Andrea Simonelli

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Introduction to QuantumChromoDynamics

INFN Istituto Nazionale di Fisica Nucleare Sezione di Roma

Road to QCD

What is inside a proton?

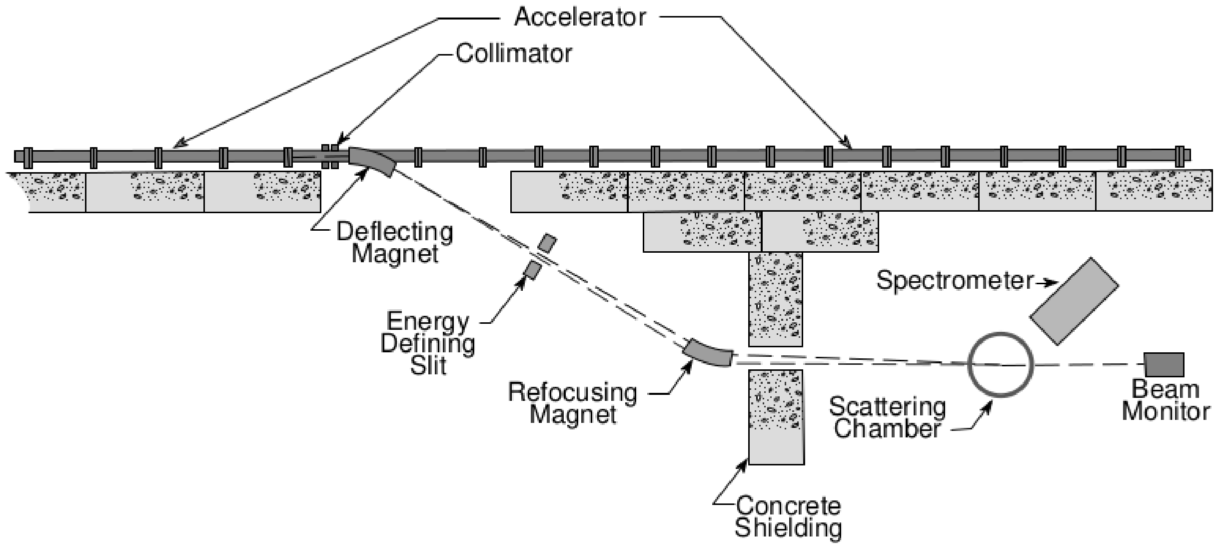
Why matter is colored?

What makes *this* theory the right one?

Structure of the Proton

This journey begins already in the 1930s with a fundamental question:

Is the proton a point-like particle, or does it have an internal structure?



Experimental set-up for electron scattering experiments at the Mark III accelerator at Stanford in the early 1950s.
[The discovery of the point–like structure of matter, R.E. Taylor (SLAC)]

NOVEMBER 1, 1936 PHYSICAL REVIEW VOLUME 50

The Scattering of Protons by Protons*

M. A. Tuve, N. P. Heydenburg and L. R. Hafstad, *Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C.*

(Received August 11, 1936)

It was known that p-p scattering did not follow the **Mott scattering formula**:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)}\left[1 + 2\tau \tan^2(\theta/2)\right]$$

$$\frac{Q^2}{4M^2}$$

Which is valid for a relativistic electron scattering against a point-like spin ½ proton.

But how can we test how far the proton is from being point-like?

Structure of the Proton

The generalization to include the possibility of a non-trivial structure is the **Rosenbluth formula**:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2(\theta/2) \right]$$

Electric form factor **Magnetic** form factor

If the proton is point-like...

$$G_E(Q^2) \equiv G_E(0) = 1$$
$$G_M(Q^2) \equiv G_M(0) = \mu_P \approx 2.79$$

...but instead (dipole structure):

$$G_E(Q^2) \approx \frac{G_M(Q^2)}{\mu_P} \approx \frac{1}{(1+Q^2/a^2)^2}$$

The proton is spread out over a small region:

$$\rho(r) \approx \rho_0 e^{-r/(0.24\ \text{fm})}$$

Phys.Rev.C 64 (2001) 038202

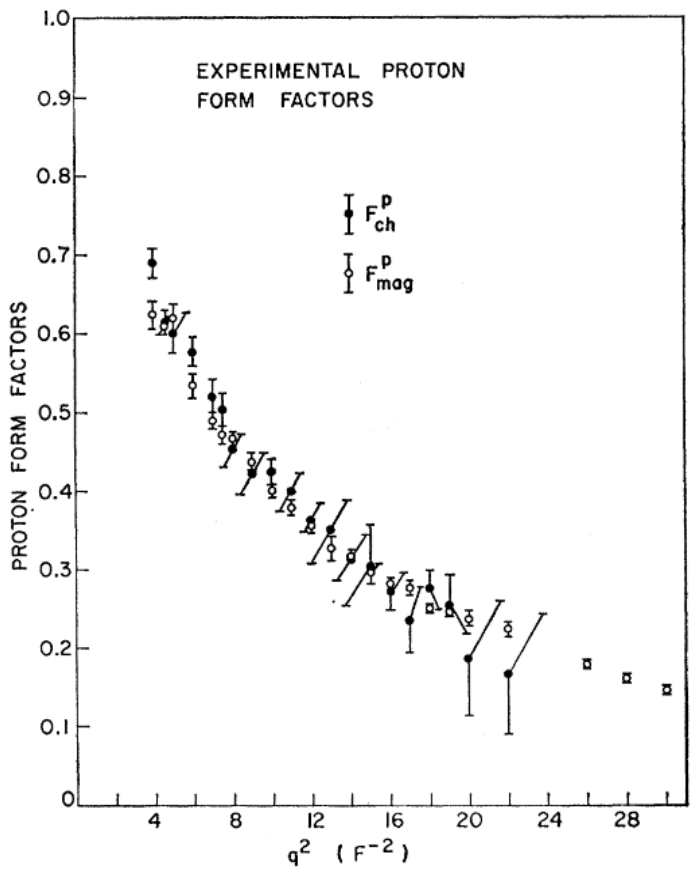


FIG. 5. The experimental values found for the charge and magnetic form factors of the proton as a function of q^2.

Structure of the Proton

The proton clearly is not point-like. But how do we determine its *inner* structure?

We have to break it!

Elastic scattering

Only 1 variable needed:

$$Q^2 = 4EE' \sin^2(\theta/2)$$

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2(\theta/2)}$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2 y^2}{Q^2}\right) F_2^{\text{elastic}}(Q^2) + y^2 F_1^{\text{elastic}}(Q^2)\right]$$

$$y = 1 - \frac{E'}{E}$$

Inelastic scattering

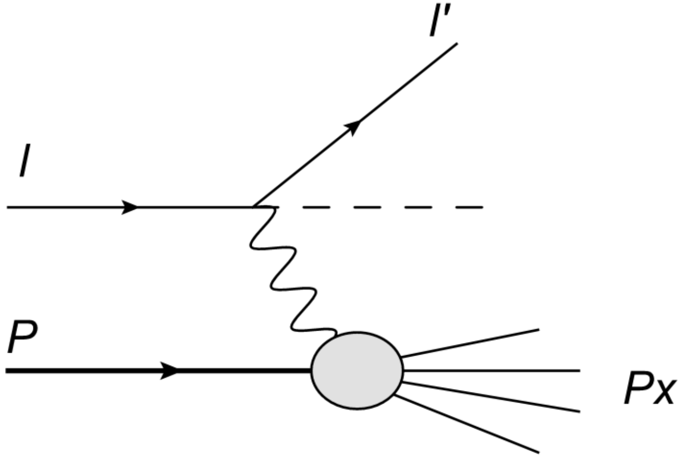
2 variables needed:

$$Q^2 = 4EE' \sin^2(\theta/2)$$

$$E' = \frac{E - \frac{W^2 - M^2}{2M}}{1 + \frac{2E}{M}\sin^2(\theta/2)}$$

$$\frac{d\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2 y^2}{Q^2}\right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right]$$

