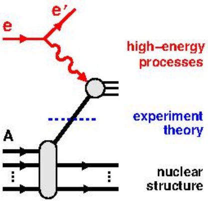
Incoherent Deeply Virtual Compton Scattering on the Deuteron

Alan Sosa

Florida International University
Miami, Florida



Generalized Parton Distribution functions (GPDs)

GPDs importance for hadron structure [1]

- Can be used to get 2D transverse spatial distribution of partons
- 1D longitudinal momentum distributions of partons
- Information of orbital angular momentum distribution
- Gives information on Energy Momentum Tensor

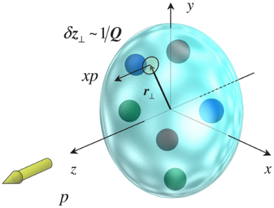


Figure: Illustration to show how IPD gotten from GPD gives 3D picture of Hadron [2]

GPDs

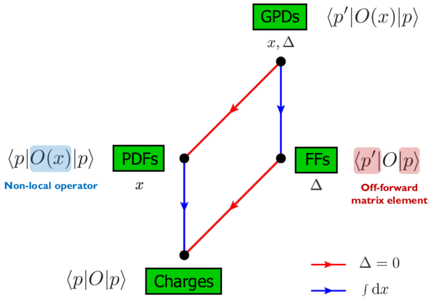


Figure: Relation between GPDs, FFs, and PDFs (took figure from Lorce slides)

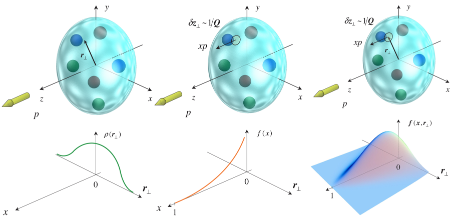
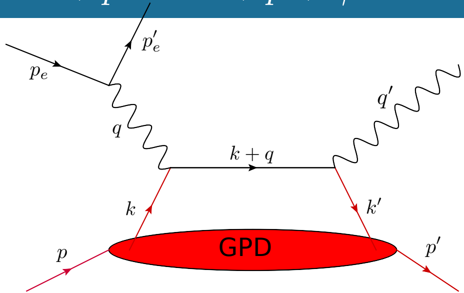


Figure: Shows relation between the Impact Parameter Distribution (IPDs) of GPDs relation to PDFs and IPDs of FFs

Processes that are sensitive to GPDs $\rightarrow$ **DVCS**, DVMP, DDVCS, TCS,etc.

Nucleon DVCS $e^- + p \to e^- + p + \gamma$



$$A_{DVCS} = \left[-\frac{i|e|^3}{2q^2} \sum_f [\bar{u}(p'_e)\gamma^\rho u(p_e)]\epsilon^*_\mu(q')g_{\rho\nu} \right] \tag{1}$$

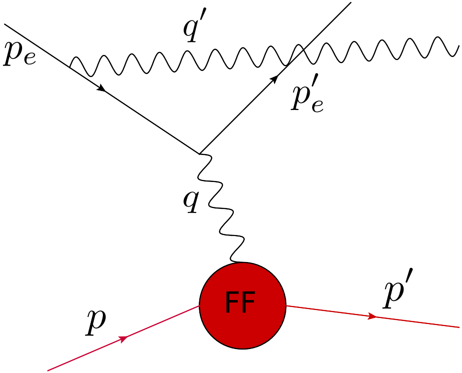
$$\times\, e_q^2 \left\{ g_\perp^{\mu\nu} \left[\mathscr{H}^q(\xi,t)\bar{u}(p'_N)\gamma^+ u(p_N) + \mathscr{E}^q(\xi,t)\bar{u}(p'_N)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p_N) \right] \right. \tag{2}$$

$$\left. +\, i\epsilon^{\mu\nu+-} \left[\tilde{\mathscr{H}}^q(\xi,t)\bar{u}(p'_N)\gamma^+\gamma_5 u(p_N) + \tilde{\mathscr{E}}^q(\xi,t)\bar{u}(p'_N)\frac{\gamma_5\Delta^+}{2m}u(p_N) \right] \right\} \tag{3}$$

$$\mathscr{H}^q(\xi,t) = \int dx \left[\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon} \right] H^q(x,\xi,t) \tag{4}$$

