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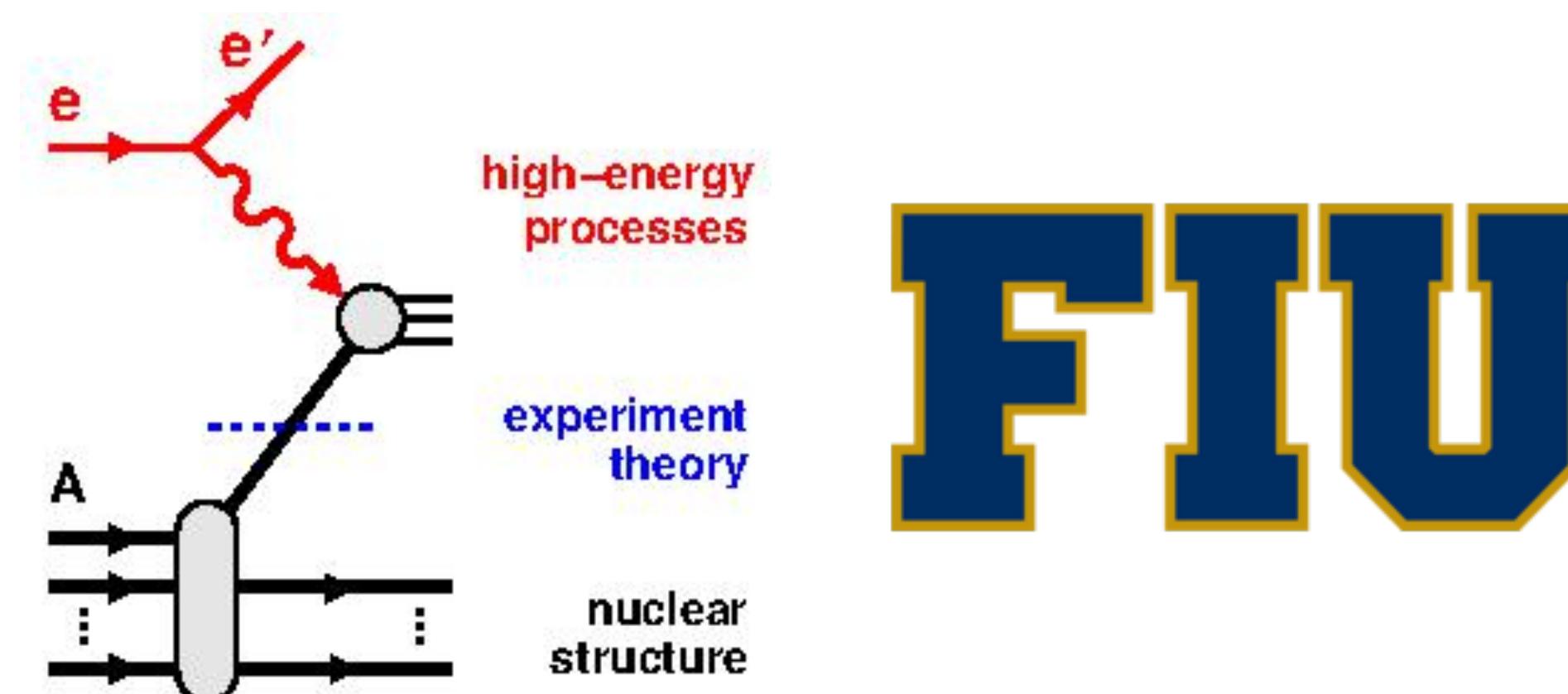
INFN  
PERUGIA  
Istituto Nazionale di Fisica Nucleare  
Sezione di Perugia

STRONG  
2020  
EU

# Nuclear structure functions within the light-front Hamiltonian dynamics

**Filippo Fornetti** (Università degli Studi di Perugia, INFN, Sezione di Perugia)

**“Light-ion physics in the EIC era: From nuclear structure to high-energy processes”**



**FIU**

# Based on

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F.F, E.Pace, M.Rinaldi, G.Salmè, S.Scopetta and M.Viviani,  
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**Phys.Lett.B 851 (2024) 138587**

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“<sup>3</sup>He spin-dependent structure functions within the relativistic light-front Hamiltonian dynamics”,  
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# Overview

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- Light-front formalism for the unpolarized Deep Inelastic Scattering (DIS) applied to the  ${}^3He$ : sizable European Muon Collaboration (EMC) effect predicted [1]
- Formalism extended to any nucleus and applied to  ${}^4He$  and  ${}^3H$  [2]
  - ${}^4He$  is a tightly bound nucleus ⇒ Challenging test to our approach
  - The formalism is generalized for the polarized DIS [3] for the  ${}^3He$

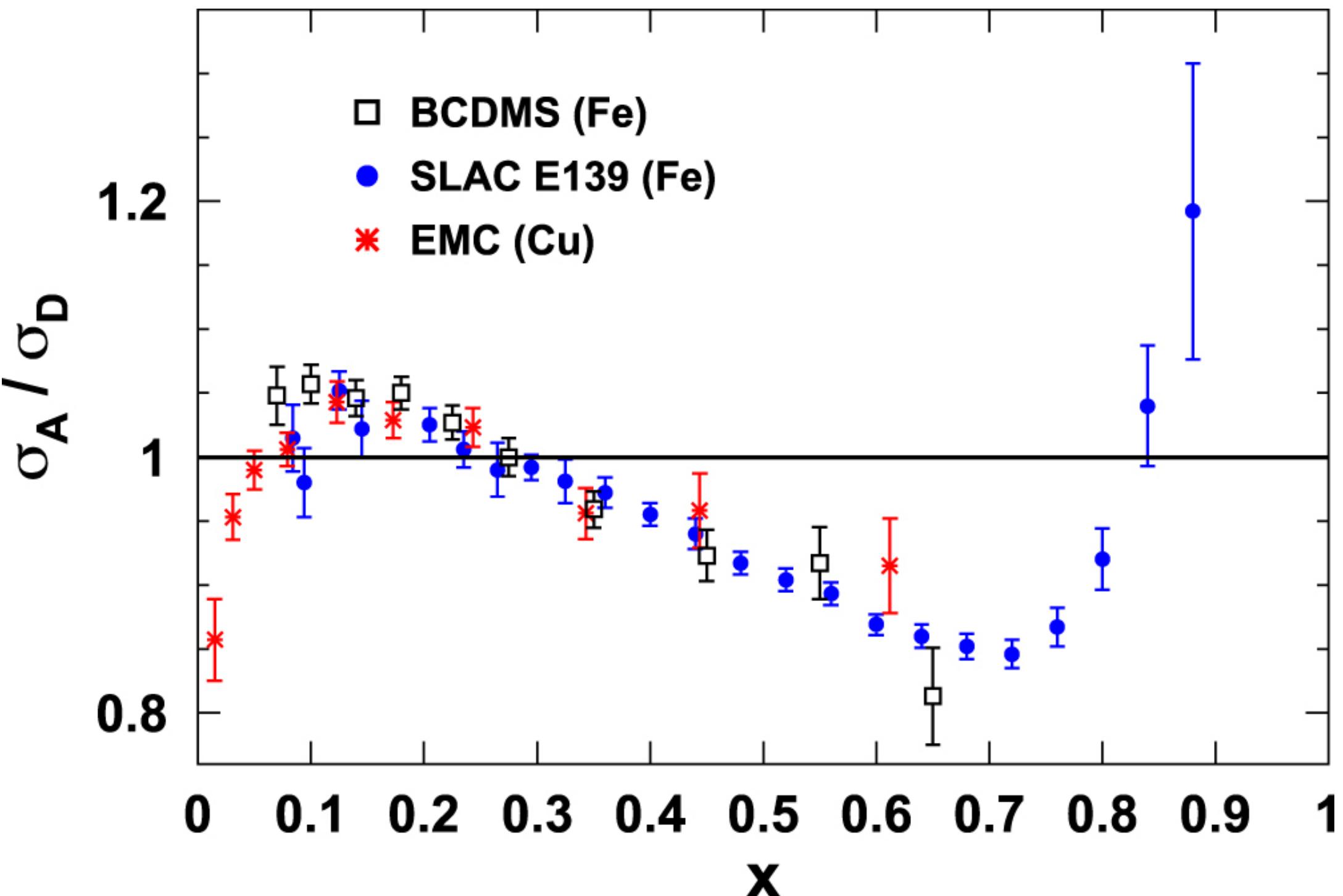
[1] E.Pace, M.Rinaldi, G.Salmè and S.Scopetta, *Phys. Lett. B* 839 (2023) 127810

[2] F.F, E.Pace, M.Rinaldi, G.Salmè, S.Scopetta and M.Viviani, *Phys.Lett.B* 851 (2024) 138587

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# The EMC effect

I.C.Cloet et al., 2019 J. Phys. G: Nucl. Part. Phys. 46 093001



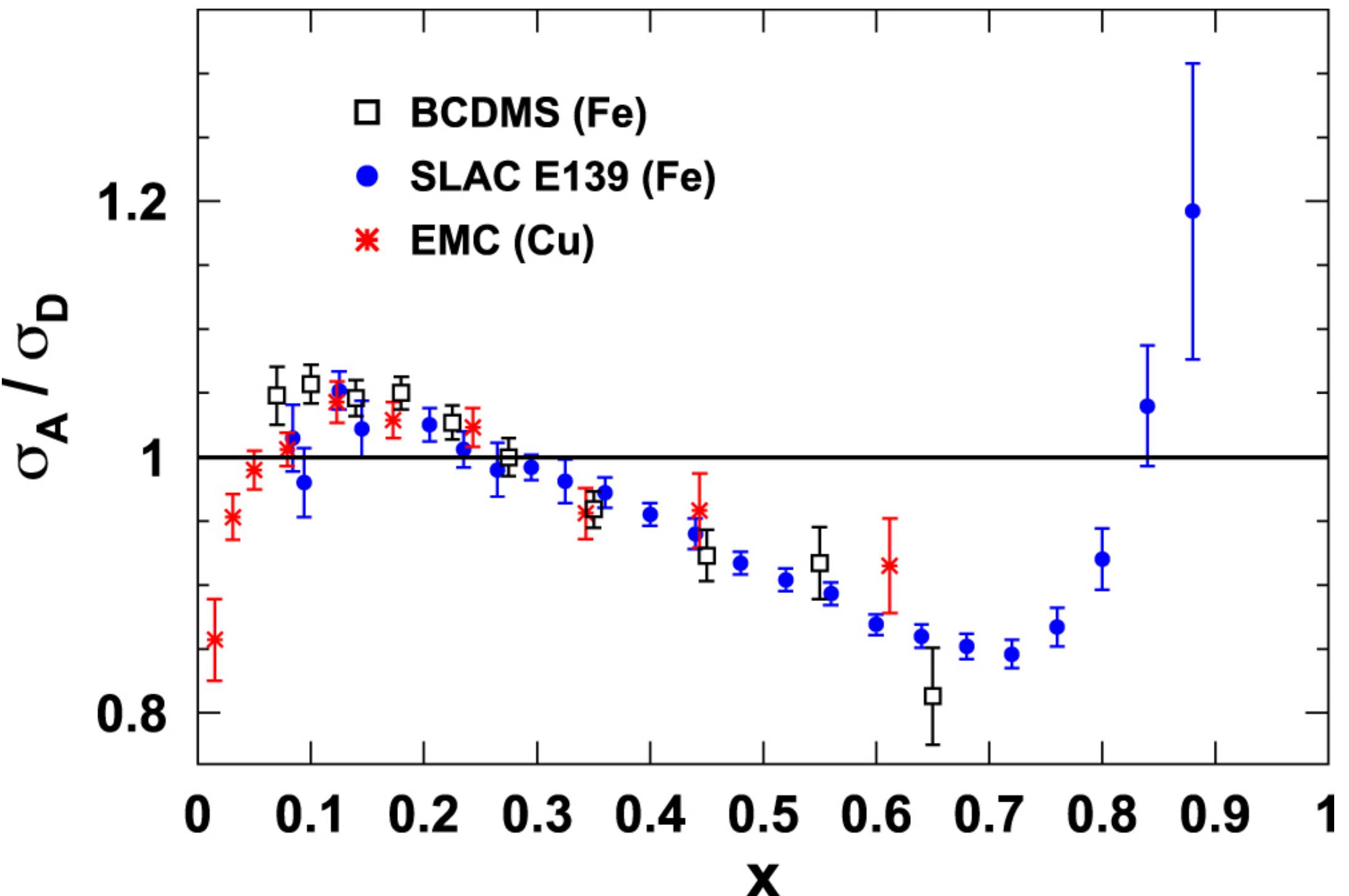
$$R_{EMC}(x) = \frac{2\sigma_A}{A\sigma_D} = \frac{2F_2^A(x)}{AF_2^d(x)} < 1$$

Naive parton model interpretation:

*Valence quarks, in the bound nucleon, are in average slower than in the free nucleon*

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Our approach is aimed to include **only nucleonic dof** through **conventional nuclear physics** in a **Poincaré-covariant** approach that preserves the **macroscopic locality**. The only way to **fulfill sum rules** while using **realistic NR nuclear potentials** is to **embed relativistic effects**

# The relativistic Hamiltonian dynamics framework

---

Our definitely preferred framework for embedding the successful **NR phenomenology**:

**Light-front Relativistic Hamiltonian Dynamics (LFRHD, fixed dof) + Bakamjian-Thomas (BT)**  
construction of the Poincaré generators for an interacting theory.