$$x = \frac{1}{1 + \frac{W^2 - M^2}{Q^2}}$$



Structure of the Proton

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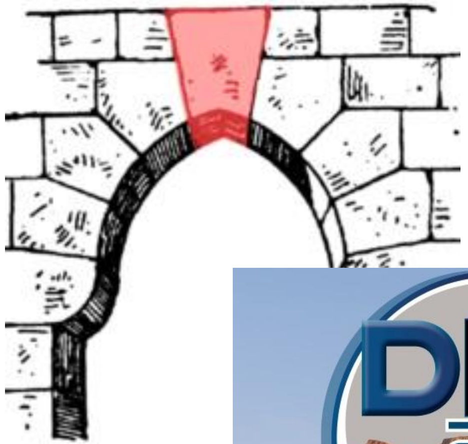
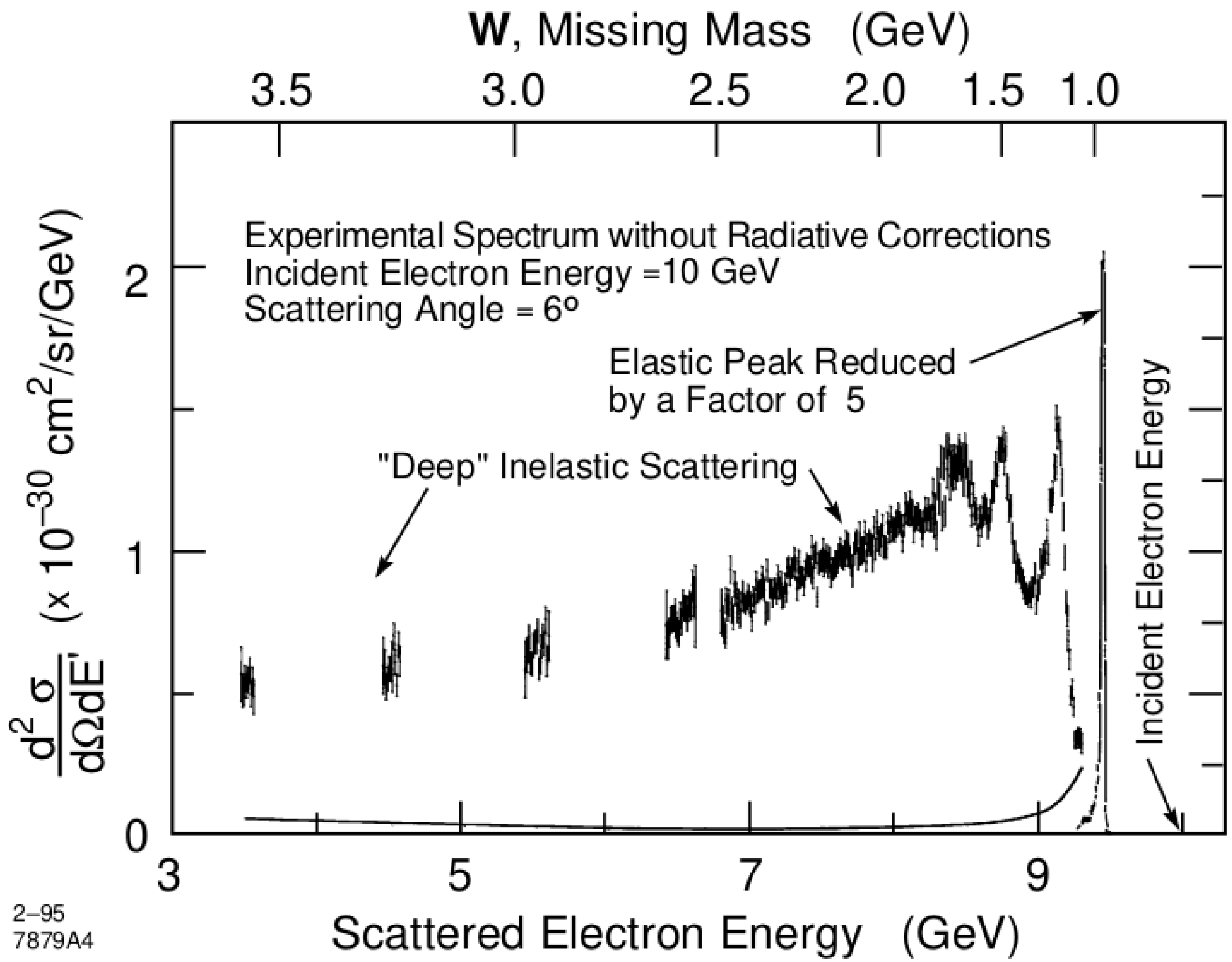
Deep Inelastic Scattering (DIS)



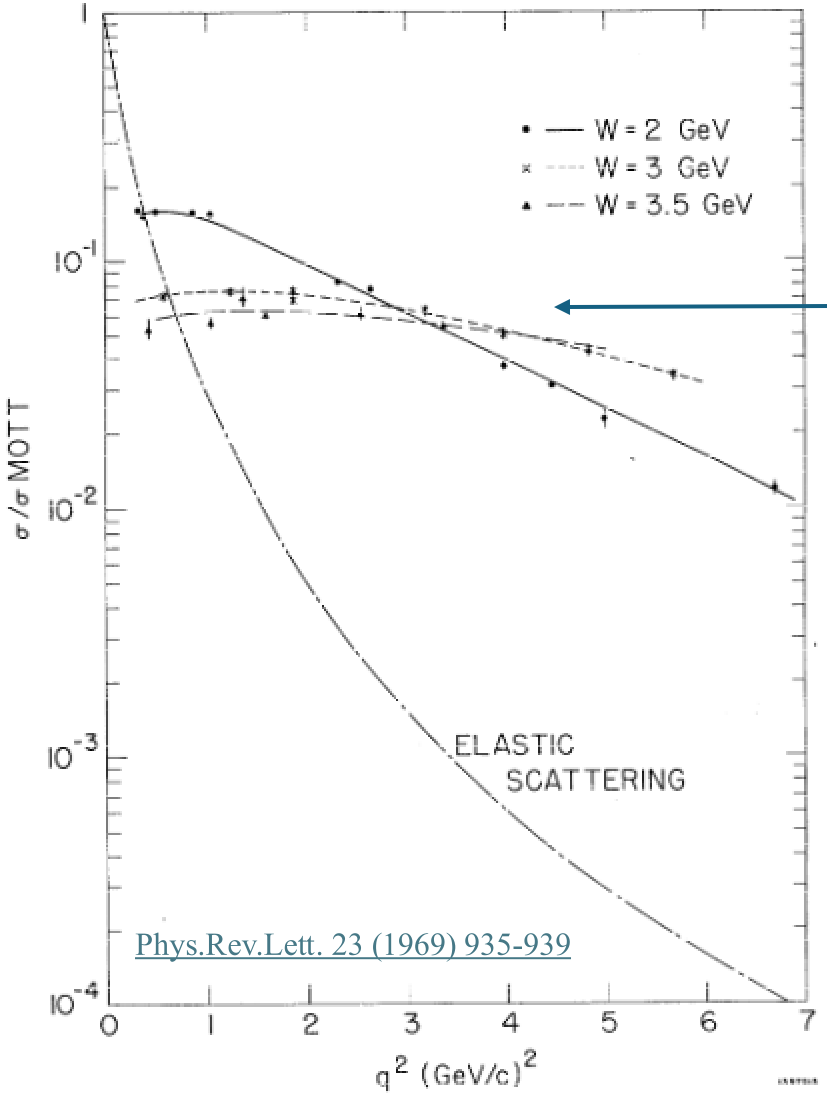
The limit in which the proton gets *really* smashed!



Experimental set-up for electron scattering experiments at the Mark III accelerator at Stanford in the early 1950s.

[The discovery of the point–like structure of matter, R.E. Taylor (SLAC)]

Structure of the Proton



[Phys.Rev.Lett. 23 (1969) 935-939]

In the DIS regime, the cross section depends *weakly* on Q^2

How is it possible?

