JOINT EICUG/ePIC COLLABORATION

July 14-18, 2025 Jefferson Lab • Newport News, VA



QCD First Inverse Problem using Maximum Likelihood Method from Exclusive Experiments.



Saraswati Pandey

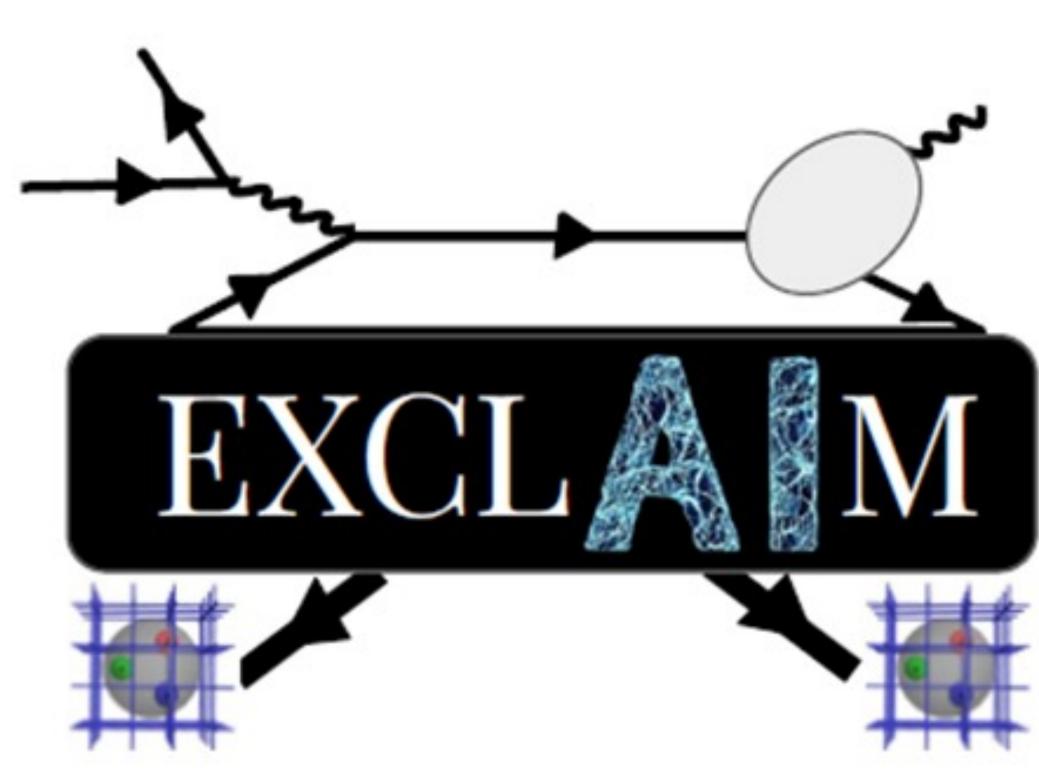
Post Doctoral Fellow

(with Simonetta Liuti)

University of Virginia





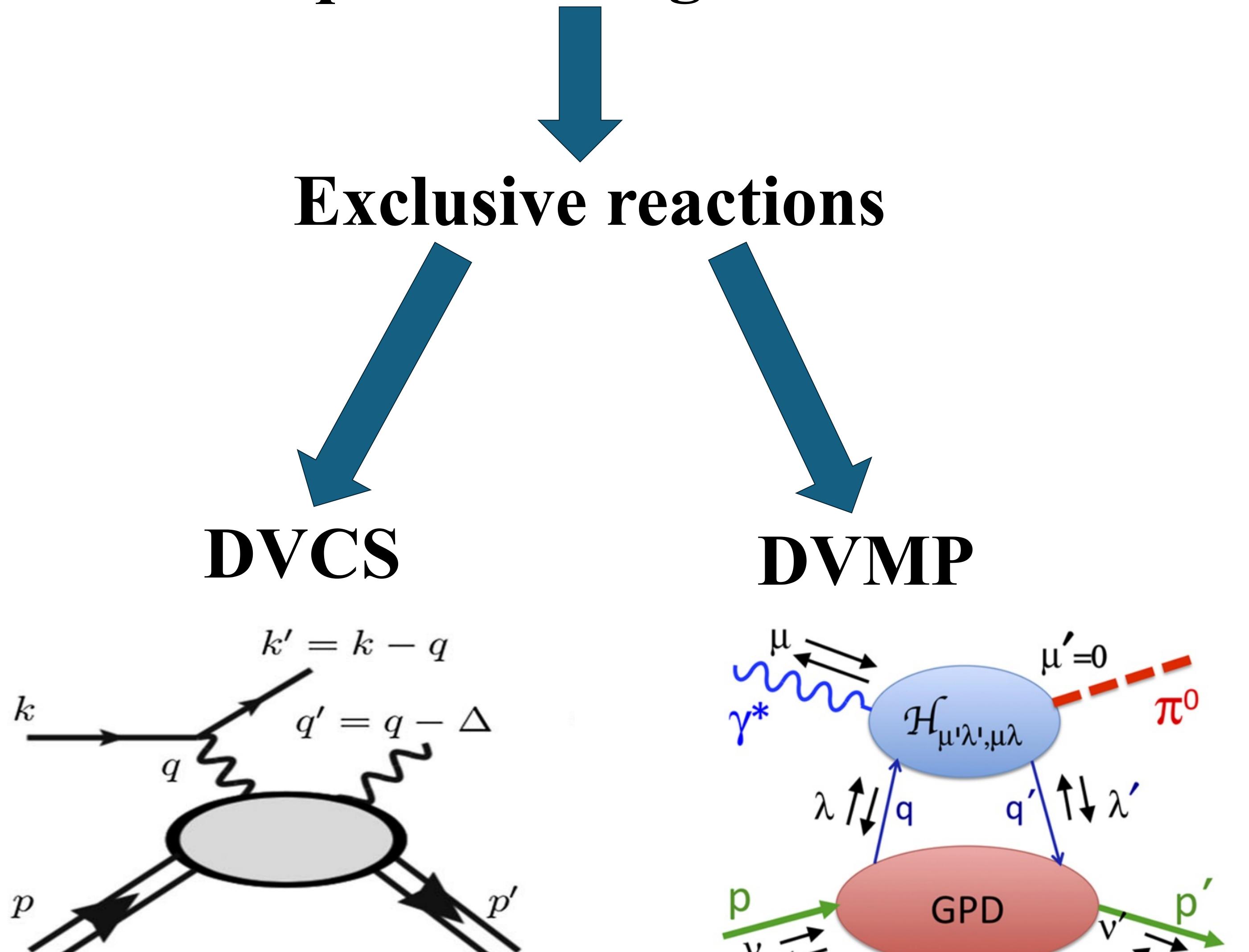


Outline

- Some of the goals of EIC
- Inverse problems in QCD
- Motivation
- □Extraction CFFs from DVCS unpolarized cross-section data
 - DApproach
 - **Likelihood Analysis**
 - Canonical Method
 - Dobtained Results
- **Conclusion and Next Steps**

Some of the goals at EIC

Imaging transverse spatial distribution of quarks and gluons



Generalized parton distributions (GPDs)

- QCD matrix element between the p and p'
 - □Non-forward limit ———— quark and gluon distributions

More information from GPDs

- □ second moment of GPDs total angular momentum of quarks and gluons in the proton.
- □ pressure and shear forces inside hadrons higher twist

Inverse problems in QCD

DFirst inverse problems QCD Theory

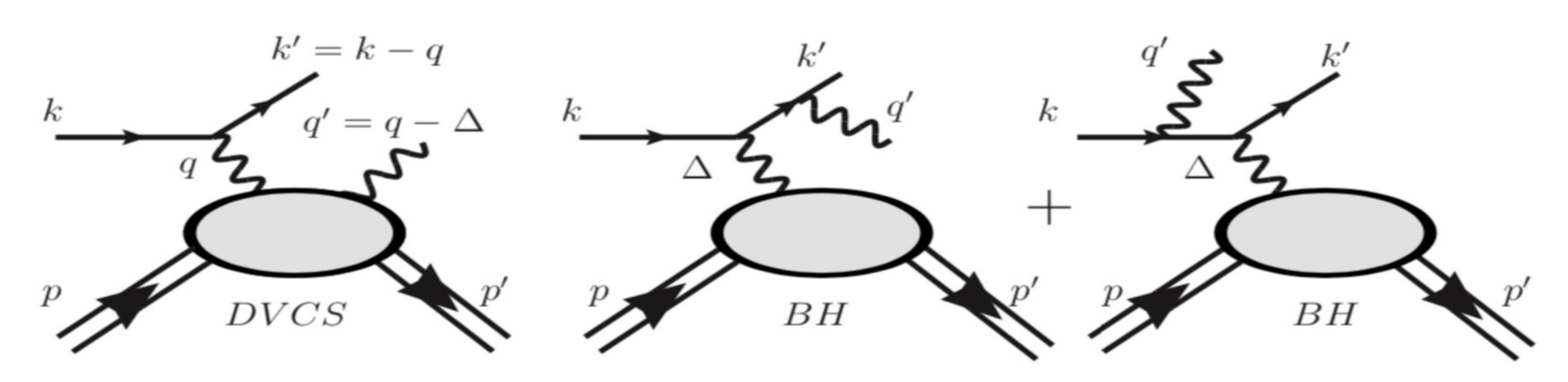
Observables
(fragmentation functions, Compton form factors,
Inverse problem etc)

Experiments (JLAB, EIC)

