Gluon TMDs

Summer School
Light-ion physics in the EIC era: from nuclear structure to high-energy process
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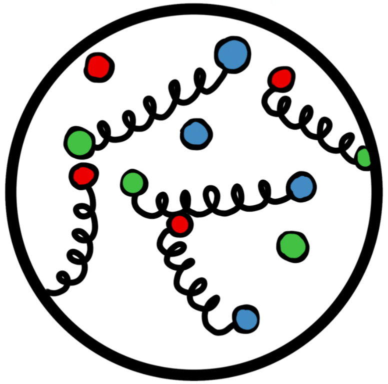
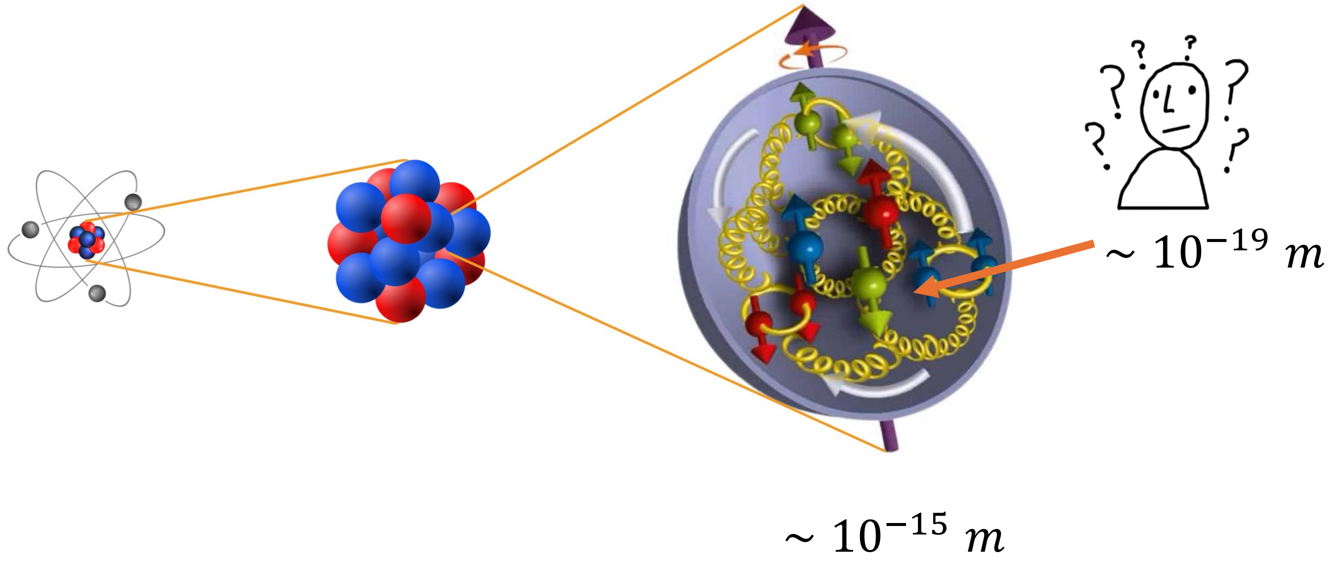
Introduction

- Goal: study the internal **structure of the hadrons** in terms of their elementary constituents, quarks and gluons

- We investigate **T**ransverse **M**omentum **D**ependent **PDFs** (**TMD PDFs**)

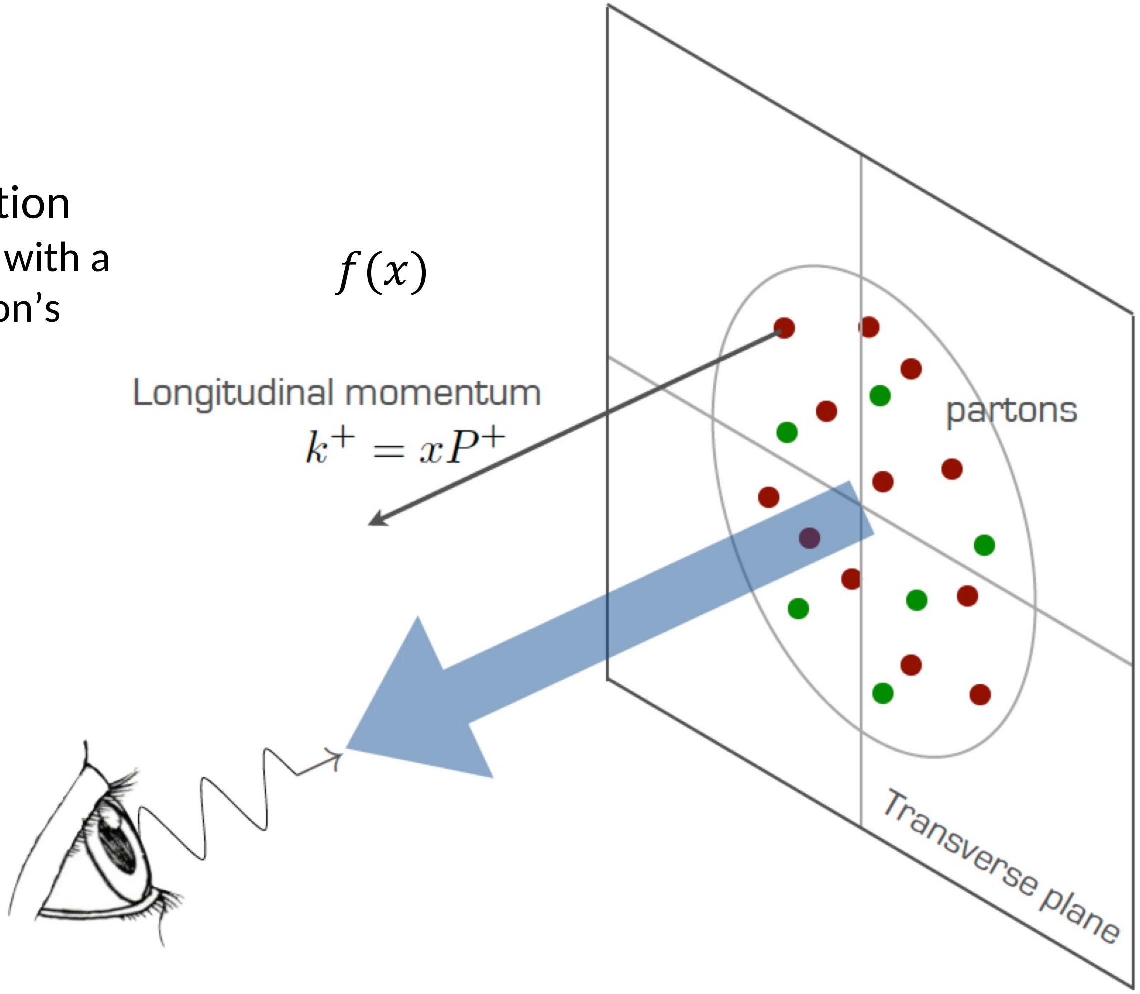
- Why focus on on **gluon TMDs**? Less studied compared to quarks, they provide crucial insights into the intrinsic motion of gluons within hadrons