It seems that the proton is constituted by elementary (point-like) constituents

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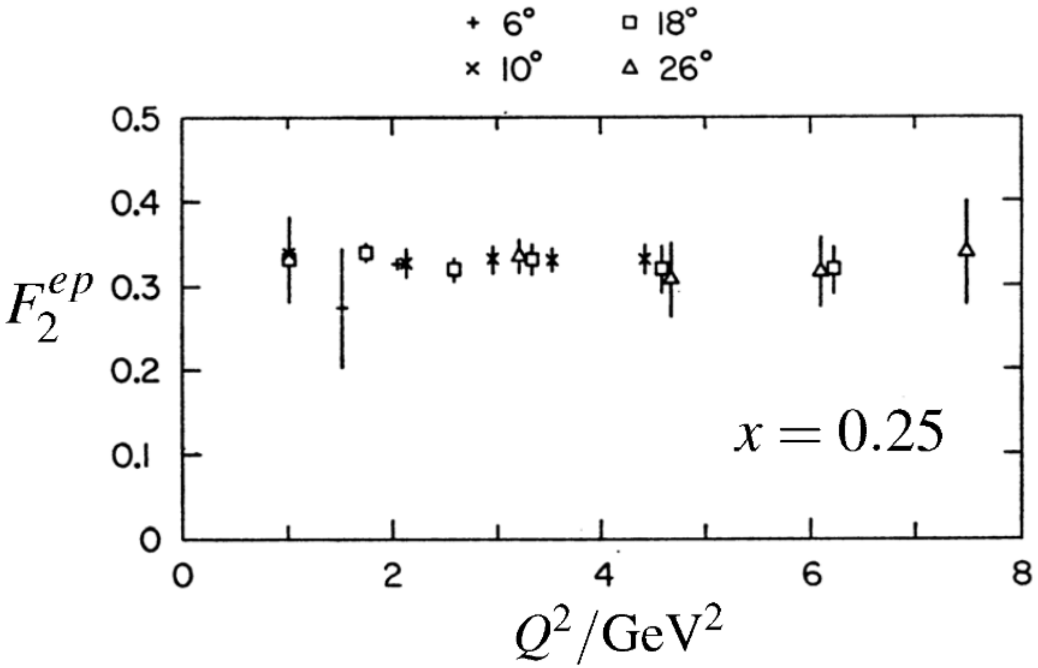
VERY HIGH-ENERGY COLLISIONS OF HADRONS

Richard P. Feynman
California Institute of Technology, Pasadena, California

(Received 20 October 1969)

Proposals are made predicting the character of longitudinal-momentum distributions in hadron collisions of extreme energies.

Structure of the Proton



Ann.Rev.Nucl.Part.Sci. 22 (1972) 203-254

Bjorken limit: $Q^2 \to \infty, x$ fixed

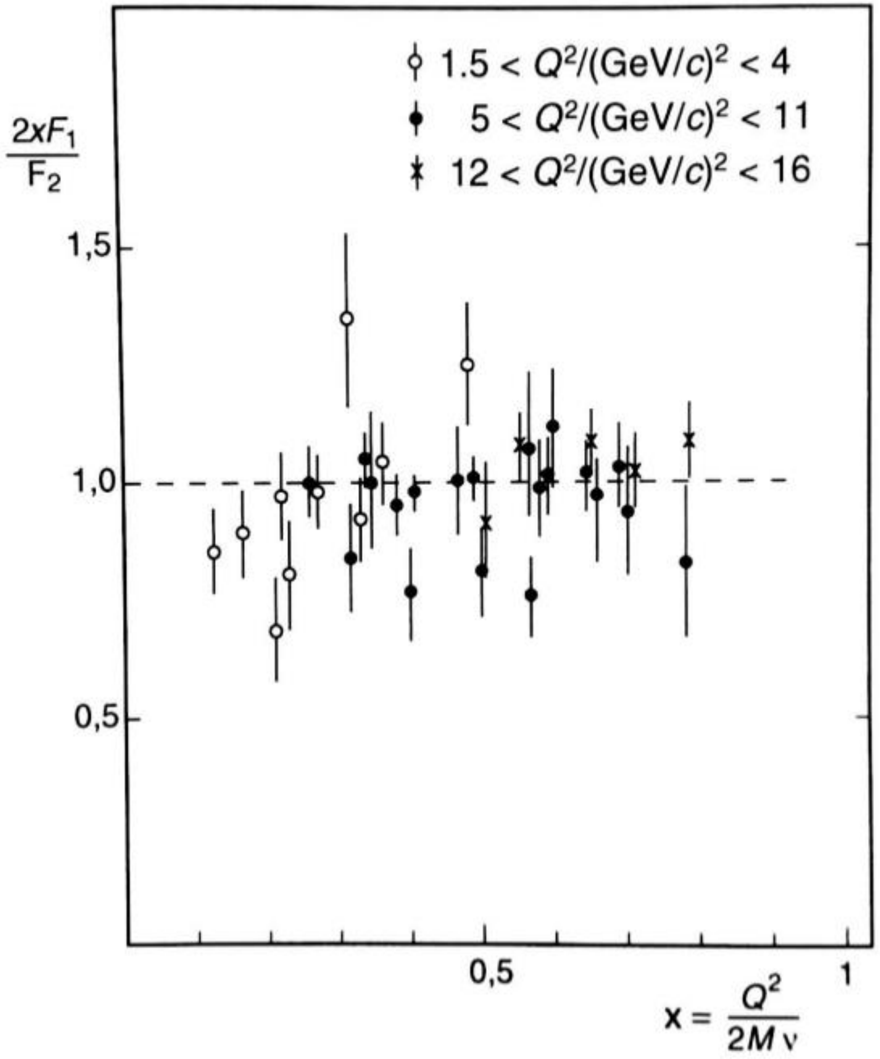
$$F_i(x, Q^2) \to F_i(x)$$

...as in a scattering against point-like particles!

But what are these particles? Why don't we observe them directly, like we do with other elementary particles (for example, electrons)?

For now, let's just refer to them as "**partons**"

Structure of the Proton



$$\frac{d\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

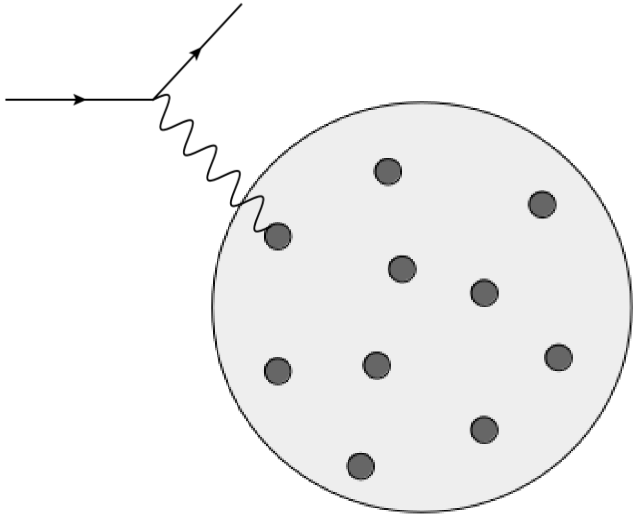
Experimental data give us more insight into partons:

$$F_1 = \begin{cases} \frac{1}{2x}F_2 & \text{if spin } 1/2 \\ 0 & \text{if spin } 0 \end{cases}$$

So, these "partons" are elementary, point-like fermions

Structure of the Proton

Feynman's parton model



Sum over all partons inside the proton

$$\frac{d\sigma}{dQ^2\,dx} = \int_x^1 \frac{d\rho}{\rho} \sum_j C_j(\rho, Q^2)\, f_j(x/\rho)$$