DSecond inverse problem

$$CFF = \int (QCDKernel) \times GPD$$

Motivation



$$\begin{split} \frac{d^{5}\sigma_{\text{DVCS}}}{dx_{Bj}dQ^{2}d|t|d\phi d\phi_{S}} &= \Gamma|T_{\text{DVCS}}|^{2} \\ &= \frac{\Gamma}{Q^{2}(1-\epsilon)} \Big\{ F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} + (2h)\sqrt{2\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \\ &\quad + (2\Lambda) \Big[\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + (2h) \Big(\sqrt{1-\epsilon^{2}} F_{LL} + 2\sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} \Big) \Big] \\ &\quad + (2\Lambda_{T}) \Big[\sin(\phi - \phi_{S}) (F_{UT,T}^{\sin(\phi - \phi_{S})} + \epsilon F_{UT,L}^{\sin(\phi - \phi_{S})}) + \epsilon \sin(\phi + \phi_{S}) F_{UT}^{\sin(\phi + \phi_{S})} + \epsilon \sin(3\phi - \phi_{S}) F_{UT}^{\sin(3\phi - \phi_{S})} \\ &\quad + \sqrt{2\epsilon(1+\epsilon)} (\sin \phi_{S} F_{UT}^{\sin \phi_{S}} + \sin(2\phi - \phi_{S}) F_{UT}^{\sin(2\phi - \phi_{S})}) \Big] \\ &\quad + (2h)(2\Lambda_{T}) \Big[\sqrt{1-\epsilon^{2}} \cos(\phi - \phi_{S}) F_{LT}^{\cos(\phi - \phi_{S})} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_{S} F_{LT}^{\cos \phi_{S}} \\ &\quad + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_{S}) F_{LT}^{\cos(2\phi - \phi_{S})} \Big] \Big\}. \end{split}$$

$$|T|^2 = |T_{\rm BH} + T_{
m DVCS}|^2 = |T_{
m BH}|^2 + |T_{
m DVCS}|^2 + \mathcal{I},$$
 $\mathcal{I} = T_{
m BH}^* T_{
m DVCS} + T_{
m DVCS}^* T_{
m BH}.$

$$\frac{d^{5}\sigma_{\text{unpol}}^{\text{BH}}}{dx_{Bj}dQ^{2}d|t|d\phi d\phi_{S}} \equiv \frac{\Gamma}{t}F_{UU}^{\text{BH}}$$

$$= \frac{\Gamma}{t}[A(y, x_{Bj}, t, Q^{2}, \phi)(F_{1}^{2} + \tau F_{2}^{2})$$

$$+ B(y, x_{Bj}, t, Q^{2}, \phi)\tau G_{M}^{2}(t)] \quad (143)$$

with,

$$A = \frac{8M^2}{t(kq')(k'q')} [4\tau((kP^2) + (k'P^2)) - (\tau + 1)((k\Delta^2) + (k'\Delta^2))]$$
(144)

$$B = \frac{16M^2}{t(kq')(k'q')}[(k\Delta^2) + (k'\Delta^2)],$$
 (145)

S. Liuti, G. R. Goldstein, et al, Phys. Rev. D 101, 054021 (2020). 5

Experimental data

E_{beam} (GeV)	x_{Bj}	$Q^2 ({ m GeV}^2)$	$t (\text{GeV}^2)$	ϕ (deg)	σ_{total}	$\Delta \sigma$
10.591	0.369	4.53	-0.2094	7.5	0.01394	0.00058
10.591	0.369	4.53	-0.2094	22.5	0.01292	0.00056
10.591	0.369	4.53	-0.2094	37.5	0.01305	0.00056
10.591	0.369	4.53	-0.2094	52.5	0.01216	0.00054
10.591	0.369	4.53	-0.2094	67.5	0.01147	0.00052
10.591	0.369	4.53	-0.2094	82.5	0.01128	0.00051
10.591	0.369	4.53	-0.2094	97.5	0.00875	0.00046
10.591	0.369	4.53	-0.2094	112.5	0.00915	0.00046
10.591	0.369	4.53	-0.2094	127.5	0.00904	0.00045
10.591	0.369	4.53	-0.2094	142.5	0.00838	0.00044
10.591	0.369	4.53	-0.2094	157.5	0.00828	0.00044
10.591	0.369	4.53	-0.2094	172.5	0.00798	0.00043
10.591	0.369	4.53	-0.2094	187.5	0.00774	0.00043
10.591	0.369	4.53	-0.2094	202.5	0.00841	0.00045
10.591	0.369	4.53	-0.2094	217.5	0.00853	0.00045
10.591	0.369	4.53	-0.2094	232.5	0.00991	0.00049
10.591	0.369	4.53	-0.2094	247.5	0.00969	0.00049
10.591	0.369	4.53	-0.2094	262.5	0.01021	0.00049
10.591	0.369	4.53	-0.2094	277.5	0.01093	0.00051
10.591	0.369	4.53	-0.2094	292.5	0.01223	0.00054
10.591	0.369	4.53	-0.2094	307.5	0.01236	0.00054
10.591	0.369	4.53	-0.2094	322.5	0.01382	0.00057
10.591	0.369	4.53	-0.2094	337.5	0.01543	0.00061
10.591	0.369	4.53	-0.2094	352.5	0.01376	0.00058

Jefferson Lab Hall A

Phys. Rev. Lett. 128, 252002 (2022), 2201.03714

Likelihood Analysis

- ☐ We use a "canonical" approach to perform a Bayesian likelihood analysis.
- ☐ The joint likelihood of the parameters of interest is calculated as a simple product of Gaussians.

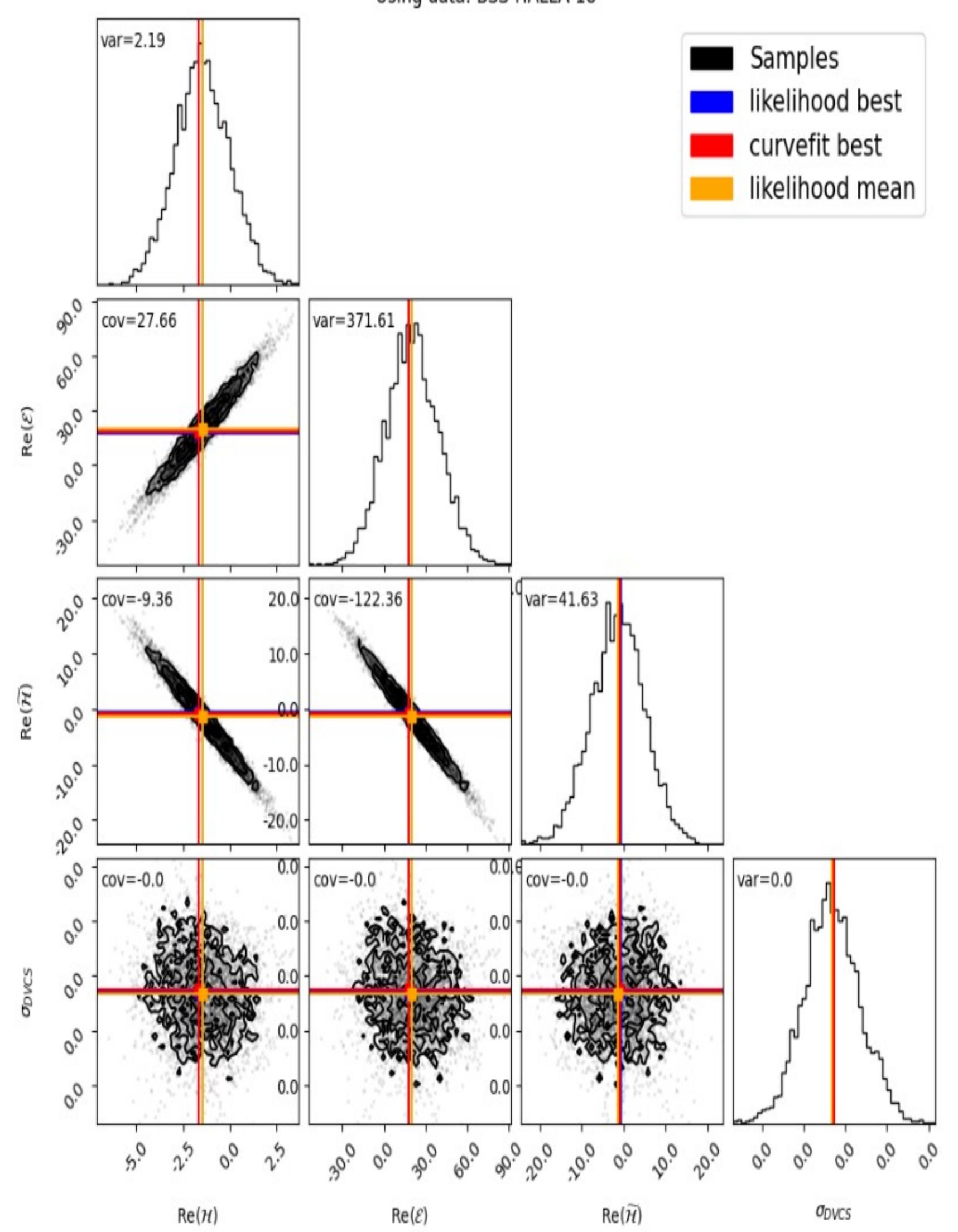
$$\mathcal{L} = \prod_{i=1}^{N} Gaussian(x, \mu, \sigma)$$

$$Gaussian(x, \mu, \sigma) \propto exp \left[-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^{2} \right]$$

- ☐ Markov Chain Monte Carlo (MCMC) algorithms are used to take multidimensional probability density functions and generate set of representative samples.
- These samples are used to create easy visualizations of the samples in the form corner plots.