Bethe-Heitler Nucleon $e^- + p \to e^- + p + \gamma$

We need to look at all terms that contribute to reaction $e^- + p \to e^- + p + \gamma$



$$A_{BH} = \left[-\frac{i|e|^3}{q^2} \bar{u}(p'_e) \not{\epsilon}^*(q') \frac{\not{p}_e - \not{q}'}{(p_e - q')^2} \gamma_\mu u(p_e) \right] \times \left[F_1(t) \bar{u}(p'_N) \gamma^\mu u(p_N) + F_2(t) \bar{u}(p'_N) \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} u(p_N) \right] \tag{5}$$

Proton Structure

Complications with this process $e^- + p \to e^- + p + \gamma$

$$\sigma \propto |A|^2 = |A_{DVCS}|^2 + |A_{BH}|^2 + A^*_{BH} A_{DVCS} + A^*_{DVCS} A_{BH} \tag{6}$$

$$A_{DVCS} \propto \text{CFFs} \tag{7}$$

$$A_{BH} \propto \text{FFs} \tag{8}$$

So $e^- + p \to e^- + p + \gamma$ does not even give you direct access to GPDs, but it gives CFFs.

For extraction of GPDs, need more data than just proton, need also neutron data

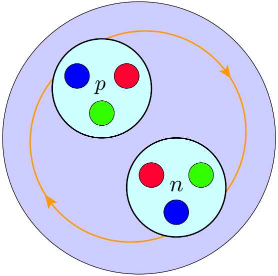
$$H_d^n(x,\xi,t) = H_u^p(x,\xi,t), \quad H_u^n(x,\xi,t) = H_d^p(x,\xi,t)$$

$$H_{\bar{d}}^n(x,\xi,t) = H_{\bar{u}}^p(x,\xi,t), \quad H_{\bar{u}}^n(x,\xi,t) = H_{\bar{d}}^p(x,\xi,t)$$

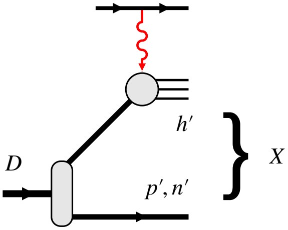
Slight problem, no free Neutron target ...

Use Deuteron

- Simplest Nucleus with one proton and one neutron
- Spin-1 particle
- Nucleons in Deuteron can be described with non-relativistic wavefunction



Inclusive eD scattering: Impulse approximation



Compute inclusive eD cross section
from eN cross section using light-front dynamics

$X = h' + p', n'$ final state in impulse approximation

Untagged scattering: Spectators summed/integrated over
Tagged scattering: Spectator identified, momentum fixed

$$\langle h' p' | J^\mu | D \rangle$$

current matrix element for $h' + p'$ final state

$$\uparrow$$

$$\int d\Gamma_p \int d\Gamma_n \, |pn\rangle\langle pn|$$

insert set of nucleon intermediate states

$$= \int d\Gamma_p \int d\Gamma_n \, \underbrace{\langle h' p' | J^\mu | pn \rangle}_{\langle h' | J^\mu | n \rangle \, \langle p' | p \rangle} \, \underbrace{\langle pn | D \rangle}_{\delta(\,..\,)\, \Psi_D}$$

current couples to neutron

Further topics

Final-state interactions

Interaction of hadrons in final state of high-energy process with spectators.
Important for tagging/breakup measurements

Initial-state interactions and non-nucleonic degrees of freedom

High-energy processes involving multiple nucleons, hadrons in NN interactions

QCD factorization and partonic structure

Methods developed here can be applied to compute nuclear partonic structure in terms of nucleon structure

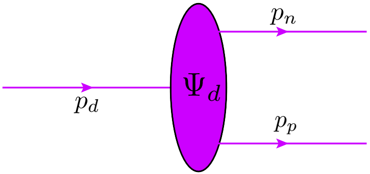
Small-x physics and nuclear shadowing

Methods developed here can be applied to nuclear shadowing in inclusive and exclusive small-x scattering on light nuclei

Exclusive processes

Applications to exclusive scattering processes, e.g. deep-virtual Compton scattering and meson production on light nuclei, in quasi-elastic or coherent scattering

