

# Nuclear Interactions at Short Distances

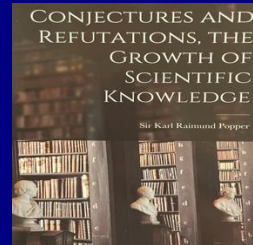
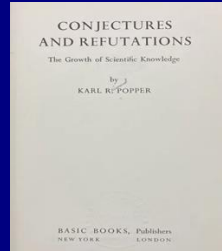
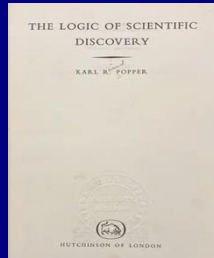
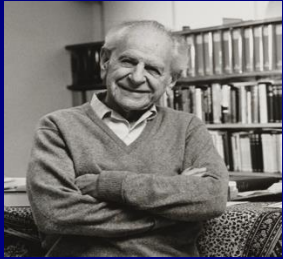
Misak Sargsian  
Florida International University, Miami



Light Ion Physics in the EIC Era, FIU, June 23rd, 2025

Scientific truth should not only describe an observed phenomenon  
it has to be able to make predictions which can be verified experimentally

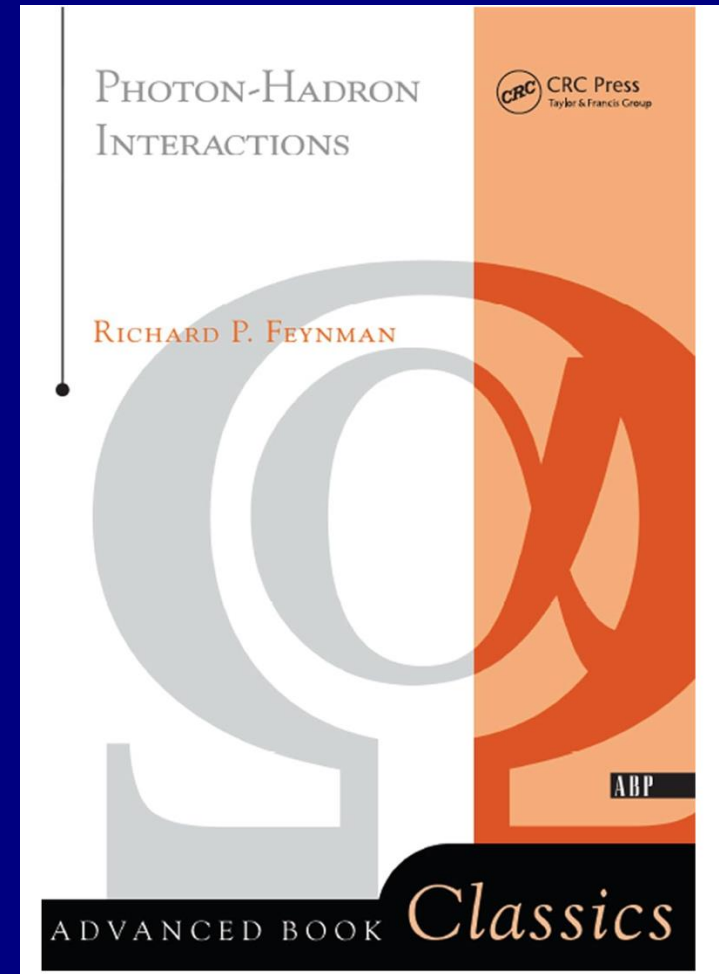
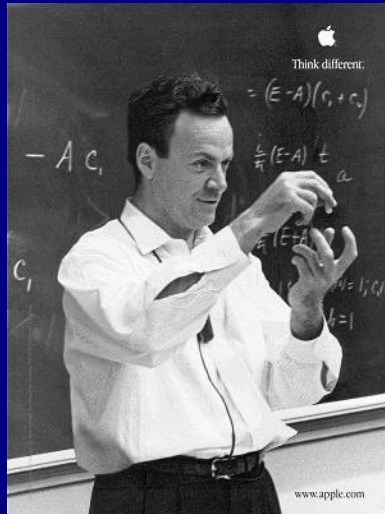
This concept was formulated in Philosophy by Sir Karl Popper



In his theory of falsification, Popper suggested that scientific theories possess potential falsifiers, and their claims about the world might later be discovered to be false. Thus, for a theory to be abandoned or refined, Popper proposed that scientists should come up with better theories by first proving them false.

- **General Principles of Electromagnetic Interaction**
- **Parton Model**

- **General Principles of Electromagnetic Interaction**
- **Parton Model**



# 1. Why matter has a structure?

- *Because Atomic Nuclei are Stable*

# 2. Why are nuclei not collapsing?

- *Because of Nuclear Repulsive Core*

# 3. What is Nuclear Repulsive Core?

- *Repulsion in the NN System*
- *Repulsion in NNN NRN + systems*

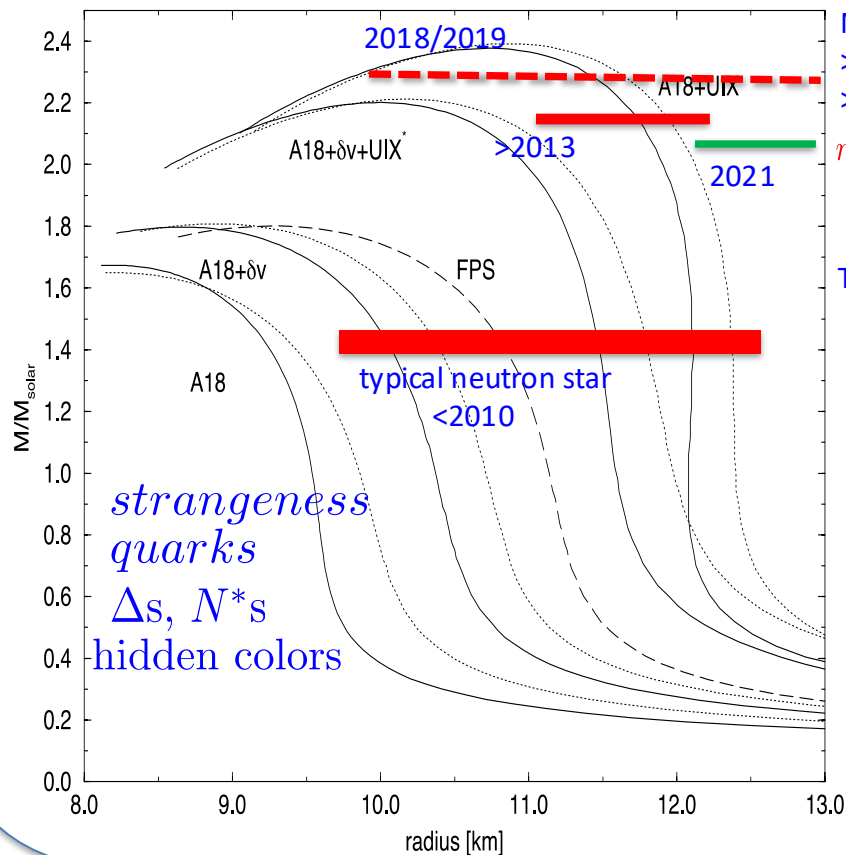
# 4. How to probe the Repulsion

- *Probe nuclei at larger and larger densities*

# 5. How to get Larger Densities @ Terrestrial Experiments

- *Probe quantum fluctuations*

# “Unreasonable” Persistence of Nucleons



Neutron star masses  
>2 solar masses  
>2010

$r_{NN} \sim 0.6 - 0.8 fm$

Typical neutron star  
1.4 Solar mass

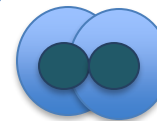
$r_{NN} \sim 0.7 - 1.0 fm$

H. Heiselberg,  
V.Pandharipande  
ARNPS 2000

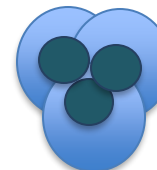


$r_c \sim 0.3 fm$

color singlet core



two color singlet  
"nucleons"



three color singlet

"baryons"

# Nuclear Dynamics at Short Distances

## Probing NN and NNN Interactions at $< 1\text{fm}$

*their role in the dynamics and structure disappearance in High Density Nuclear Matter*

### For NN interactions

- *Identification of NN interaction in the nuclear dynamics*— Since 1993
- *Intermediate – short distance tensor forces* — Since 2006
- *Isospin dependence of the tensor forces, momentum sharing*— Since 2014
- *NN repulsive core* 2024
- *Hadron-quark transition in the core* PRL2023
- *non-nucleonic components, hidden color, gluons*

### For NNN interactions

- *Identification of NNN interaction in the nuclear dynamics* PRC2019, 2023
- *Evaluation of irreducible 3N forces*
- *Evaluation of non-nucleonic component in NNN interactions*

# Probing NN Repulsive Core

One of the most fascinating properties of nuclear forces is the *nuclear repulsive core* which provides a stability for atomic nuclei, making it possible the emergence of a structure for the visible matter (Wilczek: Nature 445,2007).

In cotemporary view:

Without it nuclei will collapse to sizes  $1\text{fm}$ , followed by the onset of quark-gluon degrees of freedom and restoration of chiral symmetry with rather unimaginable consequences for the order of the universe that we know.

# Probing NN Repulsive Core

- NN force is attractive: But Nuclei are Stable

“If the two-body forces are everywhere attractive and if many-body forces are neglected then the nucleon pairs are sufficiently close to take advantage of attractive interactions and a collapsed state of nuclear matter results “

G. Breit and E.P. Wigner, Phys. Rev. 53, 998 (1938).

Many body forces keeping nucleus stable

In 1950's it was evaluated (Blatt & Weisskopf 1952) that for the case of attractive two-nucleon forces the atomic nuclei with  $A=200$  will collapse to the distances half of the NN interaction length, with per-nucleon binding energies 1600MeV (compare to actual 8MeV).

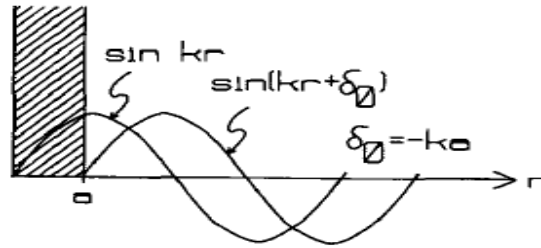
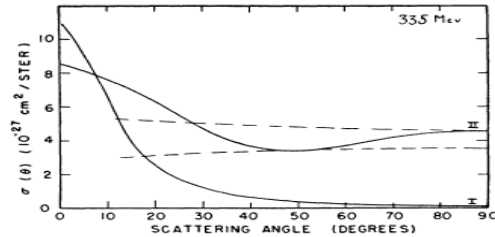
Moreover, no saturation density is possible, with the nuclear binding energy growing as  $A^2$ .

Initially it was thought that the solution lies in the exchange character of nuclear forces as well as many-body effects.

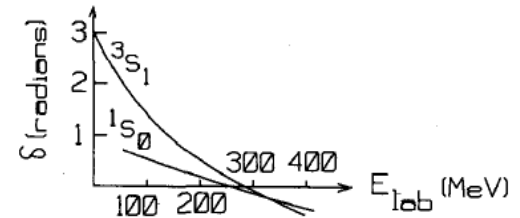


# Probing NN Repulsive Core

- However, the solution came in 1950's, from the theoretical analysis of the Berkeley 345MeV pp scattering data which exhibited almost isotropic angular distribution for  $20^\circ$ -  $90^\circ$  range of CM scattering angle.
- Jastrow 1951 assumed the existence of the infinite hard core  $r_0=0.6\text{fm}$  surrounded by an attractive well in the  $^1S_0$  channel of pp interaction.



- The further analyses of pn data demonstrated that similar repulsion exists also in  $^3S_1$  channel with the core distances estimated to be  $r_c = 0.4\text{-}0.6\text{fm}$  Arndt:1966,Walecka:1995.



**Non-monotonic NN central potential** with the repulsive core was introduced: Brueckner & Watson 1953 to obtain **nuclear density saturation**.

