Radiative Corrections for the PRad-II Experiment and the Status

Haiyan Gao

March 3, 2025

PRad-II/X17 Collaboration Meeting

Duke

PRad's estimation of the RC syst. uncertainty of r_p

- Measured radius: *Nature 575, 147 (2019)*
  - $r_p = (0.831 \pm 0.007_{stat} \pm 0.012_{syst})$ fm
- r_p uncertainties for PRad shown in the table
  - Uncertainties estimated using the rational (1,1) function

    *X. Yan et al. PRC **98**, 025204 (2018)*

Using rational (1,1)

$$f(Q^2) = \frac{1 + p_1 Q^2}{1 + p_2 Q^2}$$

Item	PRad δr_p [fm]
Stat. uncertainty	0.0075
GEM efficiency	0.0042
Acceptance	0.0026
Beam energy related	0.0022
Event selection	0.0070
HyCal response	0.0029
Beam background	0.0039
Radiative correction	0.0069
Inelastic *ep*	0.0009
G_M^p parameterization	0.0006
Total syst. uncertainty	0.0115
Total uncertainty	0.0137

Duke

PRad's estimation of the RC syst. uncertainty of r_p

- RCs one of the largest syst. uncertainty sources of r_p for PRad
  - RCs studied for both e-p and Møller scatterings
    - Event generator used, made using the results from *I. Akushevich et al, EPJA **51**, 1 (2015)*
      - Used analytical calculations for one-loop e-p and Møller RC diagrams
      - Calculated within covariant formalism and beyond ultra-relativistic limit
      - Infrared divergence extracted and cancelled by Bardin-Shumeiko approach

(ep cross checked by event generator using A. V. Gramolin et al., J. Phys. G Nucl. Part. Phys. 41(2014)115001; MØller by what's used in the Olympus experiment.)

  - PRad RC syst. uncertainty on r_p estimated
    - using the first-order RC results from *EPJA **51**, 1 (2015)*
    - using a method from *A. Arbuzov and T. Kopylova, EPJC **75,** 603 (2015)* for estimation of the contribution stemming from higher order RCs
  - Estimated correlated and Q^2 –dependent syst. uncertainties

Duke

PRad's estimation of the RC syst. uncertainty of r_p

- Two methods for forming e-p to e-e differential cross section ratio (luminosity cancellation)
  - Bin-by-bin method
    - Forms the ratio using the e-p and e-e counts from the same angular bin
    - Cancels out the energy-independent part of acceptance and GEM efficiency
    - Q^2 -dependent syst. uncertainties from the e-e process introduced
  - Integrated Møller method
    - Uses e-e counts from a selected angular range
    - Gives a common normalization factor for all e-p Q^2 bins; no effect on extracted r_p
    - Not applied to all Q^2 bins in PRad, since the GEM efficiency not precisely determined in all those bins
- Q^2-dependence much larger for Møller RC in PRad
  - Affects the cross-section results via the use of the bin-by-bin method
  - For e-p RC → $\delta r_p = 0.0020\ fm$; for Møller RC → $\delta r_p = 0.0065\ fm$
  - For total RC → $\delta r_p = 0.0069\ fm$

Duke

Independent study of the RC syst. uncertainty of r_p

- Independent study performed for the second-order RC effect on r_p
  - Followed the approach of *A. Aleksejevs et al, Physics of Atomic Nuclei, **76**, 888 (2013)*
    - Paper calculated two-loop radiative effects in the MOLLER experiment
  - Based on its mathematical framework and for PRad kinematics
    - Contribution from NNLO diagrams on the Born cross section estimated
    - For any reasonable photon energy cut for the PRad experiment
    - Q^2 -dependent syst. uncertainties smaller than that estimated in the first approach
    - The largest RC syst. uncertainty computed: $\delta r_p = 0.0047\ fm$
  - However, approximated methods and restricted number of diagrams used
    - Improved and exact NNLO calculations are much desired

