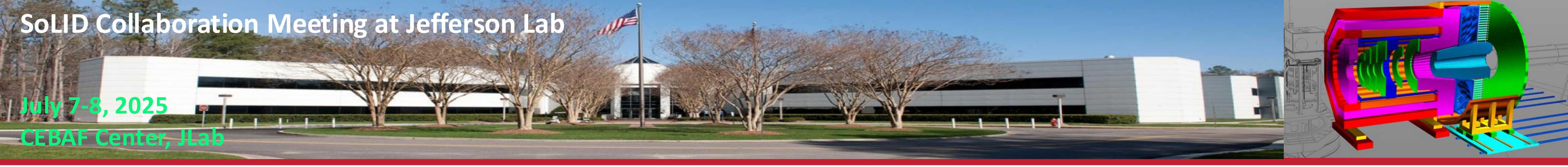
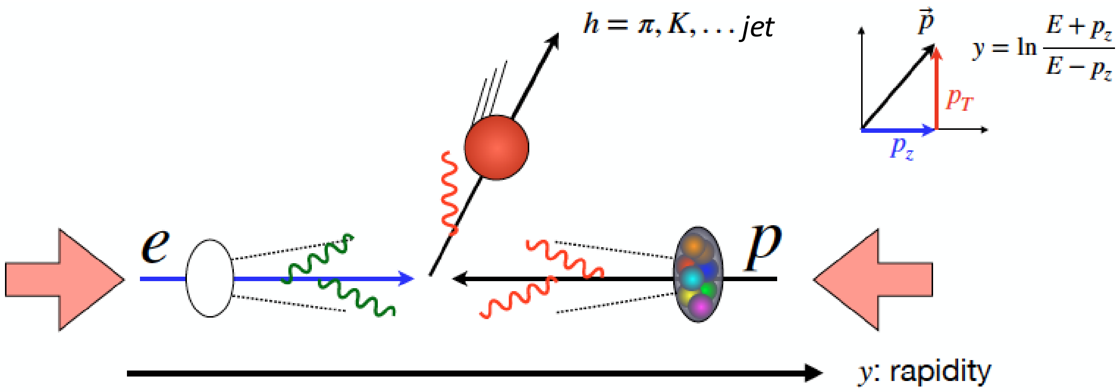
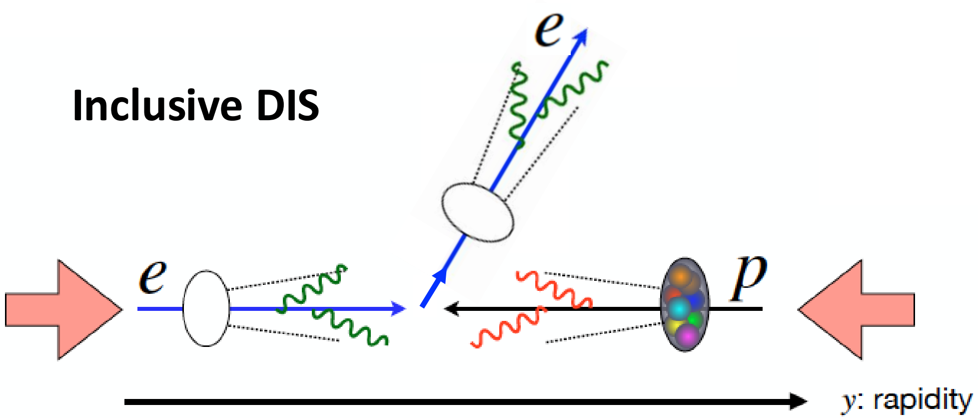


Inclusive Pion Production in Lepton-Hadron Collision

—— Prerequisite for understanding SIDIS data with current factorization formalism





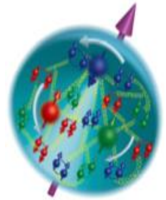
Kang, Meta, Qiu, Zhou, PRD 2011
Hinderer, Schlegel, Vogelsang, PRD 2015, 2016
Abelof, Boughezal, Liu, Petriello, PLB, 2016
Qiu, Wang, Xing, CPL, 2021

Qiu & Watanabe to be published
In collaboration with K. Watanabe, T.B. Liu, J.Y. Zhang, …

Jianwei Qiu
Jefferson Lab, Theory Center

QCD Color is Fully Entangled

- **QCD color confinement:**
  - *Do not see any quarks and gluons in isolation*
  - *The structure of nucleons and nuclei – emergent properties of QCD*



Color Confinement — Asymptotic freedom — Q (GeV)

20 MeV (10 fm) — 200 MeV (1 fm) — 2 GeV (1/10 fm)

Probing scale

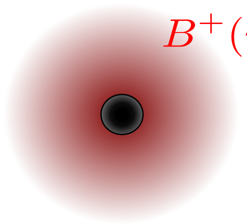
QCD at the Fermi Scale: Femto-science (0.1-10 fm)

- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

Asymptotic regime PQCD works beautifully!

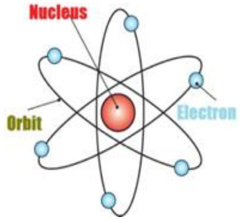
- **QCD is non-perturbative:**
  - *Any cross section/observable with identified hadron is NOT perturbatively calculable!*
  - *Color is fully entangled!*

B-meson



Brown-Muck

Atomic structure



Quantum orbits

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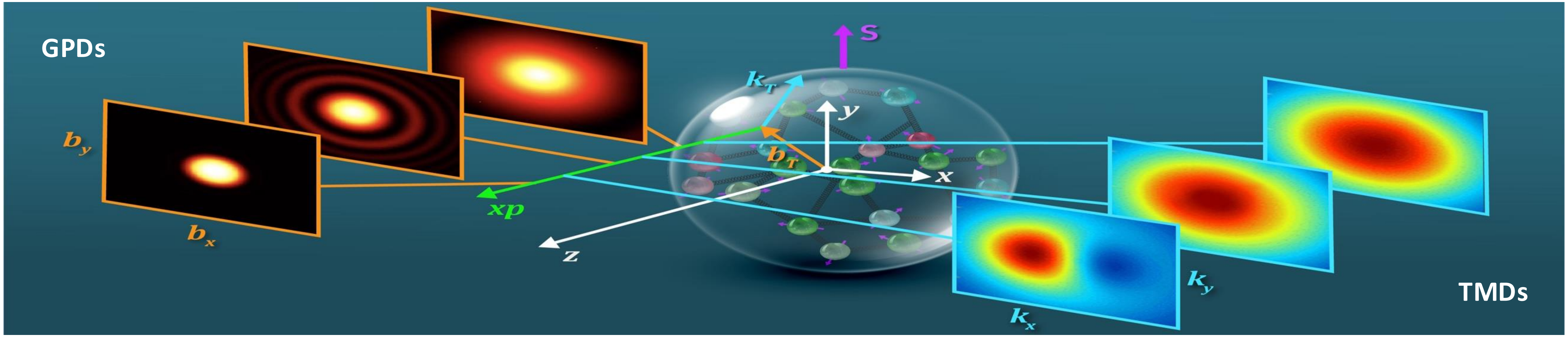
2-Scale Observables and 3-D Hadron Structure

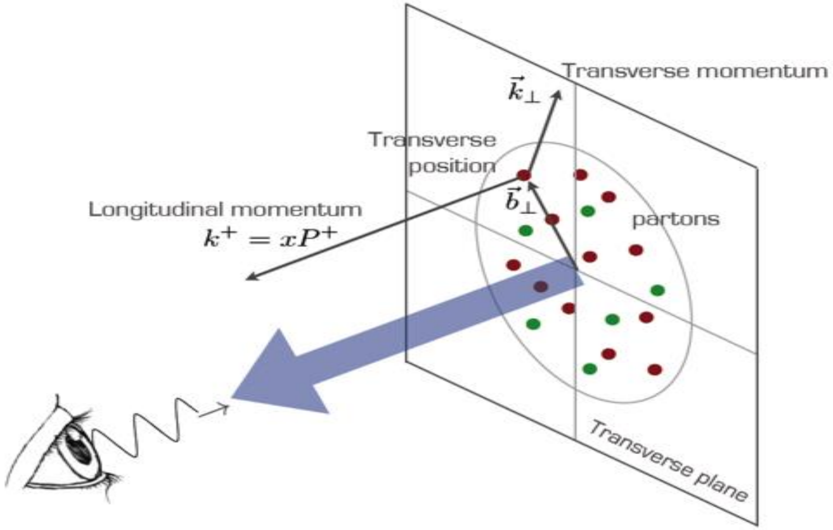
- 3-D hadron structure:



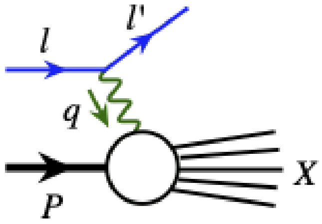
- Need new observables with 2 distinctive scales:

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

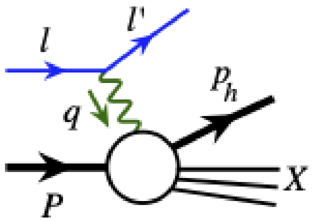
  - Hard scale: Q_1 to localize the probe to see the particle nature of quarks/gluons
  - "Soft" scale: Q_2 to be more sensitive to the emergent regime of hadron structure ~ 1/fm



Lepton-Hadron Semi-Inclusive DIS (SIDIS): $Q^2 \gg P_{hT}^2$ in Breit frame

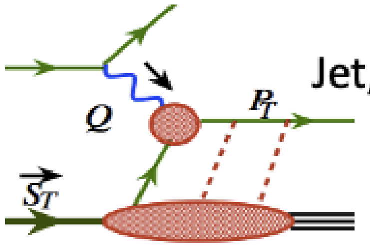


Scale: Q^2 - PDFs



$Q^2 \gg P_{h_T}^2$

In photon-hadron frame!

