System Identification of superconducting cavity parameters with recursive least-squares algorithms

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LLRF workshop, Newport News, VA

Disclaimer: presentation based on previously published work

PHYSICAL REVIEW ACCELERATORS AND BEAMS **26,** 112003 (2023)

Simulations of real-time system identification for superconducting cavities with a recursive least-squares algorithm

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PHYSICAL REVIEW ACCELERATORS AND BEAMS **27,** 032001 (2024)

New method to measure the unloaded quality factor of superconducting cavities

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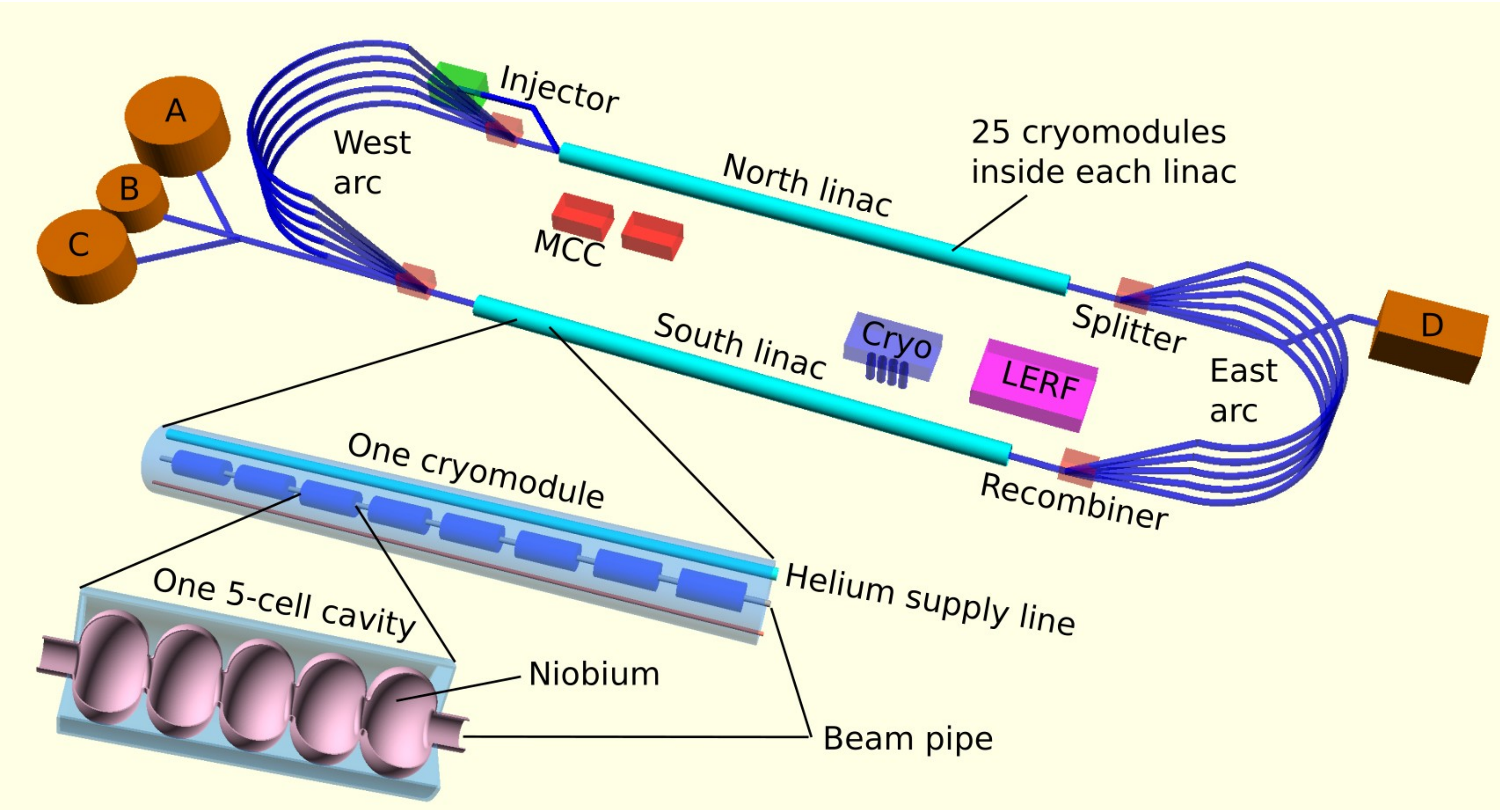
System identification