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## LF Advantages

- **7 Kinematical generators**
- The **LF boosts** have a subgroup structure
- $P^+ \geq 0 \rightarrow$  **meaningful Fock expansion**
- The **infinite-momentum frame (IMF)** description of **DIS** is easily included

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**Drawback: the transverse LF-rotations are dynamical!**

**But** within the **Bakamjian-Thomas (BT)** construction of the generators in an interacting theory, one can construct an **intrinsic angular momentum** fully kinematical

# The Bakamjian-Thomas construction

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- i) Only the **mass operator**  $M$  contains the interaction:

$$M_{BT}[1,2,3,\dots,A] = M_0[1,2,3,\dots,A] + V(\mathbf{k}^2; \mathbf{k} \cdot \mathbf{k}_i; \mathbf{k}_j \cdot \mathbf{k}_i)$$

- ii) It generates the dependence of the **3 dynamical generators** ( $P^-$  and LF transverse rotations) upon the interaction

- iii) The eigenvalue equation  $M^2 |\psi\rangle = s |\psi\rangle$  is formally equivalent to the **Schrödinger equation**

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iii) The eigenvalue equation  $M^2 |\psi\rangle = s |\psi\rangle$  is formally equivalent to the **Schrödinger equation**

The **commutation rules** impose constraints to  $V$  fulfilled by phenomenological potentials  $V^{NR}$ , so we can assume  $M_{BT}[1,2,\dots,A] \sim M^{NR}$

# LF spectral function

---

With the **impulse approximation** assumption, we have to define the **spin-dependent LF spectral function**  $P_{\sigma'\sigma}^\tau(\tilde{\kappa}, \epsilon, \mathbf{S}, M)$  to calculate the hadronic tensor  $W^{\mu\nu}$

$$P_{\sigma'\sigma}^N(\tilde{\kappa}, \epsilon, \mathbf{S}, M) = \sum_{JJ_z} \sum_{TT_z} \rho(\epsilon)_{LF} < tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa} | \Psi_{JM}; \mathbf{S}, T_A T_{Az} > < \Psi_{JM}; \mathbf{S}, T_A T_{Az} |_{LF} tT; \alpha, \epsilon; JJ_z; \tau\sigma, \tilde{\kappa} >_{LF}$$

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The **LF spectral function** contains the **effect** of the **LF boost connecting the intrinsic frames** [1; 2, 3, ..., A – 1] and [1, 2, ..., A]

# LF spectral function

---

We can express the **LF overlap** in terms of the **IF overlap** using **Melosh rotations**:

$$\langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\kappa} | \Psi_{JM}; \mathbf{S}, T_A T_{Az} \rangle_{LF} \rightarrow \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma'_c, \kappa | \Psi_{JM}; \mathbf{S}, T_A T_{Az} \rangle_{IF}$$

Then we can approximate the **IF overlap** into a **NR overlap** by using the NR wave function for the nucleus, thanks to the **BT construction**:

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**Poincarè covariance** preserved but using the **successful NR phenomenology**

We used wave functions of  $^2H$ ,  $^3H$ ,  $^3He$ ,  $^4He$  calculated through 3 different potentials: **Av18+UIX\*** and 2 versions of the **Norfolk  $\chi EFT$  interactions NVIa+3N\*\*** and **NVIb+3N\*\***

\*R. B. Wiringa, V. G. J. Stoks, R. Schiavilla, **Phys. Rev. C 51 (1995) 38–51**; R. B. Wiringa et al., **Phys. Rev. Lett. 74 (1995) 4396–4399**

\*\*M. Viviani et al., **Phys. Rev. C 107 (1) (2023) 014314**; M. Piarulli et al., **Phys. Rev. Lett. 120 (5) (2018) 052503**; M. Piarulli, S. Pastore, R. B. Wiringa, S. Brusilow, R. Lim, **Phys. Rev. C 107 (1) (2023) 014314**

# Hadronic tensor

In our approach the **symmetric part** of the **hadronic tensor** is found to be \*

$$W_A^{s,\mu\nu} = \sum_N \sum_\sigma \oint d\epsilon \int \frac{d\kappa_\perp d\kappa^+}{2(2\pi)^3 \kappa^+} \frac{1}{\xi} P^N(\tilde{\kappa}, \epsilon) W_{N,\sigma}^{s,\mu\nu}(p, q)$$

hadronic tensor of the nucleon

Unpolarized LF spectral function:

$$\rightarrow P^N(\tilde{\kappa}, \epsilon) = \frac{1}{2j+1} \sum_{\mathcal{M}} P_{\sigma\sigma}^N(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathcal{M})$$

Where  $x = \frac{Q^2}{2P_A \cdot q}$  and  $\xi = \frac{\kappa^+}{\mathcal{M}_0[1; 2, 3, \dots, A-1]}$  with  $z = \frac{Q^2}{2p \cdot q} = \frac{p}{P_A^+} \frac{x}{\xi}$

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$W_A^{s,\mu\nu}$  is parametrized by the SFs  $F_2^A(x)$  and  $F_1^A(x)$ :

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Free nucleon SF

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# LC momentum distribution

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In the Bjorken limit  $\int d\epsilon \int d\kappa^+ = \int d\kappa^+ \int d\epsilon$  so we can use the **light-cone momentum distribution** (LCMD) instead of the **LF spectral function** \*

$$\textbf{LCMD: } f_1^N(\xi) = \oint d\epsilon \int \frac{d\kappa_\perp}{(2\pi)^3} \frac{1}{2\kappa^+} P^N(\tilde{\kappa}, \epsilon) \frac{E_s}{1 - \xi} = \int d\mathbf{k}_\perp n^n(\xi, \mathbf{k}_\perp)$$

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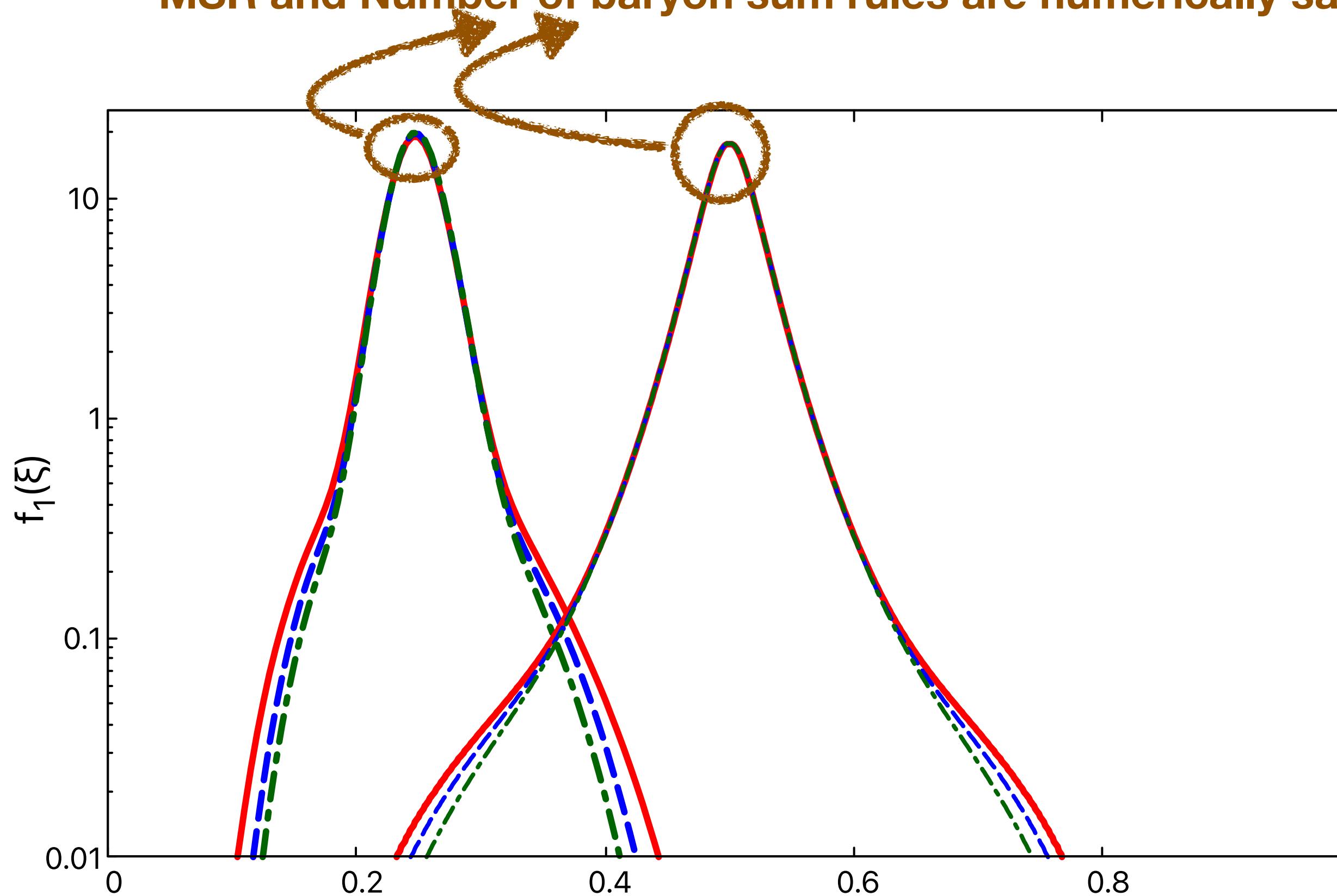
## LF momentum distribution:

$$n^N(\xi, \mathbf{k}_\perp) = \frac{1}{2\pi} \int \prod_{i=2}^{A-1} [d\mathbf{k}_i] \quad \left| \frac{\partial k_z}{\partial \xi} \right| \quad \mathcal{N}^N(\mathbf{k}, \mathbf{k}_2, \dots, \mathbf{k}_{A-1})$$

 Squared nuclear wave function. Thanks to the BT construction, one is allowed to use the NR one

# LC momentum distribution: numerical results for ${}^4He$

The distributions are peaked at  $1/A$  with an accuracy of  $1/1000$ :  
**MSR and Number of baryon sum rules are numerically satisfied**



The tails of the distributions are generated by the **short range correlations (SRC)** induced by the potentials (i.e the **high-momentum content** of the 1-body momentum distribution)

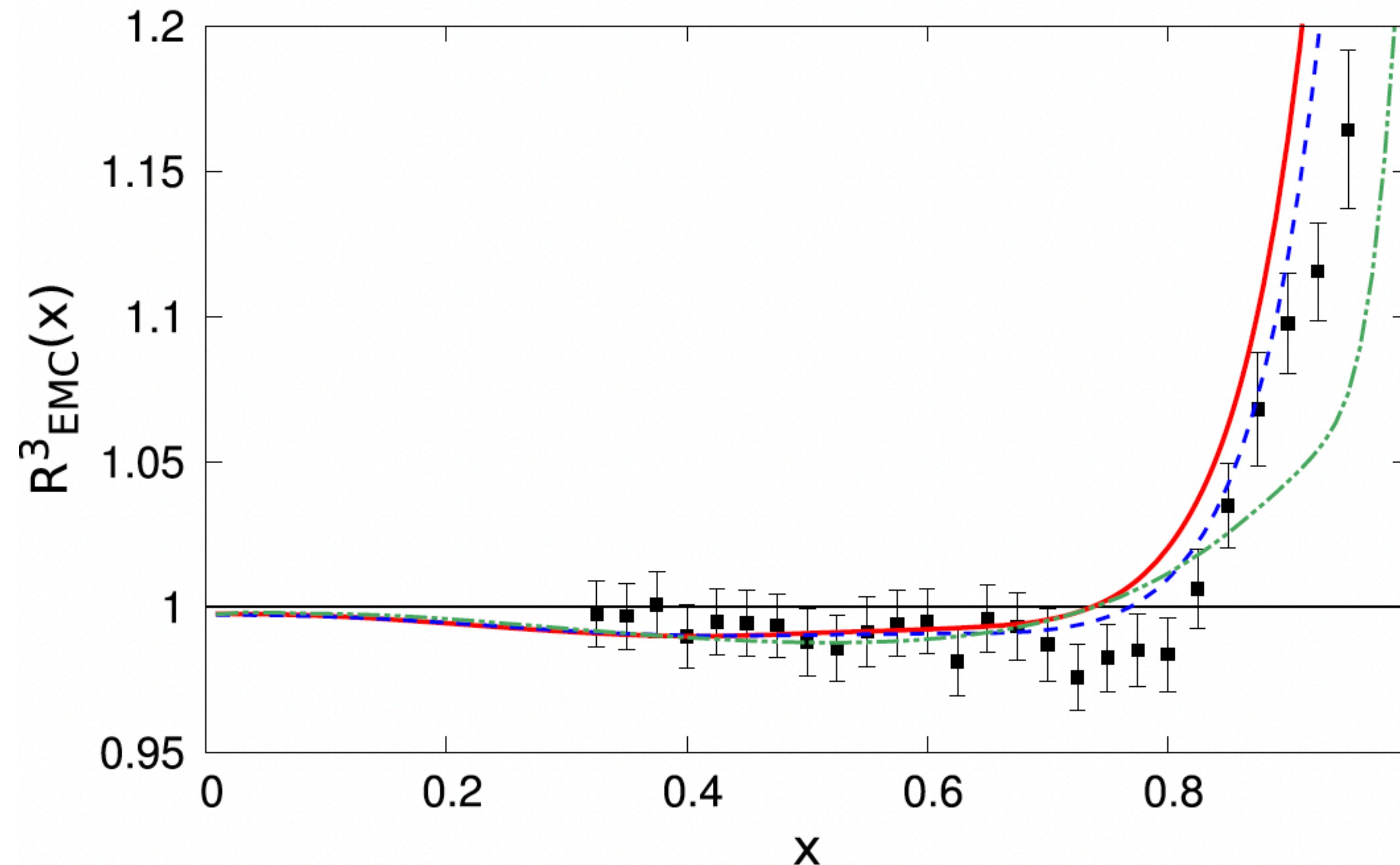
Since our approach fulfill both **macro-locality** and **Poincaré covariance** the LCMD satisfy both baryon and momentum SR

F.F. E.Pace, M.Rinaldi, G.Salmè, S.Scopetta and M.Viviani,  
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**Solid lines: Av18/UIX. Dashed lines:NVIb+3N. Dot-dashed lines: NVIa+3N**

