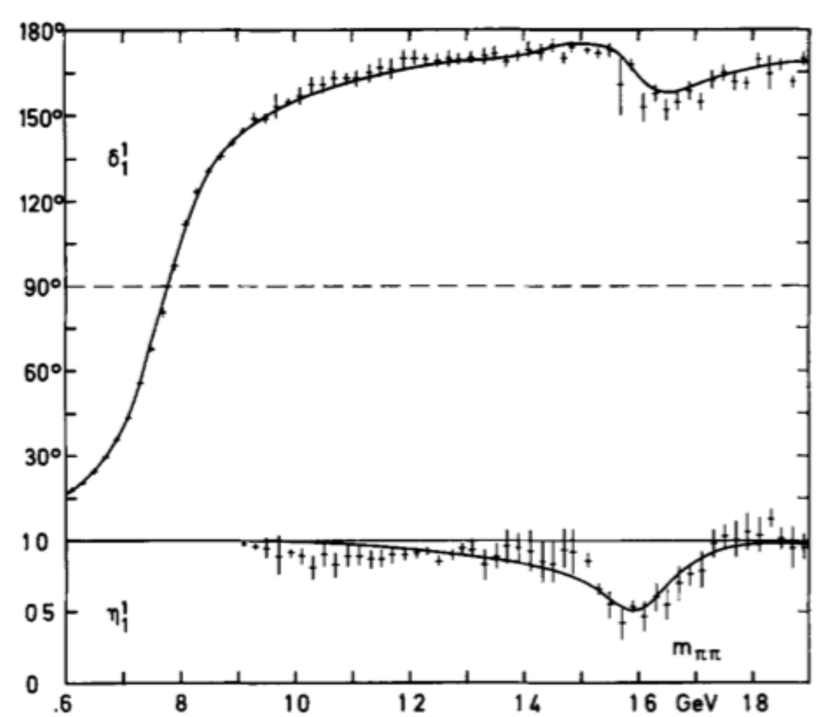
this bit is somewhat
model dependent

isospin=0



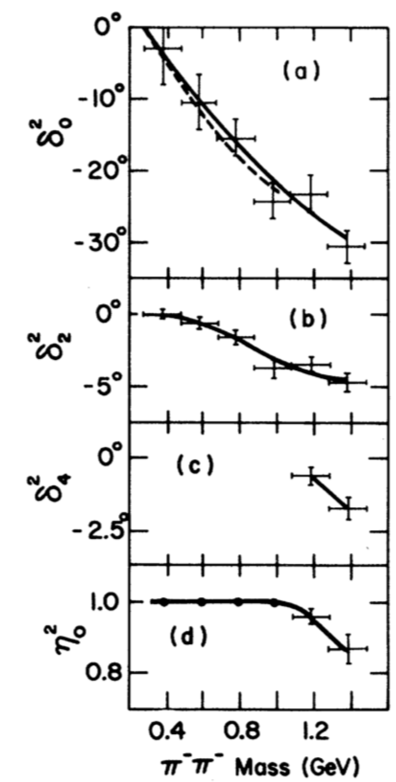
Grayer 1974

isospin=1



Hyams 1973

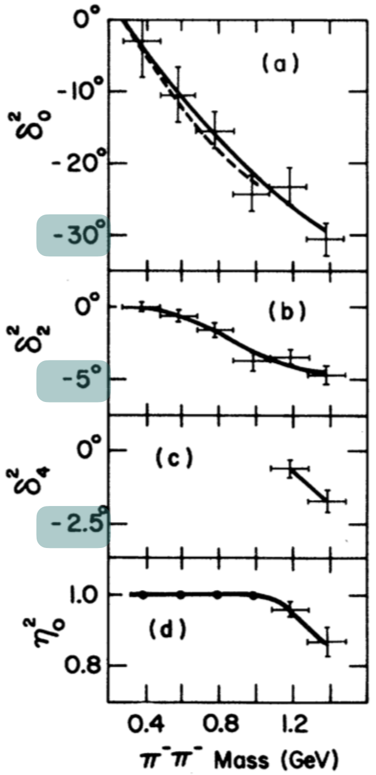
isospin=2



Cohen 1972

the "simplest" case: $\pi\pi$ elastic scattering

isospin=2



ℓ=0, *a.k.a* "*S*-wave"

ℓ=2, *a.k.a* "*D*-wave"

ℓ=4, *a.k.a* "*G*-wave"

inelasticity

— indicates other final states accessible

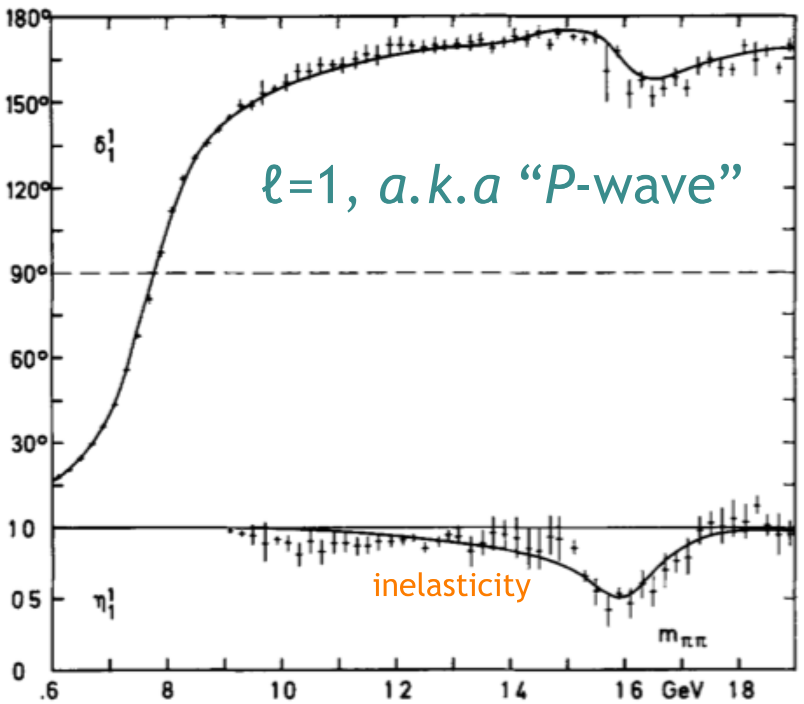
$$t_\ell(E) = \frac{\eta_\ell(E) e^{2i\delta_\ell(E)} - 1}{2i\rho(E)}$$