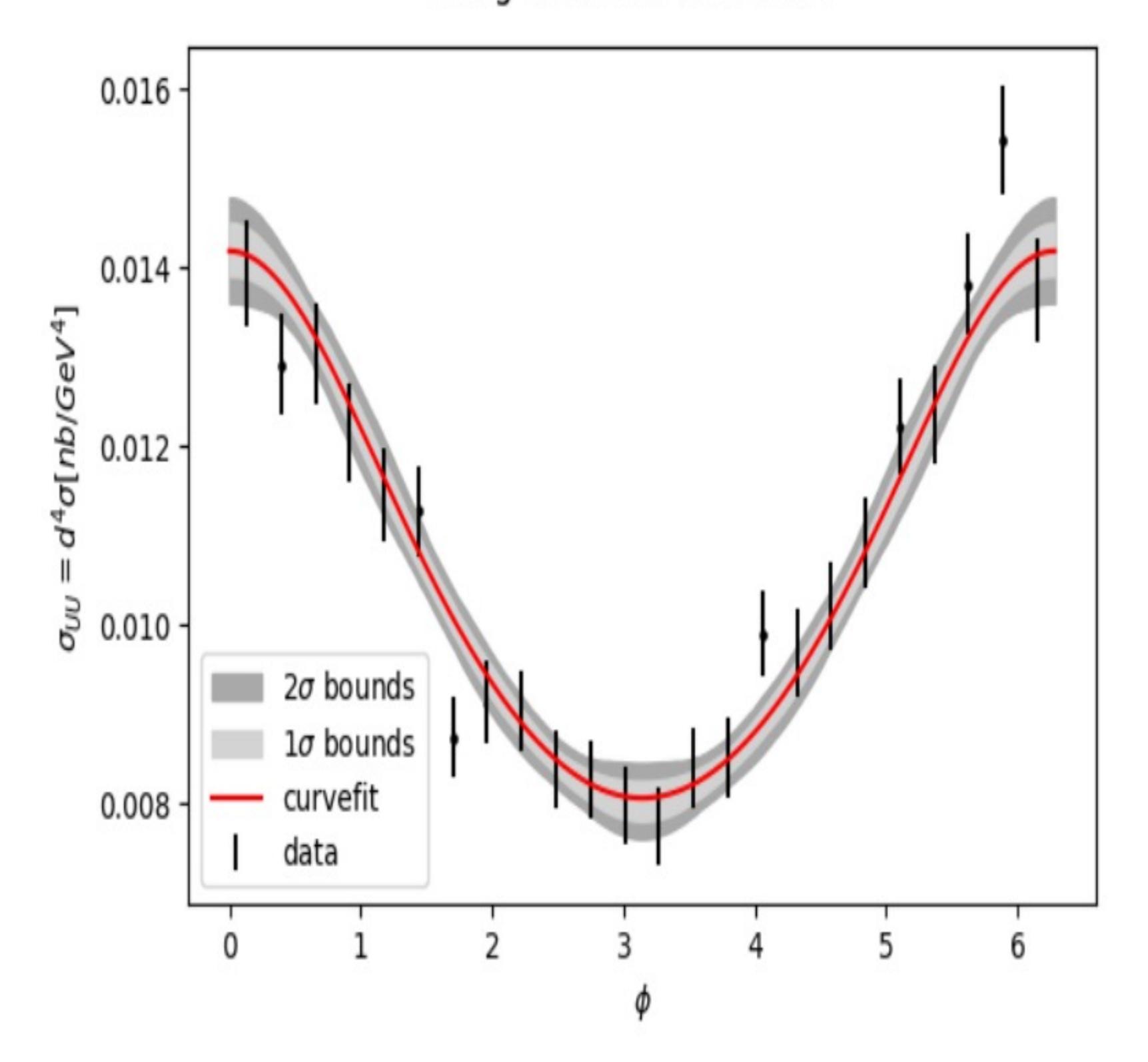
 https://arxiv.org/abs/2410.23469

Kinematic Bin: $\{E_b:10.591, x_{bj}:0.369, Q:2.1284, t:-0.2094, \}$ Max Likelihood: $\{Re(\mathcal{H}):-1.668, Re(\mathcal{E}):17.62, Re(\widetilde{\mathcal{H}}):-0.5491, \sigma_{DVCS}:0.0, \}$ Using data: BSS-HALLA-18



https://arxiv.org/abs/2410.23469

Kinematic Bin: $\{E_b:10.591, x_{bj}:0.369, Q:2.1284, t:-0.2094, \}$ Best Curve Fit: $\{Re(\mathcal{H}):-1.641, Re(\mathcal{E}):18.13, Re(\widetilde{\mathcal{H}}):-0.7673, \sigma_{DVCS}:0.0, \}$ Using data: BSS-HALLA-18



Difference likelihood result

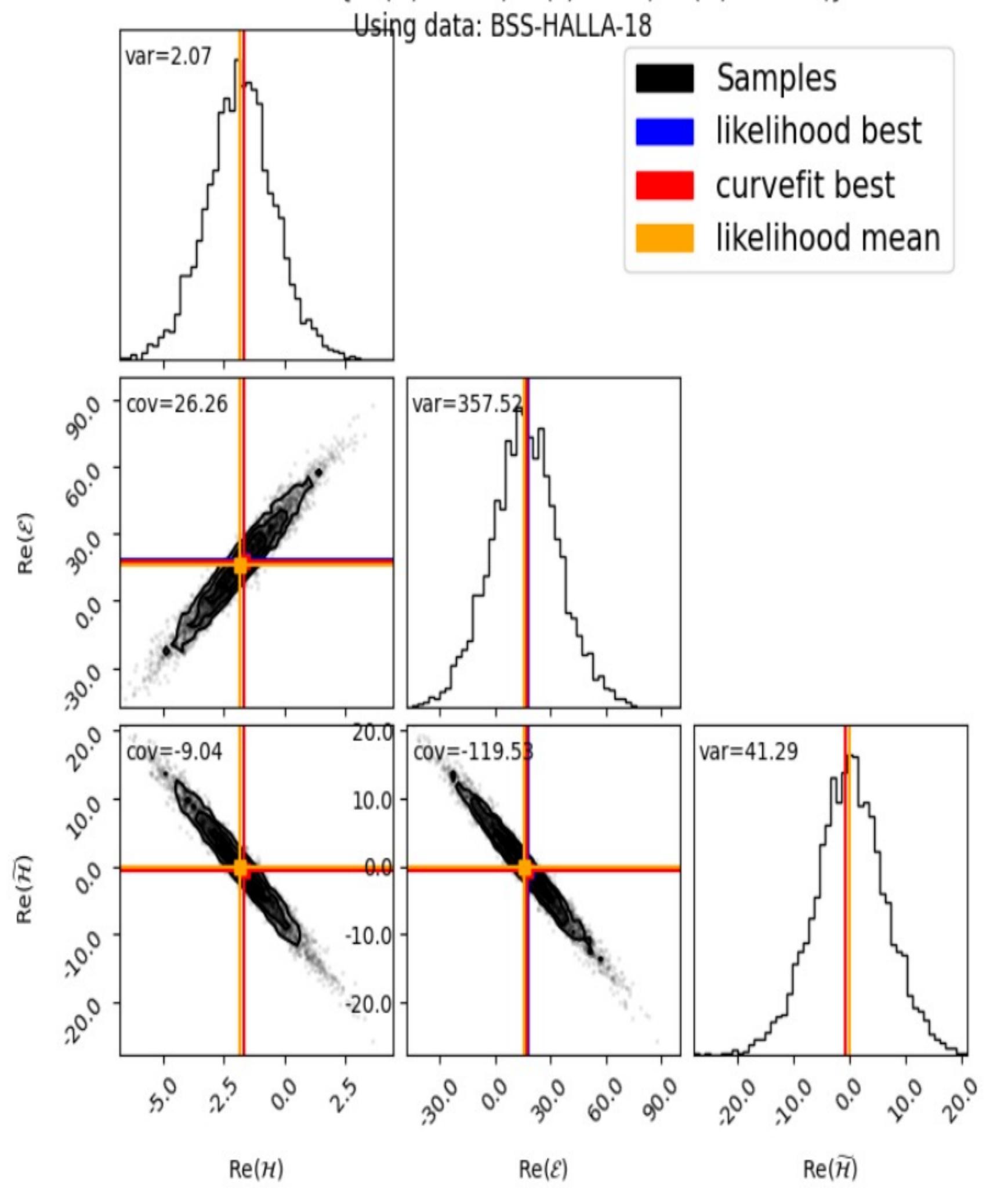
$$F_{UU,T} = 4[(1 - \xi^{2})[(\Re e\mathcal{H})^{2} + (\Im m\mathcal{H})^{2} + (\Re e\tilde{\mathcal{H}})^{2} + (\Im m\tilde{\mathcal{H}})^{2}] + \frac{t_{o} - t}{2M^{2}}[(\Re e\mathcal{E})^{2} + (\Im m\mathcal{E})^{2} + \xi^{2}(\Re e\tilde{\mathcal{E}})^{2} + \xi^{2}(\Im m\tilde{\mathcal{E}})^{2}] - \frac{2\xi^{2}}{1 - \xi^{2}}(\Re e\mathcal{H}\Re e\mathcal{E} + \Im m\mathcal{H}\Im m\mathcal{E} + \Re e\tilde{\mathcal{H}}\Re e\tilde{\mathcal{E}} + \Im m\tilde{\mathcal{H}}\Im m\tilde{\mathcal{E}})],$$

