Lattice Calculations of Nucleon Electromagnetic Form Factors at High Momentum Q^2

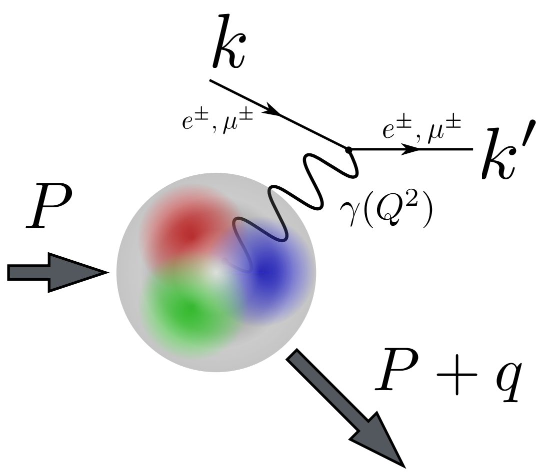
Sergey Syritsyn (Stony Brook University)
with M.Engelhardt, J.Green, S.Krieg,
S.Meinel, A.Pochinsky, J.Negele
(Lattice Hadron Physics collaboration)

JLab Hall A/C Collaboration meeting

Outline

- Large-Q^2 Nucleon vector form factors on a lattice
  - *Motivation*
  - *Methodology*
  - *Challenges*
- Dominant (Quark-Connected) contributions
  - *Status*
  - *Examining systematic effects : excited states, discretization, pion-mass*
- Other systematics: Quark-Disconnected contributions
  - *Estimating impact on FFs and ratios*
- Summary & Outlook

Nucleon Elastic E&M Form Factors



$$Q^2 = -(k-k')^2 > 0$$

Elastic e^-p amplitude

$$\langle P+q|\, \bar{q}\gamma^\mu q\, |P\rangle = \bar{U}_{P+q}\Big[\underbrace{F_1(Q^2)}_{\text{(Dirac)}}\gamma^\mu + \underbrace{F_2(Q^2)}_{\text{(Pauli)}}\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

Sachs Electric $G_E(Q^2) = F_1(Q^2) - \dfrac{Q^2}{4M^2}F_2(Q^2)$

Magnetic $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

Elastic e^-p cross-section

- $G_{E,M}$ from ϵ-dep. at fixed $\tau(Q^2)$ ("***Rosenbluth separation***")
- dominated by G_M at large Q^2
- 2γ corrections at $Q^2 \gtrsim 1$ GeV^2

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{Mott}}}{1+\tau}\left[G_E^2 + \frac{\tau}{\epsilon}G_M^2\right]$$

$$\tau = \frac{Q^2}{4M_N^2} \qquad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

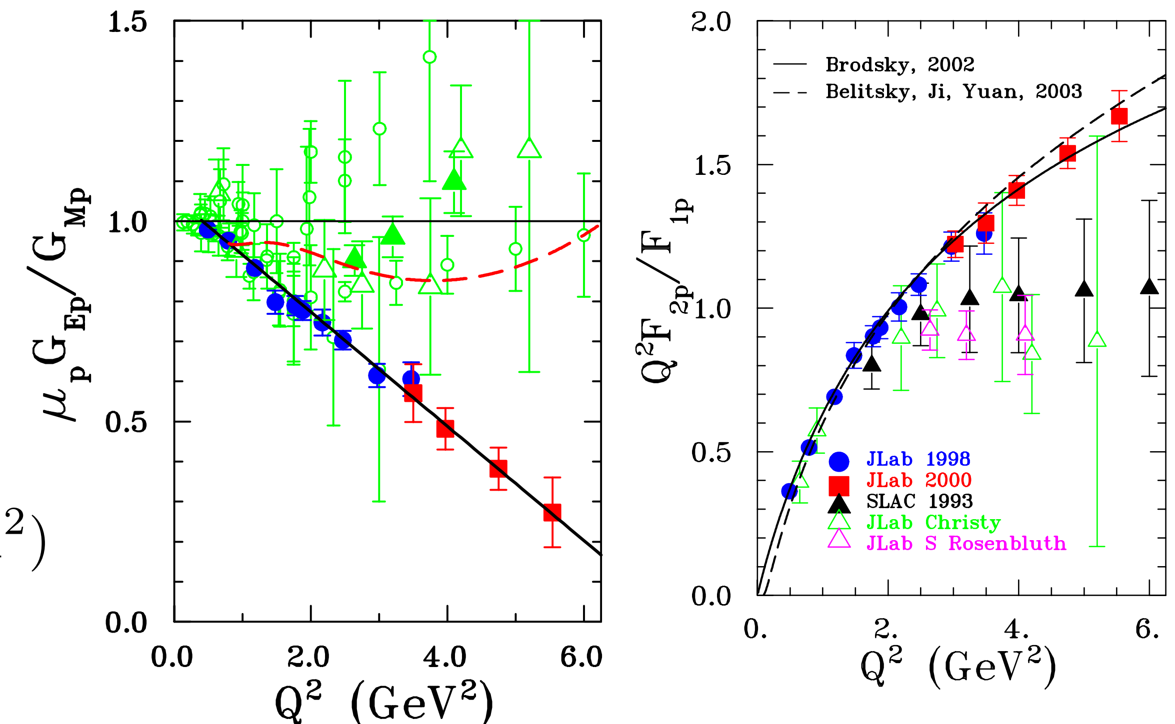
Polarization transfer: polarized e^-beam
+ detect polarization of recoil nucleon
(alt.: transverse asymmetry on pol. target)

- G_E/G_M *ratio* (only small radiative corrections)

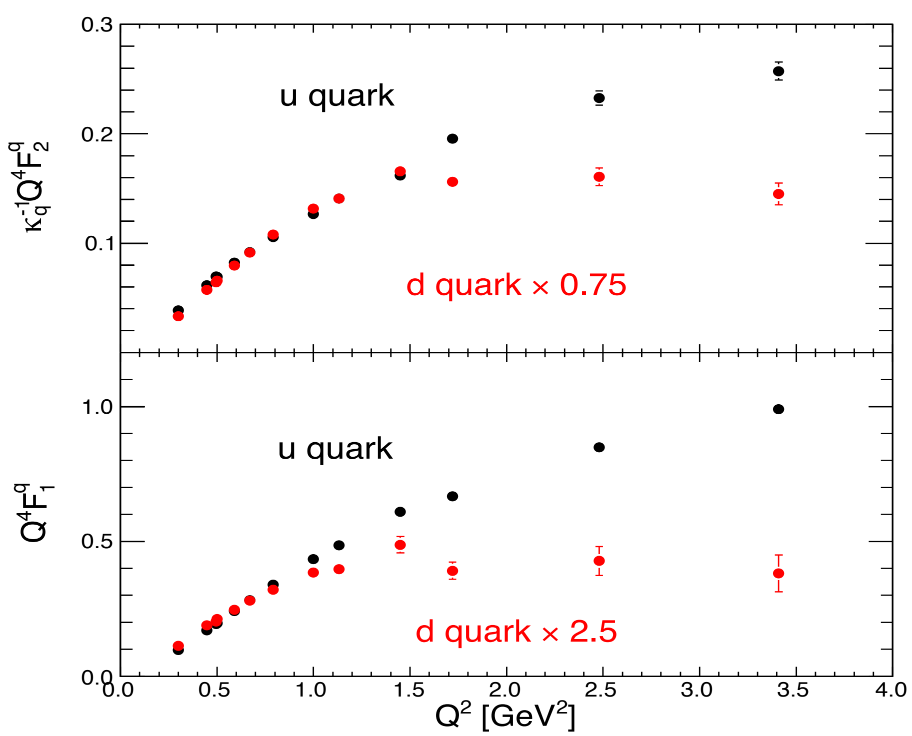
$$P_t/P_l \propto G_E/G_M$$

Why Large-Q^2 Nucleon Form Factors?

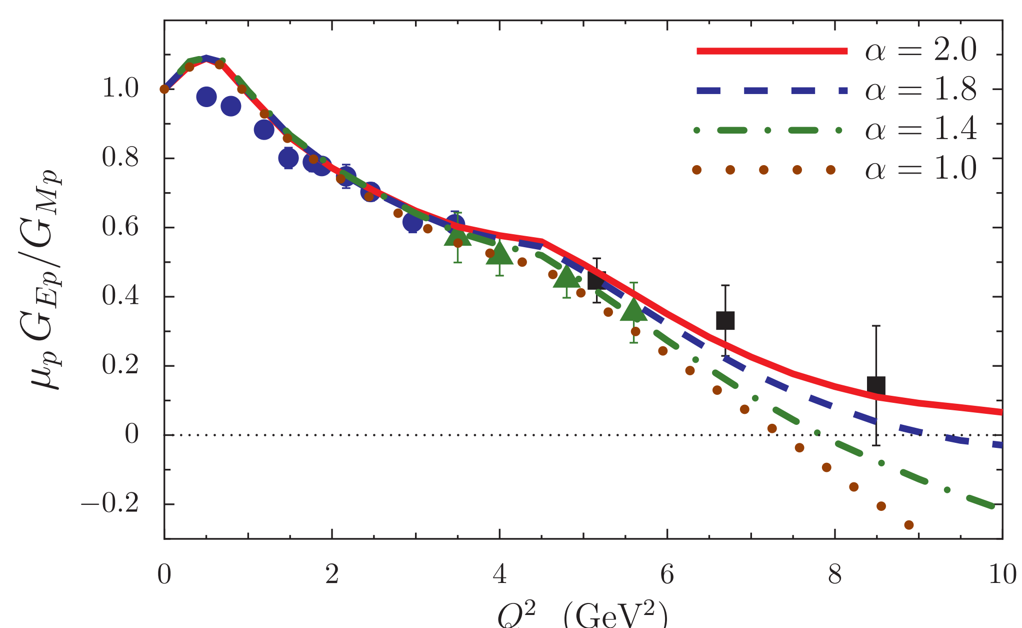
- *Rosenbluth / Pol.transfer discrepancy in G_E/G_M ?*
  Input from lattice QCD may help
- *Role of diquark correlations in elastic scattering ?*
  Neutron & proton G_E/G_M at/above $Q^2 = 8$ GeV^2
- *Scale of transition to perturbative QCD ?*
  (F_2/F_1) scaling at large Q^2 : $\quad Q^2 F_{2p}/F_{1p} \overset{?}{\propto} \log^2(Q^2/\Lambda^2)$
- *What are contributions from u and d flavors?*
  Proton <u>and</u> neutron data needed in wide Q^2 range



[Research Mgmt. Plan for SBS(JLab Hall A)]

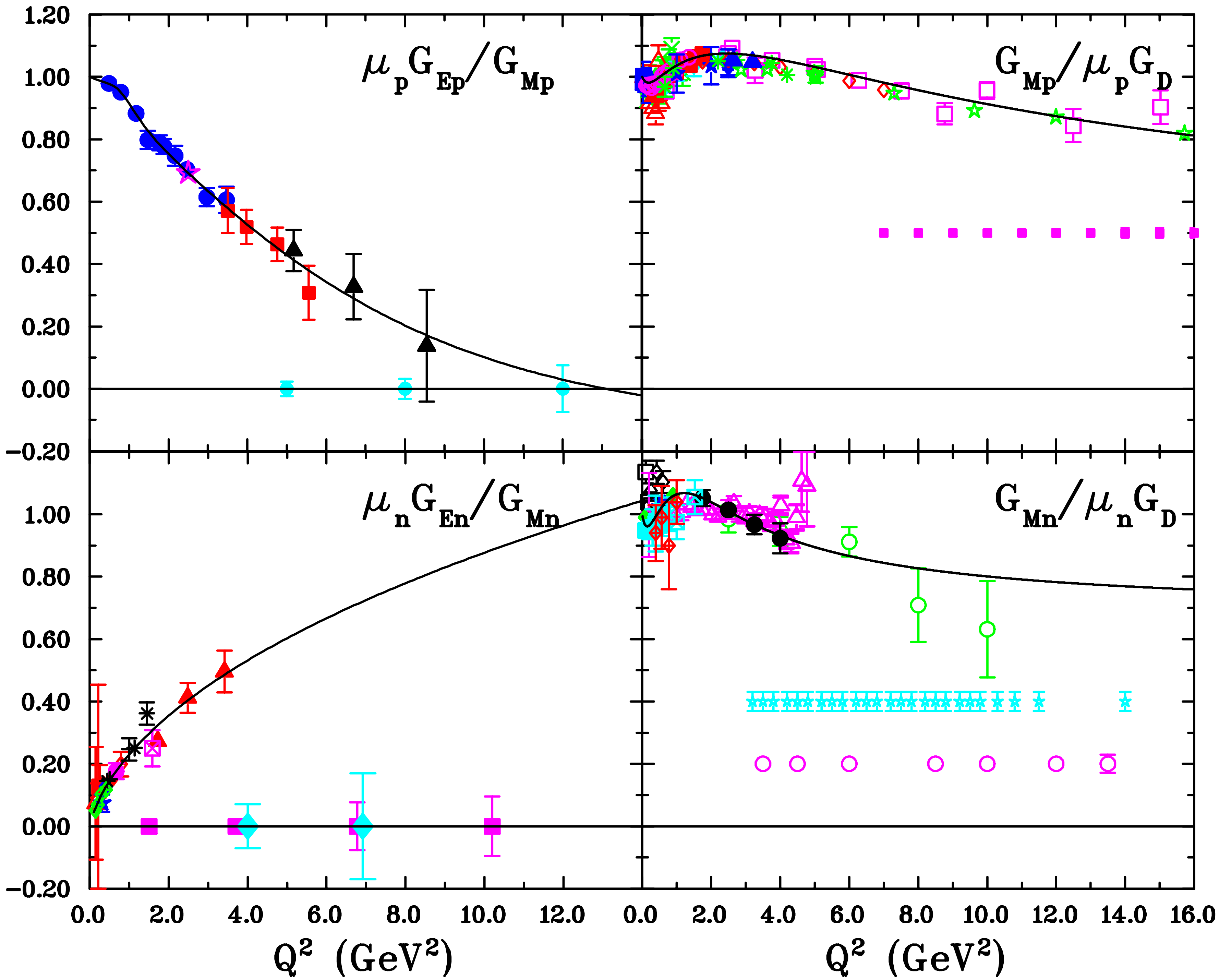


[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]



