From nuclear structure to high-energy processes

C. Weiss (JLab), Summer School “Light ion physics in the EIC era”,
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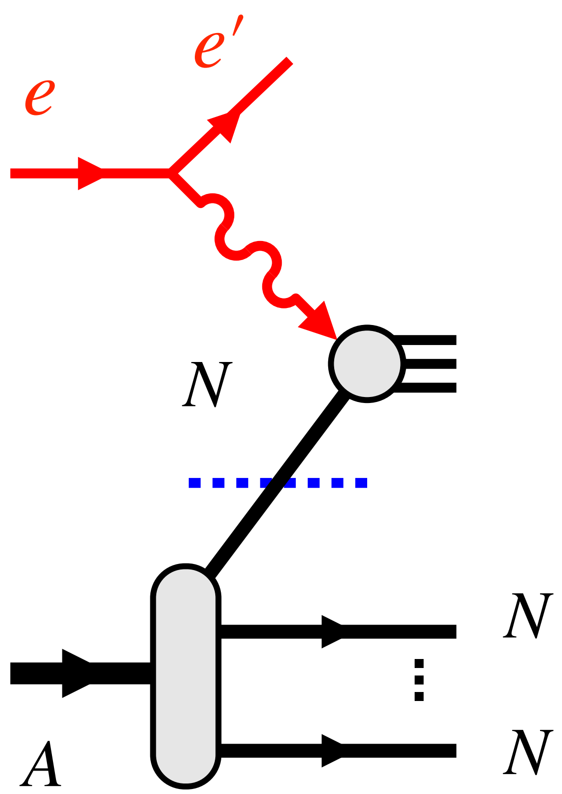
Jefferson Lab



High-energy scattering on nuclei

How to justify/implement a composite description in terms of nucleons?

How to separate/combine structure of nucleus and nucleon?

How to account for nuclear interactions - non-nucleonic DoF?

→ Relativity

→ Light-front methods

Outline

Basic considerations

High-energy scattering on nuclei

Challenges of composite description

Quantum mechanics and relativity

Essential techniques

Non-covariant representation of interactions

Light-front form of relativistic dynamics

High-energy scattering on nuclei

Energy nonconservation in intermediate states

Need for light-front form

Separating nucleus and nucleon structure

Inclusive eA scattering

Collinear frame

Inclusive eD scattering in impulse approx.

Structure functions

Sum rules

Light-front nuclear structrure

Dynamical equation

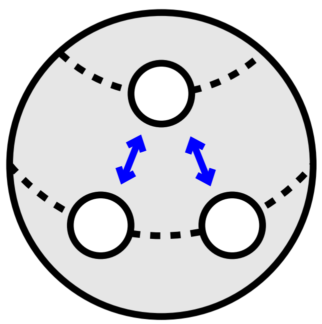
Matching with nonrelativistic structure

Rotationally symmetric representation
in 2-body sector (k-vector)

Deuteron in nonrelativistic approximation

Spin degrees of freedom

Basics: High-energy scattering on nuclei

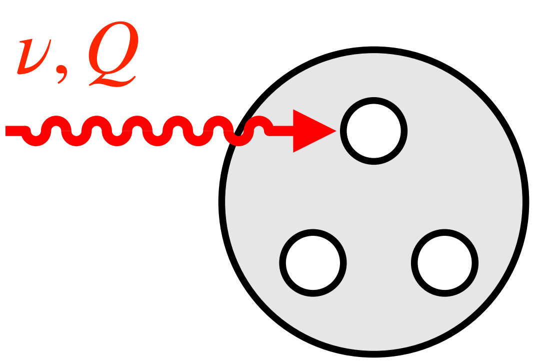


Low-energy structure and processes

Nucleus described in nucleon DoF: Motion, interactions

Other hadrons (π, vectors, Δ, ...) "integrated out" $\rightarrow NN$ interactions

Current operators describe low-energy processes $Q \sim k_{\text{bind}} \sim$ few 10 MeV



High-energy processes

Scattering processes with energy/momentum transfer $\nu, Q \gg$ 1 GeV:
Various probes, final states

How to obtain composite description in terms of nucleons?
Use nucleon-level process as input, combine with nuclear structure?

Nucleon motion?

Nucleon interactions? Non-nucleonic DoF?

This lecture: Use hadronic picture. Consider general high-energy process and focus on combining nuclear and nucleon structure. Connection with QCD (factorization, partonic structure) later.

Basics: Quantum mechanics and relativity

Quantum mechanics $\times$ Relativity

Superposition of configurations,
wave function

Transitions to intermediate states
lifetime $\Delta t \sim 1/\Delta E$

⋮

Scattering kinematics

Boost invariance,
light-front form of dynamics

Hadron creation/annihilation
in intermediate states

⋮

Consequential application of these concepts can lead to surprising conclusions

You know the basic concepts, but may not have seen them applied in this combination

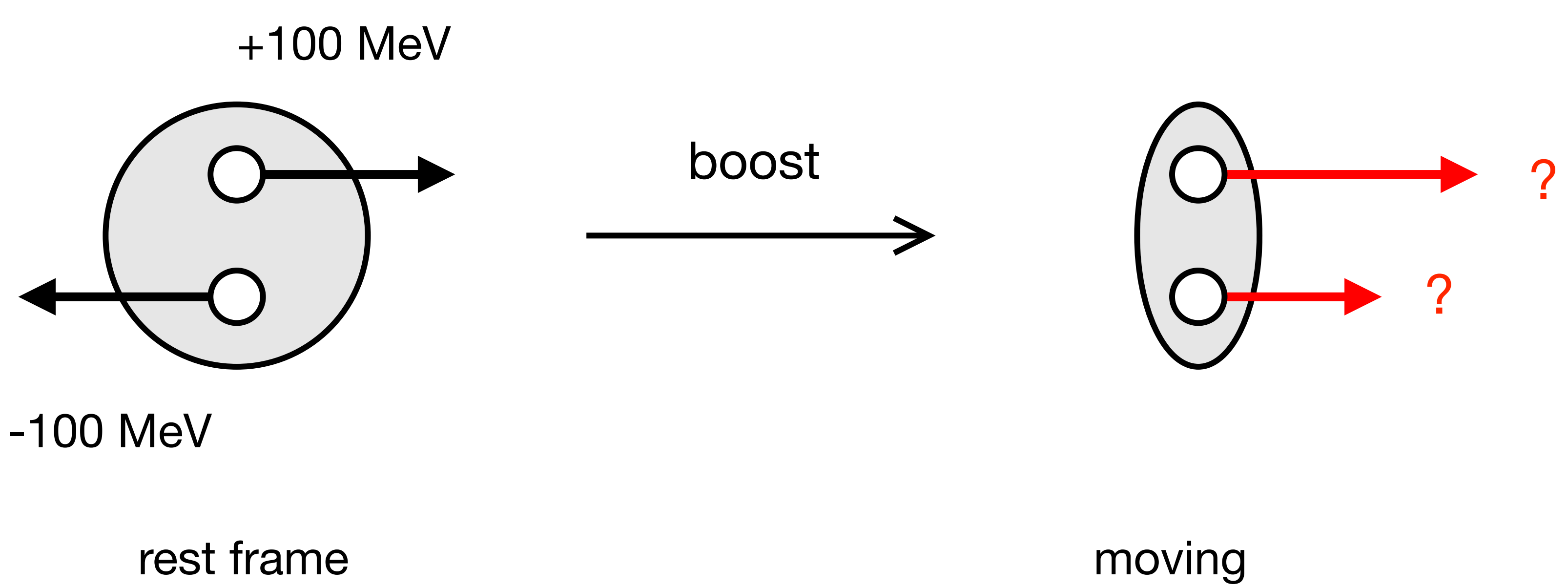
Most/all conclusions can be understood and communicated in simple form

Basics: Nuclear motion

Naive expectation: Motion of nucleons in nucleus with momenta ~ few 10 MeV
has little effect on high-energy scattering processes at $\gg$ 1 GeV

Simple example shows that this is not so!

Example: Nucleon momentum distribution in fast-moving deuteron nucleus



Deuteron moving with $E_D = 200$ GeV
Consider nucleon configuration with z-momentum $p_{1,2} = \pm$ 100 MeV in rest frame
What are the nucleon momenta in the moving deuteron?

Exercise: Perform boost of nucleon configuration in deuteron

$$\begin{pmatrix} E \\ p \end{pmatrix}_{\text{moving}} = \begin{pmatrix} \frac{1}{\sqrt{1-v^2}} & \frac{v}{\sqrt{1-v^2}} \\ \frac{v}{\sqrt{1-v^2}} & \frac{1}{\sqrt{1-v^2}} \end{pmatrix} \begin{pmatrix} E \\ p \end{pmatrix}_{\text{rest}}$$

Lorentz boost

Applies to deuteron as a whole and to individual nucleons

Deuteron: $$(E_D)_{\text{moving}} = \frac{1}{\sqrt{1-v^2}} (E_D)_{\text{rest}} = \frac{1}{\sqrt{1-v^2}} M_D$$

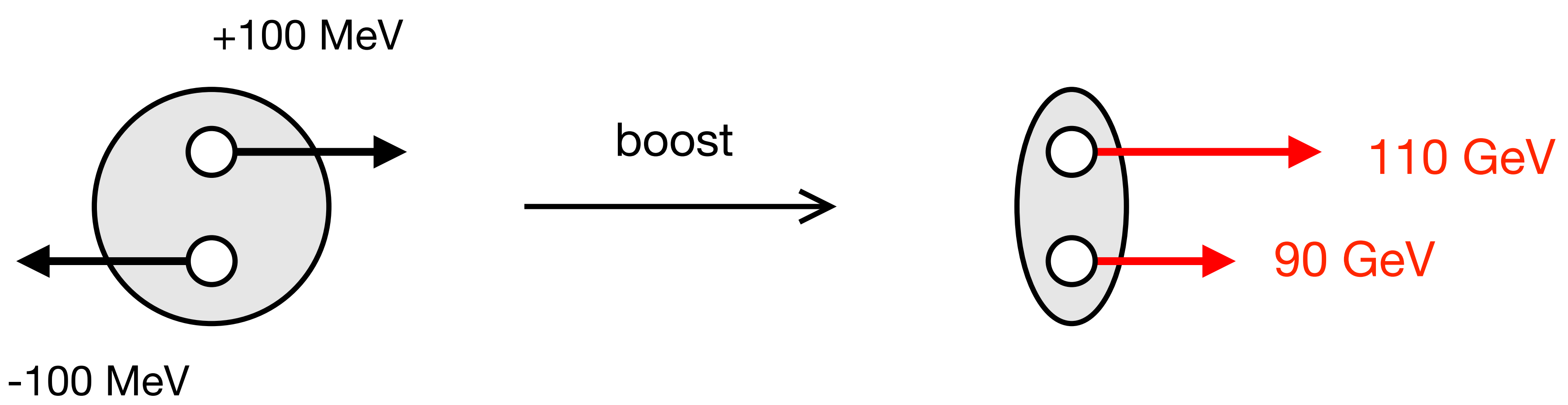
$$\frac{1}{\sqrt{1-v^2}} = \frac{(E_D)_{\text{moving}}}{M_D} = \frac{200 \text{ GeV}}{2 \text{ GeV}} = 100 \qquad \frac{v}{\sqrt{1-v^2}} \approx \frac{1}{\sqrt{1-v^2}} \quad (v \approx 1)$$

Nucleon 1, 2: $(E_{1,2})_{\text{rest}} \approx 1 \text{ GeV}$ $\quad (p_{1,2})_{\text{rest}} = \pm\, 0.1 \text{ GeV}$

$$(p_{1,2})_{\text{moving}} = 100 \times (1 \text{ GeV} \pm 0.1 \text{ GeV}) = \begin{cases} 110 \text{ GeV} \\ 90 \text{ GeV} \end{cases}$$

Large momentum difference!