Collinear PDFs

Parton **D**istribution **F**unction
Probability of finding a parton with a fraction of the parent hadron's momentum



1D

Collinear PDFs:

- ignore motion of partons on the transverse plane
- depend just on x

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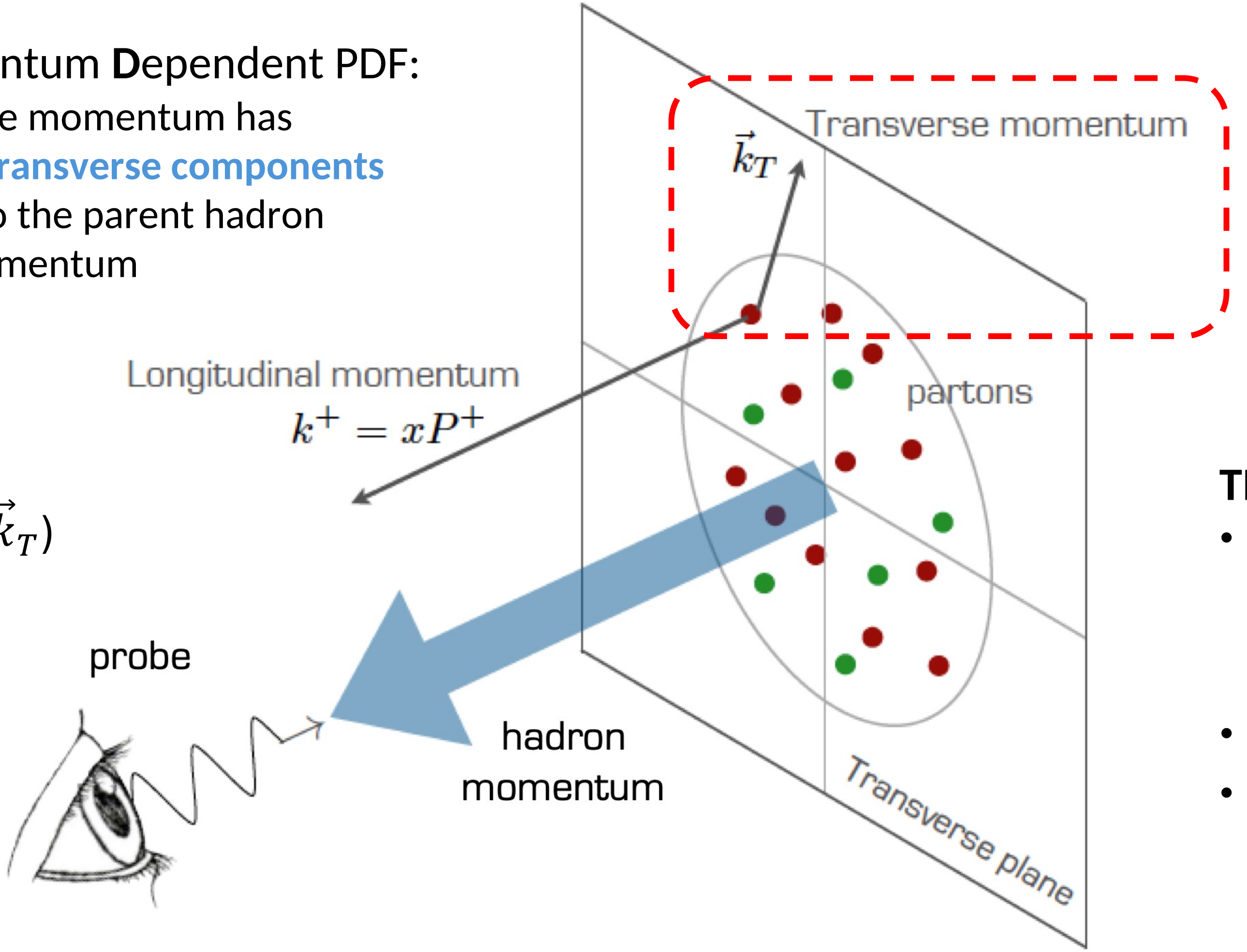
Transverse Momentum Dependent PDFs