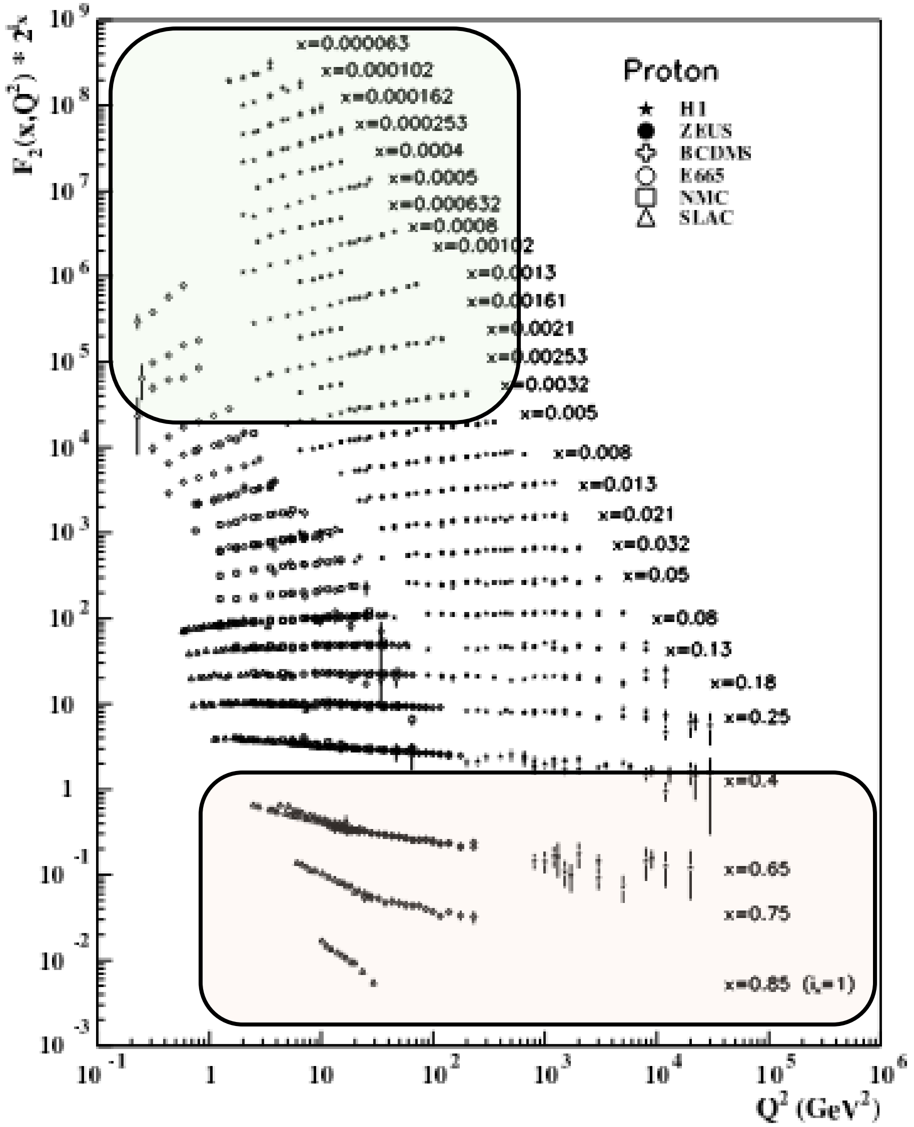
Electromagnetic interaction with a point-like parton

Probability density of finding a point-like parton inside the proton

The scattering off the proton is seen as an incoherent (i.e. independent) sum of elastic scattering processes.

...ok, but what are these partons??

Something is still elusive…scaling violation



The explanation is beyond the parton model.
We need the real QCD to understand such violations

Static quark model

Simple explanation of (that time) hadron spectroscopy, in terms of a symmetry $SU(3)_{\text{flavor}}$

THE EIGHTFOLD WAY:
A THEORY OF STRONG INTERACTION SYMMETRY

MURRAY GELL-MANN

MARCH 15, 1961

Derivation of strong interactions from a gauge invariance

Yuval Ne'eman (Imperial Coll., London)

Feb, 1961

Why 3? Because all hadrons could be classified as multiplets generated by a fundamental triplet of aces, or quarks:

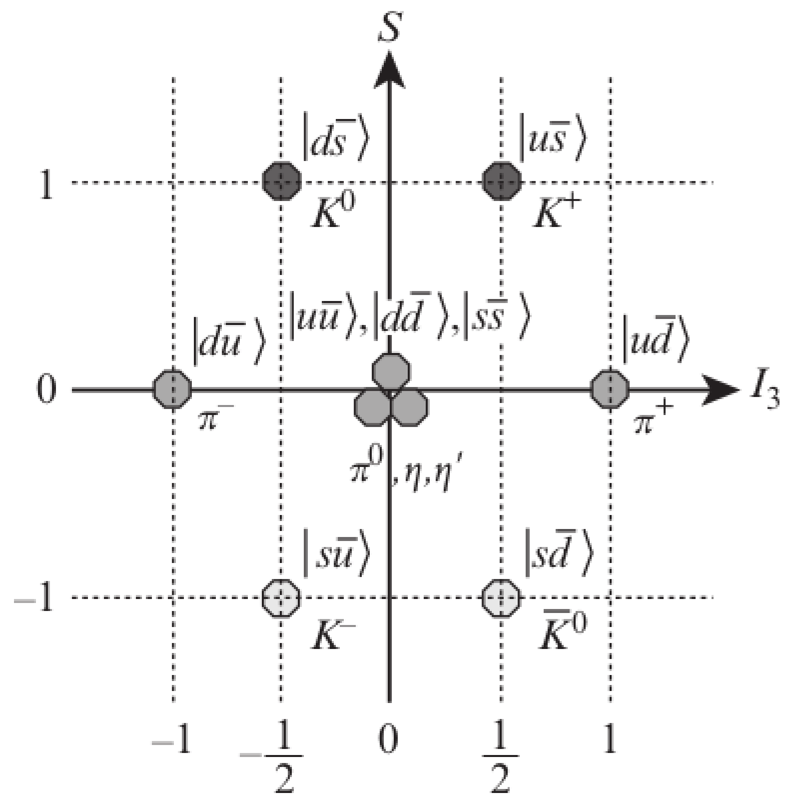
$$(u, d, s)$$

Hadrons with nearly the same mass belong to the same $SU(3)_{\text{flavor}}$ multiplet

Static quark model

Simple explanation of (that time) hadron spectroscopy, in terms of a symmetry $SU(3)_{\text{flavor}}$

Pseudo-scalar mesons



Particle Physics in the LHC Era

Why 3? Because all hadrons could be classified as multiplets generated by a fundamental triplet of aces, or quarks:

$$(u, d, s)$$

$$3 \otimes \bar{3} = 1 \oplus 8$$

$|\eta'\rangle$

mass $\approx$ 957 MeV

$$\left\{ |\pi^\pm\rangle,\ |\pi^0\rangle,\ |K^\pm\rangle,\ |K^0\rangle,\ |\overline{K}^0\rangle \right\}$$

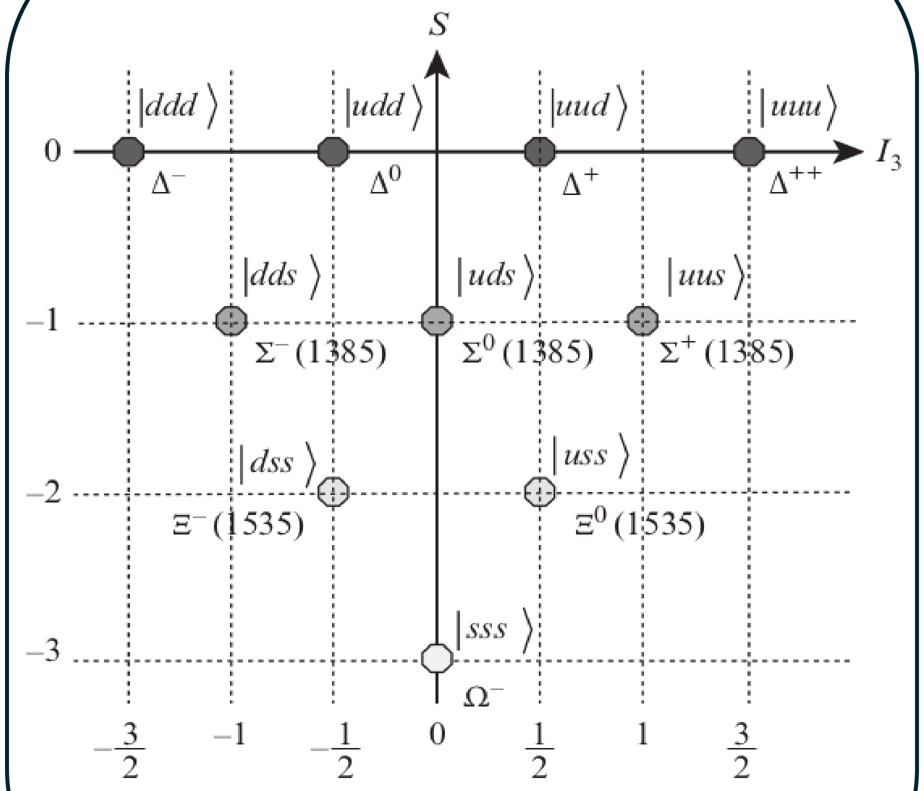
mass $\approx$ 134 – 547 MeV

Hadrons with nearly the same mass belong to the same $SU(3)_{\text{flavor}}$ multiplet