Duke

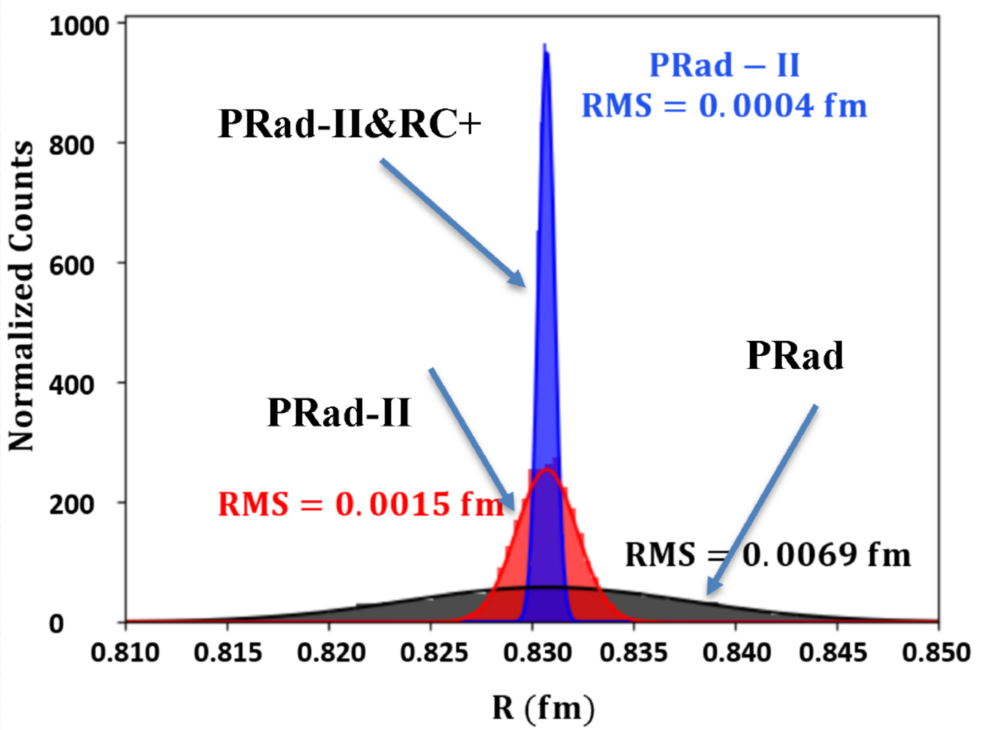
Integrated MØller method

- Limitations of GEM efficiency determination in PRad
  - Contributed indirectly to the total syst. uncertainty
    - Should be improved
- Aiming at a significantly better precision in PRad-II compared with PRad
  - Employ two planes of coordinate tracking detectors
    - Achieve a precise measurement of tracking detector efficiency (~ 0.1% level)
    - Reduce various backgrounds
    - Use the integrated Møller method for all angular bins
    - Suppress the Q^2-dependent syst. uncertainties
    - Turn all the Møller syst. uncertainties into cross section normalization uncertainties
    - δr_p from RCs will be reduced from *0.0069 fm* to *0.0015 fm*

Duke

Improvement from PRad to PRad-II

- Improvement of the RC associated syst. uncertainty on r_p
  - Black spectrum $\rightarrow$ RC δr_p for PRad
  - Red spectrum $\rightarrow$ projected RC δr_p with two planes of coordinate tracking detectors plus current RC calculations
  - Blue spectrum $\rightarrow$ projected RC δr_p with two planes of coordinate tracking detectors plus improved RC calculations at NNLO



- Outline presented
  - Whitepaper on Radiative Corrections: *arXiv:2012.09970 [nucl-th]*
  - Synergy with ongoing RC-related studies for the JLab SoLID SIDIS and the planned studies for the proposed DRad experiment

Duke

Plans for the NNLO calculations

- To achieve the PRad-II goal of total syst. uncertainty of *0.0032 fm (proposed to the PAC)*
  - Necessary to perform improved NNLO RC calculations
    - Had plans with the PRad Collaboration's theory colleagues
    - Dr. Stanislav Srednyak in close collaboration with Drs. Igor Akushevich and Alexander Ilyichev
    - Contacts/potential collaborations with the PSI and Mainz groups on the subject matter established

- Advantages and disadvantages of the original paper *EPJA* ***51****, 1 (2015)*
  - Advantages
    - Both e-p and e-e treated in the same approach
    - First-order diagrams calculated analytically
    - Dependence on the electron mass kept, accurate in $\mathbb{O}(\alpha)$
  - Disadvantages (*indicated by Andrej Arbuzov at First TPC Collaboration Meeting in Mainz*)
    - Improper treatment of higher-order effects
    - No two-photon exchange, no hadronic vacuum polarization (PRad simulation included TPE effect)
    - No radiation off proton and up-down interference,

Duke

Status for the NNLO calculations

- Submitted proposal to DOE by Akushevich (PI) and Gao (co-PI) on NNLO RC calculations for PRad-II (under review) (pending or unfunded)

Received BNL LDRD funding for RC studies focusing on PRad-II and SIDIS

- PRad-II tasks
  - Calculation of NNLO contributions to e-p and Møller scattering diagrams
    - Focus on mathematical approach of Gelfand-Kapranov-Zelevinsky
    - Develop the so-called Gamma series method
    - Calculate one- and two-loop integrals with the new method
  - Obtain all necessary results
    - Evaluate also the two-photon exchange part to hadronic corrections
    - Make a new MC event generator or update the current one (working with PRad)

Unfortunately, due to many complicated reasons, the worked by the theory postdoc did not lead to direct results for PRad-II.