LFWF for Deuteron



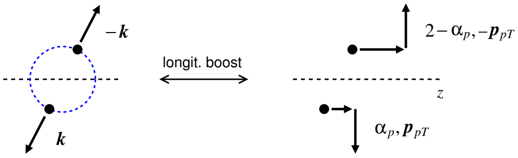
$$\Psi_d = \langle pn|d\rangle = \bar{u}_{LF}(p_n)\Gamma_\alpha v_{LF}(p_p)\epsilon^\alpha_{pn}(p_{pn}) \tag{9}$$

Where

$$\Gamma^\alpha = \gamma^\alpha G_1 + (p_p - p_n)^\alpha G_2 \tag{10}$$

Parametrization of bilinears exactly same as parametrizing form factors. To get values of G_1 and G_2 it is possible to relate these to S and D-Wave contributions in nonrelativistic wavefunctions. [3]

Light-front structure: Spherically symmetric rep



CM frame

gen. collinear frame

Here: pn configuration in deuteron

Described by proton LF momentum variables α_p, $\mathbf{p}_{pT}$

Boost invariance: Consider the config in the CM frame where the ordinary nucleon 3-momenta are back-to-back: proton $\mathbf{k}$, neutron $-\mathbf{k}$

Use 3-vector $\mathbf{k}$ as variable!

$$\alpha_p = 1 + \frac{k^z}{E(k)}, \qquad \mathbf{p}_{pT} = \mathbf{k}_T$$

$$E(k) \equiv \sqrt{|\mathbf{k}|^2 + m^2}$$

$$M_{pn}^2 = \frac{4(|\mathbf{p}_{pT}|^2 + m^2)}{\alpha_p(2-\alpha_p)} = 4(|\mathbf{k}|^2 + m^2) = 4E^2$$

invariant mass = CM energy

→ LF dynamical equation becomes 3D rotationally symmetric

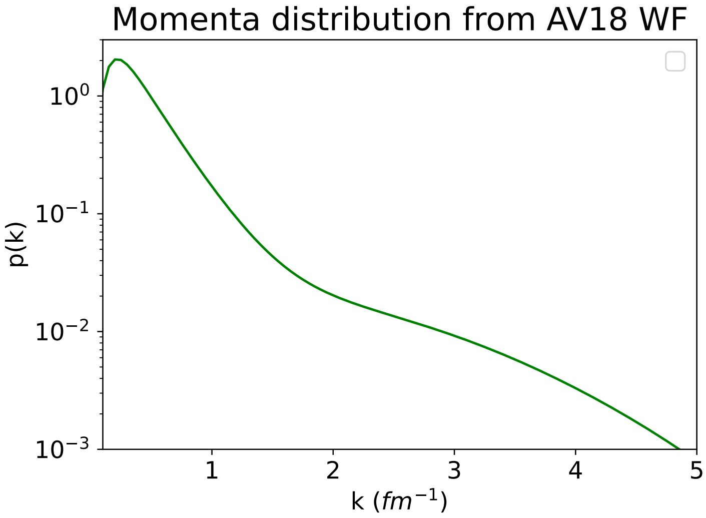
→ On-shell scattering amplitudes calculated with LF dynamics satisfy rotational invariance (“angular conditions”)