- Conceptually simplest case:
  - Determine system a in $y_i = a\, x_i$
    - from input x_i and output y_i
  - Solution: $a = \Sigma_i\, x_i y_i \,/\, \Sigma_i\, x_i^2$

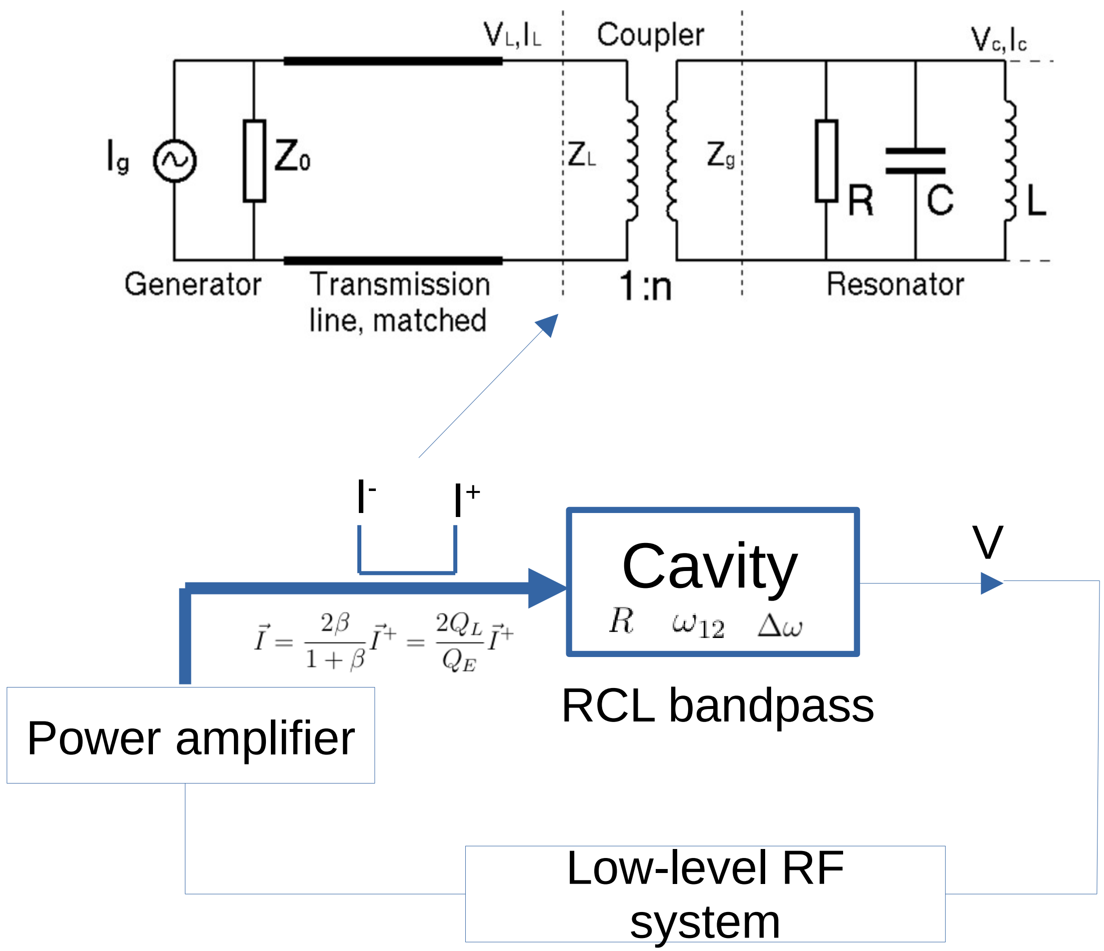
- Only a little more tricky if a is a matrix and x and y are vectors