isospin=2 phase-shift is **small** and **negative**

at low energies $|\delta_0| \gg |\delta_2| \gg |\delta_4| \ldots$

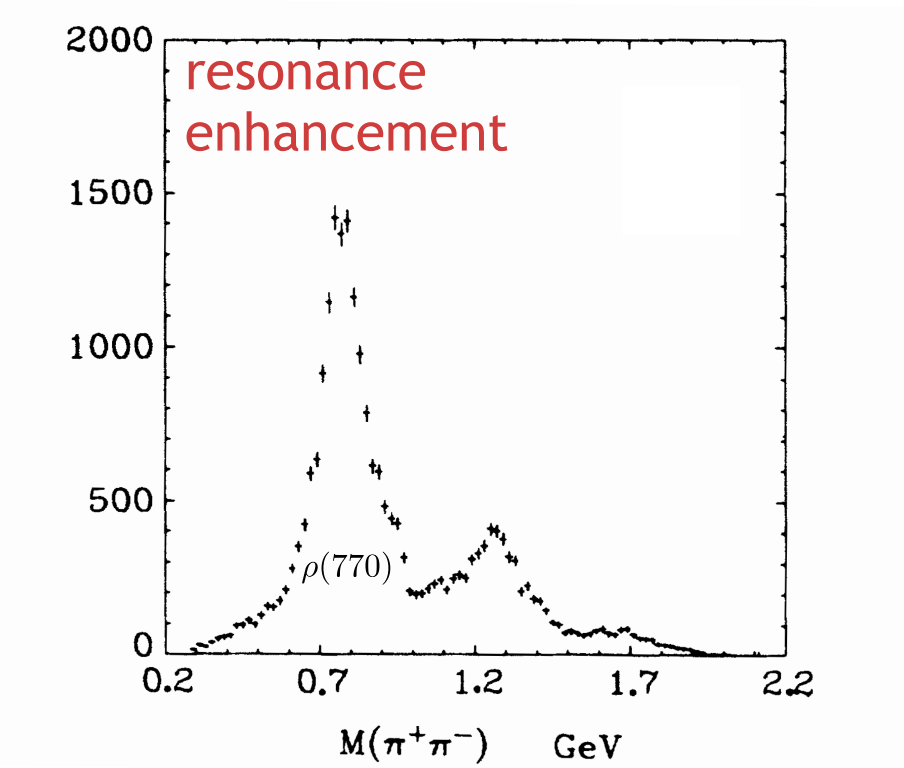
the “simplest” case: $\pi\pi$ elastic scattering

isospin=1



isospin=1 phase-shift **rises rapidly through 90°** near 770 MeV

gives rise to a ‘bump’ in the cross-section



this is the famous ρ resonance

ρ(770)

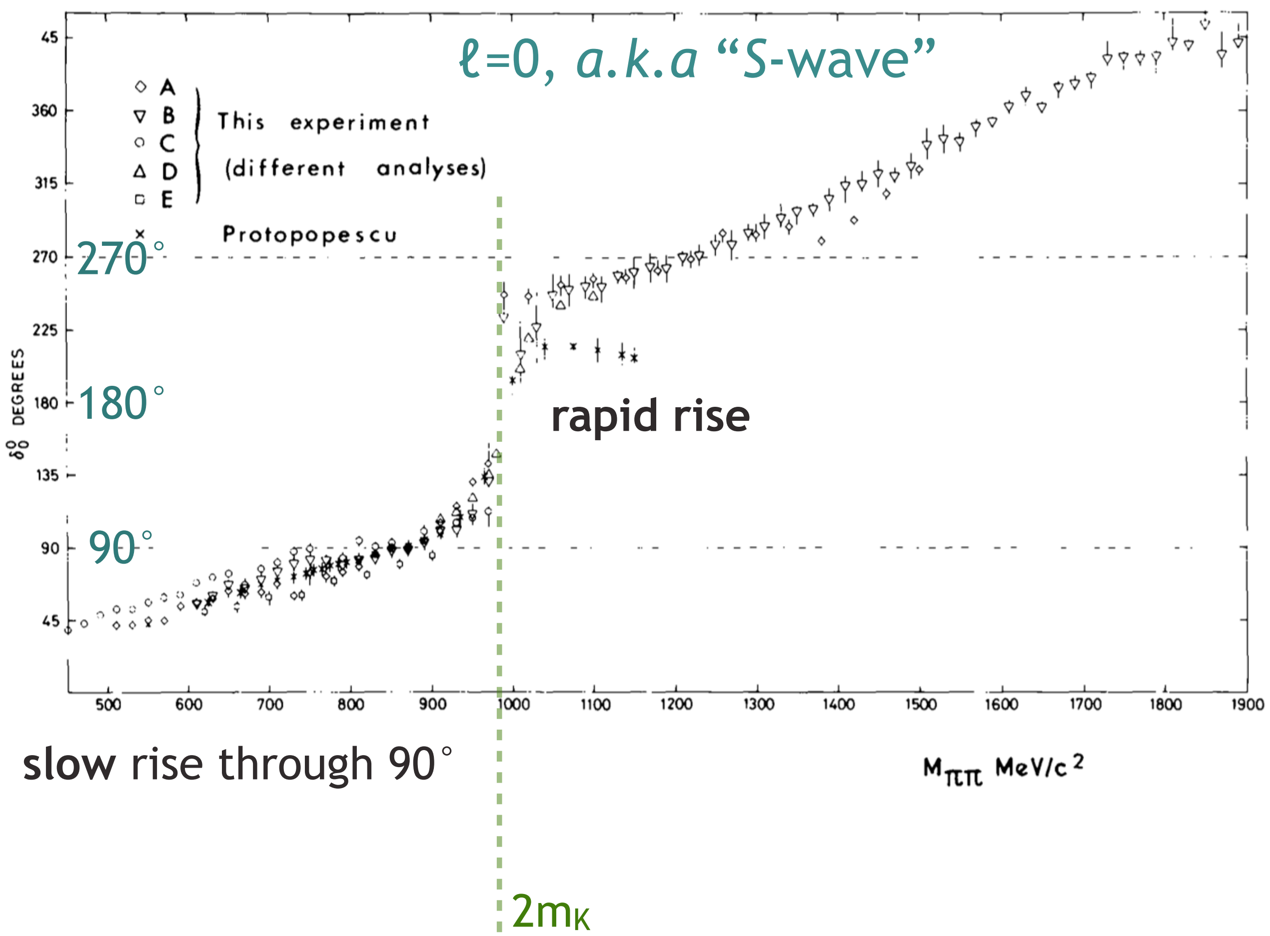
$I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV

ρ(770) DECAY MODES	Fraction (Γ_i/Γ)		p (MeV/c)
$\pi\pi$	$\sim$ 100	%	363

the "simplest" case: $\pi\pi$ elastic scattering

isospin=0



interpretation of isospin=0 phase-shift is **interesting** !

will come back to it later …

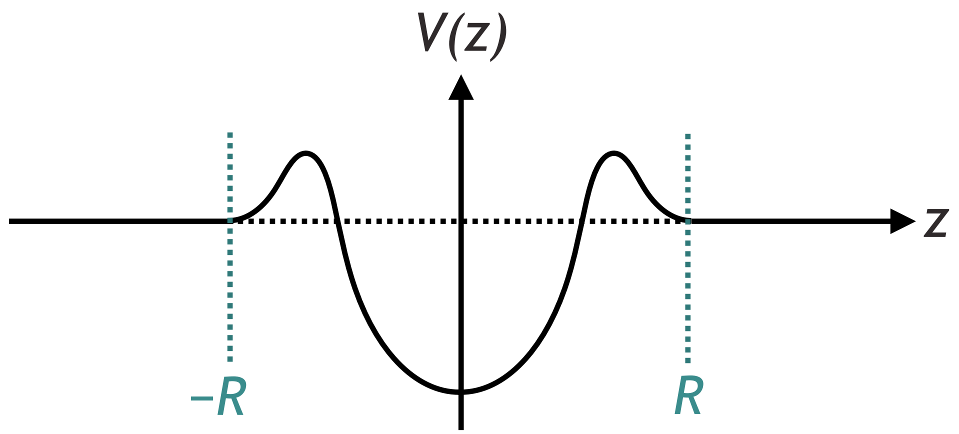
this $\pi\pi$ system is **relativistic** scattering of **composite particles**