Transverse **M**omentum **D**ependent PDF:
Parton whose momentum has
longitudinal and **transverse components**
with respect to the parent hadron
momentum

$f\,(x, \vec{k}_T)$



3D

TMD PDFs:

- encode all the possible **spin-spin** and **spin-momentum** correlations between the proton and its constituents
- more informations than PDFs
- depend on x and $\vec{k}_T$

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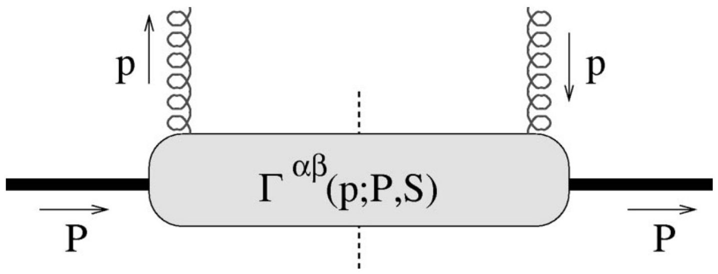
Transverse Momentum Dependent PDFs



P, S = hadron's momentum & spin
p = gluon's momentum

Gauge link:

$$\mathcal{U}^{\mathcal{C}}_{[0,\xi]} = \mathcal{P}\exp\Big(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu A^\mu(s)\Big)$$

Gluon correlator:

$$\Gamma_g^{[\mathcal{U},\mathcal{U}']\alpha\beta}(x,\boldsymbol{p}_T) \propto \langle P,S|\ \mathrm{Tr}\left[F^{\alpha+}(0)\,\mathcal{U}_{[0,\xi]}F^{\beta+}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P,S\rangle\,\Big]_{\mathrm{LF}}$$

Gluon field strength tensor

Proton state vector

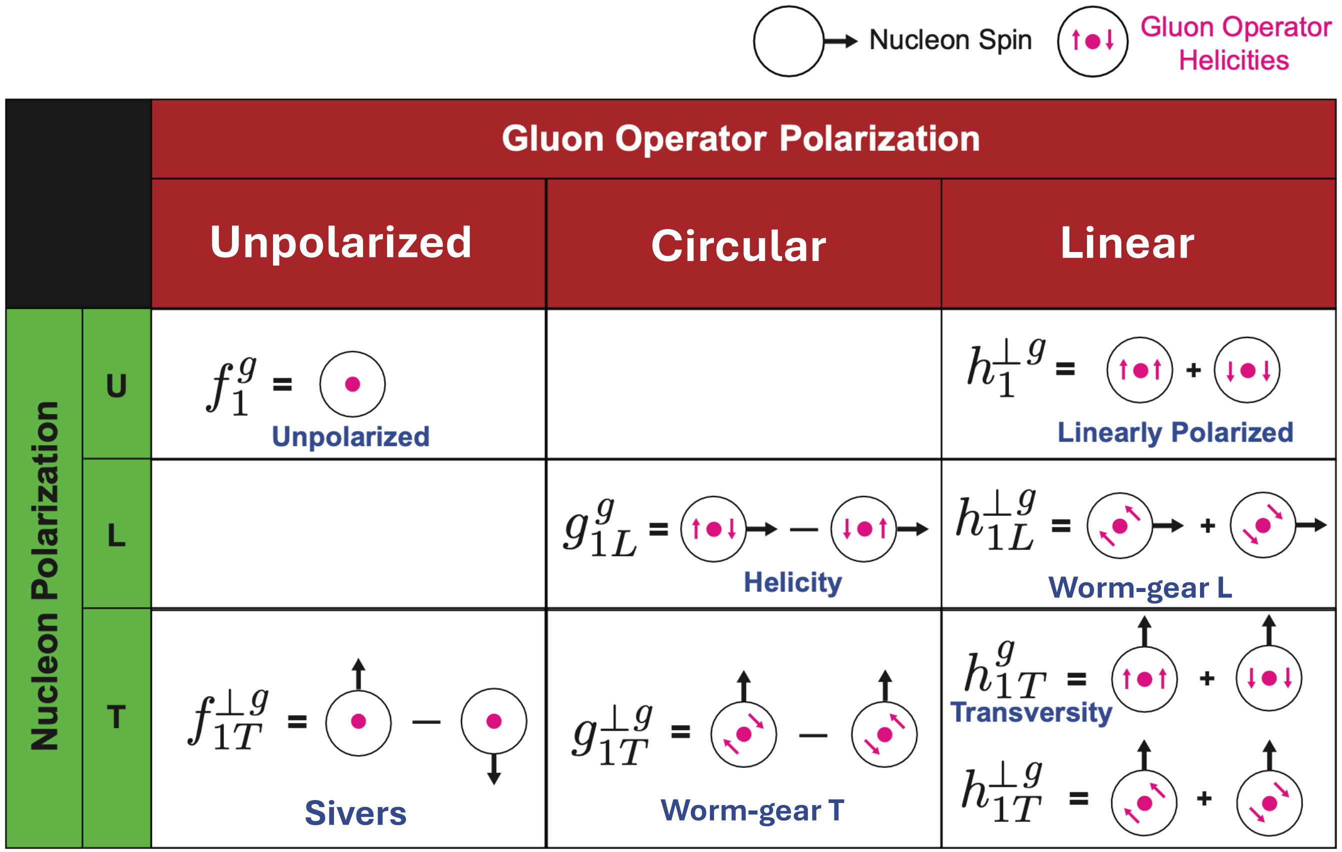
The Lorentz structure of the correlator is fixed by hermiticity and parity conservation:

$$\text{Hermicity:}\ \ \Gamma^{\rho\sigma;\alpha\beta*}(p;P,S) = \Gamma^{\alpha\beta;\rho\sigma}(p;P,S)$$

$$\text{Parity:}\ \ \Gamma^{\alpha\beta;\rho\sigma}(p;P,S) = \Gamma_{\alpha\beta;\rho\sigma}(\bar{p};\bar{P},-\bar{S})$$