Basics: Nuclear motion



Boost conserves nucleon light-cone momentum fractions

$$\frac{(E + p^z)_{1,2}}{(E + p^z)_D} = \text{invariant}$$

Internal motion of nucleons with $\sim$ few 10 MeV momenta has large effect on momentum distribution in nucleus moving with $\gg$ 1 GeV momentum

$\rightarrow$ large effect on high-energy scattering processes on nucleus (in any reference frame)

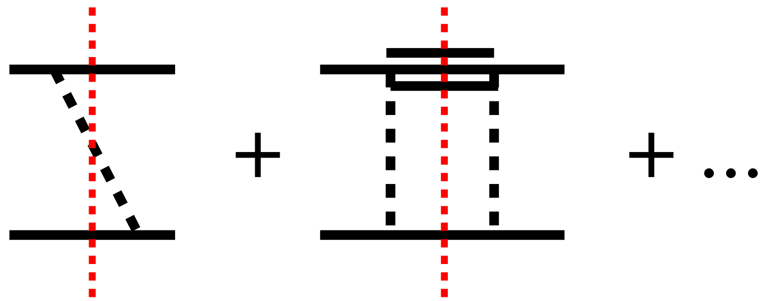
Need boost-invariant description of structure of fast-moving nucleus: Light-front wave function

Basics: Nuclear interactions

Naive expectation: Interactions of nucleons in nucleus
have little effect on high-energy scattering processes at $\gg$ 1 GeV

Simple arguments show that this is not necessarily so!

Nucleon interactions involve intermediate states
with additional hadrons: Mesons (π, σ, vector,…),
baryon resonances (Δ, …), high-mass states



In low-energy structure and reactions $q \sim k_{\text{bind}}$,
the high-mass intermediate states can be "integrated out": EFT approach

High-energy reactions $\omega \gg M_{\text{hadron}}$ can sample intermediate states up to the scale ω:
Cannot a priori assume that they are suppressed!

"Wave function" in relativistic context: Particle number not fixed, depends on scale of probe

Need to organize dynamics such that truncation of nuclear structure to nucleon constituents
becomes possible, and non-nucleonic DoF can be accounted for as corrections

Possible in light-front form of dynamics ($\rightarrow$ will be seen later)

Essential techniques

Interactions: Non-covariant representation

Form of relativistic dynamics: Instant form → light-front form

Interactions: Non-covariant representation

In describing high-energy scattering on nuclei we use the non-covariant representation of interactions: Wave function, configurations, intermediate states…

Here: Introduce/review basics using example from low-energy interactions → Lectures Pastore, Gnech

$$\langle N(p_2')N(p_1') \,|\, \hat{T} \,|\, N(p_2)N(p_1)\rangle$$

NN scattering amplitude at nuclear energies

S-matrix element, gives differential cross section

Transition between asymptotic states

$$p_2' + p_1' = p_2 + p_1$$

4-momentum conservation

$$\mathbf{p}_2' + \mathbf{p}_1' = \mathbf{p}_2 + \mathbf{p}_1$$
$$E_2' + E_1' = E_2 + E_1$$

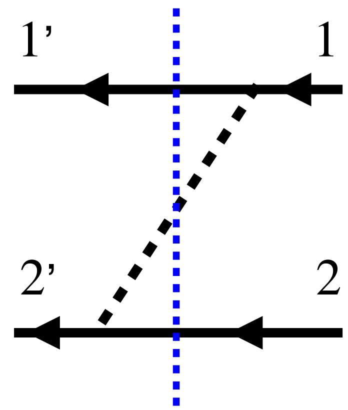
Implies 3-momentum and energy conservation

Consider transition through 1-pion exchange interaction. Apply non-covariant perturbation theory:

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} \qquad \hat{T} = \hat{H}_{\text{int}} + \hat{H}_{\text{int}} \frac{1}{E - \hat{H}_0} H_{\text{int}} + \dots$$

Interactions: Non-covariant representation

$\langle N_2' N_1' \,|\, \hat{T} \,|\, N_2 N_1 \rangle$ =



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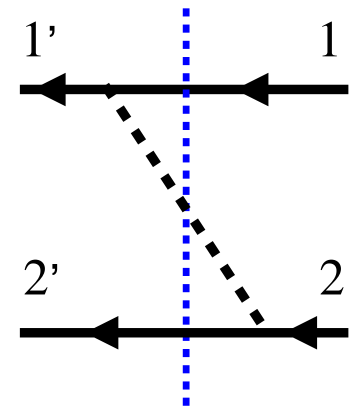


$$\frac{\langle N_2' \,|\, \hat{H}_{\text{int}} \,|\, \pi N_2 \rangle \langle \pi N_1' \,|\, \hat{H}_{\text{int}} \,|\, N_1 \rangle}{(E_\pi + E_1' + E_2) - (E_1 + E_2)} \qquad \frac{\langle N_1' \,|\, \hat{H}_{\text{int}} \,|\, \pi N_1 \rangle \langle \pi N_2' \,|\, \hat{H}_{\text{int}} \,|\, N_2 \rangle}{(E_\pi + E_2' + E_1) - (E_1 + E_2)}$$

Interactions cause transition to intermediate states

Interactions conserve 3-momentum: $\sum \mathbf{p}\text{ (interm)} = \sum \mathbf{p}\text{ (initial)} = \sum \mathbf{p}\text{ (final)}$

All particles are on their mass shell, also in intermediate states: $p_{1,2}^2 = m^2, \;\; E_{1,2} = \sqrt{|\mathbf{p}_{1,2}|^2 + m^2}, \quad p_\pi^2 = M_\pi^2, \quad E_\pi = \sqrt{|\mathbf{p}_\pi|^2 + M_\pi^2}$

Energy not conserved in intermediate states: $\sum E\text{ (interm)} \neq \sum E\text{ (initial)} = \sum E\text{ (final)}$

Transition amplitude ∝ energy denominator $1/\Delta E$

Interactions: Non-covariant representation

Properties of non-covariant representation

Particles always on mass shell, even in intermediate states

3-momentum always conserved, even in intermediate states

Energy not conserved in intermediate states, only between asymptotic states

→ 4-momentum not conserved in intermediate states!

Reasons for use

Intermediate states with well-defined particle content - configurations, constituents

Interpretation of processes as transitions

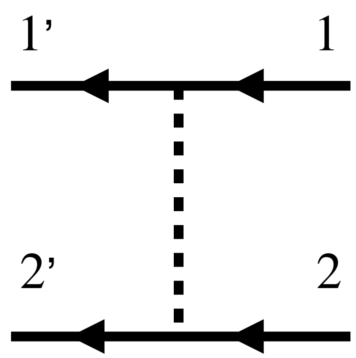
Concept of wave function defined in non-covariant representation

$$\langle N_1, N_2, \ldots N_N | A \rangle \qquad \sum_i^N \mathbf{p}_i = \mathbf{p}_A \qquad \sum_i^N E_i \neq E_A$$

Appropriate for high-energy scattering on composite systems

Interactions: Covariant representation

$$\langle N_2' N_1' \,|\, \hat{T} \,|\, N_2 N_1 \rangle \quad = $$



$$p_\pi = p_1 - p_1' = p_2' - p_2$$ pion 4-momentum

(Feynman diagram)

$$\frac{\text{couplings}}{p_\pi^2 - M_\pi^2}$$

Interactions conserve 4-momentum

Intermediate particles are off mass shell: $p_\pi^2 \neq M_\pi^2$ ("virtual particles")

Used in point-particle field theories (QCD, EFTs): Off-shell behavior of Green functions well-defined

Equivalence of non-covariant and covariant representations can be demonstrated in simple field theories

Some uses with composite systems are possible, but require additional considerations

Relativistic dynamics: Forms

In treatment of high-energy scattering on nuclei we use
the non-covariant representation of interactions

Depends on choice of time ↔ energy and coordinate ↔ momentum variables

Introduce/review forms of relativistic dynamics: Instant form, light-front form

Relativistic dynamics: Instant form

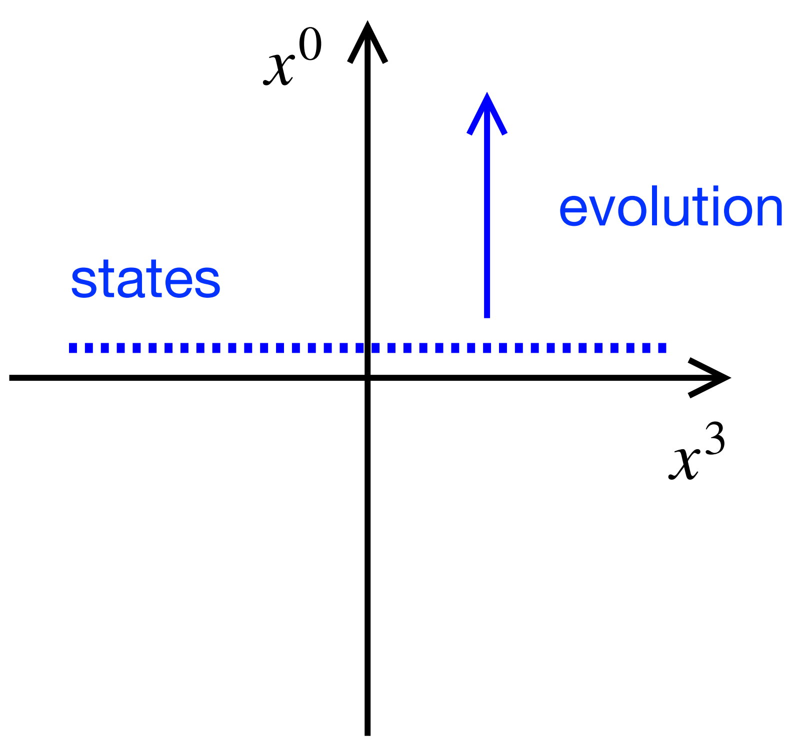
$x^\mu = (x^0, x^1, x^2, x^3)$ 4-dim spacetime

$x^0 = t$ time, $\mathbf{x} \equiv (x^1, x^2, x^3)$ space usual choice of variables

States of system defined at fixed x^0

Evolution in time x^0 described by hamiltonian P^0

Translational invariance in 3-coordinate $\mathbf{x}$ ensures conservation of 3-momentum $\mathbf{P}$



Free particle

$$p^\mu = (E, \mathbf{p}) \qquad p^2 = p^\mu p_\mu = E^2 - |\mathbf{p}|^2 = m^2 \qquad E = +\sqrt{|\mathbf{p}|^2 + m^2}$$