— we are going to need **quantum field theory**

— but first let's orient ourselves with good old fashioned quantum mechanics …

most easily illustrated considering **one-dimensional non-relativistic quantum mechanics**

imagine two identical bosons separated by a distance *z* interacting through a finite-range potential *V(z)*



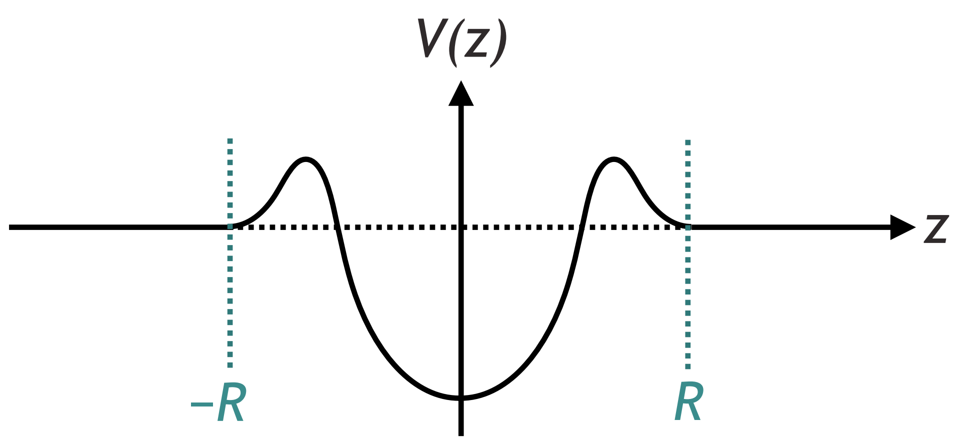
solve the Schrödinger equation

$$-\frac{1}{m}\frac{d^2\psi}{dz^2} + V(z)\psi(z) = E\psi(z)$$

scattering

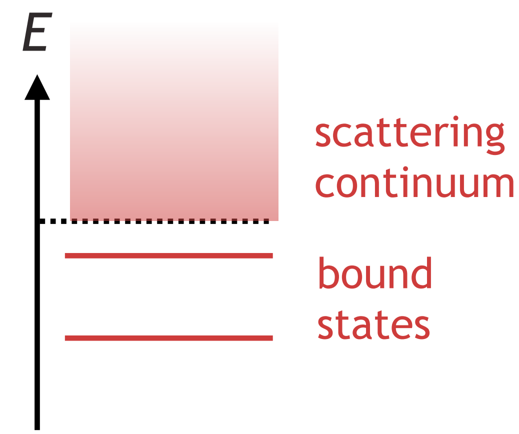
most easily illustrated considering one-dimensional non-relativistic quantum mechanics

imagine two identical bosons separated by a distance *z* interacting through a finite-range potential *V(z)*



solve the Schrödinger equation

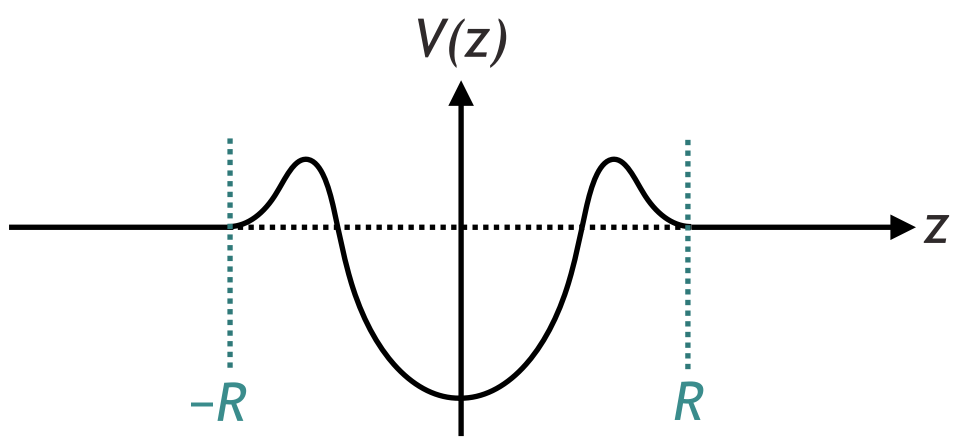
$$-\frac{1}{m}\frac{d^2\psi}{dz^2} + V(z)\psi(z) = E\psi(z)$$



$$\psi(|z| > R) \sim \cos\big(\, p\,|z| + \delta(p)\,\big)$$

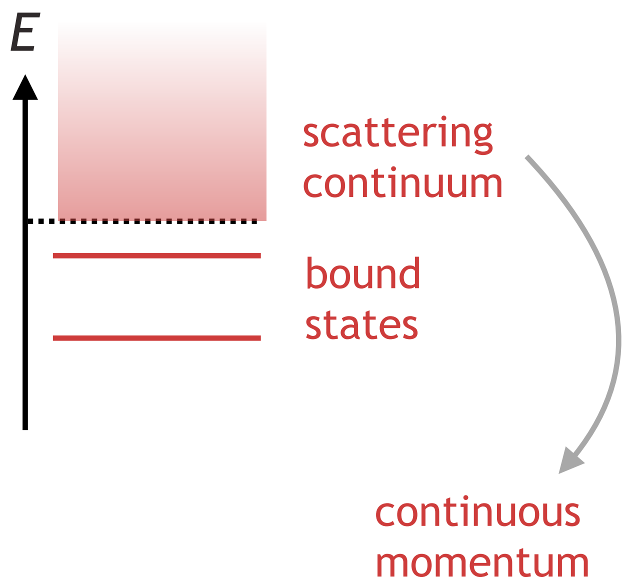
most easily illustrated considering one-dimensional non-relativistic quantum mechanics

imagine two identical bosons separated by a distance z interacting through a finite-range potential *V(z)*