Duke

Status and plan for the NNLO calculations

Implement McMule approach into PRad/PRad-II simulation code
New Duke postdoc will join us shortly and will work on this.

UNIVERSITY OF LIVERPOOL

what is McMule?

fixed-order NNLO QED framework Monte Carlo for MUons and other LEptons

- provided: matrix elements by us or others
- output: physical cross section for any physical observable
- McMule: phase space generation, subtraction, stabilisation, integration, event generation, etc.
- all leptonic $2 \to 2$ processes in QED at NNLO (+ a few others)
- stable public version is an integrator
- generator on development branch

Get the code here: `https://mule-tools.gitlab.io`
Read the docs here: `https://mcmule.readthedocs.io`

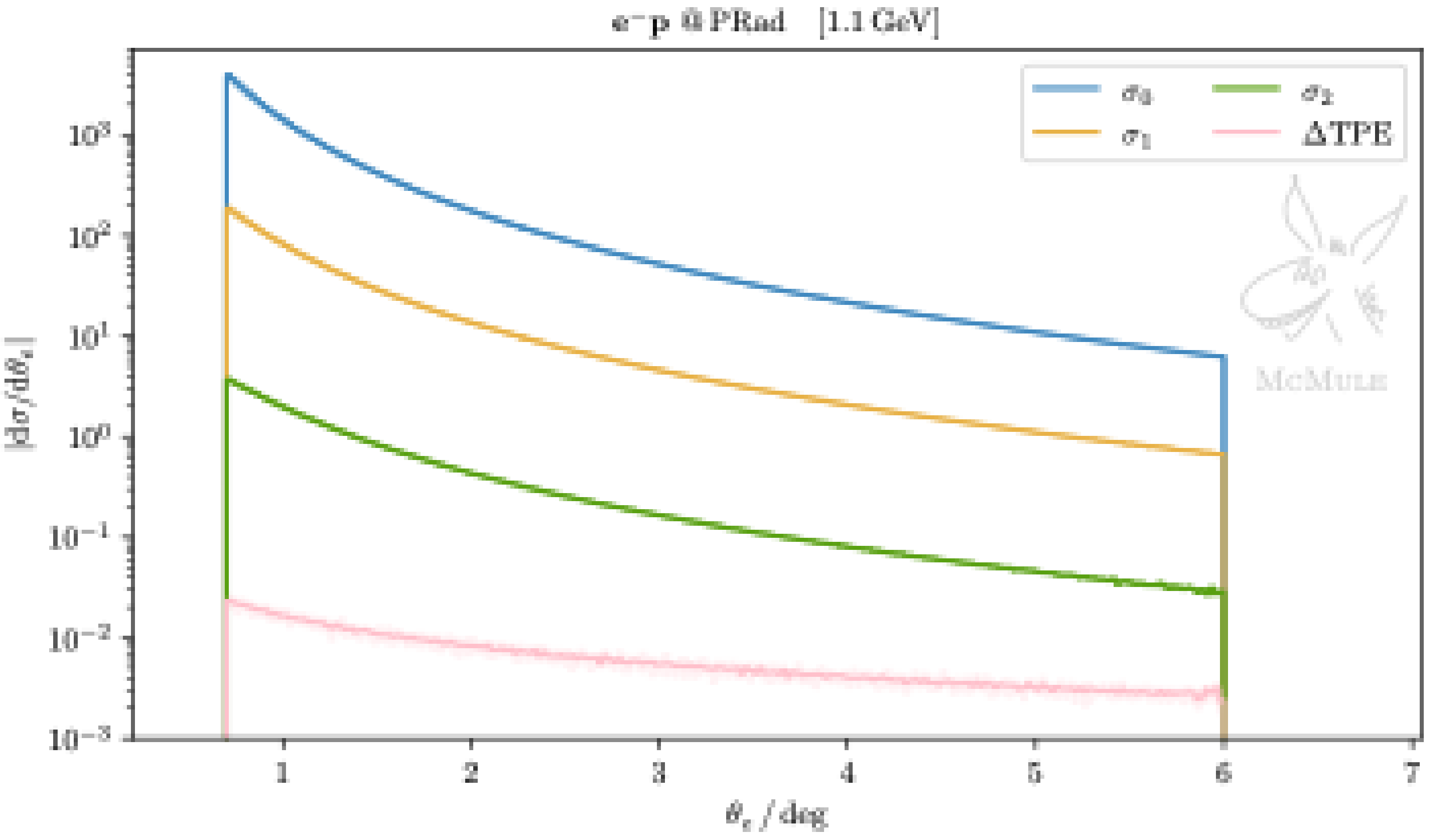
McMule