Energy-momentum relation

$|\mathbf{p}\rangle$ Particle states labeled by 3-momentum.
Sometimes write $|p\rangle$, but independent variable is always 3-momentum

$$\langle \mathbf{p}' | \mathbf{p} \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{p}' - \mathbf{p})$$

Normalization of states
Factor p^0 included for relativistic covariance

$$\int d\Gamma_p \equiv \int \frac{d^4 p}{(2\pi)^4} \, 2\pi\delta(p^2 - m^2)\, \theta(p^0) = \int \frac{d^3 p}{(2\pi)^3 2E(\mathbf{p})}$$

Invariant phase space element

Bound state

$$\langle N(\mathbf{p}_1)N(\mathbf{p}_2)\,..\,N(\mathbf{p}_A)\,|\,A(\mathbf{P})\rangle \;=\; 2E_A\,(2\pi)^3\delta^{(3)}(\sum_i^A \mathbf{p}_i - \mathbf{P})\;\Psi(\{\mathbf{p}_i\}\,|\,\mathbf{P})$$

Expand state in products of free-particle states

$$2E_A \int d\Gamma_1\,..\int d\Gamma_A\,(2\pi)^3\delta^{(3)}(\sum_i^A \mathbf{p}_i - \mathbf{P})\;\Psi^*(\{\mathbf{p}_i\}\,|\,\mathbf{P})\,\Psi(\{\mathbf{p}_i\}\,|\,\mathbf{P}) \;=\; 1$$

Normalization of wave function

$$\langle A(\mathbf{P}')\,|\,A(\mathbf{P})\rangle \;=\; 2E_A\,(2\pi)^3\delta^{(3)}(\mathbf{P}' - \mathbf{P})$$

Normalization of state (CM motion)

Wave function describes expansion of bound state in free-particle states

Here: Deal with wave function as abstract object, independent of dynamical equation.
Wave functions for specific interactions can be obtained as solution of dynamical equation.

Spin/isospin quantum numbers: Later

Useful to review: Concepts and formulas can be extended to light-front quantization

Relativistic dynamics: Light-front form

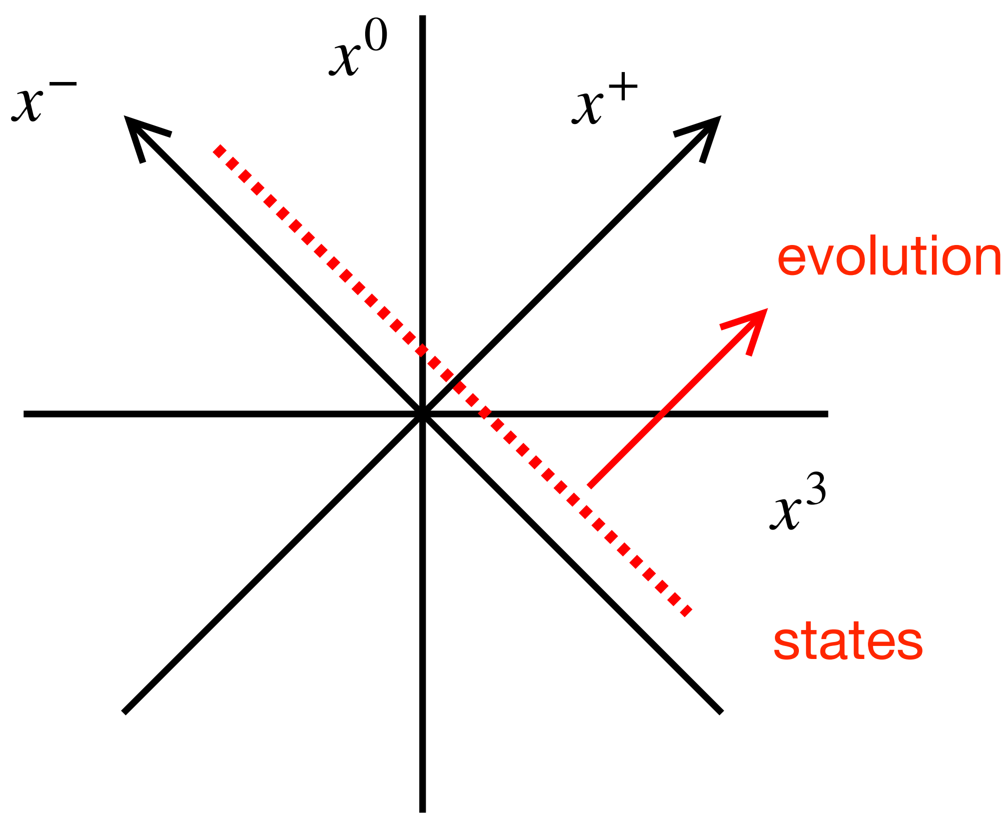
Light-cone components

$a^{\pm} \equiv a^0 \pm a^3 \qquad \mathbf{a}_T \equiv (a^1, a^2)$ light-cone components of 4-vector a^μ

$$ab = \frac{a^+b^- + a^-b^+}{2} - \mathbf{a}_T \cdot \mathbf{b}_T \qquad a^2 = a^+a^- - |\mathbf{a}_T|^2$$

scalar product and square

Light-front form of dynamics



$x^+ = x^0 + x^3 = 0$ hypersurface tangential to light-cone (3D plane)

Wave front of light wave traveling in -3 direction (= surface of constant phase)

Define states of system at light-front time $x^+ = 0$

Evolution in x^+ described by hamiltonian P^-

Relativistic dynamics: Light-front form

Light-front dynamics has many interesting formal properties: Representation of Poincare group, constrained dynamics, …

Here: Interested in applications to high-energy scattering on nuclei processes.
Take practical attitude. Start with basic features, learn about other features as needed

Boosts in light-front form

$p^+ \to e^\eta p^+$ Longitudinal boost (3-direction)

$p^- \to e^{-\eta} p^-$ Light-cone components diagonalize boost, transform multiplicatively

η rapidity = hyperbolic angle, $v = \tanh \eta$

$$\alpha = \frac{p^+}{p^+_{\text{ref}}}$$

Light-cone fractions boost-invariant

Simple technique for performing boosts of kinematic variables:
Compute fraction α in "old" frame, take p^+_{ref} in "new" frame, obtain p^+ in new frame

Exercise: Perform boost of nucleon configuration in deuteron using light-front variables

Boost-invariant momentum variables for wave functions: Light-front wave functions $\to$ following

Free particle

$p^+, \mathbf{p}_T$ momentum $\qquad p^-$ energy

$$p^2 = p^+ p^- - |\mathbf{p}_T|^2 = m^2 \qquad p^- = \frac{m^2 + |\mathbf{p}_T|^2}{p^+}$$

energy p^- fixed by mass shell condition

$p^+ > 0$ for physical particle because $\quad p^+ = p^0 + p^3 = \sqrt{m^2 + |\mathbf{p}_T|^2 + (p^3)^2} + p^3 \;>\; 0$

$p^- > 0$

regardless of sign of p^3

$|p^+, \mathbf{p}_T\rangle$ free particle state

$$\langle p'^+, \mathbf{p}'_T | p^+, \mathbf{p}_T \rangle = 2p^+ (2\pi)^3 \delta(p'^+ - p^+)\, \delta^{(2)}(\mathbf{p}'_T - \mathbf{p_T})$$

normalization of states

$$\int d\Gamma_p \;=\; \int_0^\infty \frac{dp^+}{(2\pi)\, 2p^+} \int \frac{d^2 p_T}{(2\pi)^2}$$

invariant phase space element

Relativistic dynamics: Light-front form

Bound state

$$\langle N(p_1^+, \mathbf{p}_{1T}) \, .. \, N(p_A^+, \mathbf{p}_{AT}) \, | \, A(P^+, \mathbf{P}_T) \rangle \;=\; (2\pi)^3 \, 2P^+ \, \delta(\sum_i^A p_i^+ - P^+) \, \delta^{(2)}(\sum_i^A \mathbf{p}_{iT} - \mathbf{P}_T) \, \Psi(\{p_i^+, \mathbf{p}_{iT}\} \, | \, P^+, \mathbf{P}_T)$$

Nucleon light-cone momenta satisfy $\sum_i^A p_i^+ = P^+, \qquad \sum_i^A \mathbf{p}_{iT} = \mathbf{P}_T$

Boost invariance (longitudinal): Wave function depends only on light-cone fractions

$$\alpha_i \equiv \frac{A p_i^+}{P_A^+} \qquad (i = 1,..,A) \qquad\qquad \sum_{i=1}^A \alpha_i = A$$

$\Psi \equiv \Psi(\{\alpha_i, \mathbf{p}_{iT}\} \, | \, \mathbf{P}_T)$ independent of P^+

In many applications we can use nucleus rest frame $\mathbf{P}_T = 0$

Deuteron

Two nucleons: 1,2 or p, n $\quad \alpha_1 + \alpha_2 = 2, \quad \mathbf{p}_{1T} + \mathbf{p}_{2T} = \mathbf{P}_T$

Wave function effectively depends on variables of one nucleon: $\Psi(\alpha_1, \mathbf{p}_{1T} | \mathbf{P}_T)$

Normalization:

$$\int \frac{d\alpha_1}{\alpha_1(2-\alpha_1)} \int \frac{d^2 p_{1T}}{(2\pi)^2} \Psi^*(\alpha_1, \mathbf{p}_{1T} | \mathbf{P}_T) \Psi(\alpha_1, \mathbf{p}_{1T} | \mathbf{P}_T) = 1$$

Spin/isospin quantum numbers, dynamical equation, connection with non-relativistic WF: Later

Spin degrees of freedom

Light-front helicity states: Later $\quad$ → Lecture Cosyn

High-energy scattering on nuclei

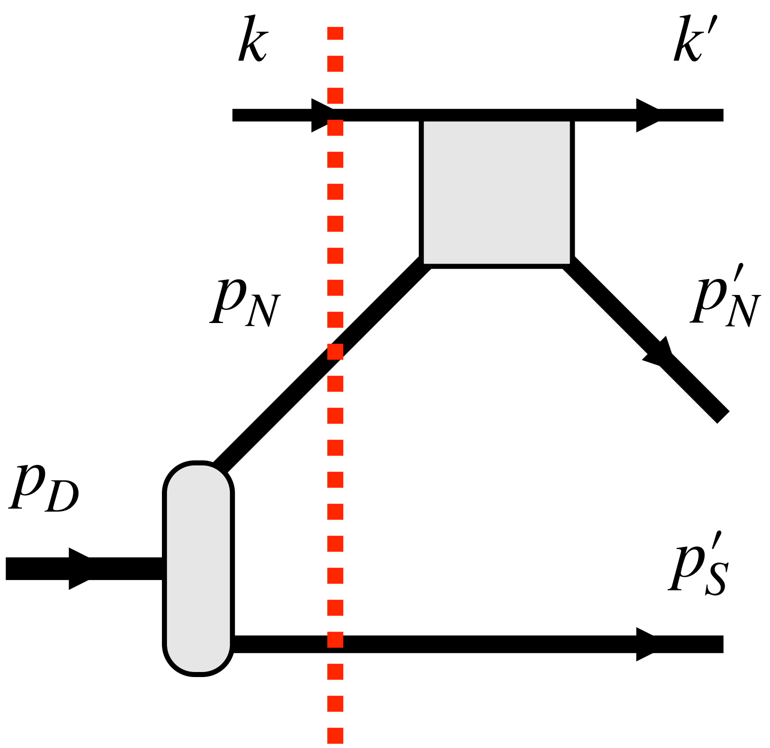
Study scattering processes at multi-GeV energy/momentum transfers