solve the Schrödinger equation

$$-\frac{1}{m}\frac{d^2\psi}{dz^2} + V(z)\psi(z) = E\psi(z)$$

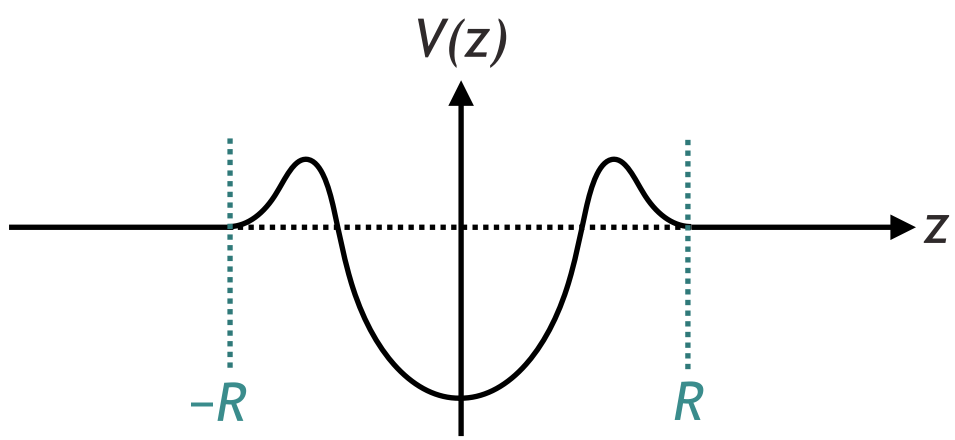


$$\psi(|z| > R) \sim \cos\big(p|z| + \delta(p)\big)$$

scattering

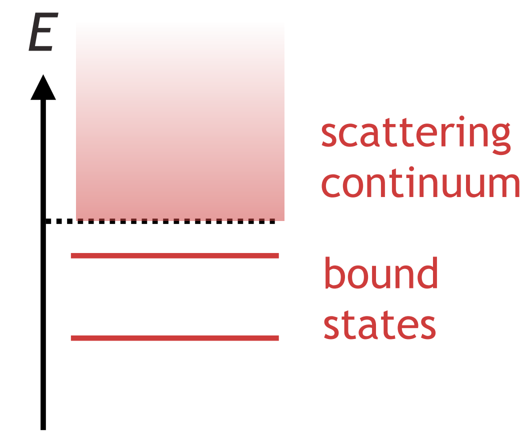
most easily illustrated considering one-dimensional non-relativistic quantum mechanics

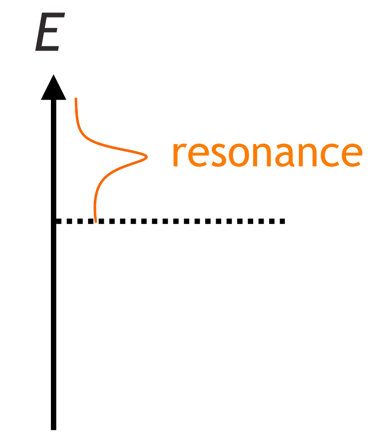
imagine two identical bosons separated by a distance *z* interacting through a finite-range potential *V(z)*



solve the Schrödinger equation

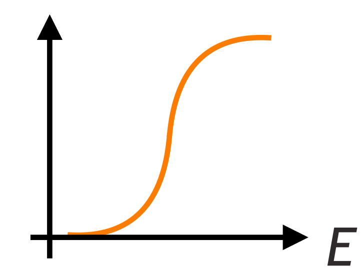
$$-\frac{1}{m}\frac{d^2\psi}{dz^2} + V(z)\psi(z) = E\psi(z)$$





$$\psi(|z| > R) \sim \cos\big(p\,|z| + \boxed{\delta(p)}\big)$$

phase-shift



scattering

$$\psi(|z| > R) \sim \cos\left(p\,|z| + \delta(p)\right)$$

phase-shift

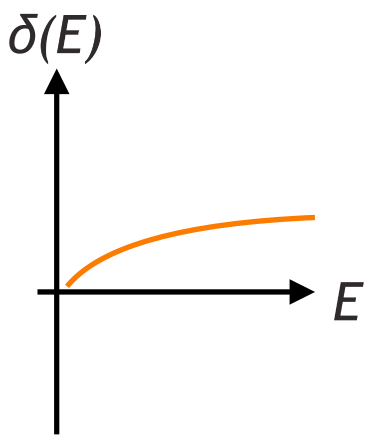
generally, consider S-matrix

$$|\text{in}\rangle = S\,|\text{out}\rangle$$

elastic scattering

$$S(E) = e^{2i\delta(E)}$$

e.g.



'weak' attraction

c.f. $\pi\pi$ isospin=0
at low energy ?

scattering

$$\psi(|z| > R) \sim \cos\left(p\,|z| + \delta(p)\right)$$

phase-shift

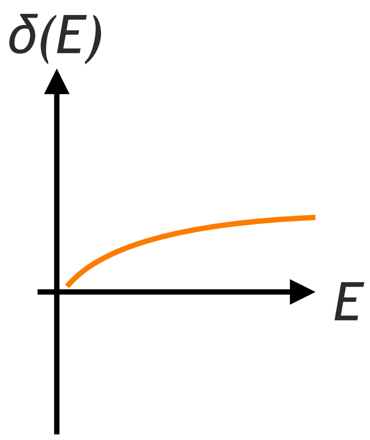
generally, consider S-matrix

$$|\mathrm{in}\rangle = S\,|\mathrm{out}\rangle$$

elastic scattering

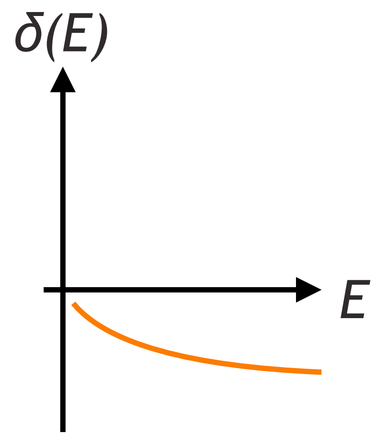
$$S(E) = e^{2i\delta(E)}$$

e.g.



'weak' attraction

c.f. $\pi\pi$ isospin=0
at low energy ?



'weak' repulsion

c.f. $\pi\pi$ isospin=2 ?

scattering

$$\psi(|z| > R) \sim \cos\left(p\,|z| + \delta(p)\right)$$

phase-shift

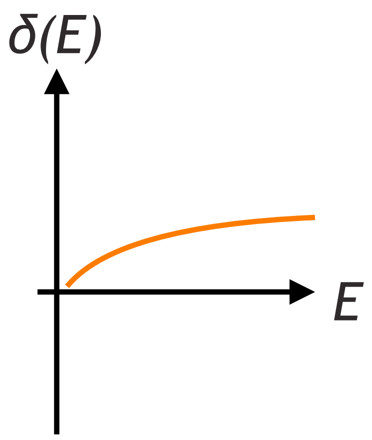
generally, consider S-matrix

$$\left|\text{in}\right\rangle = S\left|\text{out}\right\rangle$$

elastic scattering

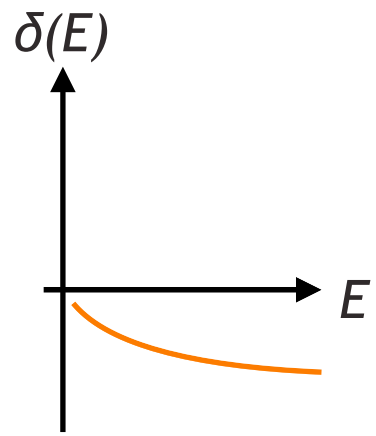
$$S(E) = e^{2i\delta(E)}$$

e.g.



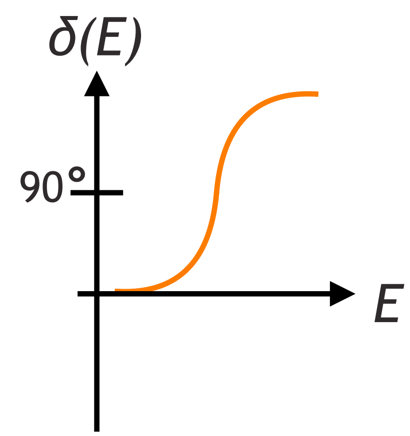
'weak' attraction

c.f. $\pi\pi$ isospin=0 at low energy ?