Static quark model

Simple explanation of (that time) hadron spectroscopy, in terms of a symmetry $SU(3)_{\text{flavor}}$

Spin 3/2 baryons



[Particle Physics in the LHC Era]

Why 3? Because all hadrons could be classified as multiplets generated by a fundamental triplet of aces, or quarks:

$$(u, d, s)$$

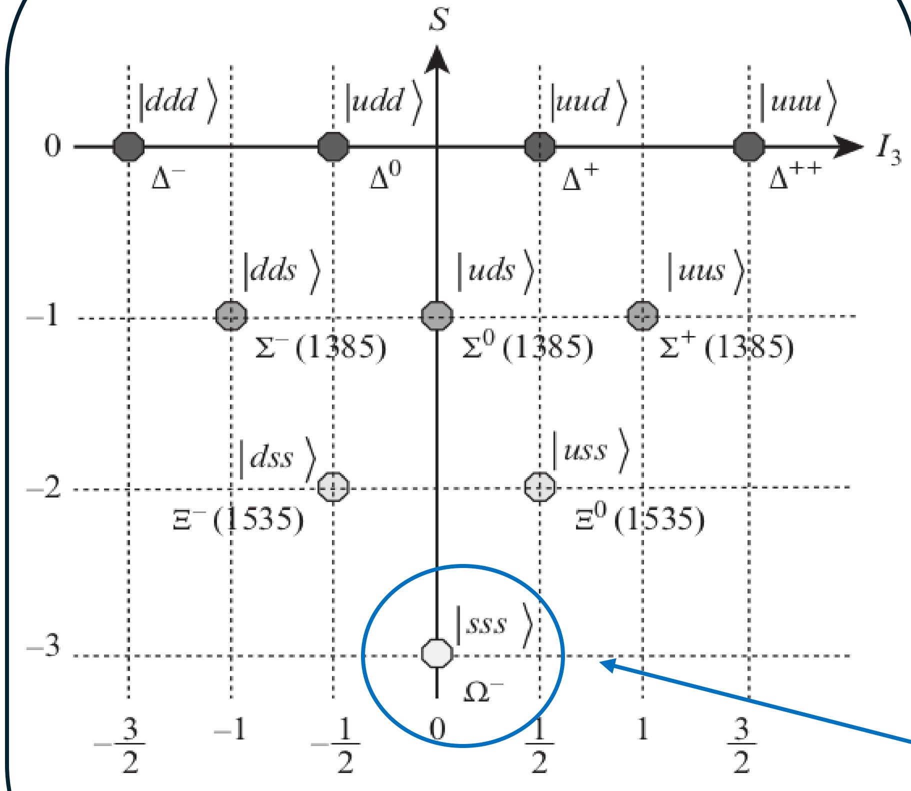
Hadrons with nearly the same mass belong to the same $SU(3)_{\text{flavor}}$ multiplet

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

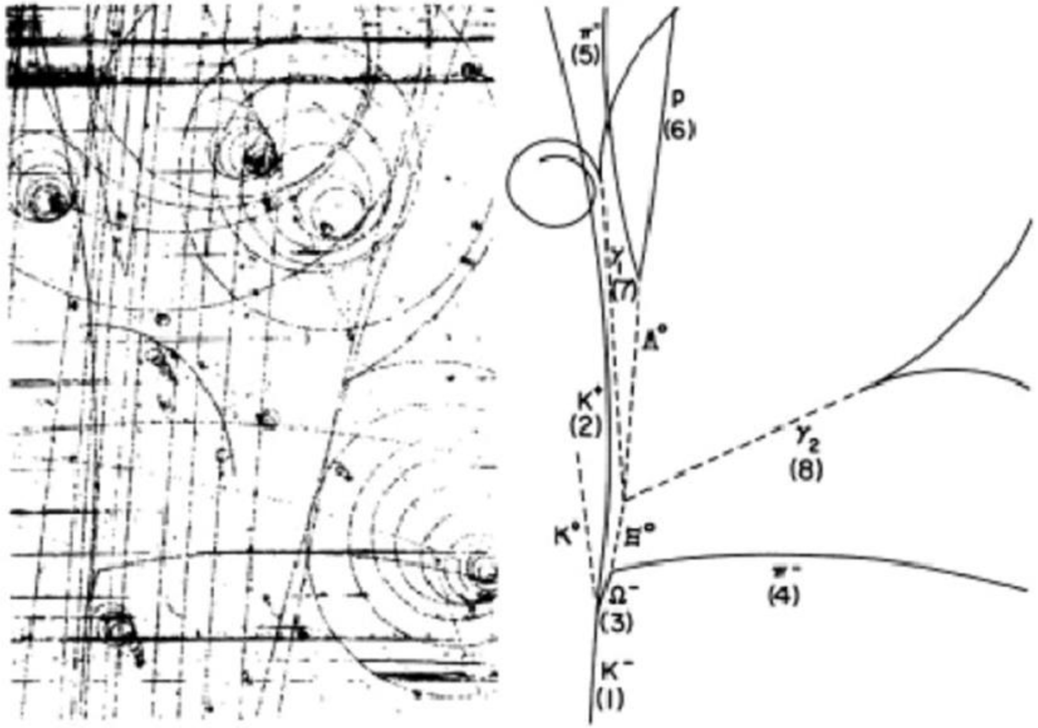
Static quark model

Simple explanation of (that time) hadron spectroscopy, in terms of a symmetry $SU(3)_{\text{flavor}}$

Spin 3/2 baryons



Particle Physics in the LHC Era

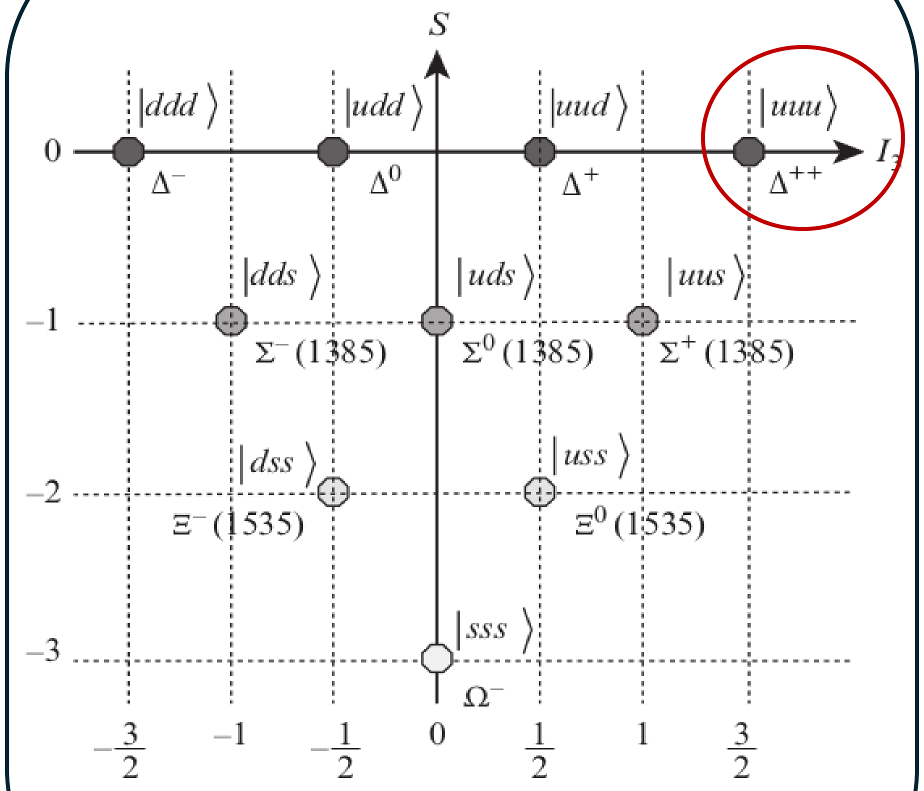


Predicted and finally found in 1964

Coloring the picture

Simple explanation of (that time) hadron spectroscopy, in terms of a symmetry SU(3)$_{\text{flavor}}$

Spin 3/2 baryons



[Particle Physics in the LHC Era]

These are fermions, they *must* have a complete antisymmetric wave function

$$\Psi = \psi_{\text{space}}\, \psi_{\text{flavor}}\, \psi_{\text{spin}}$$

$$|\Delta^{++}\rangle = |u\uparrow, u\uparrow, u\uparrow\rangle$$

This baryon does not make any sense...it has a totally symmetric wave function even though it is a fermion!

