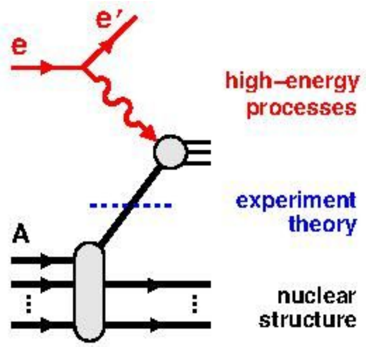
Polarization in nuclear high-energy scattering

Wim Cosyn, FIU



How did you come across spin/polarization in your research/work/studies?

<your answers here>

Plan for this lecture

- As before, we need both elements of **QM** and **SR**
- Goals
  - cover basics and important concepts
  - no detailed derivations (refs) ,but will include the abstract math
  - demystify some of the jargon
- Spin / density matrix in the preferred (rest) frame (~QM)
  →slides (review)
- How do we change frames (~boosts,SR)
  →whiteboard
- Polarization in observables/experiment
- Case study: spin 1 (if time)

Spin in a preferred frame

- Consider free massive particle + rest frame
- Spin as a feature of rotational invariance in nature
  - Particle can have degenerate states (=different polarizations)
    →irreps of rotation group SU(2)
  - If we rotate a certain state, we get back a general linear combination
  - Choose a basis of eigenstates of J_z (what the z-axis is matters)

    $$U(r)|sm\rangle = \mathscr{D}^{(s)}_{m'm}(r)|sm'\rangle$$ (Wigner D-matrices, unitary)

  - Preferred direction (magnetic field) can break this rotational symmetry (degeneracy lifted)
  - Coupling of spins with angular momentum coupling (CG coefficients)
    → see lectures Gnech

Density matrix

- Characterizes state of an **ensemble** of particles (cfr. stat.mech.)

- **Idealisation** (textbook case)
  - Pure state

$$|\psi\rangle = \sum_{m=-s}^{s} c_m |sm\rangle \qquad O_{mm'} = \langle sm|\hat{O}|sm'\rangle \qquad \left\langle \hat{O} \right\rangle_{\psi} \equiv \langle\psi|\hat{O}|\psi\rangle = \sum_{m,m'} c^*_{m'} O_{m'm} c_m$$

- **Reality**
  - Mixed states: different pure states each with a **probability** (≠ superposition)

$$\left\langle \hat{O} \right\rangle = \sum_i p^{(i)} \left\langle \hat{O} \right\rangle_{\psi^{(i)}} = \sum_{m,m'} O_{m'm} \sum_i p^{(i)} c^{(i)*}_{m'} c^{(i)}_m$$

  - Introduce **density matrix** (for that particular basis)

$$\rho_{mm'} = \sum_i p^{(i)} c^{(i)}_m c^{(i)*}_{m'} \qquad \left\langle \hat{O} \right\rangle = \sum_{m,m'} O_{m'm} \rho_{mm'} = \text{Tr}\,(O\rho)$$

↪ [[expectation values ↔ density matrix]] (polarimetry)

- In general: density matrix applies to all labels used to uniquely characterize basis states, here we focus on spin only
  ↪ **reduced** density matrix
- QFT textbook “average over all initial spin states” is the default (unpolarized) density matrix

Density matrix: general properties (exercises) $\rho_{mm'} = \sum_i p^{(i)} c_m^{(i)} c_{m'}^{(i)*}$

- Normalization of probability leads to $\text{Tr}\ \rho = 1$
- In any basis diagonal elements are positive $\rho_{mm} \geq 0$
  ↪ includes the eigenvalues
- Hermitian $\rho^*_{mm'} = \rho_{m'm}$
  ↪ can be **diagonalized** by a **unitary** transformation
  ↪ eigenvalues probabilities for orthonormal set (cfr populations in presence of **B**)
  ↪ not all those unitary transformations are rotations! (exception spin ½)
- $\text{Tr}\ \rho^2 = \text{Tr}\ \left(\rho^{\text{D}}\right)^2 = \sum_m \lambda_m^2 \leq \left(\sum_m \lambda_m\right)^2 = (\ \text{Tr}\ \rho)^2 = 1$
  ↪ Equality for pure state only
- Von Neumann entropy $S = -\,\text{tr}(\rho \ln \rho)$
  ↪ S = 0 for pure state

Density matrix: multipole decomposition

(whiteboard)

Spin + relativity: main takeaways

- Need to introduce the concept of a standard boost
  ↪ different choices, each with their unique advantages
- Boosts of general states lead to spin rotations (Wigner rotation)
- Different boosts are related by Melosh rotations
- Density matrices transform with Wigner rotations under boosts
- Covariant density matrices transform nicely, leads to covariant spin polarization vectors and tensors