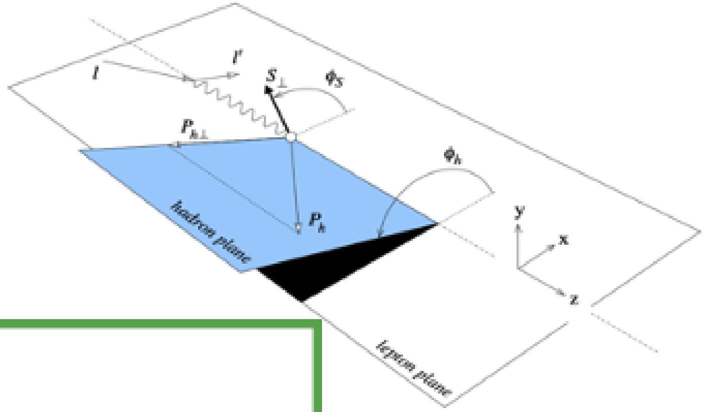


Jet, π, J/ψ, …

$f(x, k_T, Q)$ - TMDs

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = {} & \\
\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right) & \Bigg\{ \boxed{F_{UU,T}} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} \\
& + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, F_{LU}^{\sin\phi_h} \\
& + S_\parallel \left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h}\right] \\
& + S_\parallel\lambda_e \left[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\, F_{LL}^{\cos\phi_h}\right] \\
& + |\boldsymbol{S}_\perp| \Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
& \quad + \varepsilon\,\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
& + |\boldsymbol{S}_\perp|\lambda_e \Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\, F_{LT}^{\cos\phi_S} \\
& \quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

18 SIDIS Structure Functions



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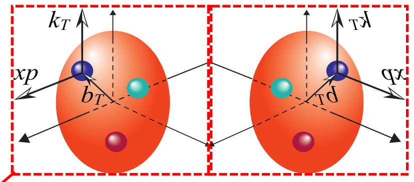
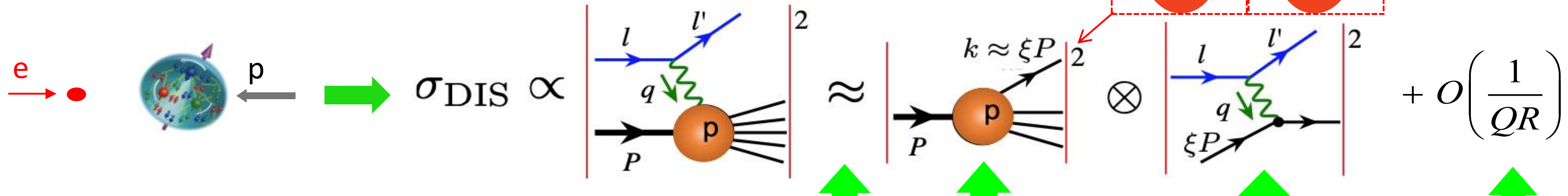
Theoretical Approaches – Approximations:

- **Perturbative QCD Factorization:**

  – Approximation at Feynman diagram level





- **Effective field theory (EFT):**

  – Approximation at the Lagrangian level

  Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

- **Lattice QCD:**

  – Approximation for finite lattice spacing, finite box, lightest quark masses, … with Euclidean time formulation (removable with increased computational cost)

  Hadron structure, hadron spectroscopy, nuclear structure, phase shift, …

- **Other approaches:**

  Light-cone perturbation theory, Dyson-Schwinger Equations (DSE), Constituent quark models, AdS/CFT correspondence, …

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Predictive Power of QCD Factorization

❑ Universality of non-perturbative hadron structure + calculable matching coefficients:

- lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{\text{EXP}}_{l+P\to l+X} = C_{l+k\to l+X} \otimes \boxed{\text{PDF}_P} + O(Q_s^2/Q^2)$$

$$\sigma^{\text{EXP}}_{l+P\to l+H+X} = \boxed{C_{l+k\to l+k+X}} \otimes \boxed{\text{PDF}_P} \otimes \boxed{\text{FF}_H} + O(Q_s^2/Q^2)$$

- hadron-hadron reactions (LHC)

$$\sigma^{\text{EXP}}_{P+P\to l+\bar{l}+X} = \boxed{C_{k+k\to l+\bar{l}+X}} \otimes \boxed{\text{PDF}_P} \otimes \boxed{\text{PDF}_P} + O(Q_s^2/Q^2)$$

- lepton-lepton reactions (Belle)

$$\sigma^{\text{EXP}}_{l+\bar{l}\to H+X} = \boxed{C_{l+\bar{l}\to k+X}} \otimes \boxed{\text{FF}_H} + O(Q_s^2/Q^2)$$

Plus other factorizable observables – Cross sections & Asymmetries
Plus LQCD calculable & factorizable hadron matrix elements

$$\sigma^{\text{LQCD}}_{n/P} = \langle P|\mathcal{O}_n|P\rangle Z^{-1}_{\mathcal{O}_n} = \boxed{K_{\mathcal{O}_n}} \otimes \boxed{\text{PDF}_P}$$

❑ Hadron structure = Theory + Experiment + Phenomenology:

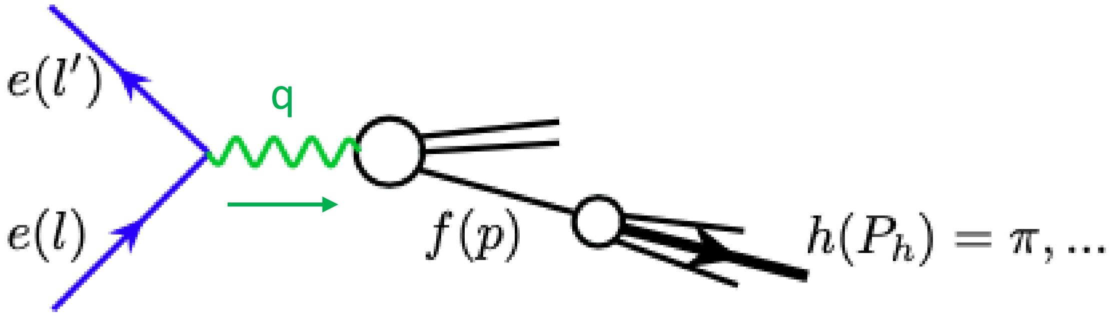
- **Factorization – Identify "Good" observables (Theory, including LQCD)**
- **Measurement – Get "Reliable" data (Experiment + LQCD)**
- **Global analysis – Extract "Universal" structure information (Phenomenology)**
  by solving an inverse problem

See talk by
Chris Cocuzza

How was the Hadron Produced in the SIDIS?

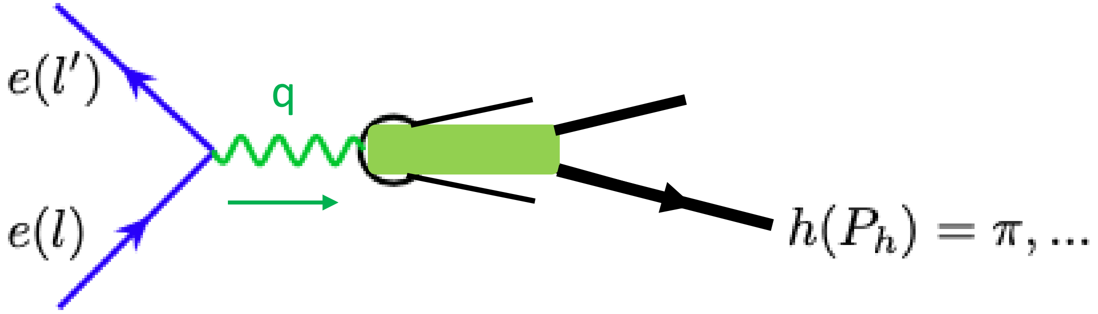
❑ Physical Process: $e(l) + N(P,\uparrow) \to e(l') + h(P_h) + X$

- **Production of a parton + parton fragmentation:**

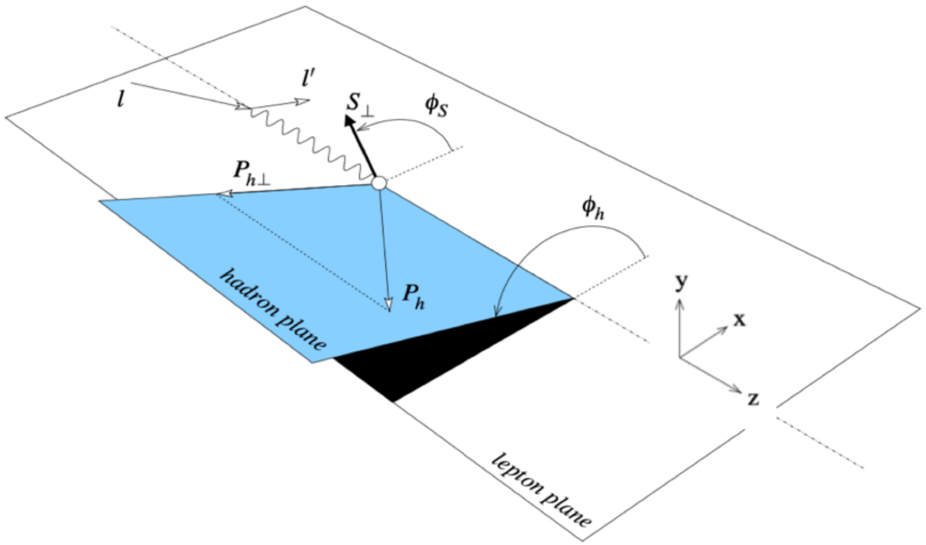


$$E_{l'} E_h \frac{\sigma^{\rm DIS}}{d^3 l' d^3 P_h} \propto \sum_{a,b} f_{a/N}(x, k_T) \otimes D_{h/b}(z, p_T)$$

- **Vector-Meson production + decay:**



$$E_{l'} E_h \frac{\sigma^{\rm DIS}}{d^3 l' d^3 P_h} \neq \sum_{a,b} f_{a/N}(x, k_T) \otimes D_{h/b}(z, p_T)$$



Experiment measures cross section & both amplitudes contribute to SIDIS cross section!

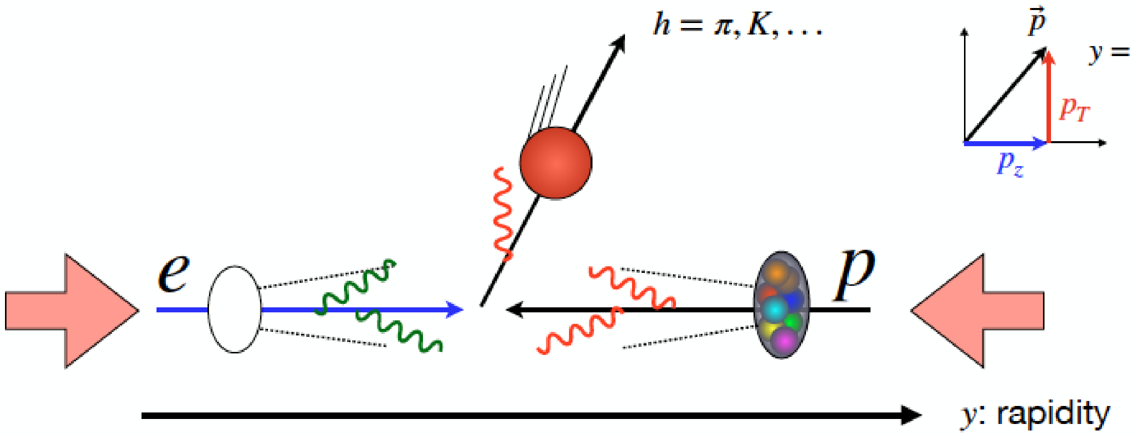
$$\sigma^{\rm DIS} \propto |\mathcal{M}_{\rm FF} + \mathcal{M}_{\rm VM} + \ldots|^2 \propto |\mathcal{M}_{\rm FF}|^2 + \mathcal{O}\left(\left[\frac{M_{\rm V}}{Q}\right]^n\right)$$

Q: At what kinematics, production via parton fragmentation dominates?