# The EMC effect: results for ${}^3He$

E.Pace, M.Rinaldi, G.Salmè and S.Scopetta, **Phys. Lett. B** 839(2023) 127810



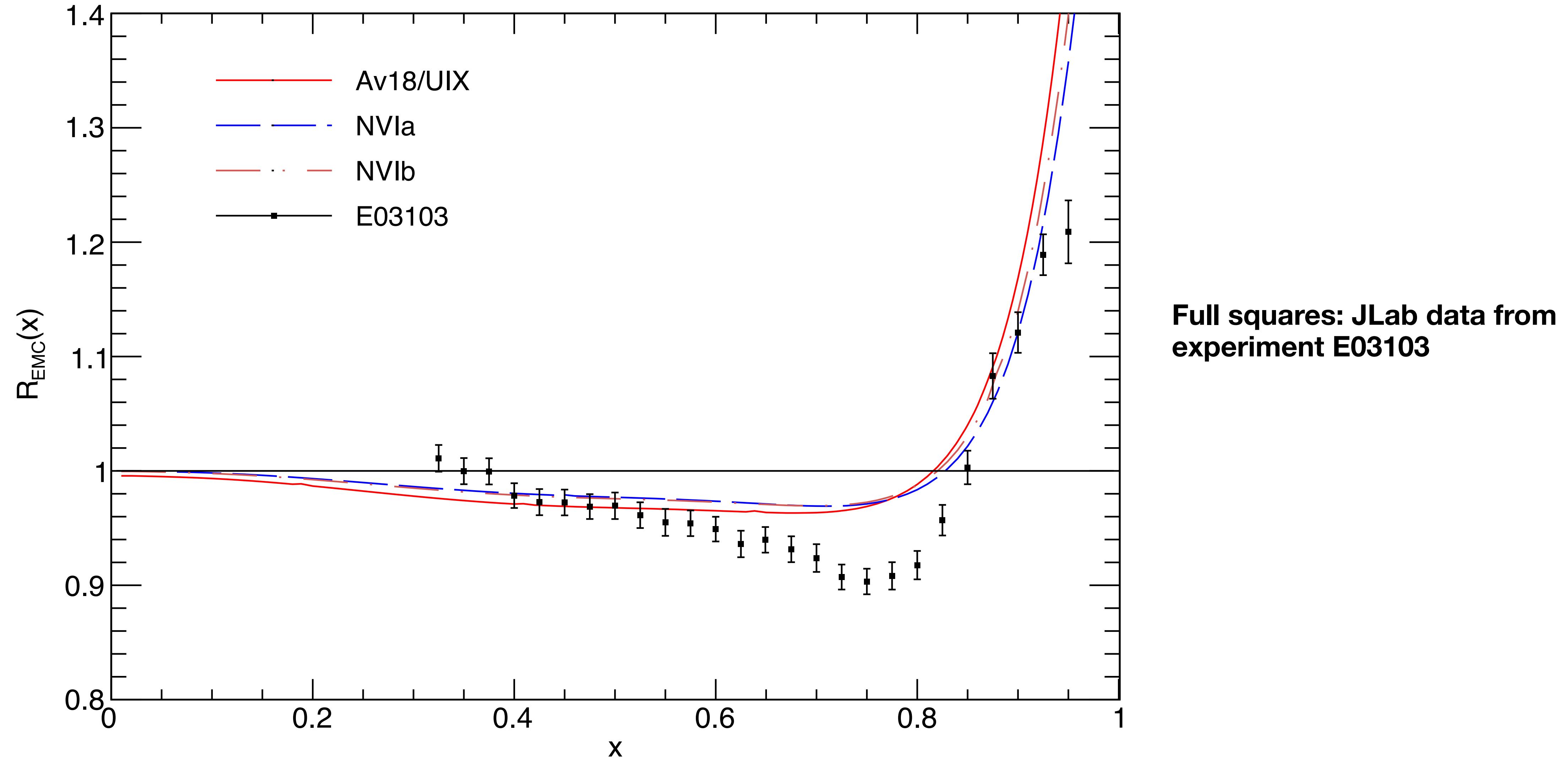
Solid line: Av18/UIX + SMC\*  
Dashed line: Av18 + SMC\*  
Dotted-dashed: Av18/UIX  
+CJ15\*\*

\*[B. Adeva, et al., **Phys. Lett. B** 412 (1997) 414–424.]

\*\*[A. Accardi, L. T. Brady, W. Melnitchouk, J. F. Owens, N. Sato, **Phys. Rev. D** 93 (11) (2016) 114017]

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# Hadronic tensor II

For the **polarized DIS** we need to calculate the **antysymmetric** part of the **hadronic tensor**:

$$W_A^{a,\mu\nu} = \sum_N \sum_\sigma \oint d\epsilon \int \frac{d\kappa dk^+}{2(2\pi)^3 \kappa^+} \frac{1}{\xi} P_\sigma^N(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathcal{M}) W_{N,\sigma}^{a,\mu\nu}(p, q)$$

hadronic tensor of the nucleon

Spin-dependent LF spectral function

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$W_A^{a,\mu\nu}$  is parametrized by the the **spin-dependent SFs (SSFs)**  $g_1^A(x, Q^2)$  and  $g_2^A(x, Q^2)$

$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi [g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi)], j = 1, 2$$

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$W_A^{a,\mu\nu}$  is parametrized by the **spin-dependent SFs (SSFs)**  $g_1^A(x, Q^2)$  and  $g_2^A(x, Q^2)$

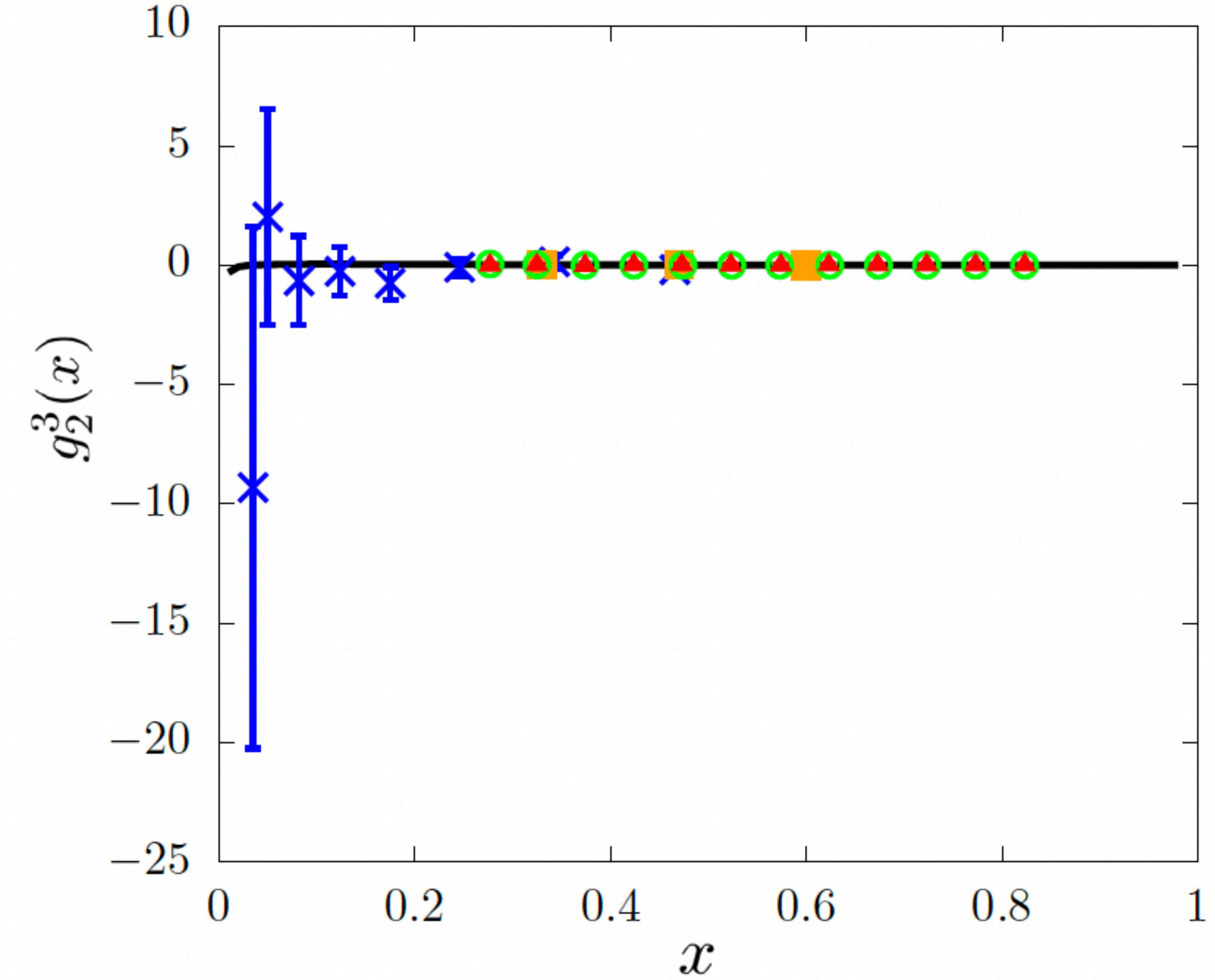
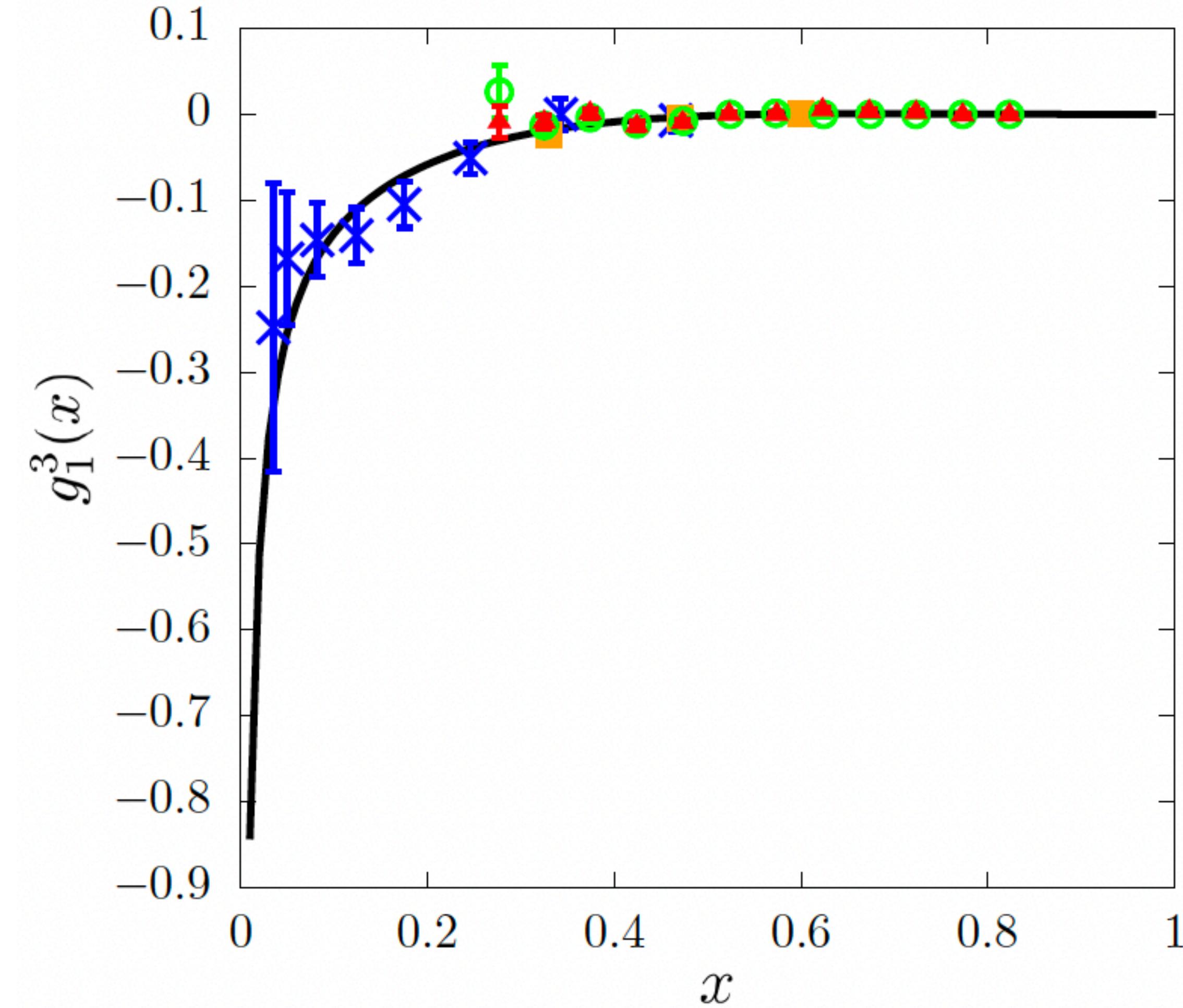
$$g_j^A(x) = \sum_N \int_{\xi_m}^1 d\xi [g_1^N(z) l_j^N(\xi) + g_2^N(z) h_j^N(\xi)], j = 1, 2$$

The **spin-dependent LCMD**  $l_j^N(\xi)$  and  $h_j^N(\xi)$  are related to the **transverse momentum-dependent distributions (TMDs)** of the nucleons  $\Delta f^N, g_{1T}^N, \Delta'_T f^N, h_{1L}^N, h_{1T}^N$ , calculated for  ${}^3He$  with the **Av18** potential in Ref. [1]

[1] R.Alessandro, A.Del Dotto, E.Pace, G.Perna, G.Salmè and S.Scopetta, **Phys.Rev.C 104(2021) 6,065204**

# $^3He$ SSFs

E.Proietti, F.F, E.Pace, M.Rinaldi, G.Salmè and S.Scopetta, *Phys.Rev.C* 110 (2024) 3, L031303



Experimental data from [1] (crosses), [2] (squares) and [3] (triangles)

[1] P. L. Anthony et al., *Phys. Rev. D* 54, 6620 (1996)

[2] X. Zheng et al., *Phys. Rev. Lett.* 92, 012004 (2004)

[3] D. Flay et al., *Phys. Rev. D* 94, 052003 (2016)

# Conclusions

- Rigorous formalism for the calculation of nuclear SFs and SSFs involving only nucleonic DOF with the conventional nuclear physics developed
- For  ${}^3He$  we obtain results in **agreement** with **experimental data** for both **EMC effect** and **SSFs**. Useful analysis for **planned experiments** in future facilities
- For  ${}^4He$  the deviations from experimental data could be ascribed to **genuine QCD effects**: our results provide a **reliable baseline** to study **exotic phenomena**