Exhibit effect of energy non-conservation in intermediate states
of scattering amplitude

Compare equal-time and light-front form of dynamics

Arguments based on: Frankfurt, Strikman, Phys. Rept. 76, 215 (1981) [INSPIRE]

High-energy scattering: Example



High-energy electron-deuteron quasi-elastic scattering

N active nucleon, S spectator nucleon

$$e(k) + D(p_D) \rightarrow e'(k') + N'(p'_N) + S'(p'_S)$$

$k + p_D = k' + p'_N + p'_S$ 4-momentum conserved in overall process (asymptotic states)

Active nucleon momentum p_N in intermediate state determined by rules of noncovariant interactions

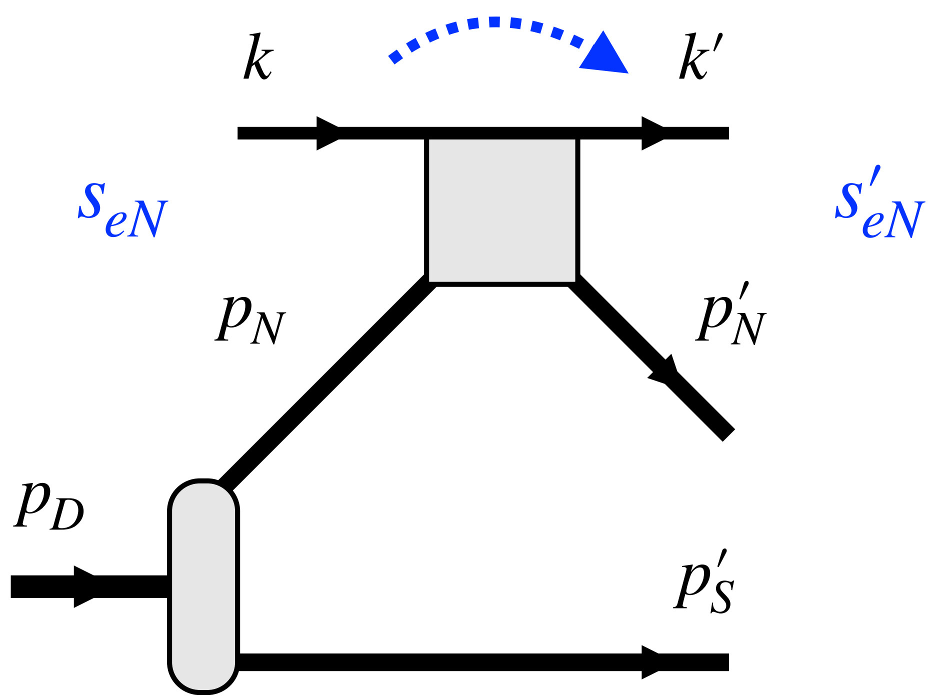
$D \rightarrow N + S$ matrix element preserves 3-momentum (wave function)

$p_N^2 = m^2$ mass shell condition fixes nucleon energy

$p_N \neq p_D - p_S \quad \curvearrowright \quad p_N \neq k - k' + p'_N$

$k + p_N \neq k' + p'_N$ 4-momentum not conserved in electron-nucleon subprocess

High-energy scattering: eN subprocess



Quantify effect of 4-momentum non-conservation in electron-nucleon subprocess

$$s_{eN} = (k + p_N)^2$$
$$s'_{eN} = (k' + p'_N)^2$$

Invariant energy before and after eN interaction

$$s'_{eN} = (k' + p'_N)^2 = (k + p_D - p_S)^2$$

using external 4-momentum conservation

Compute difference of subprocess invariant energies

$$\begin{aligned} s'_{eN} - s_{eN} &= (k + p_D - p_S)^2 - (k + p_N)^2 \\ &= \underbrace{2k \cdot (p_D - p_S - p_N)} \quad + \quad \underbrace{(p_D - p_S)^2 - p_N^2} \end{aligned}$$

involves projectile 4-momentum k, large!

related to deuteron binding energy, small

Use deuteron rest frame: $p_D = (M_D, \mathbf{0}), \quad k = (\omega, -\omega \mathbf{e}_3)$ initial electron in -3 direction

energy $\omega \gtrsim$ 1 GeV (or $\gg$)

Instant form dynamics

$(p_D - p_S - p_N)^0 \neq 0$ non-conservation in 0-component (conventional energy)

$$p_D^0 = M_D \qquad p_S^0 \approx m + \frac{|\mathbf{p}_S|^2}{2m} \qquad p_N^0 \approx m + \frac{|\mathbf{p}_N|^2}{2m}$$

$\mathbf{p}_N = -\mathbf{p}_S$

$|\mathbf{p}_{S,N}| \sim$ few 100 MeV

$$(p_D - p_S - p_N)^0 = \frac{|\mathbf{p}_S|^2}{m} + (M_D - 2m)$$ = kinetic + binding energy

$$s'_{eN} - s_{eN} = 2k^0 (p_D - p_S - p_N)^0 \approx 2\omega \frac{|\mathbf{p}_S^2|}{m}$$ grows with incident energy!

Electron-nucleon subprocess amplitude far “off energy shell” in limit of high-energy scattering

Cannot be connected with “on energy shell” amplitude measured in free eN scattering

No composite picture of high-energy scattering

High-energy scattering: Light-front dynamics

Use again deuteron rest frame

$k^+ = 0 \qquad k^- = 2\omega$ large light-front components of projectile 4-momentum

$(p_D - p_S - p_N)^- \neq 0$ non-conservation in minus component (LF energy)

$$
\begin{aligned}
s'_{eN} - s_{eN} &= 2k \cdot (p_D - p_S - p_N) \\
&= \underbrace{k^+}_{0} \cdot (p_D - p_S - p_N)^- + k^- \cdot (p_D - \underbrace{p_S - p_N)^+}_{0} = 0 + \text{terms independent of energy } \omega
\end{aligned}
$$

Use of light-front dynamics removes the term $\propto \omega$ in $s'_{eN} - s_{eN}$

Energy offshellness of eN subprocess amplitude remains finite in high-energy limit

Subprocess amplitude can be connected with "on energy shell" amplitude measured in eN scattering

Composite picture of high-energy scattering: Compute nuclear high-energy scattering amplitude from on-shell nucleon amplitudes and nuclear structure

High-energy scattering: Light-front dynamics

Light-front dynamics “aligns” the time/energy axis for nuclear dynamics with the direction of the high-energy process, in such a way that the energy nonconservation in intermediate states does not produce large effects

Light-front dynamics is the only scheme that avoids “large” energy offshellness in the nucleon subprocess amplitude and permits a composite description of high-energy scattering.
Its use is necessary, not optional, for a composite description.

In low-energy processes, there is no need to use light-front dynamics

Energy nonconservation in intermediate states is a necessary consequence of interactions and nuclear binding (→ wave function). Its manifestations in high energy scattering are physical effects, not technical artifacts.

Electroproduction and deep-inelastic scattering: Light-front direction usually aligned with momentum transfer 4-vector q. Conclusions re light-front dynamics remain the same as in example here.

High-energy scattering: eN subprocess

Result for high-energy electron-deuteron scattering amplitude (general form)

$$\mathcal{M}_{eD\to e'N'S'}(k, p_D) \quad = \underbrace{\int \frac{d\alpha_N}{\alpha_N} \int d^2 p_{NT}}_{\int d\Gamma_N} \text{Flux}(\alpha_N)\ \Psi(\alpha_N, \mathbf{p}_{NT})\ \mathcal{M}_{eN\to e'N'}(k, p_N)$$

\+ finite corrections from interactions,
non-nucleonic DoF

Ψ_D light-front wave function of deuteron, NN component

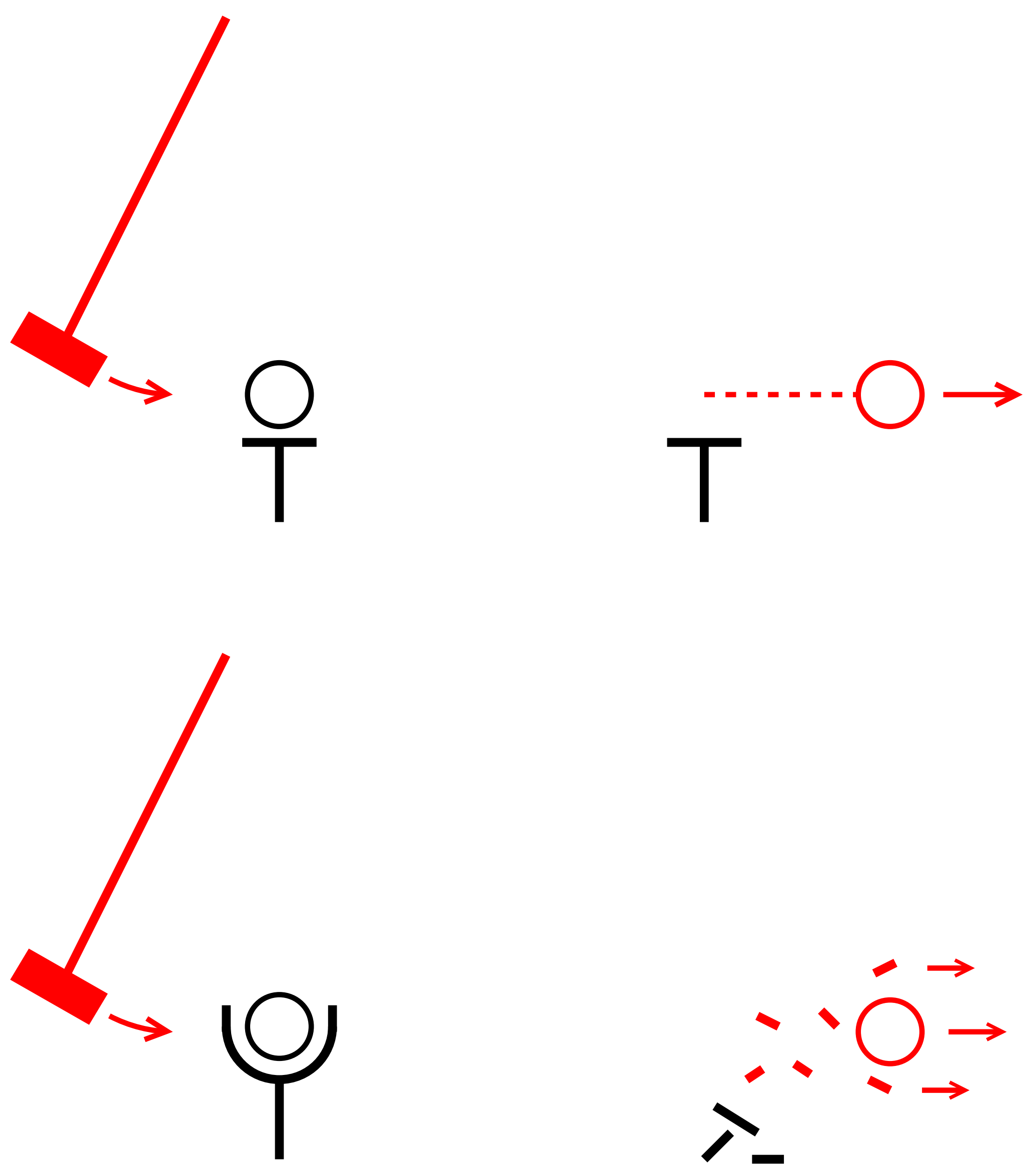
$\mathcal{M}_{eN\to e'N'}$ electron-nucleon scattering amplitude on energy shell, as measured in eN scattering