Dyson-Schwinger Eqns : quark + $0^+/1^+$ diquark
($\alpha \approx$ rate of transition const.quarks → pQCD with Q^2)
[Cloet, Roberts, Prog.Part.Nucl.Phys 77:1 (2014)]

Recent Experiments @JLab : Projections

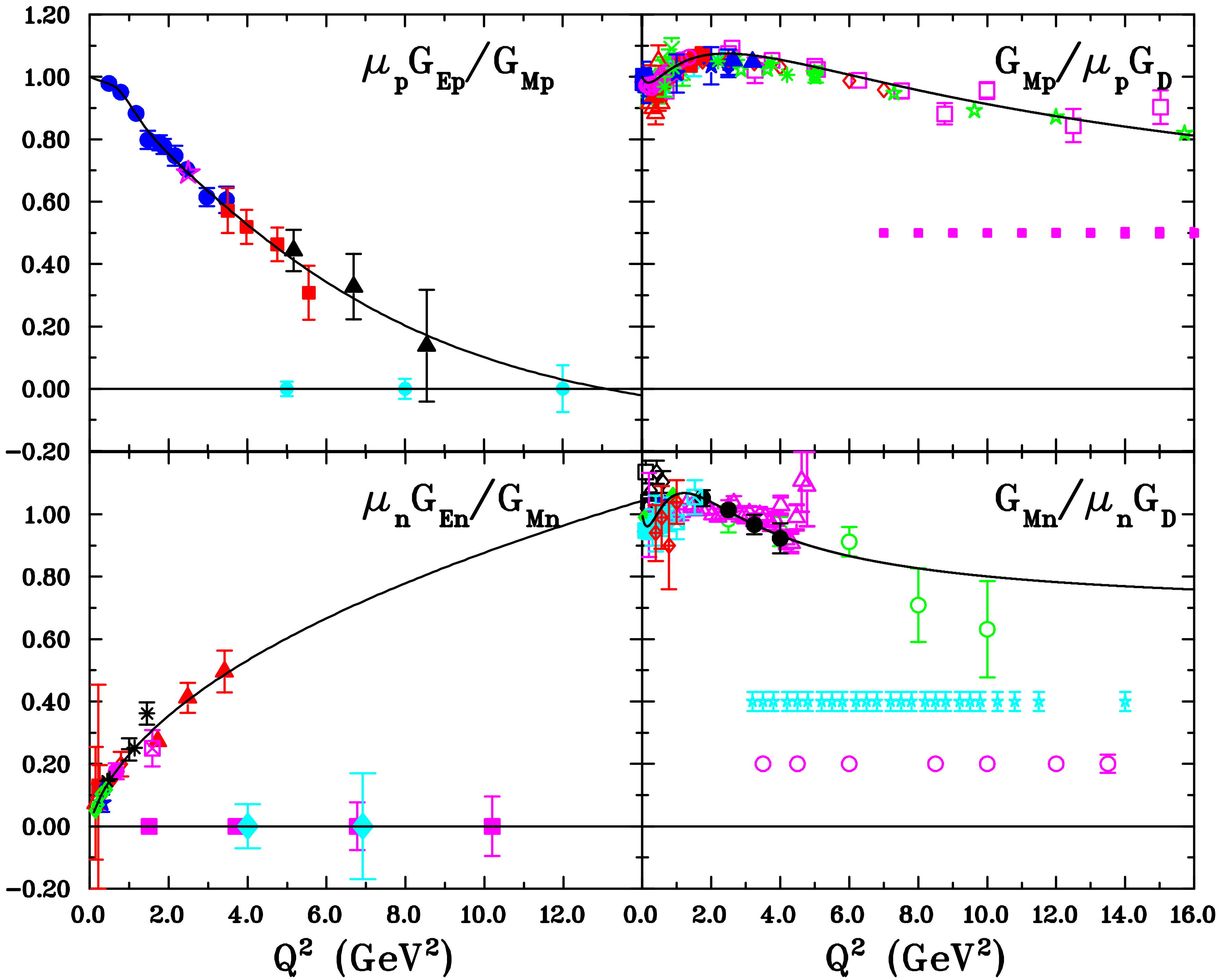


Projected new precision on proton & neutron form factors
[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]

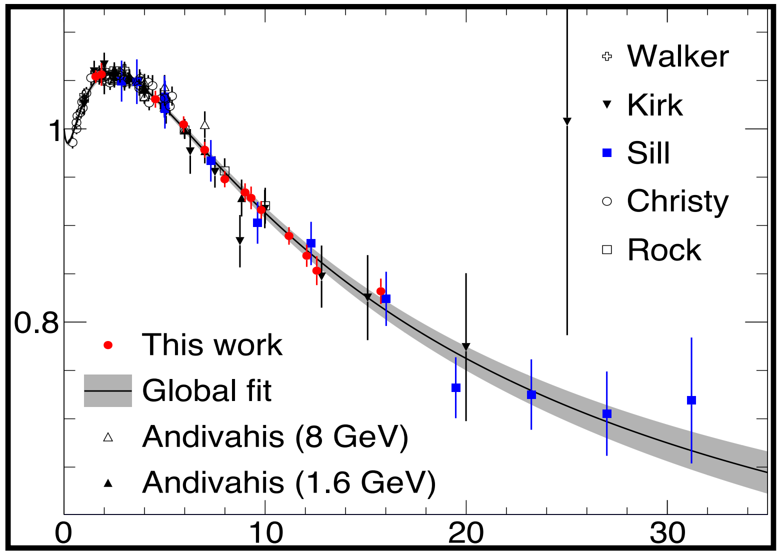
Experiments at JLab@12GeV

- Hall A (HRS, SBS):
  G_{Mp} @ $Q^2 \lesssim 17.5$ GeV^2
  G_{Ep}/G_{Mp} @ $Q^2 \lesssim 15$ GeV^2 ;
  G_{Mn} @ $Q^2 \lesssim 18$ GeV^2
  G_{En}/G_{Mn} @ $Q^2 \lesssim 10.2$ GeV^2 ;
- Hall B (CLAS12):
  G_{Mn} @ $Q^2 \lesssim 14$ GeV^2
- Hall C :
  G_{En}/G_{Mn} @ $Q^2 \lesssim 6.9$ GeV^2

Recent Experiments: Projections + G_{Mp} Result



Projected new precision on proton & neutron form factors
[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]



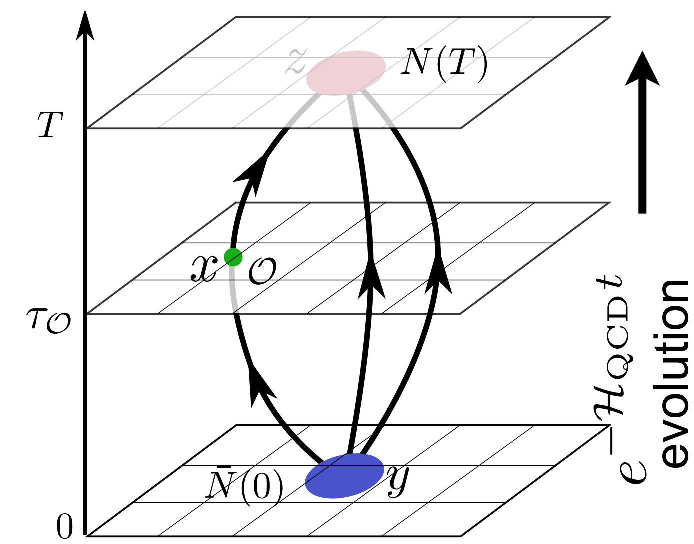
New G_{Mp} data from Hall A
[Christy et al, PRL'22]

Nucleon Structure from Lattice QCD (in a Nutshell)

- Compute nucleon correlation functions

<u>Hadron Spectrum:</u>

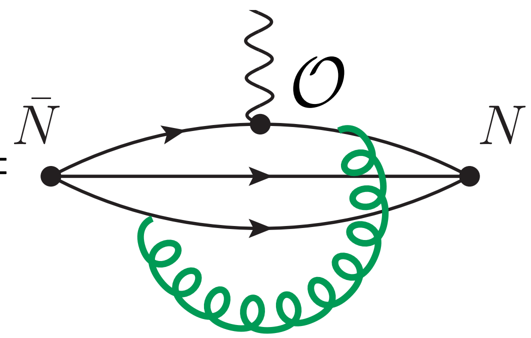
$$\langle N(\vec{p},T)\,\bar{N}(0)\rangle = \sum_{\vec{y}} \langle N(\vec{y},T)\,\bar{N}(0)\rangle$$
$$\sim |Z_0|^2 e^{-E_0 T}\left[1 + e^{-\Delta E_{10} T} + \dots\right]$$



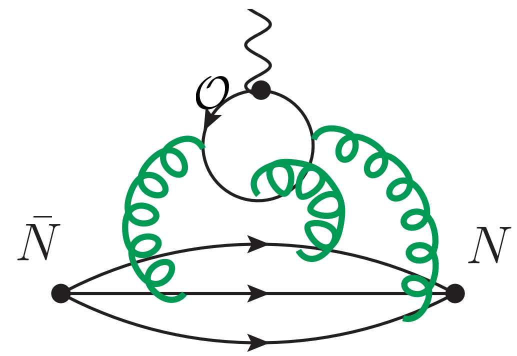
Quark lines = $(\not{D} + m)^{-1} \cdot \psi$

<u>Hadron Matrix Elements</u>

$$\langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle =$$



+



quark-connected **quark-disconnected**

- Extract ground state matrix elements

$$\langle N(T)\mathcal{O}(\tau)N(0)\rangle = \sum_{n,m} Z_m e^{-E_n(T-\tau)} \langle n|\mathcal{O}|m\rangle e^{-E_m\tau} Z_n^*$$
$$\xrightarrow{T\to\infty} Z_0 Z_{0'}^* e^{-E_0\tau - E_{0'}(T-\tau)}\Big[\langle 0'|\mathcal{O}|0\rangle + \underbrace{O\big(e^{-\Delta E\{T,\tau,(T-\tau)\}}\big)}_{\text{excited states}}\Big]$$

Ground state form factors

$$[\bar{u}'\gamma^\mu u]F_1 + [\bar{u}'\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}u]F_2$$

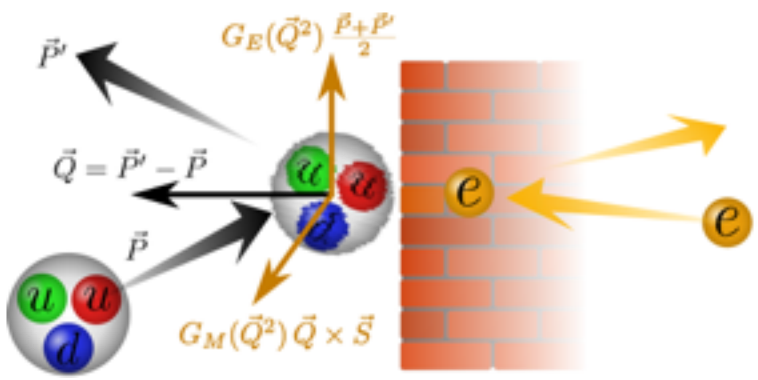
Fit and discard

Systematic uncertainties
- discretization effects
- unphysical-heavy pion mass
- finite volume
- excited state contributions
- quark-disconnected diagrams

Challenges at Large Q^2 on a Lattice

- Excited-state gaps shrink ⟹ **more contamination**
  - in Breit frame : $p_{in} = p_{out} \approx 1.6$ GeV for $Q^2 = 10$ GeV^2
  - $E_1 - E_0 = \sqrt{M_1^2 + \vec p^2} - \sqrt{M_2^2 + \vec p^2} < M_1 - M_0$

    N(~1500): ⇒ $\Delta E = 600 \to 360$ MeV



- Stochastic noise grows faster with T [Lepage'89]:

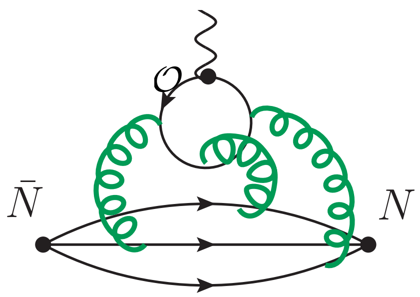
Signal	$\langle N(T)\bar N(0)\rangle$	$\sim e^{-E_N T}$
Noise	$\langle \lvert N(T)\bar N(0)\rvert^2\rangle - \lvert\langle N(T)\bar N(0)\rangle\rvert^2$	$\sim e^{-3m_\pi T}$
Signal/Noise		$\sim e^{-(E_N - \frac{3}{2}m_\pi)T}$

SNR / SNR[static] ~ 1 / 100
per Δt = 1 fm/c "time"
[$Q^2 \approx 10$ GeV^2, phys.quarks]

- Discretization effects:
  O(a) Correction to current operator
  expected to grow with momentum

$$(V_\mu)_I = [\bar q \gamma_\mu q] + c_V\, a\, \underbrace{\partial_\nu [\bar q i \sigma_{\mu\nu} q]}_{\propto Q}$$

- Quark-disconnected contributions:
  negligible (≲1%) at $Q^2 \le 1$ GeV^2, ***unknown at large Q²***



- No reliable EFT/ChPT for pion mass-, volume-extrapolation at large momentum p_N, Q

Many of the same challenges as for parton physics on a lattice
(LaMET , Quasi-/Pseudo- PDFs/DAs/TMDs)

Present QCD Calculation Parameters

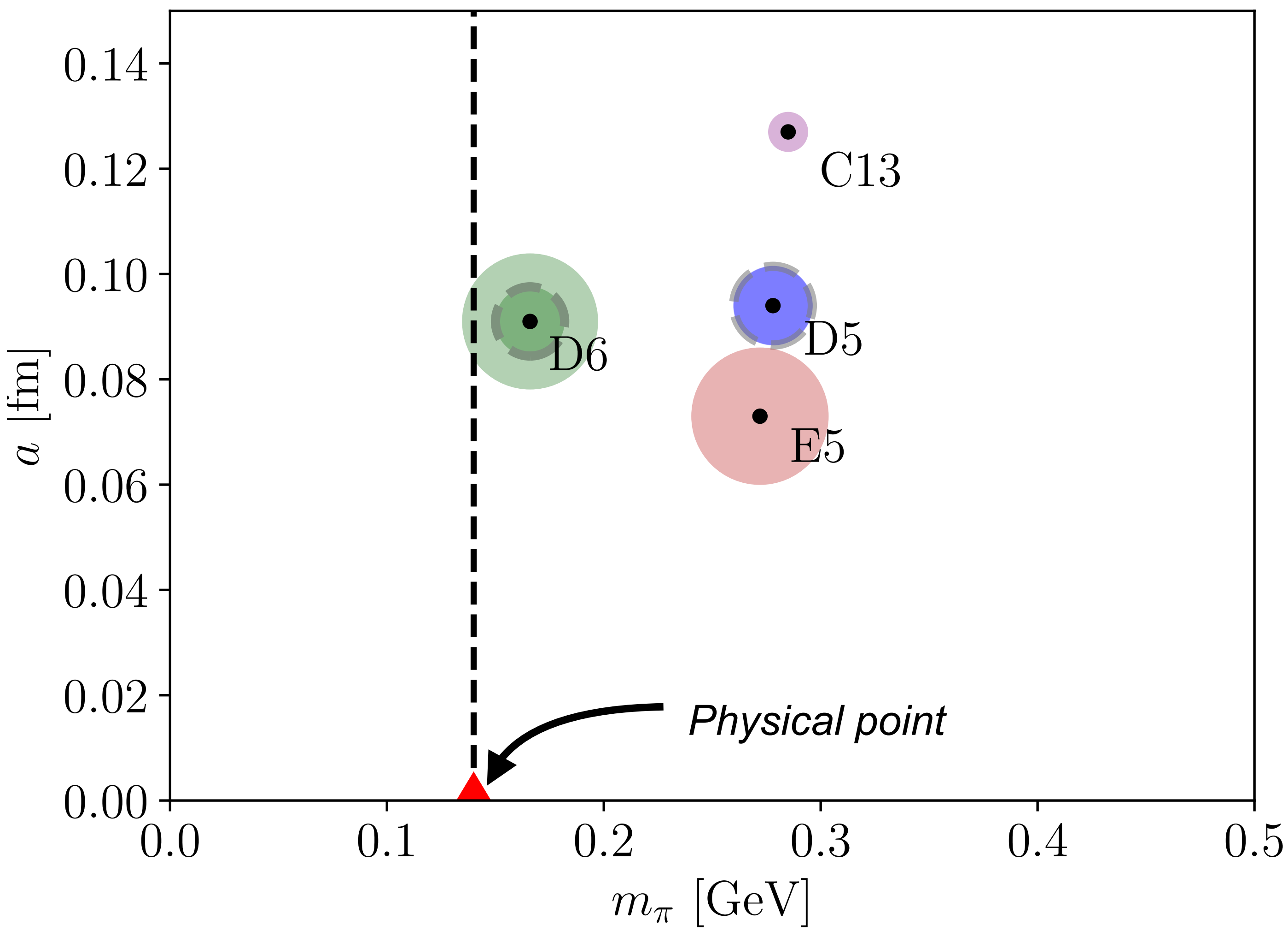
QCD action with $N_F = 2{+}1$ O(a^2)-improved dynamic quarks

Many thanks to [JLab / W&M / LANL / MIT]

- Lattice spacing $a \approx 0.073 \div 0.091$ fm
- SU(2)$_f$ -symmetric + strange quarks
- Pion mass $m\pi = 170 \div 280$ MeV
- Physical volume $L \gtrsim 3.7\,(m\pi)^{-1}$
- Euclidean time t_{sep}= 0.5 ÷ 1.1 fm

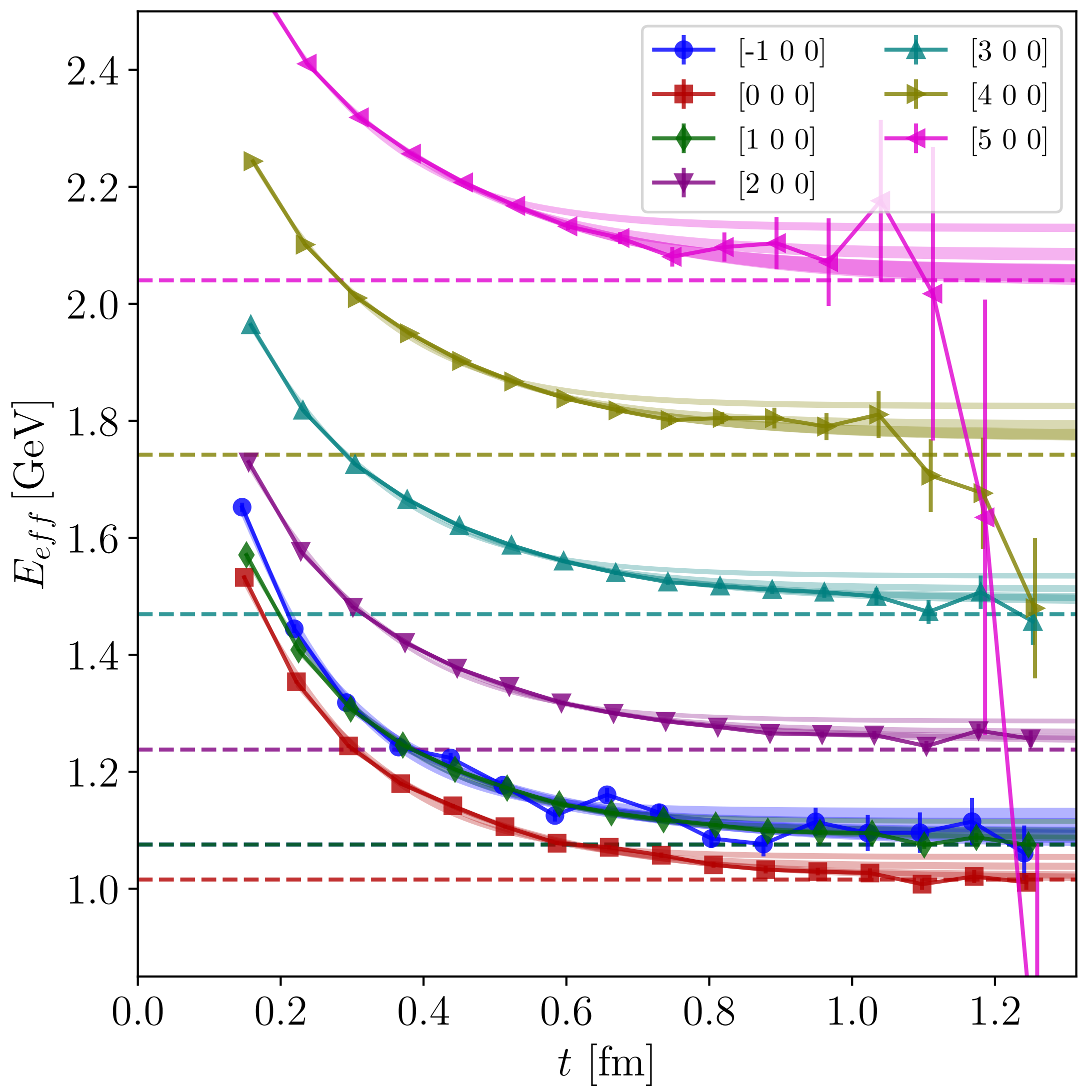
2022/24:

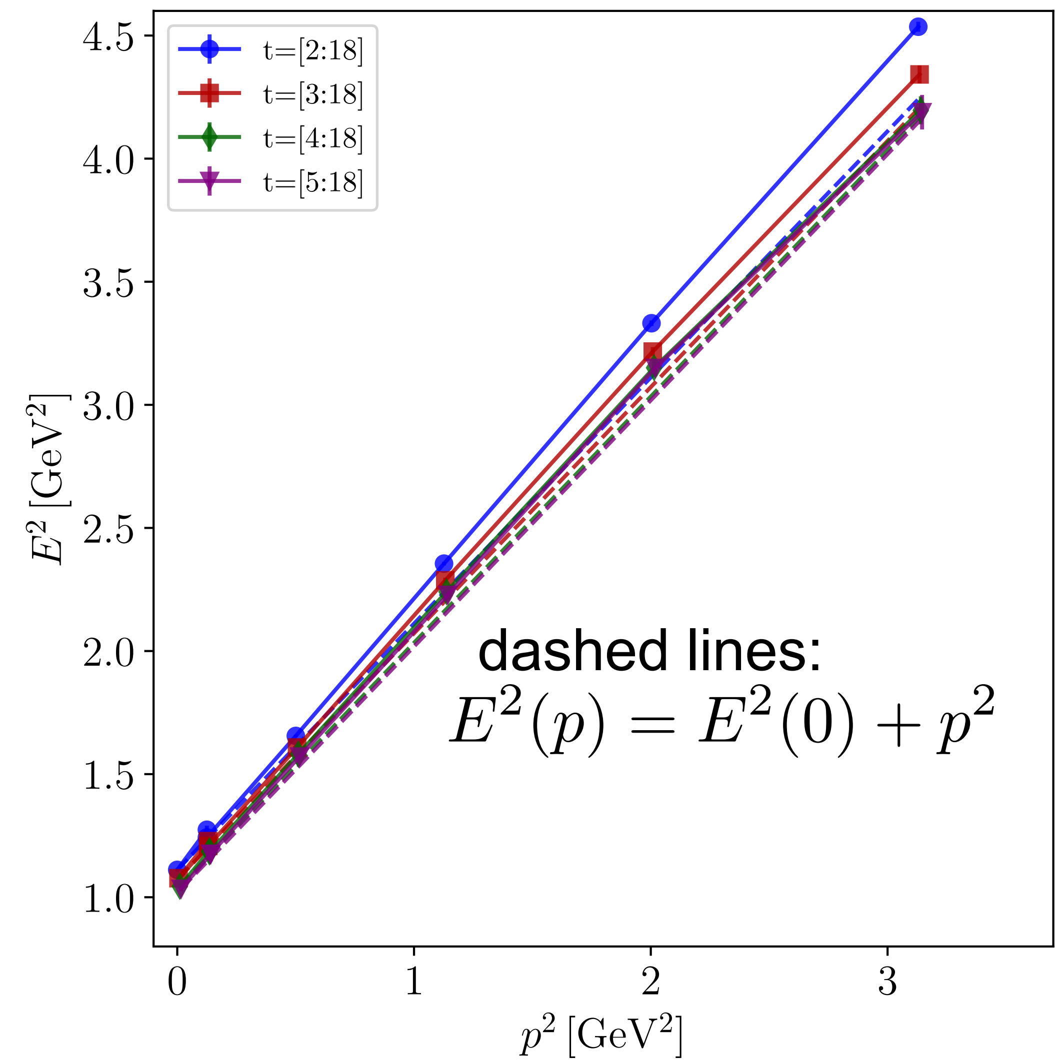
- MC Statistics ≳250k on D6 (48^3 x 96), E5 (48^3 x 128)
- Disconnected contractions on D5, D6 (1200+ configs)