Individual TMDs can be projected out of the correlator

Leading-twist gluon TMDs



The gluon correlator is parametrized in terms of **8 gluon TMDs.**

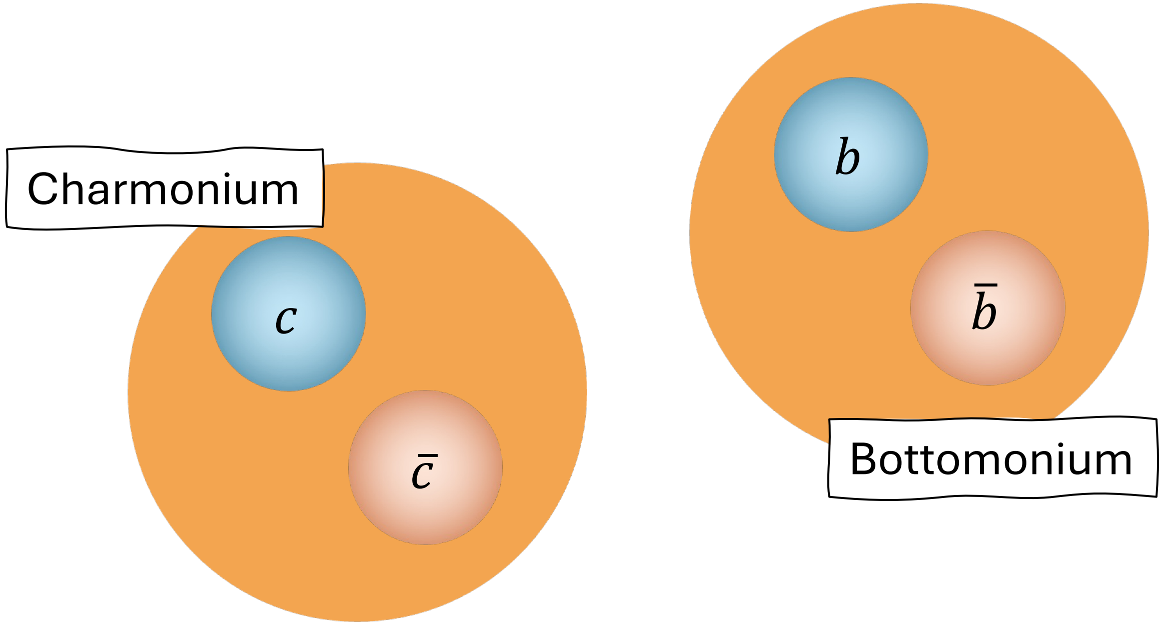
Depend on:

- **Bjorken-x**
- **Transverse momentum** of the gluon
- **Polarization** of nucleon and gluon

Angeles-Martinez et al., Acta Phys., Pol B46 (2015)
Boussarie, R, et al. "TMD handbook."

Quarkonium production

It's a bound state of heavy quark-antiquark pair

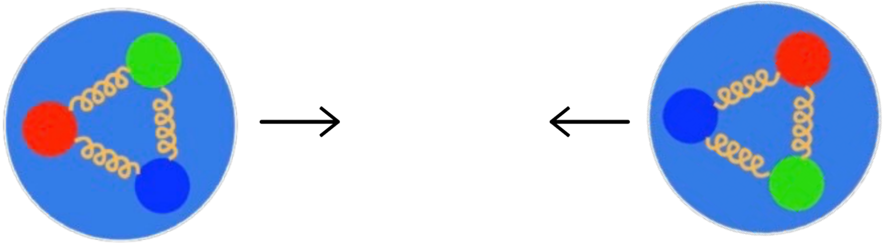


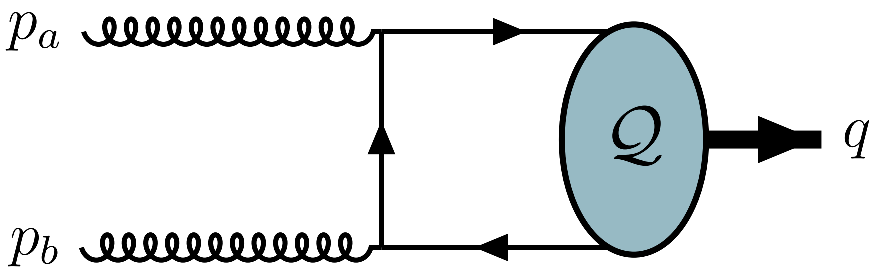
We investigated C-even quarkonia:

${}^1S_0$ states: η_c, η_b
${}^3P_0$ states: χ_{c0}, χ_{b0}
${}^3P_2$ states: χ_{c2}, χ_{b2}

Quarkonium production

$$p(P_A, S_A) + p(P_B, S_B) \;\to\; Q\overline{Q}[^{2S+1}L_J^{(1)}](q) + X$$





$$g(p_a) + g(p_b) \;\to\; Q\overline{Q}[^{2S+1}L_J^{(1)}](q)$$

In the kinematic region where $\boldsymbol{q_T} \ll \boldsymbol{M}$, we expect TMD factorization to be applicable