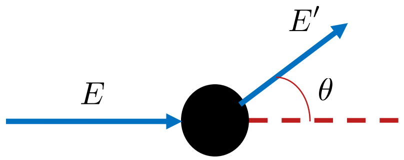
Prompt Single Hadron/Jet Production:

❑ Single hadron (Jet) production in lepton-hadron scattering:

Kang, Meta, Qiu, Zhou, PRD 2011
Hinderer, Schlegel, Vogelsang, PRD 2015, 2016
Abelof, Boughezal, Liu, Petriello, PLB, 2016
Qiu, Wang, Xing, CPL, 2021
Qiu, Watanabe, in preparation



JLab kinematics – fixed target:



$$p_T = E' \sin(\theta) \gtrsim 1\ \mathrm{GeV?}$$

HERA experience:

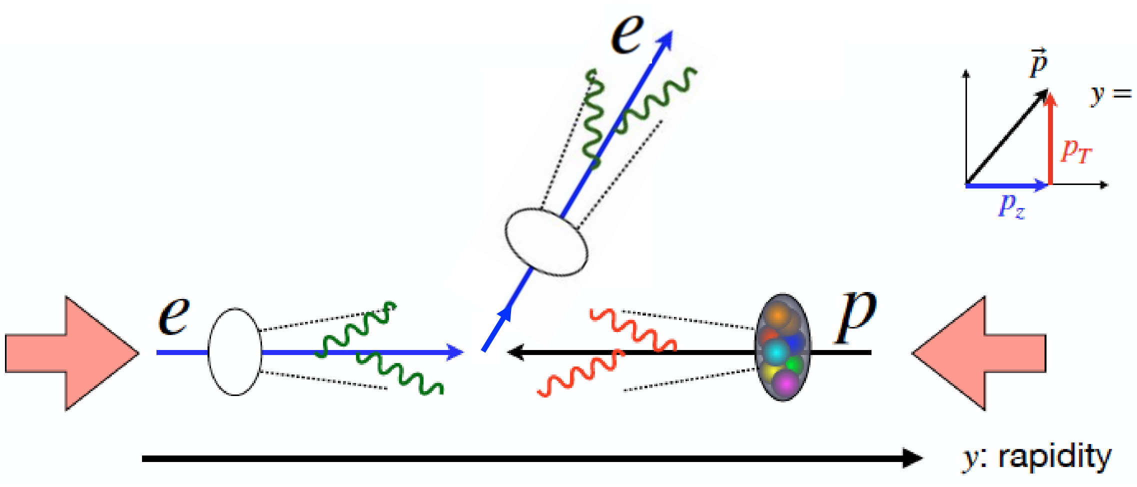
Without measuring the scattered electron, photoproduction of the observed hadron becomes a very important channel, and could be a dominated channel in some cases (e.g., quarkonium production)

Collision-induced QCD and QED radiation:

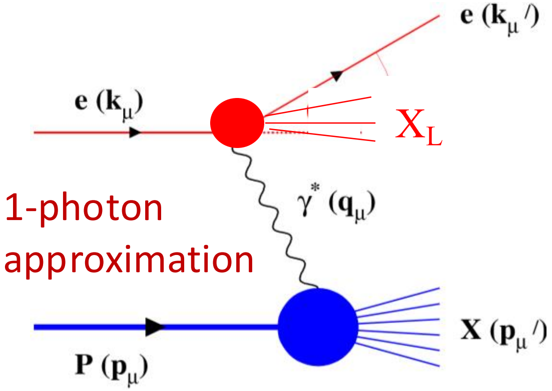
QCD: Factorization move collinear (CO) sensitive radiation to PDFs, FFs, …; include CO safe contribution to perturbatively calculable hard coefficients

QED: Change the direction and magnitude of the “exchange photon”, need unknown parameter for Radiative Corrections (RC), which is energy and process dependent, …

Hard Collision Induces both QCD and QED Radiations

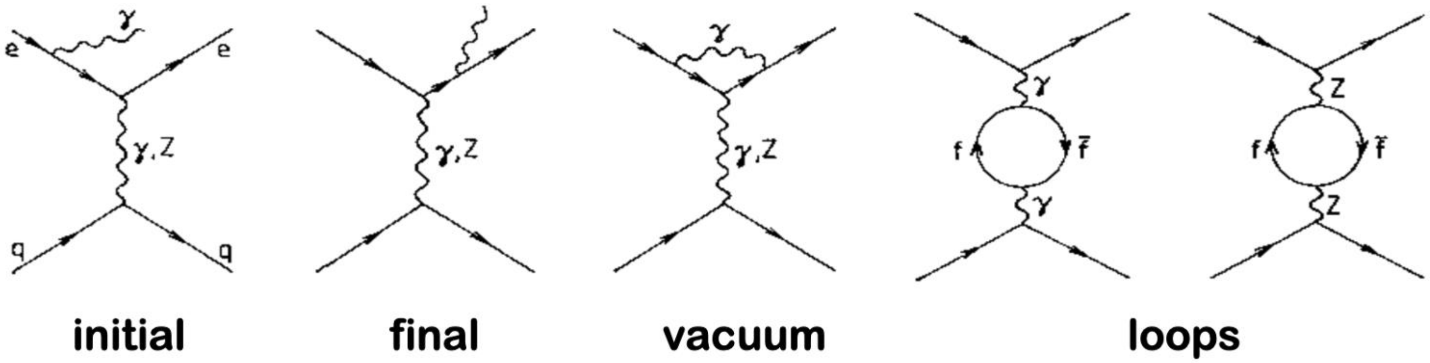


Historically, 1-photon approximation + radiative corrections (radiation from leptons)



$$(x_B, Q^2) \to (\hat{x}_B, \widehat{Q}^2)$$

"We know how to calculate QED radiation perturbatively!"

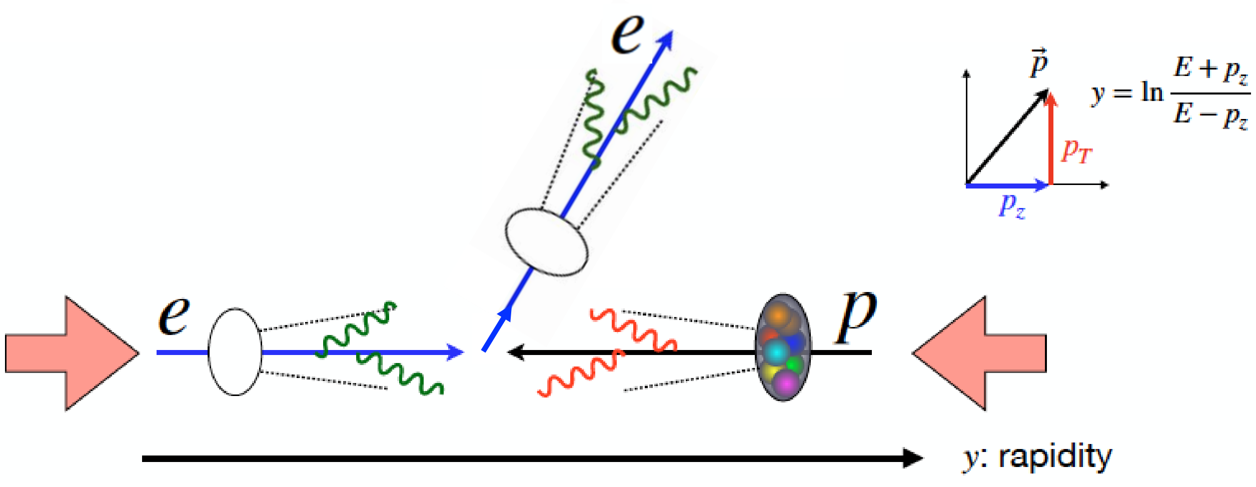


+ two-photon channel + …

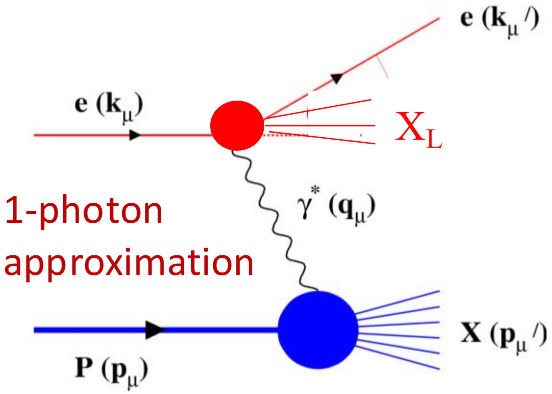
MC program(s) for the RC with "cutoff(s)"

Always keep the $\gamma*$ virtual!