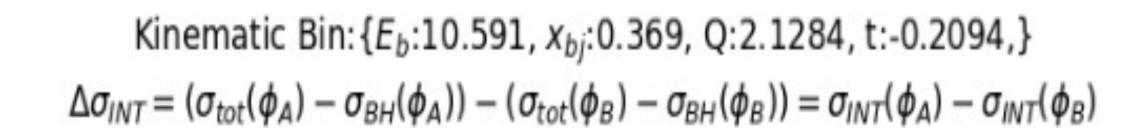
$$(56)$$

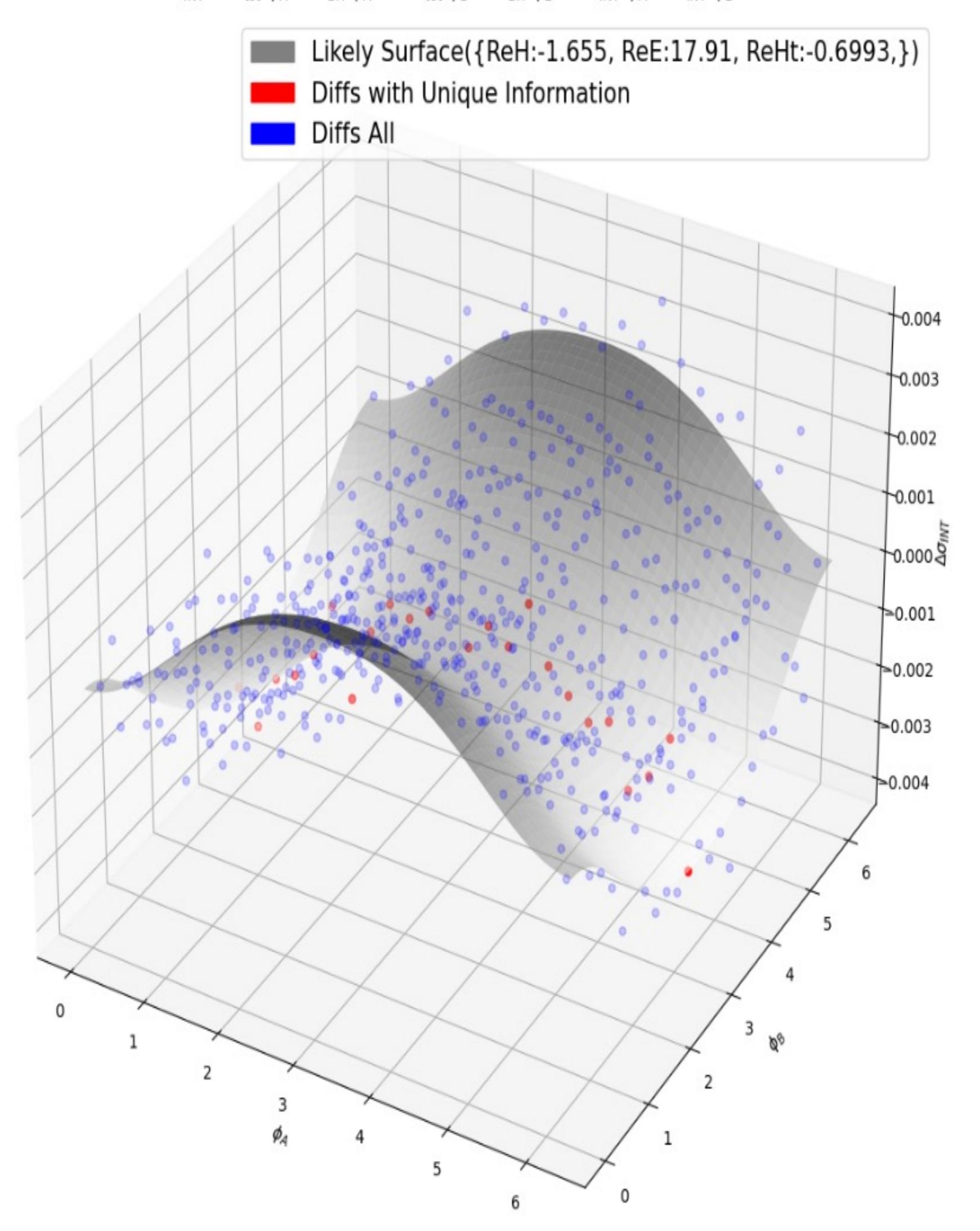
E_{beam} (GeV)	x_{Bj}	$Q^2~({ m GeV}^2)$	$t ({ m GeV}^2)$	ϕ (deg)	σ_{total}	$\Delta \sigma$
10.591	0.369	4.53	-0.2094	7.5	0.01394	0.00058
10.591	0.369	4.53	-0.2094	22.5	0.01292	0.00056
10.591	0.369	4.53	-0.2094	37.5	0.01305	0.00056
10.591	0.369	4.53	-0.2094	52.5	0.01216	0.00054
10.591	0.369	4.53	-0.2094	67.5	0.01147	0.00052
10.591	0.369	4.53	-0.2094	82.5	0.01128	0.00051
10.591	0.369	4.53	-0.2094	97.5	0.00875	0.00046

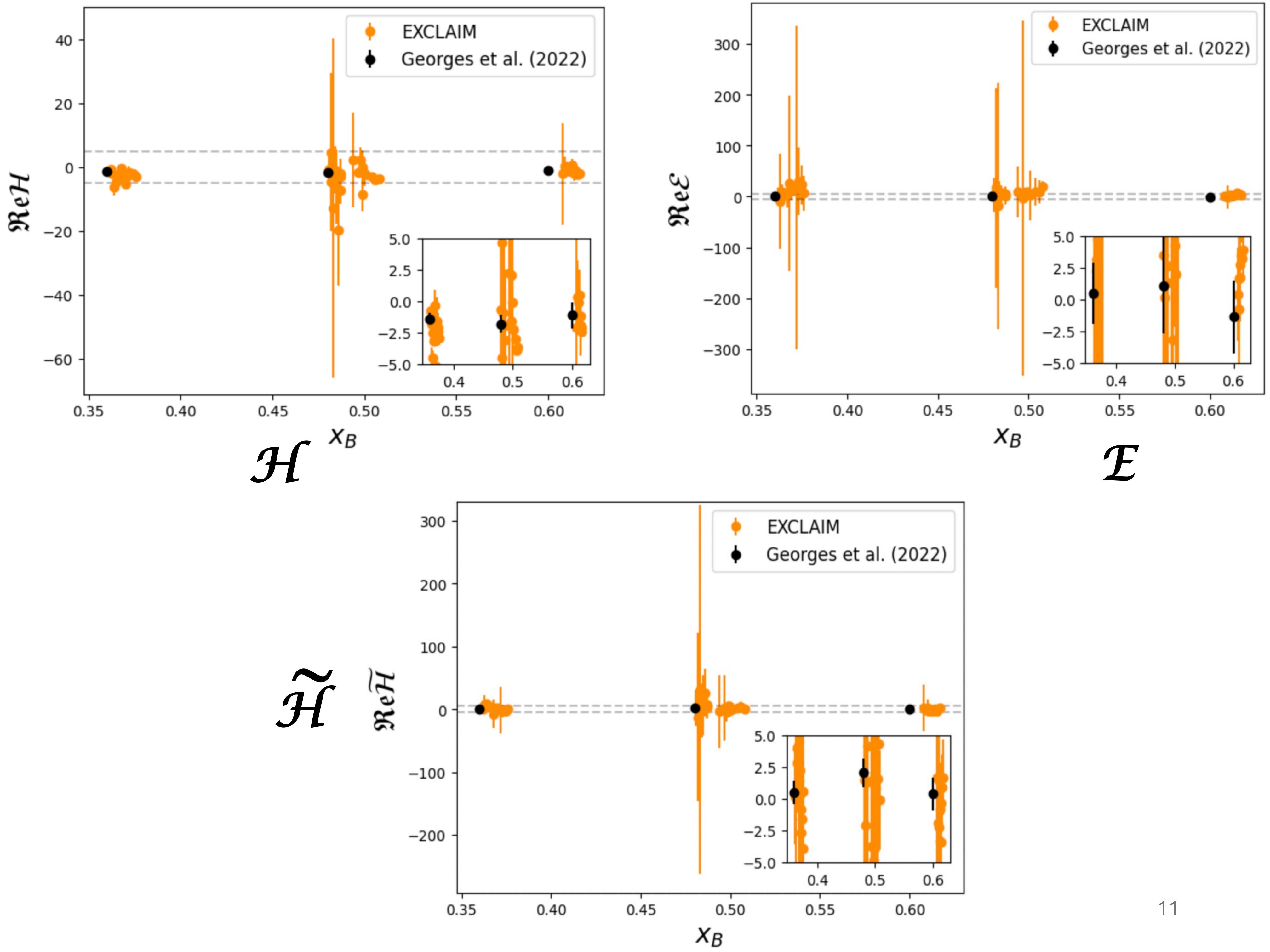
- Imore constraining method
- involves calculating some differences of cross sections for combinations choices of two angles.