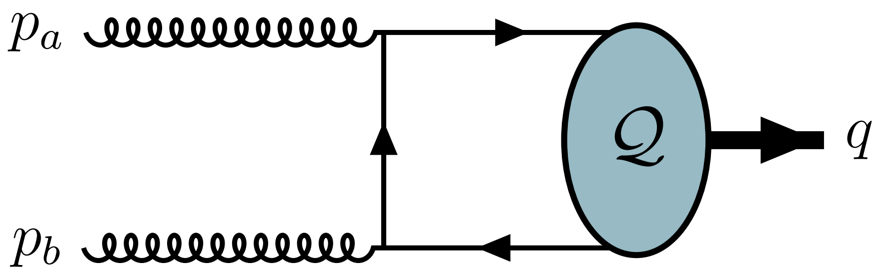
Gluon correlator

Amplitude

$$d\sigma \propto \Gamma_g^{\mu\nu}(x_a, \boldsymbol{p}_{aT})\, \Gamma_g^{\rho\sigma}(x_b, \boldsymbol{p}_{bT})\, \mathcal{A}_{\mu\rho}\, (\mathcal{A}_{\nu\sigma})^*$$

Non relativistic QCD

The model used to describe quarkonium production is called **Non-Relativistic QCD** (NRQCD)



- Double power expansion → α_S, v
- Partonic subprocess $gg \to Q\bar{Q}$ in pQCD
- Hadronization of the pair into the bound state (non perturbative) encoded in the LMDE

$$v_c^2 \simeq 0.3$$
$$v_b^2 \simeq 0.1$$

G. T. Bodwin, E. Braaten, G. P. Lepage, PRD 51 (1995)

Quarkonium production cross section

$$\frac{\mathrm{d}\sigma[\mathcal{Q}]}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{q}_T} = F_{UU}^{\mathcal{Q}} + F_{UL}^{\mathcal{Q}}\, S_{BL} + F_{LU}^{\mathcal{Q}}\, S_{AL} + F_{UT}^{\mathcal{Q},\sin\phi_{S_B}}\, |\boldsymbol{S}_{BT}| \sin\phi_{S_B} + F_{TU}^{\mathcal{Q},\sin\phi_{S_A}}\, |\boldsymbol{S}_{AT}| \sin\phi_{S_A}$$
$$+ F_{LL}^{\mathcal{Q}}\, S_{AL}\, S_{BL} + F_{LT}^{\mathcal{Q},\cos\phi_{S_B}}\, S_{AL}\, |\boldsymbol{S}_{BT}| \cos\phi_{S_B} + F_{TL}^{\mathcal{Q},\cos\phi_{S_A}}\, |\boldsymbol{S}_{AT}|\, S_{BL} \cos\phi_{S_A}$$
$$+ |\boldsymbol{S}_{AT}|\, |\boldsymbol{S}_{BT}| \left(F_{TT}^{\mathcal{Q},\cos(\phi_{S_A}-\phi_{S_B})} \cos(\phi_{S_A} - \phi_{S_B}) + F_{TT}^{\mathcal{Q},\cos(\phi_{S_A}+\phi_{S_B})} \cos(\phi_{S_A} + \phi_{S_B}) \right)$$

NK, L. Maxia, C. Pisano, PRD 110 0234028 (2024)

Every **structure function** *F* can be factorizated:

$F^{\mathcal{Q}}$ →

$H^{\mathcal{Q}} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n H_n^{\mathcal{Q}}$ — perturbative part

$\mathcal{C}[w\, F_1^g\, F_2^g]$ LDME — non-perturbative part

$\phi_{S_{A(B)}}$ = azimuthal angle of the spin vector $S_{A(B)}$

Structure functions

Unpolarized and single-transversely polarized structure functions.

$$F_{UU}^{\eta_Q} \propto \left(\mathcal{C}\left[f_1^g f_1^g\right] - \mathcal{C}\left[w_{UU}\, h_1^{\perp\, g} h_1^{\perp\, g}\right]\right)$$

$$F_{UU}^{\chi_{Q0}} \propto \left(\mathcal{C}\left[f_1^g f_1^g\right] + \mathcal{C}\left[w_{UU}\, h_1^{\perp\, g} h_1^{\perp\, g}\right]\right)$$

$$F_{UU}^{\chi_{Q2}} \propto \mathcal{C}\left[f_1^g f_1^g\right]$$

$$F_{UT}^{\eta_Q, \sin\phi_{S_B}} \propto \left(-\mathcal{C}\left[w_{UT}^f\, f_1^g f_{1T}^{\perp\, g}\right] + \mathcal{C}\left[w_{UT}^h\, h_1^{\perp\, g} h_1^g\right] - \mathcal{C}\left[w_{UT}^{h^\perp}\, h_1^{\perp\, g} h_{1T}^{\perp\, g}\right]\right)$$

$$F_{UT}^{\chi_{Q0}, \sin\phi_{S_B}} \propto \left(-\mathcal{C}\left[w_{UT}^f\, f_1^g f_{1T}^{\perp\, g}\right] - \mathcal{C}\left[w_{UT}^h\, h_1^{\perp\, g} h_1^g\right] + \mathcal{C}\left[w_{UT}^{h^\perp}\, h_1^{\perp\, g} h_{1T}^{\perp\, g}\right]\right)$$

$$F_{UT}^{\chi_{Q2}, \sin\phi_{S_B}} \propto -\mathcal{C}\left[w_{UT}^f\, f_1^g f_{1T}^{\perp\, g}\right]$$

$$w_{UU} = \frac{\boldsymbol{p}_{aT}^2 \boldsymbol{p}_{bT}^2}{4M_p^4} \cos\left[2(\phi_a - \phi_b)\right]$$

$$w_{UT}^f = \frac{|\boldsymbol{p}_{bT}|}{M_p} \cos\phi_b$$

$$w_{UT}^h = \frac{\boldsymbol{p}_{aT}^2\, |\boldsymbol{p}_{bT}|}{4M_p^3} \cos(\phi_b - 2\phi_a)$$

$$w_{UT}^{h^\perp} = \frac{\boldsymbol{p}_{aT}^2\, |\boldsymbol{p}_{bT}|^3}{8M_p^5} \cos(3\phi_b - 2\phi_a)$$