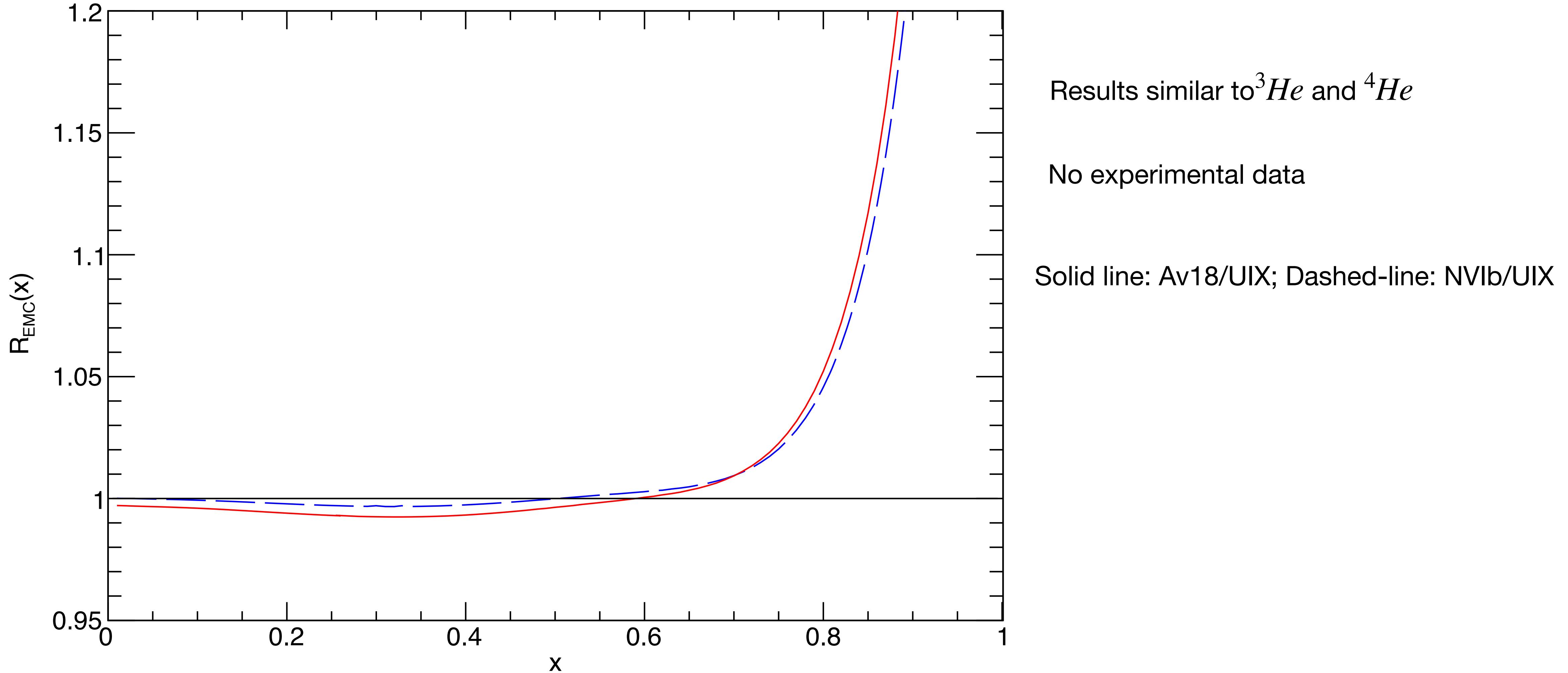
## To do next:

- Include **off-shell** corrections to our calculations
- Study the  $Q^2$  dependence
- Calculate the EMC effect for **heavier nuclei**

## In preparation:

- With the **same approach** we are developing a new formalism for the calculation of the **nuclear Double Parton Distributions (DPDs)** for **light nuclei\***

# Tritium EMC effect

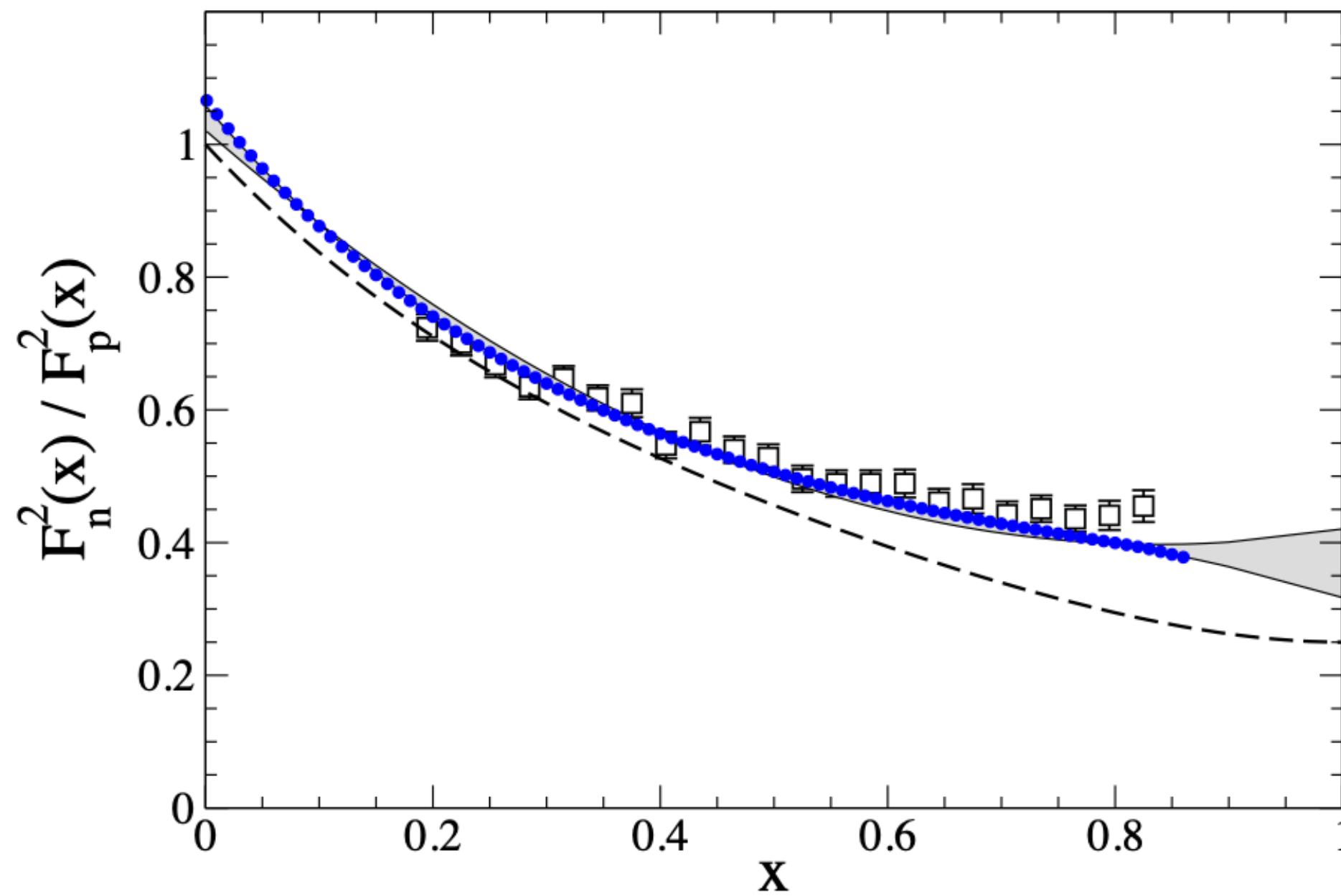


# Extraction of $F_2^n/F_2^p$ via MARATHON data

MARATHON coll. : experimental data of the super-ratio  $R^{ht}(x) = F_2^{^3He}(x)/F_2^{^3H}(x)$

${}^3He$ : 2p + n;  ${}^3H$ : n + 2p

Is possible to extract the ratio  $F_2^n(x)/F_2^p(x)$  through the super-ratio



**E.Pace, M.Rinaldi, G.Salmè and S.Scopetta Phys. Lett. B 839(2023) 127810**

Dashed line: ratio from SMC collaboration  
Empty squares: MARATHON extraction  
Solid line: cubic and conic extractions from  $F_2^p$  SMC parametrization, fitted to MARATHON data

# Canonical and LF spin

- In Instant form (initial hyperplane  $t=0$ ), one can couple spins and orbital angular momenta via Clebsch-Gordan (CG) coefficients. In this form the three rotation generators are independent of the the interaction.
- To embed the CG machinery in the LFHD one needs unitary operators, the so-called Melosh rotations that relate the LF spin wave function and the canonical one. For a particle of spin (1/2) with LF momentum

$$\tilde{\mathbf{k}} \equiv \{k^+, \vec{k}_\perp\}$$

$$|\mathbf{k}; \frac{1}{2}, \sigma\rangle_c = \sum_{\sigma'} \underbrace{D_{\sigma', \sigma}^{1/2}(R_M(\tilde{\mathbf{k}}))}_{\text{Wigner rotation for the } J=1/2 \text{ case}} |\tilde{\mathbf{k}}; \frac{1}{2}, \sigma'\rangle_{LF}$$

- $R_M(\tilde{\mathbf{k}})$  is the Melosh rotation connecting the intrinsic LF and canonical frames, reached through different boosts from a given frame where the particle is moving

$$D^{1/2}[R_M(\tilde{\mathbf{k}})]_{\sigma' \sigma} = \chi_{\sigma'}^\dagger \frac{m + k^+ - i\sigma \cdot (\hat{z} \times \mathbf{k}_\perp)}{\sqrt{(m + k^+)^2 + |\mathbf{k}_\perp|^2}} \chi_\sigma = {}_{LF}\langle \tilde{\mathbf{k}}; s\sigma' | \mathbf{k}; s\sigma \rangle_c$$

↓ two-dimensional spinor

N.B. If  $|\mathbf{k}_\perp| \ll k^+, m \rightarrow D_{\sigma' \sigma} \simeq I_{\sigma' \sigma}$

# LF spectral function and LC Correlator

The fermion correlator in terms of the LF coordinates is [e.g., Barone, Drago, Ratcliffe, Phys. Rep. 359, 1 (2002)]

$$\Phi_{\alpha,\beta}^{\tau}(p, P, S) = \frac{1}{2} \int d\xi^- d\xi^+ d\xi_{\tau} e^{i p \xi} \langle P, S, A | \bar{\psi}_{\beta}^{\tau}(0) \psi_{\alpha}^{\tau}(\xi) | A, S, P \rangle$$

isospin  
 $\Phi_{\alpha,\beta}^{\tau}$ ( $p, P, S$ ) =  $\frac{1}{2} \int d\xi^- d\xi^+ d\xi_{\tau} e^{i p \xi} \langle P, S, A | \bar{\psi}_{\beta}^{\tau}(0) \psi_{\alpha}^{\tau}(\xi) | A, S, P \rangle$

p = fermion momentum  
parent system  
(nucleus, nucleon..)

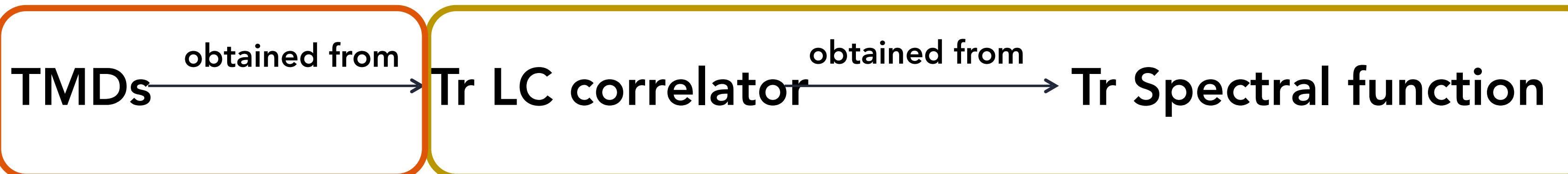
The particle contribution to the correlator in **valence approximation**, i.e. the result obtained if the antifermion contributions are disregarded, is related to the LF SF:

$$\Phi^{\tau p}(p, P, S) = \frac{(\not{p}_{on} + m)}{2m} \Phi^{\tau}(p, P, S) \frac{(\not{p}_{on} + m)}{2m} = \frac{2\pi (P^+)^2}{(p^+)^2 4m} \frac{E_S}{\mathcal{M}_0[1, (23)]} \sum_{\sigma\sigma'} \{ u_{\alpha}(\tilde{\mathbf{p}}, \sigma') \mathcal{P}_{\mathcal{M}, \sigma'\sigma}^{\tau}(\tilde{\kappa}, \epsilon, S) \bar{u}_{\beta}(\tilde{\mathbf{p}}, \sigma) \}$$

In deriving this expression it naturally appears the momentum  $\tilde{\kappa}$  in the intrinsic reference frame of the cluster [1,(23)], where particle 1 is free and the (23) pair is fully interacting.

# TMDs and LF spectral function

14



$$\text{Tr}(\gamma^+ \Phi^p) = D \text{Tr} [\hat{\mathcal{P}}_{\mathcal{M}}(\tilde{\kappa}, \epsilon, S)]$$

$$\text{Tr}(\gamma^+ \gamma_5 \Phi^p) = D \text{Tr} [\sigma_z \hat{\mathcal{P}}_{\mathcal{M}}(\tilde{\kappa}, \epsilon, S)]$$

$$\text{Tr}(\mathbf{p}_\perp \cdot \boldsymbol{\sigma} \gamma^+ \gamma_5 \Phi^p) = D \text{Tr} [\mathbf{p}_\perp \cdot \boldsymbol{\sigma} \hat{\mathcal{P}}_{\mathcal{M}}(\tilde{\kappa}, \epsilon, S)]$$

$$D = \frac{(P^+)^2}{p^+} \frac{\pi}{m} \frac{E_S}{\mathcal{M}_0[1, (23)]}$$

The integration  $\int \frac{dp^+ dp^-}{(2\pi)^4} \delta[p^+ - xP^+] P^+$  of  $\text{Tr}$  of SF



$$f(x, \mathbf{p}_\perp^2) = b_0 \quad \Delta f(x, |\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + b_{5,\mathcal{M}} \quad g_{1T}(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|} b_{4,\mathcal{M}}$$

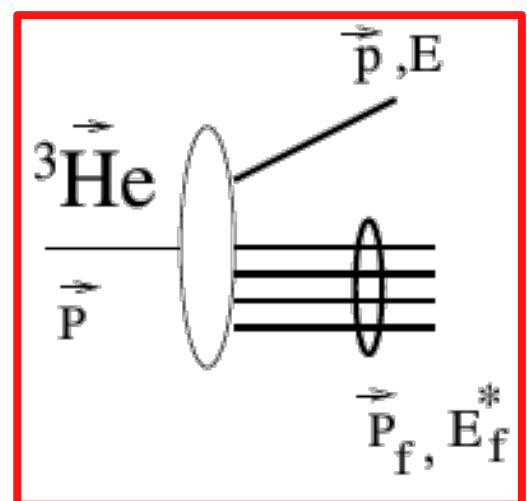
$$\Delta'_T f(x, |\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + \frac{1}{2} b_{2,\mathcal{M}} \quad h_{1L}^\perp(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|} b_{3,\mathcal{M}} \quad h_{1T}^\perp(x, |\mathbf{p}_\perp|^2) = \frac{M^2}{|\mathbf{p}_\perp|^2} b_{2,\mathcal{M}}$$

# TMDs and $^3\text{He}$ LF spectral function

The procedure works for any three-body  $J = 1/2$  system (in valence approx!)