'weak' repulsion

c.f. $\pi\pi$ isospin=2 ?



resonance

c.f. $\pi\pi$ isospin=1 ?

scattering

$$\psi(|z| > R) \sim \cos\left(p\,|z| + \delta(p)\right)$$

phase-shift

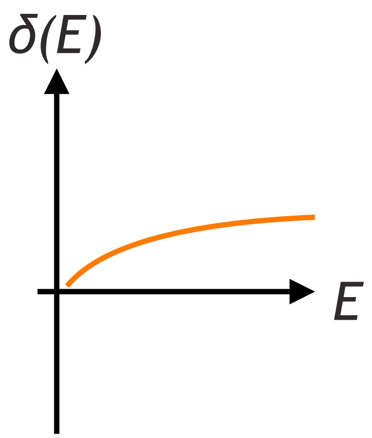
generally, consider S-matrix

$$|\mathrm{in}\rangle = S\,|\mathrm{out}\rangle$$

elastic scattering

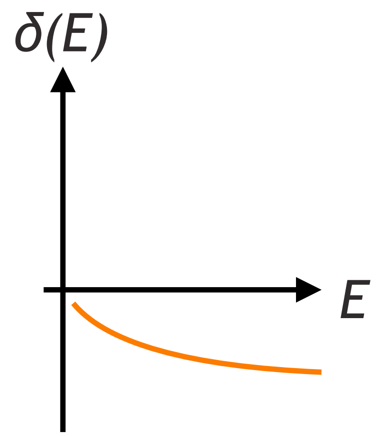
$$S(E) = e^{2i\delta(E)}$$

e.g.



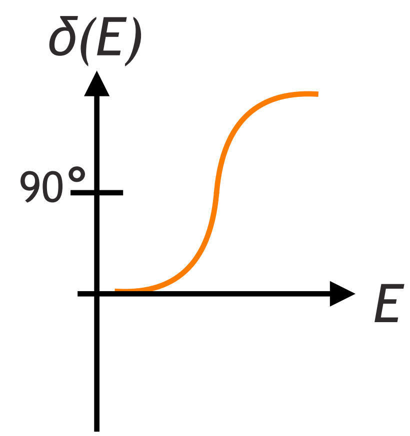
'weak' attraction

c.f. $\pi\pi$ isospin=0
at low energy ?



'weak' repulsion

c.f. $\pi\pi$ isospin=2 ?



resonance

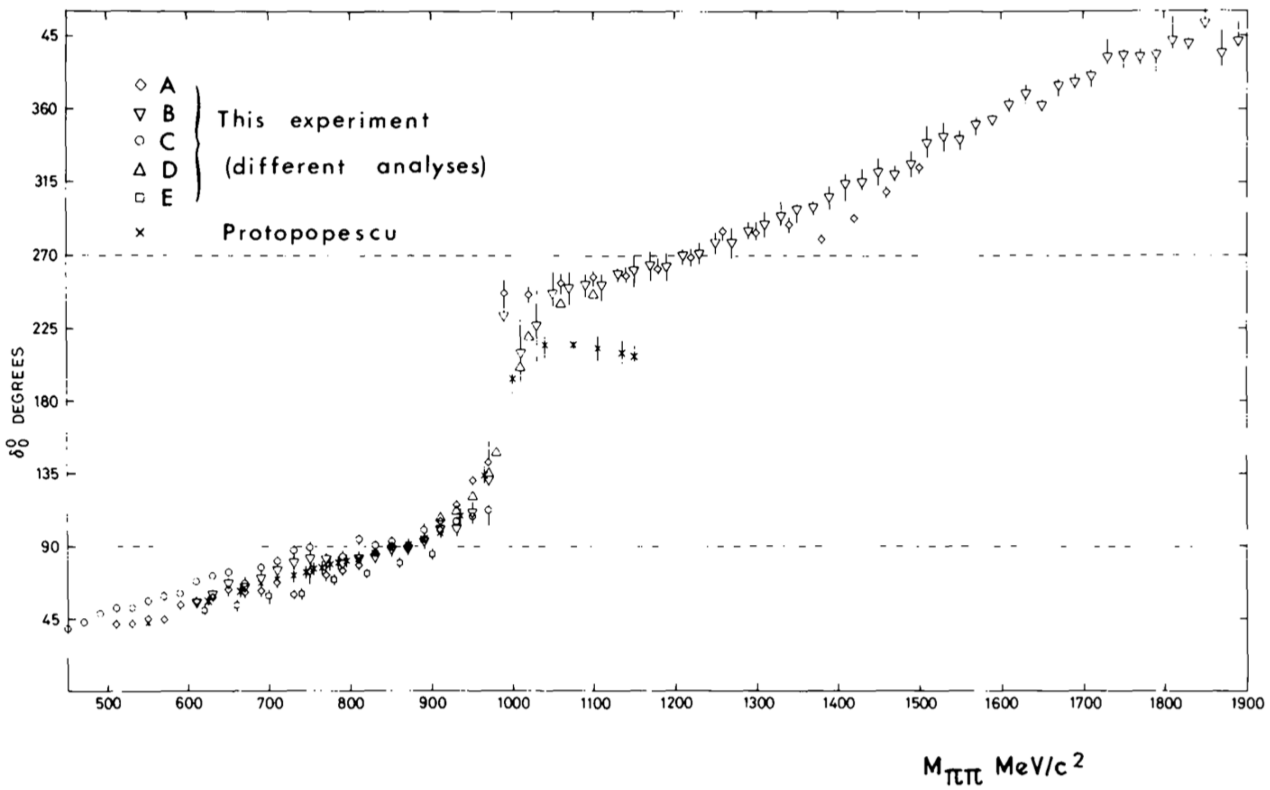
c.f. $\pi\pi$ isospin=1 ?

scattering in three-dimensions:

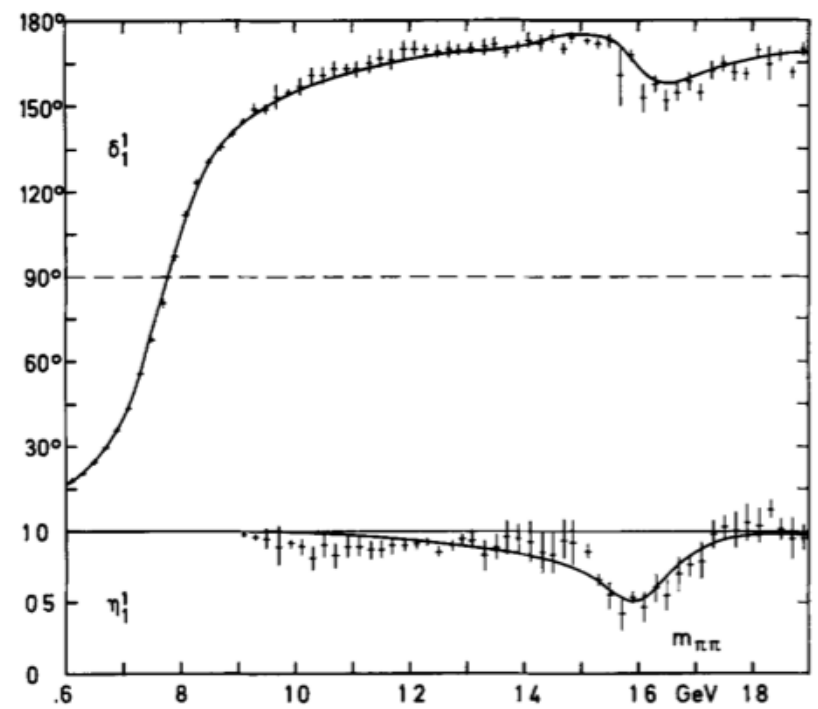
brings in the concept of the angular momentum 'barrier' $\sim \dfrac{\ell(\ell+1)}{r^2}$ which causes $\delta_\ell(p) \sim p^{2\ell+1}$