ϕ_a = azimuthal angle of p_{aT}
ϕ_b = azimuthal angle of p_{bT}

Observables measurable with **LHCSpin project**, a fixed target experiment planned at LHC

LHC spin

Single Spin Asymmetries (SSAs)

Proton A **unpolarized**
Proton B **trasv. polarized**

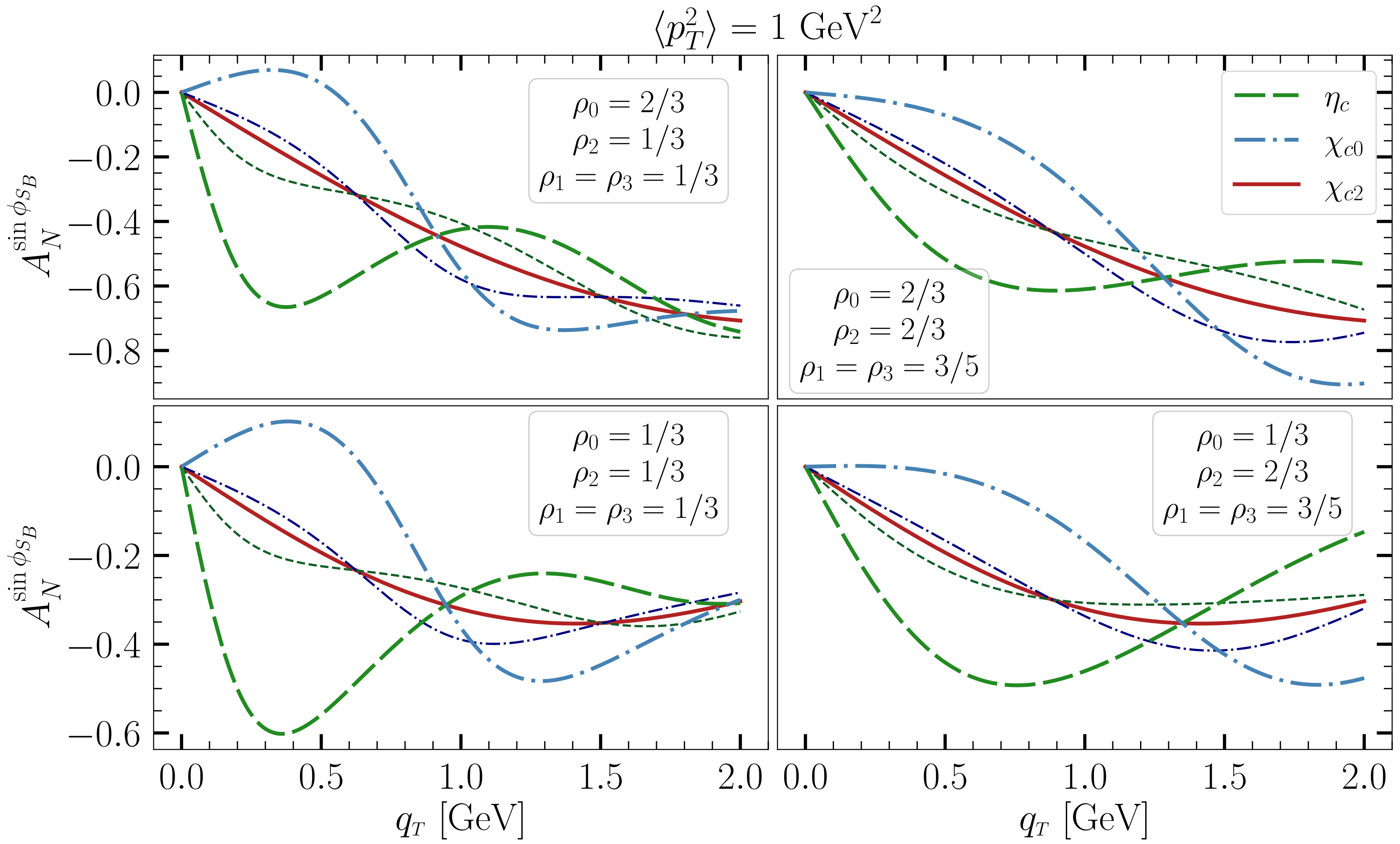
$$A_N^{\mathcal{Q},\sin\phi_S} = 2\,\frac{\int d\phi_S \sin\phi_S [d\sigma(\phi_S) - d\sigma(\phi_S+\pi)]}{\int d\phi_S [d\sigma(\phi_S) + d\sigma(\phi_S+\pi)]} = \frac{F_{UT}^{\mathcal{Q},\sin\phi_S}}{F_{UU}^{\mathcal{Q}}}$$

Gaussian parametrization of the gluon TMDs:

$$f_1^g(x,\boldsymbol{p}_T^2) = \frac{f_1^g(x)}{\pi\langle p_T^2\rangle}\,\exp\left[-\frac{\boldsymbol{p}_T^2}{\langle p_T^2\rangle}\right]$$ **unpolarized gluon TMD**

Process accessible at the future LHCSpin, the fixed target experiment planned at LHC

Numerical results: upper bounds for SSAs



$$A_N^{\eta_Q,\sin\phi_{S_B}} = \frac{-R_{UT}^f + R_{UT}^h - R_{UT}^{h^\perp}}{1 - R_{UU}}$$

$$A_N^{\chi_{Q0},\sin\phi_{S_B}} = \frac{-R_{UT}^f - R_{UT}^h + R_{UT}^{h^\perp}}{1 + R_{UU}}$$

$$A_N^{\chi_{Q2},\sin\phi_{S_B}} = -R_{UT}^f$$

SSA for the production of χ_{Q2} depends only on the Sivers function!