## Modern NN Potentials

$$V^{2N} = V_{EM}^{2N} + V_{\pi}^{2N} + V_R^{2N}$$

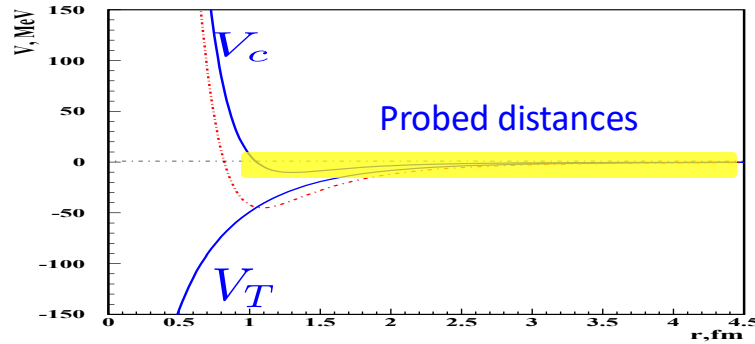
$$V_R^{2N} = V^c + V^{l2} L^2 + V^t S_{12} + V^{ls} L \cdot S + v^{ls2} (L \cdot S)^2$$

$$V^i = V_{int,R} + V_{core}$$

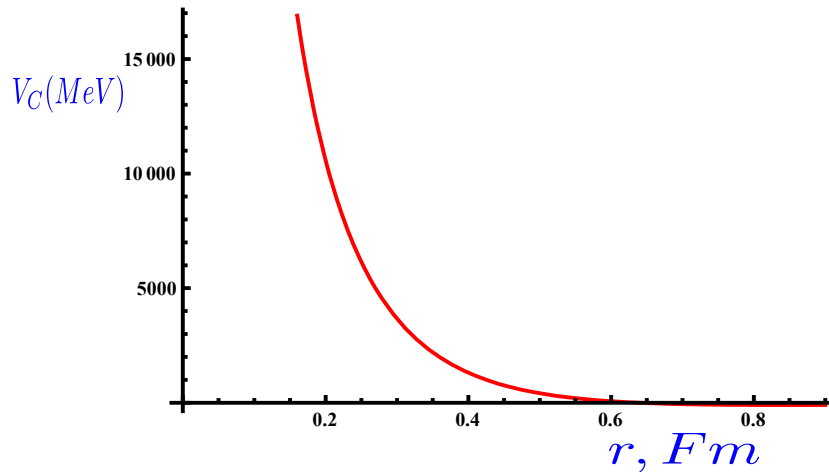
$$V_{core} = \left[ 1 + e^{\frac{r-r_0}{a}} \right]^{-1}$$

60's

Currently: Probed NN structure up to  $> 0.8\text{fm}$



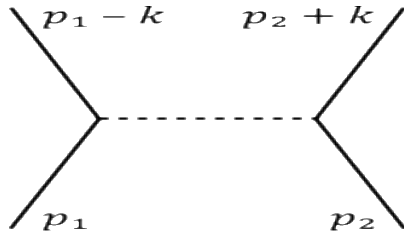
Next: NN – Repulsive Core



# Nuclear Forces and Field Theory

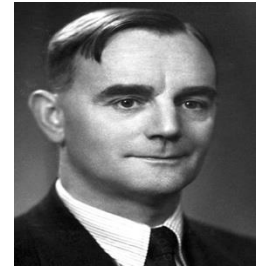
- 1935 meson exchange model:

H. Yukawa



$$V(r) = -g^2 \frac{e^{-mr}}{r}.$$

- Predicted meson with around 100 MeV :



- 1947 meson discovered:

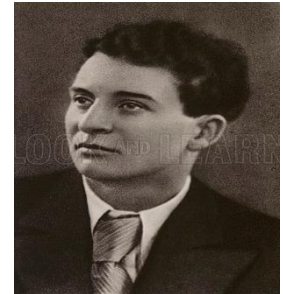
C.F. Powell

- 1943-1945 – seen by:

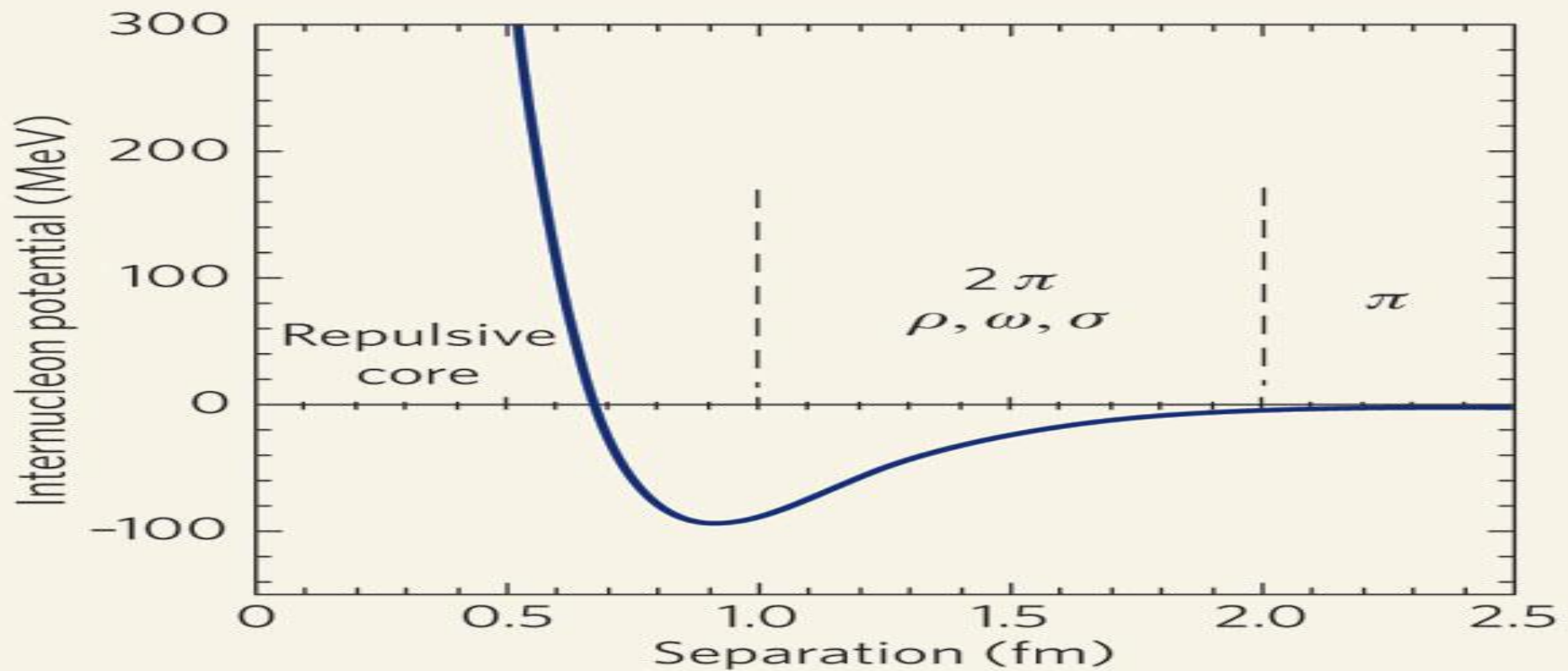
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

Artem Alikhanian

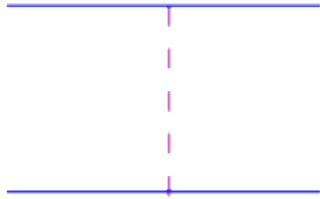
Aragats Cosmic Ray Station



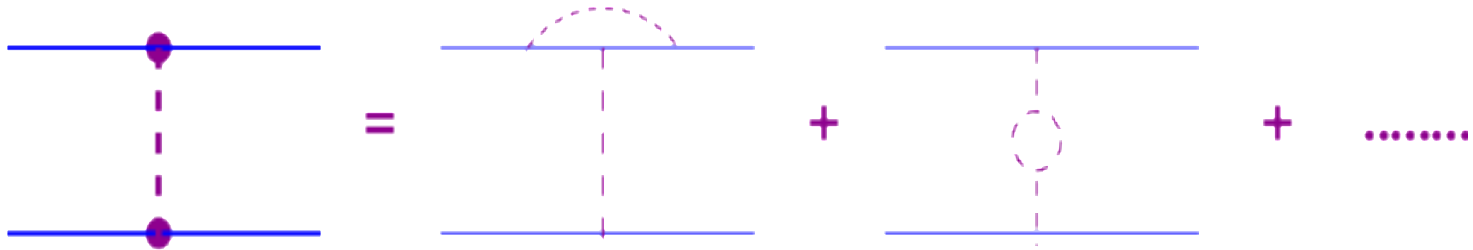
$\sigma, \pi, \rho, \omega, \dots$



# Field Theory of Nucleons & Mesons 1947-



$\sigma, \pi, \rho, \omega, \dots$



Pomeranchuk, Landau - 1950's

$$g^2(\Lambda^2) = \frac{g^2}{1 - 5\left(\frac{g^2}{4\pi}\right)\ln\left(\frac{\Lambda^2}{m^2}\right)}$$

$$\alpha_{em}(Q^2) = \frac{\alpha_{em}(\mu^2)}{1 - \frac{\alpha_{em}(\mu^2)}{3\pi}\ln\frac{Q^2}{\mu^2}}$$

$Q = m_e e^{\frac{3\pi}{2\alpha}} \sim 10^{277} \text{ GeV}$

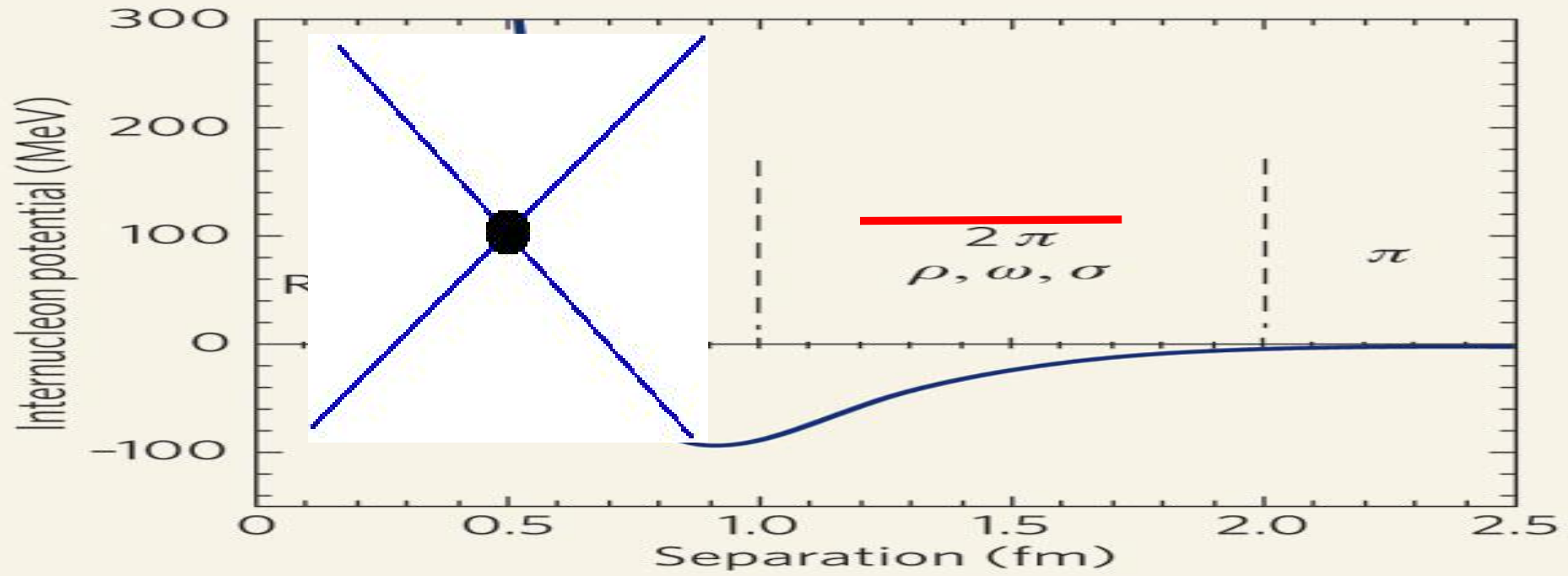
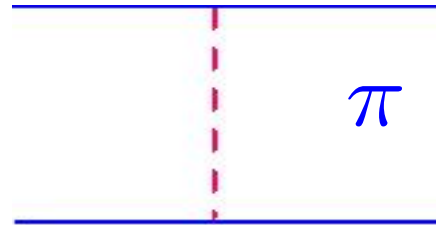
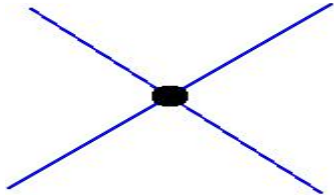
Infinite interaction occurs at transferred momenta approx **500 MeV/c** or  
at internucleon distances  $1 \sim \text{Fm}$ .