Coloring the picture

In order to make Δ^{++} antisymmetric, let's introduce a new degree of freedom, **a new "charge"** such that:

- All observed states are *neutral* with respect to it,
- It restores the Pauli principle for fermions.

$$\Psi = \underbrace{\psi_{\text{space}}\,\psi_{\text{flavor}}\,\psi_{\text{spin}}}_{\text{symmetric}}\,\underbrace{\psi_{\text{color}}}_{\text{antisymmetric}}$$

$$\Rightarrow \quad |\Delta^{++}\rangle = |u\uparrow, u\uparrow, u\uparrow + \text{antisymm. permutations}\rangle$$

Each element of the fundamental triplet of quarks needs to be charged.

Therefore (for consistency) we need **three colors**: {red, green, blue} $\longrightarrow$ Each quark is charged under $SU(3)_{\text{color}}$

Coloring the picture

What does it mean to be charged under $SU(3)_{\text{color}}$?

$$\psi = \begin{pmatrix} \color{red}{red} \\ \color{blue}{blue} \\ \color{green}{green} \end{pmatrix} \xrightarrow{SU(3)_{\text{color}}} \psi' = U\,\psi = \begin{pmatrix} \text{linear comb. of } \{\color{red}{red}, \color{blue}{blue}, \color{green}{green}\} \\ \text{linear comb. of } \{\color{red}{red}, \color{blue}{blue}, \color{green}{green}\} \\ \text{linear comb. of } \{\color{red}{red}, \color{blue}{blue}, \color{green}{green}\} \end{pmatrix}$$

$SU(3)_{\text{color}}$ matrix, i.e. unitary matrix with determinant equal to 1:

$$U\,U^\dagger = 1, \quad \det(U) = 1$$

Also: $$U = \exp\left(i\omega^a \frac{\lambda_a}{2}\right)$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

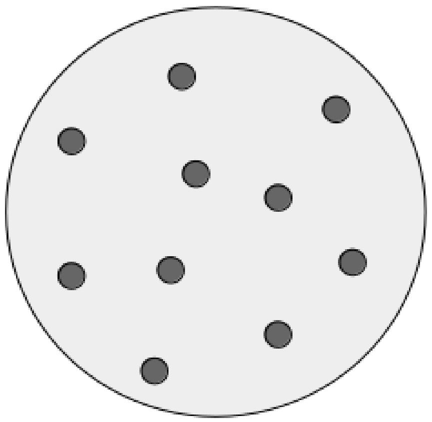
$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 \end{pmatrix}, \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix}, \qquad \lambda_8 = \sqrt{\frac{1}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

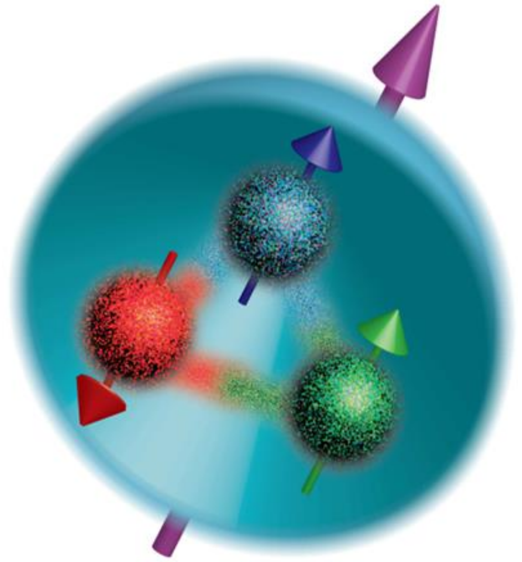
Connecting the dots

Feynman's partons

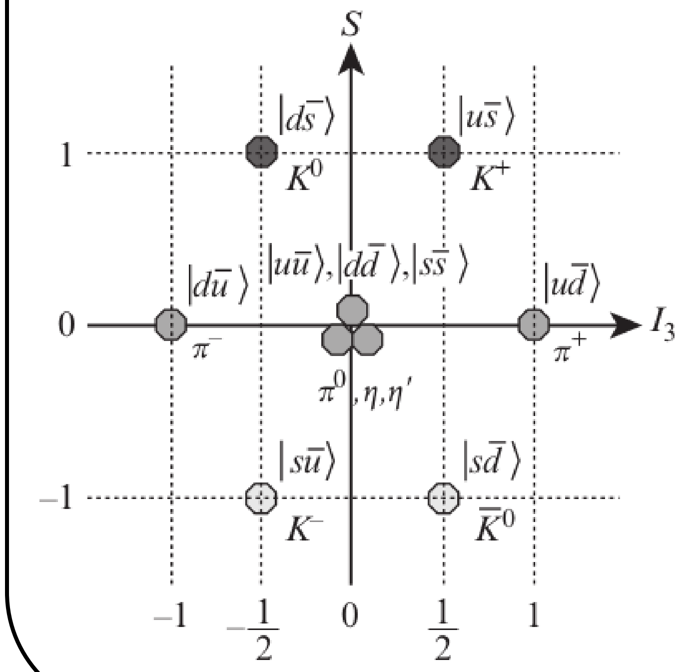


Theory with:

- Elementary fermions, spin ½
- Charged under $SU(3)_{color}$
- Confined inside the hadrons: the theory must be free at infinite energy and the hadrons must be white



Quark model



Building the theory

How to write a theory consistent with experimental evidences

What do we mean by "theory"

We need to find a **Lagrangian** of the elementary fields, consistent with:

- Elementary fermions, spin ½
- Charged under $SU(3)_{color}$
- Confined inside the hadrons: the theory must be free at infinite energy and the hadrons must be white

$$\mathcal{L} = \mathcal{L}\left(\text{fields}(t, \vec{x})\right)$$

Then we will construct the **Action**

$$S[\text{fields}] = \int d^4x \mathcal{L}(\text{fields}(x)) \longrightarrow$$ Classic field theory

Quantization

And finally, the **Partition function** (path integral)

$$Z = \int \mathcal{D}\text{fields} \exp\left\{iS[\text{fields}]\right\} \longrightarrow$$ Quantum field theory

A first attempt

Dirac Lagrangian with spinors in fundamental representation of SU(3)

$$\psi = \begin{pmatrix} red \\ blue \\ green \end{pmatrix}$$

$$\mathcal{L}_{\text{QCD}} = \overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$$

It surely works for global SU(3) transformations...

$$\psi(x) \mapsto \psi'(x) = e^{i\omega^a T_a}\psi(x)$$

...but it breaks for local SU(3) transformation:

$$\psi(x) \mapsto \psi'(x) = e^{i\omega^a(x) T_a}\psi(x)$$

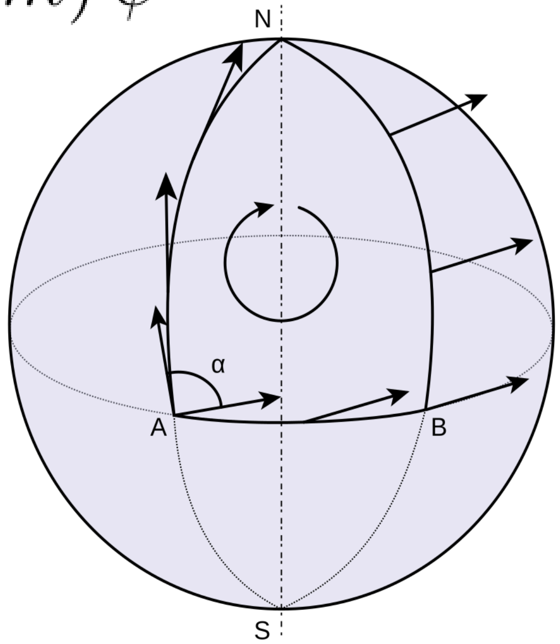
$$\overline{\psi}(x)\psi(x) \mapsto \overline{\psi}(x)e^{-i\omega^a(x)T_a}e^{i\omega^b(x)T_b}\psi(x)$$

$$\overline{\psi}(x)\partial_\mu\psi(x) \mapsto \overline{\psi}(x)e^{-i\omega^a(x)T_a}\partial_\mu\left(e^{i\omega^b(x)T_b}\psi(x)\right)$$