By comparing the SSAs for η_Q and χ_{Q0} states with those for χ_{Q2} we can reveal the impact of the combined effects of the linearly polarized gluon TMDs

Matching relations

TMDs are functions of x and b (Fourier conjugated to the transverse mometum of the parton k_T)

TMD distributions are **non-perturbative** functions and should be extracted by data

HOWEVER

They can be evaluated in terms of **collinear PDFs** in the limit of large-q_T (or small-b)

↓

Matching procedure

- Increases theoretical predictive power
- Reduces parametric freedom in model building

Matching relations

How can we find the matching relations?

At small values of b, a TMD distribution can be related to collinear distributions with a general form given by the Operator Product Expansion (OPE)

$$f(x, \boldsymbol{b}) = C(x, \ln(\mu b)) \otimes f(x, \mu)$$

Main steps:

- Expand the gluon operator
- Extract the the leading and next-to-leading twist components ⇢ spinor formalism
- Compute the gluon correlator by taking the matrix element of the gluon operator
- Find the matching relation

Scimemi, Tarasov, Vladimirov JHEP05(2019)125, Moos, Vladimirov JHEP12(2020)145, Rein, Rodini, Schäfer, Vladimirov JHEP01(2023)116

Matching relations

Contribution for twist-2 gluon TMD PDFs onto the leading and next-to-leading collinear PDFs

An example: unpolarized distribution

$$f_1^g(x,b) = f_g(x) + \sum_{k=1}^{\infty} \frac{1}{k!(k-1)!} \left(\frac{x^2 M^2 b^2}{4}\right)^k \int_0^1 du \left(\frac{\bar{u}}{u}\right)^{k-1} \delta(x-uy) f_g(y)$$

Order zero term

Mass series

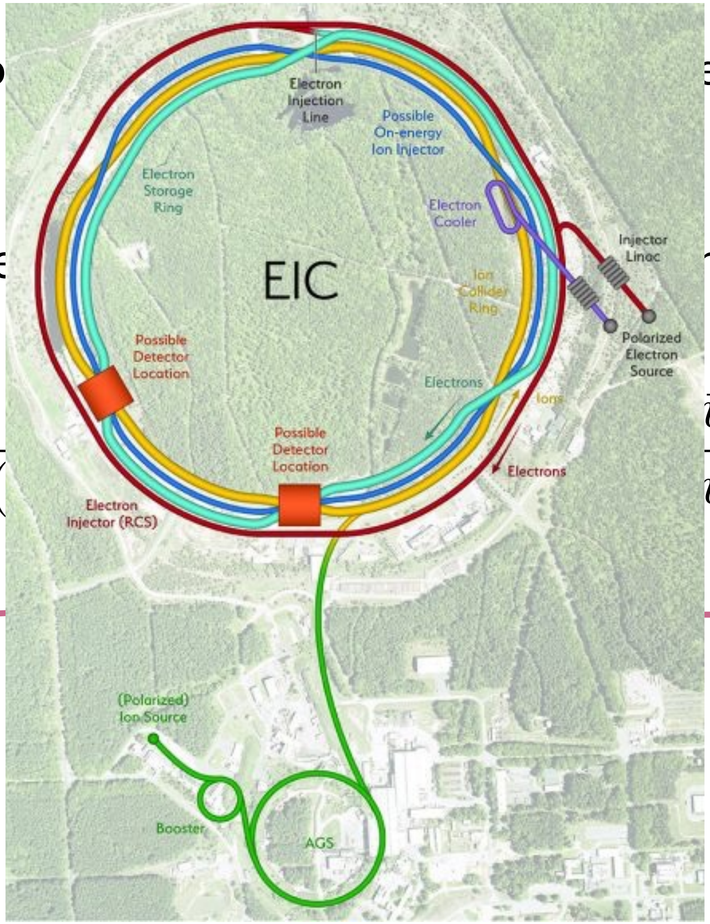
Matching relations

Contribution for twist-2 gluo … ext-to-leading collinear PDFs

An e … n

$$f_1^g(x,b) = \underbrace{f_g(x)}_{} + \sum_{k=1}^{\infty} \frac{\cdots}{k!(\cdots} \cdots \frac{\bar{u}}{u}\Big)^{k-1} \delta(x-uy) f_g(y)$$

Order zero term



Our results can be useful for the future Electron Ion Collider (EIC)

Thanks!

Backup

Gaussian parametrization of the gluon TMDs

$$f_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2) = \mathcal{N}_0(x)\,\frac{f_1^g(x)}{\pi\langle p_T^2\rangle^{3/2}}\,M_p\,\sqrt{\frac{2(1-\rho_0)}{\rho_0}}\,\exp\left[\frac{1}{2}-\frac{1}{\rho_0}\,\frac{\boldsymbol{p}_T^2}{\langle p_T^2\rangle}\right]$$

$$h_1^g(x,\boldsymbol{p}_T^2) = \mathcal{N}_1(x)\,\frac{f_1^g(x)}{\pi\langle p_T^2\rangle^{3/2}}\,M_p\,\sqrt{\frac{2(1-\rho_1)}{\rho_1}}\,\exp\left[\frac{1}{2}-\frac{1}{\rho_1}\,\frac{\boldsymbol{p}_T^2}{\langle p_T^2\rangle}\right]$$