It seems we have a problem about which the Nature is not aware of Y.Pomeranchuk

All formal quantum field theories with Yukawa type interactions contain  
the problem of the "Zero Charge"

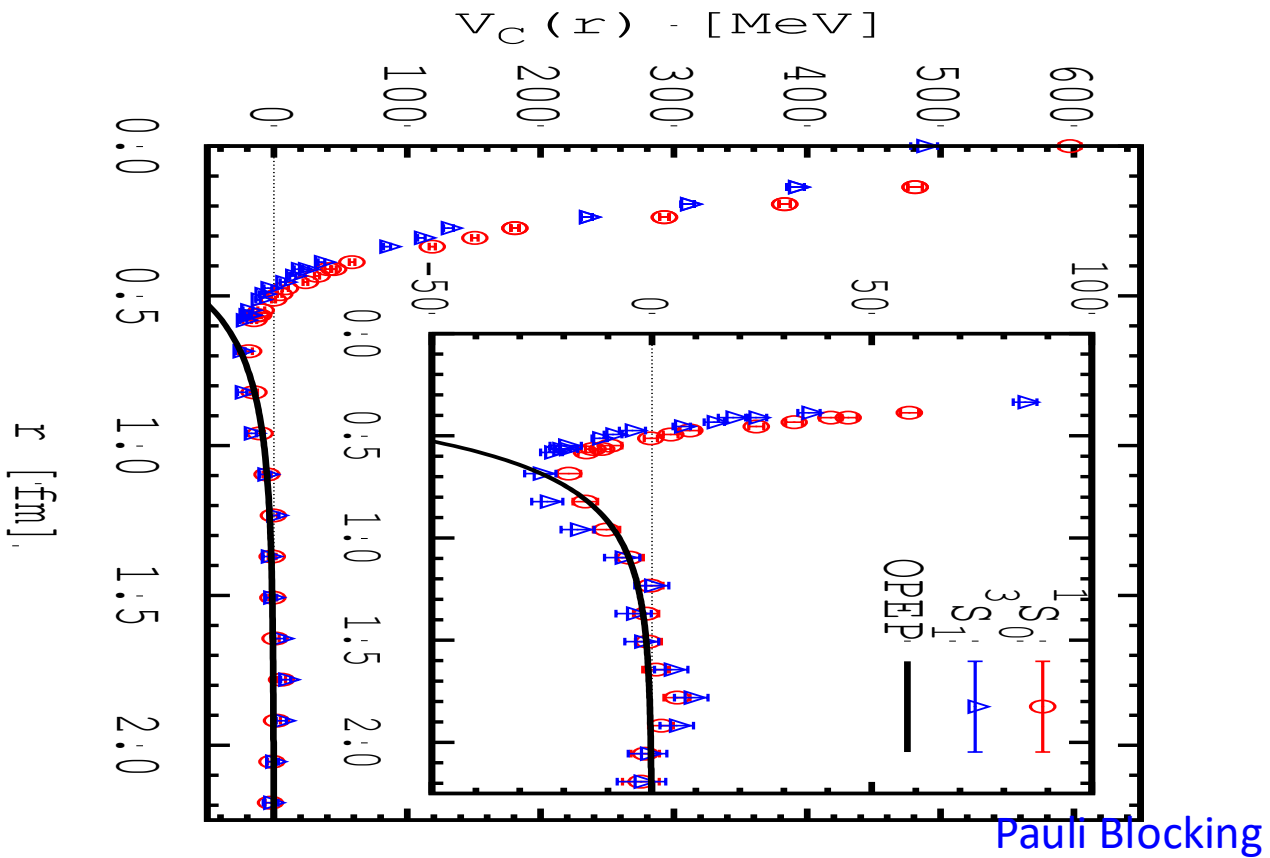
Pomeranchuk, Sudakov, Ter-Martirosyan, Phys. Rev. 1956

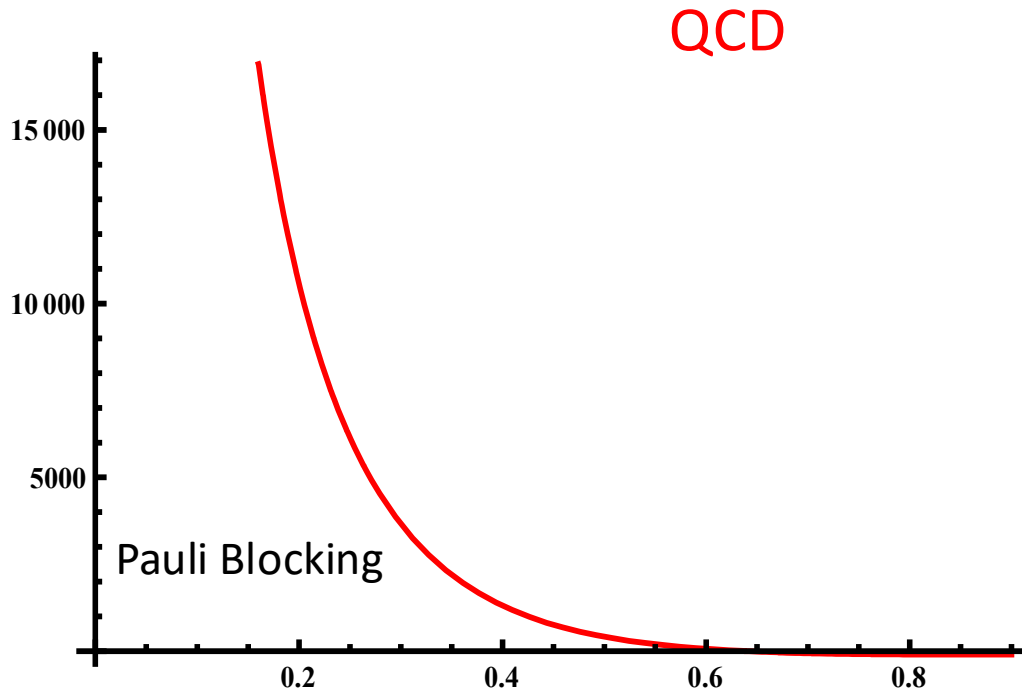
# Effective Theories 1979 -



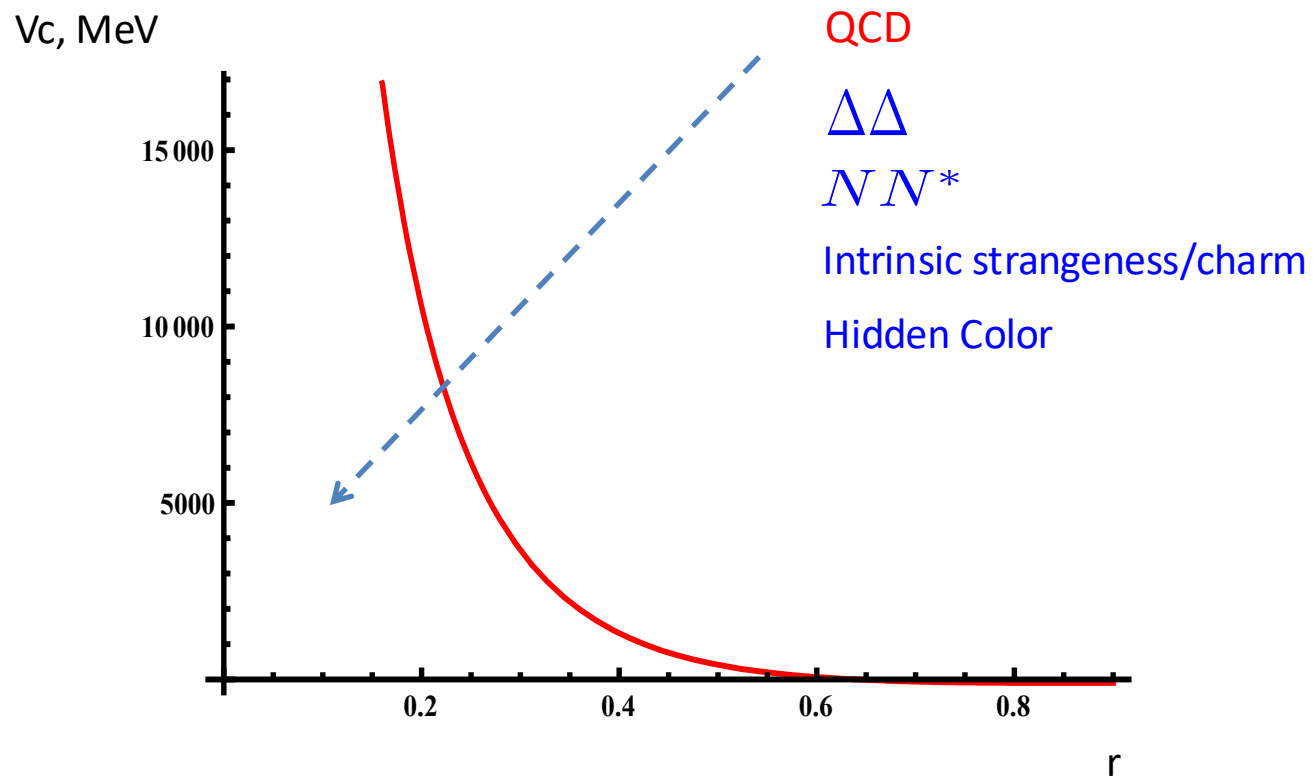


# Lattice Calculations - 2007





Contradicts Neutron Star Observations:  
will predict masses not more than 0.1 - 0.6 Solar mass



~80% hidden color  
Brodsky, Ji, Lepage, PRL 83

## Probing the Deuteron at Short Distances

$$\Psi_d = \Psi_{pn} + \Psi_{\Delta\Delta} + \Psi_{NN^*} + \Psi_{hc} \cdots$$

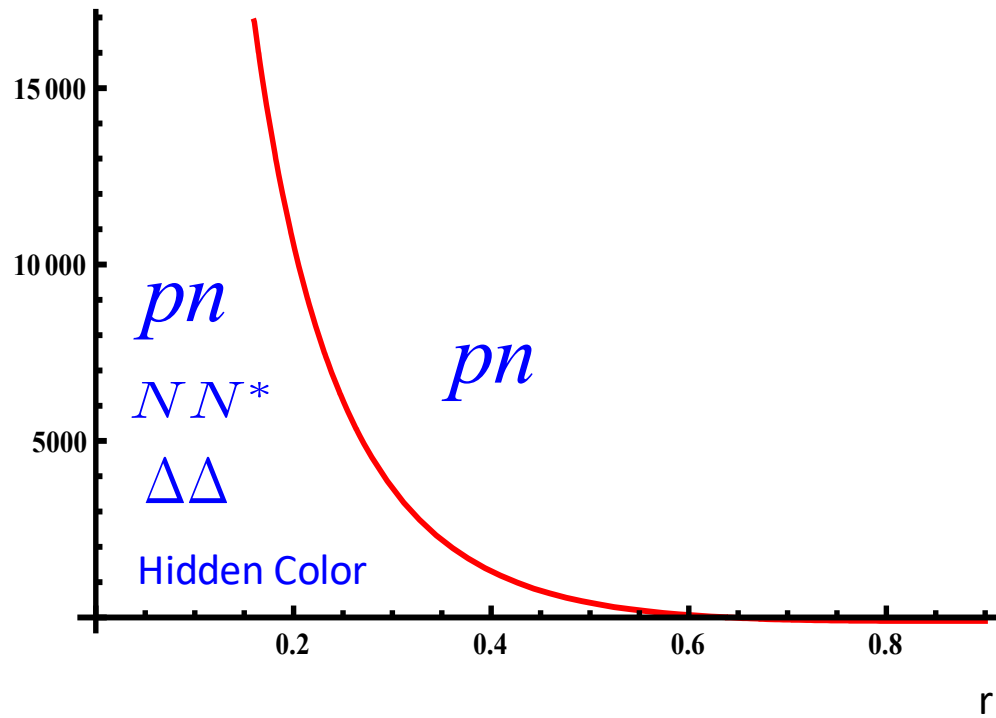
$$\Psi_{hc} = \Psi_{N_c, N_c}$$

$$\Psi_{T=0, S=1}^{6q} = \sqrt{\frac{1}{9}} \Psi_{NN} + \sqrt{\frac{4}{45}} \Psi_{\Delta\Delta} + \sqrt{\frac{4}{5}} \Psi_{CC}$$

The NN core can be due to the orthogonality of

$$\langle \Psi_{N_c, N_c} \mid \Psi_{N, N} \rangle = 0$$

Vc, MeV



# Nuclear Forces and Nuclear Structure “Standard Approach”

A-body Schroedinger equation interacting through NN -potential

$$\left[ -\sum_i \frac{\nabla_i^2}{2m} + \frac{1}{2} \sum_{ij} V(x_i - x_j) + \sum_{ijk} V(x_i, x_j, x_k) \cdots \right] \psi(x_1, \cdots, x_A) = E \psi(x_1, \cdots, x_A)$$

Mean Field Approximation

$$\left[ -\frac{\nabla_N^2}{2m} - V_{HF}(x) \right] \psi_N(x) = E_N \psi_N(x)$$

Hartree-Fock potential will smear out main properties NN potential 1990s

# Nuclear Forces and Nuclear & Nuclear Matter Structure

“Standard” Nuclear Physics Approach

A-body Schroedinger equation interacting through NN -potential

$$\left[ -\sum_i \frac{\nabla_i^2}{2m} + \frac{1}{2} \sum_{ij} V(x_i - x_j) + \sum_{ijk} V(x_i, x_j, x_k) \cdots \right] \psi(x_1, \cdots, x_A) = E \psi(x_1, \cdots, x_A)$$

Ab Initio Calculations

**NonRelativistic**

***Little Predictive Power***

## Conceptually: How to probe nuclei at short nucleon separations

- Probe bound nucleons at large internal momenta
- Need high energy probes to resolve such nucleons in nuclei  
*in high energy nuclear processes*

### # Theory of High Energy eA Scattering:

Problem! Strong Interaction is Strong!