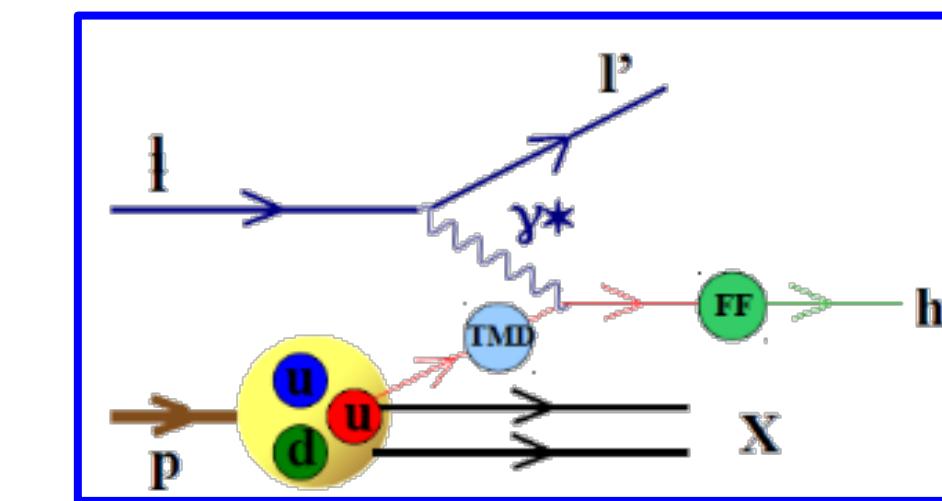
## $^3\text{He}$

- $p, p, n$
- $(e, e'p)$  reactions
- $p$  detection
- PW Impulse Approximation
- spin-dep response functions
- light-cone momentum distributions
- norms, effective polarizations



## Proton

- $u_\nu, u_\nu, d_\nu$
- SIDIS
- no  $q_\nu$  detection, fragmentation...
- leading twist
- TMDs
- PDFs
- charges (axial, tensor...)



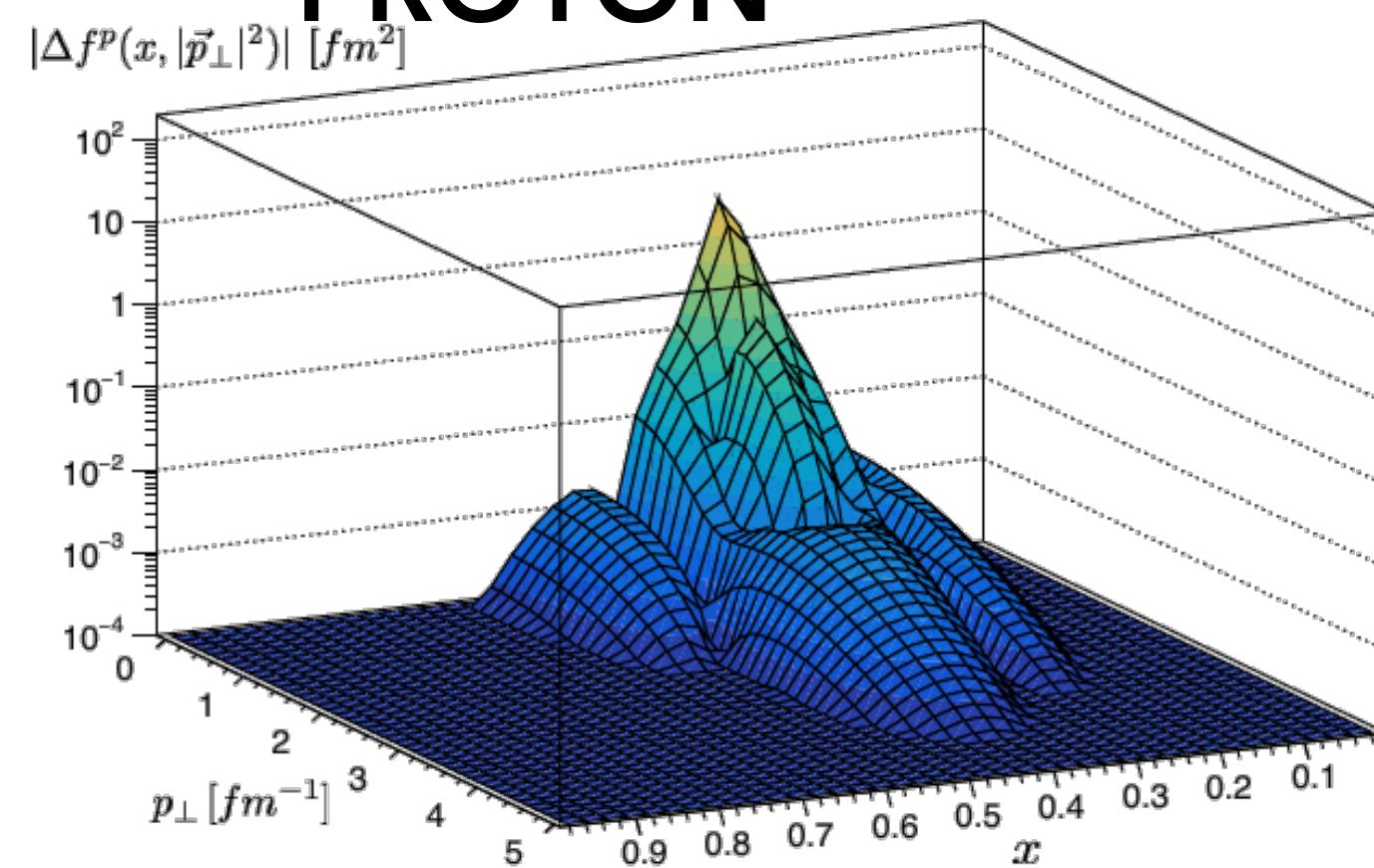
CORRESPONDENCE

- the  $^3\text{He}$  TMDs could be obtained from spin asymmetries in  $^3\vec{\text{He}}(e, e'p)$  experiments: in progress!
  - We show our calculation for the TMDs of He using Av18 + UIX wfs (A. Kievsky, M. Viviani et al.)
- Thus testing LFRHD and of the importance of Relativity in nuclear structure.

# $^3\text{He}$ TMDs

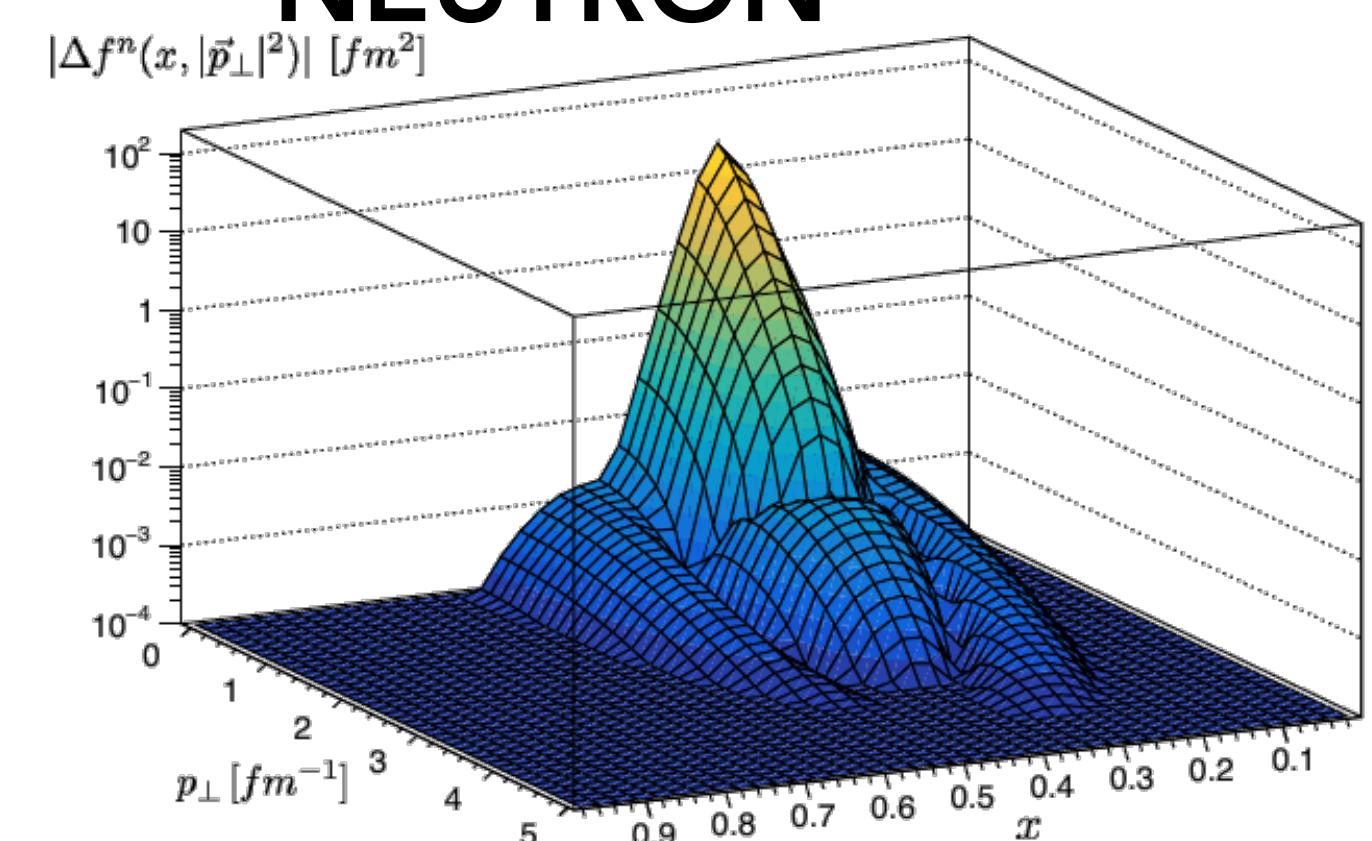
**Numerical results** A. Del Dotto, E. Pace, G. Perna, A. Rocco, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204)

