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Nuclear exclusive processes and generalized parton distributions

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Abstract

In this document I report the details of my lectures on generalized parton distributions functions for light-nuclei in Impulse Approximation with some focus on the Light-Front Fock expansion.

Keywords

Generalized parotn distributions, light-front, light nuclei

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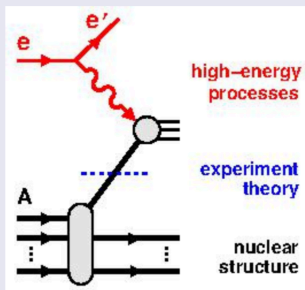

Summer School "Light-ion physics in the EIC era: From nuclear structure to high-energy processes"

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Open questions on Deep Inelastic Scattering processes.

In this section we introduce two still open questions on the Deep Inelastic Scattering (DIS) physics which motivate study o novel processes in particular for nuclear targets.

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Recap of DIS

Here I recap some features of DIS of protons. Some details can be found for example in Refs. 1, 2.

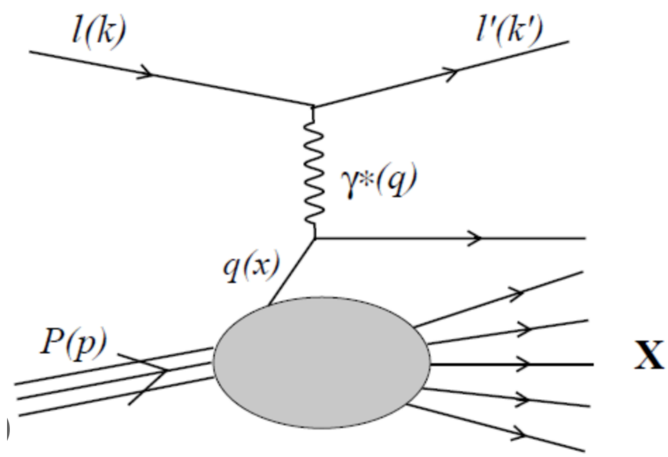

Figure 1. DIS diagram.

The cross-section is given by:

$$\frac{d\sigma}{d\Omega_{k'}dE'} = \frac{\alpha^2}{q^4}\frac{E_k}{E_{k'}}L^{\mu\nu}W_{\mu\nu} \tag{1}$$

where $q = k' - k$, $L^{\mu\nu}$ the *leptonic tensor*:

$$L^{\mu\nu} = \frac{1}{2}\sum_{s,s'}\left[\bar{u}(k',s')\gamma^\mu u(k,s)\right]^*\left[\bar{u}(k,s)\gamma^\nu u(k',s')\right] = \frac{1}{2}\sum_{s,s'}\langle k',s'|J^\mu(0)|k,s\rangle\langle k,s|J^\nu(0)|k',s'\rangle \tag{2}$$

while the *hadronic tensor*:

$$W_{\mu\nu} = \frac{1}{4\pi}\sum_{S,S_X}\int \frac{d^3P_X}{(2\pi)^3E_X}\langle P,S|J^\nu(0)|P_X,S_X\rangle\langle P_X,S_X|J^\mu(0)|PS\rangle\delta(P-P_X+k-k')(2\pi)^4 \tag{3}$$

$$= \frac{1}{4\pi M}\sum_{S,S_X}\int d^4\xi\frac{d^3P_X}{(2\pi)^3E_X}e^{i\xi(q+P-P_X)}\langle P,S|J^\nu(0)|P_X,S_X\rangle\langle P_X,S_X|J^\mu(0)|PS\rangle$$

$$= \frac{1}{4\pi M}\sum_{S,S_X}\int d^4\xi\frac{d^3P_X}{(2\pi)^3E_X}e^{i\xi q}\langle P,S|e^{i\hat{P}\xi}J^\nu(0)e^{-i\hat{P}\xi}|P_X,S_X\rangle\langle P_X,S_X|J^\mu(0)|PS\rangle \tag{4}$$

$$= \frac{1}{4\pi M}\bar{\sum}_S\int d^4\xi e^{i\xi q}\langle P,S|\underbrace{J_\mu(\xi)J_\nu(0)}_{non\ local}|P,S\rangle = \frac{1}{4\pi M}\bar{\sum}_S\int d^4\xi e^{i\xi q}\langle P,S|[J_\mu(\xi),J_\nu(0)]|P,S\rangle. \tag{5}$$

One should notice that for causality:

$$[J_\mu(\xi)J_\nu(0)] = 0 \tag{6}$$

for space-like distances, i.e. $\xi^2 < 0$. Therefore, non trivial solution are found for $\xi^2 \geq 0$. Let us define Light-Cone (LC) coordinates:

$$\xi^\pm = \frac{1}{\sqrt{2}}(\xi_0 \pm \xi_3) \tag{7}$$

therefore:

$$\xi^2 = \xi_0^2 - \xi_3^2 - \vec{\xi}_\perp^2 = (\xi_0-\xi_3)(\xi_0+\xi_3) - \vec{\xi}_\perp^2 = 2\xi_+\xi_- - \vec{\xi}_\perp^2. \tag{8}$$

In general:

$$A\cdot B = A_+B_- + A_-B_+ - \vec{A}_\perp\cdot\vec{B}_\perp \tag{9}$$

hence in the exponential we have:

$$e^{i\xi\cdot q} = e^{i(\xi_+q_- + \xi_-q_+ - \vec{\xi}_\perp\cdot\vec{q}_\perp)}. \tag{10}$$

We can choose to work in this frame: $i)$ $\vec{q}_\perp = \vec{0}_\perp$, $ii)$ $q^2 \to \infty$ and $q_3 < 0$. Within this condition we have that $q_- \to \infty$ and $q_+ << q_-$. The leadint term in the exponential is:

$$e^{i\xi_+q_-}. \tag{11}$$

This object oscillates very fast, thus the maximum contribution is given for $\xi_+ \to 0$. From the causality condition we therefore have:

$$\xi_+ \to 0; \quad \xi \geq 0. \tag{12}$$

But now $\xi^2 = -\vec{\xi}_\perp^2 \geq 0$ the only possibility is that $\vec{x}i_\perp^2 = 0 = \xi = 0$, we prove the LC dominance. In the polarized case one needs to decompose the leptonic and hadronic tensors in a symmetric and anti-symmetric parts and the cross section is now given by the product of the twon anti-symmetric terms:

$$L^{\mu,\nu} = L_S^{\mu,\nu} + L_A^{\mu,\nu} \tag{13}$$

with:

$$L_A^{\mu,\nu} = 2h\varepsilon_{\mu\nu\rho\sigma}k^\rho q^\sigma \tag{14}$$

with h helicity (conserving for $m \sim 0$). While:

$$W_A^{\mu,\nu} = i\varepsilon_{\mu\nu\rho\sigma}q^\rho \left\{ S^\sigma \frac{g_1(x,Q^2)}{M\nu} + \Big[p \cdot qS^\sigma - S \cdot qP\sigma\Big] \right\} \frac{g_2(x,Q^2)}{M^2\nu^2}. \tag{15}$$

In the Bjorken limit $Q^2/(M^2\nu^2) \to 0$, therefore the g_2 can be neglected. And now we have the spin-dependent Structure Functions (SFs). We therefore can compare the unpolarized and longitudinally polarized Parton Distribution Functions (PDFs):

$$\begin{aligned} q(x) &= \frac{1}{4\pi}\int dz_- e^{ixP^+z_-}\langle P,S|\bar{\psi}_q(0)\gamma^+\psi_q(z_-)|P,S\rangle \\ \Delta q(x) &= \frac{1}{4\pi}\int dz_- e^{ixP^+z_-}\langle P,S|\bar{\psi}_q(0)\gamma^+\gamma_5\psi_q(z_-)|P,S\rangle \end{aligned} \tag{16}$$

where $\psi_q(z)$ is the $q-$ quark operator field evaluated at the z position. The relation with the relative SFs:

$$\begin{aligned} F_2(x) &= \sum_q e_q^2 xq(x) \\ g_1(x) &= \frac{1}{2}\sum_q e_q^2\big(\Delta q(x) + \Delta\bar{q}(x)\big). \end{aligned} \tag{17}$$

The interpretation is:

$$\begin{aligned} q(x) &= q_\uparrow(x) + q_\downarrow(x) \\ \Delta q(x) &= q_\uparrow(x) - q_\downarrow(x). \end{aligned} \tag{18}$$

Moreover, the total number of quark of given flavor (charge conservation) is given by:

$$q = \int dx\; q(x) \tag{19}$$

while

$$\Delta q = n_\uparrow - n_\downarrow = \underbrace{\langle P,S|\bar{\psi}_q(0)\gamma^+\gamma_5\psi(0)|P,S\rangle}_{local\ axial\ current}. \tag{20}$$

The proton spin crsis

Some details can be found in Ref. 3 If we can measure the spin-dependent SFs one can access to the spin contriburion of quarks to the proton spin. Let us define the first moments of g_1:

$$\Gamma^p = \int dx\, g_1^p(x) = \frac{4}{9}(\Delta_u + \Delta_{\bar{u}}) + \frac{1}{9}(\Delta_d + \Delta_{\bar{d}}) + \frac{1}{9}(\Delta_s + \Delta_{\bar{s}}) \tag{21}$$

for the neutron we consider isospin symmetry:

$$\Gamma^n = \int dx\, g_1^n(x) = \frac{4}{9}(\Delta_d + \Delta_{\bar{d}}) + \frac{1}{9}(\Delta_u + \Delta_{\bar{u}}) + \frac{1}{9}(\Delta_s + \Delta_{\bar{s}}) \tag{22}$$

Let us define:

$$\begin{aligned} g_A^3 &= (\Delta_u + \Delta_{\bar{u}}) - (\Delta_d + \Delta_{\bar{d}}) \\ g_A^8 &= (\Delta_u + \Delta_{\bar{u}}) + (\Delta_d + \Delta_{\bar{d}}) - 2(\Delta_s + \Delta_{\bar{s}}) \\ g_A^0 &= (\Delta_u + \Delta_{\bar{u}}) + (\Delta_d + \Delta_{\bar{d}}) + (\Delta_s + \Delta_{\bar{s}}) \end{aligned} \tag{23}$$

therefore:

$$\Gamma^{p(n)} = \frac{1}{9}\left[\pm\frac{3}{4}g_A^3 + \frac{1}{4}g_A^8 + g_A^0\right]. \tag{24}$$

The axial currents g_A^3 and g_A^8 can be measured in the baryon β decay:

$$\begin{aligned} g_A^3 &= 1.2670 \pm 0.0035 \\ g_A^8 &= 0.585 \pm 0.025 \end{aligned} \tag{25}$$

Therefore one would get:

$$g_A^0 = g_A^8 + 3(\Delta s + \Delta \bar{s}) \tag{26}$$

therefore if we neglect the strange (or is very difficult to have polarized strange quarks in the proton) one would expect that

$$g_A^0 \sim g_A^8 \sim 0.6 \tag{27}$$

let us remind that:

$$g_A^0 \sim 2\langle S_Z^{quark}\rangle = 2\frac{1}{2}\sum_q \int dx\, \Delta q(x) \tag{28}$$

i.e. the spin contribution of quarks in the proton. Then data came and $g_A^0 \sim 0$ this is *spin crisis*, because any model suggested that this contribution should be dominant one. Now, precise data are consistent with 30%. Therefore, in order to describe the spin structure of hadron one needs to access other contributions as the orbital angular momentum of of quarks. To this aim we need new distributions such as Generalized Parton Distribution Functions (GPDs) extractedn in exclusive processes.

The nuclear EMC effect

This second topic is related to a nuclear problem when a phenomenological study started to test QCD and nuclear theory. The experimental observable providing the most direct test of nuclear effects on the parton distributions of bound nucleons, with respect to the free ones, is the ratio

$$R(x) = \frac{F_2^A(x)}{F_2^d(x)}\,, \tag{29}$$

where F_2^A is the structure function of a nucleus with A nucleons and F_2^d is referred to the deuterium nucleus, the deuteron. One should notice that the Bjorken variable x:

$$x = \frac{Q^2}{2M\nu} = \frac{M_A}{M}\frac{Q^2}{2M_A\nu} \tag{30}$$

where M_A (M) is the nucleus (nucleon) mass, ranges, for a nuclear target, between 0 and $M_A/M \sim A$. At a first sight, one could think that the parton structure of the bound nucleon is only slightly modified by the nuclear dynamics. Indeed, the *average* binding energy of a nucleon in the nucleus is of the order of MeV, and its *average* momentum is at most a few tents of MeV. In other words, the ratio Eq. (29) should be very close to one, for any x. It is not surprising therefore that, when the European Muon Collaboration at CERN published the data shown in Figure 2 4 for the Iron (A=56) target, showing effects as big as 15 % with a complicated behavior, a lot of interest and excitement arose. It seemed that conventional dynamics could not explain anything like that. Many authors tried to suggest exotic scenarios where, for example, partons were shared among different nucleons in the nuclear medium, or the QCD evolution equation themselves were different for nuclear parton distributions. The reader can find a good report on the status of the present understanding of this effect, named EMC effect after its discovery, in Ref. 5. After more than 30 years and thousands of published papers on this topic, one can summarize the situation as follows:

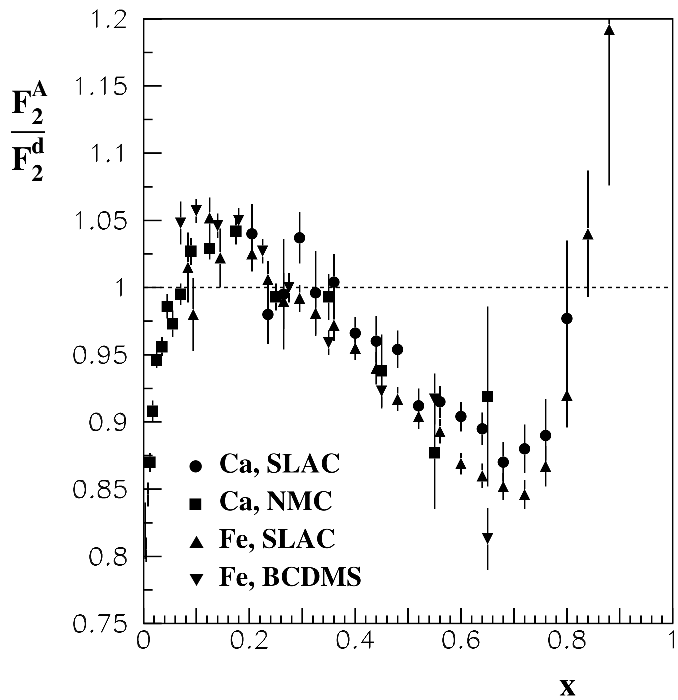

Figure 2. EMC effect shown through the ratio $R(x)$ 5.