Covariant polarization parameters

$$
\begin{aligned}
&\mathcal{F}_S \\
&= S_L\left[\sqrt{2\epsilon(1+\epsilon)}\sin\phi_h F_{US_L}^{\sin\phi_h} + \epsilon\sin 2\phi_h F_{US_L}^{\sin 2\phi_h}\right] \\
&+ S_L(2\lambda_e)\left[\sqrt{1-\epsilon^2}F_{LS_L} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_h F_{LS_L}^{\cos\phi_h}\right] \\
&+ S_T\Big[\sin(\phi_h-\phi_S)\left(F_{US_T,T}^{\sin(\phi_h-\phi_S)} + \epsilon F_{US_T,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad + \epsilon\sin(\phi_h+\phi_S)F_{US_T}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \epsilon\sin(3\phi_h-\phi_S)F_{US_T}^{\sin(3\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)}\sin\phi_S F_{US_T}^{\sin\phi_S} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)}\sin(2\phi_h-\phi_S)F_{US_T}^{\sin(2\phi_h-\phi_S)}\Big] \\
&+ S_T(2\lambda_e)\Big[\sqrt{1-\epsilon^2}\cos(\phi_h-\phi_S)F_{LS_T}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LS_T}^{\cos\phi_S} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi_h-\phi_S)F_{LS_T}^{\cos(2\phi_h-\phi_S)}\Big]\,. \quad (4.38)
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{F}_T \\
&= T_{LL}\Big[F_{UT_{LL},T} + \epsilon F_{UT_{LL},L} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h F_{UT_{LL}}^{\cos\phi_h} \\
&\qquad + \epsilon\cos 2\phi_h F_{UT_{LL}}^{\cos 2\phi_h}\Big] \\
&+ T_{LL}(2\lambda_e)\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h F_{LT_{LL}}^{\sin\phi_h} \\
&+ T_{LT}\Big[\cos(\phi_h-\phi_{T_L}) \\
&\qquad \times\left(F_{UT_{LT},T}^{\cos(\phi_h-\phi_{T_L})} + \epsilon F_{UT_{LT},L}^{\cos(\phi_h-\phi_{T_L})}\right) \\
&\qquad + \epsilon\cos(\phi_h+\phi_{T_L})F_{UT_{LT}}^{\cos(\phi_h+\phi_{T_L})} \\
&\qquad + \epsilon\cos(3\phi_h-\phi_{T_L})F_{UT_{LT}}^{\cos(3\phi_h-\phi_{T_L})} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_{T_L}F_{UT_{LT}}^{\cos\phi_{T_L}} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)}\cos(2\phi_h-\phi_{T_L})F_{UT_{LT}}^{\cos(2\phi_h-\phi_{T_L})}\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+ T_{LT}(2\lambda_e)\Big[\sqrt{1-\epsilon^2}\sin(\phi_h-\phi_{T_L})F_{LT_{LT}}^{\sin(\phi_h-\phi_{T_L})} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)}\sin\phi_{T_L}F_{LT_{LT}}^{\sin\phi_{T_L}} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)}\sin(2\phi_h-\phi_{T_L})F_{LT_{LT}}^{\sin(2\phi_h-\phi_{T_L})}\Big] \\
&+ T_{TT}\Big[\cos(2\phi_h-2\phi_{T_T}) \\
&\qquad \times\left(F_{UT_{TT},T}^{\cos(2\phi_h-2\phi_{T_T})} + \epsilon F_{UT_{TT},L}^{\cos(2\phi_h-2\phi_{T_T})}\right) \\
&\qquad + \epsilon\cos 2\phi_{T_T}F_{UT_{TT}}^{\cos 2\phi_{T_T}} \\
&\qquad + \epsilon\cos(4\phi_h-2\phi_{T_T})F_{UT_{TT}}^{\cos(4\phi_h-2\phi_{T_T})} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h-2\phi_{T_T})F_{UT_{TT}}^{\cos(\phi_h-2\phi_{T_T})} \\
&\qquad + \sqrt{2\epsilon(1+\epsilon)}\cos(3\phi_h-2\phi_{T_T})F_{UT_{TT}}^{\cos(3\phi_h-2\phi_{T_T})}\Big] \\
&+ T_{TT}(2\lambda_e)\Big[\sqrt{1-\epsilon^2}\sin(2\phi_h-2\phi_{T_T})F_{LT_{TT}}^{\sin(2\phi_h-2\phi_{T_T})} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)}\sin(\phi_h-2\phi_{T_T})F_{LT_{TT}}^{\sin(\phi_h-2\phi_{T_T})} \\
&\qquad + \sqrt{2\epsilon(1-\epsilon)}\sin(3\phi_h-2\phi_{T_T})F_{LT_{TT}}^{\sin(3\phi_h-2\phi_{T_T})}\Big]\,.
\end{aligned}
$$