I. High-Energy approximations – small parameter

II. Emergence of Effective Theory – diagrammatic method

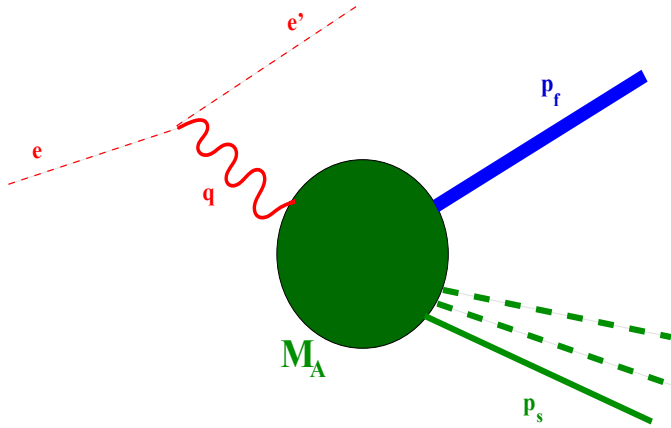
III. Light-Front Wave Function of Nucleus – relativism

IV. From Schroedinger Equation to LF Diagrams – LF-wavefunction + scattering

V. Emergence of small distance nuclear dynamics – predictions



# I. High Energy Approximations:



$$|\vec{q}| = q_3 \sim p_{f3} \gg p \sim M_N$$

$$Q^2 \geq \text{few GeV}^2$$

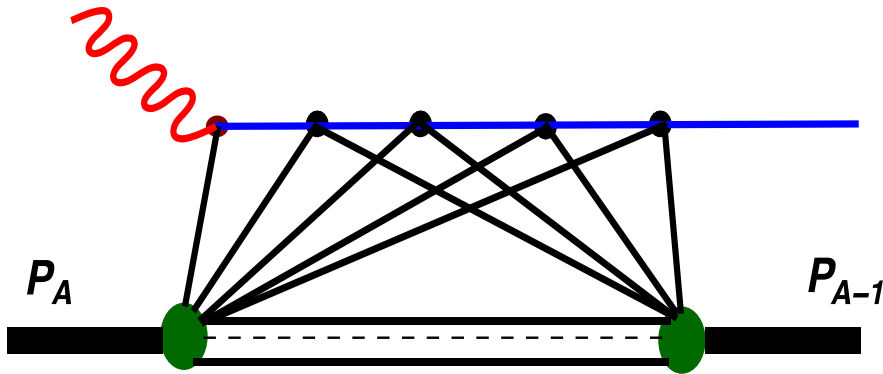
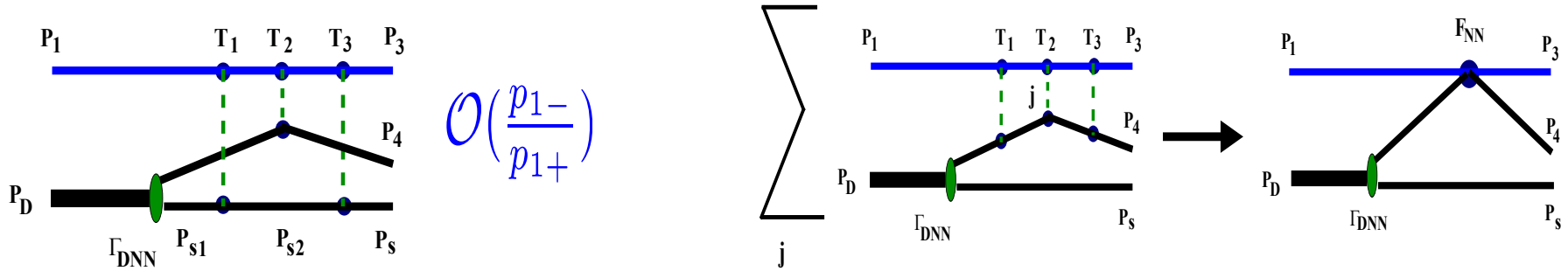
Both for QE/DIS

- Emergence of the small parameter

$$\frac{q_-}{q_+} = \frac{q_0 - q_3}{q_0 + q_3} \ll 1 \quad \mathcal{O}\left(\frac{q_-}{q_+}\right)$$

$$\frac{p_{f-}}{p_{f+}} = \frac{E_f - p_{f3}}{E_f + p_{f3}} \ll 1 \quad \mathcal{O}\left(\frac{p_{f-}}{p_{f+}}\right)$$

## II. Emergence of “effective” theory



Effective Feynman  
Diagrammatic Rules

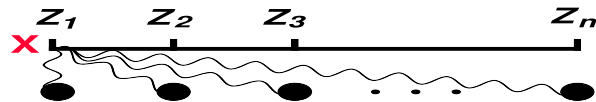
M.S. IJMS 2001

Wave function?

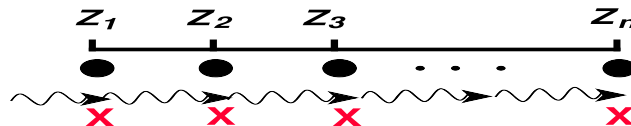
### III. Light-Front Wave Function of the Nucleus

- Emergence of the light-front dynamics

$$\tau = t - z \sim \frac{1}{q_+} \rightarrow 0$$



(a)



(b)

(a)

- nonrelativistic case: due to Galilean relativity  
observer X can probe all n-nucleons at the same time

$$\Psi(z_1, z_2, z_3, \dots, z_n, t)$$

- relativistic case: observer X probes all n-nucleons at different times

$$\Psi(z_1, t_1; z_2, t_2; z_3, t_3 \dots; z_n, t_n)$$

Example Deuteron  $z_p, t_p, z_n, t_n$

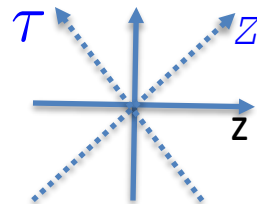
$$\Psi(z_p - z_n, t_p - t_n)$$

(b)

- observer riding the light-front X probes all n-nucleons at same light-cone time:

$$\tau = t_1 - z_1 = t_2 - z_2 = \dots = t_n - z_n$$

$$Z_i = t_i + z_i$$



# Problem with non-relativistic description: Relativistic Invariance

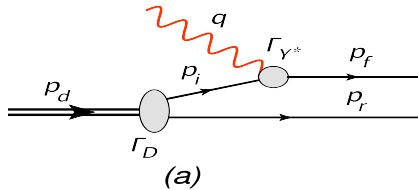
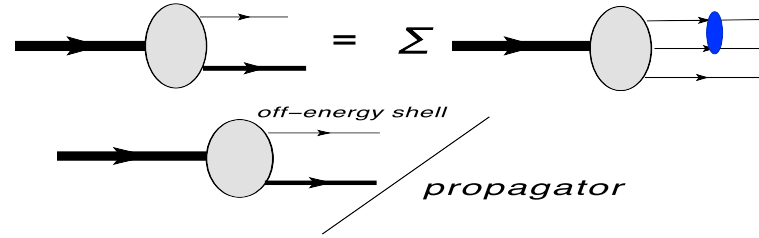
## - Relativistic Invariance

Lipmann-Schwinger Eq  $\rightarrow$

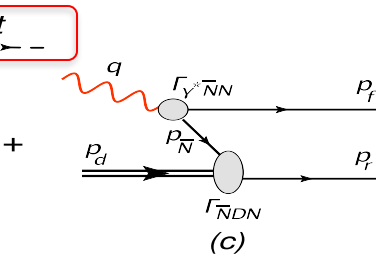
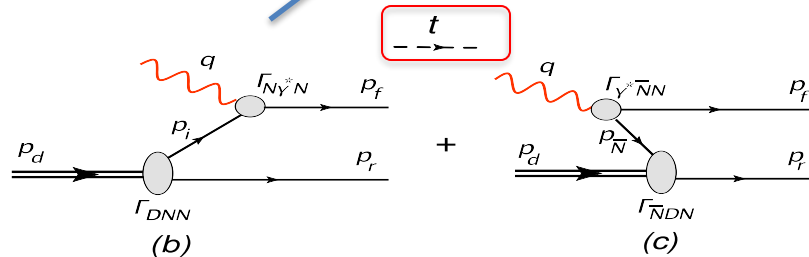
$$\left( \sum_i \frac{k_i^2}{2m} - E_b \right) \Phi(k_1, \dots, k_A) = -\frac{1}{2} \sum_{i,j} \int U(q) \Phi(k_1, \dots, k_i - q, \dots, k_j + q, \dots, k_A) d^3q$$

$$\Phi(k_1, \dots, k_A) = \frac{1}{\sum \frac{k_i^2}{2m} - E_b} \Gamma_{A \rightarrow N, A-1}$$

t- ordered diagrammatic method



=



(b)  $\sim$  (c)

Feynman/Covariant Diagram

Vacuum Fluctuations

- Normalization of the Wave Function:

$$4\pi \int_0^\infty |\Psi(k)|^2 k^2 dk = 1$$

# Light Front Description:

- Relativistic Invariance

$$\Psi_{LF}(\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \dots, \mathcal{Z}_n, \tau)$$

- in the momentum space

$$\Psi_{LF}(\alpha_1, p_{1\perp}; \alpha_2, p_{2\perp}; \alpha_3, p_{3\perp}; \dots, \alpha_n, p_{n\perp})$$

Variables

$$\alpha = \frac{p_N^+}{p_{NN}^+} \quad p_{\perp}$$

## Variables

Proper kinematic variables are not 3d momentum

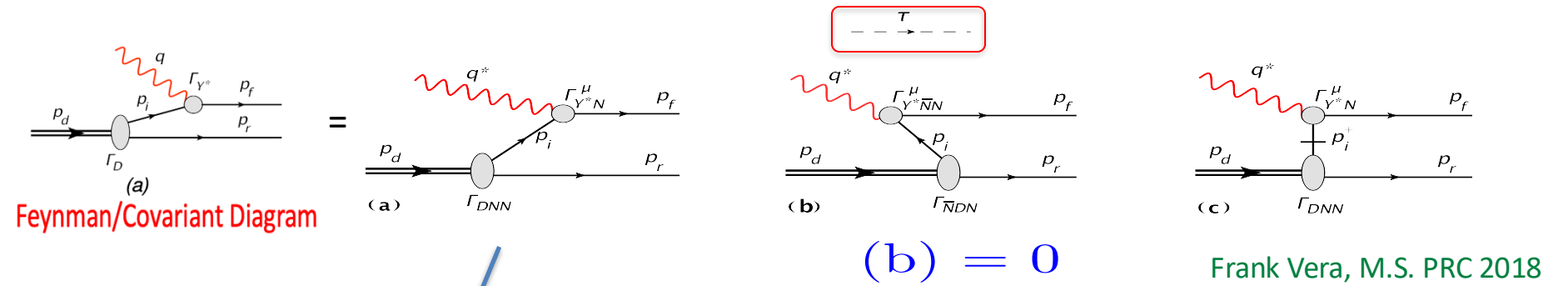
but the Light Front Momentum Fraction:

$$\alpha = \frac{p_N^+}{p_{NN}^+}$$

and transverse momentum:  $p_{\perp}$

- Lorentz invariant in z direction: Boost Invariant

- How the LF wave function appears in the scattering process

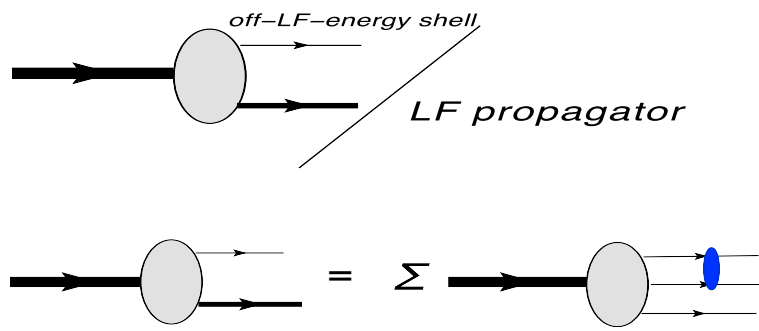


Weinberg Equation

$$\Phi_{LF}(k_1, \cdots k_A) = \frac{1}{\sum \frac{k_{i\perp}^2 + m^2}{\alpha_i} - M_A^2} \Gamma_{A \rightarrow N, A-1}$$

$$\left( \sum \frac{k_{i\perp}^2 + m^2}{\alpha_i} - M_A^2 \right) \Phi_{LF}(k_1, \cdots k_A) = \frac{1}{2} \sum_{i,j} \int U_{LF}(q) \Phi_{LF}(k_1, \cdots k_A) \prod \frac{d\alpha_i}{\alpha_i} d^2 k_{i\perp}$$

$\mathcal{T}$ - ordered diagrammatic method



- Normalization of the Wave Function to Observables:

Baryonic Number,  
Charge, etc

## IV From Schroedinger Equation -> Light-Front Wave Function

Schroedinger eq.