Kinematic Bin: $\{E_b:10.591, x_{bj}:0.369, Q:2.1284, t:-0.2094,\}$ Max Likelihood: $\{Re(\mathcal{H}):-1.655, Re(\mathcal{E}):17.91, Re(\widetilde{\mathcal{H}}):-0.6993,\}$





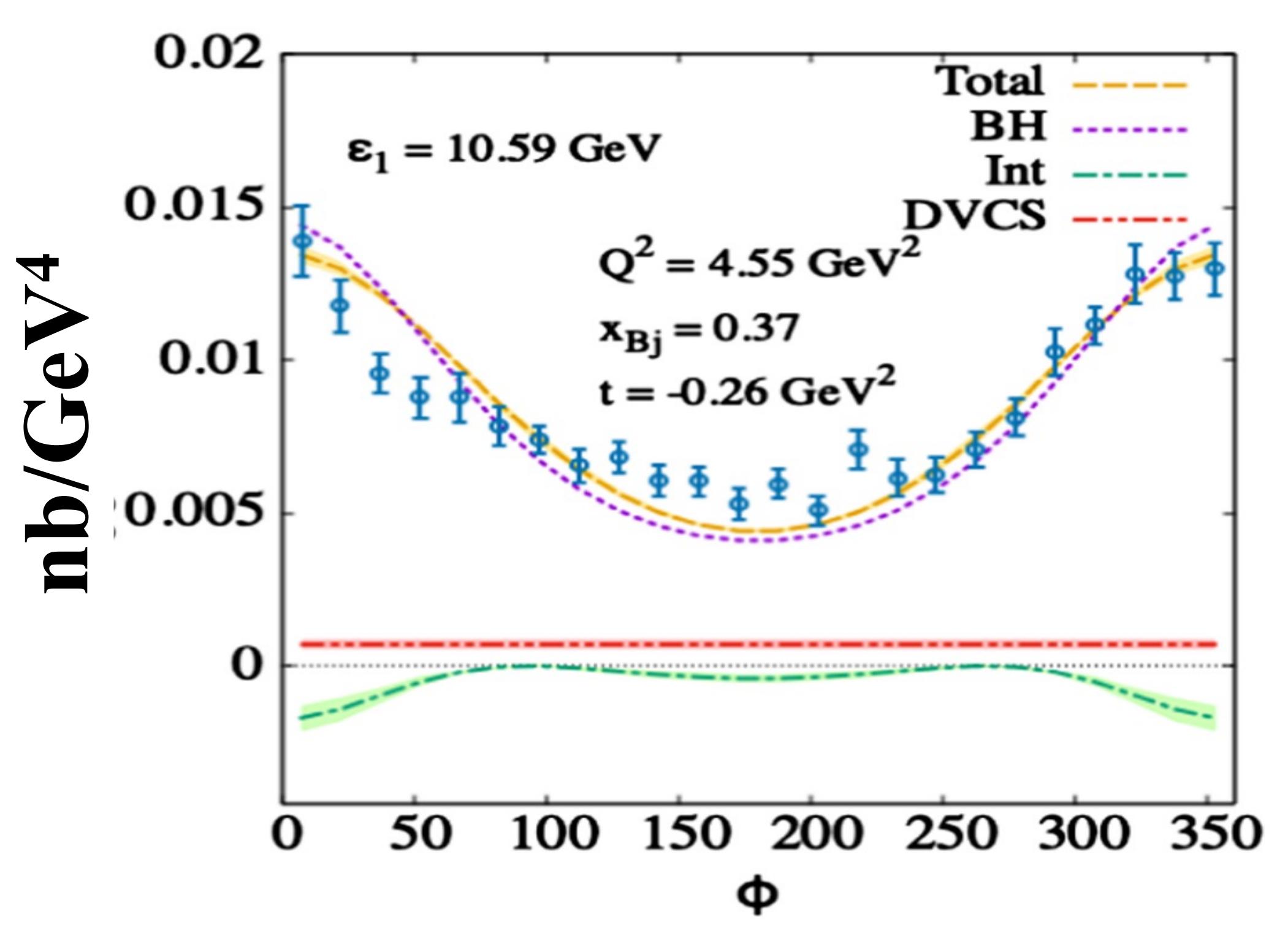




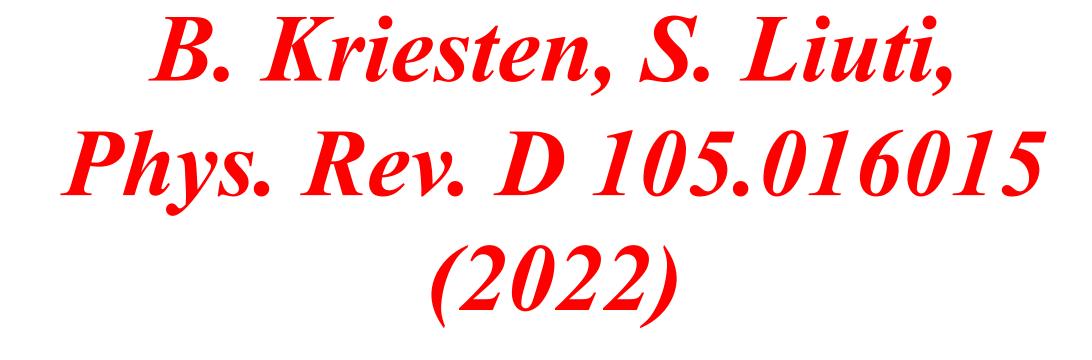
CFFs calculated using difference likelihood

T / (T T)		$\alpha / \alpha \tau$, / (C) T T T) \									
E (GeV)	x_{Bj}	Q (GeV)	$t (GeV^2)$	μ_H	μ_E	μ_{Ht}	$\Sigma_{H,H}$	$\Sigma_{E,E}$	$\Sigma_{Ht,Ht}$	$\Sigma_{H,E}$	$\Sigma_{H,Ht}$	$\Sigma_{E,Ht}$
4.487	0.483	1.646	-0.391	-4.493	0.173	5.454	9.726	35.722	55.462	18.007	-22.596	-42.475
4.487	0.483	1.646	-0.348	-12.779	-18.875	30.908	52.929	242.426	293.722	112.673	-123.436	-263.223
4.487	0.484	1.646	-0.435	-0.927	8.229	-2.092	7.424	20.815	42.313	11.453	-17.004	-27.473
4.487	0.485	1.646	-0.480	-6.142	5.385	6.293	11.723	17.263	47.518	11.581	-22.028	-25.109
4.487	0.485	1.649	-0.540	-19.579	6.592	26.070	17.562	8.300	37.734	3.584	-22.864	-10.353
7.383	0.363	1.780	-0.297	-0.776	0.393	0.186	0.197	16.700	3.797	1.284	-0.743	-7.005
7.383	0.363	1.780	-0.211	-1.867	-9.750	6.738	0.890	94.007	14.793	8.699	-3.503	-36.194
7.383	0.364	1.783	-0.586	-6.223	9.507	9.146	2.535	14.261	6.156	-2.366	-2.234	-3.500
7.383	0.365	1.783	-0.471	-4.490	2.274	4.029	0.569	12.488	4.515	0.540	-0.969	-5.578
7.383	0.365	1.783	-0.385	-2.467	3.188	2.867	0.200	11.356	3.134	0.533	-0.544	-4.787
8.521	0.367	1.910	-0.266	-3.154	4.729	4.138	0.256	27.333	5.037	2.154	-1.029	-10.802
8.521	0.367	1.910	-0.205	-0.324	24.985	-7.389	1.256	172.650	21.984	14.256	-5.124	-60.197
8.521	0.369	1.916	-0.330	-2.951	10.543	1.446	0.235	18.439	4.104	1.194	-0.774	-7.623
8.521	0.370	1.918	-0.480	-5.214	12.863	2.266	0.820	7.746	2.560	-0.098	-0.890	-2.846

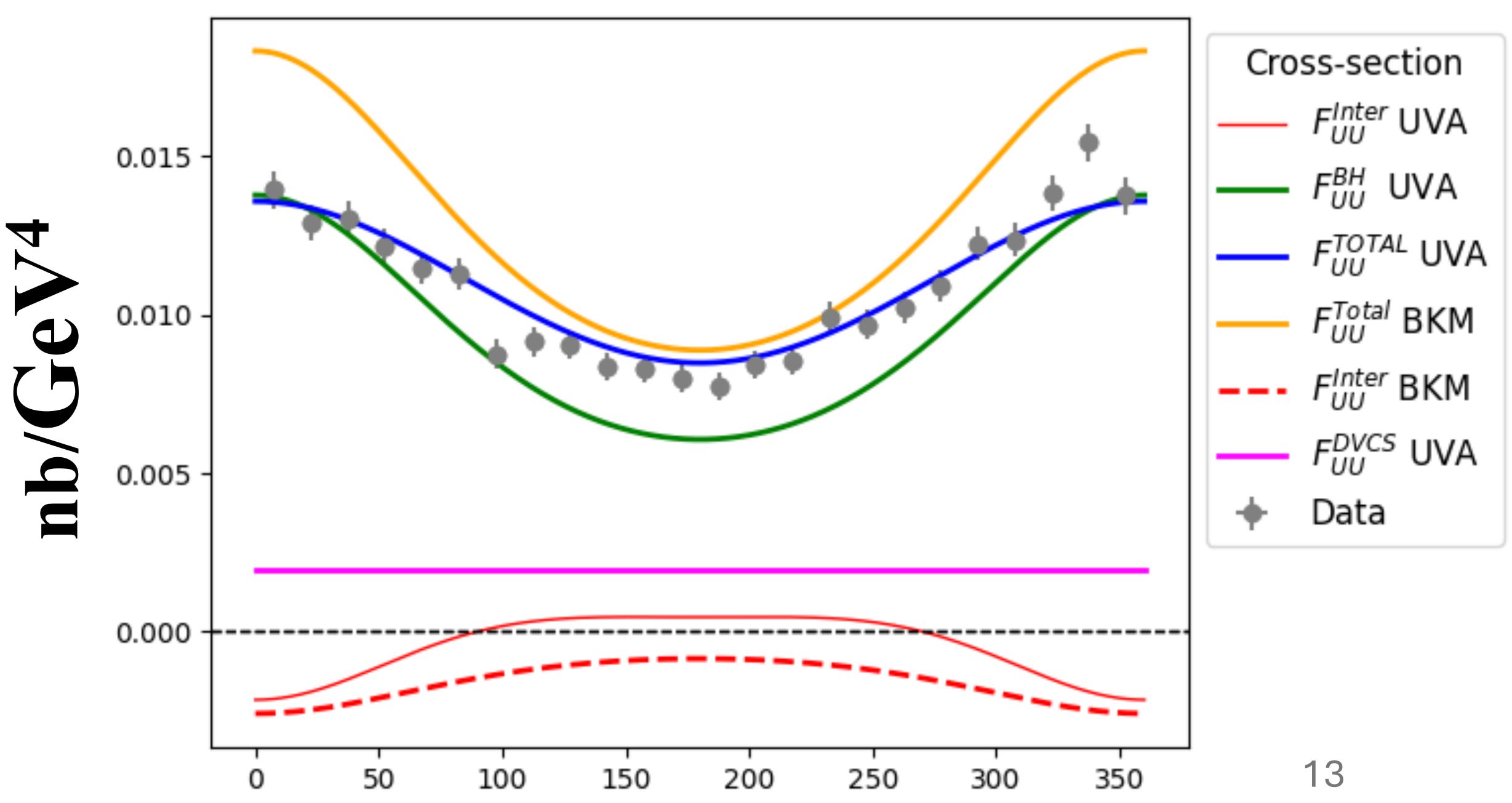
Not the complete table For full results, please see https://arxiv.org/abs/2410.23469 12