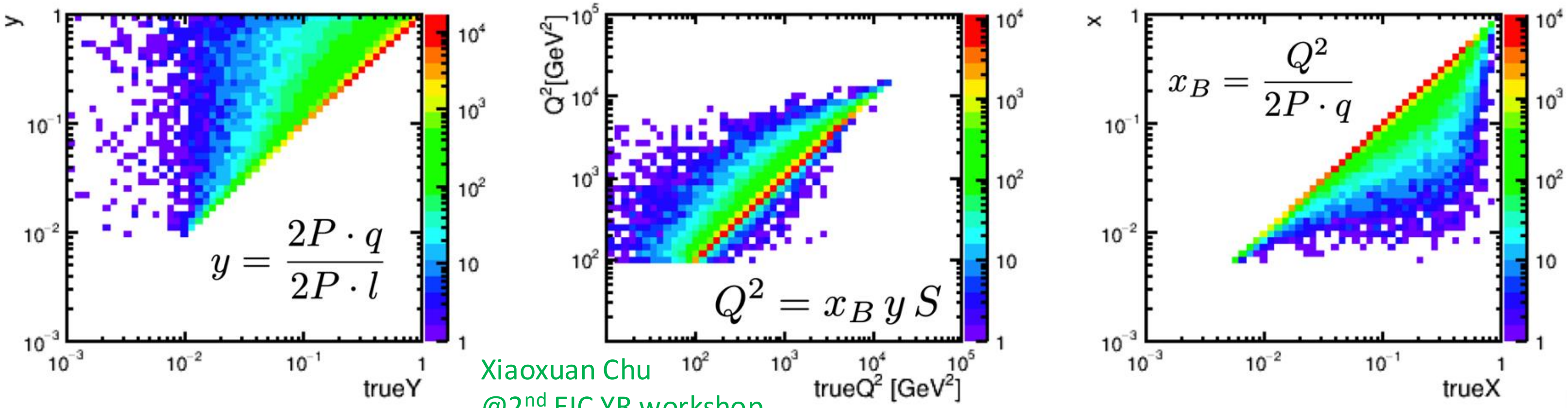
Hard Collision Induces both QCD and QED Radiations



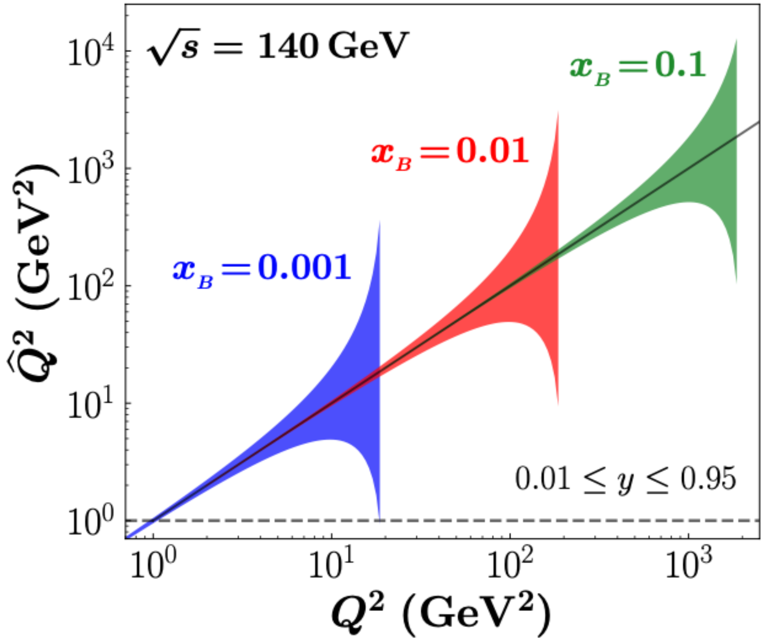
Historically, 1-photon approximation + radiative corrections (radiation from leptons)



"We know how to calculate QED radiation perturbatively!"



Xiaoxuan Chu
@2nd EIC YR workshop

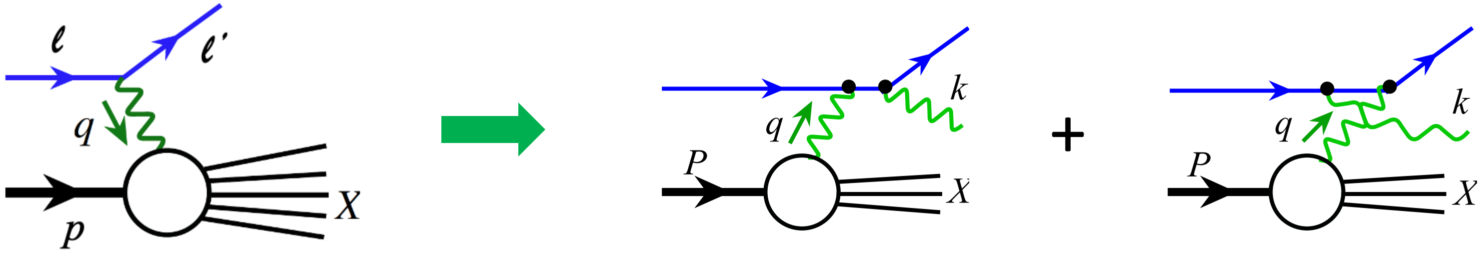


$$x_B \to \hat{x}_B \in [x_B, 1] \qquad \widehat{Q}^2_{\rm min} = Q^2\, \frac{(1-y)}{(1-x_B\, y)} \qquad \widehat{Q}^2_{\rm max} = Q^2\, \frac{1}{(1-y+x_B\, y)}$$

Why We Need Parameter(s) for the Traditional RC Approach?

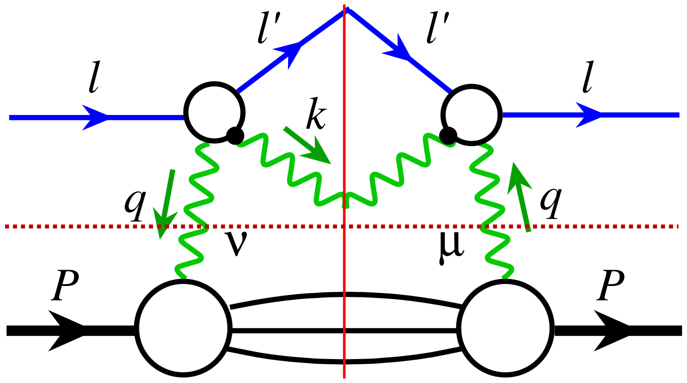
❑ Leading Order Radiative Correction:

Cammarota, Qiu, Watanabe, Zhang
arXiv: 2408.08377, 2505.23487



+ Virtual diagrams

Contribution to the cross section – cut diagram notation:



The virtuality of the exchange photon q^2 depends on the phase-space of unobserved photon of momentum k

$$E' \frac{d\sigma^{\rm RC}_{eh\to eX}}{d^3\ell'} \propto \int d^4q \left[W^{\mu\nu}(P,q)\frac{1}{q^2+i\epsilon} L^{(1)}_{\mu\nu}(\ell,\ell',q)\frac{1}{q^2-i\epsilon}\right]$$

$$\to \infty$$

The q^2 is perturbatively pinched to $q^2 = 0$ if the phase space allows!

Impose "cutoff/limit" on the radiated and un-observed photon(s) to keep it "virtual"– parameter(s)

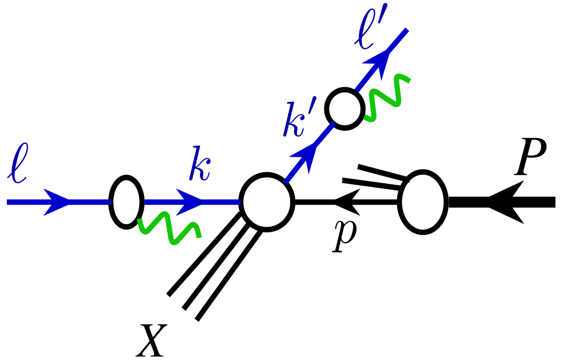
The parameter(s) must be collision energy dependent – tuning parameter(s) in MC for RC – Predictive power?

Joint QCD and QED Factorization for Deep Inelastic Scattering (DIS)

❑ Without the "one-photon" approximation:

Liu, Melnitchouk, Qiu, Sato, Phys.Rev.D 104 (2021) 094033
JHEP 11 (2021) 157
Cammarota, Qiu, Watanabe, Zhang [2408.08377]

~ Inclusive single lepton production at high transverse momentum



$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2) \times \int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2) + \cdots$$

LFFs ($D_{e/j}$), LDFs ($f_{i/e}$), PDFs ($f_{a/N}$)

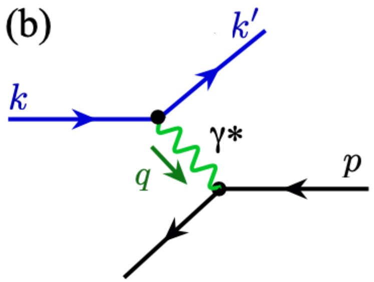
No structure functions, but have PDFs, LDFs, LFFs, …

❑ Hard parts in power of $\alpha^m\alpha_s^n$ – $\widehat{H}^{(m,n)}_{ia\to jX}$:

Hard parts are not sensitive to long-distance physics or the colliding states

- **LO – Let h(P) to be q(P):**

(b)



$$\widehat{H}^{(2,0)}_{eq\to eX} = (2\hat{s})\left[E_{k'}\frac{d\sigma^{(\mathrm{LO})}_{eq\to eq}}{d^3k'}\right] = e_q^2\left(4\alpha_{em}^2\right)\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]\delta\left(\hat{s}+\hat{t}+\hat{u}\right)$$

$$= e_q^2\left(4\alpha_{em}^2\right)\frac{x^2\zeta\left[(\zeta\xi s)^2+u^2\right]}{(\xi t)^2\,(\zeta\xi s+u)}\,\delta(x-x_{\min})$$

$$x_{\min} = \frac{\xi\, x_B\, y}{\xi\zeta+y-1},\quad \xi_{\min} = \frac{1-y}{\zeta-x_B y},\quad \zeta_{\min} = 1-(1-x_B)y,$$

$$s=(\ell+P)^2,\quad u=(\ell'-P)^2=(y-1)s,\quad t=(\ell-\ell')^2=-Q^2$$

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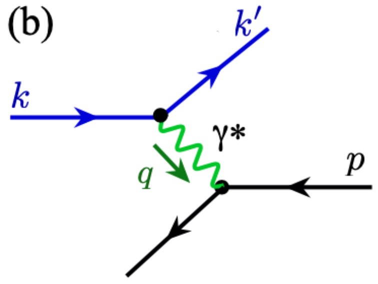
Joint QCD and QED Factorization for Deep Inelastic Scattering (DIS)

❑ Keep the "one-photon" approximation – resum collinear radiation:

Liu, Melnitchouk, Qiu, Sato, Phys.Rev.D 104 (2021) 094033
JHEP 11 (2021) 157
Cammarota, Qiu, Watanabe, Zhang [2408.08377]



$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^1 d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2)\left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right]$$

$$\times\frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2 F_1(\hat{x}_B,\widehat{Q}^2)+\left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

(the bracketed second line $=\dfrac{d^2\sigma_{kP\to k'X}}{d\hat{x}_B d\hat{y}}$)

When: $D_{e/e}(\zeta)=\delta(1-\zeta)\,\delta^{ee}$

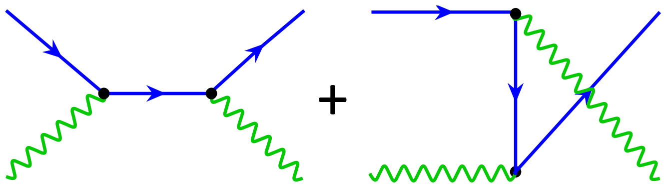
$f_{e/e}(\xi)=\delta(1-\xi)\,\delta^{ee}$