Lipmann-Schwinger Eq.

$$\left[ -\sum_i \frac{\nabla_i^2}{2m} + \frac{1}{2} \sum_{i,j} V(x_i - x_j) \right] \psi(x_1, \dots, x_A) = E \psi(x_1, \dots, x_A)$$

$$\left( \sum_i \frac{k_i^2}{2m} - E_b \right) \Phi(k_1, \dots, k_A) = -\frac{1}{2} \sum_{i,j} \int U(q) \Phi(k_1, \dots, k_i - q, \dots, k_j + q, \dots, k_A) d^3 q$$

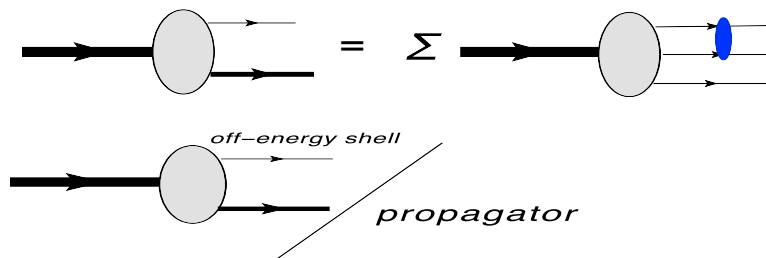
Lipmann-Schwinger Eq



t- ordered diagrammatic method

$$\left( \sum_i \frac{k_i^2}{2m} - E_b \right) \Phi(k_1, \dots, k_A) = -\frac{1}{2} \sum_{i,j} \int U(q) \Phi(k_1, \dots, k_i - q, \dots, k_j + q, \dots, k_A) d^3 q$$

$$\Phi(k_1, \dots, k_A) = \frac{1}{\sum \frac{k_i^2}{2m} - E_b} \Gamma_{A \rightarrow N, A-1}$$



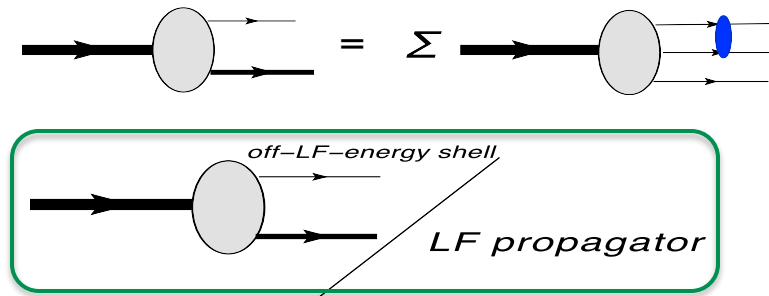
Weinberg Eq



$\mathcal{T}$ - ordered diagrammatic method

$$\left( \sum \frac{k_{i\perp}^2 + m^2}{\alpha_i} - M_A^2 \right) \Phi_{LF}(k_1, \dots, k_A) = \frac{1}{2} \sum_{i,j} \int U_{LF}(q) \Phi_{LF}(k_1, \dots, k_A) \prod \frac{d\alpha_i}{\alpha_i} d^2 k_{i\perp}$$

$$\Phi_{LF}(k_1, \dots, k_A) = \frac{1}{\sum \frac{k_{i\perp}^2 + m^2}{\alpha_i} - M_A^2} \Gamma_{A \rightarrow N, A-1}$$



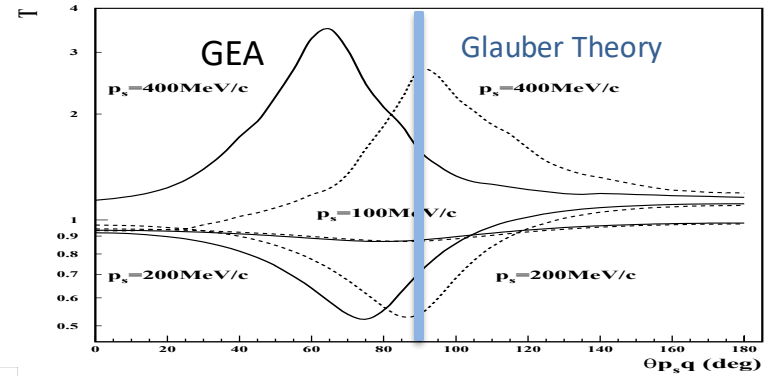
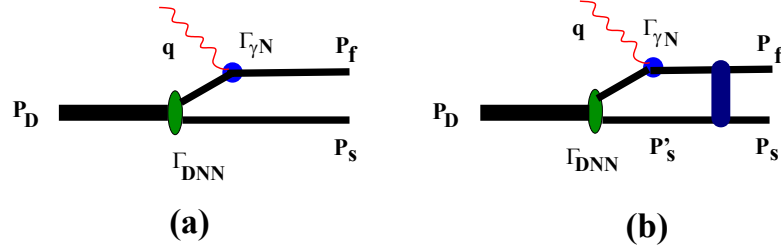
# Predictions made within the described approach

- Different location for maximum of FSI compared to Glauber model Frankfurt, MS., Strikman, PRC 1997, MS PRC2010
- Nuclear scaling due to 2N SRCs at: Frankfurt, Strikman Phys Rep.88  
Frankfurt, Day, Strikman, MS PRC1993
- pn dominance in 2N SRCs: Piasezky, MS, Frankfurt, Strikman, Watson PRL2006
- High momentum sharing in asymmetric nuclei MS PRC2014, ArXiv2012
- New Structure in the deuteron and non-nucleonic components MS & F.Vera PRL 2023
- Nuclear scaling for 3N SRCs MS, Day, Frankfurt, Strikman PRC2019  
Frankfurt, Day, MS, Strikman PRC2023
- Relation between 3N and 2N SRCs

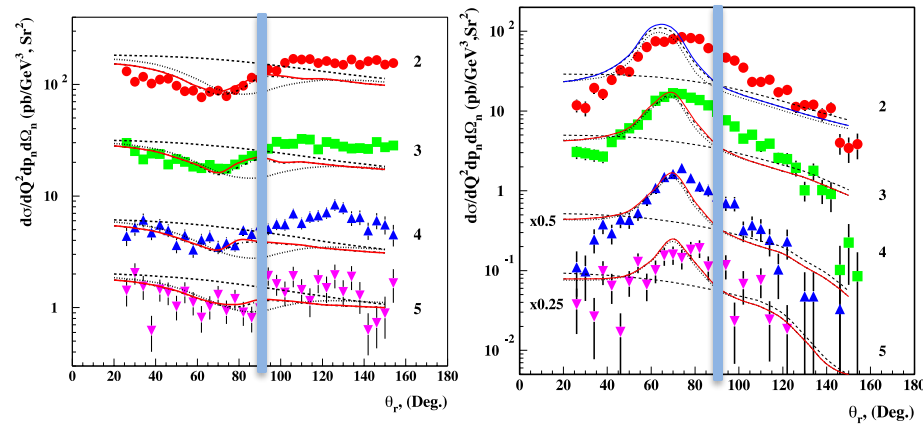


# Some Results: $e + d \rightarrow e' + p + n$

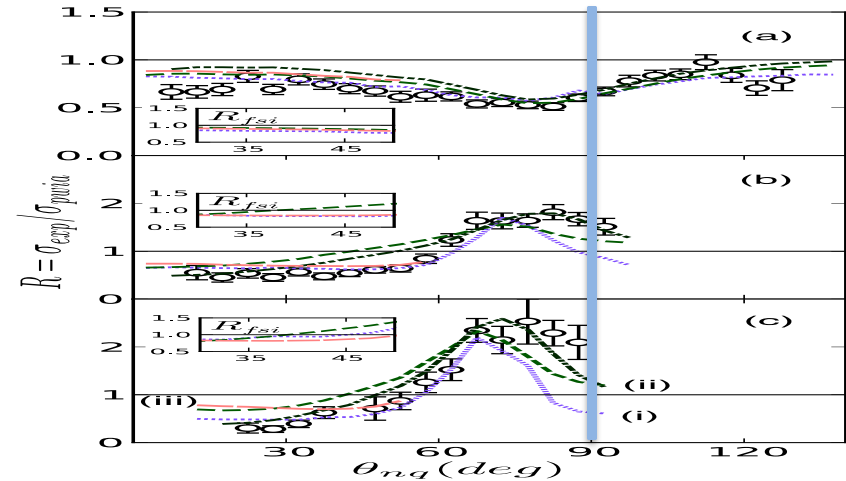
Frankfurt, M.S., Strikman, PRC 1997



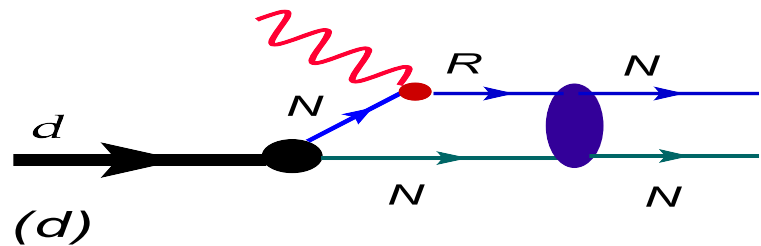
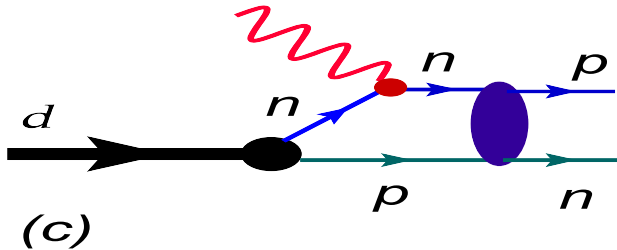
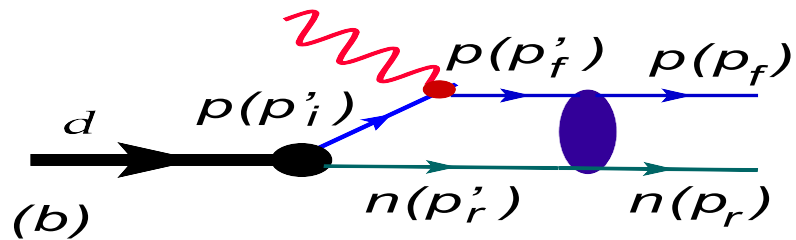
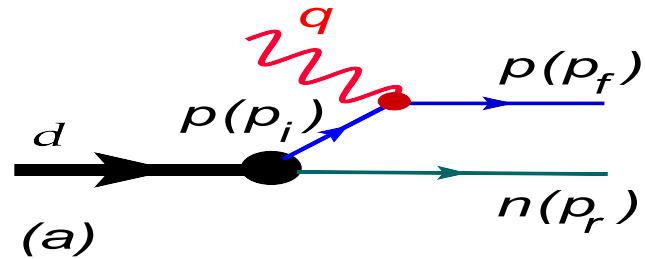
K. Egiyan et al PRL 2008



W. Boeglin et al PRL 2011



M.S. PRC 2010



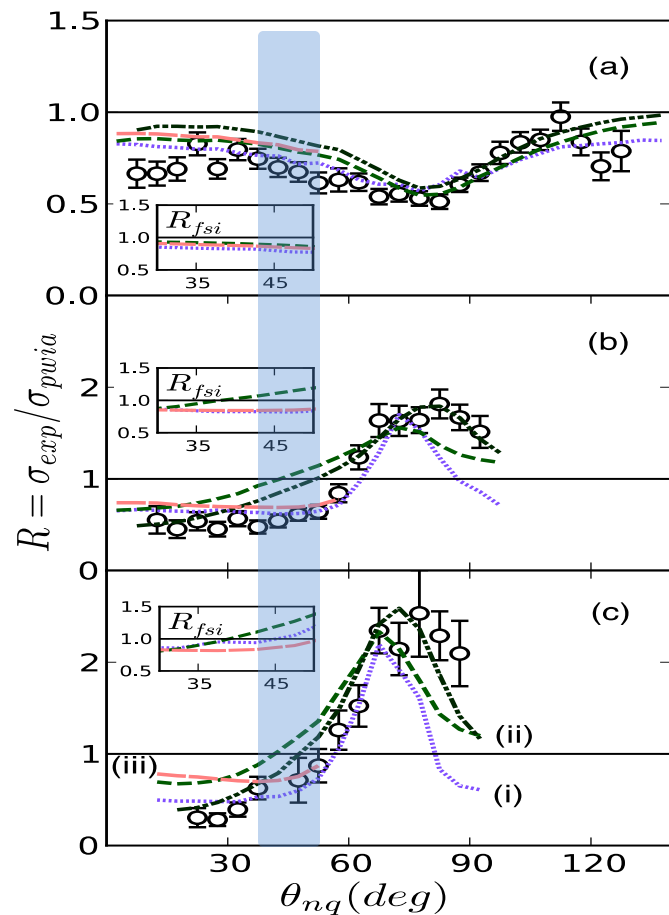
$$\langle s_f, s_r | A_0^\mu | s_d \rangle = -\bar{u}(p_r, s_r) \Gamma_{\gamma^* p}^\mu \frac{\not{p}_i + m}{p_i^2 - m^2} \cdot \bar{u}(p_f, s_f) \Gamma_{DNN} \cdot \chi^{s_d}$$

$$\begin{aligned} \langle s_f, s_r | A_1^\mu | s_d \rangle &= - \int \frac{d^4 p'_r}{i(2\pi)^4} \frac{\bar{u}(p_f, s_f) \bar{u}(p_r, s_r) F_{NN} [\not{p}'_r + m] [\not{p}_d - \not{p}'_r + \not{q} + m]}{(p_d - p'_r + q)^2 - m^2 + i\epsilon} \\ &\times \frac{\Gamma_{\gamma^* N} [\not{p}_d - \not{p}'_r + m] \Gamma_{DNN} \chi^{s_d}}{((p_d - p'_r)^2 - m^2 + i\epsilon)(p_r'^2 - m^2 + i\epsilon)}, \end{aligned} \quad (1)$$