$$h_1^{\perp\,g}(x,\boldsymbol{p}_T^2) = 2\,\mathcal{N}_2(x)\,\frac{f_1^g(x)}{\pi\langle p_T^2\rangle^{2}}\,M_p^2\,\frac{(1-\rho_2)}{\rho_2}\,\exp\left[1-\frac{1}{\rho_2}\,\frac{\boldsymbol{p}_T^2}{\langle p_T^2\rangle}\right]$$

$$h_{1T}^{\perp\,g}(x,\boldsymbol{p}_T^2) = 2\,\mathcal{N}_3(x)\,\frac{f_1^g(x)}{\pi\langle p_T^2\rangle^{5/2}}\,M_p^3\,\left[\frac{2(1-\rho_3)}{3\rho_3}\right]^{3/2}\,\exp\left[\frac{3}{2}-\frac{1}{\rho_3}\,\frac{\boldsymbol{p}_T^2}{\langle p_T^2\rangle}\right]$$

$$0<\rho_i<1$$

Gaussian parametrization of the gluon TMDs

$$R_{UU} = \mathcal{C}[w^h_{UU} h_1^{\perp g} h_1^{\perp g}]/\mathcal{C}[f_1^g f_1^g]$$
$$= \frac{1}{16\langle p_T^2\rangle^2} \frac{(1-\rho_2)^2}{\rho_2} (\boldsymbol{q}_T^4 - 8\rho_2 \langle p_T^2\rangle \boldsymbol{q}_T^2 + 8\rho_2^2 \langle p_T^2\rangle^2) \exp\left[2 - \frac{1-\rho_2}{\rho_2} \frac{\boldsymbol{q}_T^2}{2\langle p_T^2\rangle}\right],$$

$$R^f_{UT} = \mathcal{C}[w^f_{UT} f_1^g f_{1T}^{\perp g}]/\mathcal{C}[f_1^g f_1^g]$$
$$= \frac{2}{\langle p_T^2\rangle^{1/2}} \sqrt{\frac{2(1-\rho_0)}{\rho_0}} \left(\frac{\rho_0}{1+\rho_0}\right)^2 |\boldsymbol{q}_T| \exp\left[\frac{1}{2} - \frac{1-\rho_0}{1+\rho_0} \frac{\boldsymbol{q}_T^2}{2\langle p_T^2\rangle}\right],$$

$$R^h_{UT} = \mathcal{C}[w^h_{UT} h_1^{\perp g} h_1^g]/\mathcal{C}[f_1^g f_1^g]$$
$$= \frac{1}{\langle p_T^2\rangle^{3/2}} \sqrt{\frac{2(1-\rho_1)}{\rho_1}} (1-\rho_2) \frac{{\rho_1}^2 \rho_2^2}{(\rho_1+\rho_2)^4} |\boldsymbol{q}_T| (\boldsymbol{q}_T^2 - 2(\rho_1+\rho_2)\langle p_T^2\rangle) \exp\left[\frac{3}{2} - \frac{2-\rho_1-\rho_2}{\rho_1+\rho_2} \frac{\boldsymbol{q}_T^2}{2\langle p_T^2\rangle}\right],$$

$$R^{h^\perp}_{UT} = \mathcal{C}[w^{h^\perp}_{UT} h_1^{\perp g} h_{1T}^{\perp g}]/\mathcal{C}[f_1^g f_1^g]$$
$$= \frac{1}{\langle p_T^2\rangle^{5/2}} \left[\frac{2(1-\rho_3)}{3\rho_3}\right]^{3/2} (1-\rho_2) \frac{\rho_2^2 \rho_3^4}{(\rho_2+\rho_3)^6} |\boldsymbol{q}_T| (\boldsymbol{q}_T^4 - 6(\rho_2+\rho_3)\langle p_T^2\rangle \boldsymbol{q}_T^2 + 6(\rho_2+\rho_3)^2 \langle p_T^2\rangle^2)$$
$$\times \exp\left[\frac{5}{2} - \frac{2-\rho_2-\rho_3}{\rho_2+\rho_3} \frac{\boldsymbol{q}_T^2}{2\langle p_T^2\rangle}\right],$$

$0 < \rho_i < 1$

The LHCSpin project

The project: implementation of a new-generation **fixed target polarized gas** in the LHCb spectrometer allowing spin physics at LHC for the first time

Goals:

- Multi-dimensional nucleon structure in a previously unexplored kinematic region at LHC energies
- Measure experimental observables sensitive to both **polarized quarks and gluons TMDs, PDFs,** and **GPDs**
- Heavy-ion physics: probe collective phenomena in heavy-light systems through **ultra-relativistic collisions** of heavy nuclei with transv. pol. deuterons

Evolution of fixed target systems at LHCb:

SMOG → **SMOG2** → **LHCSpin**

L. Pappalardo et al., *Nuovo Cim.C* 47 (2024) 4, 235
P. Di Nezza et al., PoS (PSTP2022)001

LHCSpin timeline

SMOG2

2024 - 2026

2026 - 2029

LHC LS3
Installation of the apparatus at the IR4

2030 - 2033

LHC Run4
Data taking at the IR4

2034 - 2035

LHC LS4
Installing the target at LHCb

2036 -

LHC Run5+6
Data taking at the LHCb

The LHCspin apparatus consists of a new-generation HERMES-like polarized gaseous fixed target to be installed usptream of the VELO.
With its installation, LHCb will be the first experiment to simultaneously collect data from unpolarized beam-beam collisions at $\sqrt{s} = 14$ TeV and polarized and unpolarized beam-target collisions at $\sqrt{s} = 100$ GeV.