Separation of nucleus and nucleon structure

Finite corrections from interactions, configurations with non-nucleonic DoF

Cross section: Square amplitude, include flux factor

High-energy scattering: Analogue



Teeing up a golf ball

Golf club = high energy process

Golf ball = nucleon

Tee = low-energy nuclear structure

Light-front quantization

Low-energy structure aligned with direction of high-energy process

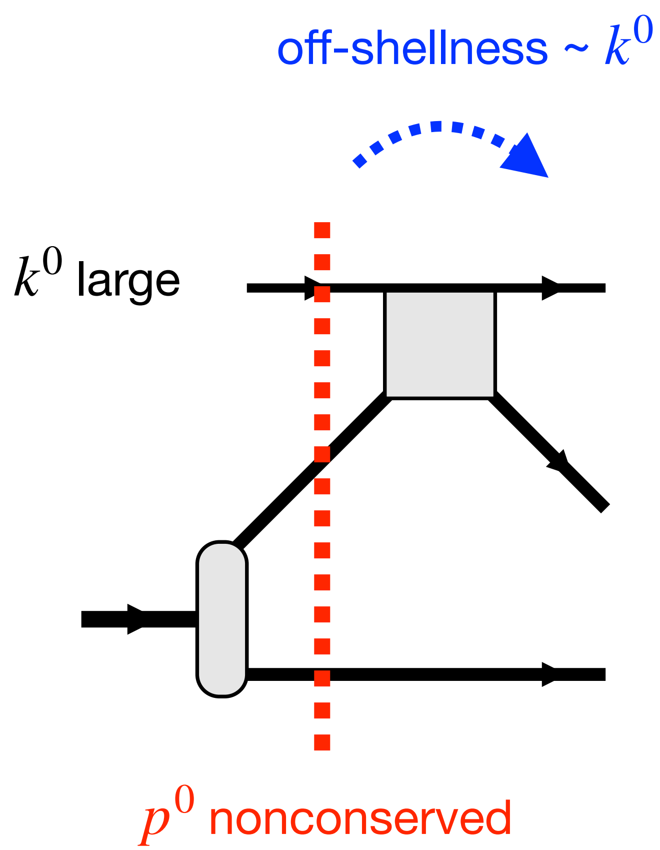
Clean separation of scales

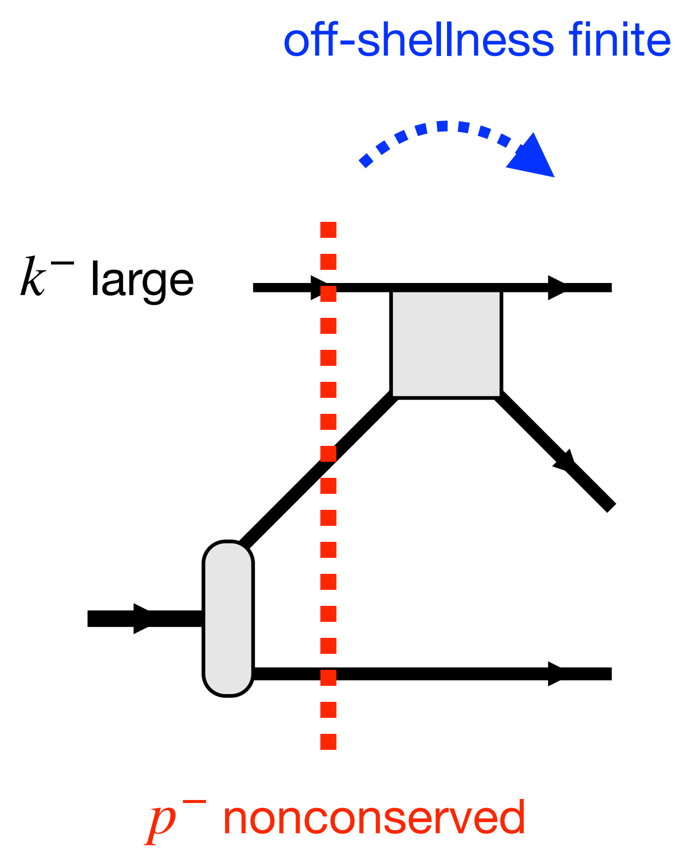
Other quantization schemes

Low-energy structure not aligned with direction of high-energy process

Low-energy structure produces effects of the order of the high collision energy

Summary: High-energy scattering





Instant form

Energy-offshellness of subprocesses amplitude grows ~ k^0

Subprocesses amplitude essentially different from free nucleon amplitude

Light-front form

Energy-offshellness of subprocesses amplitude remains finite

Subprocesses amplitude close to free nucleon amplitude

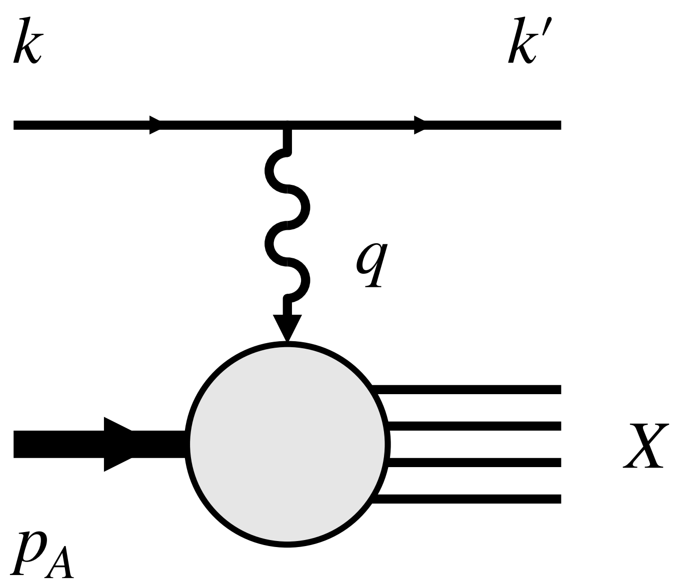
Composite picture of high-energy scattering

Use specific form of EM interaction: electron current x nuclear current

Compute cross section: Hadronic tensor, structure functions

Use reference frame aligned with momentum transfer vector q

Inclusive eA scattering: Kinematic variables



$$e(k) + A(p_A) \rightarrow e'(k') + X$$

$q^\mu = k^\mu - k'^\mu$ 4-momentum transfer, includes energy and momentum transfer

$$q^2 < 0$$

Invariant variables describing nuclear transition

$-q^2 \equiv Q^2 > 0$ invariant momentum transfer

$(q + p_A)^2 \equiv W^2 = M_X^2$ mass of hadronic final state

Scaling variables

$x_A \equiv \dfrac{-q^2}{2qp_A} = \dfrac{Q^2}{W^2 - M_A^2 + Q^2}$ $\quad 0 < x_A < 1$ standard definition

$x \equiv \dfrac{-q^2}{2qp_A/A} = A \cdot x_A$ $\quad 0 < x < A$ alt definition using p_A/A = nominal nucleon momentum

Inclusive eA scattering: Cross section

$$\mathscr{M}_{eA\to e'h} \quad = \frac{e^2}{q^2} \langle e(k') \,|\, J^\mu \,|\, e(k) \rangle \; \langle h \,|\, J_\mu \,|\, A(p_A) \rangle$$

amplitude from one-photon exchange

$$\frac{d\sigma}{dx dQ^2} \quad = \text{Flux} \times \sum_{\text{spins}} \sum_h \mathscr{M}^* \mathscr{M}$$

sum over hadrons h includes phase space integral

$$= \text{Flux} \times \frac{e^4}{(q^2)^2} \underbrace{\sum_{\text{spins}} \langle e(k') \,|\, J^\mu \,|\, e(k) \rangle \langle e(k) \,|\, J^\nu \,|\, e(k') \rangle}_{L^{\mu\nu}} \times \underbrace{\sum_h \langle A(p_A) \,|\, J_\mu \,|\, h \rangle \langle h \,|\, J_\nu \,|\, A(p_A) \rangle}_{W_A^{\mu\nu}}$$

$L^{\mu\nu}$ leptonic tensor

$W_A^{\mu\nu}$ hadronic tensor

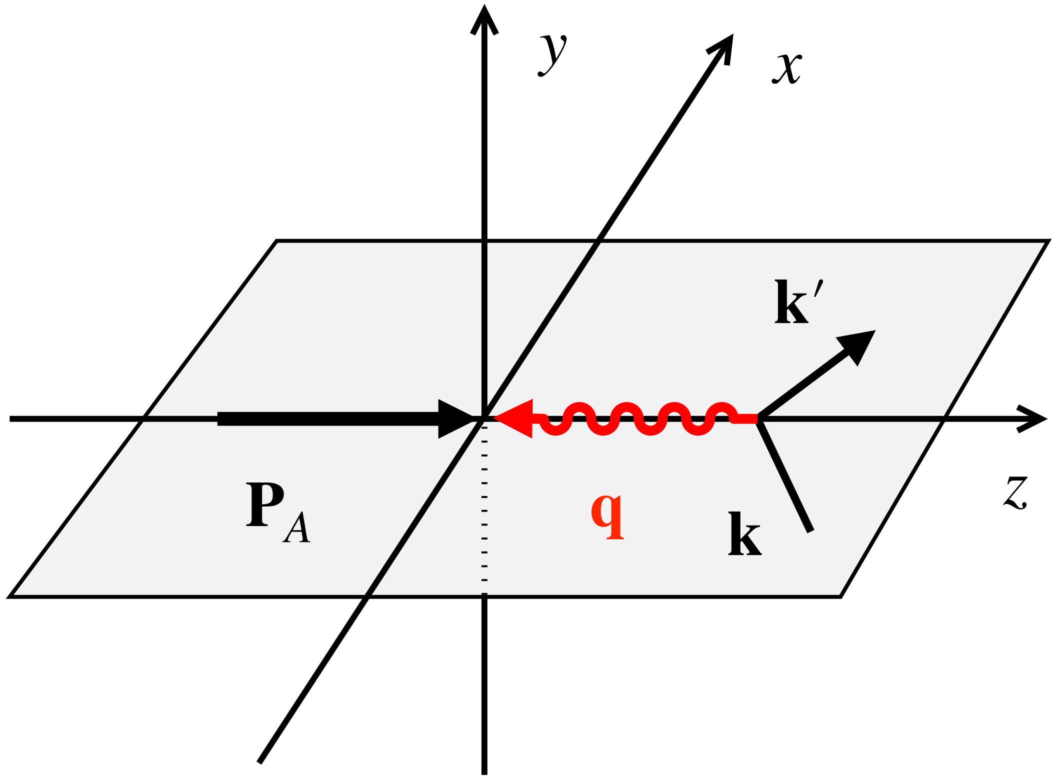
$$W_A^{\mu\nu} \;=\; e_L^{\mu\nu} F_L \;+\; e_T^{\mu\nu} F_T$$