Yannick Ulrich, 30.10.24 – p.2/17

Duke

UNIVERSITY OF LIVERPOOL

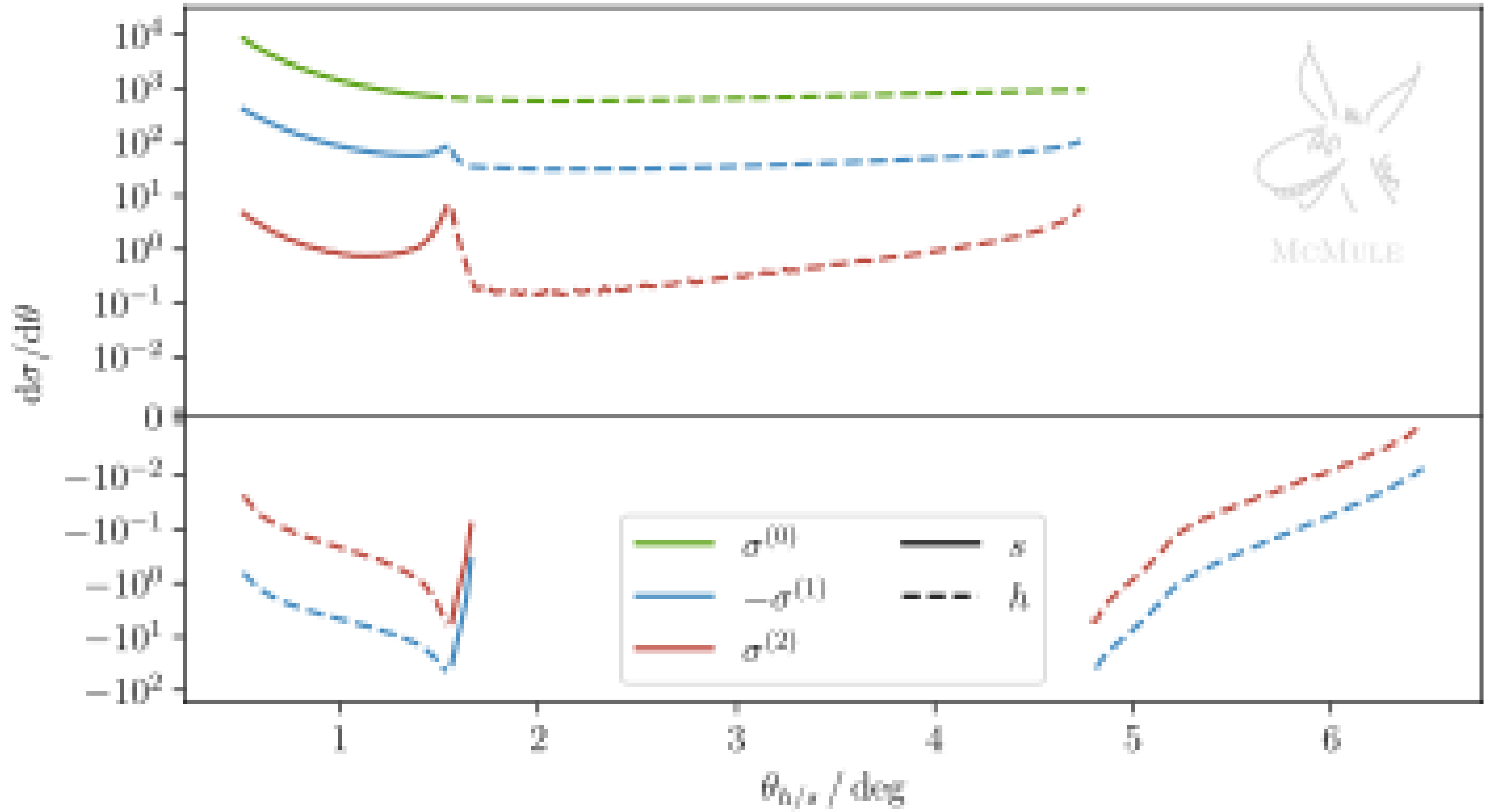
results for e-p scattering



Yannick Ulrich, 30.10.24 – p.13/17

UNIVERSITY OF LIVERPOOL

results for e-e scattering



Duke

Summary

- In PRad, the RC effect was the second largest syst. uncertainty source for r_p
- PRad-II compared to PRad
  - Employ two planes of coordinate detectors to improve the detector efficiency
    - Apply the integrated Møller method to all angular bins
    - Suppress the Møller Q^2-dependent syst. uncertainties
    - Accomplish improved NNLO calculations for e-p and Møller scatterings
    - Experimental (partial) validation of calculations of radiative effects

We will work with McMule to implement their NNLO RC calculations into PRad-II simulations.

Duke