- the behavior in the region $x < 0.2$, the so called "shadowing" region, which can be shown to describe "coherent diffusion", i.e., interactions involving partons belonging to different nucleons, is nowadays basically understood; these kind of effects are not discussed in the present work and will not be described in more detail;

- the region $0.2 < x < 0.8$, where the effect grows up logarithmically with A and depends weakly on Q^2, shows an evident minimum. This region is the genuine EMC effect, still to be really understood. In this region, it can be shown that the DIS process involves distances $d < 1fm$, smaller then the dimensions of the nucleons, so in this region the incoherent diffusion is dominant and the virtual photon interacts directly with the partons, whose momentum distributions can be modified by the nuclear medium.
- the region $x > 0.8$, where the behavior is easily understood kinematically (the deuteron parton distributions are defined up to $x = 2$).

So here is discussed now the most interesting part, the "EMC region". Two kind of modifications can be distinguished:

i) conventional Nuclear Physics effects, i.e., the ones described by realistic nuclear wave functions;

ii) exotic effects, invoking 6 quark clusters, density dependent nucleon structure, presence of other degrees of freedom to be taken into account in the nuclear Hamiltonian, with respect to those of the usual Schrödinger equation approach.

Soon after the discovery of the EMC effect, it was shown that the first kind of effects, if properly considered in a realistic framework, which is possible at least for light nuclear targets, *can describe* the EMC region, *at least in part*. All the different exotic mechanisms, with a proper set of parameters, can explain the EMC region. The problem is that these approaches are not microscopic ones, and that the available measurements cannot distinguish between them.

So, after more than 40 years, the EMC effect lacks a clear explanation. The only possibility to really understand the reaction mechanism of hard electromagnetic scattering off nucleus, is to go beyond DIS, i.e., using SiDIS or exclusive DIS. The exclusive processes, aiming at measuring nuclear GPDs, are the ones of interest in this work and will be thoroughly described in the next chapter.

Generalized Parton Distributions

For this section, most of the details can be found in this review 6.

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From the previous discussion, it is clear that processes beyond DIS can help dramatically to explain several aspects of hadron structure. A typical process, exclusive in nature, is *Deeply Virtual Compton Scattering* (DVCS), which is given in Fig.2.3. It is a generalization of DIS, with an extra real photon, so in the final state the momentum of the target changes. It is necessary to generalize the *parton distribution* and in this way GPDs are introduced in Refs.7, 8. Here is fixed the kinematics of the process thinking to Fig. 3. P and P' are the 4-momenta of the initial and of the final state of the target, k and k' those of the parton. The process is therefore: $eN \longrightarrow e'N'\gamma'$. $\Delta = P' - P$ is the momentum transferred to the target with $t = \Delta^2$. Now the considered frame is the Infinite Momentum Frame (IMF), so that the target momentum is along the z-axis and it is natural to describe the process through the light-cone components of the momenta:

$$P^{\pm} = \frac{1}{\sqrt{2}}\left(P^0 \pm P^3\right) \tag{31}$$

Defining the 4-vector:

$$\overline{P} = \frac{(P + P')}{2}\,, \tag{32}$$

another variable can be introduced :

$$\xi = \frac{P^+ - P'^+}{P^+ + P'^+} = -\frac{\Delta^+}{2\overline{P}^+}\,, \tag{33}$$

which is called *skewedness* end its range of definition is $[-1, 1]$. In this kind of processes there are two momentum transfers: the virtual photon momentum $Q^2 = -q^2$ which is very high, to distinguish details of the target, and $t = \Delta^2$, the momentum transferred between the initial and the final state of the nucleon, which has to be small because the cross-section of the process strongly decreases by increasing the momentum transfer. In other words, if the momentum transfer is quite high, then it is very difficult that the target will not break up. Now the quark plus momentum can be written (all the following formalism has been taken from Ref. 6):

$$\begin{aligned} k^+ &= (x+\xi)\overline{P}^+\,, \\ k'^+ = (k+\Delta)^+ &= (x-\xi)\overline{P}^+\,. \end{aligned} \tag{34}$$

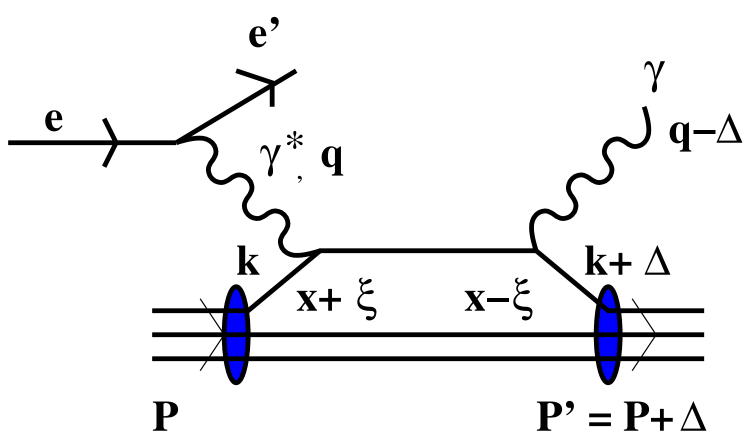

Figure 3. The DVCS processes.

These relations are shown in Fig. **??**:

In general, the variable x, which is the parton fraction of plus momentum, is different from x_{bj}.
It is possible to classify the different dynamics described by GPDs through the regions of definition of x:

- $x \in [\xi, 1]$:
  all the momentum fractions in the initial and final state are positive, so that an emission and a re-absorption

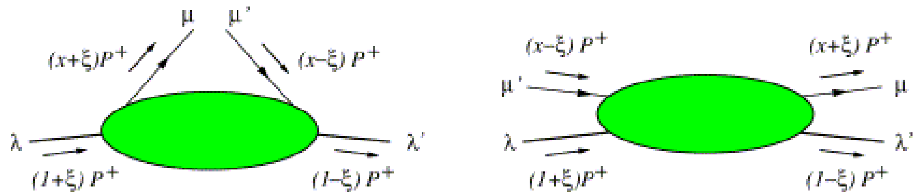

Figure 4. Definition of GPDs kinematical variables.

of a quark is described. This sector is called DGLAP region, because here the pQCD evolution equations are the ones of Ref. 9, 10, 11.

- $x \in [-\xi, \xi]$:
  $x + \xi > 0$ but $x - \xi < 0$. The quark reabsorbed by the nucleon has negative momentum, so it is an antiquark with positive momentum which is emitted together with a quark of momentum $x + \xi$. So there is an emission of pair of quark and antiquark of the "sea". This sector is called ERBL region, because here the pQCD evolution equations are the ones of Ref.12, 13.
- $x \in [-1, -\xi]$:
  All the momentum fractions are negative so there is an emission and a reabsorption of an antiquark with positive momentum, this region is called ERBL.

These three cases are represented in the figure 5:

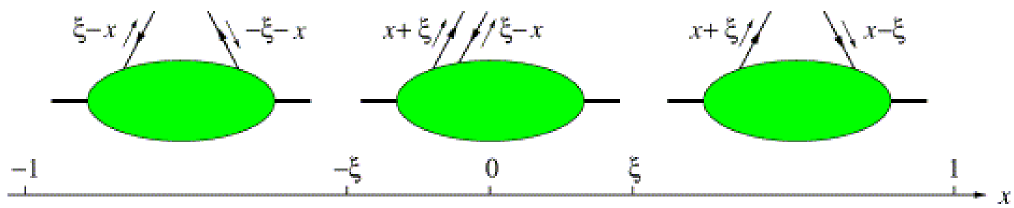

Figure 5. Different interpretations of GPDs in different kinematical regions

The GPDs can be now introduced using the light-cone correlator *.

$$F^A_{\lambda,\lambda'} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P', \lambda' | \overline{\psi}(-\frac{1}{2}z)\gamma^+\psi(\frac{1}{2}z)|P, \lambda\rangle\big|_{z^+=\vec{z}_\perp=0} \tag{35}$$

where A label the kind of target of target, λ,λ' are helicities in the initial and the final state respectively. It is important to remark that now $F^A_{\lambda,\lambda'}$ is a nondiagonal matrix so it can not be interpreted as number density as in the DIS process. It is possible to write $F^A_{\lambda,\lambda'}$ as follows:

$$F_{\lambda,\lambda'}(x,\xi,t) = \frac{1}{2P^+}\Big[H^q(x,\xi,t)\overline{u}(P',\lambda')\gamma^+u(P,\lambda) + E^q(x,\xi,t)\overline{u}(P',\lambda')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(P,\lambda)\Big] + .. \tag{36}$$

where H^q and E^q are the dominant *leading-twist* GPDs referred to the quark of flavor q, the dots (..) refer to sub-dominant (*higher-twist*) objects. They parametrize the hadronic part of the diagram in the non diagonal case. The helicity-dependent GPDs can be defined similarly, but they will not be discussed in the following.

*one should notice that, as in any hard process, in DVCS light-like distance are probed,i.e., $z^+ = \vec{z}_\perp = 0$

Extraction of GPDs from the correlator

In order to extra the GPDs from the collator, which is the quantity that usually are evaluated within some models. To this use has been made of the Gordon identity in the Breit frame where:

$$P_{in} = P - \frac{\Delta}{2}$$
$$P_{fin} = P + \frac{\Delta}{2}. \tag{37}$$

We shortly define: $u = u(P_{in}, \lambda)$ and $\bar{u}' = \bar{u}(P_{fin}, \lambda')$ In this framework:

$$\bar{u}'\gamma^+ u = \bar{u}' \left[\frac{P^+}{M} + \frac{i}{2M}\sigma^{+\mu}\Delta_\mu \right] u \tag{38}$$

from which obtain:

$$\frac{i}{2M}\Delta_\mu \bar{u}'\sigma^{+\mu} u = \bar{u}'\gamma^+ u - \frac{P^+}{M}\bar{u}' u. \tag{39}$$

. Therefore the LC correlator can be rewritten:

$$F_{H'H} = \frac{1}{2P^+}\left[(H+E)\bar{u}'\gamma^+ u - E\frac{P^+}{M}\bar{u}' u \right]. \tag{40}$$

Now one needs to use the free spinors in order to evaluate the matrix elements. We consider the following convention:

$$u(P,\uparrow) = \frac{1}{\sqrt{P^+}}\left(P^+\hat{I} + m\gamma^0 + \vec{P}_\perp \cdot \vec{a}_\perp\right)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix} u(P,\downarrow) \quad = \frac{1}{\sqrt{P^+}}\left(P^+\hat{I} + m\gamma^0 + \vec{P}_\perp \cdot \vec{a}_\perp\right)\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix} \tag{41}$$

where:

$$a_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}. \tag{42}$$

One therefore can prove that:

$$\bar{u}'\gamma^+ u = \begin{cases} 2P^+\sqrt{1-\xi^2} & \text{if } H = H' \\ \\ 0 & \text{if } H \neq H' \end{cases} \tag{43}$$

Moreover:

$$\bar{u}' u = \begin{cases} \frac{2M}{\sqrt{1-\xi^2}} & \text{if } H = H' \\ \\ -\frac{\Delta_x + i\Delta_y}{\sqrt{1-\xi^2}} & \text{if } H \neq H' \end{cases} \tag{44}$$

With this ingredients one can obtain:

$$F_{-+} = \frac{1}{2P^+}\left[E\frac{P^+}{M}\left(\frac{\Delta_x + i\delta_y}{\sqrt{1-\xi^2}}\right)\right] = E\frac{\Delta_x + i\delta_y}{2M\sqrt{1-\xi^2}} \tag{45}$$

while:

$$F_{++} = (H+E)\sqrt{1-\xi^2} - \frac{E}{\sqrt{1-\xi^2}}. \tag{46}$$

GPD properties: Forward limit

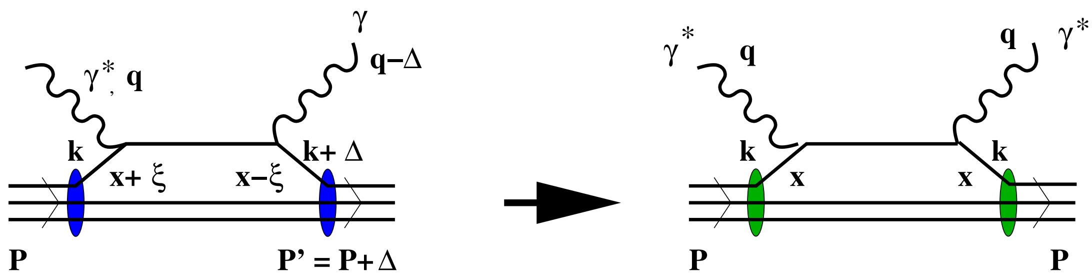

Figure 6. DVCS process and its forward limit.

When $t = 0$ and $P = P'$, $H = H'$, studying Feynman diagrams one can see that the the matrix element expressed by $F^A_{H,H'}$ has to reduce to the parton distribution. This yields a constraint for the GPDs, in particular:

$$\begin{aligned} H^q(x,0,0) &= q(x), && \text{for } x > 0, \\ H^q(x,0,0) &= -\bar{q}(-x), && \text{for } x < 0 \end{aligned} \tag{47}$$

It is very important to notice that there is not a condition like this for E^q because the Δ^α term in Eq. (36) vanishes, so there is no experimental way to access the information enclosed in E in the case of $t = \xi = 0$. It will be shown that E^q carries information about the orbital angular momentum.

GPD properties: first moment

Let us consider the following integral:

$$\begin{aligned} \int_{-1}^{1} dx\, F^A_{\lambda,\lambda'}(x,\xi,t) &= \frac{1}{2P^+}\langle P_{fin},\lambda'|\bar{\psi}(0)\gamma^+\psi(0)|P_{in},\lambda\rangle \\ &= \frac{1}{2P^+}\bar{u}'\left[\int_{-1}^{1} dx\, H^A(x,\xi,t)\gamma^+ + \int_{-1}^{1} dx\, E^A(x,\xi,t)\frac{i}{2M}\sigma^{+\mu}\Delta_\mu\right]u \end{aligned} \tag{48}$$

Recalling that the above local matrix element is parametrized in terms of elastic Form Factors (FFs):

$$\langle P_{fin},\lambda'|\bar{\psi}(0)\gamma^+\psi(0)|P_{in},\lambda\rangle = \bar{u}'\left[F_1(t)\gamma^+ + F_2(t)\frac{i}{2M}\sigma^{+\mu}\Delta_\mu\right]u \tag{49}$$

Therefore we obtain:

$$\int_{-1}^{1} dx\, H^A(x,\xi,t) = F_1(t) \tag{50}$$
$$\int_{-1}^{1} dx\, E^A(x,\xi,t) = F_2(t).$$

The dependence on ξ disappears due to Lorentz invariance (*polinomiality properties* that will not discussed here, see Ref. 6).