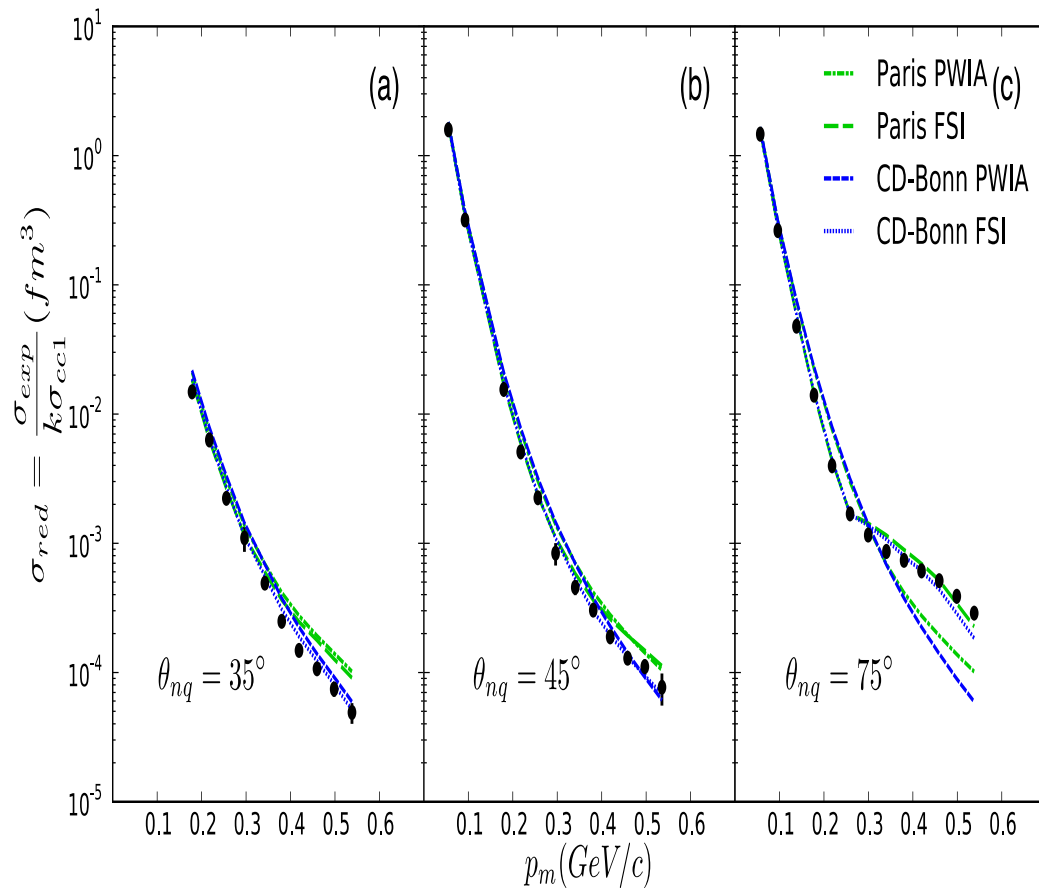
# Probing Deuteron at Small Distances at large $Q^2$

$$|p_i| = |p_f - q| \leq 550 \text{ MeV}/c$$

pn-component only



JLab,  $Q^2 = 3.5 \text{ GeV}^2$

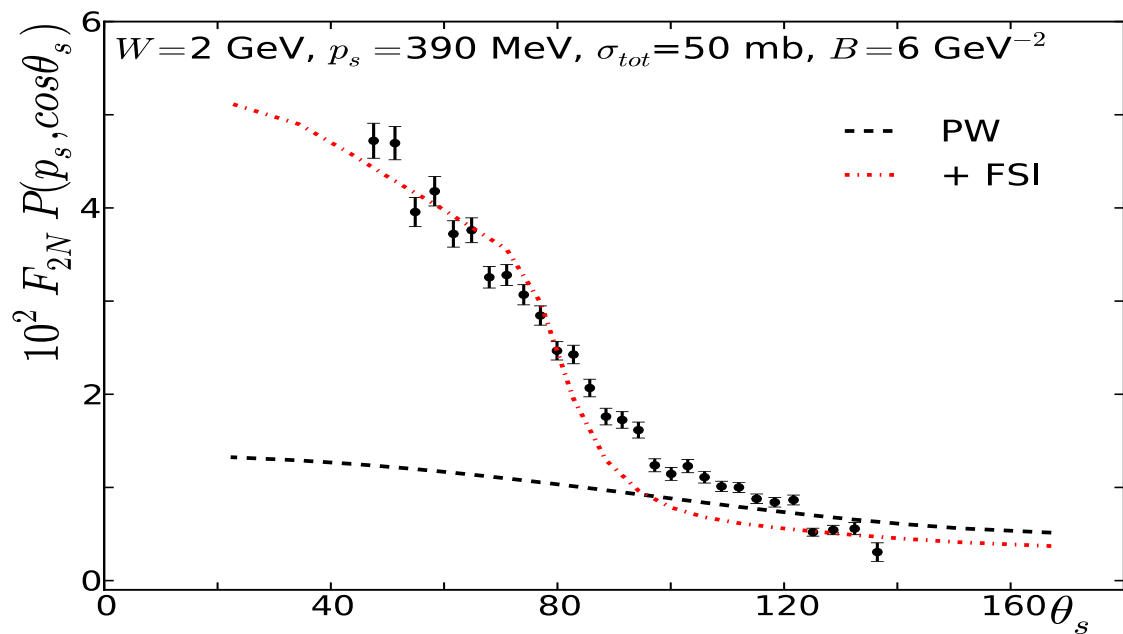


Boeglin et al PRL 2011, deuteron probed at up to 550 MeV/c

## Extension to DIS:

$$e + d \rightarrow e' + p_s + X$$

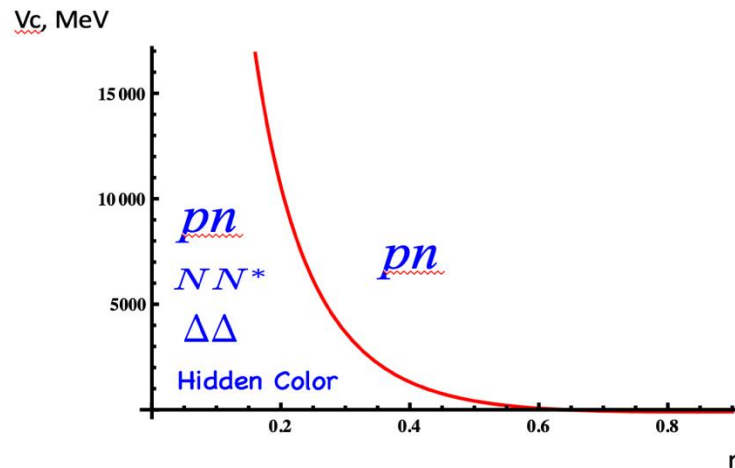
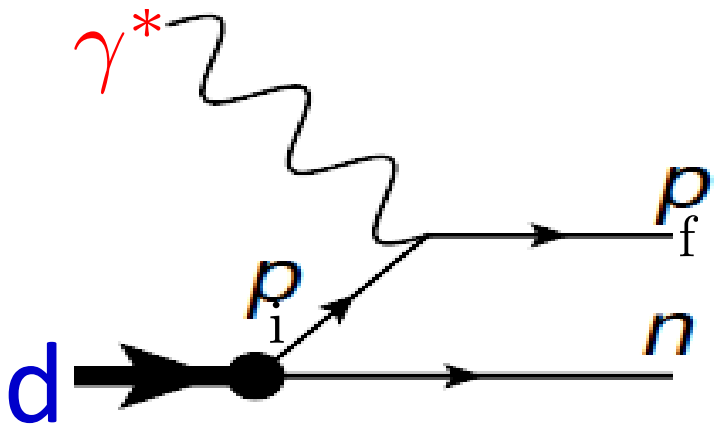
W.Cosyn & M.Sargsian, PRC 2011

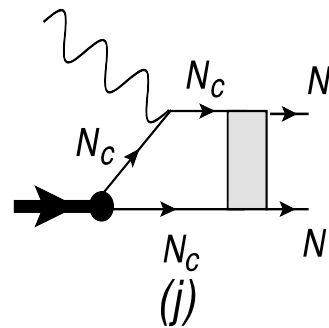
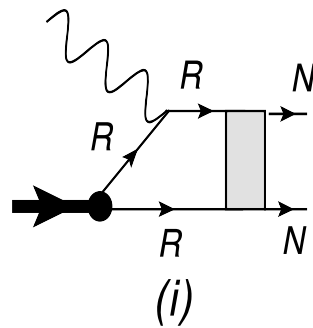
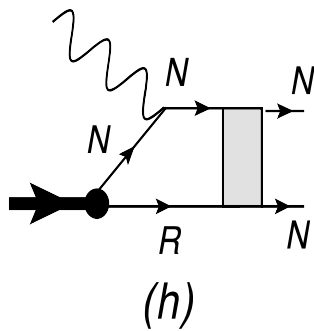
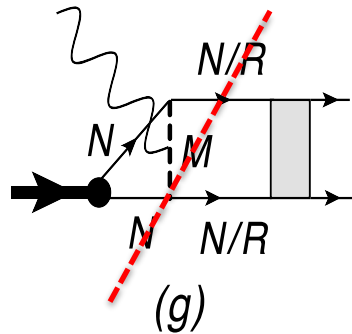
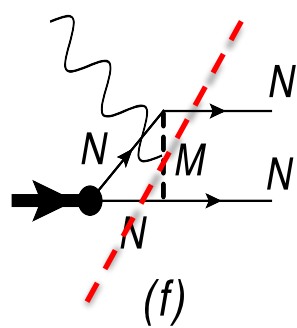
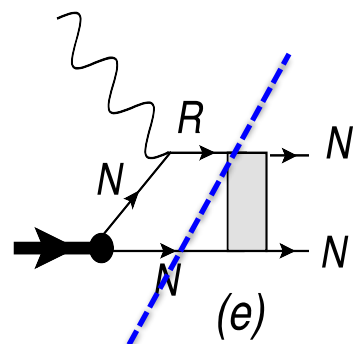
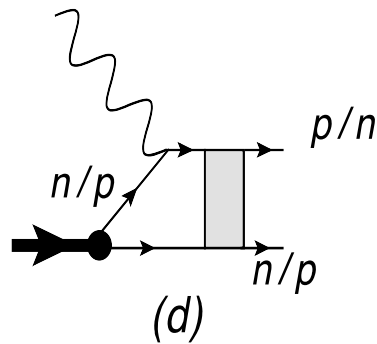
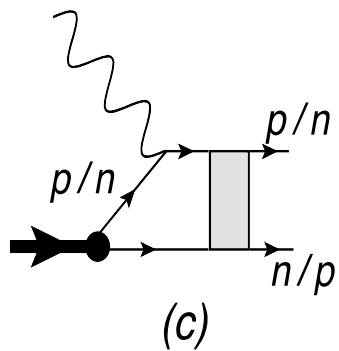
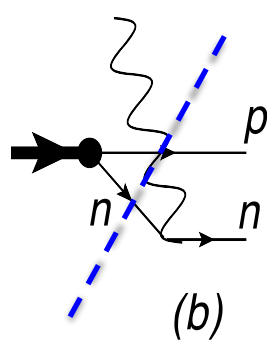
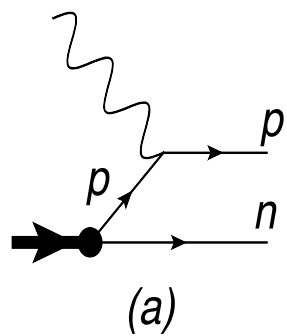


# Probing NN interaction at very short distances

Considering reaction:  $e + d \rightarrow e' + p_f + n$

$$|p_i| = |p_f - q| > 550 \text{ MeV}/c$$





# Some Paradigm Shift

Our current mindset about deuteron is fully non-relativistic, the observation that it has total spin,  $J=1$  and parity,  $P=+$ , together with the relation that for non-relativistic wave function,  $P=(-1)^l$ , one concludes that the deuteron consists of S- and D- partial waves for proton-neutron system.