System Identification of Superconducting Cavities



SysId: determine detuning $\Delta\omega$ and bandwidth ω_{12}, maybe Q_0

Cavity model



Dynamical system of envelopes real and imaginary, *I* and *Q*

$$\begin{pmatrix} \frac{dV_r}{dt} \\ \frac{dV_i}{dt} \end{pmatrix} = \begin{pmatrix} -\omega_{12} & -\Delta\omega \\ \Delta\omega & -\omega_{12} \end{pmatrix} \begin{pmatrix} V_r \\ V_i \end{pmatrix} + \begin{pmatrix} \omega_{12}R & 0 \\ 0 & \omega_{12}R \end{pmatrix} \begin{pmatrix} I_r \\ I_i \end{pmatrix}$$

Express *I* through forward current I^+

$$\begin{pmatrix} \frac{dV_r}{dt} \\ \frac{dV_i}{dt} \end{pmatrix} = \begin{pmatrix} -\omega_{12} & -\Delta\omega \\ \Delta\omega & -\omega_{12} \end{pmatrix} \begin{pmatrix} V_r \\ V_i \end{pmatrix} + \begin{pmatrix} \omega_E R & 0 \\ 0 & \omega_E R \end{pmatrix} \begin{pmatrix} I_r^+ \\ I_i^+ \end{pmatrix}$$

$$\omega_{12} = \hat{\omega}/2Q_L \qquad \omega_E = \hat{\omega}/Q_E$$

Discretize with time step *Δt*

$$\vec{V}'_{t+1} = (\mathbf{1} + F)\,\vec{V}'_t + B\vec{I}^+_t \qquad \text{with} \qquad F = \begin{pmatrix} -\omega_{12}\Delta t & -\Delta\omega\Delta t \\ \Delta\omega\Delta t & -\omega_{12}\Delta t \end{pmatrix}$$

$$B = \omega_E \Delta t R$$

Isolate the known and the unknown

$$\boxed{\vec{y}_{t+1}} = \vec{V}'_{t+1} - \vec{V}'_t - B\vec{I}^+_t = \boxed{F\vec{V}'_t}$$

Least squares

Rewrite the mixture of unknowns and known voltages to separate the unknowns

$$\boxed{\vec{y}_{t+1}} = \vec{V}'_{t+1} - \vec{V}'_t - B\vec{I}^+_t \quad = \boxed{F\vec{V}'_t}$$

$$\boxed{\vec{y}_{t+1}} = \boxed{G_t\vec{q}}$$

$$\boxed{F\vec{V}'_t} = \begin{pmatrix} -\omega_{12}\Delta t & -\Delta\omega\Delta t \\ \Delta\omega\Delta t & -\omega_{12}\Delta t \end{pmatrix} \begin{pmatrix} V'_r \\ V'_i \end{pmatrix}_t = \begin{pmatrix} -V'_r & -V'_i \\ -V'_i & V'_r \end{pmatrix}_t \begin{pmatrix} \omega_{12}\Delta t \\ \Delta\omega\Delta t \end{pmatrix} = \boxed{G_t\vec{q}}$$

Stack multiple copies, on for each time step

$$\begin{pmatrix} \vec{y}_2 \\ \vec{y}_3 \\ \vdots \\ \vec{y}_{T+1} \end{pmatrix} = U_T \begin{pmatrix} \omega_{12}\Delta t \\ \Delta\omega\Delta t \end{pmatrix} \quad \text{with} \quad U_T = \begin{pmatrix} G_1 \\ G_2 \\ \vdots \\ G_T \end{pmatrix}$$