Lattice Nucleon Energy & Dispersion Relation (E5)

- E5 : mπ = 272 MeV , spacing a = 0.073 fm , 266k MC samples
- Ground-state energy follows disp.relation for fits with $t_{sep} \gtrsim$ **0.3 fm**





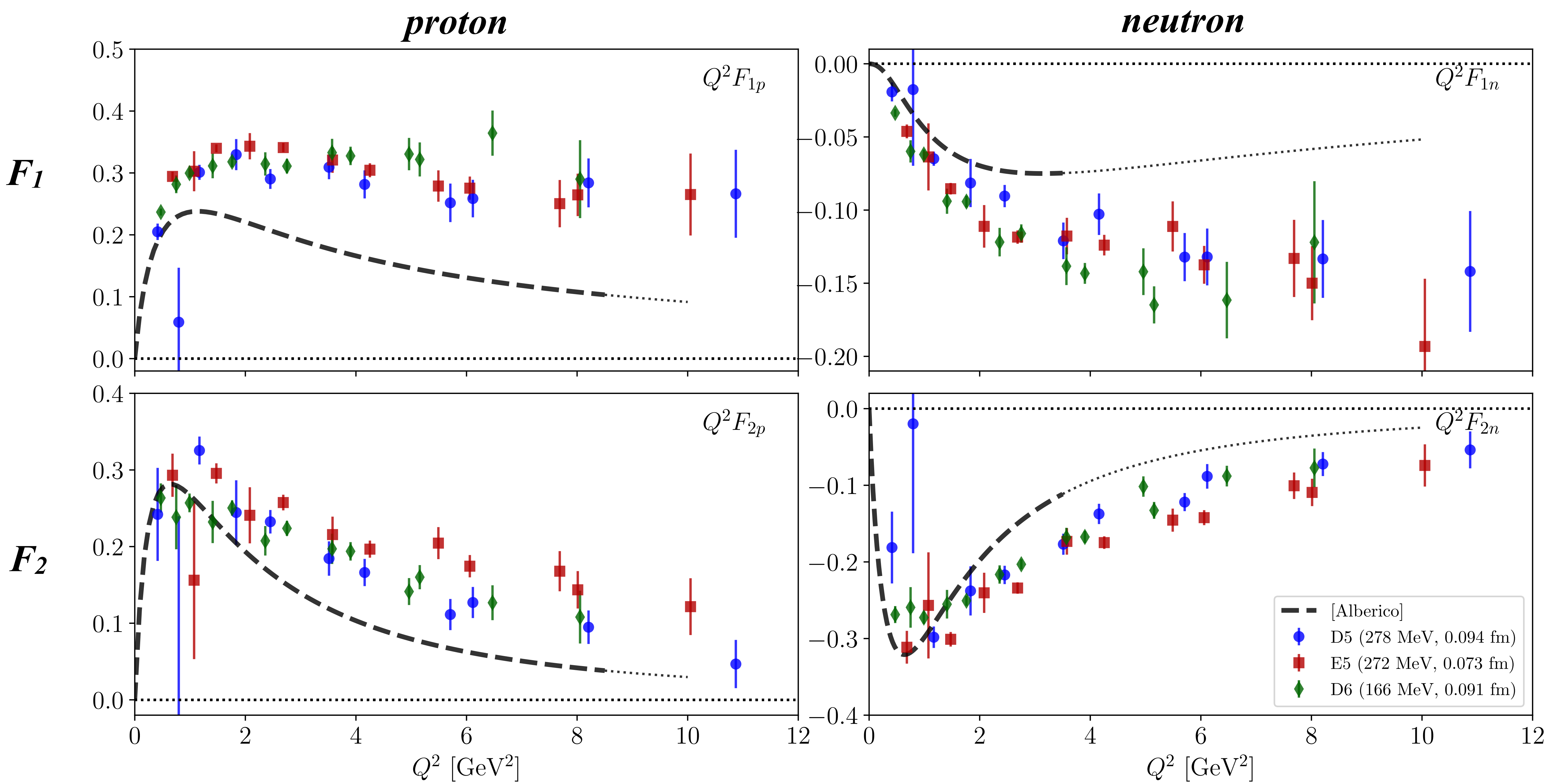
- 2-state fits compared to Effective energy

$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$

- Lattice energies vs. continuum dispersion relation (dashed)

Nucleon Form Factors: Ensemble Comparison

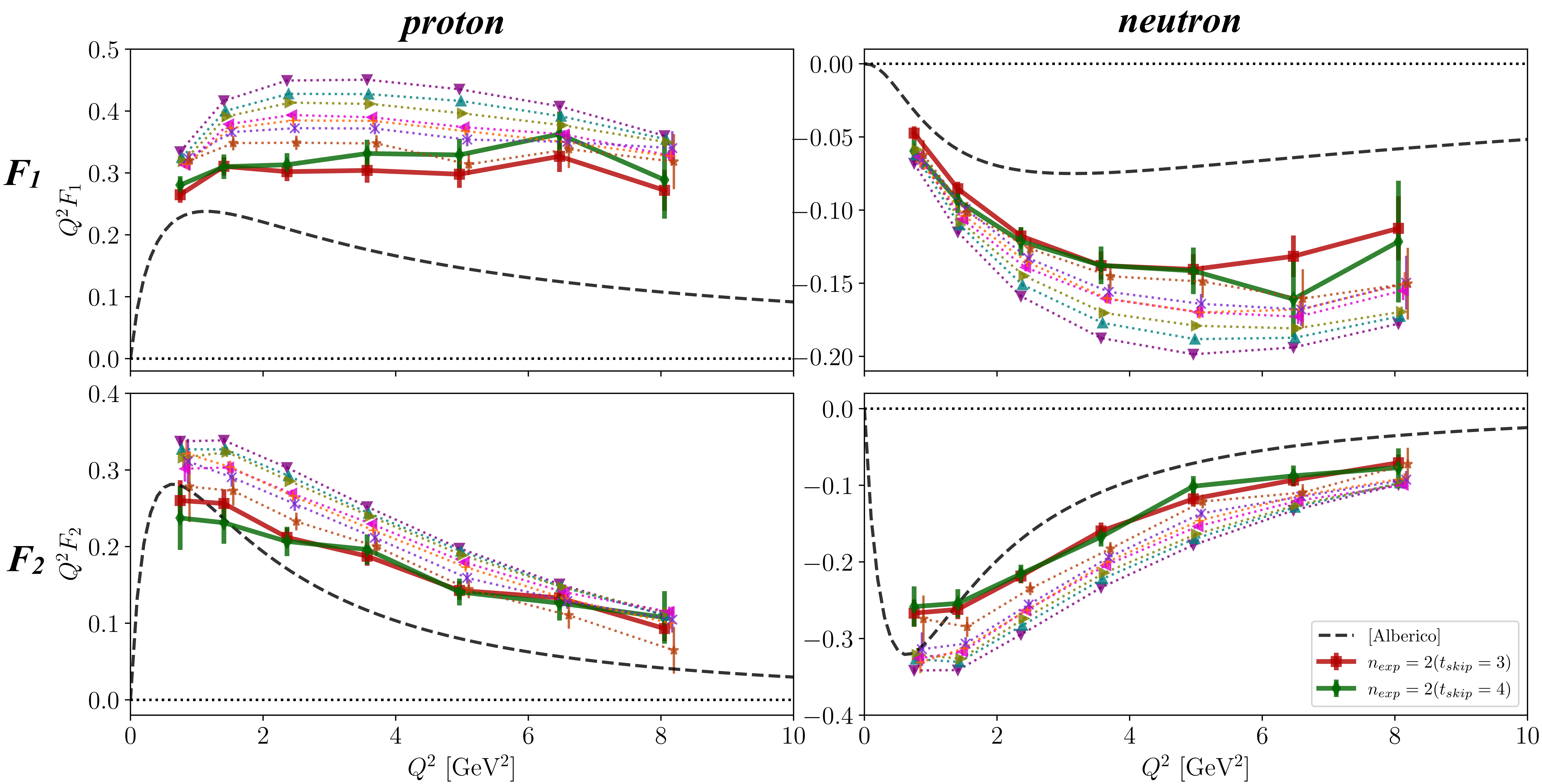
- "Ground" state from **CONSERVATIVE** 2-state fits with t_{sep}= 0.7÷1.1 fm
- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples)
- *Phenomenology (dashed)* : [Alberico et al, PRC79:065204 (2008)]



- *No disconnected diagrams*

Nucleon Form Factors: 2-state fit vs. fixed T_{sep}(D6)

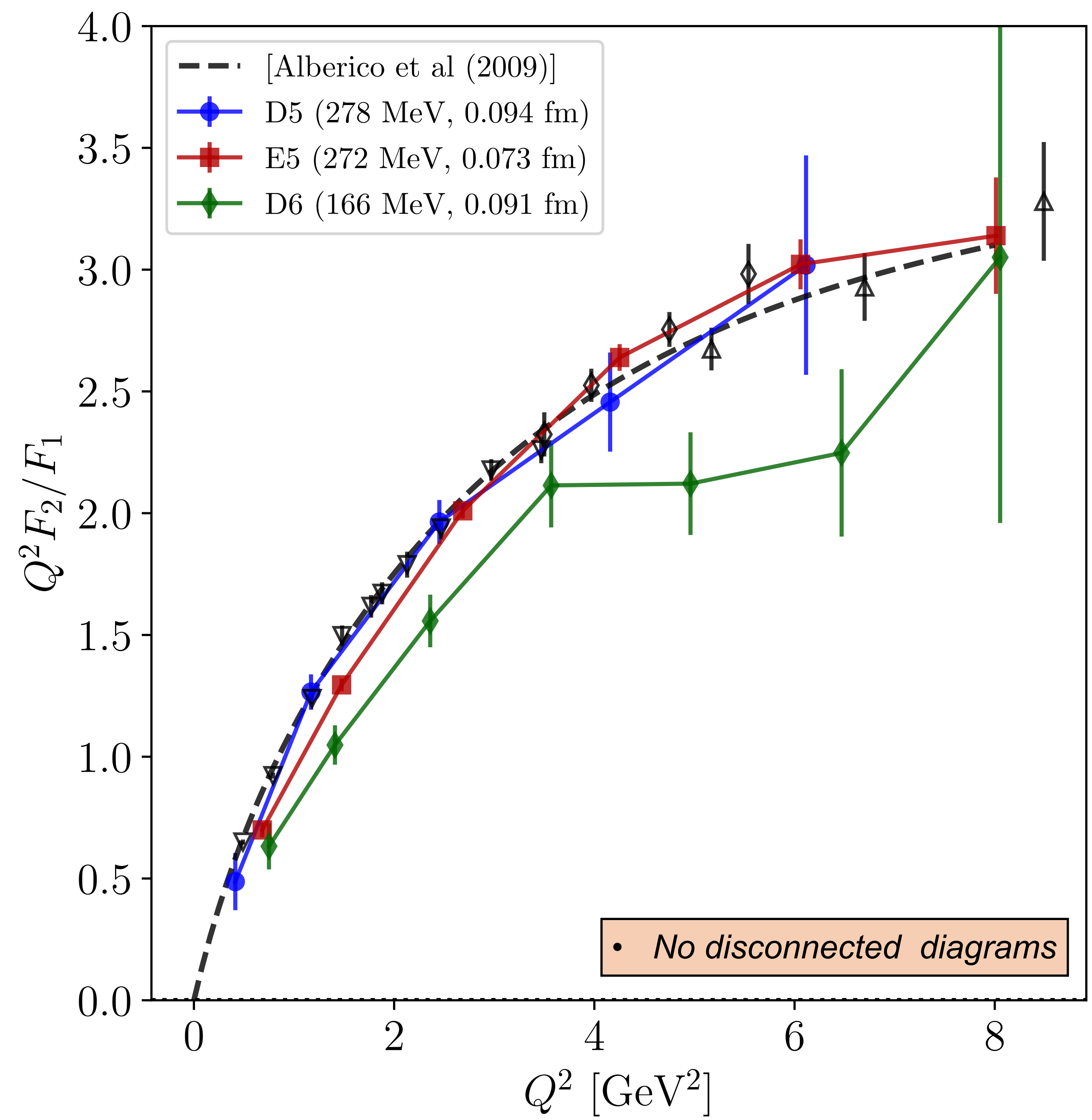
- D6 ensemble (260k samples) : fits with t_{sep}= 0.7÷1.1 fm and t_{sep}=0.5÷1.1 fm range
- Dotted lines: comparison of 1-state approx. for various fixed t_{sep}
- *Phenomenology (dashed)* : [Alberico et al, PRC79:065204 (2008)]