suppresses higher
partial-waves
at low energies

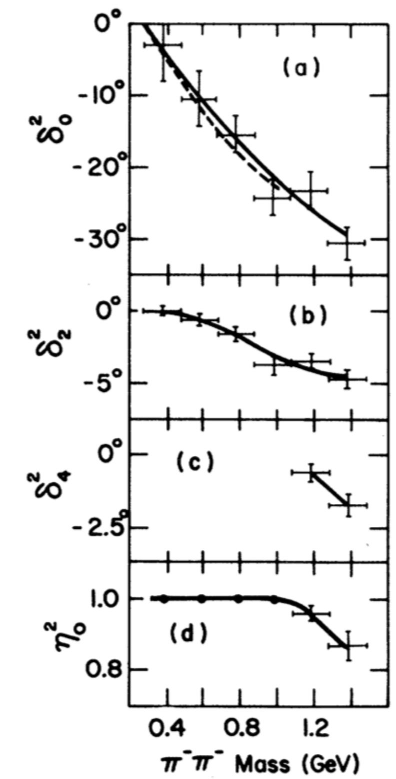
isospin=0



isospin=1



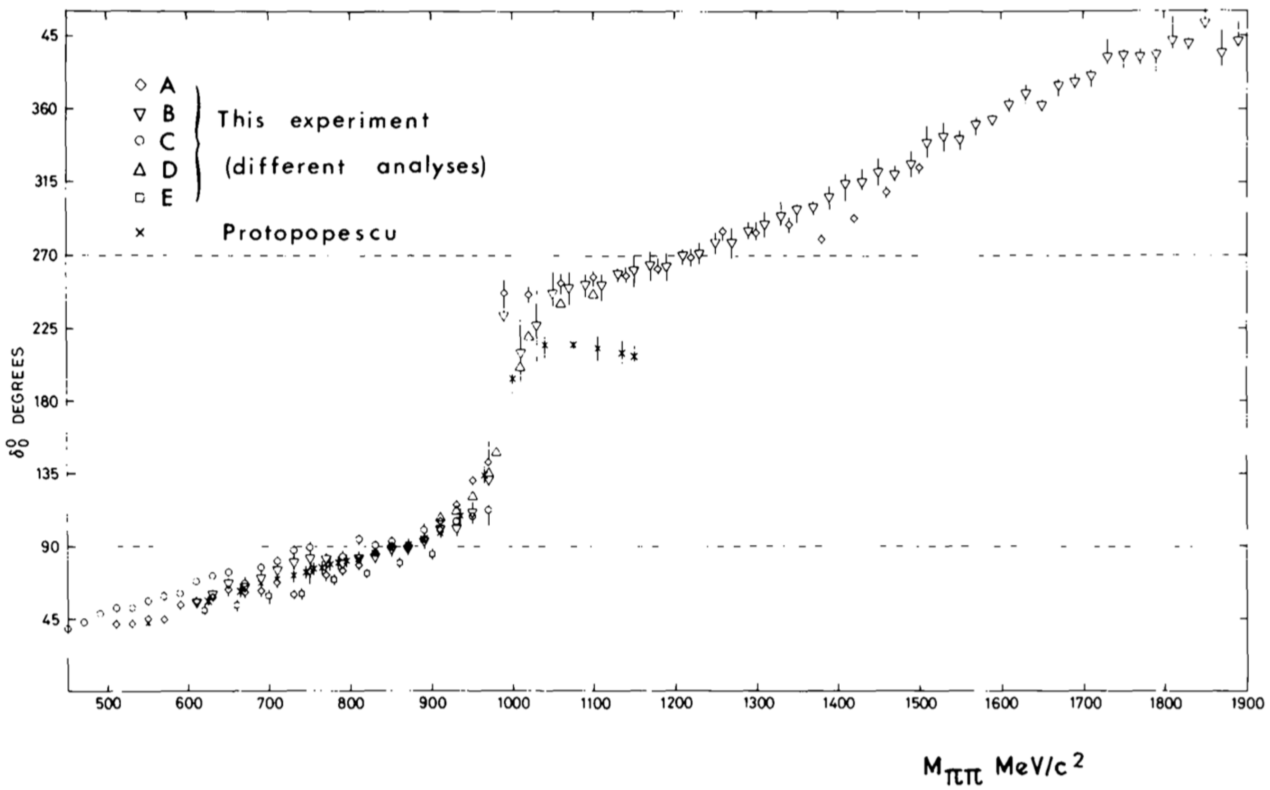
isospin=2



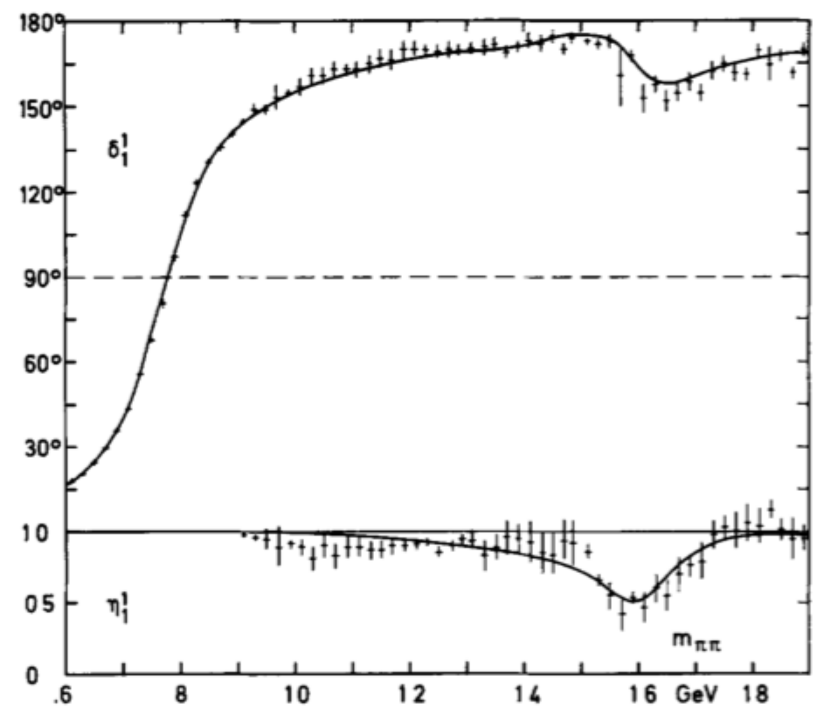
the “simplest” case: $\pi\pi$ elastic scattering

contribution to origin of the ‘quark model’ for mesons: “mesons are $q\bar{q}$ bound-states”