Solve in the least-squares sense with the Moore-Penrose pseudo inverse

$$\vec{q}_T = \begin{pmatrix} \omega_{12}\Delta t \\ \Delta\omega\Delta t \end{pmatrix}_T = (U_T^\top U_T)^{-1} U_T^\top \begin{pmatrix} \vec{y}_2 \\ \vec{y}_3 \\ \vdots \\ \vec{y}_{T+1} \end{pmatrix}$$

Tedious to calculate $(U_T^\top U_T)^{-1} U_T^\top$ for quickly growing U_T

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Recursive least squares

Introduce $P_T^{-1} = U_T^\top U_T$, the inverse of empirical covariance matrix.

$$\begin{aligned}P_{T+1}^{-1} &= U_{T+1}^\top U_{T+1} \\ &= p_0\mathbf{1} + G_1^\top G_1 + G_2^\top G_2 + \cdots + G_T^\top G_T + G_{T+1}^\top G_{T+1} \\ &= P_T^{-1} + G_{T+1}^\top G_{T+1}.\end{aligned}$$

$$G_t^\top G_t = (V_r'^2 + V_i'^2)_t\mathbf{1} = \vec{V}_t'^2\mathbf{1}$$

Obtain the updated quantities p_{T+1} and q_{T+1} at the next time step by absorbing the new information from *V'* and *y*.

$$p_{T+1} = \left[\frac{1}{1 + p_T\vec{V}_T'^2}\right]p_T$$

$$\begin{aligned}\vec{q}_{T+1} &= p_{T+1}(G_1^\top\vec{y}_2 + G_2^\top\vec{y}_3 + \cdots + G_T^\top\vec{y}_{T+1} + G_{T+1}^\top\vec{y}_{T+2}) \\ &= \left[\frac{1}{1 + p_T\vec{V}_T'^2}\right]p_T\left(\sum_{t=1}^{T} G_t^\top\vec{y}_{t+1} + G_{T+1}^\top\vec{y}_{T+2}\right) \\ &= \left[\frac{1}{1 + p_T\vec{V}_T'^2}\right](\vec{q}_T + p_T G_{T+1}^\top\vec{y}_{T+2}).\end{aligned}$$

Forgetting factor α
for time-varying systems

- Time horizon N_f and $\alpha = 1 - 1/N_f$

"discount factor"

$$P_{T+1}^{-1} = \alpha P_T^{-1} + G_{T+1}^{\top} G_{T+1}$$

- Update equations for p_T and $\vec{q}_T$ from one time-step to the next

New y_{T+2} and G_{T+1} come from new currents I and voltages V'

$$p_{T+1} = \left[\frac{1}{\alpha + p_T \vec{V}_T'^2}\right] p_T$$

$$\vec{q}_{T+1} = \left[\frac{1}{\alpha + p_T \vec{V}_T'^2}\right] (\alpha \vec{q}_T + p_T \hat{G}_{T+1}^{\top} \vec{y}_{T+2})$$