- *No disconnected diagrams*

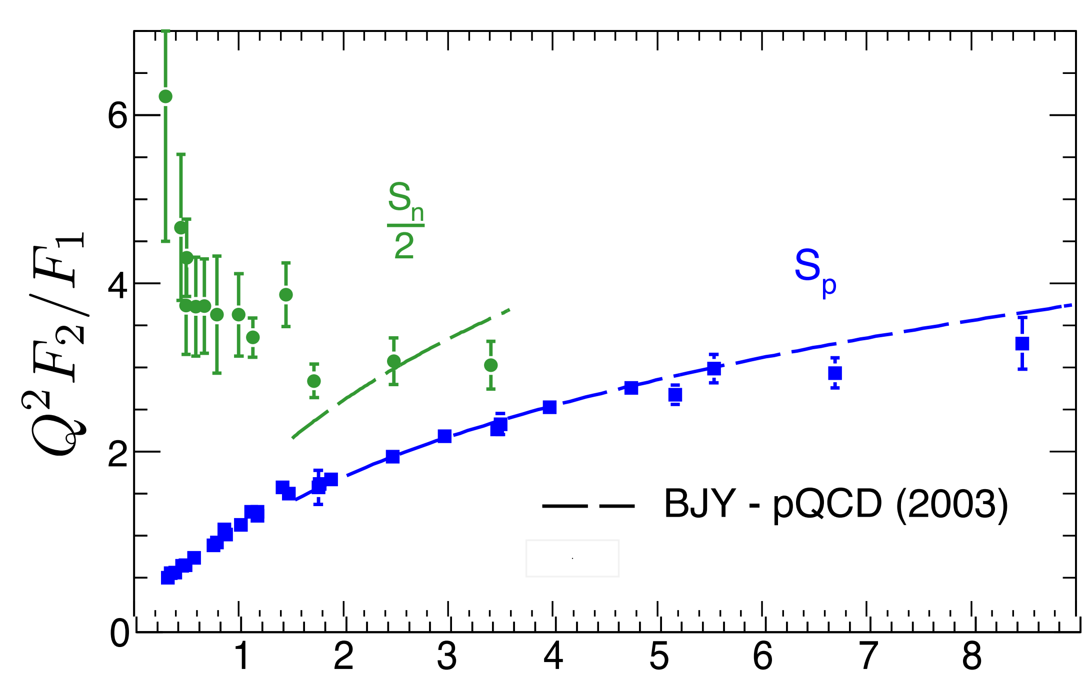
Proton F_2/F_1 Ratio

- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples) ; fit **t_{sep}= 0.7÷1.1 fm**
- *Phenomenology (dashed)* : [Alberico et al, PRC79:065204 (2008)]
- Proton experimental data $Q^2 \lesssim 8.5$ GeV^2 (black points)



- Prediction from pQCD + quark OAM [Balitsky, Ji, Yuan (2003)]

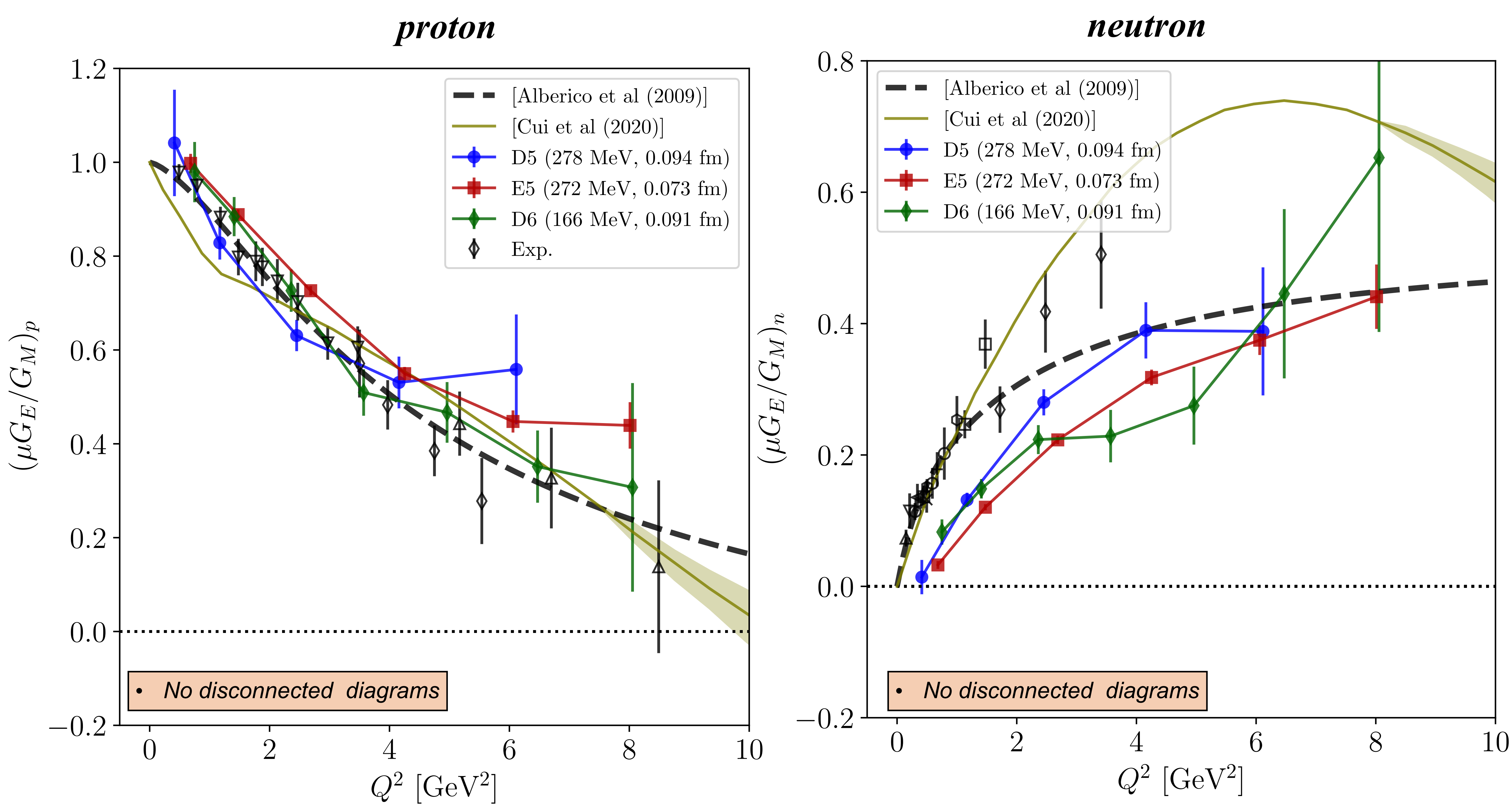
$$Q^2 F_{2p}/F_{1p} \overset{?}{\propto} \log^2(Q^2/\Lambda^2)$$



[G.D.Cates, et al, PRL106:252003 (2011)]

Proton & Neutron G_E/G_M Ratio (min. t_{sep}= 0.5 fm)

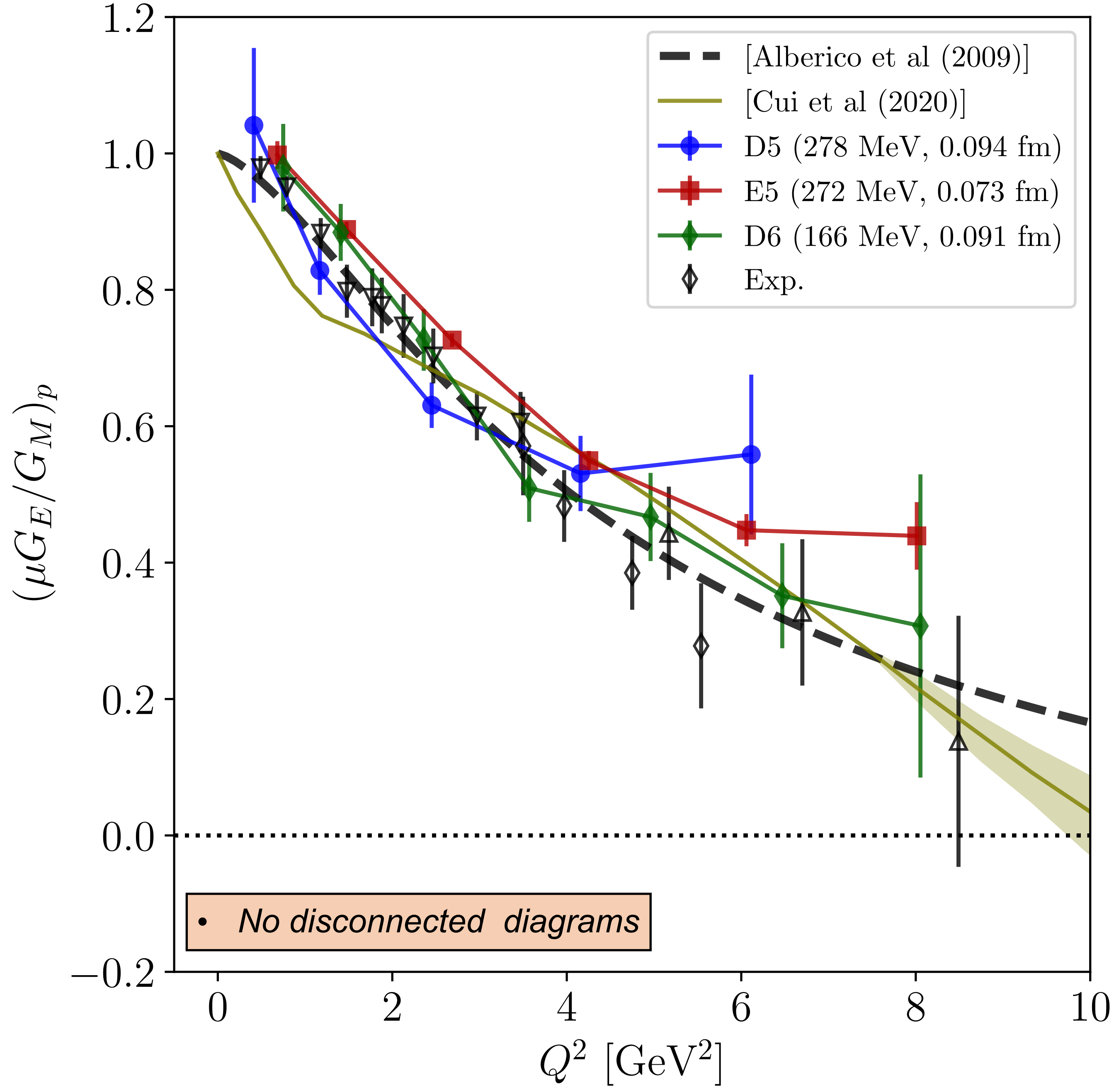
- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples) ; fit t_{sep}**= 0.5÷1.1 fm**
- Phenomenology : [Alberico et al, PRC79:065204 (2008)] ;
- Experimental data (black points) $Q^2 \lesssim 8.5$ GeV^2 (proton) and $Q^2 \lesssim 3.4$ GeV^2 (neutron)

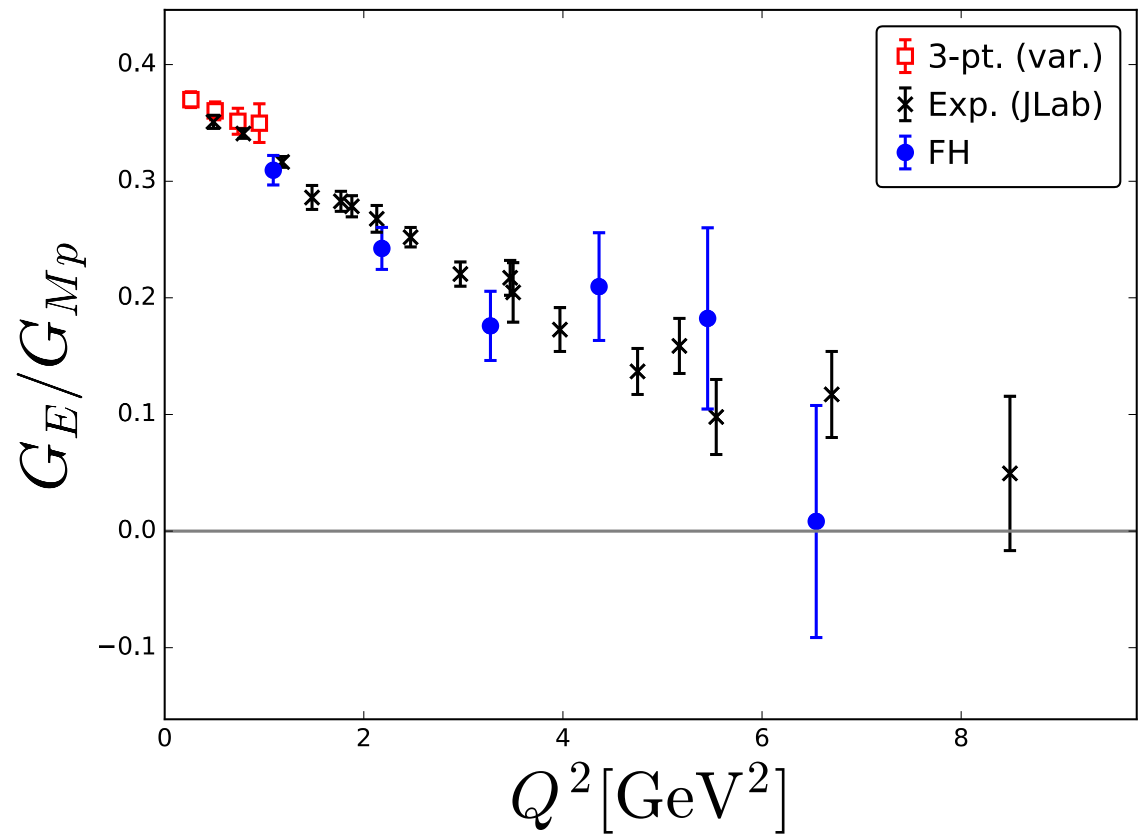


Proton & Neutron G_E/G_M Ratio (min. t_{sep}= 0.5 fm)

- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples) ; fit **t_{sep}= 0.5÷1.1 fm**
- Phenomenology : [Alberico et al, PRC79:065204 (2008)] ;
- Experimental data (black points) $Q^2 \lesssim 8.5$ GeV^2 (proton) and $Q^2 \lesssim 3.4$ GeV^2 (neutron)

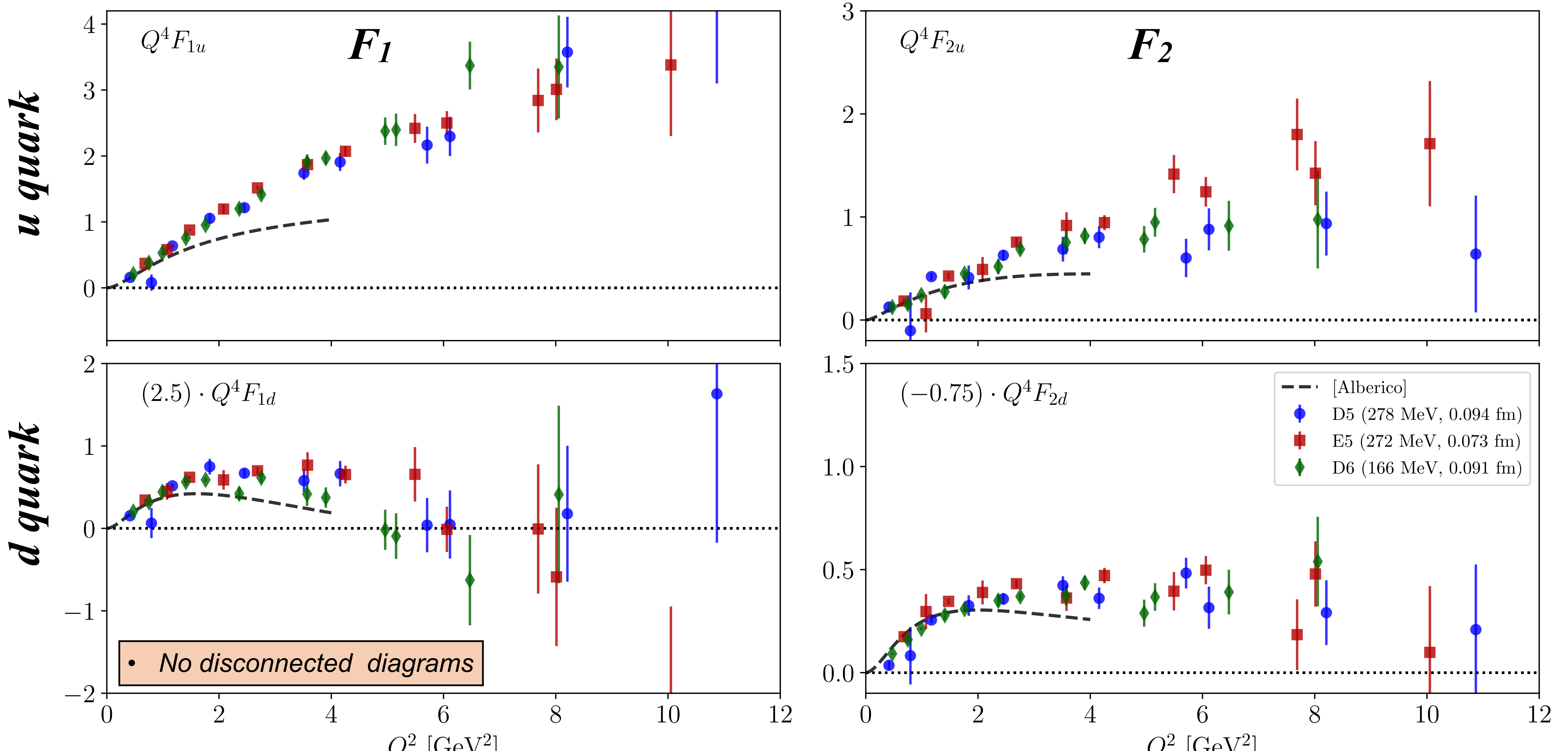
proton

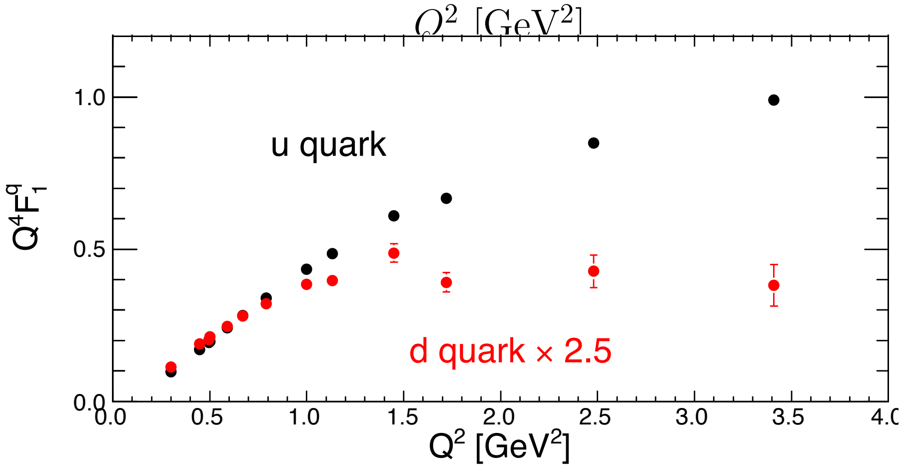


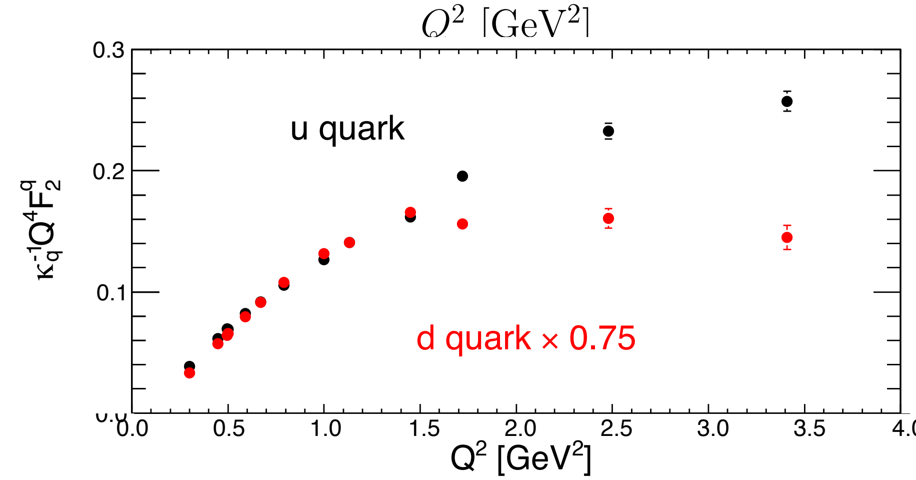


Earler calculation: (a=0.074 fm, mπ=470MeV)
Feynman-Hellman method
[Chambers et al (CSSM), PRD96: 114509]

Light-Flavor Decomposition (Proton)







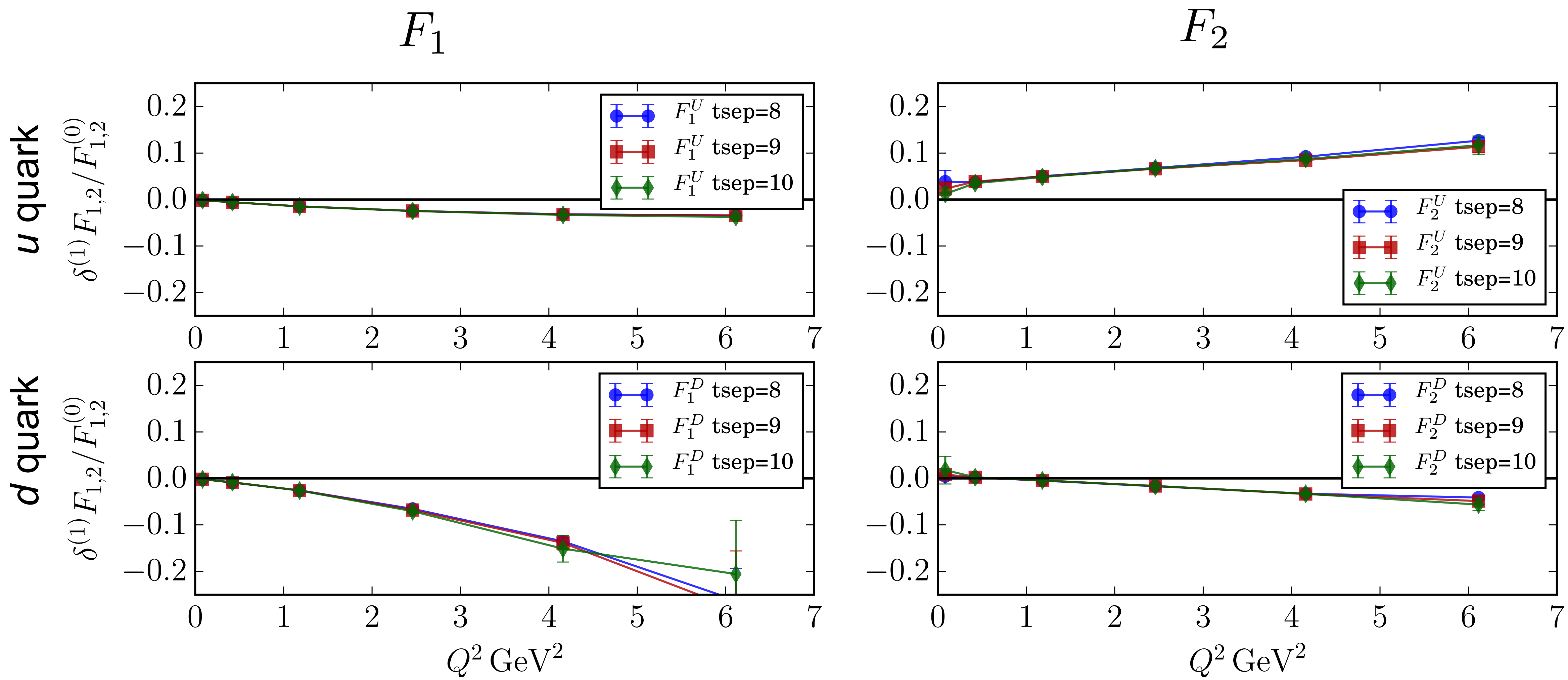
- Similar qual.features of flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]

O(a) Vector Current Correction

- *No disconnected diagrams*

Improved vector current $(V_\mu)_I = \bar{q}\gamma_\mu q + c_V\, a\partial_\nu \bar{q} i\sigma_{\mu\nu} q$

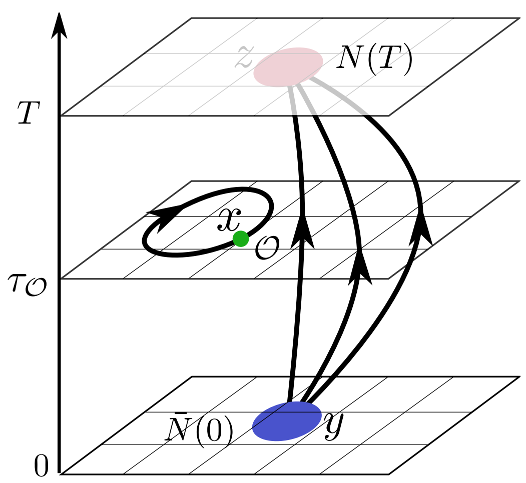
O(a¹) correction : form factors of $a\,\langle N|\partial_\nu(\bar{q}i\sigma^{\mu\nu}q)|N\rangle$