$e_{L,T}^{\mu\nu}$ tensors formed from 4-momenta q, p_A — geometry

$F_{L,T} \equiv F_{L,T}(x, Q^2)$ invariant structure functions — dynamics

alt. def: $F_{1,2}$

Inclusive eA scattering: Collinear frame



For analysis and structure calculations use reference frame where

$\mathbf{P}_A, \mathbf{q}$ along z-axis, $\mathbf{q}$ in -z direction

$\mathbf{k}, \mathbf{k}'$ in xz plane

Light-front components of kinematic 4-vectors

$\mathbf{P}_{AT} = 0$

P_A^+ arbitrary - can by changed by boost

$P_A^- = M_A^2 / P_A^+$

$\mathbf{q}_T = 0$

q^+, q^- determined by invariant kinematic variables

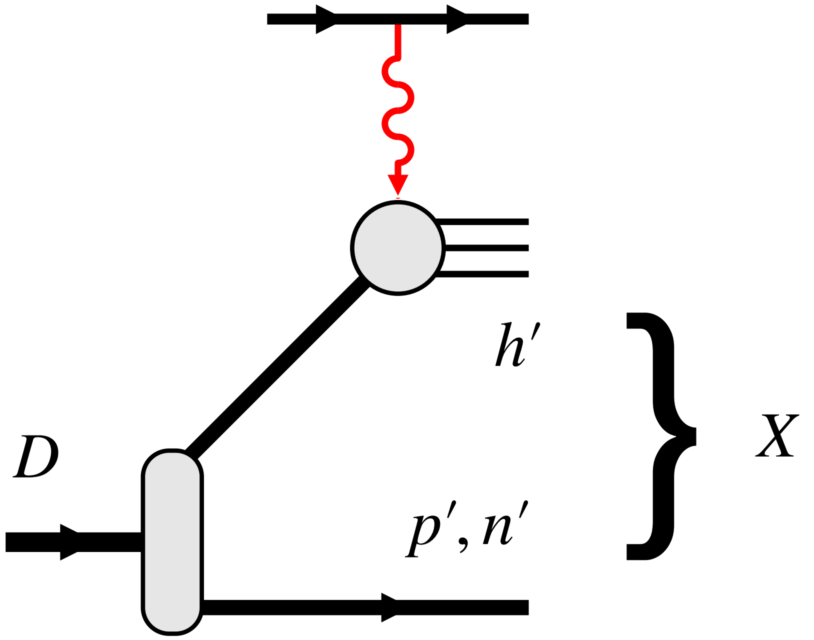
$$Q^2 = -q^2 = -q^+q^- \qquad Q^2/x_A = 2qP_A = q^+P_A^- + q^-P_A^+$$

$$q^+ = -\xi_A P_A^+ \qquad \xi_A = \frac{2x_A}{1 + \sqrt{1 + 4x_A^2 M_A^2 / Q^2}} = x_A + \mathcal{O}(x_A^2 M_A^2 / Q^2)$$

$$q^- = -Q^2 / P_A^+$$

q^- large in DIS limit $Q^2 \to \infty, x_A$ fixed

Inclusive eD scattering: Impulse approximation



Compute inclusive eD cross section
from eN cross section using light-front dynamics

$X = h' + p', n'$ final state in impulse approximation

Untagged scattering: Spectators summed/integrated over
Tagged scattering: Spectator identified, momentum fixed

$$\langle h'p' | J^\mu | D \rangle$$

current matrix element for $h' + p'$ final state

$$\uparrow$$

$$\int d\Gamma_p \int d\Gamma_n \, | pn \rangle \langle pn |$$

insert set of nucleon intermediate states

$$= \int d\Gamma_p \int d\Gamma_n \, \underbrace{\langle h'p' | J^\mu | pn \rangle}_{\langle h' | J^\mu | n \rangle \, \langle p' | p \rangle} \, \underbrace{\langle pn | D \rangle}_{\delta(\,..\,) \, \Psi_D}$$

current couples to neutron

Inclusive eD scattering: Impulse approximation

Take current matrix element times complex conjugate

Combine nucleon and deuteron factors

Perform phase space intergrals over momenta of final proton p' and intermediate proton/neutron p, n using up delta functions

$$W_D^{\mu\nu} = \int d\Gamma_{p'} \sum_{h'} \langle D \,|\, J^\mu \,|\, h'p' \rangle \langle h'p' \,|\, J^\mu \,|\, D \rangle \quad + \; (p' \leftrightarrow n')$$

$$= \int_x^2 \frac{d\alpha_n}{\alpha_n} \int d^2 p_{nT} \; \underbrace{\frac{2\,|\,\Psi_D(\alpha_n, \mathbf{p}_{nT})\,|^2}{\alpha_n(2-\alpha_n)}}_{\frac{\rho_D(\alpha_n, \mathbf{p}_{nT})}{\alpha_n}} \; \underbrace{\sum_h{}' \langle n(\alpha_n, \mathbf{p}_{nT}) \,|\, J^\mu \,|\, h' \rangle \langle h' \,|\, J^\nu \,|\, n(\alpha_n, \mathbf{p}_{nT}) \rangle}_{W_n^{\mu\nu}(p_n, \tilde{q})}$$

neutron light-front momentum distribution in deuteron

neutron hadronic tensor

q^- shifted by nonconservation in intermediate state - small effect

Inclusive eD scattering: Impulse approximation

$$W_D^{\mu\nu}(p_D, q) \quad = \quad \int_x^2 \frac{d\alpha_n}{\alpha_n} \int d^2 p_{nT} \, \frac{2\,\rho_D(\alpha_n, \mathbf{p}_{nT})}{\alpha_n} \quad W_n^{\mu\nu}(p_n, \tilde{q}) \qquad + \; (n \leftrightarrow p)$$

Deuteron hadronic tensor expressed through neutron hadronic tensor and neutron light-front momentum distribution in deuteron

Integration limit on α_n follow from condition that in electron-neutron subprocess $q^+ + p_n^+ > 0$, which implies $\alpha_n > 2\xi_D \approx 2x_D = x$ (we use the simplified value in the DIS limit $Q^2 \gg x_D^2 M_D^2$)

Here: Untagged scattering, integration over active nucleon momentum, spectator momentum fixed by active nucleon momentum

For tagged scattering, write result as phase space integral over spectator nucleon momentum, obtain different flux factor. See

Light-front impulse approximation: Finite corrections from interactions, deuteron configurations beyond NN

$$\rho_N(\alpha_N) \;=\; \frac{|\,\Psi_D^*(\alpha_N, \mathbf{p}_{NT})\,|^2}{2 - \alpha_N}$$

LF momentum distribution of nucleons $(N = p, n)$ in deuteron with $\mathbf{P}_T = 0$

Spin: Averaged over deuteron spin, summed over nucleon spin. LF spin states discussed later

$$\int \frac{d\alpha_N}{\alpha_N} \int d^2 p_{NT} \; \rho_N(\alpha_N) \;=\; 1$$

Total baryon number

from $$\int \frac{d\alpha_N}{\alpha_N(2-\alpha_N)} \int d^2 p_{NT} \,|\,\Psi_D\,|^2 \;=\; 1$$

normalization of wave function

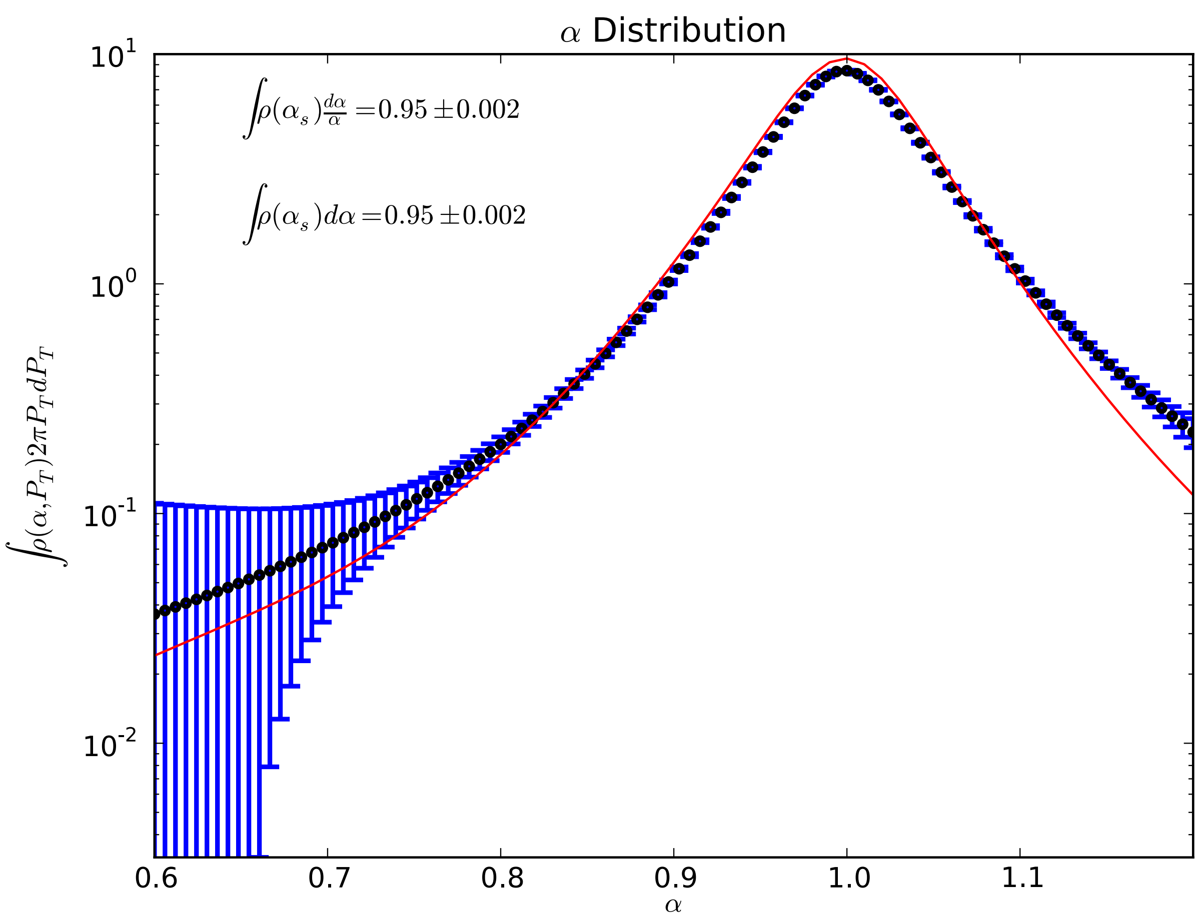
$$\int \frac{d\alpha_N}{\alpha_N} \int d^2 p_{NT} \; \alpha_N \; \rho_N(\alpha_N) \;=\; 1$$

Total LF momentum

from $\Psi_D(\alpha_N, \mathbf{p}_{NT}) = \Psi_D(2 - \alpha_N, -\,\mathbf{p}_{NT})$

symmetry of wave function

Inclusive eD scattering: N momentum distributions



LF momentum distribution of nucleons are “universal” (= independent of high-energy process) can be extracted from quasi-elastic scattering and other processes