**PROTON**

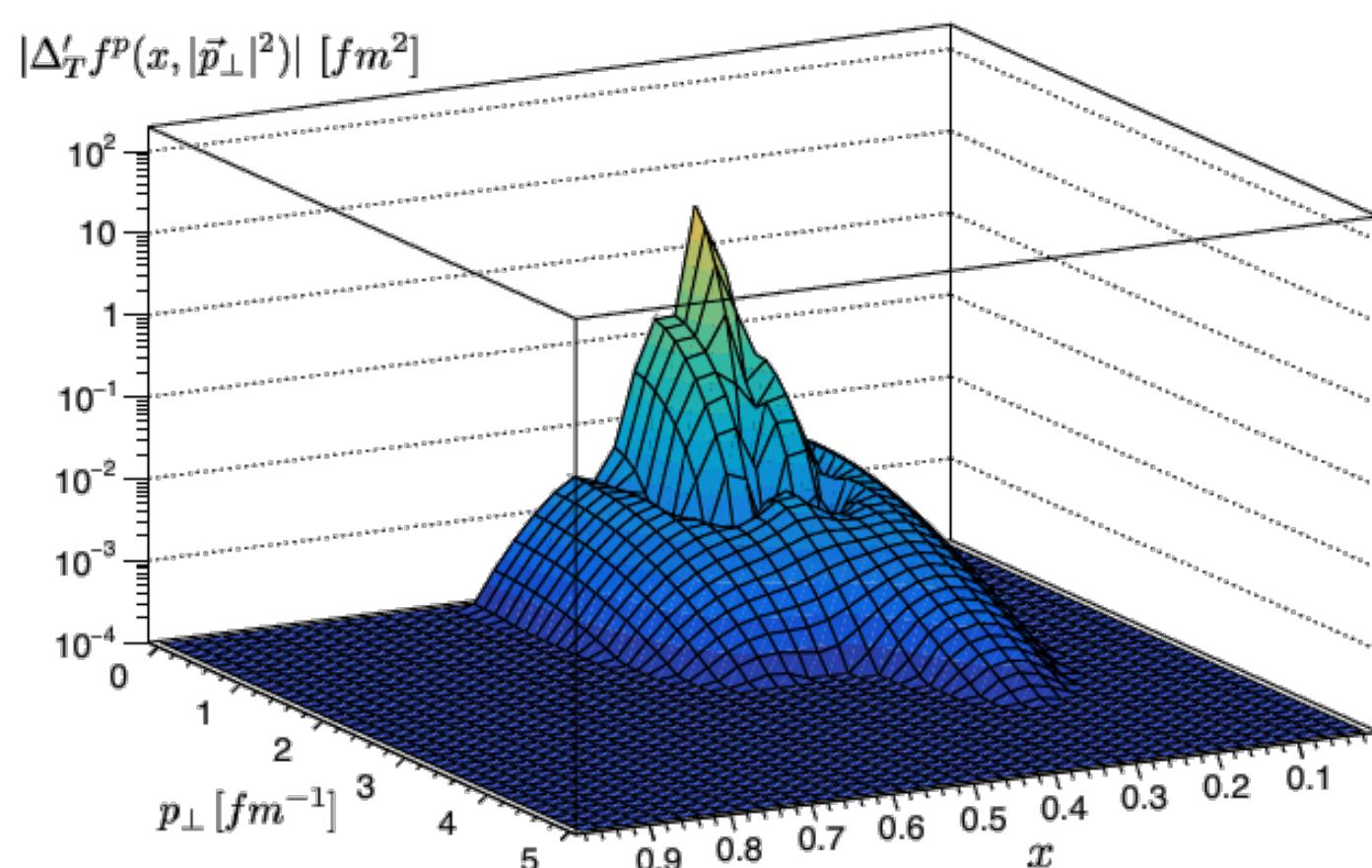


$$\Delta f^\tau(x, |\mathbf{p}_\perp|^2)$$

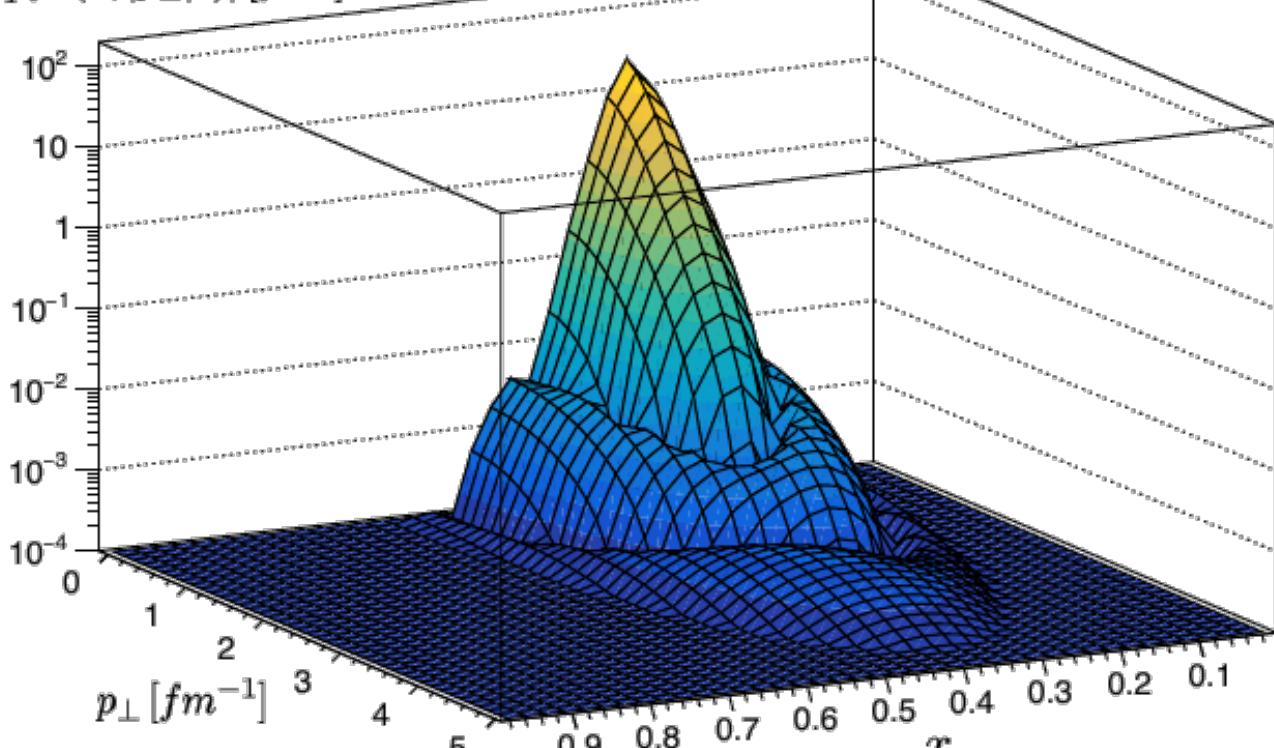
**NEUTRON**



$$\Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2)$$



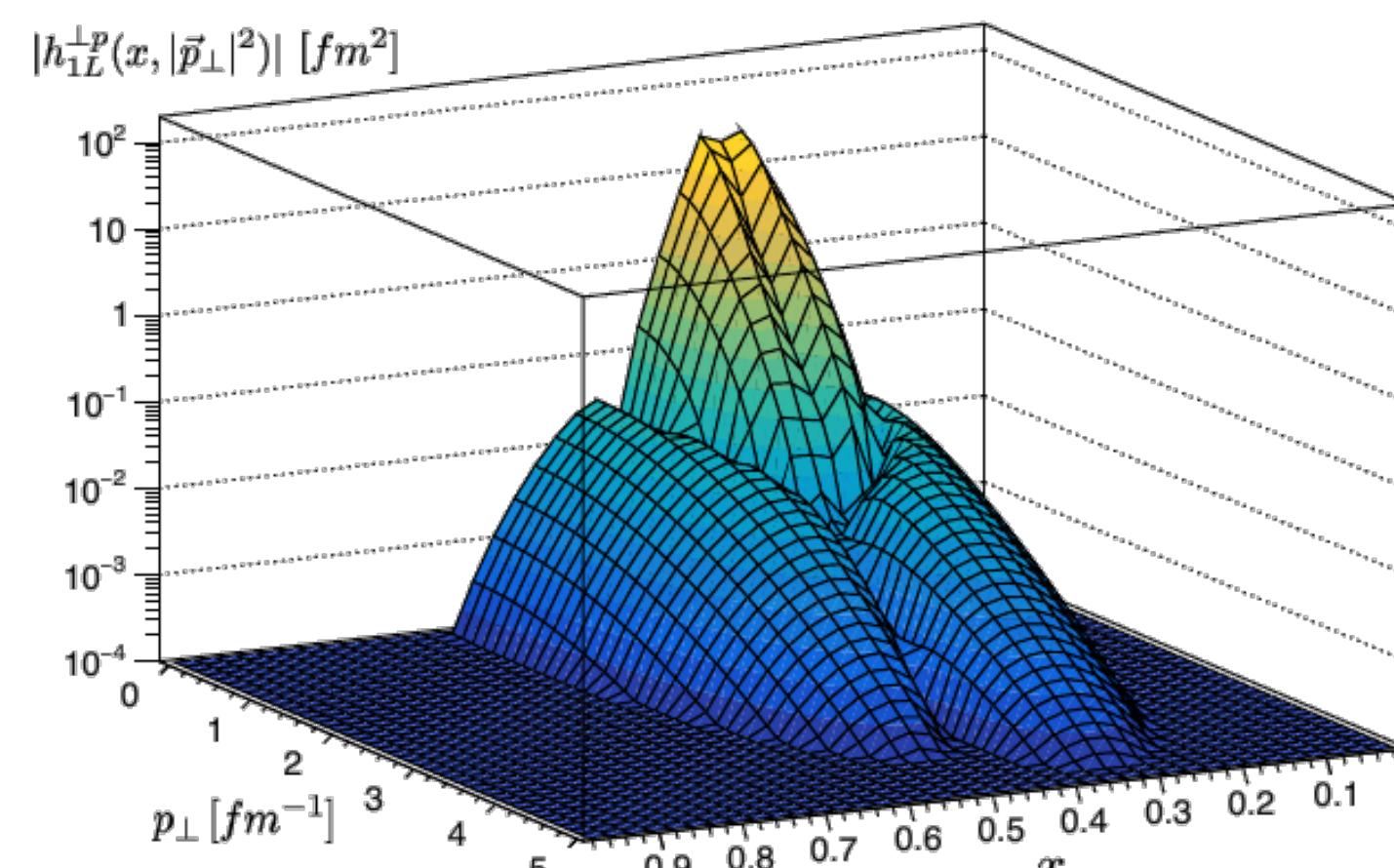
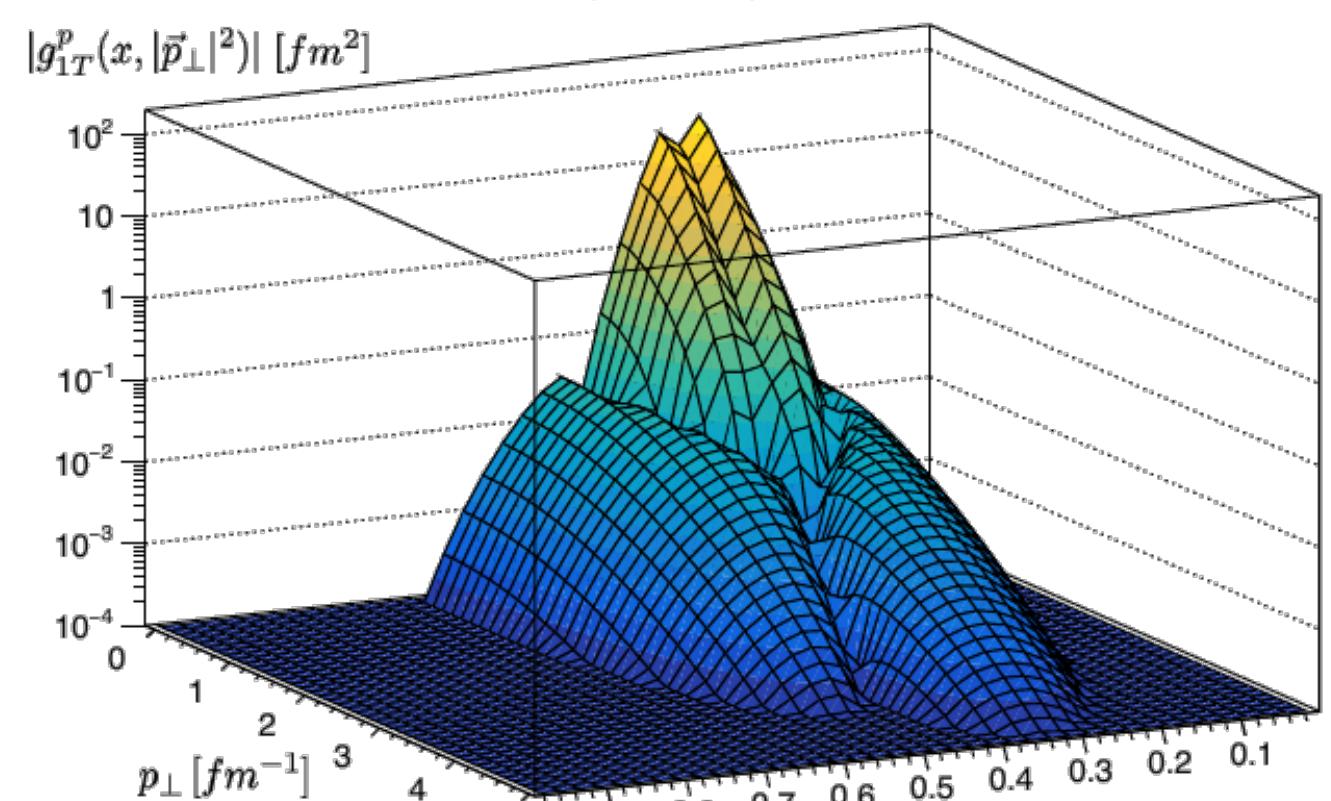
$$|\Delta'_T f^n(x, |\mathbf{p}_\perp|^2)| [fm^2]$$



# $^3\text{He}$ TMDs

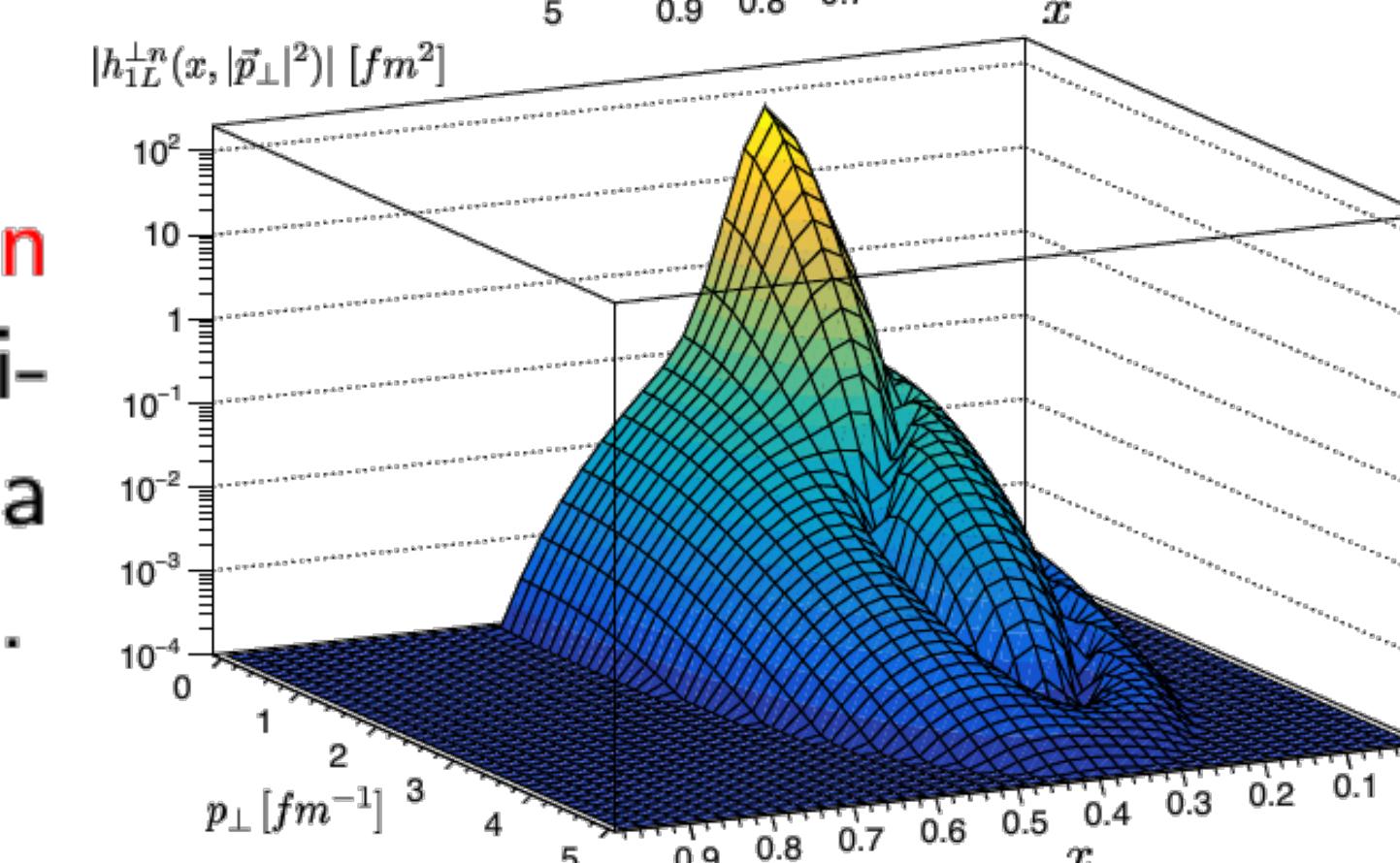
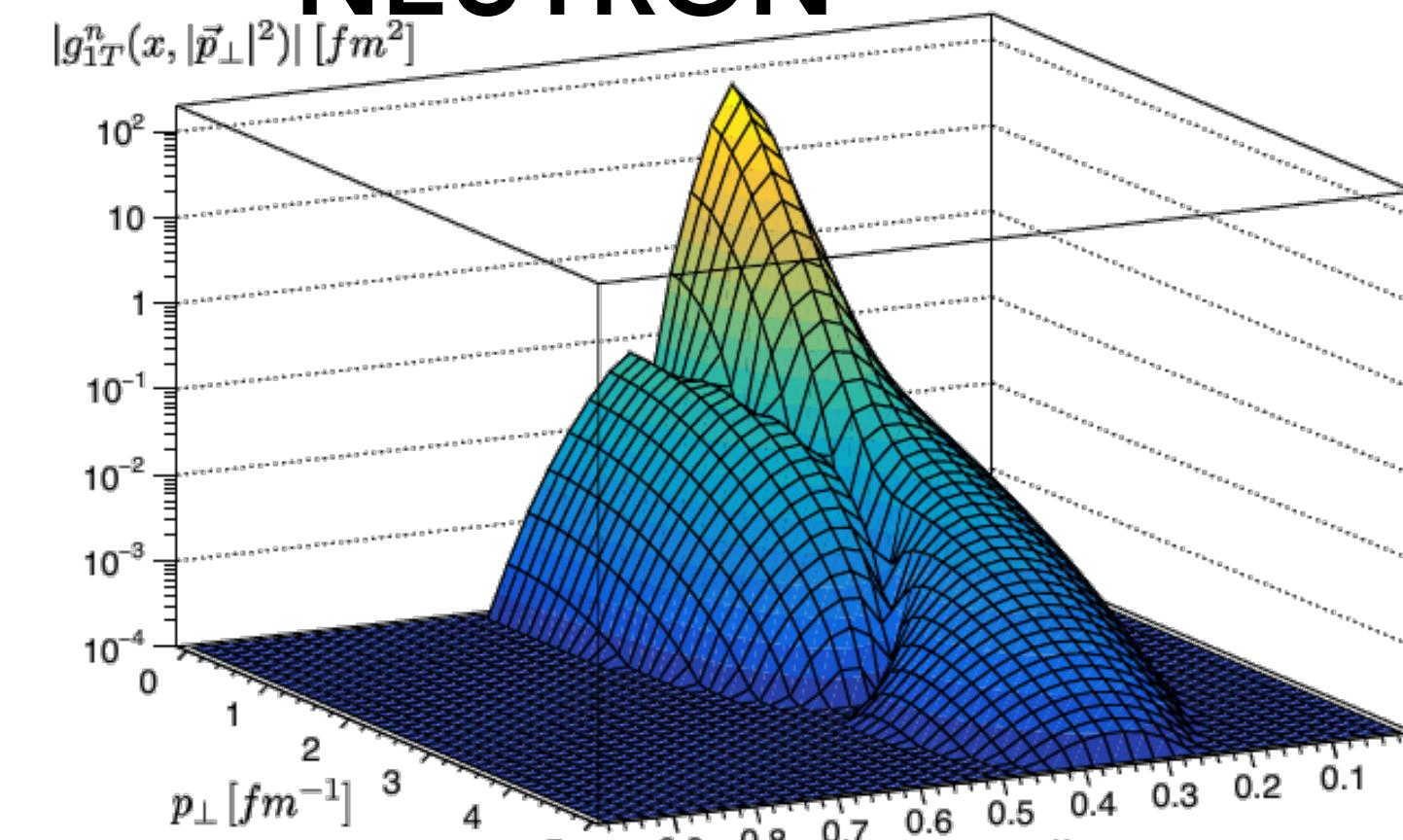
**Numerical results** A. Del Dotto, E. Pace, G. Perna, A. Rocco, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204)