Generalization to 3N and beyond: Possible but much more complex

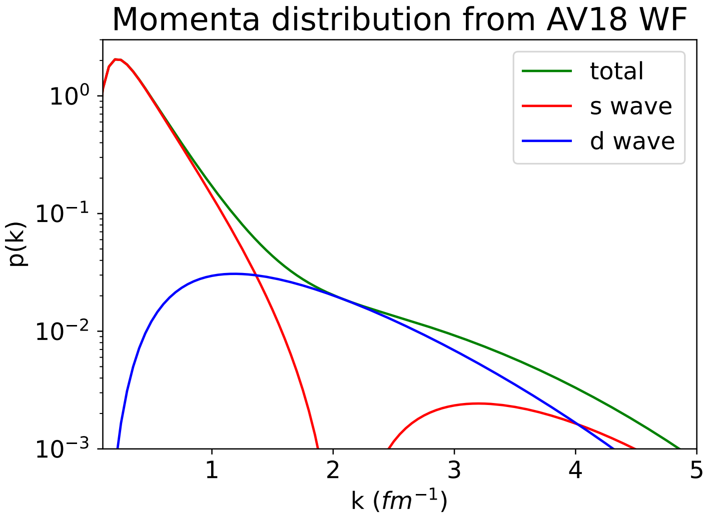
Terentev 76, Kondratyuk, Strikman 84

Lev 1993

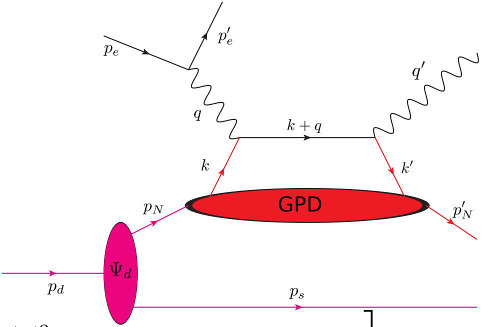
Deuteron Wavefunction



Deuteron Wavefunction



Incoherent DVCS PWIA $e^- + D \to e^- + p + n + \gamma$



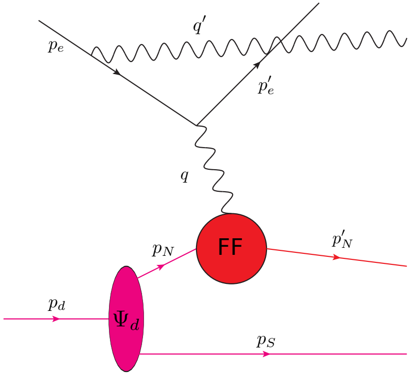
$$A_{NDVCS} = \left[-\frac{i|e|^3}{2q^2} \sum_f [\bar{u}(p'_e)\gamma^\rho u(p_e)]\epsilon^*_\mu(q')g_{\rho\nu} \right] \tag{11}$$

$$\times\, e_q^2 \left\{ g_\perp^{\mu\nu} \left[\mathscr{H}_N^q(\xi,t)\bar{u}(p'_N)\gamma^+ u(p_N) + \mathscr{E}_N^q(\xi,t)\bar{u}(p'_N)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p_N) \right] \right. \tag{12}$$

$$\left. +\, i\epsilon^{\mu\nu+-} \left[\tilde{\mathscr{H}}_N^q(\xi,t)\bar{u}(p'_N)\gamma^+\gamma_5 u(p_N) + \tilde{\mathscr{E}}_N^q(\xi,t)\bar{u}(p'_N)\frac{\gamma_5\Delta^+}{2m}u(p_N) \right] \right\}$$

$$\times\, \bar{u}_{LF}(p_N)\Gamma_\alpha v_{LF}(p_S)\epsilon^\alpha_{pn}(p_{NS})$$

Incoherent BH $e^- + D \to e^- + p + n + \gamma$



$$
\begin{aligned}
A_{BH} = & \left[-\frac{i|e|^3}{q^2} \bar{u}(p'_e) \not{\epsilon}^*(q') \frac{\not{p}_e - \not{q}'}{(p_e - q')^2} \gamma_\mu u(p_e) \right] \\
& \times \left[F_1(t) \bar{u}(p'_N) \gamma^\mu u(p_N) + F_2(t) \bar{u}(p'_N) \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} u(p_N) \right] \\
& \times \bar{u}_{LF}(p_N) \Gamma_\alpha v_{LF}(p_S) \epsilon^\alpha_{pn}(p_{NS})
\end{aligned}
\tag{13}
$$

Final State Interactions

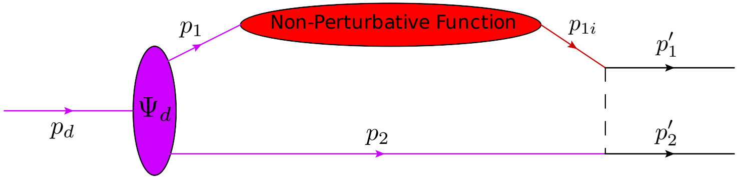


Figure: Dash Line corresponds to interaction between nucleons

$$\mathscr{M}_{\lambda'_1\lambda'_2;\lambda_1\lambda_2} = (\bar{u}_{\lambda'_1}(p'_1))_a(\bar{u}_{\lambda'_2}(p'_2))_b M_{ab;cd}(u_{\lambda_1}(p_1))_c(u_{\lambda_2}(p_2))_d \tag{14}$$

$$M_{ab;cd} = F_S(s,t)\delta_{ac}\delta_{bd} + F_V(s,t)\gamma_{ac}\cdot\gamma_{bd} + F_T(s,t)\sigma^{\mu\nu}_{ac}(\sigma_\mu\nu)_b d \tag{15}$$

$$+ F_P(s,t)\gamma^5_{ac}\gamma^5_{bd} + F_A(s,t)(\gamma^5\gamma)_{ac}\cdot(\gamma^5\gamma)_{bd} \tag{16}$$