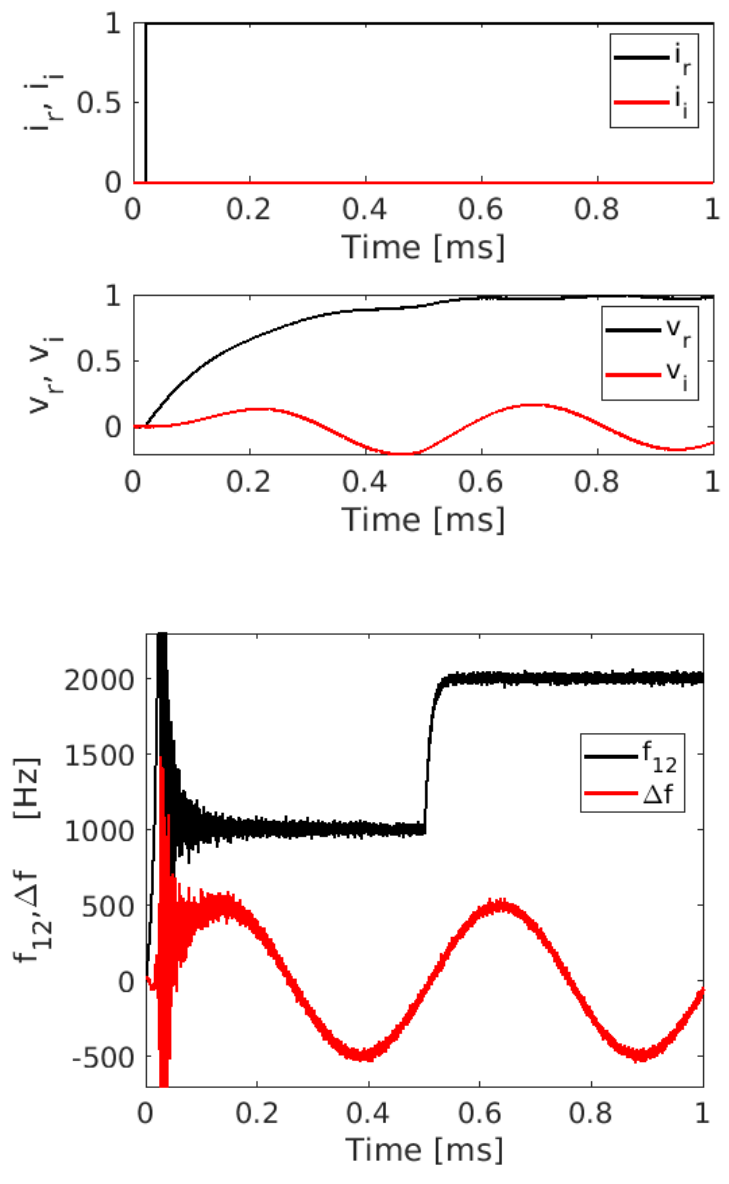
Simulations

Cavity: 1 GHz, Q_L=5x10^5,
f_s=10 MHz, N_f=100,
$\sigma_p=10^{-4}\,V_{max}$, $\sigma_m=10^{-3}\,V_{max}$
Step in ω_{12}
2 kHz oscillation of $\Delta\omega$

Current step
- Cavity starts charging
- observe wiggle after 0.5 ms
- Q-phase oscillates

Reconstructed parameters
- Bandwidth ω_{12} doubles
- Frequency $\Delta\omega$ varies (2 kHz)

Behavior is reconstructed faithfully



Measuring the unloaded Q_0

Hunting the little discrepancy between Q_E and Q_L

$$\omega_E = \hat{\omega}/Q_E$$
$$\omega_0 = \hat{\omega}/Q_0'$$
$$1/Q_0' = (1/Q_0 + 1/Q_t)$$

$$\vec{y}_{t+1} = \vec{V}'_{t+1} - \vec{V}'_t = F\vec{V}'_t + \omega_E \Delta t R\vec{I}^+_t$$

with

$$F = \begin{pmatrix} -\frac{1}{2}(\omega_E+\omega_0)\Delta t & -\Delta\omega\Delta t \\ \Delta\omega\Delta t & -\frac{1}{2}(\omega_E+\omega_0)\Delta t \end{pmatrix}$$

Rewrite the right-hand side

$$F\vec{V}'_t + \omega_E \Delta t R\vec{I}^+_t$$
$$= \omega_E\Delta t \begin{pmatrix} -\frac{1}{2}V'_r + RI^+_r \\ -\frac{1}{2}V'_i + RI^+_i \end{pmatrix}_t + \Delta\omega\Delta t \begin{pmatrix} -V'_i \\ V'_r \end{pmatrix}_t + \omega_0\Delta t \begin{pmatrix} -\frac{1}{2}V'_r \\ -\frac{1}{2}V'_i \end{pmatrix}_t$$
$$= \underbrace{\begin{pmatrix} -\frac{1}{2}V'_r + RI^+_r & -V'_i & -\frac{1}{2}V'_r \\ -\frac{1}{2}V'_i + RI^+_i & V'_r & -\frac{1}{2}V'_i \end{pmatrix}_t}_{G_t} \underbrace{\begin{pmatrix} \omega_E\Delta t \\ \Delta\omega\Delta t \\ \omega_0\Delta t \end{pmatrix}}_{q}$$