## PROTON



Absolute value of the nucleon longitudinal-polarization distribution,  $g_{1T}^\tau(x, |\mathbf{p}_\perp|^2)$ , in a transversely polarized  $^3\text{He}$ .

## NEUTRON



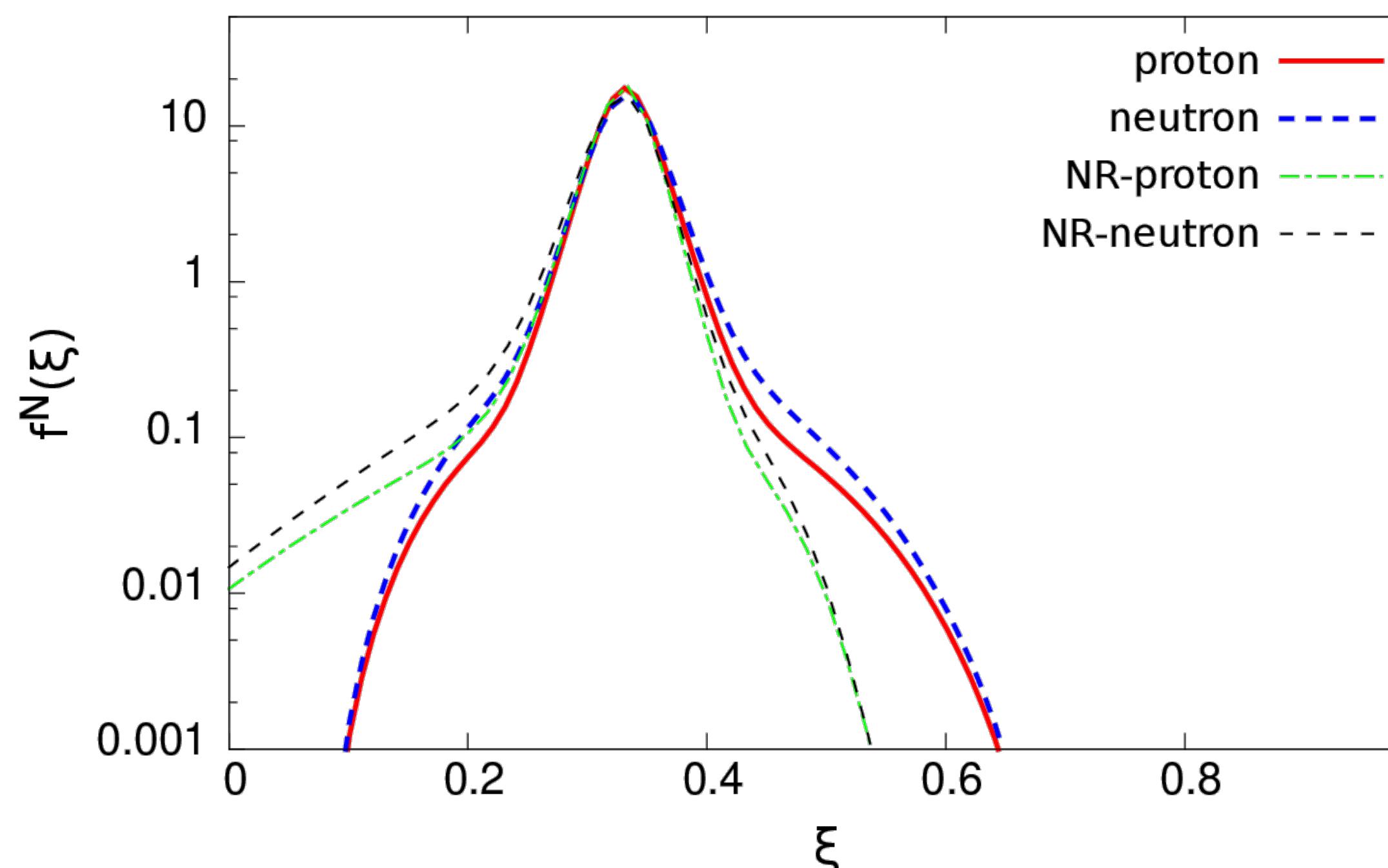
Absolute value of the nucleon transverse-polarization distribution,  $h_{1L}^{\perp\tau}(x, |\mathbf{p}_\perp|^2)$  in a longitudinally polarized  $^3\text{He}$ .

# LC momentum distributions

From the normalization of the Spectral Function one has

$$f_{\tau}^A(\xi) = \int d\mathbf{k}_{\perp} n^{\tau}(\xi, \mathbf{k}_{\perp})$$

$$\longrightarrow \int_0^1 d\xi f_{\tau}^A(\xi) = 1$$

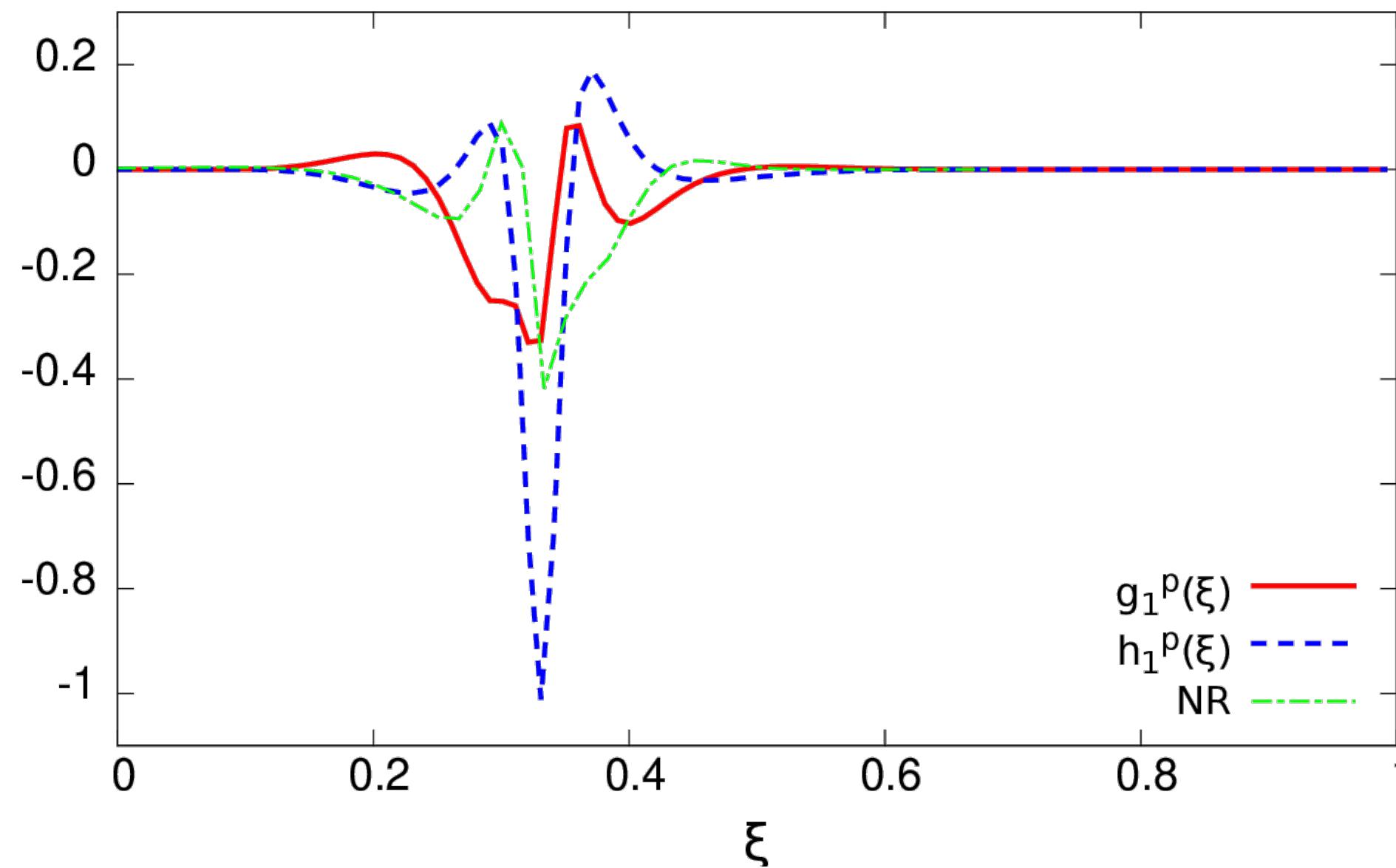


unpolarized distribution

E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485

# LC momentum distributions

PROTON

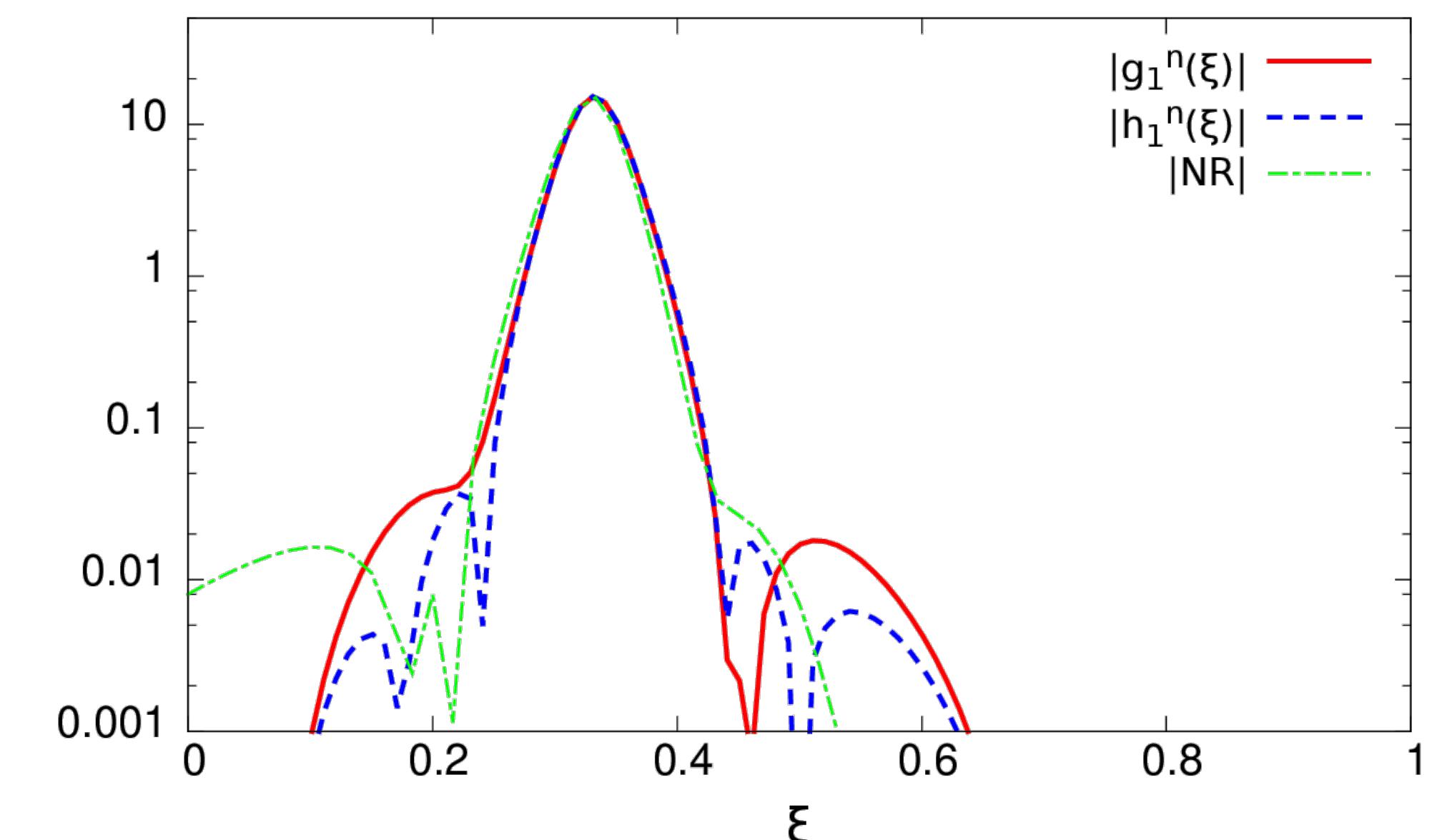


$g_1^n(\xi)$  longitudinal-polarization distribution

$h_1^n(\xi)$  transverse-polarization distribution

- They would be the same in a NR framework;
- Crucial for the extraction of the neutron information from DIS and SIDIS off  ${}^3\text{He}$ .  
Work in progress to LF update our NR results —> important for JLab12, EIC

NEUTRON



E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485

# Backup Slides: effective polarizations

## Effective polarizations

Key role in the extraction of **neutron polarized structure functions** and **neutron Collins and Sivers single spin asymmetries**, from the corresponding quantities measured for  ${}^3\text{He}$

Effective longitudinal polarization (axial charge for the nucleon)

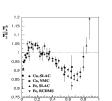
$$p_{||}^\tau = \int_0^1 dx \int d\mathbf{p}_\perp \Delta f^\tau(x, |\mathbf{p}_\perp|^2)$$

Effective transverse polarization (tensor charge for the nucleon)

$$p_\perp^\tau = \int_0^1 dx \int d\mathbf{p}_\perp \Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2)$$

Effective polarizations	proton	neutron
LF longitudinal polarization	-0.02299	0.87261
LF transverse polarization	-0.02446	0.87314
non relativistic polarization	-0.02118	0.89337

- The difference between the LF polarizations and the non relativistic results are **up to 2% in the neutron case** (larger for the proton ones, but it has an overall small contribution), and should be **ascribed to the intrinsic coordinates**, implementing the **Macro-locality**, and not to the Melosh rotations involving the spins.
- N.B. Within a NR framework:  $p_{||}^\tau(\text{NR}) = p_\perp^\tau(\text{NR})$



# Backup Slides: effective polarizations

The BT Mass operator for A=3 nuclei - II

The NR mass operator is written as

$$M^{NR} = 3m + \sum_{i=1,3} \frac{k_i^2}{2m} + V_{12}^{NR} + V_{23}^{NR} + V_{31}^{NR} + V_{123}^{NR}$$

and must obey to the commutation rules proper of the Galilean group, leading to translational invariance and independence of total 3-momentum.