GPD properties: Moments of GPDs

From polynomiality one gets:

$$\int_{-1}^{1} dx\, xH(x,\xi,t) = A(t) + \xi^2 D(t) \tag{51}$$
$$\int_{-1}^{1} dx\, xE(x,\xi,t) = B(t) - \xi^2 D(t) \tag{52}$$

where $A(t),\ B(t)$ and $D(t)$ are Gravitational Form Factors (gFFs). In order to see that let us evaluate:

$$\begin{aligned}\int_{-1}^{1} dx\, x^m F^A_{\lambda,\lambda'}(x,\xi,t) &= \frac{1}{2}\int_{-1}^{1} dx\, x^m \int dz_- \frac{e^{ixP^+z_-}}{2\pi} \underbrace{\langle P', H'|\bar\psi(0)\gamma^+\psi(z_z)|P, H\rangle}_{J(z_-)} \\ &= \frac{1}{2}\int_{-1}^{1} dx \int dz_- \frac{1}{(iP^+)^m}\frac{d^m}{dz_-^m}\left[\frac{e^{ixP^+z_-}}{2\pi}\right] J(z_-) \\ &= \frac{1}{2}\int_{-1}^{1} dx \int dz_- \frac{1}{(iP^+)^m}\frac{d}{dz_-}\left[\frac{d^{m-1}}{dz_-^{m-1}}\frac{e^{ixP^+z_-}}{2\pi}\right] J(z_-)\end{aligned} \tag{53}$$

if we assume that

$$\int dz_-\, J(z_-) = 0, \tag{54}$$

then we can integrate by parts:

$$\begin{aligned}\int_{-1}^{1} dx\, x^m F^A_{\lambda,\lambda'}(x,\xi,t) &= -\frac{1}{2}\int_{-1}^{1} dx \int dz_- \frac{1}{(iP^+)^m}\left[\frac{d^{m-1}}{dz_-^{m-1}}\frac{e^{ixP^+z_-}}{2\pi}\right]\frac{d}{dz_-}J(z_-) \qquad (55) \\ &= \frac{1}{2(iP^+)^m}(-1)^m \int_{-1}^{1} dx \int dz_-\, \frac{e^{ixP^+z_-}}{2\pi}\frac{d^m}{dz_-^m}J(z_-) \\ &= \frac{i^m}{2(P^+)^m}\int dz_-\, \delta(P^+z_-)\frac{d^m}{dz_-^m}J(z_-) \\ &= \frac{i^m}{2(P^+)^m}\frac{d^m}{dz_-^m}\langle P', H'|\bar\psi(0)\gamma^+\psi(z_-)|P, H\rangle\Big|_{z=0} \\ &= \frac{1}{2(P^+)^{m+1}}\langle P', H'|\bar\psi(0)\gamma^+(i\overleftrightarrow{\partial}^+)^m\psi(0)|P, H\rangle \qquad (56)\end{aligned}$$

This is one of the matrix elements of the Energy Momentum Tensor (EMT).

Energy Momentum Tensor

A nice review on the relation between EMT and mechanical properties of hadrons can be found in Ref. 14. Let us consider the quark EMT:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{\psi}(x) i \left[\gamma^{\mu} \overleftrightarrow{\partial}^{\nu} + \gamma^{\nu} \overleftrightarrow{\partial}^{\mu} \psi(x) \right]. \tag{57}$$

Therefore $T^{++}(0)$ is related to the integral of GPDs, so that one needs to parametrize de EMT. In the Bright frame:

$$\langle P + \frac{\Delta}{2}| T^{\mu\nu}(0) | P - \frac{\Delta}{2}\rangle = \bar{u}' \left[A(t) \frac{1}{2}\left(\gamma^{\mu} P^{\nu} + \gamma^{\nu} P^{\mu}\right) + B(t) \frac{i}{4M}\left(\sigma^{\mu\alpha} P^{\nu} + \sigma^{\nu\alpha} P^{\mu}\right)\Delta_{\alpha} + D(t) \frac{\Delta^{\mu}\Delta^{\nu} - \eta^{\mu\nu}\Delta^2}{4M} \right] u. \tag{58}$$

For the momentum and the Ji's (spin) sum rules, we can neglect the last term (which is fundamental for other properties of hadrons). By using again again Gordon identity:

$$\langle P + \frac{\Delta}{2}| T^{\mu\nu}(0) | P - \frac{\Delta}{2}\rangle = \bar{u}' \left[A(t) \frac{P^{\mu}P^{\nu}}{M} + [A(t) + B(t)] \frac{i}{4M}(P^{\mu}\sigma^{\nu\alpha} + P^{\nu}\sigma^{\mu\alpha})\Delta_{\alpha} \right] u. \tag{59}$$

In order to prove the sum rule to interpret the role of these gFFs, let us introduce the following definitions. We start with:

$$\langle \int d\vec{r}\, \hat{O}(r) \rangle = \frac{\langle P | \int d\vec{r}\, \hat{O}(r) | P \rangle}{\langle P | P \rangle} \tag{60}$$

where in principle:

$$\langle P | P \rangle = 2(2\pi)^3 P^0 \delta^3(\vec{0}) \tag{61}$$

of course can be useful to consider the limit:

$$\langle \int d\vec{r}\, \hat{O}(r) \rangle = \lim_{\Delta \to 0} \frac{\langle P' | \int d\vec{r}\, \hat{O}(r) | P \rangle}{\langle P' | P \rangle} = \lim_{\Delta \to 0} \frac{\int d\vec{r}\, \langle P' | \hat{O}(r) | P \rangle}{\langle P' | P \rangle} \tag{62}$$

$$= \lim_{\Delta \to 0} \frac{\int d\vec{r}\, \langle P' | e^{-i\hat{P}\cdot r} \hat{O}(0) e^{i\hat{P}\cdot\vec{r}} | P \rangle}{\langle P' | P \rangle} = \lim_{\Delta \to 0} \int d\vec{r}\, e^{i\vec{r}\cdot\vec{\Delta}} \frac{\langle P' | \hat{O}(0) | P \rangle}{\langle P' | P \rangle}$$

$$= \lim_{\Delta \to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \frac{\langle P' | \hat{O}(0) | P \rangle}{2(2\pi)^3 P^0 \delta^{(3)}(\vec{\Delta})} = \lim_{\Delta \to 0} \frac{\langle P' | \hat{O}(0) | P \rangle}{2P^0} \tag{63}$$

The other quantity that we need to evaluate is:

$$\langle \int d\vec{r}\, r_i \hat{O}(r) \rangle = \lim_{\Delta \to 0} \frac{1}{2P^0} \left[-i \frac{\partial}{\partial \Delta^i} \langle P' | \hat{O}(0) | P \rangle \right]. \tag{64}$$

We start with **Momentum Sum Rule**. One has to evaluate:

$$\langle \hat{P}^{\mu} \rangle = \lim_{\Delta \to 0} \frac{\langle P + \frac{\Delta}{2} | T^{0\mu} | P - \frac{\Delta}{2} \rangle}{2P^0} = \lim_{\Delta \to 0} A(t) \frac{P^0 P^{\nu}}{2P^0 M} \bar{u}' u = P^{\nu} A(0) \tag{65}$$

therefore $A(0) = 1$. Of course, in general these quantities depend on the parton flavor, hence the full sum rule is:

$$\sum_q A_q(0) = 1. \tag{66}$$

We continue with the **Ji's Sum Rule** (spin sum rule). We simplify the procedure by considering the rest frame: $P^\mu = (m, \vec{0})$ In this case we have:

$$\langle \hat{J}_z \rangle = \lim_{\Delta \to 0} \frac{\langle P + \frac{\Delta}{2} | \int d\vec{r}\, \varepsilon_{ik3} r^i T^{0k}(r) | P - \frac{\Delta}{2} \rangle}{\langle P + \frac{\Delta}{2} | P - \frac{\Delta}{2} \rangle} = \varepsilon_{ik3} \lim_{\Delta \to 0} \frac{-i}{2M} \left[\frac{\partial}{\partial \Delta^i} \langle P + \frac{\Delta}{2} | T^{0j}(0) | P - \frac{\Delta}{2} \rangle \right] \tag{67}$$

if we consider $\Delta = (\Delta_0, \Delta_x, 0, 0)$. Then:

$$\langle \hat{J}_z \rangle = \lim_{\Delta \to 0} \frac{-i}{2M} \frac{\partial}{\partial \Delta_x} \langle P + \frac{\Delta}{2} | T^{02}(0) | P - \frac{\Delta}{2} \rangle. \tag{68}$$

Let us consider the on-shell condition:

$$\left(P \pm \frac{\Delta}{2} \right)^2 = M^2 = M^2 + P + \frac{\Delta^2}{4} \pm \Delta \cdot P \tag{69}$$

Therefore we get the condition:

$$\Delta^2 = \Delta_0^2 - \Delta_x^2 = \mp 4M\Delta_0 \tag{70}$$

For which we get:

$$\Delta_0 = \pm 2M \mp \sqrt{4M^2 + \Delta_x^2} \tag{71}$$

in this way if $\Delta_x \to 0$ then also $\Delta_0 = 0$. In addition, one can prove that:

$$\bar{u}' u = \frac{-\Delta_0^2 + \Delta_x^2 + 16M^2}{2\sqrt{4M - \Delta_0}\sqrt{4M - \Delta_0}} \tag{72}$$

this term goes to 0 after the derivation and sending $\Delta \to 0$. Therefore, the only non trivial term which contributes is given by (recall that $\vec{P} = 0$):

$$\bar{u}' \left\{ [A(t) + B(t)] \frac{i}{4M} P^0 \sigma^{2\nu} \Delta_\nu \right\} = \bar{u}' \left\{ [A(t) + B(t)] \frac{i}{4M} \left(M\sigma^{20}\Delta_0 - M\sigma^{21}\Delta_x \right) \right\} u \tag{73}$$

since both $\Delta_0 \to 0$ and $\Delta_x \to 0$, the only chance to have a non-zero result is that the derivative acts the term proportional to Δ_x. Finally:

$$\langle \hat{J}_z \rangle = -\frac{i}{2M}\frac{i}{2M}[A(0)+B(0)]M\bar{u}\sigma^{12}u = [A(0)+B(0)]\frac{1}{4M^2}M2M = [A(0)+B(0)]\frac{1}{2} = \frac{1}{2} \qquad (74)$$

So that:

$$\sum_q A_q(0) + B_q(0) = 1 = 2J(0) = \int dx\, x[H(x,0,0)+E(x,0,0)]. \qquad (75)$$

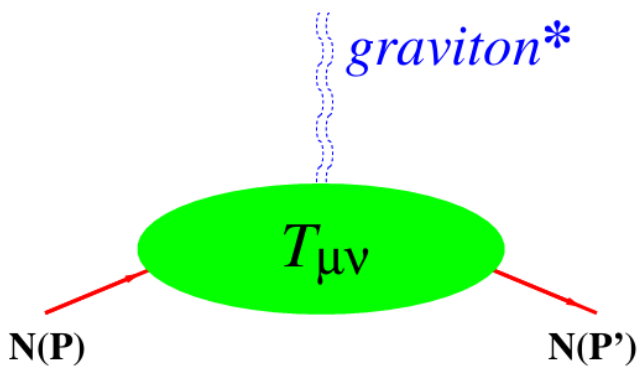

Figure 7. Schematic graviton interaction with matter.

GPDs and hadron tomography

The main details here presented can be found in Ref. 15. In order to get a probabilistic interpretation let us consider GPDs at in the NR limit.

We start from the correlator:

$$f_\xi(x,t) = \int \frac{dz_-}{4\pi} e^{ixz_-P^+} \langle P'|\bar{\psi}(0)\gamma^+\psi(z_-)|P\rangle \qquad (76)$$

We consider the wave-packet:

$$|\psi\rangle = \int d\vec{p}\, \frac{\psi(\vec{p})}{\sqrt{2(2\pi)^3 E_p}} |\vec{p}\rangle \qquad (77)$$

with $E_p = \sqrt{m^2+p^2}$ and the normalization condition is:

$$\langle P'|P\rangle = 2E_p\, \delta^3(\vec{P}' - \vec{P}) \qquad (78)$$

moreover (we consider a scalar particle for simplicity):

$$\langle \vec{p}'|\rho(0)|\vec{p}\rangle = (E_p + E_p')F(q') \qquad (79)$$

with $q = p' - p$. We evaluate the following quantity:

$$
\begin{aligned}
\mathcal{F}_\psi(q^2) &= \int d\vec{z}\, e^{-i\vec{q}\cdot\vec{z}} \langle\psi|\rho(\vec{z})|\psi\rangle = \int d\vec{z}\, e^{-i\vec{q}\cdot\vec{z}} \int \frac{d\vec{p}\, d\vec{p}'}{2(2\pi)^3} \frac{\psi(p)}{\sqrt{E_p}} \frac{\psi^*(p')}{\sqrt{E_{p'}}} \langle\vec{p}'|\rho(\vec{z})|\vec{p}\rangle \\
&= \int d\vec{z}\, e^{-i\vec{q}\cdot\vec{z}} \int \frac{d\vec{p}\, d\vec{p}'}{2(2\pi)^3} \frac{\psi(p)}{\sqrt{E_p}} \frac{\psi^*(p')}{\sqrt{E_{p'}}} \langle\vec{p}'|e^{i\hat{P}\cdot\vec{z}}\rho(0)e^{-i\hat{P}\cdot\vec{z}}|\vec{p}\rangle \\
&= \int d\vec{z} \int \frac{d\vec{p}d\vec{p}'}{2(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{z}} e^{i(\vec{p}'-\vec{p})\cdot\vec{z}}\, \frac{\psi(p)}{\sqrt{E_p}} \frac{\psi^*(p')}{\sqrt{E_{p'}}} \langle\vec{p}'|\rho(0)|\vec{p}\rangle = \frac{1}{2}\int d\vec{p}\, \frac{\psi(p)}{\sqrt{E_p}} \frac{\psi^*(p+q)}{\sqrt{E_{p+q}}} \langle\vec{p}+\vec{q}|\rho(0)|\vec{p}\rangle \\
&= \frac{1}{2}\int d\vec{p}\, \frac{\psi(p)}{\sqrt{E_p}} \frac{\psi^*(p+q)}{\sqrt{E_{p+q}}} (E_p + E_{p+q}) F(q^2)
\end{aligned} \tag{80}
$$

We remind that $q^2 = (E_p - E_{p+q})^2 - \vec{q}^2$, moreover in the NR limit we have:

$$
\begin{aligned}
\frac{E_p + E_{p+q}}{2\sqrt{E_p E_{p+q}}} &= \frac{\sqrt{m^2+\vec{p}^2} + \sqrt{m^2 + (\vec{p}+\vec{q})^2}}{2(m^2+\vec{p}^2)^{1/4}(m^2+(\vec{p}+\vec{q})^2)^{1/4}} = \frac{m\left(\sqrt{1+\frac{\vec{p}^2}{m^2}}\right) + \left(\sqrt{1+\frac{(\vec{p}+\vec{q})^2}{m^2}}\right)}{2m\left[\left(1+\frac{\vec{p}^2}{m^2}\right)\left(1+\frac{\vec{p}^2}{m^2}\right)\right]^{1/4}} \\
&\sim \frac{2+\frac{\vec{p}^2}{2m^2}+\frac{\vec{p}^2}{2m^2}}{2\left[1+\frac{\vec{p}^2}{4m^2}+\frac{\vec{p}^2}{4m^2}\right]} = 1
\end{aligned} \tag{81}
$$

finally:

$$
\mathcal{F}_\psi(q^2) = F(q^2)\int d\vec{p}\, \psi^*(\vec{p}+\vec{q})\psi(\vec{p}) \sim F(q^2) \tag{82}
$$

if the package is localized in coordinate space (so broad in momentum space). So in the NR as long as you have this wave packets, the FT of your mean density is the form factor. If ones include relativistic corrections, then we have extra terms depending on these factors:

i) $\vec{q}\cdot\vec{p}/E_p^2$
ii) q^2/E_p^2.