$\mathscr{M}_{\lambda'_1\lambda'_2;\lambda_1\lambda_2}$ is a helicity amplitude that is accessible via SAID parametrizations of nucleon nucleon scattering data

Final State Interactions

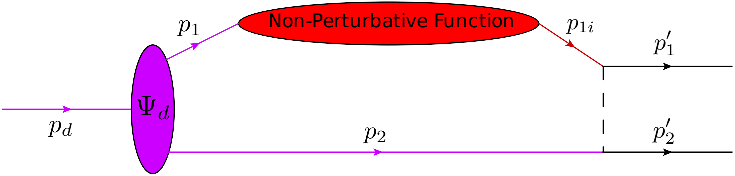


Figure: Dash Line corresponds to interaction between nucleons in Final State

$$A_{FSI,1} = \int \frac{d^3p_2}{2E_2} N_N [\bar{u}(p_e')O_{L2}u(p_e)][\bar{u}(p_1')\bar{u}(p_2')Mu(p_2)u(p_{1i})] \tag{17}$$

$$\times [\bar{u}(p_{1i})O_1u(p_1)][\bar{u}(p_1)\Gamma_\alpha v(p_2)]\epsilon^\alpha \tag{18}$$

Inputs for Model

- Proton GPDs
- Neutron GPDs
- Proton Form Factors
- Neutron Form Factors
- Deuteron Wavefunction
- Parametrizations of Final State Interactions

$$A_{N,PWIA} \propto (\text{Perturbative Part})(\text{Non-Perturbative Function})(\text{Deuteron WF}) \tag{19}$$

$$A_{N,FSIs} \propto (\text{Perturbative Part}) \int (\text{Non-Perturbative Function})(\text{Deuteron WF}) \times (\text{Amplitude for nucleon-nucleon scattering})$$

$$\sigma \propto |A|^2 \tag{20}$$

Steps for unpolarized cross section

Model Inputs

↓

Amplitudes of $e^- + D \to e^- + p + n + \gamma$ in PWIA

↓

Take into account Final State Interactions

↓

Model unpolarized Cross section and compare to experiment

↓

Look into kinematics where Neutron tagging is suppressed and vice versa

Thanks!

Figure: Work done in collaboration with Wim Cosyn

References

A. Belitsky, A. Radyushkin, Physics Reports **418**(1–6), 1–387 (2005).
DOI 10.1016/j.physrep.2005.06.002.
URL http://dx.doi.org/10.1016/j.physrep.2005.06.002

D. Müller, Nuclear Physics A **755**, 71 (2005).
DOI https://doi.org/10.1016/j.nuclphysa.2005.03.151.
URL https://www.sciencedirect.com/science/article/pii/S0375947405003933.
Proceedings of the 10th International Conference on the Structure of Baryons

W. Cosyn, C. Weiss, Physical Review C **102**(6) (2020).
DOI 10.1103/physrevc.102.065204.
URL http://dx.doi.org/10.1103/PhysRevC.102.065204

J.W. Qiu, Z. Yu, Physical Review D **107**(1) (2023).
DOI 10.1103/physrevd.107.014007.
URL http://dx.doi.org/10.1103/PhysRevD.107.014007

S. Wallon.
Hard exclusive processes (2013).
URL https://arxiv.org/abs/1302.2888

Back Up Slides

EMC Effect

Nucleon parton distributions are changed while bound in nucleus ...