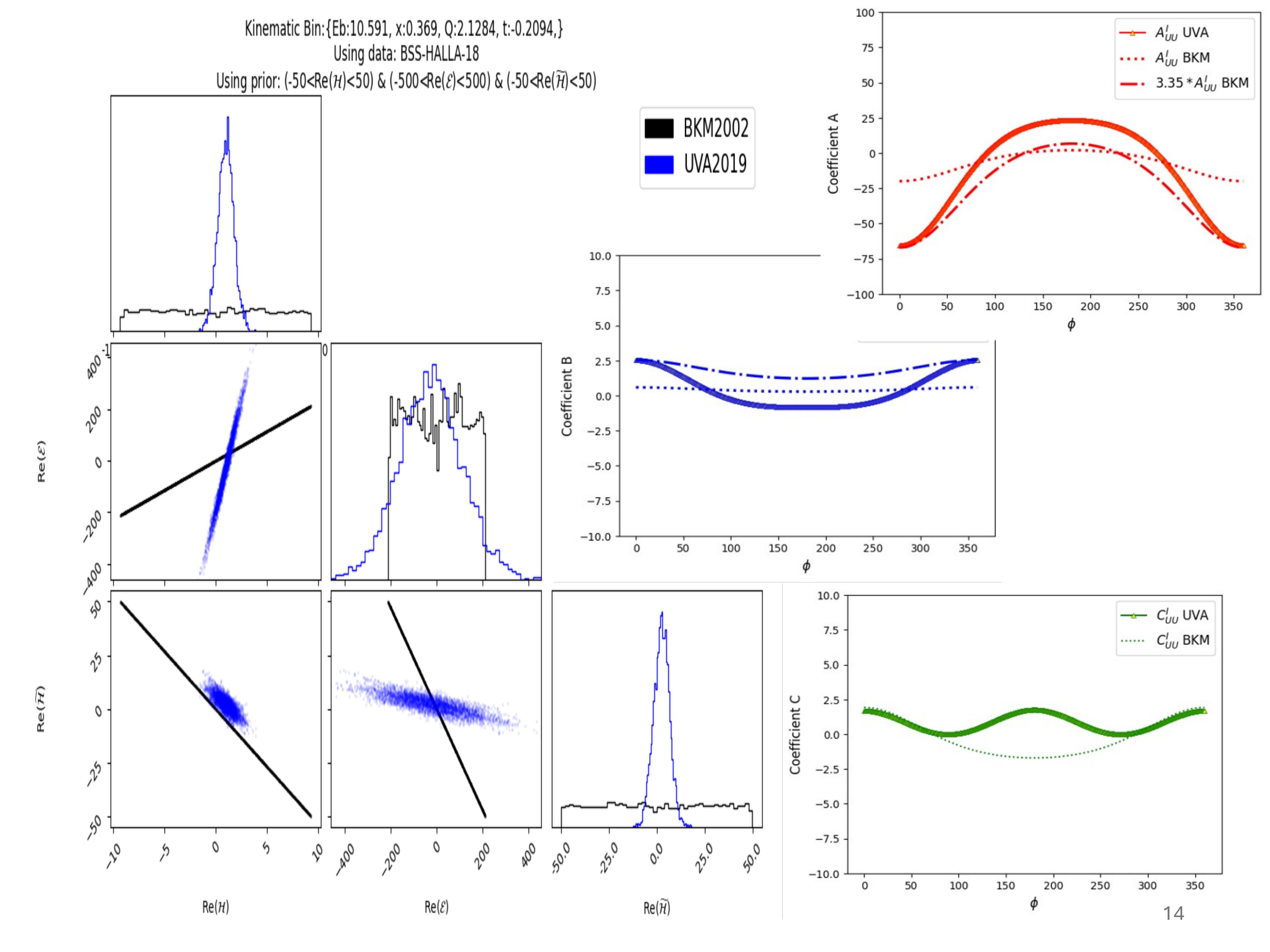


E = 10.591 GeV $Q^2 = 4.53 \text{ GeV}^2$ $x_{Bj} = 0.369$ $t = -0.20942 \text{ GeV}^2$



A. V. Belitsky, D. Mueller, and A. Kirchner,
Nucl. Phys. B629, 323 (2002).





What next can be done!

Possibility to improve the Analysis

More Observables

$$\begin{split} F_{LL} &= 4[2(1-\xi^2)(\Re e\mathcal{H}\Re e\tilde{\mathcal{H}} + \Im m\mathcal{H}\Im m\tilde{\mathcal{H}}) + 2\frac{t_o - t}{2M^2}(\Re e\mathcal{E}(\xi\Re e\tilde{\mathcal{E}}) + \Im m\mathcal{E}(\xi\Im m\tilde{\mathcal{E}})) \\ &+ \frac{2\xi^2}{1-\xi^2}(\Re e\mathcal{H}\Re e\tilde{\mathcal{E}} + \Im m\mathcal{H}\Im m\tilde{\mathcal{E}} + \Re e\tilde{\mathcal{H}}\Re e\mathcal{E} + \Im m\tilde{\mathcal{H}}\Im m\mathcal{E})], \end{split}$$

$$F_{LL}^{\mathcal{I},tw2} = A_{LL}^{\mathcal{I}} \Re e(F_1(\tilde{\mathcal{H}} - \xi \tilde{\mathcal{E}}) + \tau F_2 \tilde{\mathcal{E}}) + B_{LL}^{\mathcal{I}} G_M \Re e \tilde{\mathcal{H}} + C_{LL}^{\mathcal{I}} G_M \Re e(\mathcal{H} + \mathcal{E})$$

$$\begin{split} \frac{d^4\sigma}{dx_{Bj}dyd\phi dt} &= \Gamma \left\{ \left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} + h \sqrt{\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \right] \right. \\ &+ \left. S_{||} \left[\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + h \left(\sqrt{1-\epsilon^2} F_{LL} + \sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} \right) \right] \right. \\ &- \left. S_{\perp} \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) + \frac{\epsilon}{2} \left(\sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right) \right. \\ &+ \left. \sqrt{\epsilon(1+\epsilon)} \left(\sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right) \right] \\ &+ \left. S_{\perp} h \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} + \sqrt{\epsilon(1-\epsilon)} \left(\cos \phi_S F_{LT}^{\cos \phi_S} + \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right) \right] \right\} \end{split}$$

Conclusion

- ☐ Attempt to have a possible extraction of Compton form factors from unpolarized DVCS cross-section.
- Need of more observables
- ☐ Improve the extracted CFFs by having one more observable such as polarized cross-section.

$$\epsilon = \frac{1 - y - \frac{1}{4}y^2\gamma^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}y^2\gamma^2} = \frac{\sum_h |A_h^0|^2}{\sum_h \sum_{\Lambda_{\gamma^*} = \pm 1} |A_h^{\Lambda_{\gamma^*}}|^2}$$

$$\sigma_{LL} = (\sigma_{++} - \sigma_{+-}) - (\sigma_{-+} - \sigma_{--}) = \sigma_{LL}^{DVCS} + \sigma_{LL}^{BH} + \sigma_{LL}^{T},$$

$\sigma_{\rm UU}$, $\sigma_{\rm LL}$, ϵ , Φ and the interference terms

Dworking on the formalism (soon to be on arxiv)

