Topics

- Introduction to GPDs:
 - 1. Open questions in nuclear high-energy physics: Proton spin crisis and EMC effect
 - 2. Introduction to DVCS and GPDs
 - 3. Forward limit and first moment of GPDs
 - 4. Tomography
 - 5. Momentum and Ji's sum rule
 - 6. Cross-section and Asymmetries
 - 7. Multi-dimensional picture of hadrons
- Nuclear GPDs:
 - 1. Why nuclear GPDs and DVCS in IA: the 3 He case
 - 2. Coherent DVCS on spin 1/2 nuclei in IA and NR limit
 - 3. Some Numerical results
 - 4. Some experiments

- Examples:
 - 1. Megnetic FF
 - 2. Fock expansion for hadronic states
 - 3. Use of the Fock expansion as a tool to evaluate nuclear distributions in IA. The case of PDFs.
 - 4. If I will have time also Double Parton Distributions.





Elastic Scattering



Light-Front wave-function





Multidimensional picture of hadrons $\int \frac{\mathrm{d}\mathbf{z}^{-}}{4\pi} \mathrm{e}^{\mathsf{x}\mathsf{P}^{+}\mathbf{z}^{-}} \langle \mathsf{P}|\bar{\psi}_{\mathsf{q}}(0)\gamma^{+}\psi_{\mathsf{q}}(\mathbf{z}^{-})|\mathsf{P}\rangle$

Deep Inelastic Scattering



Light-Front wave-function













The EMC effect

In DIS off a nuclear target with A nucleons:

$$0 \le x = \frac{Q^2}{2M\nu} \le \frac{M_A}{M} \simeq A$$

 $0.2 \le x \le 0.8$ "EMC (binding) region": mainly valence quarks involved

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 $\frac{F_2^A}{F_2^d}$



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Naive parton model interpretation:

"Valence quarks, in the bound nucleon, are in average slower that in the free nucleon"

Is the bound proton bigger than the free one??

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The EMC effect

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Explanation (exotic) advocated: confinement radius bigger for bound nucleons, quarks in bags with 6, 9,..., 3A quark, pion cloud effects... Alone or mixed with conventional ones...

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Nuclear SFs and EMC ratio

To calculate the EMC ratio $R_{EMC}^A(x) = \frac{F_2^A(x)}{F_2^d(x)}$ for any nucleus A, we need the nuclear SFs.

Within our approach we have:



1) in the Bjorken limit we have the LCMD: $f_1^N(\xi) =$

Since our approach fulfill both macro-locality and Poincaré covariance the LC momentum distribution satisfies 2 essential sum rules at the same time:

 $A = \int_{-1}^{1} d\xi [Zf_1^p(\xi) + (A - Z)f^n(\xi)]: \text{Baryon number SR}$

$$1 = Z < \xi >_p + (Z - N) < \xi >_n; < \xi >_N = \int_0^\infty d\xi \,\xi f_1^N$$

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$$F_2^N\left(\frac{mx}{\xi M_A}\right) \quad f_A^N(\xi)$$

 $\xi =$ longitudinal momentum fraction carried by a nucleon in the nucleus

$$\oint d\epsilon \int \frac{d\kappa_{\perp}}{(2\pi)^3} \frac{1}{2\kappa^+} P^N(\tilde{\kappa},\epsilon) \frac{E_s}{1-\xi}$$

Unpolarized LF spectral function: $P^{N}(\tilde{\kappa}, \epsilon) = \frac{1}{2j+1} \sum_{\mathscr{M}} P^{N}_{\sigma\sigma}(\tilde{\kappa}, \epsilon, \mathbf{S}, \mathscr{M})$

$N(\xi)$: Momentum SR (MSR)



Nuclear DVCS

In the nuclear case we have two channels:

Coherent channel — we access the GPDs of the nucleus Tomography of the nucleus

→ we access the GPDs of the bound nucleons **Incoherent channel-**Same distribution of the free one? Tomography of the bound nucleon







Double Parton Scattering

Multiparton interaction (MPI) can contribute to the, pp, pA and AA cross section @ the LHC:



Transverse distance between two partons

$$\begin{split} F_{ij}^{\lambda_{1},\lambda_{2}}(x_{1},x_{2},\vec{k}_{\perp}) &= (-8\pi P^{+})\frac{1}{2}\sum_{\lambda}\int d\vec{z}_{\perp} \, e^{\mathrm{i}\vec{z}_{\perp}\cdot\vec{k}_{\perp}} \\ &\times \int \left[\prod_{l}^{3}\frac{dz_{l}^{-}}{4\pi}\right] e^{ix_{1}P^{+}z_{1}^{-}/2} e^{ix_{2}P^{+}z_{2}^{-}/2} e^{-ix_{1}P^{+}z_{3}^{-}/2} \\ &\times \langle\lambda,\vec{P}=\vec{0}|\hat{\mathcal{O}}_{i}^{1}\left(z_{1}^{-}\frac{\bar{n}}{2},z_{3}^{-}\frac{\bar{n}}{2}+\vec{z}_{\perp}\right)\hat{\mathcal{O}}_{j}^{2}\left(z_{2}^{-}\frac{\bar{n}}{2}+\vec{z}_{\perp}\right) \end{split}$$

 $+ \vec{z_{\perp}}, 0$

 $d\sigma \propto \int d^2 z_{\perp} F_{ij}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z}_{\perp}, \mu_A, \mu_B) F_{kl}(\mathbf{x}_3, \mathbf{x}_4, \mathbf{z}_{\perp}, \mu_A, \mu_B)$

Double Parton Distribution (DPD) N. Paver and D. Treleani, Nuovo Cimento 70A, 215 (1982) Mekhfi, PRD 32 (1985) 2371 M. Diehl et al, JHEP 03 (2012) 089

$$\hat{\mathcal{O}}_i^k(z,z') = \bar{q}_i(z)\hat{\mathcal{O}}(\lambda_k)q_i(z')$$
 $\hat{\mathcal{O}}_i^k(z,z') = \bar{q}_i(z)\hat{\mathcal{O}}(\lambda_k)q_i(z')$
 $\hat{\mathcal{O}}(\lambda_k) = \frac{\bar{n}}{2}\frac{1+\lambda_k\gamma_5}{2}$.



DPS in pA collisions

$$egin{aligned} \mathsf{F}_{\mathsf{a}_1\mathsf{a}_2}(\mathsf{x}_1,\mathsf{x}_2,\mathbf{y}_{ot}) &= 2p^+ \int rac{dz_1^-}{2\pi} rac{dz_2^-}{2\pi} dy^- e^{i(x_1z_1^-+x_2z_2^-)p^+} \ & imes \langle \mathsf{A} ig| \mathcal{O}_{\mathsf{a}_2}(\mathsf{0},\mathsf{z}_2) \mathcal{O}_{\mathsf{a}_1}(\mathsf{y},\mathsf{z}_1) ig| \mathsf{A}
angle \end{aligned}$$



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In this case we have two mechanisms that contribute:

B. Blok et al, EPJC (2013) 73:2422



$$egin{aligned} \mathsf{F}_{\mathsf{a}_1\mathsf{a}_2}(\mathsf{x}_1,\mathsf{x}_2,\mathbf{y}_{ot}) &= 2p^+ \int rac{dz_1^-}{2\pi} rac{dz_2^-}{2\pi} dy^- e^{i(x_1z_1^-+x_2z_2^-)p^-} \ & imes \langle \mathsf{A} ig| \mathcal{O}_{\mathsf{a}_2}(\mathsf{0},\mathsf{z}_2) \mathcal{O}_{\mathsf{a}_1}(\mathsf{y},\mathsf{z}_1) ig| \mathsf{A}
angle \end{aligned}$$

DPS 2: The two partons belong to the DIFFERENT nucleons in the nucleus!

$$\begin{split} \widetilde{\mathsf{F}}_{\mathsf{a}_1\mathsf{a}_2}^2(\mathsf{x}_1,\mathsf{x}_2,ec{\mathsf{k}}_\perp) \propto & \int rac{1}{\xi_1\xi_2} \prod_{i=1}^{i=\mathsf{A}} rac{\mathsf{d}\xi_i\mathsf{d}^2\mathsf{p}_{\mathsf{t}i}}{\xi_i} \deltaigg(\sum_i \xi_i - \mathsf{A}igg) \delta^{(2)} igg(\sum_i \xi_i) \\ & imes \psi_\mathsf{A}\Big(\xi_1,\xi_2,\mathsf{p}_{\mathsf{t}1}+ec{\mathsf{k}}_\perp,\mathsf{p}_{\mathsf{t}2}-ec{\mathsf{k}}_\perp,\ldots\Big) \mathsf{G}_{\mathsf{a}_1}^{\mathsf{N}_1}\Big(\mathsf{x}_i) \end{split}$$