➡ Recover the Born expression

❑ Next-to-Leading Order (NLO) to the leading subprocess: $e(k)+q(p)\to e(k')+X$

- **NLO – Let h(P) to be q(P):**

$$\widehat{H}^{(3,0)}_{eq\to eX} = \sigma^{(3,0)}_{eq\to eX} - D^{(1)}_{e/e}\otimes_\zeta \widehat{H}^{(2,0)}_{eq\to eX} - f^{(1)}_{e/e}\otimes_\xi \widehat{H}^{(2,0)}_{eq\to eX}$$
$$-f^{(1)}_{q/q}\otimes_x \widehat{H}^{(2,0)}_{eq\to eX} - \boxed{f^{(1)}_{\gamma/q}\otimes_x \widehat{H}^{(2,0)}_{e\gamma\to eX}}$$



Similar to the Bethe-Heitler subprocess for Exclusive Processes

Completely UV, IR, CO safe!
No need for any free parameter other than the standard factorization scale

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NLO QED contributions – beyond 1-vector boson exchange

❑ NLO QED contribution – IR & CO safe:

$$\hat{H}^{(3,0)}_{eq\to eX} \propto \alpha^3 e_q^2 \left\{ e_l^2 \frac{2(1+\hat{v}^2)}{9\hat{v}} \left[3\ln\frac{(1-\hat{v})s}{\mu^2} - 5 \right] \delta(1-\hat{w}) \right.$$

⬅ $e_l^2 = \sum_f N_c^f e_f^2$ Sum over the flavors appeared in the photon vacuum polarization

$$+ e_q \left[a_1 \delta(1-\hat{w}) + \frac{a_7}{(1-\hat{w})_+} + a_6 \right]$$

⬅ Two-photon exchange, IR & CO safe, no μ dependence

$$+ e_q^2 \left[b_1 \delta(1-\hat{w}) + b_2 \left(\frac{1}{1-\hat{w}}\right)_+ + b_3 \left(\frac{\ln(1-\hat{w})}{1-\hat{w}}\right)_+ + b_4 \right]$$

⬅ Radiation from quark lines Same as QCD NLO correction

$$\left. + c_1 \delta(1-\hat{w}) + c_2 \left(\frac{1}{1-\hat{w}}\right)_+ + c_3 \left(\frac{\ln(1-\hat{w})}{1-\hat{w}}\right)_+ + c_4 \right\}$$

⬅ RC - Radiation from electron Need photon distribution of the proton

$$\hat{v} = 1 - \frac{x_B}{x}\frac{y}{\zeta}$$

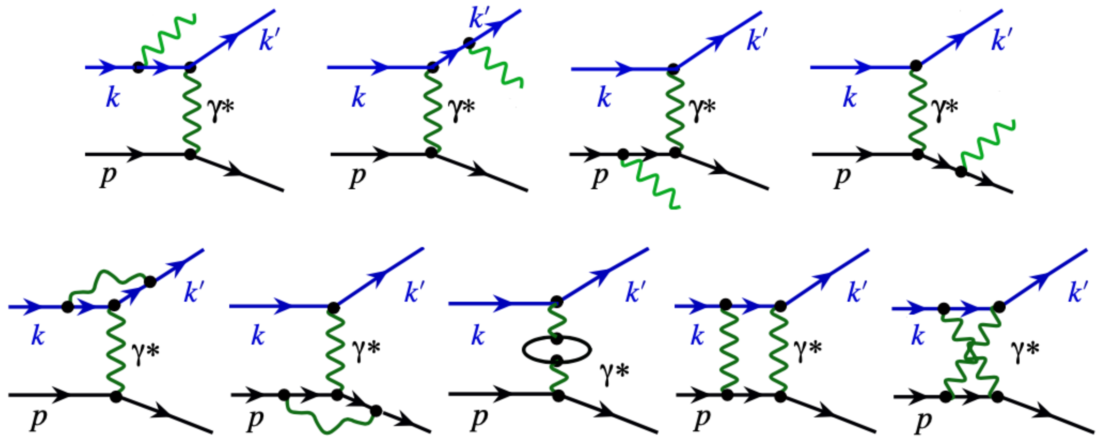
$$\hat{w} = \frac{1-y}{\xi\,(\zeta - (x_B/x)\,y)}$$

a_1, a_6, a_7

b_1, b_2, b_3, b_4

c_1, c_2, c_3, c_4

are analytic functions of $\hat{v}$ and $\hat{w}$.



In joint QCD & QED factorization:

Lepton-distributions are not pure QED !
Hadron's parton distributions are not pure QCD !

Impact of Factorized QED Contribution to Lepton-Hadron Scattering

❑ Perturbative lepton distributions:

$$f_{e/e}^{(\mathrm{NLO})}(\xi, \mu^2) = \delta(1-\xi) + \frac{\alpha_{em}}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$$

$$D_{e/e}^{(\mathrm{NLO})}(\zeta, \mu^2) = \delta(1-\zeta) + \frac{\alpha_{em}}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$$

Electron mass to regularize the CO divergence
– Only defined under the integration.

❑ Nonperturbative model distributions:

LDFs: Very different from PDFs, peaked
LFFs: at larger momentum fraction

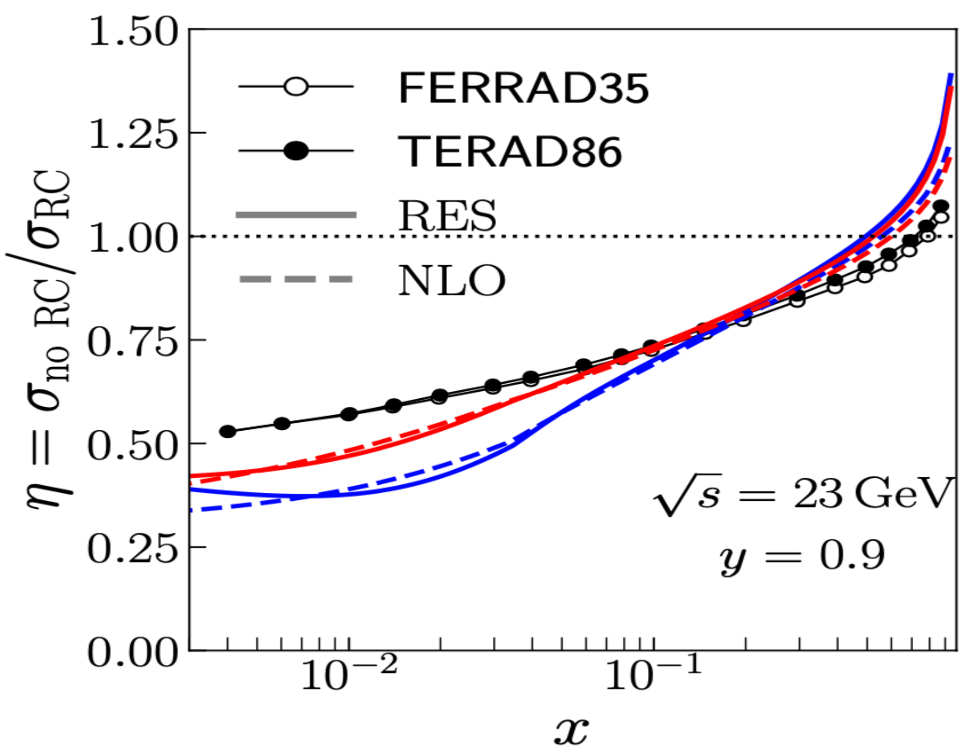
$$f_{e/e}(x) \approx D_{e/e}(x) = N_e \frac{x^\alpha (1-x)^\beta}{B(1+\alpha, 1+\beta)}$$

with $N_e = 1$

$$(\alpha, \beta) = (5, 1/2),\ (50, 1/8)$$

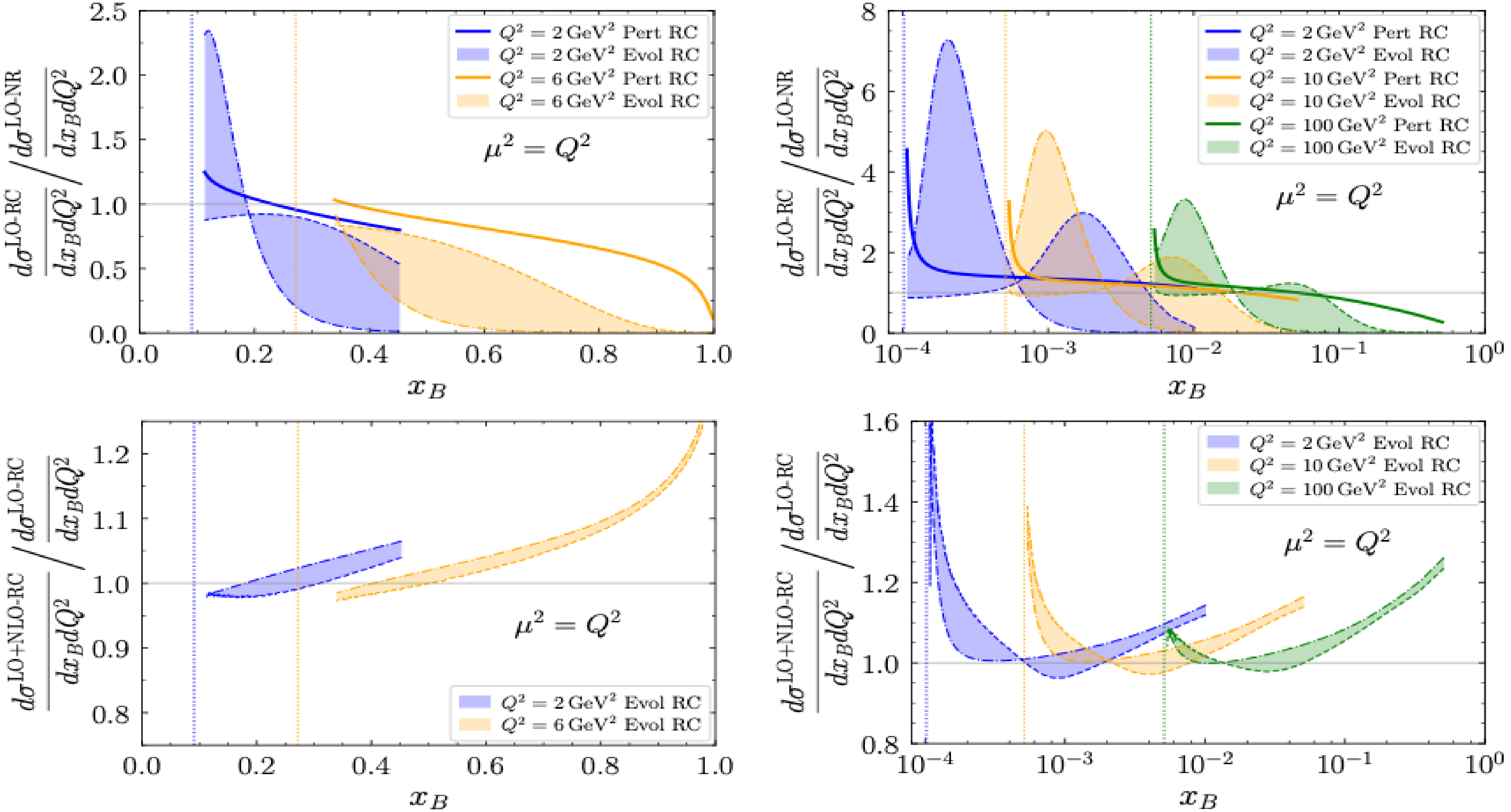
Cover the range of perturbative lepton distributions