isospin=0



isospin=1

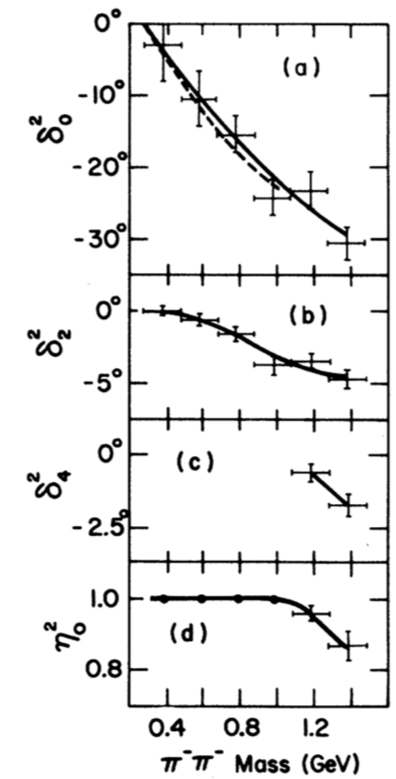


narrow resonance

can make isospin=1 from $q\bar{q}$

$u\bar{d}, d\bar{u}, u\bar{u} - d\bar{d}$

isospin=2



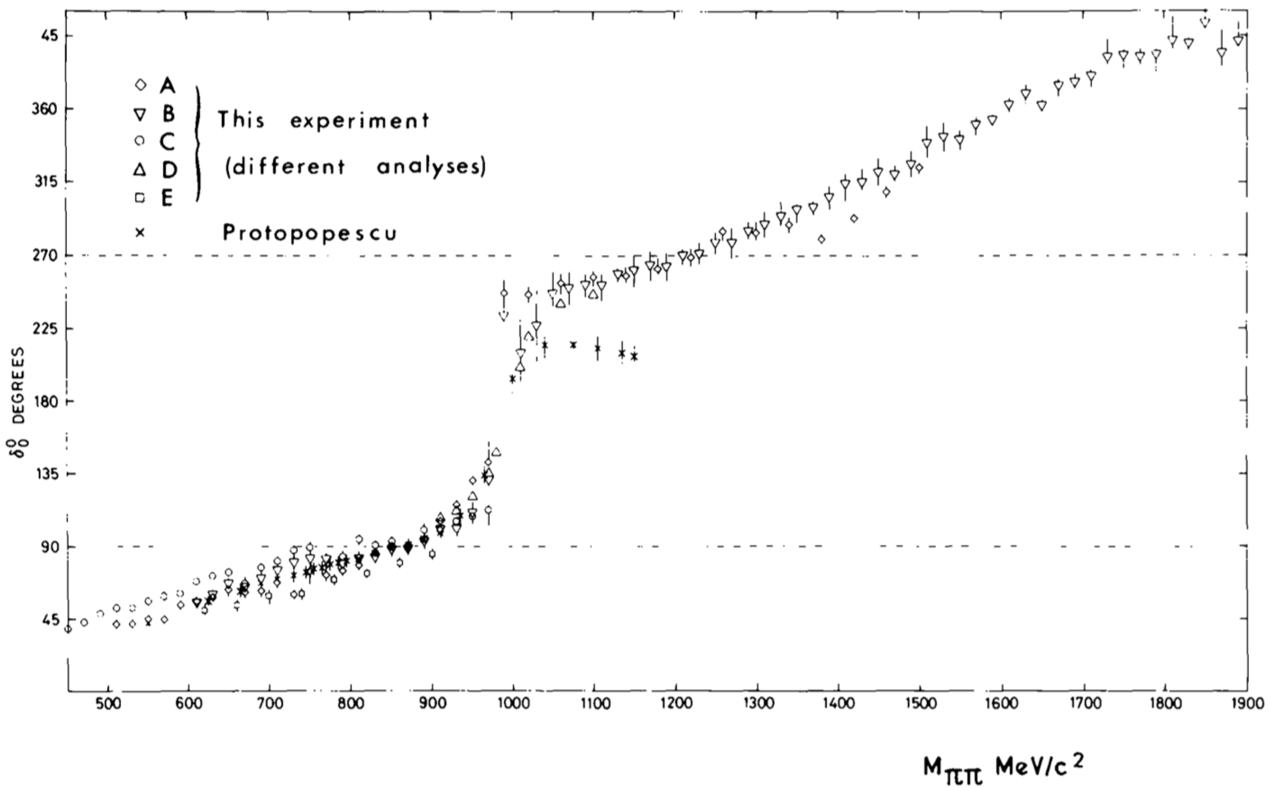
no resonance

cannot make isospin=2 from $q\bar{q}$

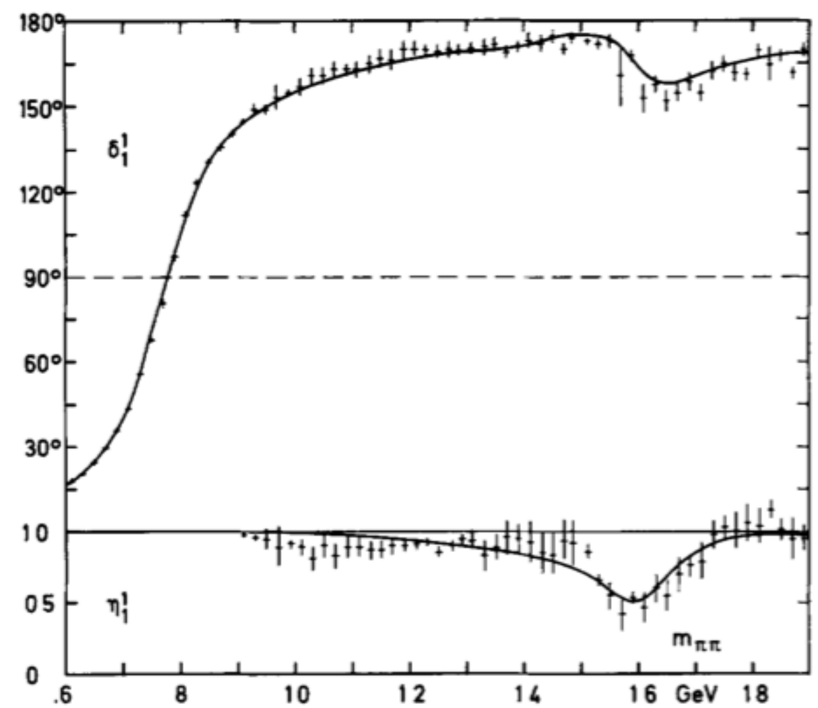
the “simplest” case: $\pi\pi$ elastic scattering

contribution to origin of the ‘quark model’ for mesons: “mesons are $q\bar{q}$ bound-states”