Possible solution at EIC

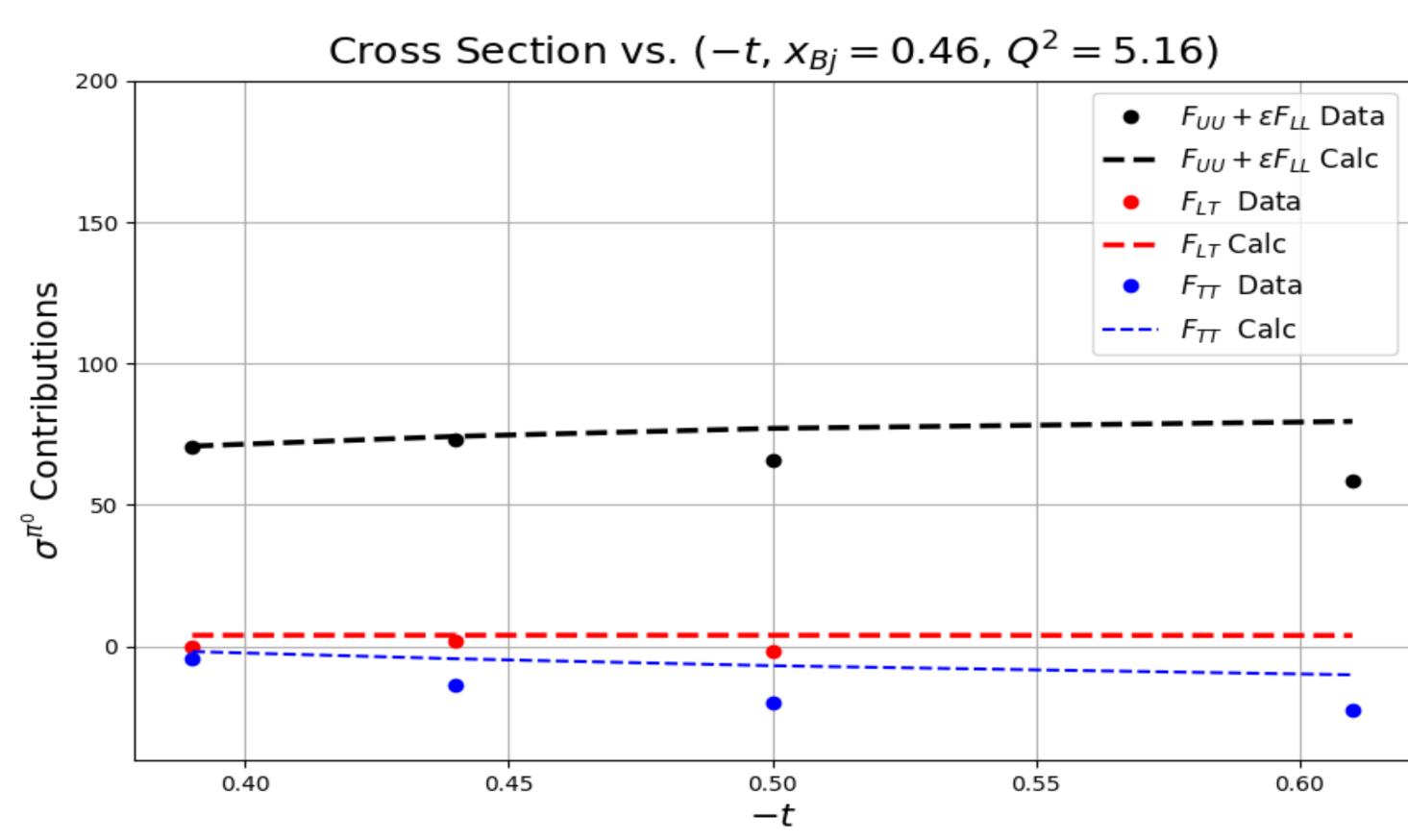
- Low x
- High Q²
- more polarizations



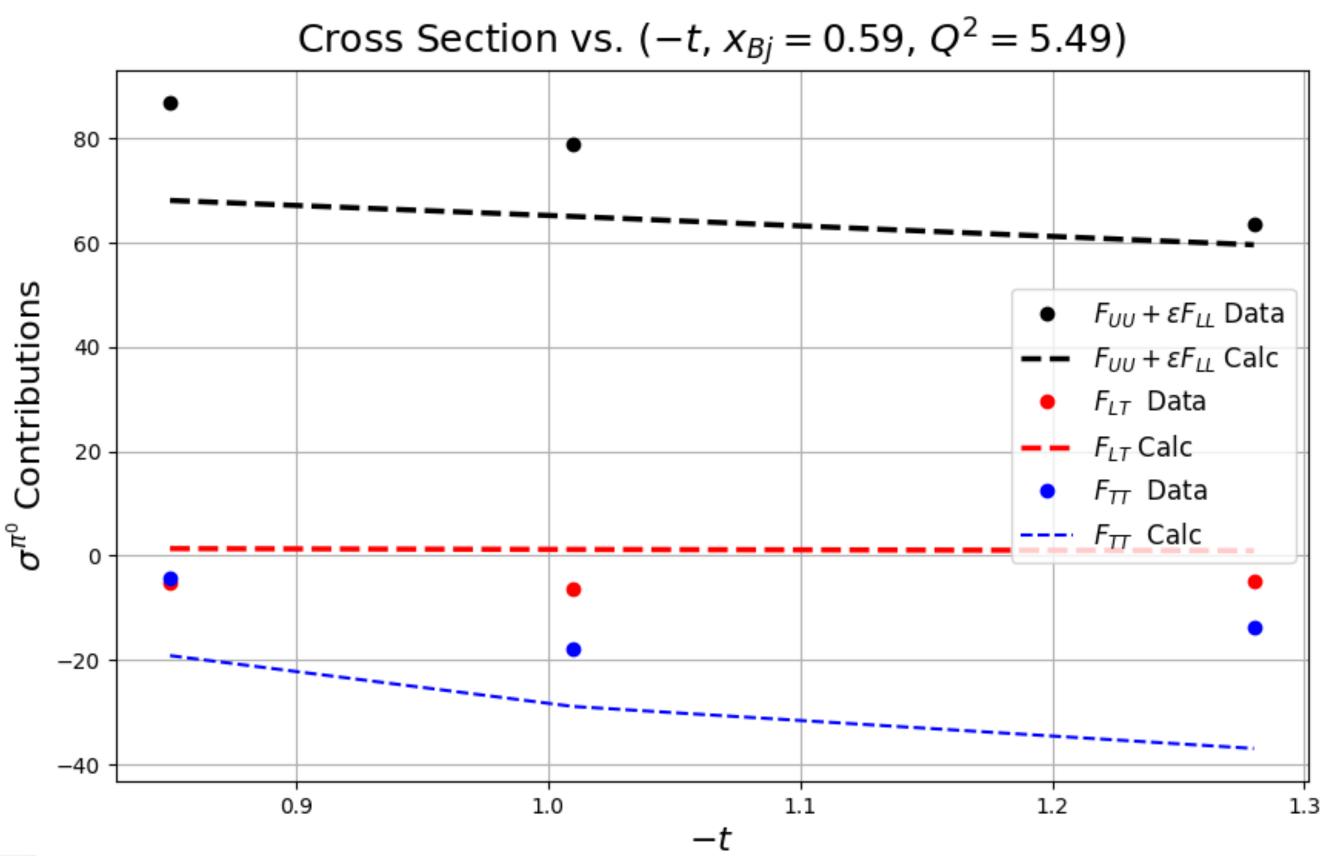
Next/Future Steps

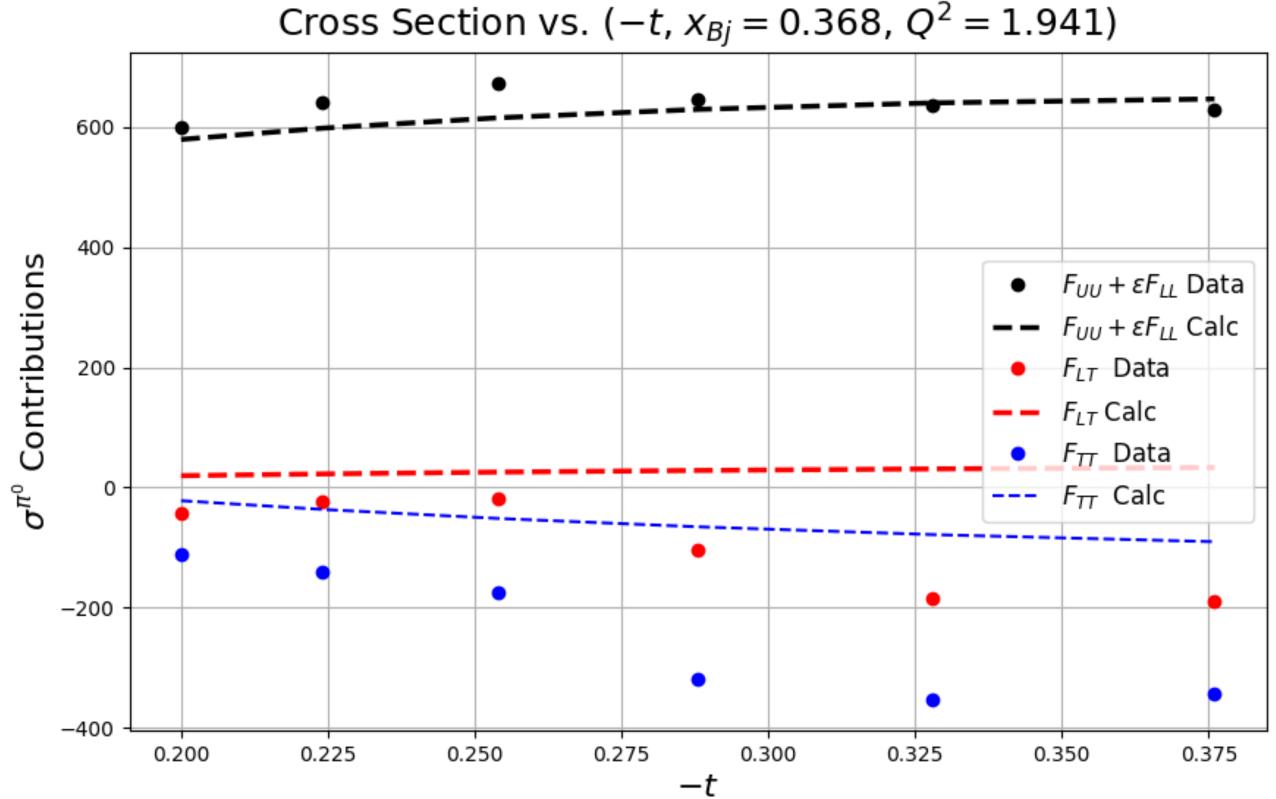
- Possible Analysis for the Chiral odd sector
 - DExclusive meson production