PDFs: CTEQ CT18
not much difference from using other set of PDFs



Liu, Melnitchouk, Qiu, Sato, JHEP 11 (2021) 157

Impact of Factorized QED Contribution to Lepton-Hadron Scattering



Impact of Factorized QED Contribution to Lepton-Hadron Scattering

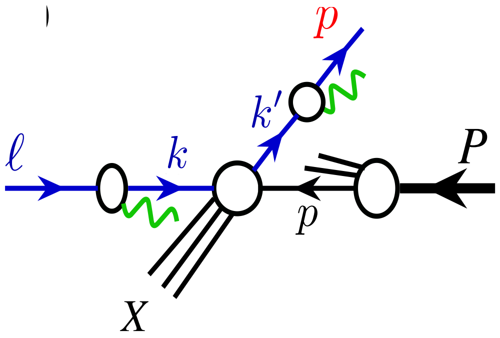
- Q^2 is NOT an ideal hard scale at small x_B and/or beyond LO in QED:

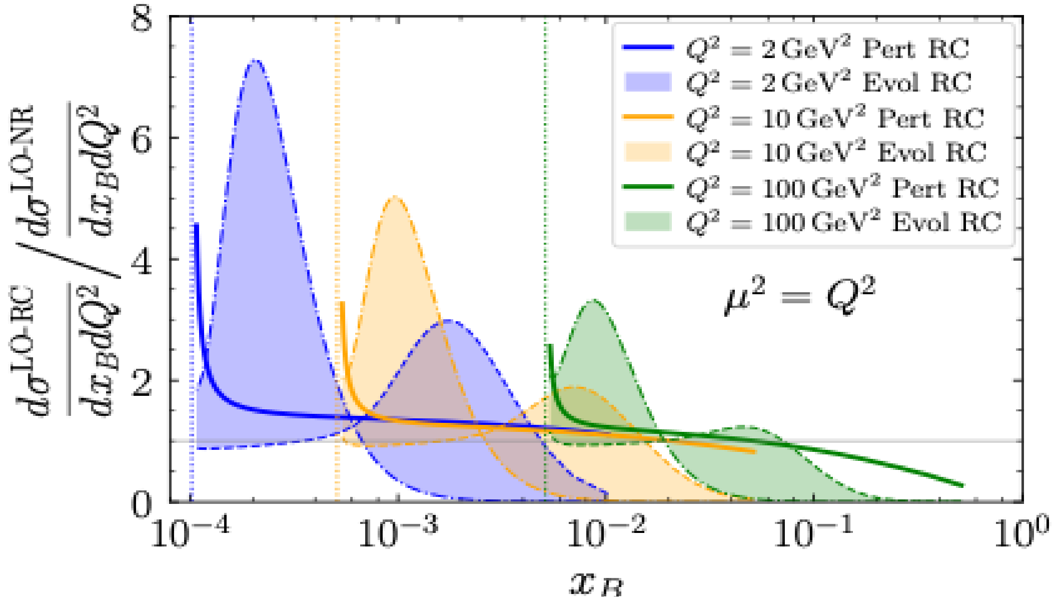
$$p_T^2 = Q^2(1-y) = Q^2\left(1-\frac{Q^2}{x_B s}\right)$$

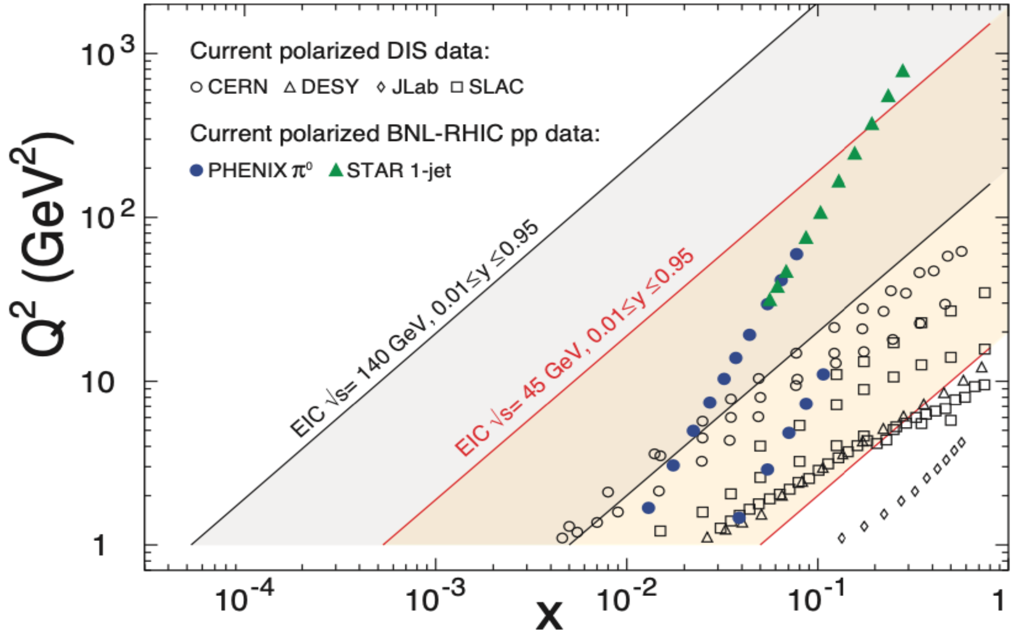
For EIC: $y \leq 0.95$

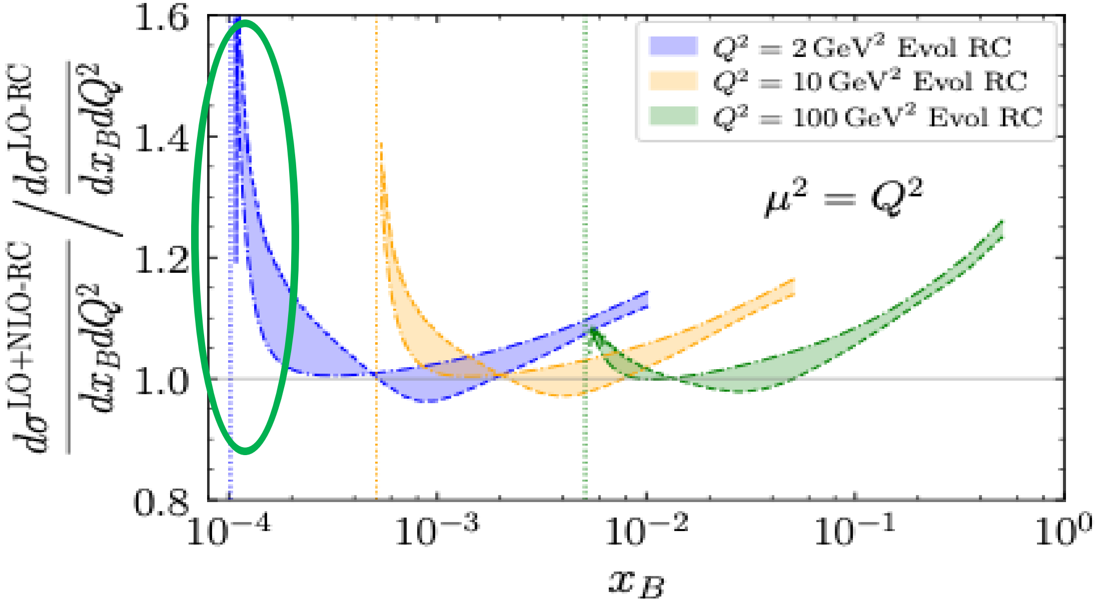
$$p_{T_{\rm min}}^2 = Q^2(1-y_{\rm max}) = \frac{Q^2}{20}!!!$$

Factorization does not work if p_T^2 is too small!









Impact on Extracting Non-perturbative Hadron Structure

❑ Additional universal & non-perturbative LDFs and LFFs:

- lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{\mathrm{EXP}}_{l+P\rightarrow l+X} = C_{l+k\rightarrow l+X} \otimes \boxed{\mathrm{PDF}_P} \otimes \boxed{\mathrm{LDF}_e} \otimes \boxed{\mathrm{LFF}_e} + \mathcal{O}(Q_s^2/Q^2)$$

$$\sigma^{\mathrm{EXP}}_{l+P\rightarrow l+H+X} = \boxed{C_{l+k\rightarrow l+k+X}} \otimes \boxed{\mathrm{PDF}_P} \otimes \boxed{\mathrm{FF}_H} \otimes \boxed{\mathrm{LDF}_e} \otimes \boxed{\mathrm{LFF}_e} + \mathcal{O}(Q_s^2/Q^2)$$

- hadron-hadron reactions (LHC)

$$\sigma^{\mathrm{EXP}}_{P+P\rightarrow l+\bar{l}+X} = \boxed{C_{k+k\rightarrow l+\bar{l}+X}} \otimes \boxed{\mathrm{PDF}_P} \otimes \boxed{\mathrm{PDF}_P} + O(Q_s^2/Q^2)$$

- lepton-lepton reactions (Belle)

$$\sigma^{\mathrm{EXP}}_{l+\bar{l}\rightarrow H+X} = \boxed{C_{l+\bar{l}\rightarrow k+X}} \otimes \boxed{\mathrm{FF}_H} \otimes \boxed{\mathrm{LDF}_e} \otimes \boxed{\mathrm{LDF}_{\bar{e}}} + \mathcal{O}(Q_s^2/Q^2)$$

❑ Additional physical processes sensitive to LDFs and/or LFFs:

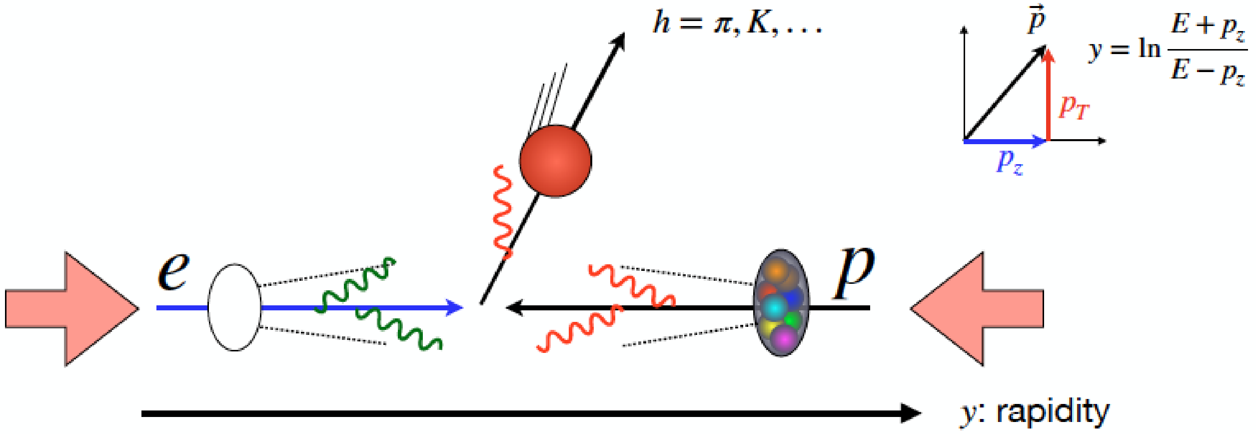
- lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{\mathrm{Exp}}_{l+P\rightarrow H+X} = \boxed{C_{l+k\rightarrow k+X}} \otimes \boxed{\mathrm{LDF}_s} \otimes \boxed{\mathrm{PDF}_P} \otimes \boxed{\mathrm{FF}_H} + \mathcal{O}(Q_s^2/Q^2)$$

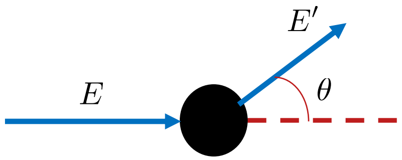
Prompt Single Hadron/Jet Production:

❑ Single hadron (Jet) production in lepton-hadron scattering:

Kang, Meta, Qiu, Zhou, PRD 2011
Hinderer, Schlegel, Vogelsang, PRD 2015, 2016
Abelof, Boughezal, Liu, Petriello, PLB, 2016
Qiu, Wang, Xing, CPL, 2021
Qiu, Watanabe, in preparation



JLab kinematics – fixed target:



$$p_T = E' \sin(\theta) \gtrsim 1 \text{ GeV?}$$

Factorization:

$$E\frac{d\sigma_{\ell P\to pX}}{d^3p} \approx \frac{1}{2s}\sum_{i,a,b}\int_{z_{\min}}^{1}\frac{dz}{z^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{h/b}(z,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to bX}(\xi\ell, xP, p/z, \mu^2) + (1/p_T)^\alpha$$

Single hard scale
Collinear factorization

Nayak, Qiu, Sterman, PRD72, 114012 (2005)

Hard parts are valid at NLO
Partially available at NNLO
in QCD

The new unknown is $f_{i/e}(\xi,\mu^2)$
lepton distribution functions (LDFs)

Hadron fragmentation functions (FFs):

Known, but, limited knowledge, in particular, at large z !

Is also a pre-requisite for applying the factorization for SIDIS - validity of fragmentation

Evolution of lepton distribution functions (LDFs)

❑ Modified DGLAP equation for LDFs:

Qiu, Watanabe
In preparation

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix} = \left(\begin{array}{ccc|ccc} P_{ee}^{(1,0)} & P_{e\bar{e}}^{(2,0)} & P_{e\gamma}^{(1,0)} & P_{eq}^{(2,0)} & P_{e\bar{q}}^{(2,0)} & P_{eg}^{(2,1)} \\ P_{\bar{e}e}^{(2,0)} & P_{\bar{e}\bar{e}}^{(1,0)} & P_{\bar{e}\gamma}^{(1,0)} & P_{\bar{e}q}^{(2,0)} & P_{\bar{e}\bar{q}}^{(2,0)} & P_{\bar{e}g}^{(2,1)} \\ P_{\gamma e}^{(1,0)} & P_{\gamma\bar{e}}^{(1,0)} & P_{\gamma\gamma}^{(1,0)} & P_{\gamma q}^{(1,0)} & P_{\gamma\bar{q}}^{(1,0)} & P_{\gamma g}^{(1,1)} \\ \hline P_{qe}^{(2,0)} & P_{q\bar{e}}^{(2,0)} & P_{q\gamma}^{(1,0)} & P_{qq}^{(0,1)} & P_{q\bar{q}}^{(0,2)} & P_{qg}^{(0,1)} \\ P_{\bar{q}e}^{(2,0)} & P_{\bar{q}\bar{e}}^{(2,0)} & P_{\bar{q}\gamma}^{(1,0)} & P_{\bar{q}q}^{(0,2)} & P_{\bar{q}\bar{q}}^{(0,1)} & P_{\bar{q}g}^{(0,1)} \\ P_{ge}^{(2,1)} & P_{g\bar{e}}^{(2,1)} & P_{g\gamma}^{(1,1)} & P_{gq}^{(0,1)} & P_{g\bar{q}}^{(0,1)} & P_{gg}^{(0,1)} \end{array}\right) \otimes \begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix}$$

- **Factorization scale:**

  $$\mu^2 \sim m_c^2$$

- **Input LDFs at μ^2:**
  - **Perturbatively generated by solving QED evolution from lepton mass threshold**
  - **With perturbatively calculated fixed-order MSbar LDFs**
  - **Test the size of non-perturbative hadronic contribution**
  - **…**

Evolution kernels in both QCD and QED:

$$P_{ij}(\xi,\mu^2) = \sum_{n,m=0}^{\infty} \left(\frac{\alpha_{em}(\mu^2)}{2\pi}\right)^n \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^m \hat{P}_{ij}^{(n,m)}(\xi) = \sum_{n,m=0}^{\infty} P_{ij}^{(n,m)}(\xi,\mu^2)$$

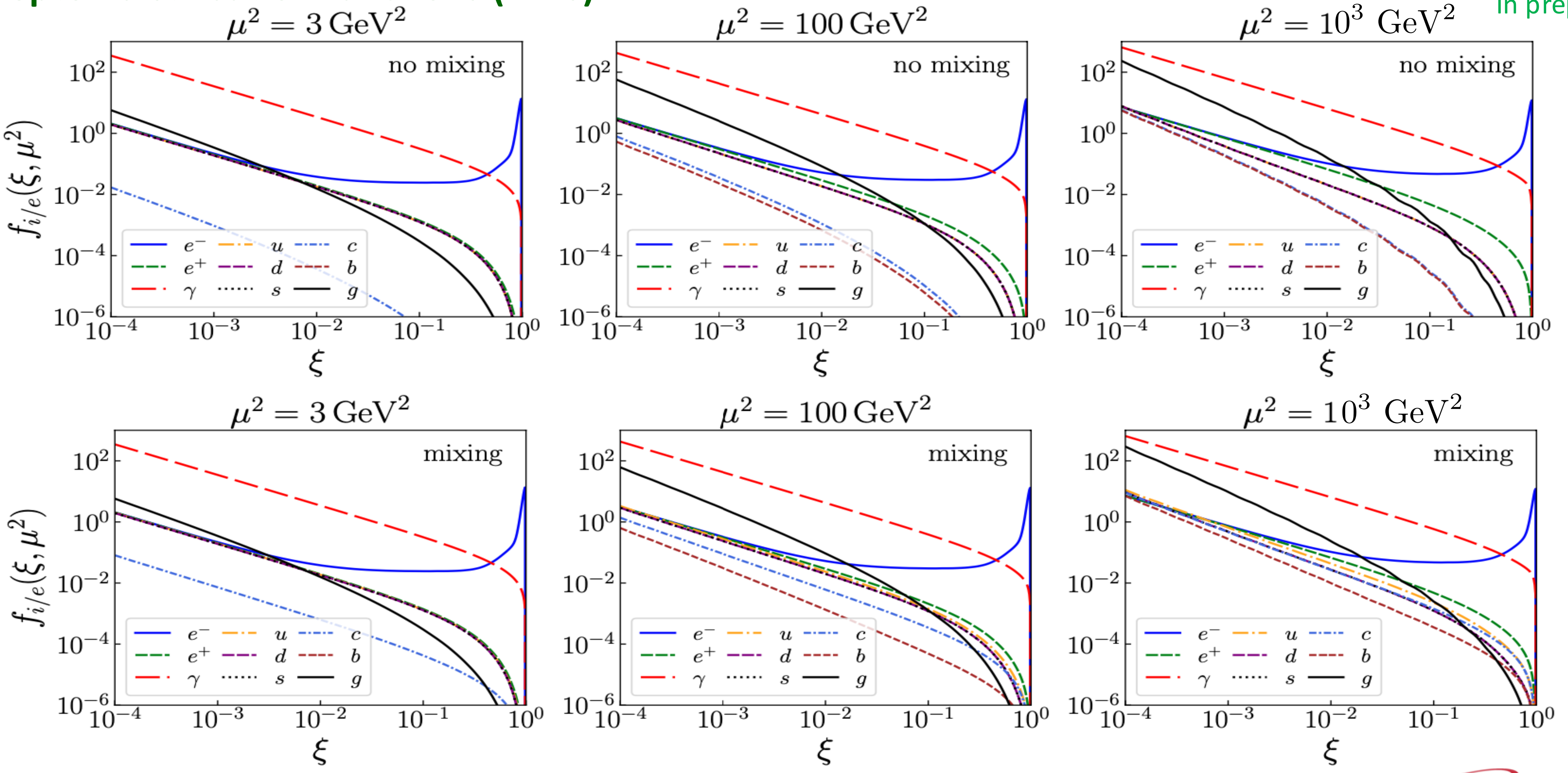
with $\quad P_{ij}^{(0,0)} = 0\,, \quad N_F\,, \quad N_l$

The QCD/QED mixing is critically important for canceling all collinear divergence between LO!