Empirical LF momentum distributions

[Sargsian, Boeglin, Int.J.Mod.Phys.E 24 (2015) 03, 1530003]

Inclusive eD scattering: Structure functions

$$W_D^{\mu\nu}(p_D, q) \;=\; \sum_N \int_x^2 \frac{d\alpha_N}{\alpha_N} \int d^2 p_{NT} \; \frac{2\,\rho_D(\alpha_N, \mathbf{p}_{NT})}{\alpha_N} \; W_N^{\mu\nu}(p_N, \tilde{q})$$

LF impulse approximation result for hadronic tensor

$$W_D^{\mu\nu} \;=\; e_{LD}^{\mu\nu} F_{LD} \;+\; e_{TD}^{\mu\nu} F_{TD}$$

Deuteron tensor and structure functions (from p_D, q)

$$W_N^{\mu\nu} \;=\; e_{LN}^{\mu\nu} F_{LN} \;+\; e_{TN}^{\mu\nu} F_{TN}$$

Nucleon tensor and structure functions (from $p_N, \tilde{q}$)

Structure functions obtained by projection of result for hadronic tensor (e.g. W^{++}, W^{TT})

$$F_{LD}(x, Q^2) \;=\; \sum_N \int_x^2 \frac{d\alpha_N}{\alpha_N} \int d^2 p_{NT} \; \frac{2\,\rho_D(\alpha_N, \mathbf{p}_{NT})}{\alpha_N} \; F_{LN}(\tilde{x}, \tilde{Q}^2)$$

L, T structure functions

$$F_{2D}(x, Q^2) \;=\; [\ldots] \quad \rho_D(\alpha_N, \mathbf{p}_{NT}) \quad F_{2N}(\tilde{x}, \tilde{Q}^2)$$

Here F_2 structure function

$\tilde{x} \equiv \dfrac{x}{\alpha_N}$ effective scaling variable in eN subprocess, includes motion of nucleon

$\tilde{Q}^2$ effective momentum transfer in eN subprocess, $= Q^2$ in DIS limit

Momentum sum rule for F_2 structure function (fixed Q^2)

$$\int_0^2 dx\, F_{2D}(x) \;=\; \sum_N \int_0^2 dx \int_x^2 \frac{d\alpha_N}{\alpha_N} \int d^2p_{NT} \;\; \rho_D(\alpha_N, \mathbf{p}_{NT}) \;\; F_{2N}\left(\frac{x}{\alpha_N}\right)$$

Interchange orders of integration in x and α_N

$$= \sum_N \int_x^2 \frac{d\alpha_N}{\alpha_N} \underbrace{\int d^2p_{NT} \;\; \alpha_N\, \rho_D(\alpha_N, \mathbf{p}_{NT})} \;\; \int_0^{\alpha_n} \frac{dx}{\alpha_n} F_{2N}\left(\frac{x}{\alpha_N}\right) \;\;=\;\; \sum_N \int_0^1 d\tilde{x}\, F_{2N}(\tilde{x})$$

= 1 by momentum sum rule for ρ_D

Momentum sum rule for nucleon distribution ρ_D and for nucleon structure function F_{2N} ensure momentum sum rule for deuteron structure function F_{2D}

Weak nuclear binding approximation

$$F_{2D}(x) \;=\; \sum_N \int_x^2 \frac{d\alpha_N}{\alpha_N} \int d^2p_{NT} \;\; \rho_D(\alpha_N, \mathbf{p}_{NT}) \;\; \underbrace{F_{2N}\left(\frac{x}{\alpha_N}\right)}_{\approx\; F_{2N}(x)} \qquad \approx \; \sum_N \; F_{2N}(x)$$

Nucleon momentum distribution peaked around $\alpha_N = 1$

Integral dominated by region $\alpha_N \approx 1$

Evaluate nucleon structure function under integral at $\alpha_N = 1$ (peaking approximation)

Confirms naive expectation: Nuclear structure function F_{2A} becomes sum of nucleon structure functions $\sum_N F_N$ in weak nuclear binding approximation

Note: Simple addition applies only to structure function F_2, not F_1

Comments on impulse approximation for A > 2

Nuclear remnant system has internal degrees of freedom, can absorb excitation energy

Impulse approximation includes summation over remnant variables differential in excitation energy:
Spectral function = function$(\alpha_N, \mathbf{p}_{NT}; M_{A-1})$

For the deuteron the remnant system is the spectator proton/neutron and has no nuclear excitations:
Spectral function coincides with momentum density

Light-front structure

Light-front nuclear structure is low-energy structure in nucleon DoF,
only presented "as seen" by a high-energy process: $x^+ =$ const, boost-invariant

Need nuclear light-front wave functions in nucleon DoF

→ momentum densities, spectral functions, breakup/tagging cross sections, spin observables, …

Two basic methods:

A) Construct effective NN interactions in LF dynamics and solve dynamical equation for nuclear LF wave function

B) Determine nuclear LF wave functions by matching with nonrelativistic nuclear wave functions

→ import results and experience in nonrelativistic nuclear physics

Light-front structure: Dynamical equation

$$\hat{\mathscr{P}}^{-}\,\Psi_A(\ldots\,|\,P) \;=\; P^{-}\,\Psi_A(\ldots\,|\,P)$$

Hamiltonian $\hat{\mathscr{P}}^{-}$ is one of Poincare generators,
describes LF time evolution $\partial/\partial x^{+}$

$$\hat{\mathscr{M}}^2 = \hat{\mathscr{P}}^{+}\hat{\mathscr{P}}^{-} - \hat{\mathscr{P}}_T^2$$

$\hat{\mathscr{P}}^{-}$ is operator in nucleon LF momentum/spin vars,
Eigenvalues are total LF energy of nucleus P^{-}

$$\hat{\mathscr{M}}^2\,\Psi_A(\ldots\,|\,P) \;=\; M_A^2\,\Psi_A(\ldots\,|\,P)$$

Alt form: Eigenvalue eqn for invariant mass of bound state

CM and internal motion separated as in nonrel dynamics

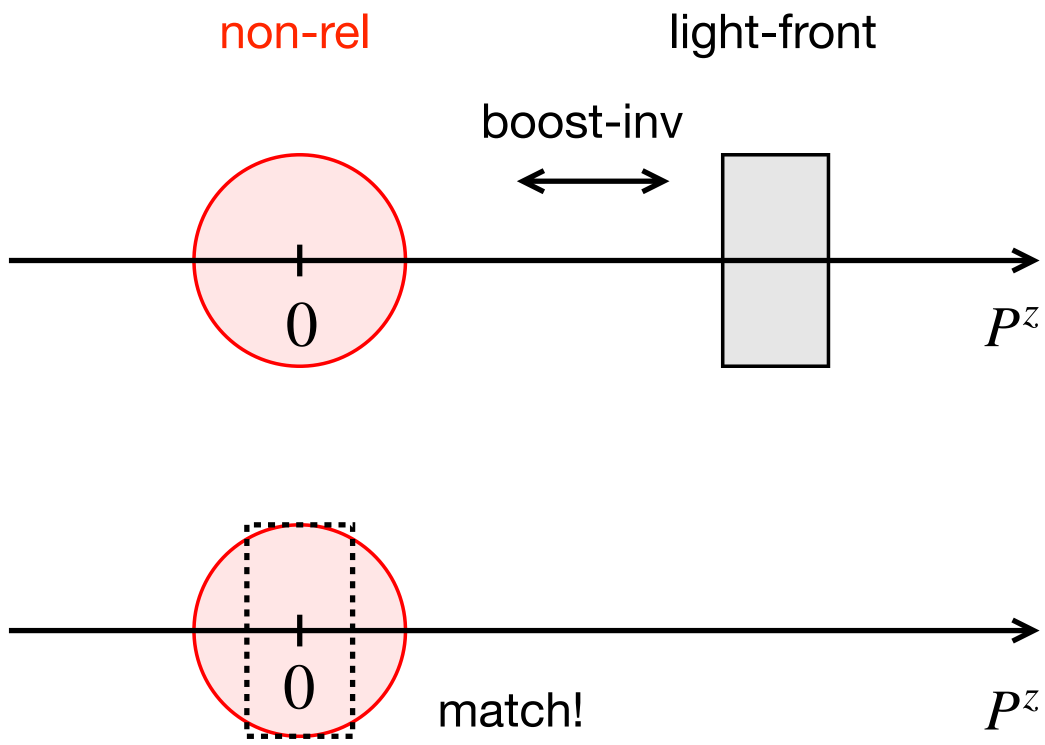
Mass operator contains kinetic energy of nucleons and interactions.
Possible form of interactions restricted by Poincare invariance

Also integral equation: Weinberg equation, analogue of Lippmann-Schwinger equation
Applications to nuclei: Frankfurt, Strikman 1981/1993

Technical issues in description of interacting systems:
Cluster separation, spin and partial waves, definition of potential, …
Specialized literature: Lev, Pace, Salme et al; Polyzou et al

Effective NN interactions in LF dynamics?
Miller, Cooke 2001

Light-front structure: Matching with nonrelativistic



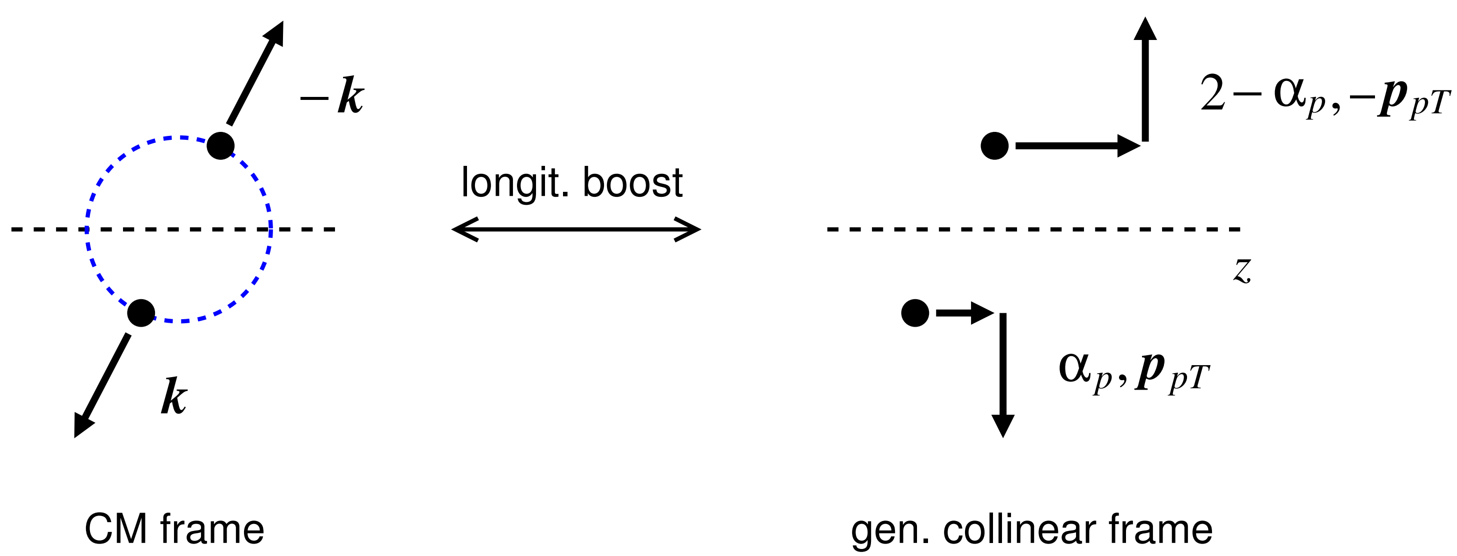
Light-front description: Boost invariant, axially symmetric