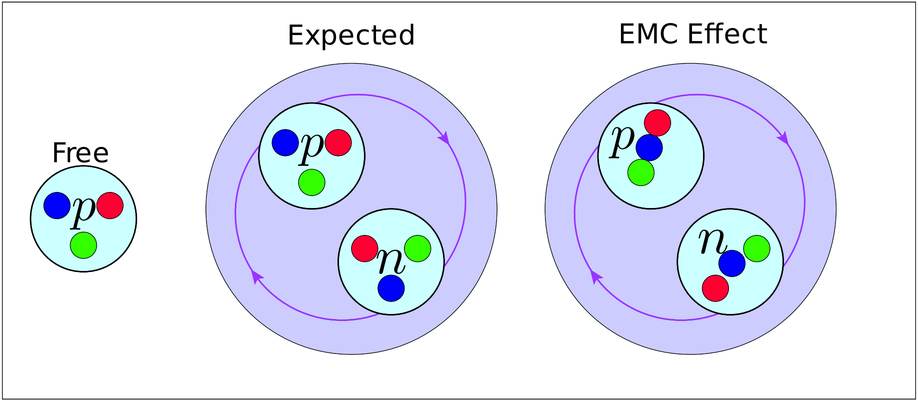


Figure: Dramatized Cartoon Illustration of EMC Effect

Mellin Moments of GPDs

Moments of GPDs [1]

$$\int dx\, x^n \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \bar{q}(0)\gamma^+ q(z^-) = \frac{1}{(P^+)^{n+1}} \bar{q}(0)\gamma^+ (\frac{i}{2}D^+)^n q(0) \quad (21)$$

With $n = 0$ which is first moment we have

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \bar{q}(0)\gamma^+ q(z^-) = \frac{1}{P^+} \bar{q}(0)\gamma^+ q(0) \quad (22)$$

Remember though these are matrix elements for Electromagnetic Form Factors

$$\langle p'|\bar{q}(0)\gamma^+ q(0)|p\rangle = F_1^q(t)\bar{u}(p')\gamma^+ u(p) + F_2^q(t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p) \quad (23)$$

Therefore we have

$$\int H_q(x,\xi,t)dx = F_1^q(t) \quad (24)$$

$$\int E_q(x,\xi,t)dx = F_2^q(t) \quad (25)$$

Second moment and GFFs

$$\int dx\ x \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \bar{q}(0)\gamma^+ q(z^-) = \frac{1}{(P^+)^2}\bar{q}(0)\gamma^+(\frac{i}{2}D^+)q(0) \quad (26)$$

Which the operator $i\gamma^\mu D^\nu = T^{\mu\nu}$ is the Energy Momentum tensor. Parametrize this matrix element with gravitational form factors.

$$\langle p'|\bar{q}(0)T^{\mu\nu}q(0)|p\rangle = \frac{P^\mu P^\nu}{M}A_a(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C_a(t) + Mg^{\mu\nu}\bar{C}_a(t) \quad (27)$$

$$+ \frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_\lambda}{2M}J_a(t) - \frac{P^{[\mu}i\sigma^{\nu]\lambda}\Delta_\lambda}{2M}S_a(t) \quad (28)$$

Therefore

$$\int H_q(x,\xi,t)xdx = A_q(t) + 4\xi^2 C_q(t) \quad (29)$$

$$\int \frac{1}{2}[H_q(x,\xi,t) + E_q(x,\xi,t)]xdx = J_q(t) \quad (30)$$

Coordinate Systems

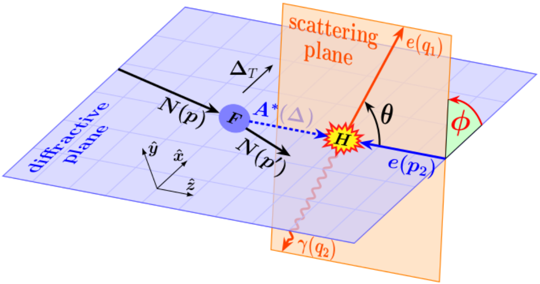


Figure: SDHEP for Nucleon

We look at SDHEP frame where separated into Soft and Hard parts of interaction. More information in Qiu and Yu [4]

- Hard part we have 3 independent 4 momenta Δ, k, k'
- Soft part we have 3 independent 4 momenta Δ, p_d, p_p