**Paradigm Shift:** The above reaction at high  $Q^2$ , measures the probability of observing proton and neutron in the deuteron at very large relative momenta. In such a formulation the deuteron is not a composite system consisting of proton and neutron but it is a composite pseudo - vector ( $J=1, P=+$ ) "particle" from which one extracts proton and neutron.

How such a proton and neutron produced at such extremal conditions is related to the dynamical structure of Light-Front deuteron wave function, which may include internal elastic  $pn \rightarrow pn$  as well as inelastic  $\Delta\Delta \rightarrow pn$ ,  $N^*N \rightarrow pn$  or  $N_C N_C \rightarrow pn$  transitions.

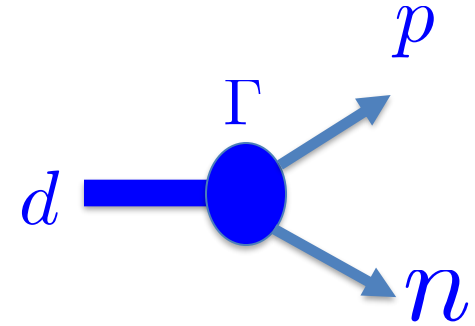
# New Structure in the Deuteron and possible non-nucleonic components

M.S & Frank Vera PRL 2023

## Paradigm shift:

- consider a deuteron not a nucleus that consist of proton and neutron
- but **pseudovector composite particle** from which we **extract** proton and neutron
- Light-Front Deuteron wave function

$$\psi_d^{\lambda_d}(\alpha_i, p_{\perp}, \lambda_1 \lambda_2) = - \frac{\bar{u}(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) \Gamma_d \chi^{\lambda_d}}{\frac{1}{2} (m_d^2 - 4 \frac{m_N^2 + p_{\perp}^2}{\alpha_i (2 - \alpha_i)}) \sqrt{2(2\pi)^3}}$$



- Absorbing the energy denominator into the vertex function and using crossing symmetry

$$\psi_d^{\mu}(\alpha_i, p_{\perp}, \lambda_1, \lambda_2) = -\bar{u}(p_2, \lambda_2) \Gamma_d^{\mu}(k) \frac{(i\gamma_2 \gamma_0)}{\sqrt{2}} \bar{u}(p_1, \lambda_1)^T = - \sum_{\lambda'_1} \bar{u}(p_1, \lambda_1) \Gamma_d^{\mu} \gamma_5 \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} u(p_1, \lambda'_1)$$



$$\psi_d^\mu(\alpha_i, p_\perp, \lambda_1, \lambda_2) = -\bar{u}(p_2, \lambda_2) \Gamma_d^\mu(k) \frac{(i\gamma_2 \gamma_0)}{\sqrt{2}} \bar{u}(p_1, \lambda_1)^T = - \sum_{\lambda'_1} \bar{u}(p_1, \lambda_1) \Gamma_d^\mu \gamma_5 \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} u(p_1, \lambda'_1)$$

-  $\Gamma_d^\mu$  - is a four-vector, which can be constructed in a most general form satisfying time reversal, parity and charge conjugate symmetries

- Because the deuteron is a bound system, in addition to on-shell  $p_1$  and  $p_2$  four momenta one introduces

$$\Delta^\mu \equiv p_1^\mu + p_2^\mu - p_d^\mu \equiv (\Delta^-, \Delta^+, \Delta_\perp) = (\Delta^-, 0, 0)$$

$$\Delta^- = p_1^- + p_2^- - p_d^- = \frac{m_N^2 + k_\perp^2}{p_1^+} + \frac{m_N^2 + k_\perp^2}{p_2^+} - \frac{M_d^2}{p_d^+} = \frac{1}{p_d^+} \left[ \frac{4(m_N^2 + k_\perp^2)}{\alpha_1(2 - \alpha_1)} - M_d^2 \right] = \frac{4}{p_d^+} \left[ m_N^2 - \frac{M_d^2}{4} + k^2 \right]$$

- Constructed vertex:

$$\Gamma_d^\mu = \Gamma_1 \gamma^\mu + \Gamma_2 \frac{(p_1 - p_2)^\mu}{2m_N} + \Gamma_3 \frac{\Delta^\mu}{2m_N} + \Gamma_4 \frac{(p_1 - p_2)^\mu \not{\Delta}}{4m_N^2} + i\Gamma_5 \frac{1}{4m_N^3} \gamma_5 \epsilon^{\mu\nu\rho\gamma} (p_d)_\nu (p_1 - p_2)_\rho (\Delta)_\gamma + \Gamma_6 \frac{\Delta^\mu \not{\Delta}}{4m_N^2}$$

## High Momentum Transfer Kinematics

For large  $Q^2$  limit, Light-Front momenta for the reaction are chosen as follows:

$$p_d^\mu \equiv (p_d^-, p_d^+, p_{d\perp}) = \left( \frac{Q^2}{x\sqrt{s}} \left[ 1 + \frac{x}{\tau} - \sqrt{1 + \frac{x^2}{\tau}} \right], \frac{Q^2}{x\sqrt{s}} \left[ 1 + \frac{x}{\tau} + \sqrt{1 + \frac{x^2}{\tau}} \right], 0_\perp \right)$$
$$q^\mu \equiv (q^-, q^+, q_\perp) = \left( \frac{Q^2}{x\sqrt{s}} \left[ 1 - x + \sqrt{1 + \frac{x^2}{\tau}} \right], \frac{Q^2}{x\sqrt{s}} \left[ 1 - x - \sqrt{1 + \frac{x^2}{\tau}} \right], 0_\perp \right)$$

where  $s = (q + p_d)^2$ ,  $\tau = \frac{Q^2}{M_d^2}$  and  $x = \frac{Q^2}{M_d q_0}$ , with  $q_0$  being virtual photon energy in the deuteron rest frame.

- One observes that for fixed  $x$ ,  $p_d^+ \sim \sqrt{Q^2} \gg m_N$

$$\Delta^\mu \equiv p_1^\mu + p_2^\mu - p_d^\mu \equiv (\Delta^-, \Delta^+, \Delta_\perp) = (\Delta^-, 0, 0),$$

where

$$\begin{aligned} \Delta^- &= p_1^- + p_2^- - p_d^- = \frac{m_N^2 + k_\perp^2}{p_1^+} + \frac{m_N^2 + k_\perp^2}{p_2^+} - \frac{M_d^2}{p_d^+} \\ &= \frac{1}{p_d^+} \left[ \frac{4(m_N^2 + k_\perp^2)}{\alpha_1(2 - \alpha_1)} - M_d^2 \right] = \frac{4}{p_d^+} \left[ m_N^2 - \frac{M_d^2}{4} + k^2 \right]. \end{aligned}$$

In high  $Q^2$  limit

$$\frac{\Delta^-}{2m_N} \ll 1$$

$$\begin{aligned} \Gamma_d^\mu &= \Gamma_1 \gamma^\mu + \Gamma_2 \frac{(p_1 - p_2)^\mu}{2m_N} + \Gamma_3 \frac{\cancel{\Delta^\mu}}{2m_N} + \Gamma_4 \frac{(p_1 - p_2)^\mu \cancel{\Delta}}{4m_N^2} \\ &\quad + i\Gamma_5 \frac{1}{4m_N^3} \gamma_5 \epsilon^{\mu\nu\rho\gamma} (p_d)_\nu (p_1 - p_2)_\rho (\Delta)_\gamma + \Gamma_6 \frac{\cancel{\Delta^\mu} \cancel{\Delta}}{4m_N^2} \end{aligned}$$

Consider:  $\epsilon^{\mu,+,\perp,-} p_{d,-} k_{\perp} \Delta_{+}$

Since:  $p_{d,-} = \frac{1}{2} p_d^{+}$  and  $\Delta_{+} = \frac{1}{2} \Delta^{-}$  then  $p_d^{+} \Delta^{-} = p_d^{+} \frac{1}{p_d^{+}} \left[ \frac{4(m_N^2 + k_{\perp}^2)}{\alpha_1(2-\alpha_1)} - M_d^2 \right] = \left[ \frac{4(m_N^2 + k_{\perp}^2)}{\alpha_1(2-\alpha_1)} - M_d^2 \right]$

$\epsilon^{\mu,+,\perp,-} p_{d,-} k_{\perp} \Delta_{+} = \frac{1}{4} \epsilon^{\mu,+,\perp,-} p_d^{+} k_{\perp} \Delta^{-}$  **Leading Order!**

$$\Gamma_d^{\mu} = \Gamma_1 \gamma^{\mu} + \Gamma_2 \frac{(p_1 - p_2)^{\mu}}{2m_N} + \cancel{\Gamma_3 \frac{\Delta^{\mu}}{2m_N}} + \Gamma_4 \frac{(p_1 - p_2)^{\mu} \cancel{\Delta}}{4m_N^2} \\ + i\Gamma_5 \frac{1}{4m_N^3} \gamma_5 \epsilon^{\mu\nu\rho\gamma} (p_d)_{\nu} (p_1 - p_2)_{\rho} (\Delta)_{\gamma} + \Gamma_6 \frac{\cancel{\Delta^{\mu}} \cancel{\Delta}}{4m_N^2}$$

$$\psi_d^{\lambda_d}(\alpha_i, k_{\perp}) = - \sum_{\lambda_2, \lambda_1, \lambda'_1} \bar{u}(-k, \lambda_2) \left\{ \Gamma_1 \gamma^{\mu} + \Gamma_2 \frac{\tilde{k}^{\mu}}{m_N} + \sum_{i=1}^2 i\Gamma_5 \frac{1}{8m_N^3} \epsilon^{\mu+i-} p_d'^{+} k_i \Delta'^{-} \right\} \gamma_5 \frac{\epsilon_{\lambda_1, \lambda'_i}}{\sqrt{2}} u(k, \lambda'_1) s_{\mu}^{\lambda_d}$$

where  $\tilde{k}^{\mu} = (0, k_z, k_{\perp})$

$$\psi_d^{\lambda_d}(\alpha_i, k_{\perp}) = - \sum_{\lambda_2, \lambda_1, \lambda'_1} \bar{u}(-k, \lambda_2) \left\{ \Gamma_1 \gamma^{\mu} + \Gamma_2 \frac{\tilde{k}^{\mu}}{m_N} + \sum_{i=1}^2 i \Gamma_5 \frac{1}{8m_N^3} \epsilon^{\mu+i-} p_d^{\prime+} k_i \Delta^{\prime-} \right\} \gamma_5 \frac{\epsilon_{\lambda_1, \lambda'_i}}{\sqrt{2}} u(k, \lambda'_1) s_{\mu}^{\lambda_d}$$

$$\psi_d^{\lambda_d}(\alpha_1, k_t, \lambda_1, \lambda_2) = \sum_{\lambda'_1} \phi_{\lambda_2}^{\dagger} \sqrt{E_k} \left[ \frac{U(k)}{\sqrt{4\pi}} \sigma \mathbf{s}_{\mathbf{d}}^{\lambda_d} - \frac{W(k)}{\sqrt{4\pi} \sqrt{2}} \left( \frac{3(\sigma \mathbf{k})(\mathbf{k} \mathbf{s}_{\mathbf{d}}^{\lambda})}{k^2} - \sigma \mathbf{s}_{\mathbf{d}}^{\lambda} \right) + \right. \\ \left. (-1)^{\frac{1+\lambda_d}{2}} P(k) Y_1^{\lambda_d}(\theta, \phi) \delta^{1, |\lambda_d|} \right] \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} \phi_{\lambda'_1}$$

$$U(k) = \frac{2\sqrt{4\pi}\sqrt{E_k}}{3} \left[ \Gamma_1 (2 + \frac{m_N}{E_k}) + \Gamma_2 \frac{k^2}{m_N E_k} \right]$$

$$W(k) = \frac{2\sqrt{4\pi}\sqrt{2E_k}}{3} \left[ \Gamma_1 (1 - \frac{m_N}{E_k}) - \Gamma_2 \frac{k^2}{m_N E_k} \right]$$

Where:  $Y_1^{\pm}(\theta, \phi) = \mp i \sqrt{\frac{3}{4\pi}} \sum_{i=1}^2 \frac{(k \times s_d^{\pm 1})_z}{k}$