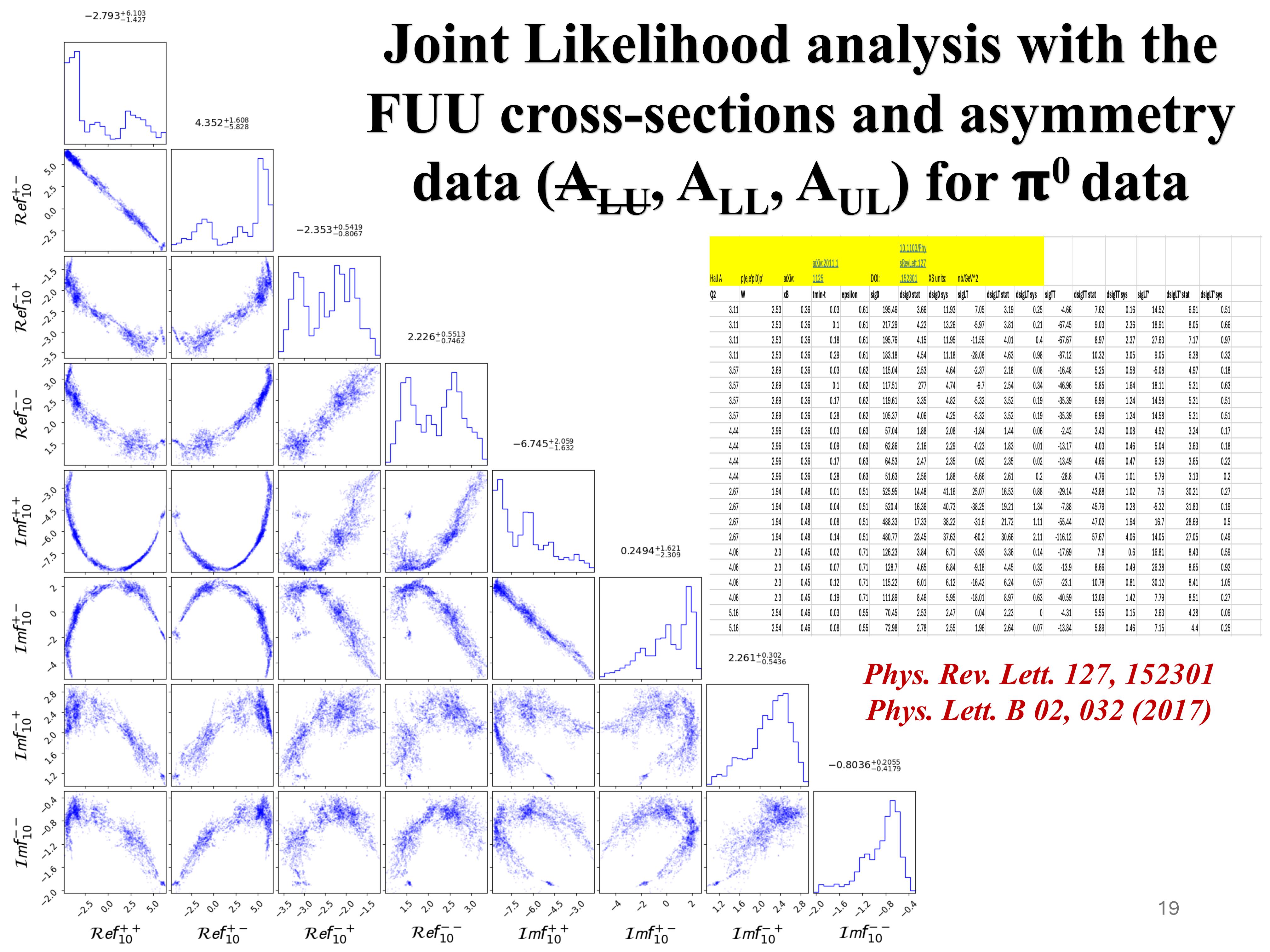
$$\begin{split} \frac{d^4\sigma}{dx_{Bj}dyd\phi dt} &= \Gamma \left\{ \left[F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} + h \sqrt{\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \right] \right. \\ &+ \left. S_{||} \left[\sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + h \left(\sqrt{1-\epsilon^2} \, F_{LL} + \sqrt{\epsilon(1-\epsilon)} \, \cos \phi \, F_{LL}^{\cos \phi} \right) \right] \right. \\ &- \left. S_{\perp} \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) + \frac{\epsilon}{2} \left(\sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right) \right. \\ &+ \left. \sqrt{\epsilon(1+\epsilon)} \left(\sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right) \right] \\ &+ \left. S_{\perp} h \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} + \sqrt{\epsilon(1-\epsilon)} \left(\cos \phi_S F_{LT}^{\cos \phi_S} + \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right) \right] \right\} \end{split}$$

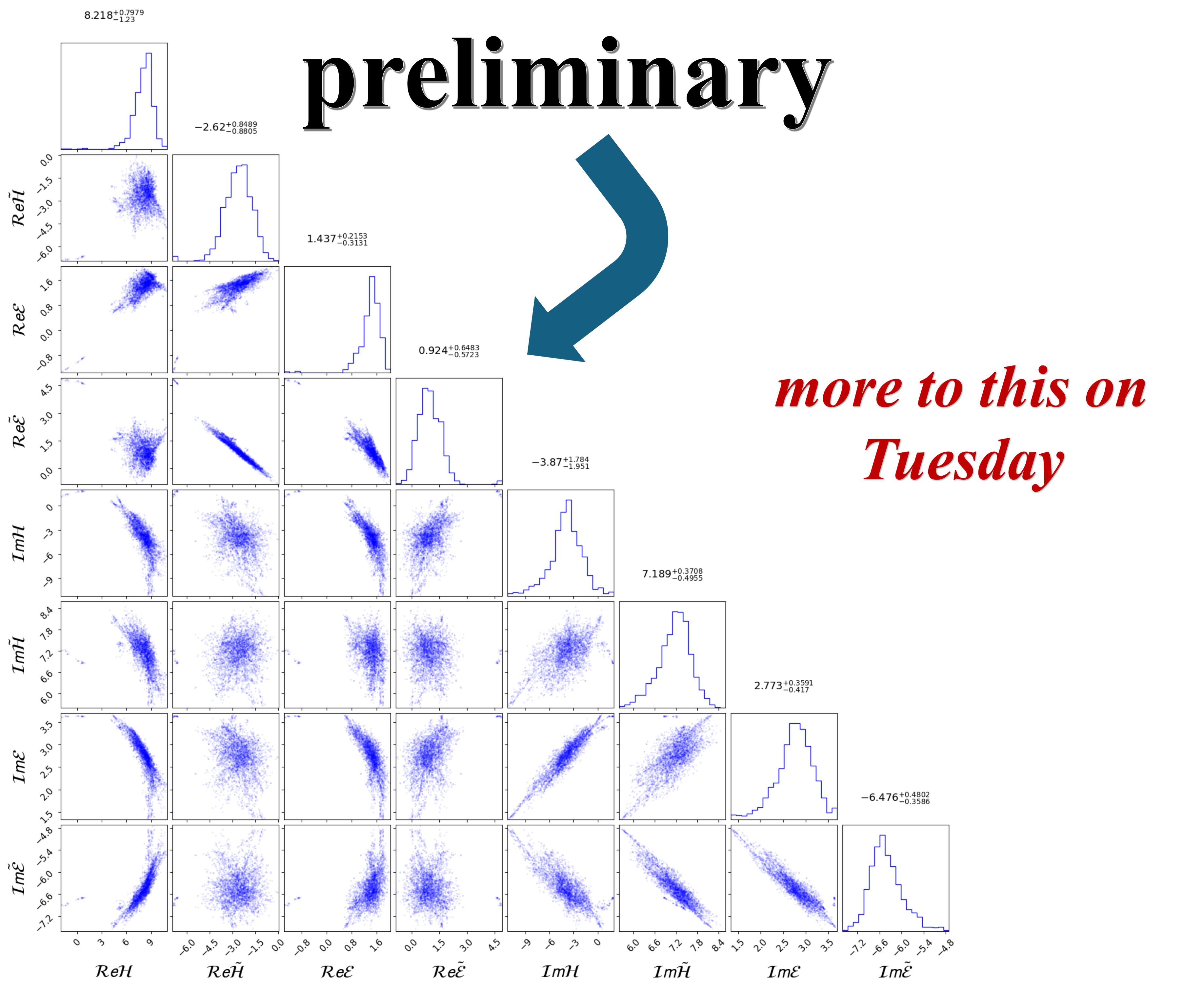


HALL A data









Thankyou

Backup slides