Therefore we needs some extra-conditions:

- if $\vec{q}$ is finite then we need $E_p \to \infty$
- if $\vec{p} \to \infty$ then for the first term, in order to remove that term we need to request that, assuming again $\vec{q} = (0, 0, q_z)$ then $\vec{p}(p_x, p_y, 0)$.

in other words the IMF. So the proton is moving very fast in the transverse plane w.r.t. $\vec{q}$. Therefore, qualitatively, in the GPDs case we need to require that $\Delta^+ = 0 = \xi$. *Why IMF is so nice?*. Let us consider the NR energy:

$$
E = \sum_i \frac{\vec{p}_i^2}{2m_i} + Binding \tag{83}
$$

In the relativistic case we have in the free case:

$$E = \sum_i \sqrt{m_i^2 + \vec{p}_i^2} = \sum_i m_i \sqrt{1 + \frac{\vec{p}_i^2}{m^2}} \longrightarrow \underbrace{\sum_i m_i}_{shift} + \sum_i \frac{\vec{p}_i^2}{2m_i} \tag{84}$$

Let us separate the longitudinal and transverse parts and we make a longitudinal boost where the z component of the momentum is almost infinity:

$$\begin{aligned} E &= \sum_i \sqrt{m_i^2 + \vec{p}_{\perp,i}^2 + p_{z,i}^2} = \sum_i \sqrt{p_{z,i}^2 + \underbrace{m_i^2 + \vec{p}_{\perp,i}^2}_{as\ in\ rest}} = \sum_i p_{z,i} \sqrt{1 + \frac{m_i^2 + \vec{p}_{\perp,i}^2}{p_{z,i}^2}} \\ &\sim \underbrace{\sum_i p_{z,i}}_{(1)} + \underbrace{\sum_i \frac{\vec{p}_{\perp,i}^2 + m_i^2}{2p_{z,i}}}_{(2)} \end{aligned} \tag{85}$$

So now:

i) can be arbitrary big and it is like inertia (eq. of motion will not change)
ii) The evolution operator will be the conjugate to

$$\frac{\vec{p}_{\perp,i}^2 + m_i^2}{2p_{z,i}} \tag{86}$$

therefore if p_z is big we have slow changes.

Therefore in IMF we have a relativistic Physics which is NR in the transverse part. Very useful.

Cross-section

For this section and convention please consider the following reference 16. For the cross-section, the DVCS is one of the possible diagrams that contribute to the final state (electron+photon+proton). The dominant term is the Bethe-Hidler process:

From the previous discussion it is clear that GPDs are very useful to understand the parton structure of the nucleon. Nevertheless the experimental situation is quite unsatisfactory, due to several problems:

. the cross-section for this event is very small, of the order $\left(10^{-2}\frac{nb}{Gev^4}\right)$, this happens because the scattered electron and the real photon must measured at the same time and also the target should not be broken, despite the fact that Q^2 is very high.
. DVCS events can be confused with Bethe-Heitler events because they have the some initial and final state (see Figure:8), with the last process dominant for high Q^2. The observed cross-section can be sketched as follows:

$$\begin{aligned} d\sigma &\sim |T^2| \sim \left|T_{BH} + T_{DVCS}\right|^2 = \left|T_{BH}\right|^2 + \left|T_{DVCS}\right|^2 + I \\ &\sim \left|T_{BH}\right|^2 + I. \end{aligned} \tag{87}$$

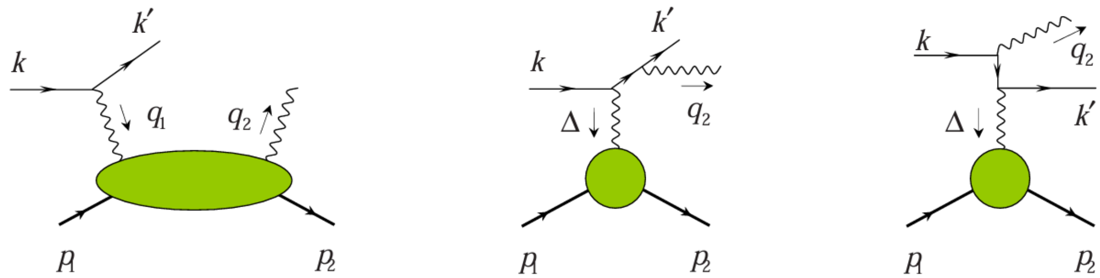

Figure 8. The processes contributing to the reaction $A(e, e'\gamma)A$: DVCS (a) and BH (b)

Usually asymmetries of the cross-sections are measured in order to eliminate $\left|T_{BH}\right|^2$ and to extract the relevant information, hidden in $\left|T_{DVCS}\right|$. I is the interference term which should extracted. Let us remind that the BH term contains only FFs (pure QED) and the DVCS part includeds CFFs.

. An experimental problem on extracting GPDs from $\left|T_{DVCS}\right|$ arises from the fact that this scattering amplitude is not proportional to the GPDs directly, as in the forward limit where the cross section itself is proportional to the parton distribution, but in this case, one has:

$$T_{DVCS} \propto \int_{-1}^{1} dx \frac{H(x, \xi, \Delta^2)}{x - \xi + i\epsilon} + .. \tag{88}$$

i.e., the amplitude is given by a convolution, over the variable x, of the GPDs with a propagator. These integrals are called Compton Form Factors (CFF).

One should notice that the total amplitude depends on the charge (hidden in the interference term). As reference for the momenta look at Fig. 8.

The cross-section:

$$\frac{d^5\sigma}{dx_B\, dy\, dt\, d\phi\, d\varphi} = \frac{\alpha^3 x_B y}{16\pi^2 Q^2 \sqrt{1+\varepsilon^2}} \left|\frac{T}{e^3}\right|^2 \tag{89}$$

where:

$$\begin{aligned} x_B &= \frac{Q^2}{2p_1 \cdot q_1} \\ Q^2 &= -q_1^2 \\ q_1 &= k_1 - k \\ \Delta &= p_2 - p_1 \\ y &= \frac{p_1 \cdot q_1}{p_1 \cdot k} \\ \varepsilon &= 2x_B \frac{M}{Q} \end{aligned} \tag{90}$$

For the angular definition see Fig. 9.

The CFF can be spitted in its real and imaginary part. For a given GPD $F(x, \xi, t)$:

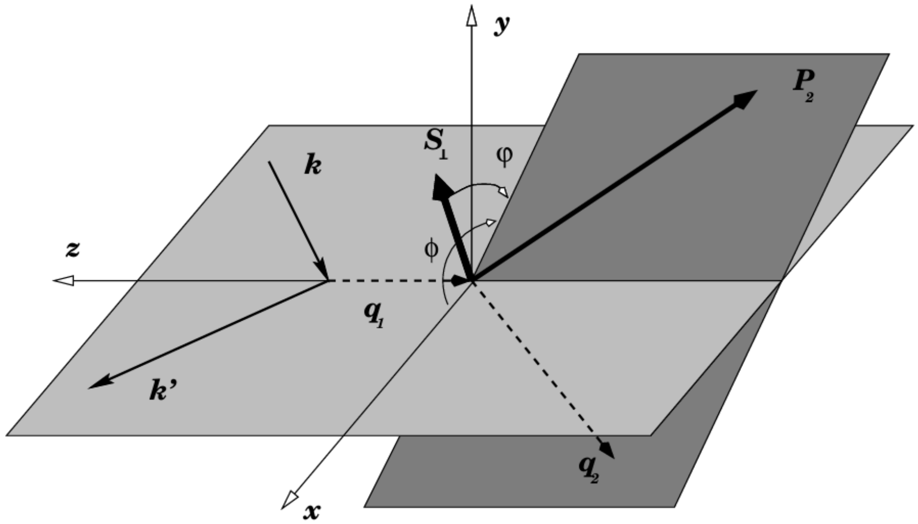

Figure 9. The kinematics of the leptoproduction in the target rest frame. The z-direction is chosen counter-along the three-momentum of the incoming virtual photon. The lepton threemomenta form the lepton scattering plane, while the recoiled proton and outgoing real photon define the hadron scattering plane. In this reference system the azimuthal angle of the scattered lepton is $\phi_l = 0$, while the azimuthal angle between the lepton plane and the recoiled proton momentum is $\phi_N = \phi$. When the hadron is transversely polarized (in this reference frame) $S_\perp = (0, cos\Phi, sin\Phi, 0)$, the angle between the polarization vector and the scattered hadron is denoted as $\varphi = \Phi - \phi_N$.

$$Re\mathcal{F}(\xi,t) = P\int_0^1 dx\, F_+(x,\xi,t)C_+(x,\xi) \tag{91}$$

$$Im\mathcal{F}(\xi,t) = -\pi F_+(\xi,\xi,t) \tag{92}$$

with:

$$F_+(x,\xi,t) = F(x,\xi,t) - F(-x,\xi,t) \tag{93}$$

$$C_+ = \frac{1}{x+\xi} + \frac{1}{x-\xi}.$$

Let us mention the dispersion relation:

$$Re\mathcal{F}(\xi,t) = P\int_0^1 F_+(x,x,t)C_+(x,\xi) - \delta(t) \tag{94}$$

One should notice that the integrand is directly proportional to the imaginary part of CFF and $\delta(t)$ is related to the D-term in EMT which encodes information on the mechaanical proprties of hadrons.

Let us now discuss how amplitudes depend on CFFs and FFs:

$$|T_{BH}|^2 = \frac{e^6}{x_B^2 y^2 (1+\varepsilon^2)^2 \Delta^2 j(\phi)}\Big\{c_0^{BH} + \sum_{n=1}^{2} c_n^{BH}\cos(n\phi) + s_1^{BH}\sin(\phi)\Big\} \tag{95}$$

$$I_\pm = \frac{\pm e^6}{x_B y^3 \Delta^2 j(\phi)}\Big\{c_0^I + \sum_{n=1}^{3} c_n^I \cos(n\phi) + s_n^I \sin(n\phi)\Big\}$$

where $\pm$ in the interference term is related to the charge of the beam and the function $j(\phi)$ defined in Ref. **<empty citation>** is not needed here since it simplifies where studying asymmetries. The coefficients c_i^{BH} and s_i^{BH} include the FF while c_i^I and s_i^I depend on both CFFs and FFs. Let us start with the :

$$BCA = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} \tag{96}$$

where:

$$d\sigma^+ - d\sigma^- = \frac{2\alpha^3 x_B y}{16\pi^2 Q^2 \sqrt{1+\varepsilon^2}} \frac{I}{e^6} \tag{97}$$

$$d\sigma^+ + d\sigma^- \sim \frac{2\alpha^3 x_B y}{16\pi^2 \sqrt{1+\varepsilon^2}} \frac{|T_{BH}|^2}{e^6} \tag{98}$$

hence

$$BCA = \frac{x_B(1+\varepsilon^2)^2}{y} \frac{c_1^I \cos(\phi)}{c_0^{BH} + c_1^{BH} \cos(\phi)} \tag{99}$$

where in uthe unpolarized case and for quarks:

- s_n^I are proportional to spin, therefore in unpolarized case it does not contribute
- c_3^I is related to gluons
- c_0^I and c_2^I and the others are proportional to Δ^2/Q^2, which is small

and:

$$c_1^I \propto Re\left\{F_1\mathcal{H} + \frac{x_B}{2-x_B}(F_1+F_2)\tilde{\mathcal{H}} - \frac{\Delta^2}{4M^2}F_2\mathcal{E}\right\}. \tag{100}$$

Finally the Beam Spin Asymmetry (BSA):

$$BSA \sim \frac{x_B}{y}\frac{s_1^I}{c_0^{BH}} \sin(\phi). \tag{101}$$

Two final remarks are in order in closing this section. The first one is that all the described formalism can be applied also to the production of a vector meson, with the same quantum numbers of the real photon. Also these kind of experiments, important to obtain the flavor structure of GPDs, are being planned. The second remark concerns the pQCD evolution of these quantities. This is known at next-to-leading order and even codes are available to explain to perform the evolution. This topic, not relevant in this work, will not be farther discussed.

Nuclear GPDs in Impulse Approximation

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For ^{3}He, the proposed approach is described in Refs. 17, 18, while for ^{4}He see Ref. 19. One of the main differences between DVCS of protons and nuclei is that in the nuclear case we have two mechanisms:

- *Coherent* In this case the probe interacts with the full nucleus and one accesses nuclear GPDs and tomography, see Fig. 10.

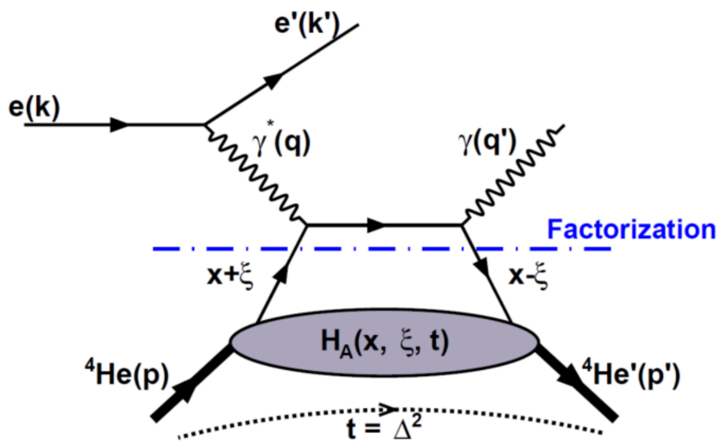

Figure 10. Coherent nuclear DVCS

- *Incoherent* In this case, the probe interact with a nucleon that will be detected, see Fig. 11.