Local symmetry brings the glue

The problem is in the derivative (kinetic) term

$$\mathcal{L}_{\mathrm{QCD}} = \overline{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi$$



$$n^\mu\partial_\mu\psi(x) = \lim_{\epsilon\to 0}\frac{\psi(x+n\,\epsilon)-\psi(x)}{\epsilon}$$

$\mapsto U(x+\epsilon\,n)\psi(x+\epsilon+n)$ $\qquad$ $\mapsto U(x)\psi(x)$

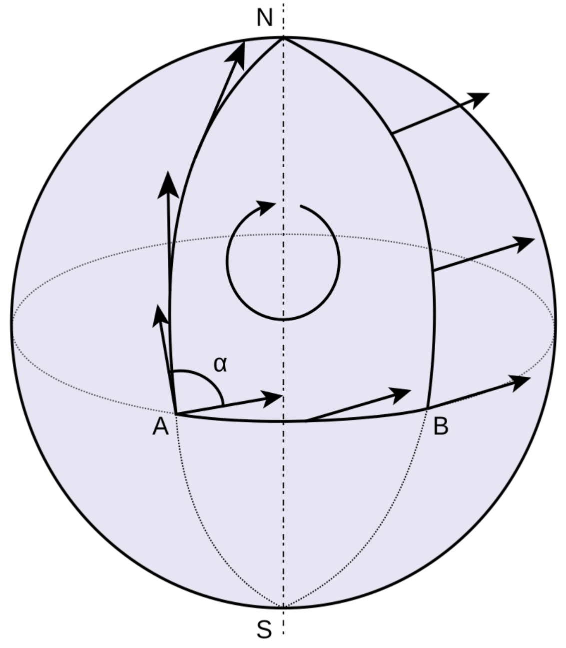
We need to transport the information from x to $x+\epsilon\,n$ while at the same time preserving it.
Let's define the (infinitesimal) **parallel transport** operator:

$$W_{x\to x+n\,\epsilon} = 1 + ig\epsilon n^\mu A^a_\mu(x)T_a + \mathcal{O}(\epsilon^2)$$

Local symmetry brings the glue

This SU(3) operator transforms as:

$$W_{x\to x+n\,\epsilon} \mapsto U(x+n\,\epsilon)\,W_{x\to x+n\,\epsilon}\,U^\dagger(x)$$



Ensuring the correct transformation for the derivative term, given the new definition:

$$n^\mu\partial_\mu\psi(x) = \lim_{\epsilon\to 0}\frac{\psi(x+n\,\epsilon) - W_{x\to x+n\,\epsilon;n}\psi(x)}{\epsilon}$$

$$= n^\mu \underbrace{\left(\partial_\mu - igA^a_\mu(x)T_a\right)}_{D_\mu}\psi(x)$$

Covariant derivative

We had to introduce a new spin 1 field: **the gluon**, belonging to the adjoint representation of SU(3)

$$A_\mu(x) \to U(x)\left(A_\mu(x) + \frac{i}{g}\partial_\mu\right)U^\dagger(x)$$

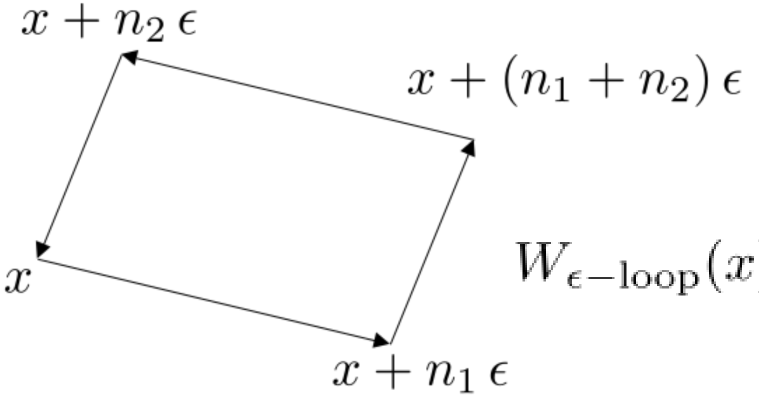
A second attempt

$$\mathcal{L}_{\mathrm{QCD}} = \overline{\psi}\,(i\gamma^{\mu}D_{\mu} - m)\,\psi = \mathcal{L}_{\mathrm{Dirac}} \underbrace{- g\overline{\psi}\gamma^{\mu}A_{\mu}(x)\psi(x)}_{\text{quark-gluon interaction term}}$$

So far, gluons have no dynamics. How do we make them move?

In other words, we need a kinetic term for the gluon fields.

Consider again the parallel transport operator, but this time let's move the SU(3) information along a (infinitesimal) loop:



$$W_{\epsilon-\mathrm{loop}}(x) = W_{x \to x+n_1\,\epsilon}\, W_{x+n_1\,\epsilon \to x+(n_1+n_2)\,\epsilon}\, W_{x+(n_1+n_2)\,\epsilon \to x+n_2\,\epsilon}\, W_{x+n_2\,\epsilon \to x}$$

Loops and gluons

$$W_{\epsilon-\mathrm{loop}}(x) = W_{x\to x+n_1\,\epsilon}\, W_{x+n_1\,\epsilon\to x+(n_1+n_2)\,\epsilon}\, W_{x+(n_1+n_2)\,\epsilon\to x+n_2\,\epsilon}\, W_{x+n_2\,\epsilon\to x}$$

Using:

1. Exponentiation (infinitesimal) $\mathrm{W}_{x\to x+n\,\epsilon} = e^{ig\epsilon n^\mu A_\mu(x+n\,\epsilon)+\dots}$

2. Bacher-Campbell-Hausdorff

We get:

$$W_{\epsilon-\mathrm{loop}}(x) = \exp\left\{ig\epsilon^2\left[\overbrace{n_1^\mu n_2^\nu}^{\text{Loop area}}\underbrace{(\partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu])}_{G_{\mu\nu}}\right] + \dots\right\}$$

$G_{\mu\nu}$ Gluon Field Strength

Also defined as:

$$[D_\mu, D_\nu] = -igG_{\mu\nu}$$

$$G_{\mu\nu}(x) \mapsto U(x)G_{\mu\nu}(x)U^\dagger(x)$$

Loops and gluons

We learned that the parallel transport operator is non-trivial, even though we eventually return to the starting point.