$$\boxed{\vec{y}_{t+1} = G_t\vec{q}}$$

Stack consecutive measurements on top of each other
→ least-squares problem

$$\begin{pmatrix} \vec{y}_2 \\ \vec{y}_3 \\ \vdots \\ \vec{y}_{T+1} \end{pmatrix} = U_T \begin{pmatrix} \omega_E\Delta t \\ \Delta\omega\Delta t \\ \omega_0\Delta t \end{pmatrix} \quad \text{with} \quad U_T = \begin{pmatrix} G_1 \\ G_2 \\ \vdots \\ G_T \end{pmatrix}$$

Solve by Moore-Penrose

$$\vec{q}_T = \begin{pmatrix} \omega_E\Delta t \\ \Delta\omega\Delta t \\ \omega_0\Delta t \end{pmatrix}_T = (U_T^\top U_T)^{-1}U_T^\top \begin{pmatrix} \vec{y}_2 \\ \vec{y}_3 \\ \vdots \\ \vec{y}_{T+1} \end{pmatrix}$$

Recursive ...

Technical difficulty: need to invert P_{T+1}^{-1} if new term is added

$$
\begin{aligned}
P_{T+1}^{-1} &= U_{T+1}^{\top} U_{T+1} \\
&= p_0 \mathbf{1} + G_1^{\top} G_1 + G_2^{\top} G_2 + \cdots + G_T^{\top} G_T + G_{T+1}^{\top} G_{T+1} \\
&= P_T^{-1} + G_{T+1}^{\top} G_{T+1}. \qquad (12)
\end{aligned}
$$

Woodbury matrix identity to the rescue

$$(A + VW^{\top})^{-1} = A^{-1} - A^{-1}V(\mathbf{1} + W^{\top}A^{-1}V)^{-1}W^{\top}A^{-1}$$

$$P_{T+1} = [\mathbf{1} - P_T G_{T+1}^{\top}(\mathbf{1} + G_{T+1} P_T G_{T+1}^{\top})^{-1} G_{T+1}] P_T$$

Substitutions

$$A^{-1} = P_T$$

$$V = G_{T+1}^{\top}$$

$$W^{\top} = G_{T+1}$$

With forgetting factor α

$$P_{T+1} = \frac{1}{\alpha}[\mathbf{1} - P_T G_{T+1}^{\top}(\alpha + G_{T+1} P_T G_{T+1}^{\top})^{-1} G_{T+1}] P_T$$

$$
\begin{aligned}
\vec{q}_{T+1} = {} & [\mathbf{1} - P_T G_{T+1}^{\top}(\alpha \mathbf{1} + G_{T+1} P_T G_{T+1}^{\top})^{-1} G_{T+1}] \\
& \times \left(\vec{q}_T + \frac{1}{\alpha} P_t G_{T+1}^{\top} \vec{y}_{T+2} \right).
\end{aligned}
$$