Those properties are analogous to the ones in the BT construction. This allows us to consider the standard non-relativistic mass operator as a sensible BT mass operator, and embed it in a Poincaré covariant approach.

$$M_{BT}(123) = M_0(123) + V_{12,3}^{BT} + V_{23,1}^{BT} + V_{31,2}^{BT} + V_{123}^{BT} \sim M^{NR}$$

The 2-body phase-shifts contain the relativistic dynamics, and the Lippmann-Schwinger equation, like the Schrödinger one, has a suitable structure for the BT construction.

Therefore what has been learned till now about the nuclear interaction, within a non-relativistic framework, can be re-used in a Poincaré covariant framework.

The eigenfunctions of  $M^{NR}$  do not fulfill the cluster separability, but we take care of Macro-locality in the spectral function.

# LF spectral function decomposition

The LF spin-dependent spectral function (SF), for a nucleus with polarization  $S$ , can be macroscopically decomposed in terms of the available vectors:

- the unit vector  $\hat{n}$ ,  $\perp$  to the hyperplane  $n^\mu x_\mu = 0$ . Our choice is  $n^\mu \equiv \{1, 0, 0, 1\} \Rightarrow \hat{n} \equiv \hat{z}$
- the polarization vector  $\mathbf{S}$
- the transverse (wrt the  $\hat{z}$  axis) momentum component of the constituent, i.e.  $\mathbf{k}_\perp(123) = \mathbf{p}_\perp(Lab) = \kappa_\perp(1;23)$

$$\mathcal{P}_{\mathcal{M},\sigma'\sigma}^\tau(\tilde{\kappa}, \epsilon, S) = \frac{1}{2} [\mathcal{B}_{0,\mathcal{M}}^\tau + \boldsymbol{\sigma} \cdot \mathcal{F}_{\mathcal{M}}^\tau(\tilde{\kappa}, \epsilon, \mathbf{S})]_{\sigma'\sigma}$$

unpolarized SF  $\mathcal{B}_{0,\mathcal{M}}^\tau = Tr[\mathcal{P}_{\mathcal{M},\sigma'\sigma}^\tau(\tilde{\kappa}, \epsilon, S)]$

$\mathcal{F}_{\mathcal{M}}^\tau(\tilde{\kappa}, \epsilon, \mathbf{S}) = Tr[\hat{\mathcal{P}}_{\mathcal{M}}^\tau(\tilde{\kappa}, \epsilon, S) \boldsymbol{\sigma}]$  pseudovector

$$\mathcal{F}_{\mathcal{M}}^\tau(x, \mathbf{k}_\perp; \epsilon, \mathbf{S}) = \mathbf{S} \mathcal{B}_{1,\mathcal{M}}^\tau(\dots) + \hat{\mathbf{k}}_\perp (\mathbf{S} \cdot \hat{\mathbf{k}}_\perp) \mathcal{B}_{2,\mathcal{M}}^\tau(\dots) + \hat{\mathbf{k}}_\perp (\mathbf{S} \cdot \hat{z}) \mathcal{B}_{3,\mathcal{M}}^\tau(\dots) + \hat{z} (\mathbf{S} \cdot \hat{\mathbf{k}}_\perp) \mathcal{B}_{4,\mathcal{M}}^\tau(\dots) + \hat{z} (\mathbf{S} \cdot \hat{z}) \mathcal{B}_{5,\mathcal{M}}^\tau(\dots)$$

$\downarrow$

$x = \kappa^+(1;23)/\mathcal{M}_0(1;23)$

The scalar functions  $\mathcal{B}_{i,\mathcal{M}}^\tau(\dots)$  depend, for  $\mathcal{J} = 1/2$ , on  $|\mathbf{k}_\perp|$ ,  $x$ ,  $\epsilon$

# LF spectral function and momentum distribution

By integrating the LF SF on  $\kappa^-$ , equivalent to the integration on the  $\epsilon = \text{internal energy of the spectator system}$ , one straightforwardly gets the **LF spin-dependent momentum distribution**

$$\mathcal{N}_{\sigma'\sigma}^{\tau}(x, \mathbf{k}_\perp; \mathcal{M}, \mathbf{S}) = \frac{1}{2} \{ b_{0,\mathcal{M}}(\dots) + \sigma \cdot \mathbf{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S}) \}_{\sigma'\sigma}$$
$$\int d\epsilon \mathcal{B}_{i,\mathcal{M}}^{\tau}(\dots) \quad \int d\epsilon \mathcal{F}_{\mathcal{M}}^{\tau}(x, \mathbf{k}_\perp; \epsilon, \mathbf{S})$$

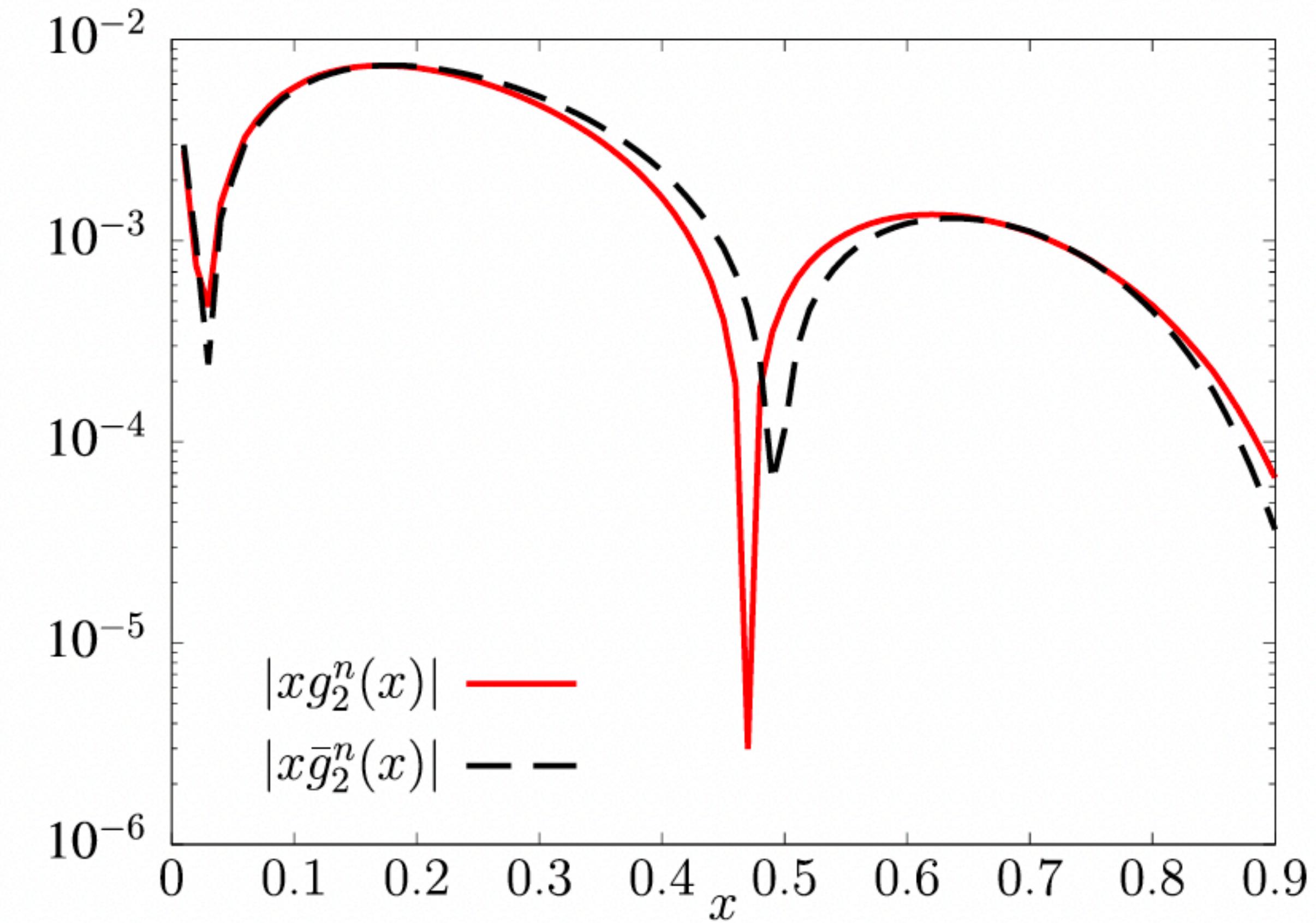
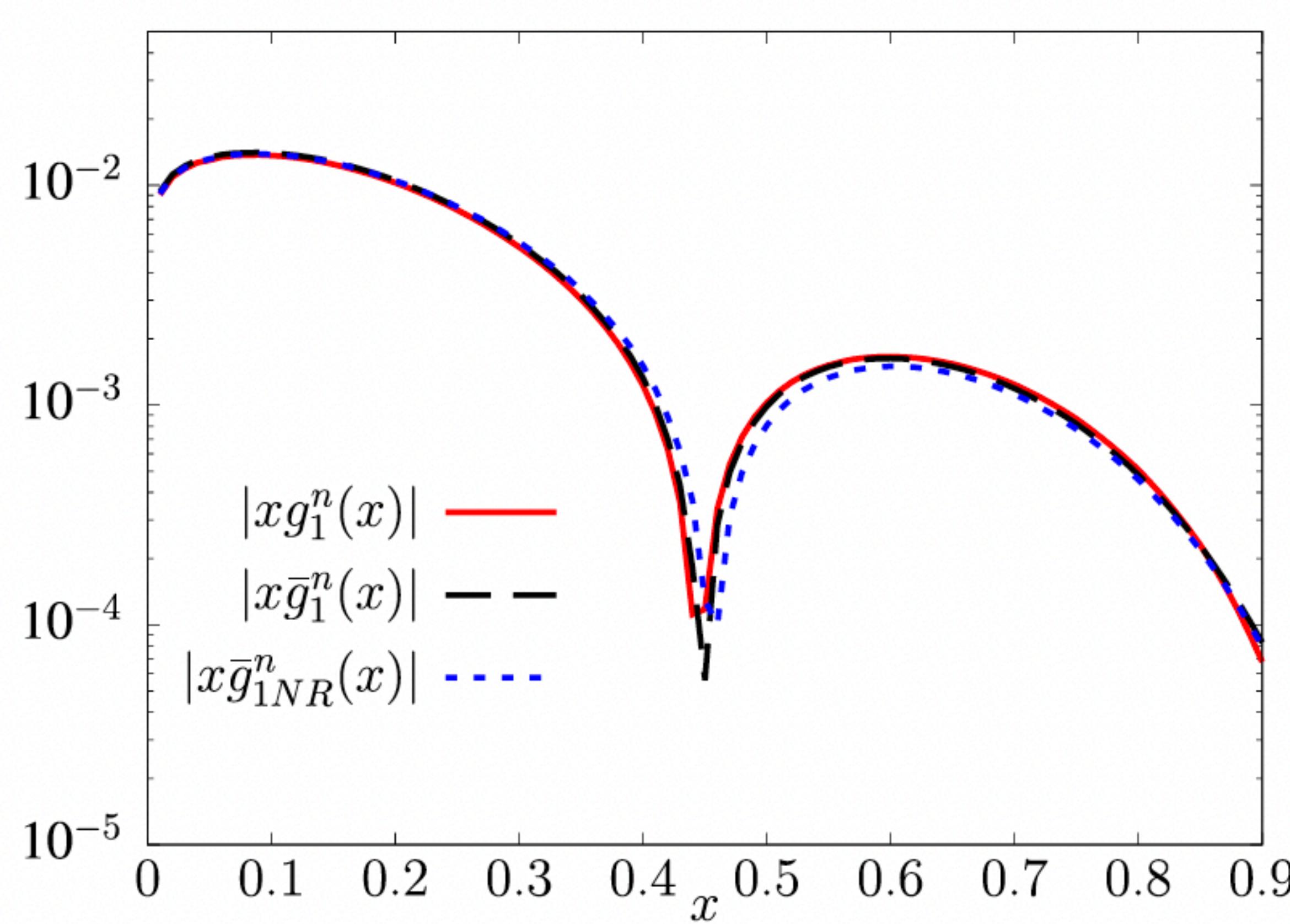
The decomposition is useful to get:

an explicit interplay between transverse momentum component and spin dofs

relations between Transverse-momentum distributions (TMDs) in the *valence sector*

# Neutron SSFs

E.Prietti, F.F, E.Pace, M.Rinaldi, G.Salmè and S.Scopetta, *Phys.Rev.C* 110 (2024) 3, L031303



**Solid lines:** GRSV parametrization of the free neutron SSFs

**Dashed lines:** extraction of the free neutron SSFs from **relativistic** effective polarizations

**Dotted line:** extraction of the free neutron SSFs from **non-relativistic** effective polarizations