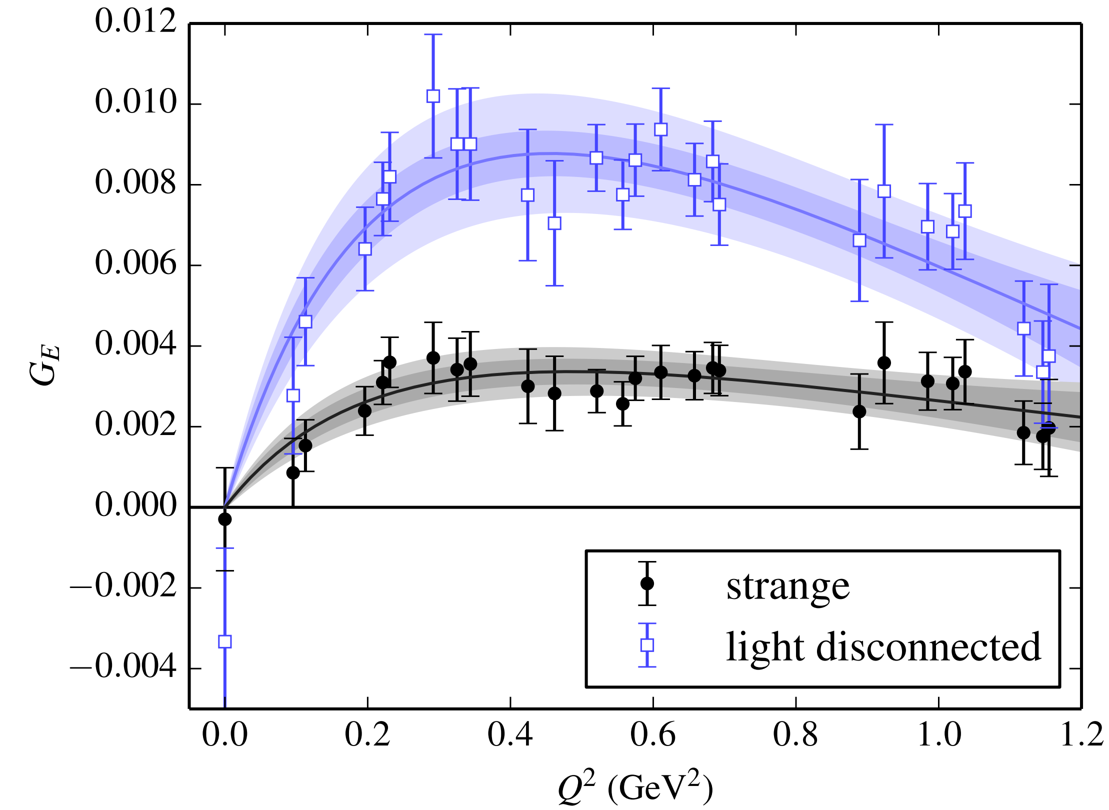
Relative magnitude of *O(a¹)* effects : {*O(a¹)* } / {*O(a⁰)* } form factors
(assuming c_V=0.05)

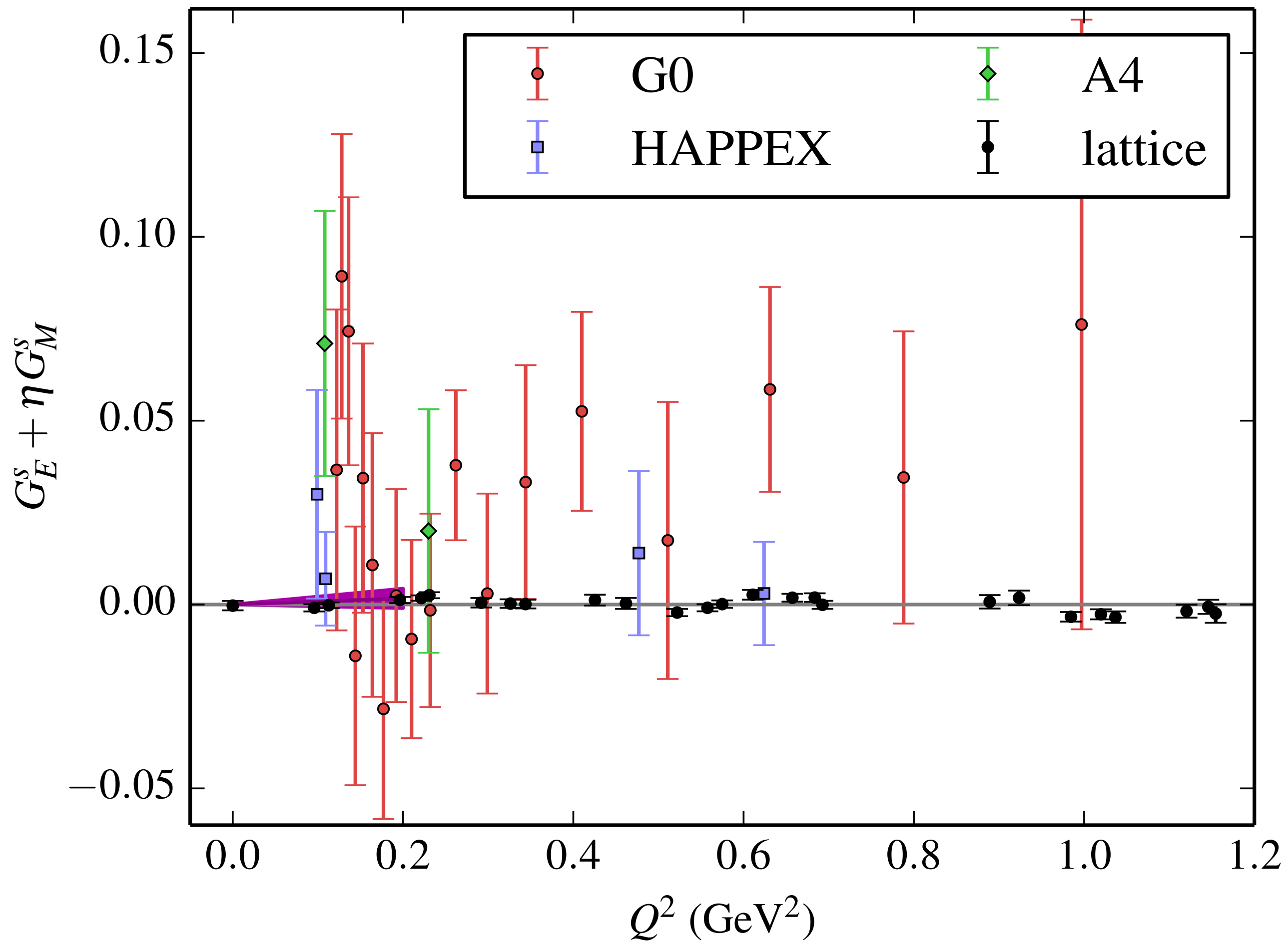
- improvement coefficient c_V : must be computed on lattice from WI
- perturbation theory: cV ≈ –0.01$C_F(g_0)^2$

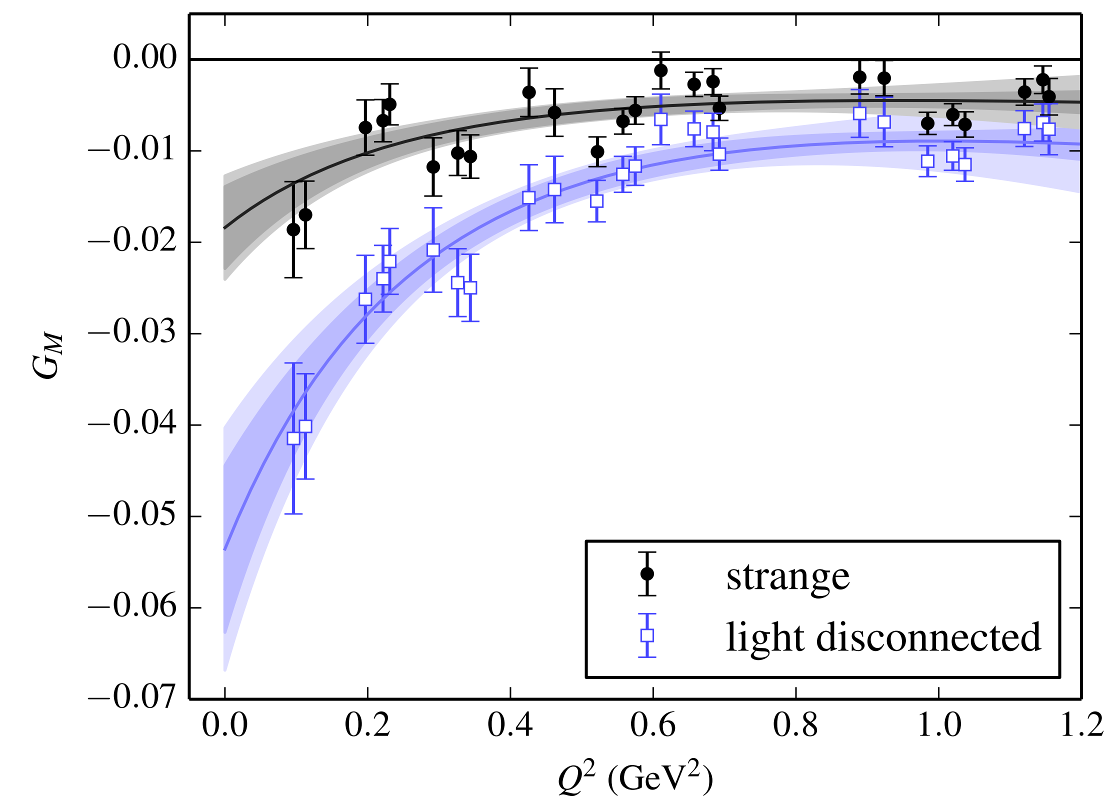
Disconnected Contributions to Vector FFs?



Quark-Disconnected diagrams contribute ≲ 1% to nucleon FFs
[J. Green, S. Meinel, S.S. et al; PRD92:031501 (2015)]







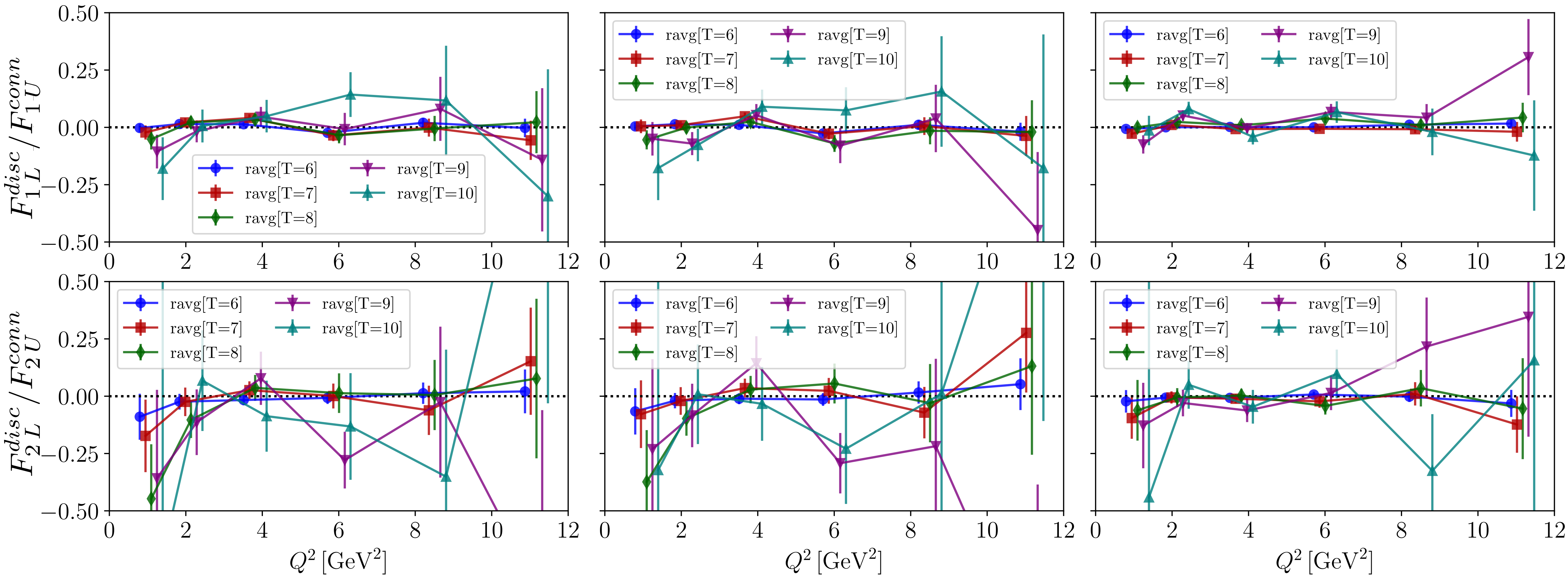
Disconnected Light & Strange vs. Connected (D5)