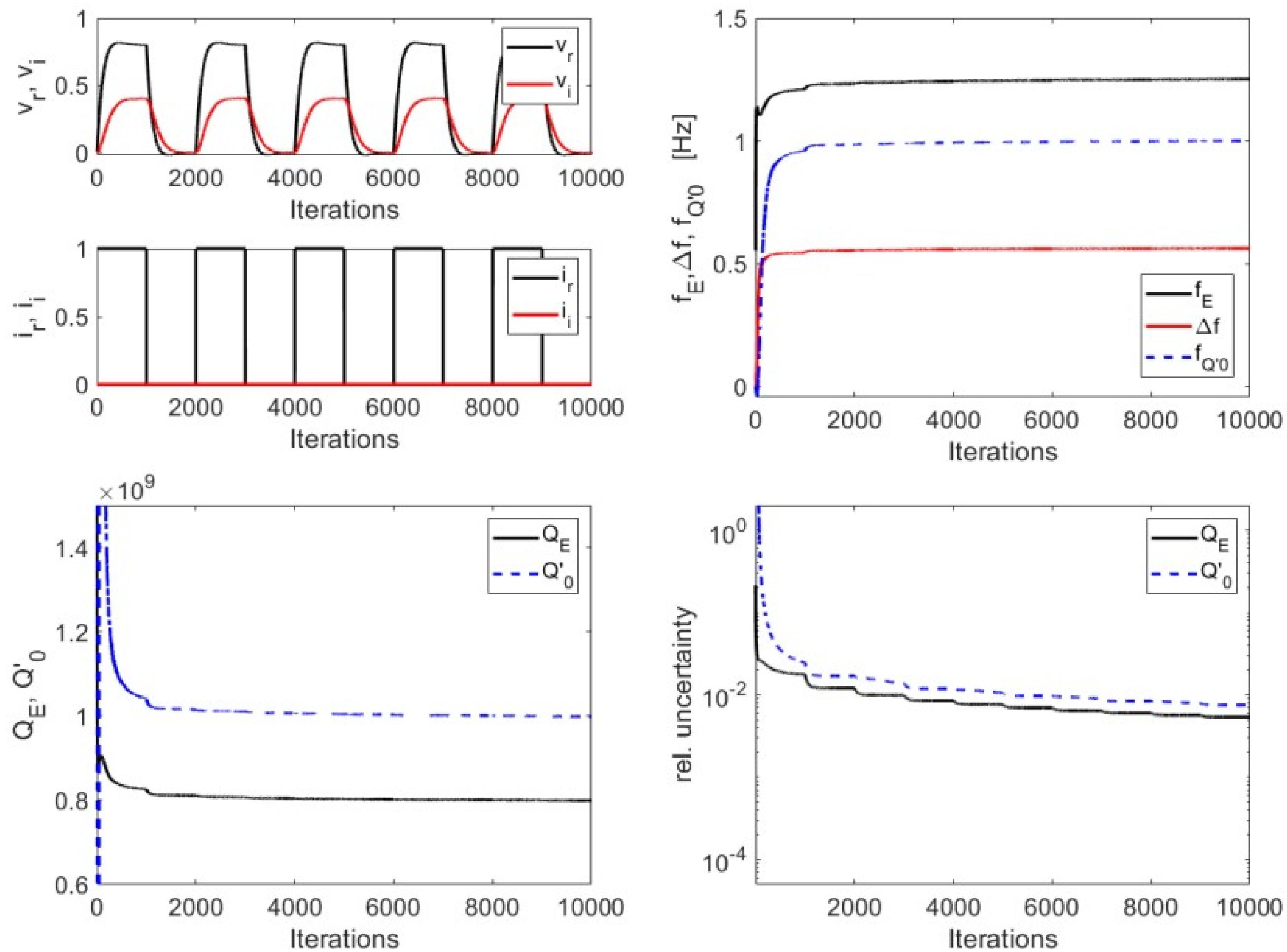
Critically coupled cavity

f=1GHz, $Q_0=10^9$, $Q_E=0.8\times10^9$, $N_f=\infty$

Algorithm does not work in steady-state, where the equations are degenerate
→ pulsed