Evolution of lepton distribution functions (LDFs)

❑ Lepton distribution functions (LDFs):

Qiu, Watanabe
In preparation



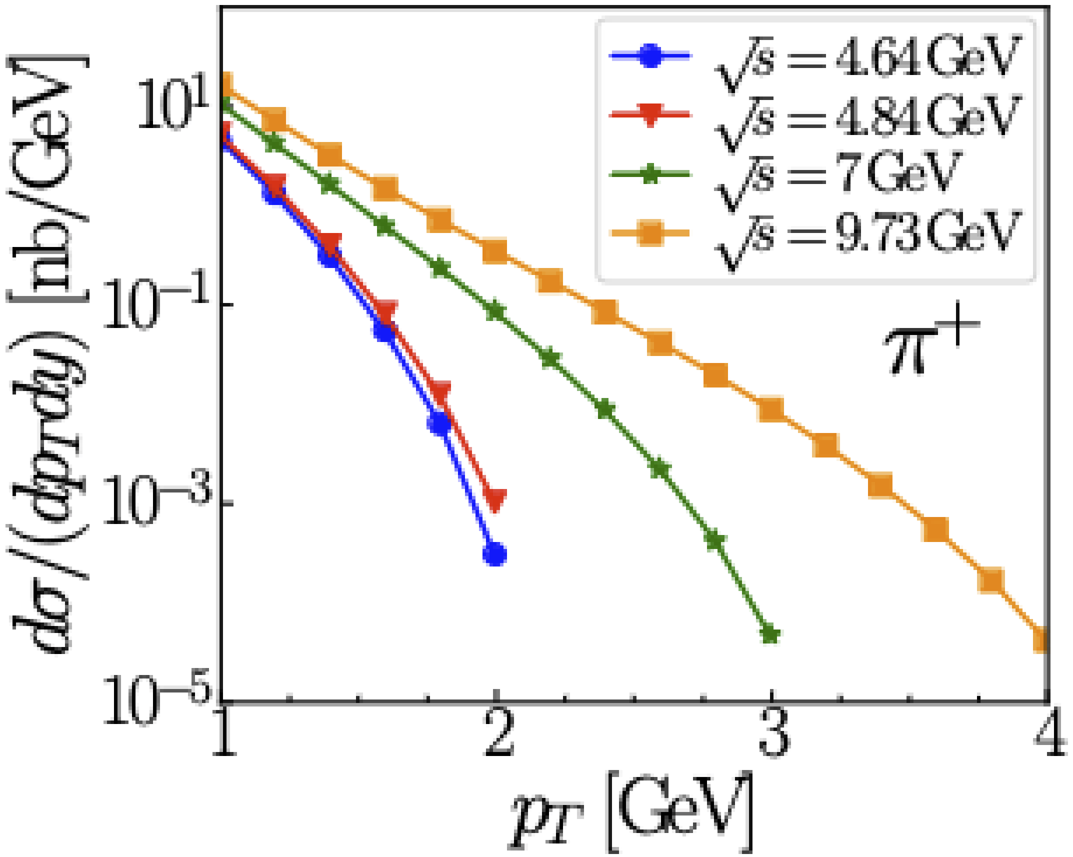
With LDFs, we calculated single hadron production, including J/ψ production at the EIC

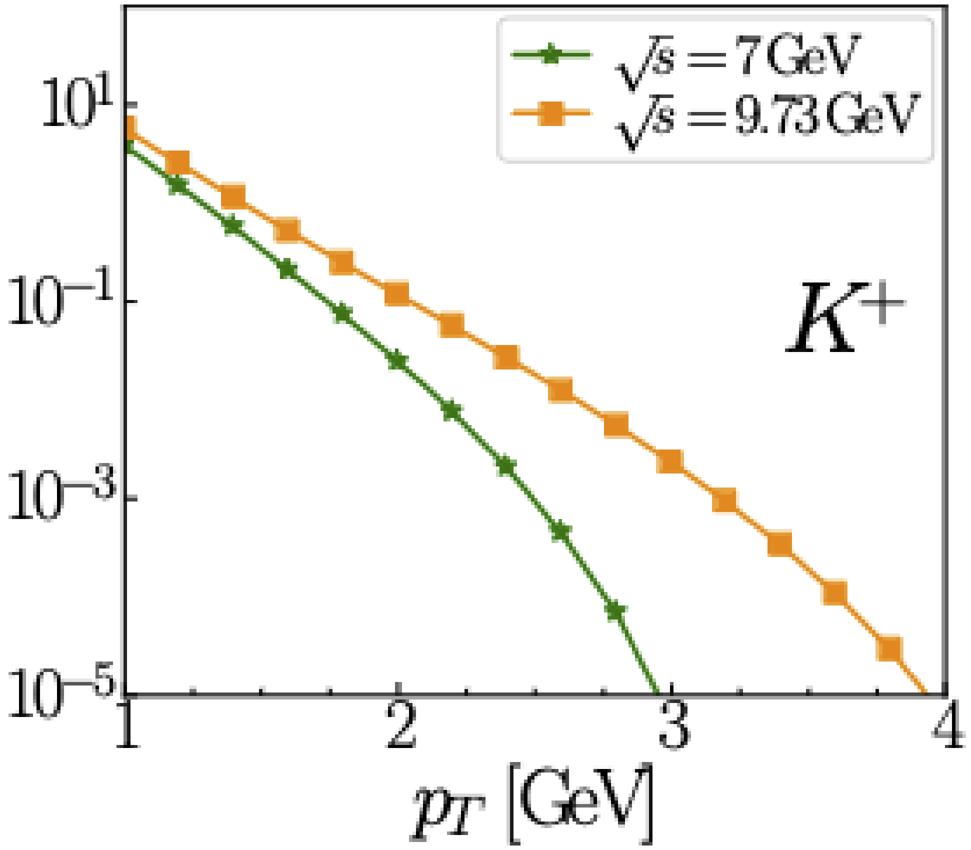
Calculations for various Fixed Target Energies

- **FFs – JAM20:**

Qiu, Watanabe
In preparation

Preliminary





Explore Uncertainties of Fragmentation Functions

Qiu, Watanabe
In preparation

- **Parameterize the JAM20 FFs:**

Preliminary

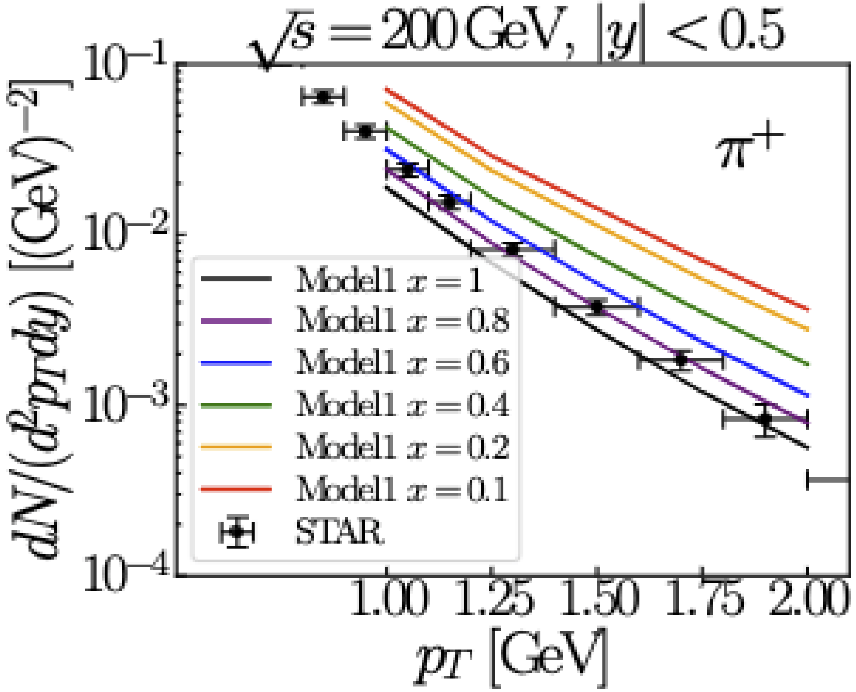
$$(\text{Model1}): zD_i^{\pi^+}(z,x) = N_i \frac{z^{\alpha_i}(1-z)^{\beta_i x}}{B[1+\alpha_i, 1+\beta_i x]},$$

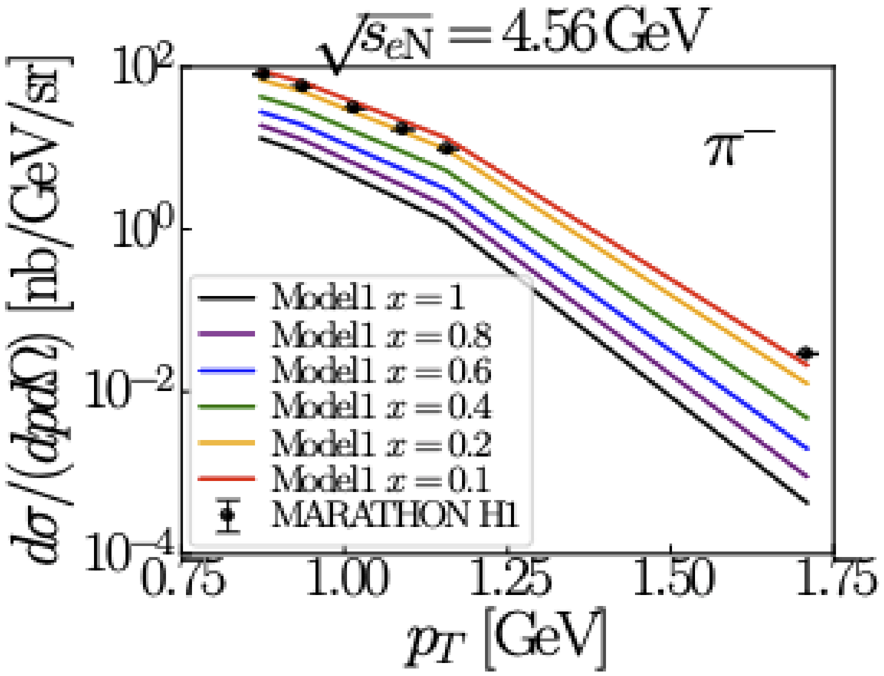
$$(\text{Model2}): zD_i^{\pi^+}(z,x) = N_i \frac{z^{\alpha_i}(1-z)^{\beta_i x}(1+c_i z^{\gamma_i})}{B[1+\alpha_i, 1+\beta_i x] + c_i B[1+\alpha_i+\gamma_i, 1+\beta_i x]}.$$

With a parameter: $x \in [0,1]$

- Smaller x = Larger contribution from large z region
- e+e- has weak constraint on large z

- **Compare with data:**





Role of power corrections

Jefferson Lab

Summary and Outlook – Thank you!

- Proposed to use the prompt single hadron inclusive production to test the fragmentation picture in particular at JLab energy – Prerequisite to fitting SIDIS data with current factorization formalism
  - Current fragmentation functions with the NLO perturbative coefficients can fit the RHIC data
  - But, have a difficulty to fit the JLab data, which could have included "non-prompt" pions (those from rho decay, …)
  - Same problem is much less important due the more steep falling spectrum of rho and other VMs.

  Need to consistently subtract the "background" from "non-prompt" source of "leading hadron" when studying SIDIS at JLab energies

- Collision induced QED radiation is an integrated part of the lepton-hadron collision
  - Radiative correction approach is difficult for a consistent treatment beyond the inclusive DIS
  - No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC

  Factorization approach to include both QCD and QED radiative contributions provides a consistent and controllable approximation to high-energy lepton-hadron scattering processes