- *relative correction* $F_{1,2}^{disc} / F_{1,2}^{conn}$ from plateau averages $\mathbf{t_{sep}}$**= 0.5÷0.9 fm,** $Q^2 \lesssim 11$ GeV^2
- **D5 ensemble (mπ=280 MeV, a=0.094 fm)**, 1346 configs ⊗ 64 samples of ⟨NN̄⟩
- partial noise cancellation between L=U/D and S in proton & neutron

$$P = \frac{1}{3}\left[2U - D\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

$$N = \frac{1}{3}\left[2D - U\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

disconnected L=U or D ***disconnected S*** ***disconnected (L–S)***



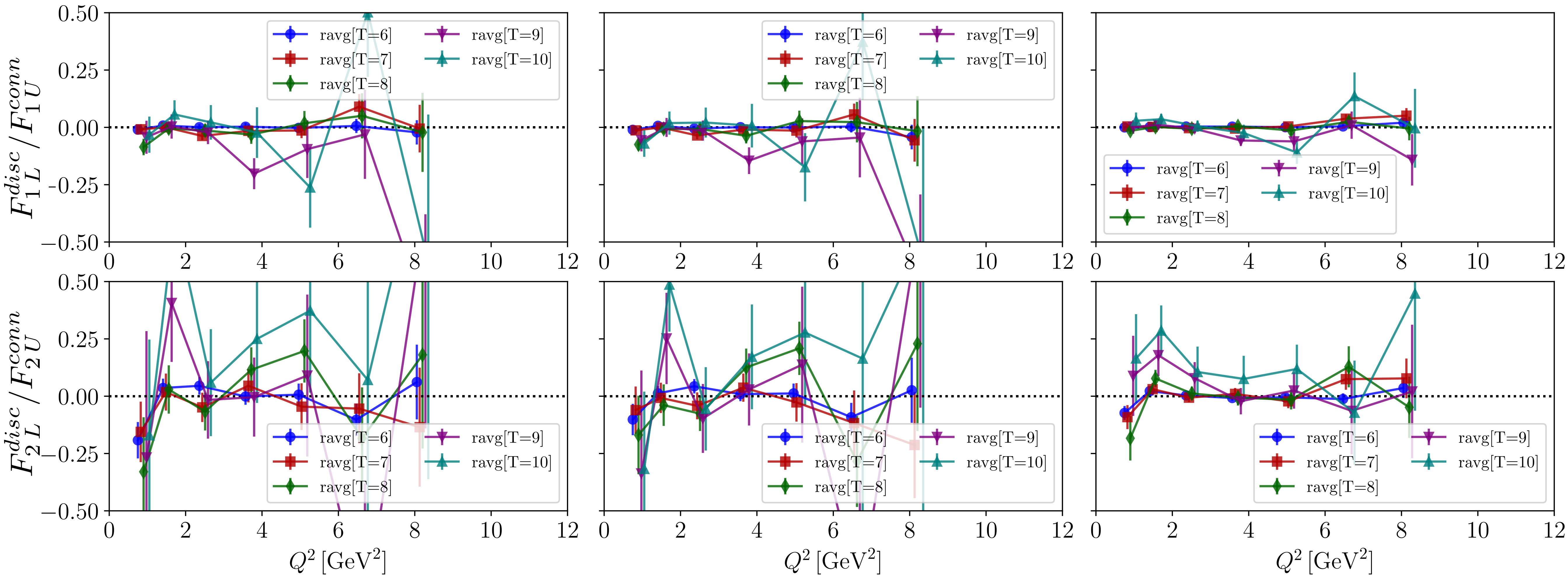
Disconnected Light & Strange vs. Connected (D6)

- *relative correction* $F_{1,2}^{disc}$ / $F_{1,2}^{conn}$ from plateau averages **t_{sep}= 0.5÷0.74 fm,** $Q^2 \lesssim 8$ GeV^2
- **D6 ensemble (m_π=170 MeV, a=0.092 fm)**, 727 configs ⊗ 128 samples of $\langle N\bar{N}\rangle$
- partial noise cancellation between L=U/D and S in proton & neutron

$$P = \frac{1}{3}\left[2U - D\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

$$N = \frac{1}{3}\left[2D - U\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

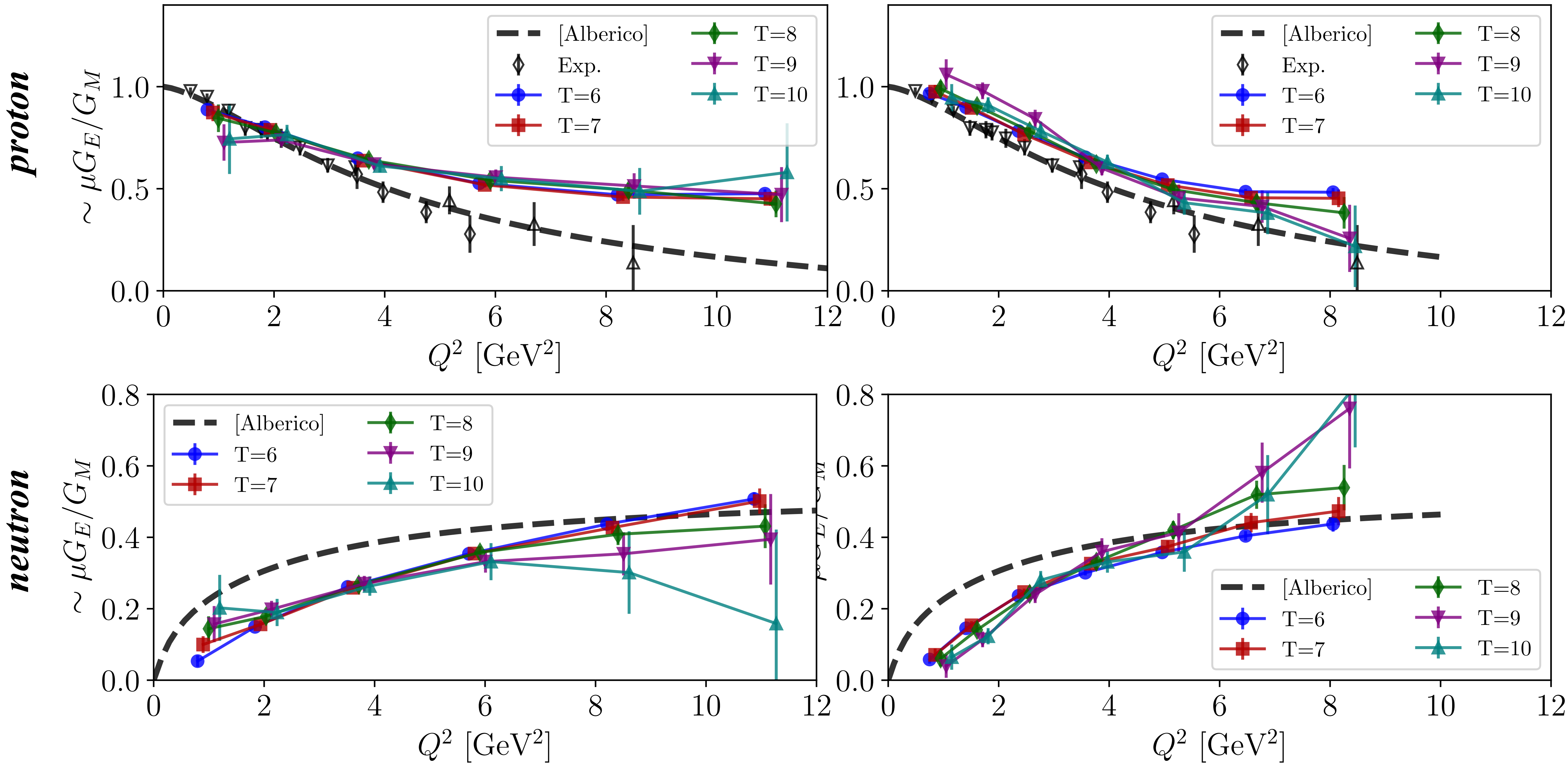
disconnected L=U or D ***disconnected S*** ***disconnected (L–S)***



Proton&Neutron G_E/G_M : Connected-only

$$\left(\frac{\sinh\frac{\lambda'-\lambda}{2}}{\cosh\frac{\lambda'+\lambda}{2}}\right)\frac{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_t(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_y(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}\overset{T\to\infty}{=}G_E/G_M \quad \text{where} \quad \begin{pmatrix} p^{(\prime)} & = m_N\sinh\lambda^{(\prime)} \\ E^{(\prime)} & = m_N\cosh\lambda^{(\prime)} \end{pmatrix}$$

D5(mπ =278 MeV, a = 0.094 fm) **D6(mπ =166 MeV, a = 0.094 fm)**



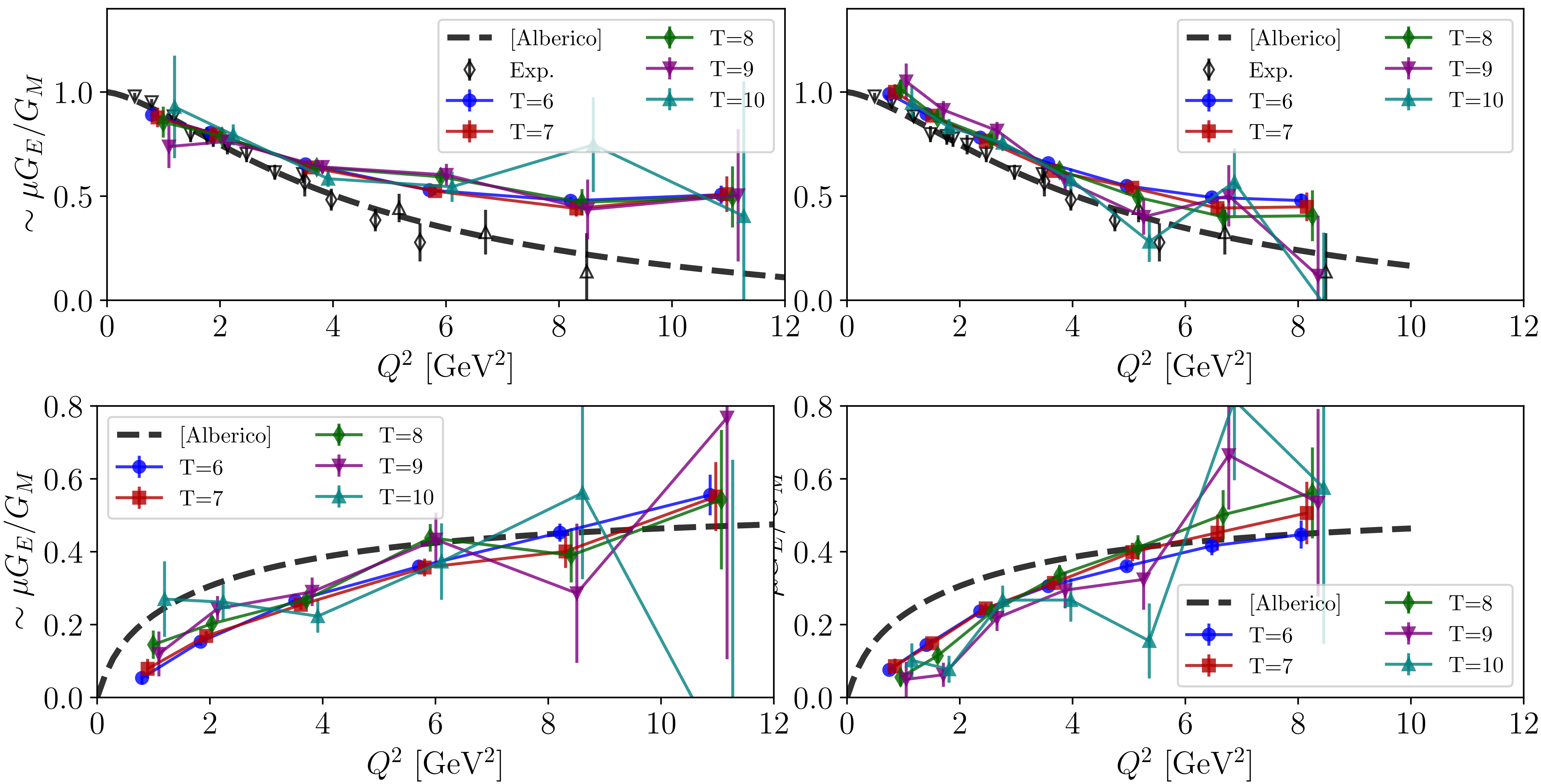
Proton&Neutron G_E/G_M : Connected+Disconnected

$$\left(\frac{\sinh\frac{\lambda'-\lambda}{2}}{\cosh\frac{\lambda'+\lambda}{2}}\right)\frac{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_t(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_y(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle} \overset{T\to\infty}{=} G_E/G_M \quad \text{where} \quad \left(\begin{array}{ll} p^{(\prime)} & = m_N \sinh\lambda^{(\prime)} \\ E^{(\prime)} & = m_N \cosh\lambda^{(\prime)} \end{array}\right)$$

D5($m\pi$ =278 MeV, a = 0.094 fm) **D6($m\pi$ =166 MeV, a = 0.094 fm)**

proton

neutron



Summary

- Preliminary results for high MC-statistics high-momentum form factors
  - up to $Q^2 \lesssim 10\ \mathrm{GeV}^2$
  - two lattice spacings $a \gtrsim 0.07$ fm
  - two pion masses $m\pi \gtrsim 170\ \mathrm{MeV}$

- Quark-disconnected contributions (only a ≈ 0.09 fm)
  - consistent with zero but is a major source of uncertainty (esp. above $Q^2 \gtrsim 8\ \mathrm{GeV}^2$)
  - little impact below $Q^2 \lesssim 6\ \mathrm{GeV}^2$ (except in G_{Ep}/G_{Mp} and G_{En})
  - very unlikely to be significant, but longer t_{sep}, more stat. data needed

- Form factor results overshoot experimental data x(2 ... 2.5) ;
  F_2/F_1 , G_E/G_M ratios are in qualitative agreement with pheno. fits to experiment
  - Non-physical quarks masses?
  - ~~Discretization?~~ (less likely)
  - Excited states (most likely) — *large additional statistics will be needed for analysis*

Important cross-check with experiments,
relevant for calculations of relativistic nucleon matrix elements
as well as TMDs, PDFs, DAs ...

BACKUP