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In this case we have two mechanisms that contribute:

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DPS2 in yA collisions with light nuclei

For example in DPS2:

$$\begin{split} D_{ij}^{A,2}(x_1, x_2, \mathbf{k}_{\perp}) &= A(A-1) \sum_{\tau_1, \tau_2 = n, p} \sum_{\lambda_1, \lambda_2} \sum_{\lambda'_1, \lambda'_2} \int d\xi_1 \ \frac{\xi}{\xi_1} \\ &\times \Phi_{\lambda_1, \lambda'_1}^{\tau_1, i} \left(x_1 \frac{\bar{\xi}}{\xi_1}, 0, \mathbf{k}_{\perp} \right) \Phi_{\lambda_2, \lambda'_2}^{\tau_2, j} \left(x_2 \frac{\bar{\xi}}{\xi_2}, 0, -\mathbf{k}_{\perp} \right) \ . \end{split}$$

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 $\frac{\xi}{1} \int d\xi_2 \, \frac{\xi}{\xi_2} \, \rho^A_{\tau_1 \tau_2}(\xi_1, \xi_2, \mathbf{k}_\perp, \lambda_1, \lambda_2, \lambda_1', \lambda_2')$





S. WWW.KANOT

DPS2 in γ A collisions with light nuclei

For example in DPS2:

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Standard LC correlator parametrized by GPDs





Off-forward LC momentum distribution

We address the possible role of nucleon spin-flip effects for the first time!

1) the Off-forward LCMDs which depends

of the deuteron obtained within the Av18

Potential + LF approach to properly fulfill

the Poincaré covariance

2) the role of spin effects could be important to make **Realistic predictions**



GPDs properties

- Forward limit: $\Delta^{\mu} = 0$

 $\begin{array}{c} \mathsf{H}(\mathsf{x},\xi,\mathsf{t}) \longrightarrow \mathsf{f}(\mathsf{x}) \\ \Delta^{\mu} \rightarrow 0 \end{array}$

- First moment: relations between GPDs and form factors

$$\int_{-1}^{1} dx H_{q}(x,\xi,t) = F_{1}^{q}(t) \qquad \int_{-1}^{1} dx H_{q}(x,\xi,t) = F_{1}^{q}(t)$$



f(x) = Parton Distribution Function (PDF)

 $dx \ E_q(x,\xi,t) = F_2^q(t)$

 ξ -independence is a consequence of Lorentz invariance



GPDs properties

- Forward limit: $\Delta^{\mu} = 0$

 $H(x, \xi, t) \longrightarrow_{\Delta^{\mu} \to 0} f(x) \qquad f(x) = Parton Distribution Function (PDF)$

- First moment: relations between GPDs and form factors $\int_{-1}^{1} dx H_{q}(x,\xi,t) = F_{1}^{q}(t) \qquad \int_{-1}^{1} dx E_{q}(x,\xi,t) = F_{2}^{q}(t)$

- Lorentz invariance implies polynomiality:

$$egin{array}{lll} \displaystyle \int_{-1}^1 dx\, x^n H^q(x,\xi,t) &=& \displaystyle \sum_{\substack{i=0\ \mathrm{even}}}^n \ \displaystyle \int_{-1}^1 dx\, x^n E^q(x,\xi,t) &=& \displaystyle \sum_{\substack{i=0\ \mathrm{even}}}^n \ \displaystyle \sum_{\substack{i=0\ \mathrm{even}}}^n \end{array}$$

 $(2\xi)^{i} A_{n+1,i}^{q}(t) + \operatorname{mod}(n,2) (2\xi)^{n+1} C_{n+1}^{q}(t),$

 $(2\xi)^{i}B_{n+1,i}^{q}(t) - \mathrm{mod}(n,2) (2\xi)^{n+1}C_{n+1}^{q}(t).$