The result obtained for an infinitesimal loop extends to a generic loop as:

$$W_{x\to y;\gamma} = \mathcal{P}\exp\left\{-ig\int_\gamma dx^\mu A_\mu(x)\right\} = \mathcal{P}\exp\left\{-i\frac{g}{2}\int_\Sigma d\sigma^{\mu\nu}G_{\mu\nu}(x)\right\}$$

Path ordering

Such that we always have:

$$W_{\text{loop}}(x) = 1 + \text{area}\times\text{Field Strenght} + \ldots$$

The gluon kinetic term

In SU(3) components:

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g A^b_\mu A^c_\nu f^{bca}$$

Thus, the field strength has 1 derivative w.r.t. time $\longrightarrow$ G-square is a perfect operator candidate for the gluon kinetic term!

$$-\frac{1}{4}G^2(x) = -\frac{1}{4}G^a_{\mu\nu}(x)G^{\mu\nu}_a(x)$$

$\longrightarrow$ It contains 2 derivatives w.r.t. time and also interactions between gluons

Trace over color for SU(3)-invariance

$$G_{\mu\nu}(x) \mapsto U(x)G_{\mu\nu}(x)U^\dagger(x)$$

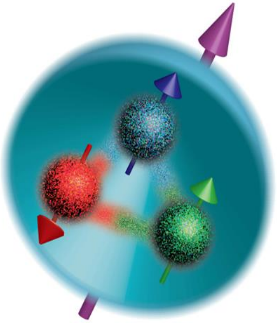
The QCD Lagrangian

Finally:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G^2(x) + \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi$$

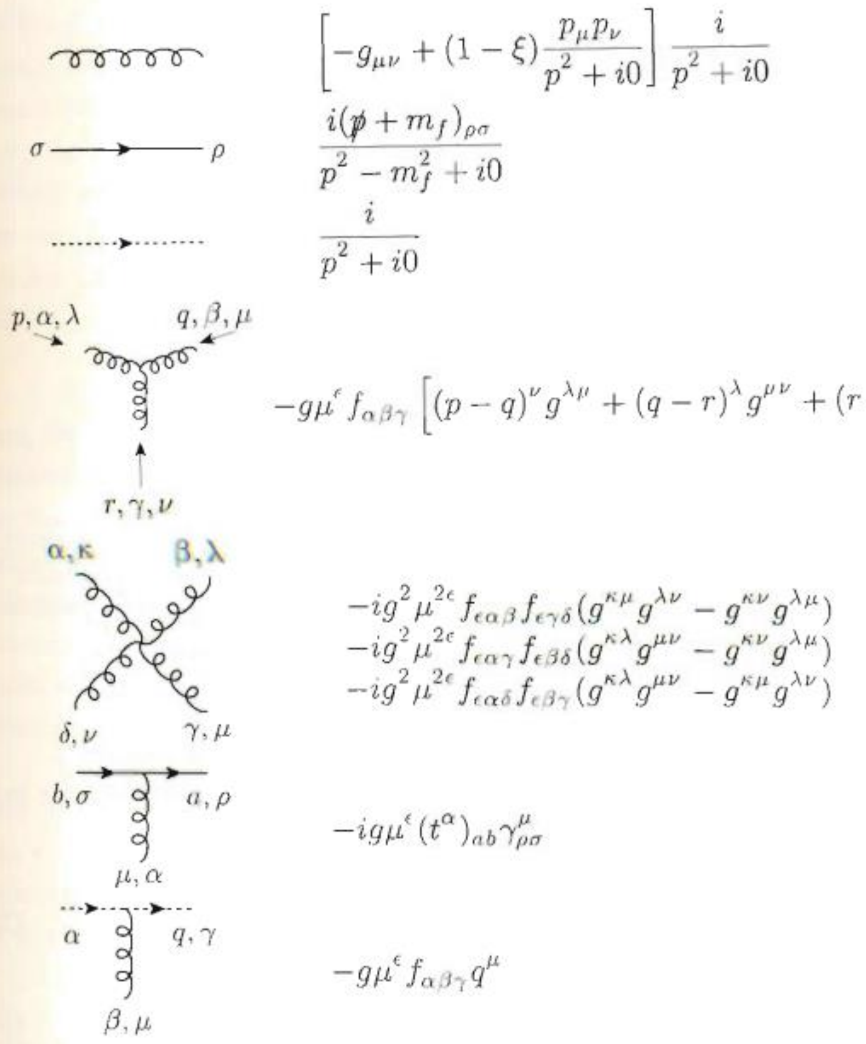
Theory with:

- Elementary fermions, spin ½
- Charged under $SU(3)_{\text{color}}$
- Free at infinite energy, hadrons must be white



Is this considered?

From Collin's book



$$\left[-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2+i0}\right]\frac{i}{p^2+i0}$$

$$\frac{i(\not p + m_f)_{\rho\sigma}}{p^2 - m_f^2 + i0}$$

$$\frac{i}{p^2+i0}$$

$$-g\mu^\epsilon f_{\alpha\beta\gamma}\left[(p-q)^\nu g^{\lambda\mu} + (q-r)^\lambda g^{\mu\nu} + (r-p)^\mu g^{\nu\lambda}\right]$$

$$\begin{aligned}&-ig^2\mu^{2\epsilon} f_{e\alpha\beta} f_{e\gamma\delta}(g^{\kappa\mu}g^{\lambda\nu} - g^{\kappa\nu}g^{\lambda\mu})\\ &-ig^2\mu^{2\epsilon} f_{e\alpha\gamma} f_{e\beta\delta}(g^{\kappa\lambda}g^{\mu\nu} - g^{\kappa\nu}g^{\lambda\mu})\\ &-ig^2\mu^{2\epsilon} f_{e\alpha\delta} f_{e\beta\gamma}(g^{\kappa\lambda}g^{\mu\nu} - g^{\kappa\mu}g^{\lambda\nu})\end{aligned}$$

$$-ig\mu^\epsilon (t^\alpha)_{ab}\gamma^\mu_{\rho\sigma}$$

$$-g\mu^\epsilon f_{\alpha\beta\gamma} q^\mu$$

Fig. 3.1. Basic Feynman rules of QCD. The coupling has been replaced by $g\mu^\epsilon$, according to the standard convention for use in $4-2\epsilon$ dimensions. Propagators and vertices are diagonal in any indices (flavor or color) that are not explicitly indicated. For the renormalization counterterm vertices, see Fig. 3.2.

Asymptotic freedom

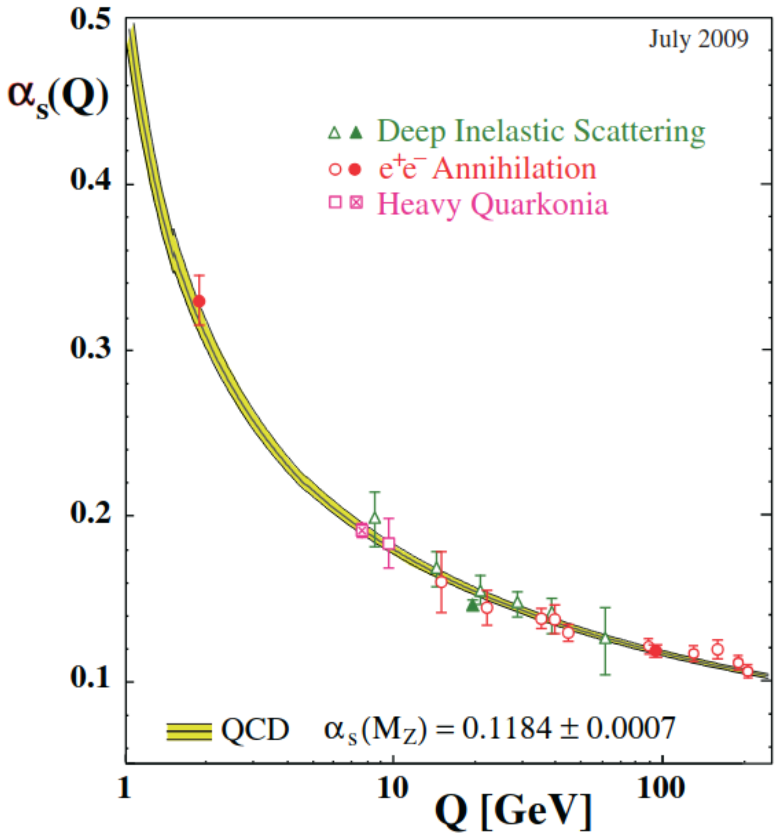
The intensity of the strong force g is not constant: it acquires an "anomalous" (due to quantum corrections) dependence on the energy scale.
Let's define the coupling:

$$\alpha_S = \frac{g^2}{4\pi}$$

$$\frac{d\alpha_S}{d\log\mu^2} = \beta(\alpha_S) = -\boxed{\beta_0}\alpha_S^2 + \dots$$

QCD beta function

$$\frac{1}{4\pi}\left(11 - \frac{2}{3}n_{\text{flavors}}\right)$$



The intensity decreases at high energies if the beta function is *negative*: $n_{\text{flavors}} \leq 16$

On the next episode

Considering that the QCD fundamental fields (quarks and gluons) do not correspond to the observable asymptotic states (hadrons), how do we make predictions?