isospin=0

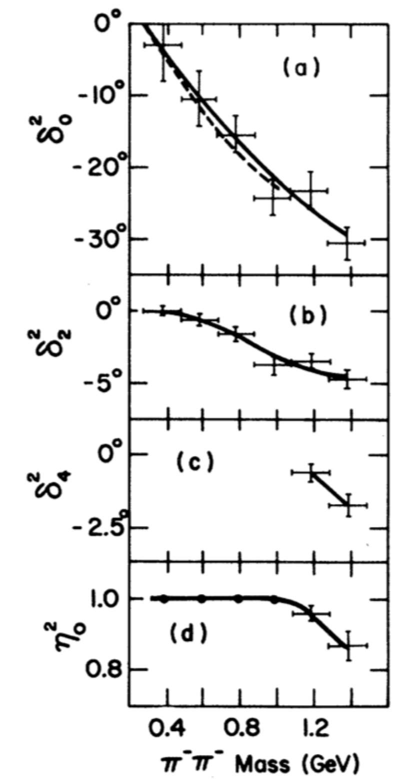


isospin=1



narrow resonance

isospin=2



no resonance

can make isospin=0 from $q\bar{q}$

$u\bar{u} + d\bar{d}, s\bar{s}$

can make isospin=1 from $q\bar{q}$

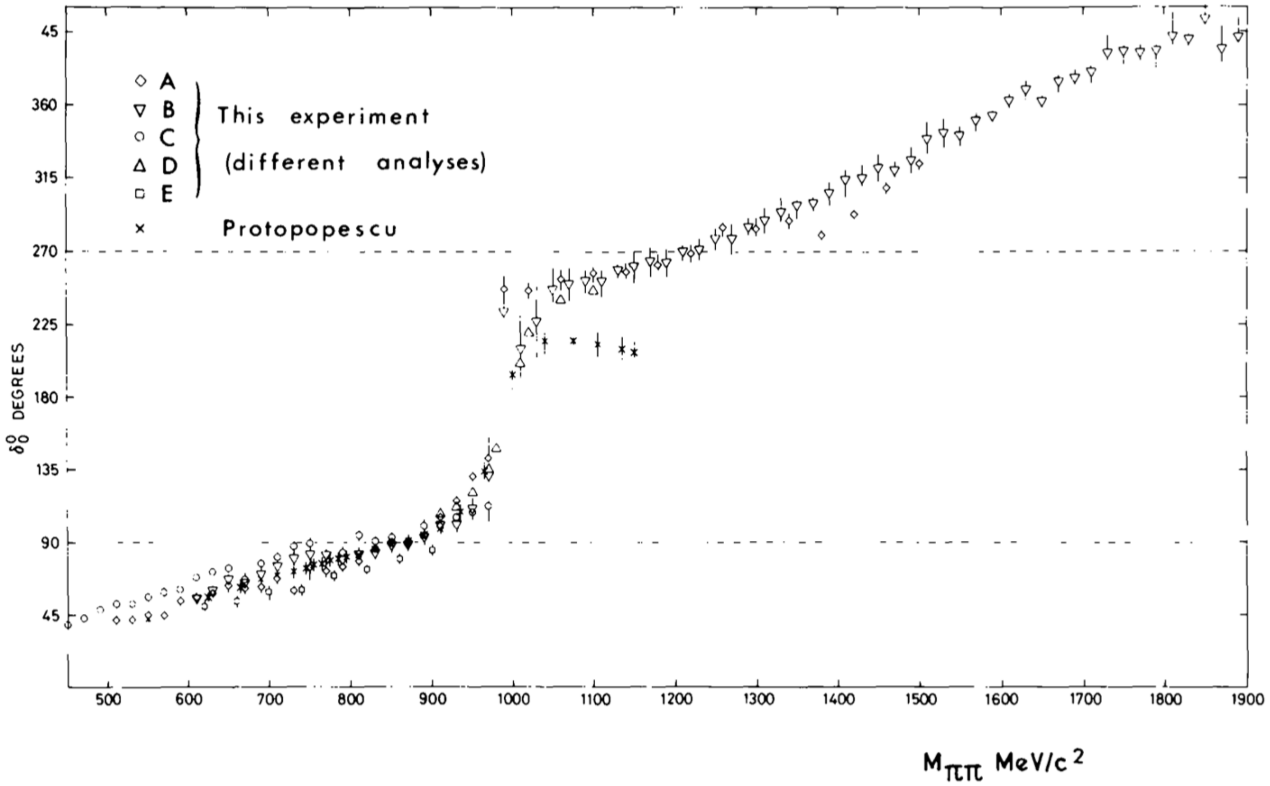
$u\bar{d}, d\bar{u}, u\bar{u} - d\bar{d}$

cannot make isospin=2 from $q\bar{q}$

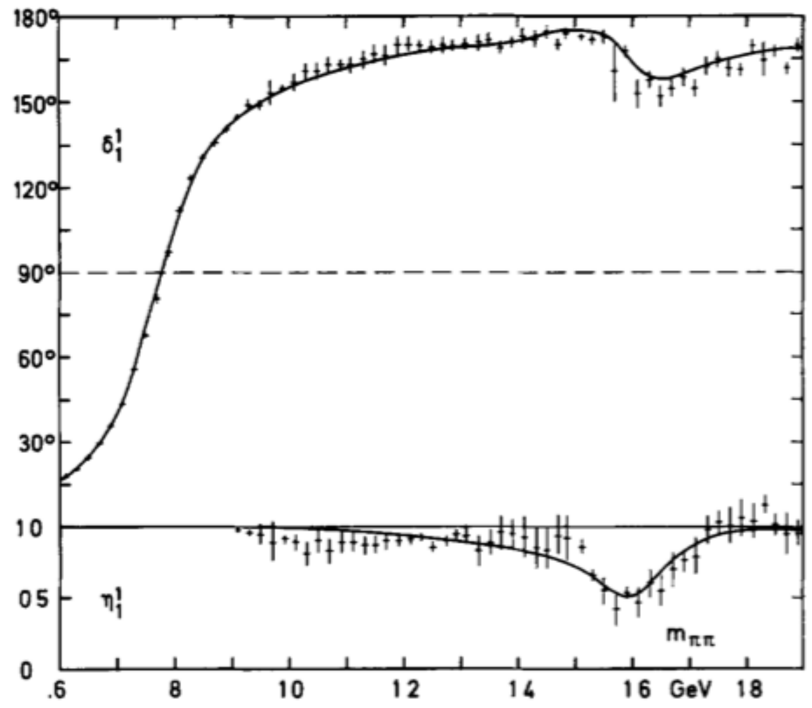
but can this weird scattering amplitude
be explained that way ?

the “simplest” case: $\pi\pi$ elastic scattering

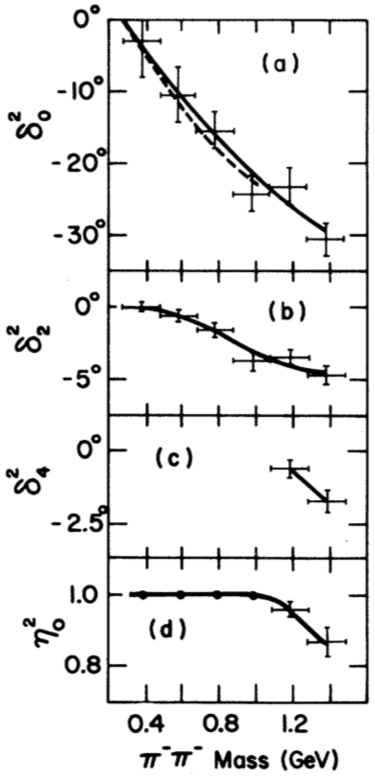
isospin=0



isospin=1



isospin=2



a first target: can a **first-principles QCD** calculation lead to these kinds of behaviour ?

a next target: can we understand these behaviours in terms of **resonances** ?

an ultimate target: can we understand the **quark-gluon make-up** of these resonances ?