Track small changes:
→ need larger sample time Δt~ms
→ 10 seconds real time

Fairly robust against small errors in shunt resistance *R*



Over-coupled cavity

$Q_0=10^9$, $Q_E=10^6$

Sampling time f_s=0.1 ms
six seconds real time

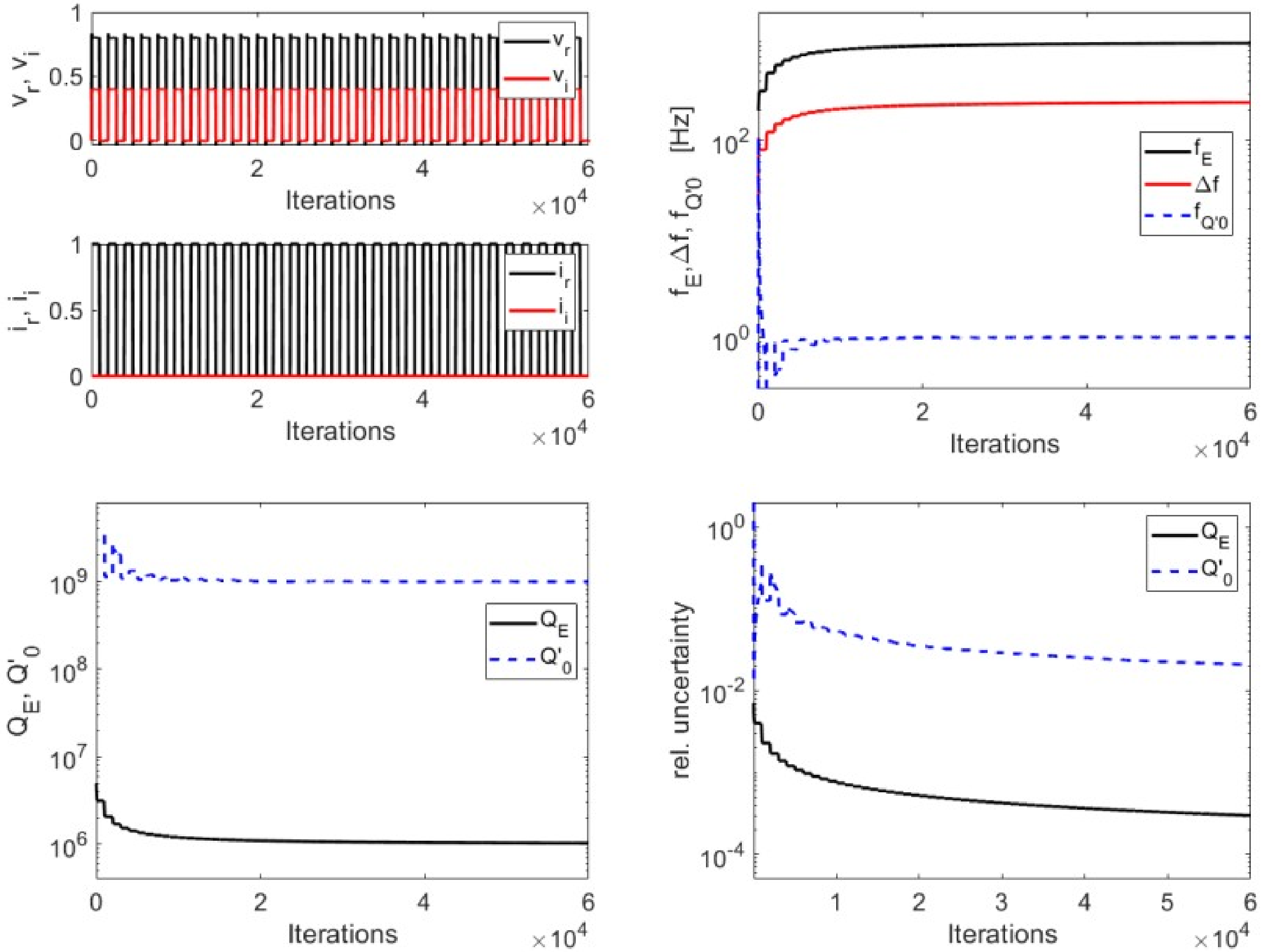
Also works with random errors in percent range

But!!!
Extreme sensitivity on systematic error of R

Use RLS calibration of R
In steady-state

$$V'_t = R\,(I_t^{+}+I_t^{-})$$

Use RLS to continuously update R for many interations



Conclusions

- Recursive least-squares algorithms are a powerful tool to continuously improve estimates in real time
  - Computational effort per time step is moderate
  - Colleague in Uppsala has a working system running close to 10 MHz