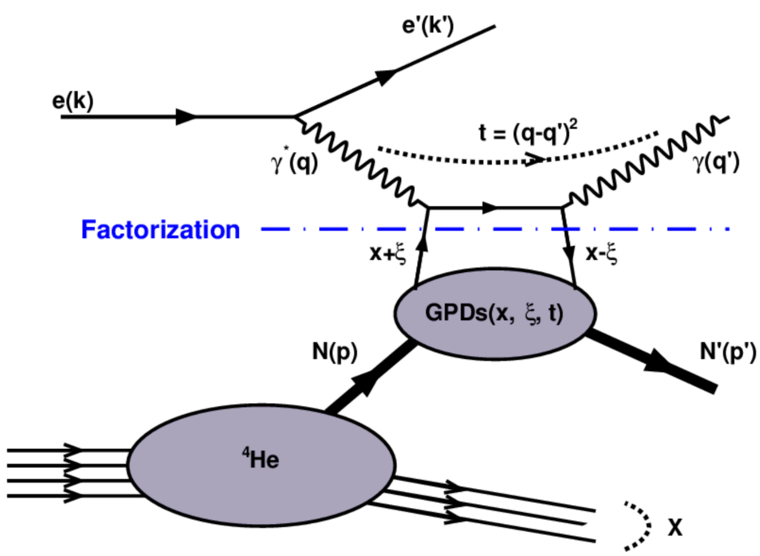

Figure 11. Incoherent nuclear DVCS

In this lecture I will focus in the coherent case.

In this section, witin an Impulse Approximation (IA) approach the nuclear GPDs will be obtained in terms of those of the constituent nucleons. When IA is used, the following conditions are assumed to be valid:

- the nuclear target with A nucleons is described by two systems, one of $A-1$ nucleons, which could be either in the ground or in an excited state, and an off-shell nucleon leaving the target;
- the virtual photon interacts with the off-shell nucleon and the produced hadronic state does not further interact in the final state with the $A-1$ system, which is called, for this reason, *spectator*. The interacting nucleon is treated therefore as a free particle, so that its spatial wave function is a plane wave;
- the interacting nucleon is *off shell* only *kinematically*. This means that its internal structure does not change in the nuclear medium.

In this scenario, typical for example of the parton model, the scattering off a compound system is given by the summation of incoherent scattering off the constituents. Despite of its simplicity, IA has been proven to describe the bulk of nuclear effects in high-energy electron scattering off nuclei. The treatment of nuclear GPDs in IA requires a preliminary kinematical study.

Kinematics for GPDs in IA

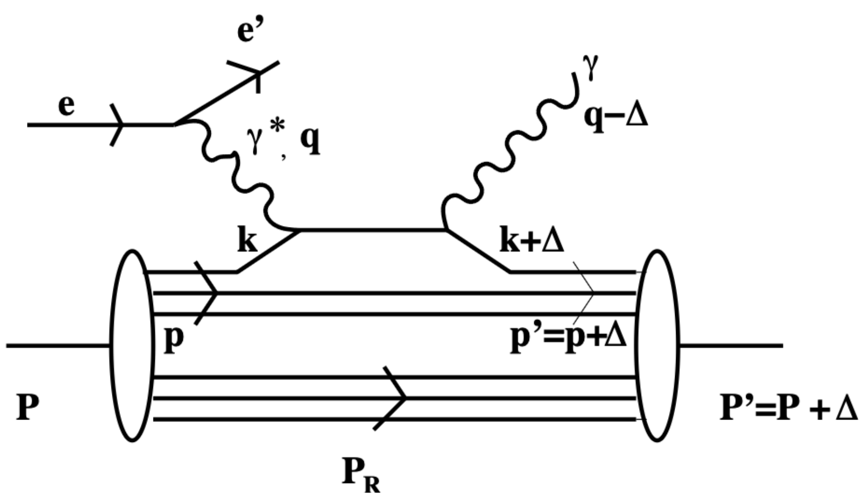

Figure 12. DVCS process off nuclear target in Impulse Approximation.

In order to describe a hard exclusive process, such as DVCS, new kinematical variables have to be introduced. For this purpose, it is useful to look at Fig.4.1. The "plus" parton momentum can be related to the nucleon and nucleus "plus" momenta as follows

$$
\begin{aligned}
k^+ &= (x+\xi)\overline{P}^+ = (x'+\xi')\overline{p}^+ \,, \\
k'^+ = (k+\Delta)^+ &= (x-\xi)\overline{P}^+ = (x'-\xi')\overline{p}^+ \,,
\end{aligned}
\tag{102}
$$

where $\bar{P}=(P+P')/2$, $\bar{p}=(p+p')/2$, $P, P'(p,p')$ are nuclear (nucleon) momenta and k, k' the parton ones. One can notice two different skewednesses, and two "plus" momentum fraction, x and x'. Using explicitly light-cone 4-vectors, one can define the skewedness and the plus momentum fraction carried by the parton. In the nuclear case, the fraction of "plus" momentum carried by the parton can be written with respect to the nucleus or the nucleon momenta. The skewednesses are:

$$\begin{aligned}\xi &= -\frac{\Delta^+}{2\overline{P}^+}\,,\\ \xi' &= -\frac{\Delta^+}{2\overline{p}^+}\,.\end{aligned}\qquad(103)$$

The above equation reflects an obvious, peculiar feature of IA: the momentum transfer is the same for the nucleus and the internal nucleon. Now relations between the variables associated to the nucleus and to the nucleon can be found:

$$x' = \frac{\xi'}{\xi}x\,, \qquad \tilde{z} = \frac{p^+}{P^+}\,,$$

$$\xi' = \frac{\xi}{\tilde{z}(1+\xi)-\xi}\,,\qquad(104)$$

so that the GPDs of the nucleus will depend on the variable $x,\ \xi,\ \Delta^2$, while the one of the bound nucleon on $x',\ \xi',\ \Delta^2$. As previously stated, the nucleons are off-shell. By analyzing energy conservation at the electromagnetic nuclear vertex, one has:

$$p^0 = M_A - p^0_{A-1} = M_A - \sqrt{M^{f,2}_{A-1} + \vec{P}^2_{A-1}} \approx M_N - E - K_R\,,\qquad(105)$$

where N is referred to the nucleon and A to the nucleus, while $E = |E_A| - |E_{A-1}| + E^*_{A-1}$. In the last expression the first and the second terms are binding energies, while the last is the excitation energy of the recoiling $A-1$ system with kinetic energy K_R. All this information is contained in the argument of the structure functions of the bound nucleon, whose functional form is taken to be same of the free nucleon, being therefore the off-shellness a kinematical one. One should notice that, from the above equation, one has $p^2 \neq M^2$. Another important topic which has to be discussed when nuclear targets are involved, due to the lack of a relativistic description of them, is the non relativistic limit of the Eq. (36). This point is fundamental to find direct relations between the correlator $F^A_{H,H'}$ and the GPDs in the non-relativistic case (NR).

The light-cone correlators F_{++} and F_{+-}, in IA

In order to find formulae for the GPDs of ^{3}He, it is convenient to start from the definition Eq. (35) for the nuclear target with $A=3$:

$$F^{A,q}_{S,S'}(x,\xi,\Delta^2) = \int \frac{dz^-}{2(2\pi)^3} e^{ixP^+z^-} \langle PS|\widehat{O}^+_q|P'S'\rangle\qquad(106)$$

where $\widehat{O}^+_q$ is the one-body operator for the quark of flavor q:

$$\widehat{O}^+_q = \overline{\Psi}_q(0,-\frac{z^-}{2},0)\gamma^+\Psi_q(0,\frac{z^-}{2},0)\qquad(107)$$

The state $|PS\rangle$ is normalized as follows:

$$\langle P'S'|PS\rangle = (2\pi)^3 P^+\delta(P'^+ - P^+)\delta^2(\vec{P}_\perp - \vec{P}'_\perp)\delta_{S'S}.\qquad(108)$$

Following a standard procedure, two complete sets of states, corresponding to the nucleon interacting with the virtual photon in an IA scenario, and to a recoiling interacting system, are properly inserted to the left and right-hand sides of the quark operator. With this procedure one will obtain convolution-like formulae between the light-cone correlator $F_{S,S'}$ of the nucleus and the corresponding quantity, $f_{S,S'}$, of the bound nucleon, i.e., relations of the following type:

$$F^{A,q}_{S'S} = \sum_{s,s',N} \mathcal{F}^{ss'}_{S'S} f^{N,q}_{ss'} \tag{109}$$

where N refers to the nucleon and $+$ $(-)$ corresponds to positive (negative) eigenvalue of the third-component of the spin of the system. Now, IA describes the interaction of a virtual photon with a parton of *one* nucleon. Therefore, an interacting two-body recoiling system, either bound or in a scattering state, together with a free nucleon, has to be originated from the nucleus. Introducing complete sets of states for these systems and an auxiliary variable ζ, one gets:

$$\begin{aligned} F^{A,q}_{S,S'}(x,\xi,\Delta^2) = & \sum_\alpha \sum_\beta \int d\zeta \delta\left(\zeta - \frac{p^+}{P^+}\right) \int \frac{dz^-}{2(2\pi)^3} e^{i(\frac{x}{\zeta}p^+z^-)} \\ \times & \langle P|\{|\overrightarrow{P}'_R S'_R\rangle|\overrightarrow{t}' s'_{t'}\rangle|\overrightarrow{p}' s'\rangle\}\{\langle\overrightarrow{P}'_R S'_R|\langle\overrightarrow{t}' s'_{t'}|\langle\overrightarrow{p}' s'|\} \\ \times & \widehat{O}^+_q\{|\overrightarrow{P}_R S_R\rangle|\overrightarrow{t} s_t\rangle|\overrightarrow{p} s\rangle\}\{\langle\overrightarrow{P}_R S_R|\langle\overrightarrow{t} s_t|\langle\overrightarrow{p} s|\}|P'\rangle \, , \end{aligned} \tag{110}$$

where $\vec{P}_R$ and S_R are the momentum and the spin of the recoiling two-body R system in the nuclear center of mass, $\vec{t}$ and s_t are the relative momentum and the spin of the same object, $\vec{p}$ and s refer to the nucleon. The summations (and integral) run over the sets of indexes (and continuous variables):

$$\begin{aligned} \alpha &= \overrightarrow{P}'_R, S'_R, \overrightarrow{p}', s', \overrightarrow{t}', s'_{t'} \, ; \\ \beta &= \overrightarrow{P}_R, S_R, \overrightarrow{p}, s, \overrightarrow{t}, s_t \, . \end{aligned} \tag{111}$$

Since later the nuclear matrix elements have to be evaluated by means of NR wave functions, the inserted states, which are now spin states, not helicity ones as in Eq. (106), are normalized in a NR manner:

$$\langle \vec{P}' S' | \vec{P} S\rangle = (2\pi)^3 \delta^3(\overrightarrow{P}' - \overrightarrow{P})\delta_{S'S} \, . \tag{112}$$

Using this normalization, together with Eq. (108), one gets the general relation:

$$|P\rangle = \sqrt{2P_0}|\overrightarrow{P}\rangle \, . \tag{113}$$

Due to the normalization (112), using intrinsic coordinates and assuming in IA that the recoiling system does not interact further with the interacting nucleon, one obtains the following condition:

$$\langle \vec{P}_R S_R|\langle \vec{t} s_t|\langle\overrightarrow{p} s|\overrightarrow{P} S\rangle = \langle\overrightarrow{p} s, \overrightarrow{t} s_t|\overrightarrow{P} S\rangle(2\pi)^3\delta^3(\overrightarrow{P} - \overrightarrow{P}_R - \overrightarrow{p})\delta_{S,S_R,s,s_t} \, . \tag{114}$$

where the global motion has been separated from the intrinsic one. The latter is described by the first term in r.h.s. of the above equation, describing an intrinsic overlap. Besides, since $\widehat{O}_q^+$ is a one-body operator, one also gets:

$$\begin{aligned}
&\{\langle \overrightarrow{P}'_R S'_R | \langle \overrightarrow{t}' s'_t | \langle \overrightarrow{p}' s' |\} \widehat{O}_q^\alpha \{| \overrightarrow{P}_R S_R \rangle | \overrightarrow{t} s_t \rangle | \overrightarrow{p} s \rangle\} = \langle \overrightarrow{P}'_R S'_R | \overrightarrow{P}_R S_R \rangle \langle \overrightarrow{t}' s'_{t'} | \overrightarrow{t} s_t \rangle \langle \overrightarrow{p}' s' | \widehat{O}_q | \overrightarrow{p} s \rangle \\
&= (2\pi)^6 \delta^3(\overrightarrow{P}'_R - \overrightarrow{P}_R) \delta^3(\overrightarrow{t} - \overrightarrow{t}') \delta_{S_R, S'_R} \delta_{s_t, s'_t} \langle \overrightarrow{p}' s' | \widehat{O}_q | \overrightarrow{p} s \rangle
\end{aligned} \tag{115}$$

which, using Eqs.(113),(115), can be written:

$$\begin{aligned}
F_{S,S'}^{A,q}(x, \xi, \Delta^2) = \sum_N \int \frac{d\overrightarrow{p}}{(2\pi)^3} \int_x^1 d\zeta \delta\left(\zeta - \frac{p^+}{P^+}\right) \int \frac{d\overrightarrow{t}}{(2\pi)^3} \sqrt{\frac{P_0 P'_0}{p_0 p'_0}} f_{s,s'}^{N,q}\left(\frac{x}{\zeta}, \frac{\xi}{\zeta}, \Delta^2\right) \\
\times \sum_{s,s',s_t} \langle \overrightarrow{P} S | \overrightarrow{p} s, \overrightarrow{t} s_t \rangle \langle \overrightarrow{p}' s', \overrightarrow{t} s_t | \overrightarrow{P}' S' \rangle
\end{aligned} \tag{116}$$

where it has been used the fact that, in IA, also the momentum transferred to the nucleon is $\Delta = p' - p$, and $f_{s,s'}^{N,q}\left(\frac{x}{\zeta}, \frac{\xi}{\zeta}, \Delta^2\right)$ is the light-cone correlator for the nucleon:

$$f_{s,s'}^{N,q}\left(\frac{x}{\zeta}, \frac{\xi}{\zeta}, \Delta^2\right) = \int \frac{dz^-}{2(2\pi)^3} e^{i(\frac{x}{\zeta} p^+ z^-)} \langle ps | \widehat{O}_q^+ | p' s' \rangle \ . \tag{117}$$

In the above equation, $N = p, n$, specifies the nucleon type. It is useful to define the intrinsic relative energy of the two-body system as $E = t^2/M$, so that one gets:

$$\int d\overrightarrow{t} = \int dE d\Omega_t \frac{M\sqrt{ME}}{2} \ . \tag{118}$$