Nucleon DVCS

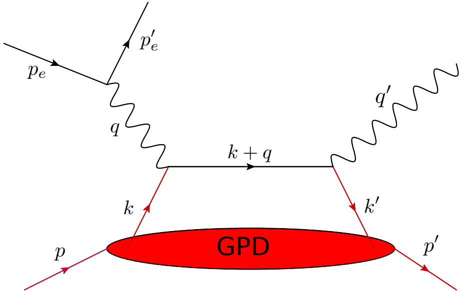


Figure: One of the DVCS diagrams for Nucleon

The Amplitude for this diagram can be written as

$$A = -i \sum_f \int \frac{d^4k}{(2\pi)^4} \; H_{ab} S_{ab} \tag{31}$$

H is the perturbative part of process (hard) and S is nonperturbative (soft) and ab are explicit spinor indices. Also note that $k = (x + \xi)p$

Hard and Soft Parts

Hard part of diagram gives

$$H_{ab} = -i|e|^3 e_f^2 [\epsilon_\rho^*(q')\gamma^\rho_{bc}] \left[\frac{\not{k} + \not{q} + m}{(k+q)^2 - m^2} \gamma^\mu \right]_{ca} \frac{g_{\mu\nu}}{q^2} [\bar{u}_e(p'_e)\gamma^\nu u_e(p_e)] \tag{32}$$

The soft part gives

$$S_{ab} = \int d^4z \, e^{ikz} \langle p' | \bar{q}_b(0) W(0,z) q_a(z) | p \rangle \tag{33}$$

How is the soft part of this process related to GPDs?

Matrix Elements for Soft Part

Looking at the matrix elements that define soft part of diagram

$$S_{ab} = \int d^4z\, e^{ikz} \langle p'|\bar{q}_b(0) W(0,z) q_a(z)|p\rangle \tag{34}$$

We use Fierz Identity [5]

$$\bar{q}_b q_a = \frac{1}{4}\gamma^{\lambda}_{ab}\bar{q}\gamma_\lambda q + \frac{1}{4}(\gamma_5\gamma^\lambda)_{ab}\bar{q}\gamma_\lambda\gamma_5 q + \frac{1}{4}(I)_{ab}\bar{q}q + \frac{1}{4}(\gamma_5)_{ab}\bar{q}\gamma_5 q + \frac{1}{4}\sigma^{\alpha\beta}_{ab}\bar{q}\sigma_{\alpha\beta}q \tag{35}$$

For when we do trace later and knowing odd number of gamma matrix trace=0 we have the following left over elements

$$S_{ab} = \frac{1}{4}\int d^4z\, e^{iz\cdot k}[\gamma^{\lambda}_{ab}\langle p'|\bar{q}(0)\gamma_\lambda W(0,z)q(z)|p\rangle \tag{36}$$

$$+ (\gamma_5\gamma^\lambda)_{ab}\langle p'|\bar{q}(0)\gamma_\lambda\gamma_5 W(0,z)q(z)|p\rangle \tag{37}$$

$$+ \sigma^{\alpha\beta}_{ab}\langle p'|\bar{q}(0)\sigma_{\alpha\beta}W(0,z)q(z)|p\rangle] \tag{38}$$

Leading twist matrix elements

Considering mass of quarks=0 then Leading twist matrix elements are

$$S_{ab} = \frac{1}{4}\int d^4z\, e^{iz\cdot k}[\gamma^-_{ab}\langle p'|\bar{q}(0)\gamma^+ q(z)|p\rangle \tag{39}$$

$$+ (\gamma_5\gamma^-)_{ab}\langle p'|\bar{q}(0)\gamma^+\gamma_5 W(0,z)q(z)|p\rangle \tag{40}$$

Using the leading twist matrix elements, neglecting mass, taking the collinear limit $k^- = k_\perp = 0$ and using Lightcone gauge $A^+ = 0$ so $W(0,z) = 1$

$$A \propto \int dx H(x,\xi)\left[\frac{1}{2}\int \frac{dz^+}{2\pi} e^{ixP^+z^-}\langle p'|\bar{q}(0)\gamma^+ q(z)|p\rangle\right] + ... \tag{41}$$

Where we can get [1],[5]

$$\frac{1}{2}\int \frac{dz^+}{2\pi} e^{ixP^+z^-}\langle p'|\bar{q}(0)\gamma^+ q(z)|p\rangle = \tag{42}$$

$$\frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right] \tag{43}$$