Non-relativistic description: Valid at momenta ~few 100 MeV in nucleus rest frame $P^z = 0$, spherically symmetric

Take light-front wave function in/near nucleus rest frame and match with nonrel wave function

→ import structure (angular momentum conservation) and dynamics

Light-front structure: Spherically symmetric rep



Here: pn configuration in deuteron

Described by proton LF momentum variables α_p, $\mathbf{p}_{pT}$

Boost invariance: Consider the config in the CM frame where the ordinary nucleon 3-momenta are back-to-back: proton $\mathbf{k}$, neutron $-\mathbf{k}$

Use 3-vector $\mathbf{k}$ as variable!

$$\alpha_p = 1 + \frac{k^z}{E(k)}, \qquad \mathbf{p}_{pT} = \mathbf{k}_T$$

$$E(k) \equiv \sqrt{|\mathbf{k}|^2 + m^2}$$

$$M_{pn}^2 = \frac{4(|\mathbf{p}_{pT}|^2 + m^2)}{\alpha_p(2-\alpha_p)} = 4(|\mathbf{k}|^2 + m^2) = 4E^2$$

invariant mass = CM energy

Terentev 76, Kondratyuk, Strikman 84

→ LF dynamical equation becomes 3D rotationally symmetric

→ On-shell scattering amplitudes calculated with LF dynamics satisfy rotational invariance (“angular conditions”)

Generalization to 3N and beyond: Possible but much more complex

Lev 1993

Light-front structure: Nonrelativistic approximation

$$\Psi_D(\alpha_N, \mathbf{p}_{NT}) \quad = \quad \sqrt{E(k)}\; \tilde{\Psi}_D(\mathbf{k})$$

Deuteron LF wave function in spherically symmetric variables

Here: No spin, S-wave only. Spin DoF later

$$\int \frac{d\alpha_p}{\alpha_p(2-\alpha_p)} \int d^2 p_{pT}\, |\Psi_D(\alpha_N, \mathbf{p}_{NT})|^2$$

$$= \quad \int d^3k\, |\tilde{\Psi}_D(\mathbf{k})|^2$$

Normalization condition

$$\tilde{\Psi}_D(\mathbf{k}) \overset{\text{approx.}}{=} \Phi_D(\mathbf{k}) \text{ [nonrel]}$$

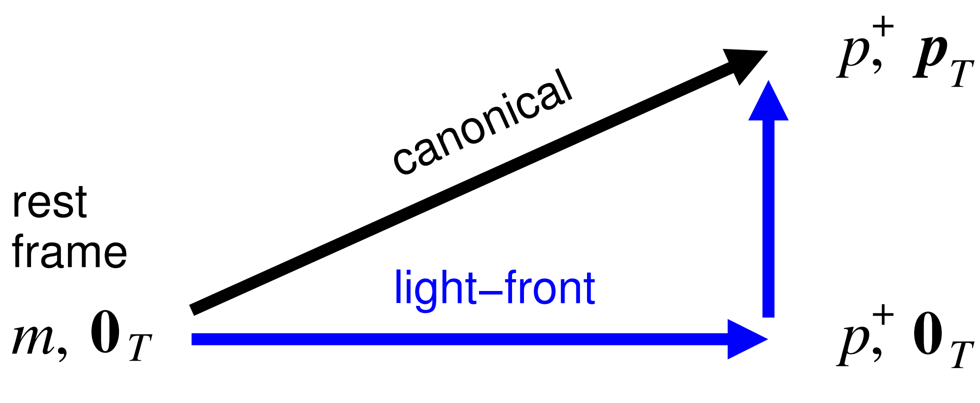
Spherically symmetric deuteron LF wave function approximated by nonrelativistic wave function

Good approximation up to $|\mathbf{k}|$ ~ 300-400 MeV

Used in most theoretical/experimental applications of deuteron LF structure

Frankfurt, Strikman 81

Light-front structure: Spin



Light-front helicity states

Obtained from rest-frame spin states by LF boosts: longitudinal + transverse

Simple transformation properties:
Invariant under longitudinal boosts,
transform kinematically under transverse boosts

Differ from canonical spin states by spin rotation (Melosh rotation)

Nucleon spin states and deuteron spin structure formulated as LF helicity states

Nonrelativistic approximation to deuteron wave function can be constructed including spin DoF

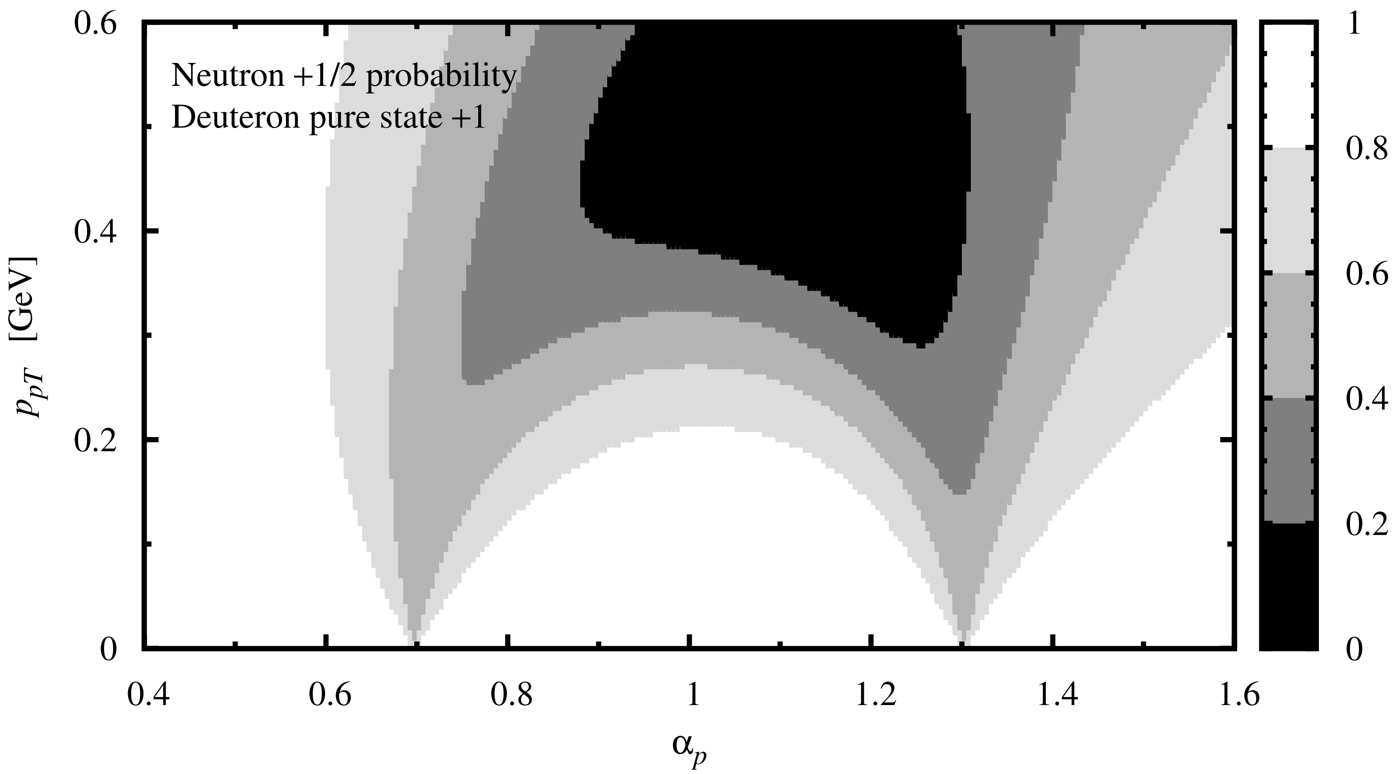
S, D partial waves
LF helicity

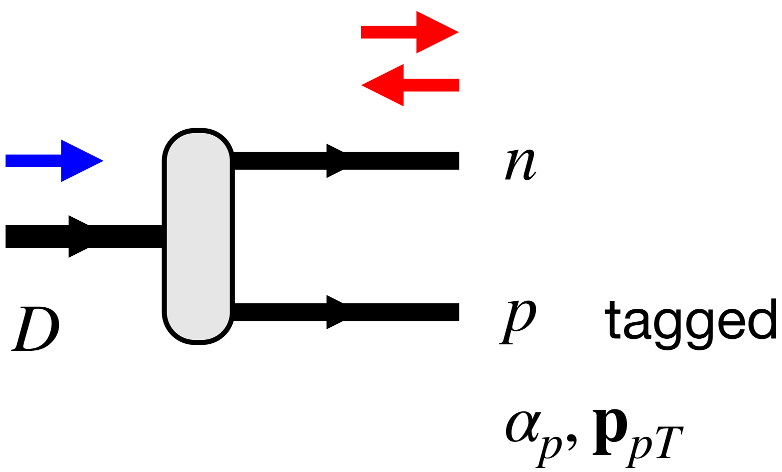
spin rotation ⟷

S, D partial waves
CM frame, canonical spin

→ Lecture Wim Cosyn

Light-front structure: Deuteron spin structure





Example: Effective neutron polarization in spectator proton tagging

Deuteron polarized longitudinally (LF helicity +1)

What is probability that neutron in polarized along/opposite to deuteron spin (LF helicity +/- 1/2)?

Probability depends on spectator proton momentum α_p, p_{pT}: Controls S/D wave ration

Cosyn, Weiss, Phys.Rev.C 102 (2020) 065204

Summary

Light-front dynamics is an essential tool in high-energy scattering from nuclei

- Provides boost-invariant description
- Keeps off-shell effects from nuclear binding finite in high-energy limit
- Enables composite description in terms of nucleus and nucleon structure

Inclusive eA scattering in light-front impulse approximation

- Nuclear structure functions obtained from nucleon LF momentum distribution and nucleon structure functions
- Approximation preserves baryon number and LF momentum sum rules

Light-front nuclear structure

- Low-energy nuclear structure “as seen” by high-energy scattering process
- Can be inferred by matching with nonrelativistic nuclear structure
- Deuteron: Rich structure, spin and orbital motion entangled, observables in spectator tagging
- A > 2 nuclei: Rich structure, many processes. Needs studies and calculations.
  Needs expertise of low-energy nuclear structure community

Further topics

Final-state interactions

Interaction of hadrons in final state of high-energy process with spectators.
Important for tagging/breakup measurements

Initial-state interactions and non-nucleonic degrees of freedom

High-energy processes involving multiple nucleons, hadrons in NN interactions

QCD factorization and partonic structure

Methods developed here can be applied to compute nuclear partonic structure
in terms of nucleon structure

Small-x physics and nuclear shadowing

Methods developed here can be applied to nuclear shadowing in inclusive
and exclusive small-x scattering on light nuclei

Exclusive processes

Applications to exclusive scattering processes, e.g. deep-virtual Compton scattering and meson
production on light nuclei, in quasi-elastic or coherent scattering

Supplemental material