Using Eq. (118), one finds finally:

$$F_{S,S'}^{A,q}\left(x, \xi, \Delta^2\right) = \sum_N \sum_{ss'} \int dE \int d\vec{p}\, P_{SS'ss'}^N(\vec{p}, \vec{p} + \vec{\Delta}, E) \sqrt{\frac{P_0 P'_0}{p_0 p'_0}} f_{s,s'}^{N,q}\left(x', \xi', \Delta^2\right) \ , \tag{119}$$

where $x' = x/\zeta$, $\xi' = \xi/\zeta$ and $P_{SS'ss'}^N(\vec{p}, \vec{p} + \vec{\Delta}, E)$ is the one-body non-diagonal spin-dependent spectral function for the nucleon N in the nucleus, which can be written:

$$P_{SS'ss'}^N(\vec{p}, E, \vec{p} + \vec{\Delta}) \quad = \quad \frac{1}{(2\pi)^6} \int d\Omega_t \frac{M\sqrt{ME}}{2} \sum_{S_R, s_t} \langle \vec{P}' S' | \vec{p} + \vec{\Delta} s', \vec{t} s_t \rangle \langle \vec{p} s, \vec{t} s_t | \vec{P} S \rangle \tag{120}$$

Now, considering proper values of SS', ss':

$$F_{++}^{A,q} = \sum_{s_t} \frac{1}{(2\pi)^6} \int dE \frac{M\sqrt{ME}}{2} \int d\overrightarrow{p}\, d\Omega_t \sqrt{\frac{P_0 P_0'}{p_0 p_0'}} f_{++}^{N,q}(x', \xi', \Delta^2)$$
$$\times \left[\langle \vec{P}' + |\vec{t}s_t, \vec{p}' + \rangle \langle \vec{p}+, \vec{t}s_t | \vec{P}+ \rangle \, + \, \langle \vec{P}' + |\vec{t}s_t, \vec{p}' - \rangle \langle \vec{p}-, \vec{t}s_t | \vec{P}+ \rangle \right] \tag{121}$$

where here $+$ $(-)$ is the eigenvalue of the z component of the spin.
Expressions like $\langle \vec{p}s, \vec{t}s_t | \vec{P}S \rangle$ are the intrinsic *overlap* of the wave function of the nucleus with those of the recoiling system and of the nucleon, which have to be specified within a dynamical description of the two- and three-body system. Using symmetry properties the above Eq. (121) can be rewritten:

$$F_{++}^{A,q}(x, \xi, \Delta^2) = \sum_{s_t,\sigma} \overline{\sum_{S}} \frac{1}{(2\pi)^6} \int dE \frac{M\sqrt{ME}}{2} \int d\overrightarrow{p}\, d\Omega_t \sqrt{\frac{P_0 P_0'}{p_0 p_0'}} f_{++}^{N,q}(x', \xi', \Delta^2)$$
$$\times \left[\langle \vec{P}' S | \vec{t}s_t, \vec{p}' \sigma \rangle \langle \vec{p}\sigma, \vec{t}s_t | \vec{P}S \rangle \right]. \tag{122}$$

In order to find E_q^A, it is necessary to define F_{+-}^A:

$$F_{+-}^{A,q}(x, \xi, \Delta^2) = \sum_{s_t} \frac{1}{(2\pi)^6} \int dE \frac{M\sqrt{ME}}{2} \int d\vec{p} d\Omega_t \sqrt{\frac{P_0 P_0'}{p_0 p_0'}} f_{+-}^{N,q}(x', \xi', \Delta^2)$$
$$\times \left[\langle \vec{P}' + |\vec{t}s_t, \vec{p}' + \rangle \langle \vec{p}-, \vec{t}s_t | \vec{P}- \rangle \right.$$
$$\left. - \, \langle \vec{P}' + |\vec{t}s_t, \vec{p}' - \rangle \langle \vec{p}+, \vec{t}s_t | \vec{P}- \rangle \right] \, . \tag{123}$$

As one can see the spin structure of the coefficients are more complicated than in the case of the GPD H_q^A, because that calculation was diagonal in the spins, so that many of the simplifications found in the diagonal case will not work in this second case. The explicit formulae which relate the light-cone correlator matrices to the GPDs will be obtained in the next section. If one use:

$$|P\rangle = \sqrt{2P^+} |\vec{P}\rangle \tag{124}$$

then in all the above formulas:

$$\sqrt{\frac{P_0 P_0'}{p_0 p_0'}} \to \sqrt{\frac{P^+ P'^+}{p^+ p'^+}} = \sqrt{\frac{(P^+ - \frac{\Delta^+}{2})(P^+ + \frac{\Delta^+}{2})}{(p^+ - \frac{\Delta^+}{2})(p^+ + \frac{\Delta^+}{2})}} \sim \frac{P^+}{p^+} = \frac{\xi'}{\xi} \tag{125}$$

Therefore we finally get:

$$H^{A,q}(x, \xi, \Delta^2) = \sum_{N} \overline{\sum_{S}} \sum_{s} \int dE \int d\vec{p}\, \frac{\xi'}{\xi} H^{N,q}(x', \xi', \Delta^2) P_{SSss}^{N}(\vec{p}, \vec{p} + \vec{\Delta}, E).$$

Moreover:

Table 1. Light nuclei features: atomic number Z, mass number A, Spin – Parity $\mathcal{J}^\pi$, binding energy E, dipole magnetic moment μ, quadrupole electric moment Q.

Nucleus	Z	A	$\mathcal{J}^\pi$	$E\,(MeV)$	$\mu\,(\mu_N)$	Q (barns)
^{2}H	1	2	1^+	- 2.225	0.857	0.0028
^{3}H	1	3	$\frac{1}{2}^+$	- 8.482	2.979	–
^{3}He	2	3	$\frac{1}{2}^+$	- 7.718	- 2.128	–

$$H^{A,q}(x,\xi,t) + E^{A,q}(x,\xi,t) = \sum_N \overline{\sum_S} \sum_s \int dE \int d\vec{p}\, \frac{\xi'}{\xi} \left[H^{N,q}(x',\xi',t) + E^{N,q}(x',\xi',t)\right] \times \left[P^N_{+-+-}(\vec{p},\vec{p}+\vec{\Delta},E) - P^N_{+--+}(\vec{p},\vec{p}+\vec{\Delta},E)\right]. \tag{126}$$

Why ^{3}He

The study of GPDs for ^{3}He is interesting for many aspects. In fact, ^{3}He is a well known nucleus so conventional nuclear effects can be calculated, and it is extensively used as an effective neutron target. As a matter of facts, free neutron target do not exist and the properties of the free neutron are being investigated through experiments with nuclei , whose data are analyzed taking nuclear effects properly into account. For example, it has been shown that unpolarized DIS off trinucleons (^{3}H and ^{3}He) can provide relevant information on (parton distributions) PDFs at large x_{Bj}, while it is known since a long time that its particular spin structure suggests the use of ^{3}He as an effective polarized neutron target, see Refs. 20, 21. Polarized ^{3}He is therefore the first candidate for experiments aimed at the study of angular momentum properties of the free neutron, such as the GPD E_q^N. Moreover it is well known that 90 % of the nucleus spin comes from the neutron one, as a is consequence of its internal dynamics.
Summarizing, ^{3}He is a unique target for GPDs studies, for two main reasons,i.g., the investigation of nuclear effects on GPDs and the access to the neutron information. Here are discussed these two issues in this order.

Nuclear effects on GPDs studied with a ^{3}He target

The H_q^3 GPD has been obtained in terms of the H_q^N of the nucleon, using a realistic non-diagonal spectral function, so that momentum and binding effects have been rigorously estimated. The scheme proposed was valid for $\Delta^2 \ll Q^2, M^2$ and it permitted to calculate GPDs in the kinematical range relevant to the coherent, no break-up channel of deep exclusive processes off ^{3}He. In fact, the latter channel can be hardly studied at large Δ^2, due to the vanishing cross section, see Ref. 22. Nuclear effects were found to be larger than in the forward case and to increase with Δ^2 at fixed skewedness, and with the skewedness at fixed Δ^2. In particular the latter Δ^2 dependence did not simply factorize.

A detailed study of the flavor dependence of the nuclear effects, due to the fact that ^{3}He is non isoscalar was also presented. One should notice that the other few-body targets, such as ^{2}H and ^{4}He, are isoscalar and the flavor dependence of GPDs can be hardly accessed. In this sense ^{3}He is unique.

^{3}He as an effective neutron target

Another reason to choose ^{3}He as a target for future experiments is that it can be considered a source of polarized neutrons. As a matter of facts corrections coming from the proton polarization are smaller than for deuteron target. This property can be understood thinking simply to the independent particle (shell) model where nuclear spin and parity are the same of the unpaired nucleon. For ^{3}He , it would be therefore $J^\pi = \frac{1}{2}^+$, the same of the neutron. In this model, the Orbital Angular Momentum (OAM) of each particle is 0, so that proton-proton pair total spin must be zero, due to the Pauli principle, so the nucleus spin comes only from that of the neutron. Now experiments show that this model is valid up to a 10% error. Nowadays, a rigorous theoretical description of three-body nuclear systems is available. The solution of the corresponding Schrödinger equation can be obtained exactly even using realistic potentials †, using the Faddeev technique. Variational calculations, although approximated, provide the results of a similar quality. For the aim of this work it is enough to say that all the realistic calculations show that the ^{3}He wave-function can be written as follows:

$$\psi_{\frac{1}{2},M} \approx a_{\mathcal{S}}\Phi_{\mathcal{S}}(^2\mathcal{S}_{\frac{1}{2}}) + a_{\mathcal{S}'}\Phi_{\mathcal{S}'}(^2\mathcal{S}'_{\frac{1}{2}}) + a_{\mathcal{D}}\Phi_{\mathcal{D}}(^4\mathcal{D}_{\frac{1}{2}}) \ , \tag{127}$$

where the usual spectroscopic notation: $^{2\mathcal{S}+1}\mathcal{L}_J$ is used and $\mid a_{\mathcal{L}} \mid^2 = P(\mathcal{L})$ the probability of having the components with OAM $\mathcal{L}$. It is useful to examine all these terms. The first is the dominant part and coincides with the model previously exposed. It is totally symmetric for the spatial term, so that it is totally antisymmetric for the spin-isospin part. This is the contribution which would be present if the interaction were central and Isospin-independent. Anyway the Deuteron properties show that strong interactions depend on the spin and the isospin, and, these effects are contained in the other terms of the wave-function. $\Phi_{\mathcal{S}'}$ has $\mathcal{L} = 0$, too, but it is of mixed spatial symmetry. $\Phi_{\mathcal{D}}$ depends on the non-central nature of the interaction. This term is related to the tensor force active between symmetric spin states with different orbital angular momenta. These two terms of the wave-function are found to give small contributions, so the total nuclear spin and dipole magnetic momentum are essentially given by neutron. The probabilities for the $\mathcal{L}$ waves obtained in realistic calculations lie in the range:

$$P(\mathcal{S}) \sim 85 - 95\% \qquad P(\mathcal{S}') \sim 0.5 - 2\% \qquad P(\mathcal{D}) \sim 4 - 10\% \ . \tag{128}$$

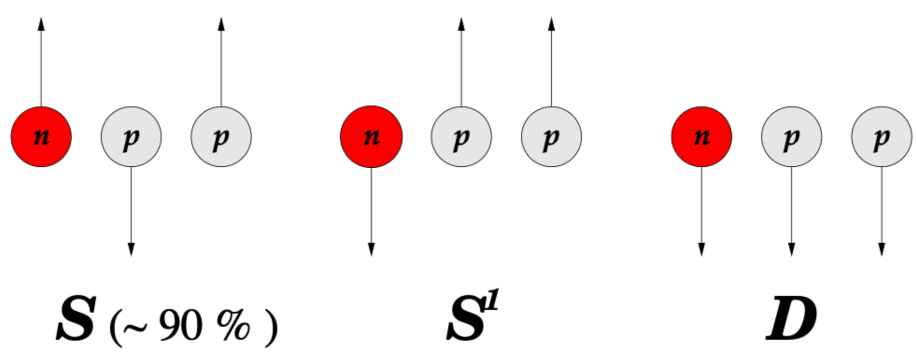

Figure 13. Polarization of protons and neutron in the main components of the wave function of ^{3}He

As already stressed, GPDs are not densities, and they cannot be related to static wave function properties. In any case, in the forward limit, such as an identification is possible. Since one of the main interests in GPDs study is the forward limit of the GPD E_q^3, whose integral is the magnetic form factor, yielding the dipole magnetic moment at $t = 0$, at least in this limit, this static ^{3}He feature could be used. One should remember that the relation between the GPDs and OAM (**??**,**??**), holds in the forward limit. This should allow a direct connection between the measurement of E_q^3 for ^{3}He and the OAM content of the neutron. This is one of the reasons why the present analysis started.

A crucial observation is the following: the others “light” nuclear targets, such as ^{2}H and ^{4}He, are useless for this purpose. Indeed, the anomalous magnetic moment of ^{2}H is extremely small (and such is the contribution of the GPD E to DVCS close to the forward limit), while ^{4}He, being an isoscalar-scalar nucleus, does not have a E GPD in the current describing the DVCS process. The numerical results of the calculations and relative discussion can be found in Ref. 18.

†Technically, a realistic 2-body potential is the one which is able to reproduce the properties of the 2-body nuclear systems, in the bound (deuteron) and elastic scattering state. A few potentials are available nowadays with a $\frac{\chi^2}{datum} \approx 1$

Examples

Here I collect the examples that I discussed in the final lecture.

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QCD Light-Front

We use LF quantization to get a relativistic quantum mechanim description of strong interacting systems. The evolution will not be in term of ordinary time coordinate but with LF time $x^+ = x^0 + x^3$. As already pointed out in other lectures, within the LF dynamics you can get mass equations similar to the Schrödinger. An important result is that the vacum is trivial so that a Fock expansion of composite systems is allowed in terms of constituents, see Ref. 13, 23. For any hadronic system, the LF quantization in term of $\tau = t + z/c$, leads to the relativistic equation:

$$H_{LF}^{QCD} = P^2 = P^- P^+ - \mathbf{P}_\perp^2, \tag{129}$$

where

$$P^- = \frac{\mathbf{P}_\perp^2 + M^2}{P^+}, \quad P^+ > 0. \tag{130}$$

Now there is no square root of any operator leading to sign problems and the dependence of the $P \perp$ operator is similar to the non relativist case, as in the IMF. Clearly, P^- is the conjugate variable to x^+ so that it will represent the time evolution operator:

$$i\frac{\partial}{\partial x^+}|\psi(P)\rangle = P^-|\psi(P)\rangle, \tag{131}$$

One should recall that the generators P^+ and $P_\perp$ are kinematical and do not depend on the interaction. Clearly, from the usual relativistic interpretation one gets $P^\mu P_\mu = M^2$, i.e. the total invariant mass which is related to the spctrum of the hadron. Therefore one gets the following equation:

$$H_{LF}|\psi_h\rangle = M_h^2|\psi_h\rangle \tag{132}$$

where $|\psi_h\rangle$ is the hadron state that can be now expanded (in the gauge $A^+ = 0$) as a coherent sum of Fock states solution of the free hamiltonian:

$$|\psi_h\rangle = \sum_n \psi_{nh}|\psi_h\rangle. \tag{133}$$

The LF wave-sfunction ψ_{n/h_n} do not depend on the relative frame and they own the usual probabilistic interpretation. Moreover, the Fock states $|n\rangle$ are obtained as useful, by applying the creation and annihilation operators on the vacum $|0\rangle$:

$$
\begin{aligned}
&n = 0 : |0\rangle, \\
&n = 1 : |q\bar{q}; k_i^+, k_{\perp i}, \lambda_i\rangle = b^\dagger(q_1) d^\dagger(q_2)|0\rangle, \\
&n = 2 : |q\bar{q}g; k_i^+, k_{\perp i}, \lambda_i\rangle = b^\dagger(q_1) d^\dagger(q_2) a^\dagger(q_3)|0\rangle, \\
&\quad\vdots
\end{aligned}
\tag{134}
$$

where the operators $b^\dagger(q)$, $d^\dagger(q)$ and $a^\dagger(q)$ create quark, anti quatks and gluons, respectively, with momentum k_i^+ and $k_{\perp i}$ and, for quarks, LF helicity λ_i. These states are eigenstates of the operators P^+ e $P_\perp$. Since these quantities are kinematical, the following relations hold for composite systems:

$$
P_\perp = \sum_{i \in n} k_{\perp i}, \quad P^+ = \sum_{i \in n} k_i^+, \quad k_i^+ > 0. \tag{135}
$$

The vacuum is given by: $P_\perp|0\rangle = 0$ e $P^+|0\rangle = 0$. One can define the LF boost invariant longitudinal momentum fraction carried by a parton i:

$$
x_i = \frac{k_i^+}{P^+}, \quad 0 < x_i < 1, \tag{136}
$$

Therefore, the full four momentum vector of a given parton is:

$$
k_i^\mu = (k_i^+, k_i^-, \vec{k}_{\perp,i}) = \left(x_i P^+, \frac{m_i^2 + k_{\perp,i}^2}{x_i P^+}, \vec{k}_{\perp,i} \right), \quad i = 1, \ldots, N_n. \tag{137}
$$

and the on-shell condition is fulfilled: $(k_i^\mu)^2 = m_i^2$. Now if we consider non interacting partons then we would get the following condition:

$$
\left(\sum_{i=1}^{N_n} k_i^- \right) P^+ - P_\perp^2 = \sum_{i=1}^{N_n} \left(\frac{k_{\perp i}^2 + m_i^2}{x_i} \right) - P_\perp^2 = M_0^2. \tag{138}
$$

where M_0^2 is the free mass, i.e. the mass of the hadron if made of free partons. In terms of LF variable we have:

$$
\sum_{i=1}^{N_n} x_i = 1, \quad \sum_{i=1}^{N_n} \mathbf{k}_{\perp i} = 0. \tag{139}
$$

Within these ingredients, the hadron state, expanded in terms of a basis of Fock states reads:

$$
\begin{aligned}
|\psi_h(P^+, \mathbf{P}_\perp, S_z)\rangle = \sum_n \prod_{i=1}^n \int \frac{dx_i d^2\mathbf{k}_{\perp i}}{2x_i(16\pi^3)} \, \delta\left(1 - \sum_{j=1}^n x_j\right) \delta^{(2)}\left(\sum_{j=1}^n \mathbf{k}_{\perp j}\right) \\
\times \psi_{n/h}(x_i, \mathbf{k}_{\perp i}, \lambda_i)|n; P^+, x_i\mathbf{P}_\perp + \mathbf{k}_{\perp i}, \lambda_i\rangle.
\end{aligned}
\tag{140}
$$

with the normalization which is fixed by the following condition:

$$
\langle \psi_h(P^+, \mathbf{P}_\perp, S_z)|\psi_h(P'^+, \mathbf{P}'_\perp, S'_z)\rangle = 2P^+(2\pi)^3 \delta_{S_z S'_z} \delta(P^+ - P'^+) \delta^{(2)}(\mathbf{P}_\perp - \mathbf{P}'_\perp). \tag{141}
$$

Let us define:

$$\left[dx_i d^2k_{\perp i}\right] = \delta\left(1 - \sum_{j=1}^{N_n} x_j\right)\delta^{(2)}\left(\sum_{j=1}^{N_n} \mathbf{k}_{\perp j}\right) dx_1...dx_{N_n} d^2k_{\perp 1}...d^2k_{\perp N_n}. \tag{142}$$

then:

$$\sum_n \int \left[dx_i d^2k_{\perp i}\right] |\psi_{n/h}(x_i, k_{\perp i})|^2 = 1. \tag{143}$$

For practical purposes let us mention that when you need to study parton distributions or quantities depending on quark field operators, in particular one can define the good and bad components:

$$\psi_\pm = \Lambda_\pm \psi \tag{144}$$

$$\Lambda_\pm = \gamma^0 \gamma^\pm. \tag{145}$$

So the phsyical fields will be:

$$\psi_+(x^-, \mathbf{x}_\perp)_a = \sum_\lambda \int_{q^+>0} \frac{dq^+ \, d^2q_\perp}{\sqrt{2q^+}(2\pi)^3}[a(q,\lambda)u_a(q,\lambda)e^{-iq\cdot x} + d^\dagger(q,\lambda)v_a(q,\lambda)e^{iq\cdot x}], \tag{146}$$

and

$$A_\perp(x^-, \mathbf{x}_\perp) = \sum_\lambda \int_{q^+>0} \frac{dq^+ \, d^2q_\perp}{\sqrt{2q^+}(2\pi)^3}[a(q,\lambda)\epsilon_\perp(q,\lambda)e^{-iq\cdot x} + a^\dagger(q,\lambda)\epsilon^*_\perp(q,\lambda)e^{iq\cdot x}], \tag{147}$$

where the spinors:

$$u^\uparrow(p) = \frac{1}{\sqrt{p^+}}(p^+ + \beta m + \bar{\alpha}_\perp \cdot p_\perp)\chi^\uparrow, \tag{148}$$

$$v^\uparrow(p) = \frac{1}{\sqrt{p^+}}(p^+ - \beta m + \bar{\alpha}_\perp \cdot p_\perp)\chi^\uparrow. \tag{149}$$

with this convention for the matrices:

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \tag{150}$$

Spinors are

$$\chi_\uparrow = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi_\downarrow = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \tag{151}$$

and

$$\chi_\uparrow = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi_\downarrow = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}. \tag{152}$$

The commutation rules are:

$$[a(q), a^\dagger(q')] = \{b(q), b^\dagger(q')\} = \{d(q), d^\dagger(q')\} = (2\pi)^3 \delta(q^+ - q'^+)\delta^{(2)}(q_\perp - q'_\perp). \tag{153}$$

and the single state particle is:

$$|q\rangle = \sqrt{2q^+} b^\dagger(q)|0\rangle \tag{154}$$

with the orthogonality condition:

$$\langle q|q'\rangle = 2q^+(2\pi)^3\delta(q^+ - q'^+)\delta^{(2)}(q_\perp - q'_\perp). \tag{155}$$

Therefore, each Fock state in the expansion satisfies:

$$\langle p_i^+, p_{\perp i}, \lambda_i | p_i'^+, p'_{\perp i}, \lambda'_i\rangle = 2p_i^+(2\pi)^3\delta(p_i^+ - p_i'^+)\delta^{(2)}(P_{\perp i} - P'_{\perp i})\delta_{\lambda,\lambda'}. \tag{156}$$

Fock expansion applied to Nuclear Physics

In this final part we apply the Fock expansion to the nuclear case. In fact, now it is natural to apply it since we have a system with a fixed number of constituent (as a first approximation). The convertion would be the following:

- the free partonic state are replace with nucleon states. Therefore our creation and annihilation operators create and destroy nucleon states;
- $x \to \xi$ the longitudinal momentum fraction carried by a nucleon w.r.t. the nucleus momentum;
- we will have the nuclear LF wave-function;

Nuclear PDFs

Let us first recall the PDFs for the nucleon case:

$$f_q^\tau(x) = 2p^+ \int \frac{dz^-}{2\pi}\, e^{ixp^+z^-} \langle p;\ \tau|\theta_q(0,z)|\tau;\ p\rangle\Big|_{z^+=0}^{\mathbf{z}_\perp=\mathbf{0}}, \tag{157}$$

where q is the flavors of the involved quarks, $|\tau; p\rangle$ is the nucleon state with intrinsic dofs, $p^\mu \equiv \{p^\pm, \mathbf{p}_\perp\}$ the nucleon four-momentum with $p^\pm = (p^0 \pm p^3)/\sqrt{2}$ and $x_\ell = q_\ell^+/p^+$ the longitudinal momentum fraction carried by the ℓ-th parton w.r.t. the nucleon momentum, being q_ℓ^μ the momentum of the ℓ-parton. Clearly, one must have $x \leq 1$. The bi-linear operators appearing in the above expression read:

$$\theta_i(y,z) = \bar{q}_i\left(y - \frac{1}{2}z\right)\frac{1}{2}\gamma^+ q_i\left(y + \frac{1}{2}z\right), \tag{158}$$

where $q_i(y \pm z/2)$ is the quark field operator for a parton of flavor i. The generalization to a nucleus A is:

$$f_q^A(x) = 2P^+ \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle A|\theta_q(0,z)|A\rangle\Big|_{z^+=0}^{\mathbf{z}_\perp=\mathbf{0}}. \tag{159}$$

Now P^+ is the plus component of the nucleus momentum in the lab frame, whereas $|A\rangle$ denotes the nuclear state. We consider the above LF approach since it remarkably allows us to easily separate the centre of mass motion from the intrinsic one (as in the non relativistic case), both at the level of the nucleus and the nucleon, given the subgroup properties of the LF-boosts. Let us define the conventions for the momenta of partons, nucleons and nuclei, respectively: i) q^μ_ℓ is the four-momentum of the $\ell-$th parton in the *Lab frame*, ii) p^μ_r (k^μ_r) the four-momentum of the $r-$th nucleon inside the nucleus in the Lab (intrinsic) frame with $r = 1, 2 \ldots, A$ and M the nucleon mass (notice that $p^2_r = M^2$ in LF formalism), iii) M_A the nucleus mass with four-momentum $P^\mu \equiv \{P^\pm, \mathbf{P}_\perp\}$ in the Lab frame (recall $P^\mu \equiv \{P^\pm = M_A, \mathbf{0}_\perp\}$ in the intrinsic frame). One can also define the longitudinal-momentum fractions carried by the r-th nucleon w.r.t. the parent nucleus:

$$\xi_r = \frac{p^+_r}{P^+} \,. \tag{160}$$

With these definitions, the nucleon momentum vector can be expressed as follows in terms of the intrinsic LF coordinates $\{\xi_r, \mathbf{k}_{\perp,r}\}$:

$$p^+_r = \xi_r P^+; \quad \mathbf{p}_{\perp,r} = \xi_r \mathbf{P}_\perp + \mathbf{k}_{\perp,r} \,, \tag{161}$$

and the following constraints from the four-momentum conservation are found:

$$\sum_r^A p^+_r = P^+ \Rightarrow \sum_r^A \xi_r = 1 \,, \tag{162}$$

$$\sum_r \mathbf{p}_{\perp,r} = \mathbf{P}_\perp \ \Rightarrow \sum_r \mathbf{k}_{\perp,r} = \mathbf{0} \,. \tag{163}$$

Analogously, the longitudinal momentum fractions carried by the ℓ-th parton w.r.t. the nucleus can be introduced (N.B. in Eq. 157 the same notation indicates the longitudinal fraction w.r.t. the nucleon):

$$x_\ell = \frac{q^+_\ell}{P^+} \,. \tag{164}$$

In the present analysis, we assume a factorized form of the nuclear state $|A\rangle$, as the Cartesian product of i) the nuclear that takes into account only nucleonic dofs and ii) an intrinsic part, with partonic dofs. In this framework, one gets:

$$|A\rangle = \sum_{\tau_1,..,\tau_A = n,p} \sum_{\lambda_1,..,\lambda_A} \frac{1}{[2(2\pi)^3]^{(A-1)/2}} \int \left[\prod_{r=1}^A \frac{d\xi_r d^2k_{\perp,r}}{\sqrt{\xi_r}}\right] \delta\left(1 - \sum_{l=1}^A \xi_l\right) \delta\left(\sum_{l=1}^A \mathbf{k}_{\perp,l}\right) \tag{165}$$
$$\times \psi(\xi_1, .., \xi_A, \mathbf{k}_{1,\perp}, ..., \mathbf{k}_{\perp,A}, \tau_1, ..., \tau_A, \lambda_1, ..., \lambda_A)|n_1\rangle \cdots |n_A\rangle \,,$$

where, ψ is the LF nuclear wf that describes the dynamics of A nucleons (i.e. the collection of centres of masses of QCD singlet-states) in the frame where $\mathbf{P}_\perp = 0$, and λ_r is the projection of the spin of r-th nucleon along the z axis. Of course, the nuclear wf is completely antisymmetric under the exchange of two nucleons.

In analogy with the Fock decomposition of the hadronic states in terms of free parton states the above independent nucleons state, $|n_i\rangle$, should consist only of plane-waves describing the centre-of-mass motion of each nucleon, as whole. However, in order to completely describe the nuclear state, one should also include the intrinsic partonic part, where the introduction of a large scale, i.e. the nucleus radius, suggests to separate the effects of nucleonic and partonic dofs, obtaining a workable approximation. In particular, we simply assume that:

$$|n_r\rangle = \underbrace{|\xi_r P^+, \xi_r \mathbf{P}_\perp + \mathbf{k}_{\perp,r}, \tau_r, \lambda_r\rangle}_{CM\ plane-wave} \otimes \underbrace{|\phi_r\rangle}_{intrinsic} \,. \tag{166}$$

Since our calculations will parametrize the intrinsic dependence of the nucleon state entirely through parton distribution functions, as discussed in what follows, we will henceforth omit explicit reference to such a dependence. Here, τ_r represents the nucleon isospin. Once the intrinsic part is properly normalized, the orthonormalization rule of the nucleon states is:

$$\begin{aligned}\langle n_r|n'_k\rangle &= 2(2\pi)^3 p_r^+ \delta(p_r^+ - p_k'^+)\delta^{(2)}(\mathbf{p}_{\perp,r} - \mathbf{p}'_{\perp,k})\delta_{\tau_r,\tau'_k}\delta_{\lambda_r,\lambda'_k}\delta_{r,k} \\ &= 2(2\pi)^3 \xi_r \delta\left(\xi_r - \frac{P'^+}{P^+}\xi'_k\right)\delta^{(2)}(\xi_r \mathbf{P}_\perp + \mathbf{k}_{\perp r} - \xi'_k \mathbf{P}'_\perp - \mathbf{k}'_{\perp k})\delta_{\tau_r,\tau'_k}\delta_{\lambda_r,\lambda'_k}\delta_{r,k} \,,\end{aligned} \tag{167}$$

Where we denote with $\delta_{r,k}$ the orthogonality over all the intrinsic quantum numbers. The above expression can be generalized to the nuclear case, viz.:

$$\langle A|A'\rangle = 2(2\pi)^3 P^+ \delta(P^+ - P'^+)\delta^{(2)}(\mathbf{P}_\perp - \mathbf{P}'_\perp) \,, \tag{168}$$

therefore, after inserting Eq. 165, the orthonormalization rule of the nuclear wfs reads:

$$\begin{aligned}\langle A|A'\rangle = &\sum_{\tau_1,\dots,\tau_A=n,p}\sum_{\tau'_1,\dots,\tau'_A=n,p}\int\left[\prod_{r=1}^A \frac{d\xi_r d^2k_{\perp,r}}{\sqrt{\xi_r}}\right]\left[\prod_{r=1}^A \frac{d\xi'_r d^2k'_{\perp,r}}{\sqrt{\xi'_r}}\right]\left[\prod_{r=1}^A\sum_{\lambda_r}\right]\left[\prod_{r=1}^A\sum_{\lambda'_r}\right] \\ &\times \frac{1}{[2(2\pi^3)]^{A-1}}\langle n_1|n'_1\rangle\langle n_2|n'_2\rangle\dots\langle n_A|n'_A\rangle \\ &\times \psi^\dagger(\xi_1,..,\xi_A,\mathbf{k}_{1,\perp},...,\mathbf{k}_{\perp,A},\tau_1,...,\tau_A,\lambda_1,...,\lambda_A)\delta\left(1-\sum_{k=1}^A\xi_k\right)\delta\left(\sum_{l=1}^A\mathbf{k}_{\perp,l}\right) \\ &\times \psi(\xi'_1,..,\xi'_A,\mathbf{k}'_{\perp,1},...,\mathbf{k}'_{\perp,A},\tau'_1,...,\tau'_A,\lambda'_1,...,\lambda'_A)\delta\left(1-\sum_{k=1}^A\xi'_k\right)\delta\left(\sum_{l=1}^A\mathbf{k}'_{\perp,l}\right) \,.\end{aligned} \tag{169}$$

$$\begin{aligned}\langle A|A'\rangle = &\frac{1}{[2(2\pi^3)]^{A-1}}\sum_{\tau_1,\dots,\tau_A=n,p}\int\left[\prod_{r=1}^A \frac{d\xi_r d^2k_{\perp,r}}{\sqrt{\xi_r}}\right]\left[\prod_{r=1}^A\sum_{\lambda_r}\right] \\ &\times \psi^\dagger(\xi_1,..,\xi_A,\mathbf{k}_{1,\perp},...,\mathbf{k}_{\perp,A},\tau_1,...,\tau_A,\lambda_1,...,\lambda_A)\delta\left(1-\sum_{k=1}^A\xi_k\right)\delta\left(\sum_{l=1}^A\mathbf{k}_{\perp,l}\right) \\ &\times \left\{\sum_{\tau'_1,\dots,\tau'_A=n,p}\int\left[\prod_{r=1}^A \frac{d\xi'_r d^2k'_{\perp,r}}{\sqrt{\xi'_r}}\right]\left[\prod_{r=1}^A\sum_{\lambda'_r}\right]\langle n_1|n'_1\rangle\langle n_2|n'_2\rangle\dots\langle n_A|n'_A\rangle\right. \\ &\times \left.\psi(\xi'_1,..,\xi'_A,\mathbf{k}'_{\perp,1},...,\mathbf{k}'_{\perp,A},\tau'_1,...,\tau'_A,\lambda'_1,...,\lambda'_A)\delta\left(1-\sum_{k=1}^A\xi'_k\right)\delta\left(\sum_{l=1}^A\mathbf{k}'_{\perp,l}\right)\right\} .\end{aligned} \tag{170}$$

By inserting the result of Eq. 167, the quantity in the curly brackets becomes:

$$\int \left[\prod_{r=1}^{A} \frac{d\xi'_r d^2k'_{\perp,r}}{\sqrt{\xi'_r}} \sum_{\lambda'_r,\tau'_r}\right] \left[\prod_{r}^{A} \langle n_r|n'_r\rangle\right] \psi(\xi'_1,..,\xi'_A,\mathbf{k}'_{\perp,1},...,\mathbf{k}'_{\perp,A},\tau'_1,...,\tau'_A,\lambda'_1,...\lambda'_A)\delta\left(1-\sum_{k=1}^{A}\xi'_k\right)\delta\left(\sum_{l=1}^{A}\mathbf{k}'_{\perp,l}\right) = \tag{171}$$

$$= [2(2\pi^3)]^A \left[\prod_{r=1}^{A}\sum_{\lambda'_r,\tau'_r}\sqrt{\xi_r}\right] P^+\delta\left(P^+-P'^+\right)\delta\left(\mathbf{P}_\perp-\mathbf{P}'_\perp\right)\psi(\xi_1,..,\xi_A,\mathbf{k}_{1,\perp},...,\mathbf{k}_{\perp,A},\tau'_1,...,\tau'_A,\lambda'_1,...,\lambda'_A)$$

$$\times \prod_{i=r}^{A}\delta(\tau_r-\tau'_r)\delta(\lambda'_r-\lambda_r)\,,$$

and therefore:

$$\langle A|A'\rangle = 2(2\pi^3)P^+\delta\left(P^+-P'^+\right)\delta\left(\mathbf{P}_\perp-\mathbf{P}'_\perp\right) \tag{172}$$

$$\times \int \left[\prod_{r=1}^{A} d\xi_r d^2k_{\perp,r}\sum_{\lambda_r}\sum_{\tau_r=n,p}\right] |\psi(\xi_1,..,\xi_A,\mathbf{k}_{1,\perp},...,\mathbf{k}_{\perp,A},\tau_1,...,\tau_A,\lambda_1,...,\lambda_A)|^2\delta\left(1-\sum_{k=1}^{A}\xi_k\right)\delta\left(\sum_{l=1}^{A}\mathbf{k}_{\perp,l}\right)\,.$$

Thus the normalization of the nuclear LF wf reads:

$$\sum_{\tau_1,...,\tau_A=n,p}\int \left[\prod_{r=1}^{A} d\xi_r d^2k_{\perp,r}\sum_{\lambda_r}\right] |\psi(\xi_1,..,\xi_A,\mathbf{k}_{1,\perp},...,\mathbf{k}_{\perp,A},\tau_1,...,\tau_A,\lambda_1,..,\lambda_A)|^2\delta\left(1-\sum_{k=1}^{A}\xi_k\right)\delta\left(\sum_{l=1}^{A}\mathbf{k}_{\perp,l}\right) = 1\,. \tag{173}$$

Once the nuclear state has been properly defined in terms of its constituents and the normalization of the wave function is established, the expression given in Eq. (165) can be used to evaluate nuclear distributions in Eq. (159).

Nuclear PDFs in impulse approximation

In order to properly allow the bilinear operators in Eq. (159) to act on the intrinsic part of the nucleonic states which describe the nucleon internal structure, we adopt the decomposition of the nuclear operators in impulse approximation. In this framework, each operator in Eq. (159) is given, for each quark flavor, by an incoherent sum over operators acting on nucleon states:

$$\theta_q(0,z) \Longrightarrow \Theta_q(0,z) = \sum_{l}^{A}\theta_q^l\,(0,z)\,, \tag{174}$$

where the superscript l in θ specifies that the operator acts only on the partons pertaining to the nucleon l. From now on, taking care of antisymmetry in the nuclear wf:

$$\Theta_q(0,z) = A\theta_q^1(0,z). \tag{175}$$

Once the above operator is considered in Eq. (159) and the nuclear expansion of Eq. (165) for the state $|A\rangle$ is taken into account one can contract all the nucleon states that are not ivolved in the matrix element. We live just two nucleon to show how it works:

$$f_q^A(x) = \frac{2P^+}{[2(2\pi)^3]} \sum_{\tau_1,\tau_2=n,p} \sum_{\lambda_1,\lambda_2} \sum_{\lambda_1',\lambda_2'} \int \frac{dz^-}{2\pi} \frac{d\xi_1 d\xi_2 d\xi_1'}{\sqrt{\xi_1\xi_2\xi_1'(\xi_1+\xi_2-\xi_1')}} d^2k_{1,\perp} d^2k_{2,\perp} d^2k'_{\perp,1} \tag{176}$$
$$\times\, e^{ixP^+z^-} \psi(\xi_1,\xi_2,\mathbf{k}_{1,\perp},\mathbf{k}_{2,\perp},\tau_1,\tau_2,\lambda_1,\lambda_2)\psi^\dagger(\xi_1',\xi_2',\mathbf{k}'_{\perp,1},\mathbf{k}'_{\perp,2},\tau_1,\tau_2,\lambda_1',\lambda_2')$$
$$\times\, \langle n_1'|\langle n_2'|\Theta_i(y,z_1)\,\Theta_j(0,z_2)\,|n_1\rangle|n_2\rangle\Big|_{z^+=0}^{\mathbf{z}_\perp=\mathbf{0}},$$

where $\xi_2' = \xi_1+\xi_2-\xi_1'$, $\mathbf{k}'_{\perp,2} = \mathbf{k}_{1,\perp}+\mathbf{k}_{2,\perp}-\mathbf{k}'_{\perp,1}$ and

$$\psi(\xi_1,\xi_2,\mathbf{k}_{1,\perp},\mathbf{k}_{2,\perp},\tau_1,\tau_2,\lambda_1,\lambda_2)\psi^\dagger(\xi_1',\xi_2',\mathbf{k}'_{\perp,1},\mathbf{k}'_{\perp,2},\tau_1,\tau_2,\lambda_1',\lambda_2') \tag{177}$$
$$\equiv \sum_{\tau_3,..,\tau_A=n,p} \sum_{\lambda_3,..,\lambda_A} \int \left[\prod_{r=3}^{A-1} d\xi_r d^2k_{\perp,r}\right] \psi(\xi_1,..,\xi_A,\mathbf{k}_{1,\perp},...,\mathbf{k}_{\perp,A},\tau_1,...,\tau_A,\lambda_1,...,\lambda_A)$$
$$\times\, \psi^\dagger(\xi_1',\xi_2',\xi_3,..,\xi_A,\mathbf{k}'_{\perp,1},\mathbf{k}'_{\perp,2},\mathbf{k}_{\perp,3},...,\mathbf{k}_{\perp,A},\tau_1,...,\tau_A,\lambda_1',\lambda_2',\lambda_3,...,\lambda_A).$$

In Eq. (177), one has $\xi_A = 1-\sum_{r=1}^{A-1}\xi_r$ and $\mathbf{k}_{\perp,A} = -\sum_{r=1}^{A-1}\mathbf{k}_{\perp,r}$. The normalization of the above quantity reads:

$$\sum_{\tau_1,\tau_2}\sum_{\lambda_1,\lambda_2}\int d\xi_1 d\xi_2 d^2k_{1,\perp} d^2k_{2,\perp} |\psi(\xi_1,\xi_2,\mathbf{k}_{1,\perp},\mathbf{k}_{2,\perp},\tau_1,\tau_2,\lambda_1,\lambda_2)|^2 = 1\,. \tag{178}$$

Notice that in order to compare the results here presented to those usually obtained, for example for the EMC effect, it is necessary to evaluate PDFs as function of:

$$x_l \equiv \frac{q_l^+}{P^+}\frac{M_A}{M} = \frac{q_l^+}{P^+}\frac{1}{\bar\xi} \tag{179}$$

with:

$$\bar\xi = \frac{M}{M_A} \sim \frac{1}{A}\,. \tag{180}$$

N.B. In order to avoid a heavy notation, in the following we adopt the same symbol for the longitudinal fraction of the parton momentum w.r.t. to the nucleus introduced in Eq. 164, but with a different normalization, i.e. $\sum_\ell x_\ell = M_A/M \sim A$ and not equal to 1. This amounts to have an average nucleon plus-component given by $\sim P_A^+/A$. One should notice that:

$$x_1 + x_2 \le (q_1^+/P_A^+)(M_A/M) + [(P_A^+ - q_1^+)/P_A^+](M_A/M) = M_A/M\,. \tag{181}$$

Let us discuss separately the two possibilities where the operators act on partons belonging to i) the same nucleon, i.e. the DPS1 mechanism, or ii) two different nucleons, i.e. the DPS2 mechanism.

The final formula

In this case one can exploit also the inner product $\langle n_2'|n_2\rangle$ in Eq. (176). Thus, by using x_l of Eq. 179, one can simplify Eq. (176):

$$f_q^A(x) = A\,2P^+ \sum_{\tau,\tau_2=n,p} \int \frac{dz^-}{2\pi}\int\frac{d\xi_1}{\xi_1}\int d\xi_2 \int d^2k_{1,\perp}\int d^2k_{2,\perp}\, e^{ixP^+z^-/\bar\xi} \tag{182}$$
$$\times\, \bar\xi^2 |\psi(\xi_1,\xi_2,\mathbf{k}_{1,\perp},\mathbf{k}_{2,\perp},\tau,\tau_2)|\,\langle n_1|\theta_q(0,z)\,|n_1\rangle\Big|_{z^+=0}^{\mathbf{z}_\perp==\mathbf{0}}.$$

where the $\bar{\xi}$ is needed to preserve the normalization of the distribution and you keep integrating you get:

$$f_q^A(x) = A \sum_{\tau=n,p} \int d\xi_1 \, \frac{\bar{\xi}}{\xi_1} \, \rho_\tau^A(\xi_1) \, f_q^\tau \left(x \frac{\bar{\xi}}{\xi_1} \right) \, , \tag{183}$$

where $\rho_\tau^A(\xi_1)$ is the one-body LCMD of a nucleus with A nucleons, given by

$$\rho_\tau^A(\xi_1) \equiv \sum_{\tau_2=n,p} \int d\xi_2 \int d^2 k_{1,\perp} \int d^2 k_{2,\perp} |\psi(\xi_1, \xi_2, \mathbf{k}_{1,\perp}, \mathbf{k}_{2,\perp}, \tau, \tau_2)|^2 \tag{184}$$

and normalized as follows

$$\sum_\tau \int d\xi \, \rho_\tau^A(\xi) = 1 \, . \tag{185}$$

To compare with other results in literature one can use:

$$\bar{\rho}_\tau(\xi) \equiv A \rho_\tau(\xi) \, . \tag{186}$$

Acronym Index

BSA Beam Spin Asymmetry. 21

CFF Compton Form Factors. 19

DIS Deep Inelastic Scattering. 3

EMT Energy Momentum Tensor. 13

FFs Form Factors. 12

gFFs Gravitational Form Factors. 13

GPDs Generalized Parton Distribution Functions. 6

IA Impulse Approximation. 23

IMF Infinite Momentum Frame. 9, 17, 18

LC Light-Cone. 4

OAM Orbital Angular Momentum. 28

PDFs Parton Distribution Functions. 5

SFs Structure Functions. 5

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