$P(k) = \sqrt{4\pi} \frac{\Gamma_5(k) \sqrt{E_k}}{\sqrt{3}} \frac{k^3}{m_N^3}$ 

fully relativistic: in addition to

has additional

$\frac{k^{l=1}}{m_N}$  term  
 $\frac{k^2}{m_N^2}$  term

## Light Front Density Matrix and Momentum Distribution

$$\psi_d^{\lambda_d}(\alpha_1, k_t, \lambda_1, \lambda_2) = \sum_{\lambda'_1} \phi_{\lambda'_1}^\dagger \sqrt{E_k} \left[ \frac{U(k)}{\sqrt{4\pi}} \sigma \mathbf{s}_d^{\lambda_d} - \frac{W(k)}{\sqrt{4\pi}\sqrt{2}} \left( \frac{3(\sigma \mathbf{k})(\mathbf{k} \mathbf{s}_d^{\lambda_d})}{k^2} - \sigma \mathbf{s}_d^{\lambda_d} \right) + \right. \\ \left. (-1)^{\frac{1+\lambda_d}{2}} P(k) Y_1^{\lambda_d}(\theta, \phi) \delta^{1,|\lambda_d|} \right] \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} \phi_{\lambda'_1}$$

$$\rho_d(\alpha, k_\perp) = \frac{n_d(k, k_\perp)}{2-\alpha}$$

$$n_d(k, k_\perp) = \frac{1}{3} \sum_{\lambda_d=-1}^1 | \psi_d^{\lambda_d}(\alpha, k_\perp) |^2 = \frac{1}{4\pi} \left( U(k)^2 + W(k)^2 + \frac{k_\perp^2}{k^2} P^2(k) \right)$$

Baryonic and Momentum Sum Rules  $\int \rho_d(\alpha, k_\perp) \frac{d\alpha}{\alpha} = 1$  and  $\int \alpha \rho_d(\alpha, k_\perp) \frac{d\alpha}{\alpha} = 1$

$$\int \left( U(k)^2 + W(k)^2 + \frac{2}{3} P^2(k) \right) k^2 dk = 1.$$

## Non-Nucleonic Components and the New Structure

$$n_d(k, k_{\perp}) = \frac{1}{3} \sum_{\lambda_d=-1}^1 | \psi_d^{\lambda_d}(\alpha, k_{\perp}) |^2 = \frac{1}{4\pi} \left( U(k)^2 + W(k)^2 + \frac{k_{\perp}^2}{k^2} P^2(k) \right)$$

- Momentum distribution depends on  $k_{\perp}$  separately
- *This is impossible for non-relativistic quantum mechanics of the deuteron since in this case the potential of the interaction is real (no inelasticities) and the solution of Lippmann-Schwinger (or Schroedinger) equation for partial S- and D-waves satisfies "angular condition", according to which the momentum distribution in unpolarized deuteron depends on the magnitude of relative momentum only.*
- On the other hand, in the relativistic domain the definition of the interaction potential is not straightforward to allow to use quantum-mechanical arguments in claiming that momentum distribution should satisfy the angular condition (i.e. depends on magnitude of  $k$  only).

- However, for the Light-Front, there is a remarkable theorem by Frankfurt and Strikman which states that if **one considers only pn component in the deuteron**, then for most acceptable forms of NN potential – constructed from elastic pn → pn scattering, the angular condition should be satisfied also for LF momentum distribution.

$$T_{NN}(\alpha_i, k_{i\perp}, \alpha_f, k_{f\perp}) \equiv T_{NN}(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f\perp}) = V(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f\perp}) + \int V(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m\perp}) \times \frac{d^3 k_m}{(2\pi)^3 \sqrt{m^2 + k_m^2}} \frac{T_{NN}(k_{m,z}, k_{m\perp}, k_{f,z}, k_{f\perp})}{4(k_m^2 - k_f^2)}$$

- The realization of the angular condition for relativistic case will require that light-front potential to satisfy a condition

$$V(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m\perp}) = V(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2)$$

- Lorentz invariance for on-shell NN amplitude requires

$$T_{NN}^{on\ shell}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m\perp}) = T_{NN}^{on\ shell}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2)$$



- Existence of the Born term indicates that

$$T_{NN}^{on\ shell}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2) = V_{NN}^{on\ shell}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2) + \int V_{NN}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) \times \frac{d^3 k_m}{(2\pi)^3 \sqrt{m^2 + k_m^2}} \frac{T_{NN}(k_{m,z}, k_{m\perp}, k_{f,z}, k_{f,\perp})}{4(k_m^2 - k_f^2)}$$

- Iterating the equation around the on-shell kinematic point.

$$T_{NN}(\alpha_i, k_{i\perp}, \alpha_f, k_{f,\perp}) \equiv T_{NN}(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f,\perp}) = V(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f,\perp}) + \int V(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) \times \frac{d^3 k_m}{(2\pi)^3 \sqrt{m^2 + k_m^2}} \frac{T_{NN}(k_{m,z}, k_{m\perp}, k_{f,z}, k_{f,\perp})}{4(k_m^2 - k_f^2)}$$

- will result in:

$$T_{NN}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) = T_{NN}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2)$$

$$V_{NN}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) = V_{NN}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2) \quad \text{for the general case}$$

-  $V_{NN}$  – analytic function of angular momentum and it does not diverge exponentially in the complex-angular momentum space it was shown that also for the off-shell case

- This results in a light front momentum distribution that satisfies the angular condition

- For Non-nucleonic components no such iteration can be done

$$T_{NN}(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f,\perp}) = \int V_{NN^*}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) \frac{d^3 k_m}{(2\pi)^3 \sqrt{m_m^2 + k_m^2}} \frac{T_{N^*N}(k_{m,z}, k_{m\perp}, k_{f,z}, k_{f,\perp})}{4(k_m^2 - k_f^2 + m_m^2 - m_N^2)}$$

- transition amplitudes such as  $T_{\Delta\Delta\rightarrow NN}$ ,  $T_{N^*,N\rightarrow NN}$  or  $T_{N^c,N^c\rightarrow NN}$  where  $N^cN^c$  represents a hidden color component in the deuteron could not be described with any combination of interaction potentials that satisfies angular condition
- if  $\Gamma_5$  term is not zero then it should originate from non-nucleonic component in the deuteron.
- Our prediction is that the observation of LF momentum distribution depending on the center of mass  $k$  and  $k_\perp$  separately will indicate the presence of non-nucleonic component in the deuteron

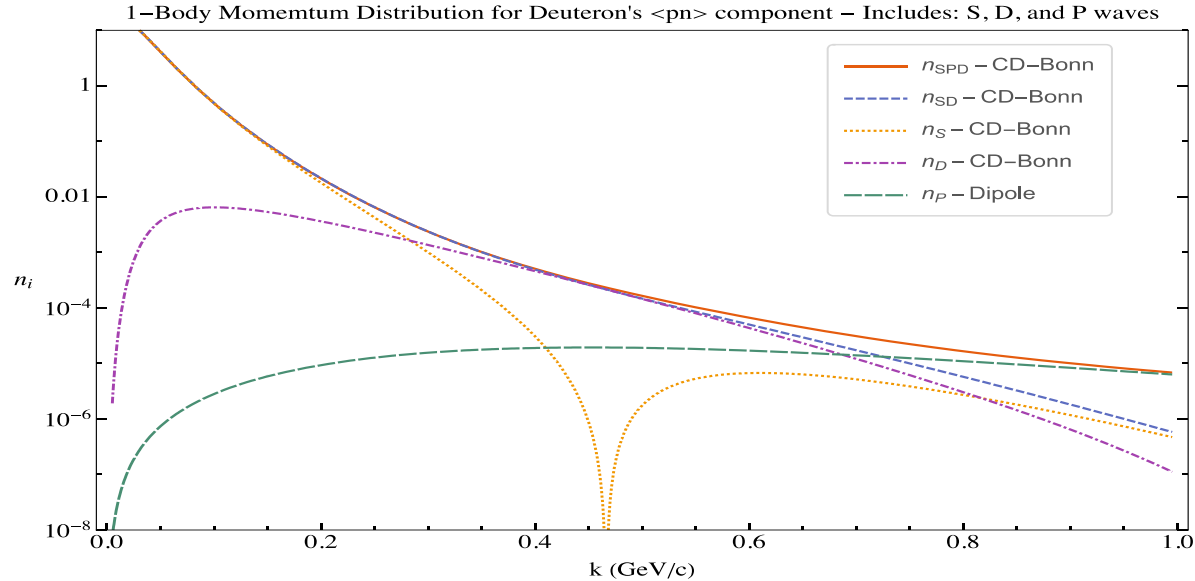
## Estimate of the effect

$$n_d(k, k_{\perp}) = \frac{1}{3} \sum_{\lambda_d=-1}^1 |\psi_d^{\lambda_d}(\alpha, k_{\perp})|^2 = \frac{1}{4\pi} \left( U(k)^2 + W(k)^2 + \frac{k_{\perp}^2}{k^2} P^2(k) \right)$$

$$P(k) = \sqrt{4\pi} \frac{\Gamma_5(k) \sqrt{E_k}}{\sqrt{3}} \frac{k^3}{m_N^3}$$

$$\Gamma_5(k) = \frac{A}{(1 + \frac{k^2}{0.71})^2}$$

$A$  is estimated by assuming 1% contribution to the total normalization.



Frank Vera, PhD Thesis 2021

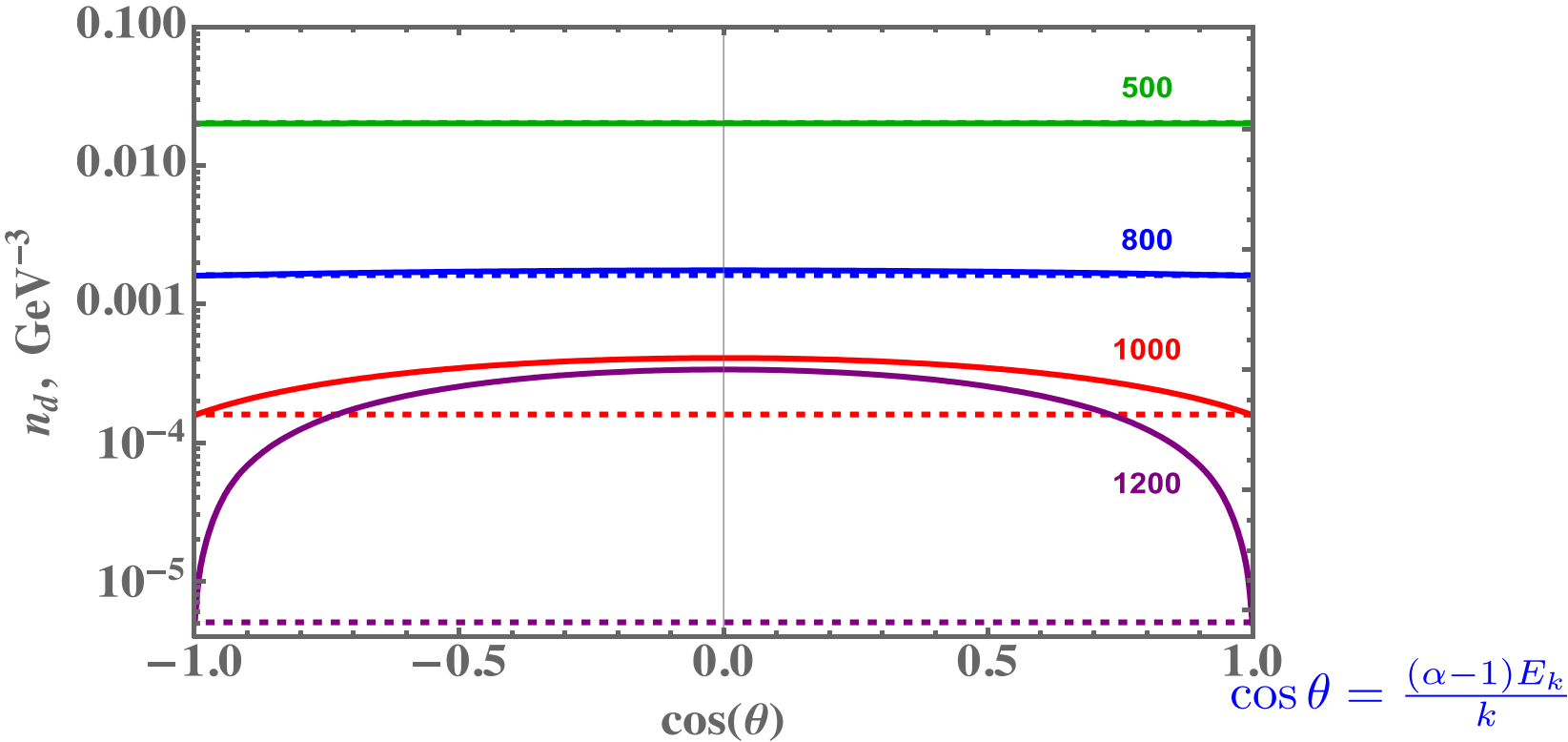
**Estimate of the effect**

$$n_d(k,k_{\perp}) = \frac{1}{3} \sum_{\lambda_d=-1}^1 \left| \psi_d^{\lambda_d}(\alpha,k_{\perp}) \right|^2 = \frac{1}{4\pi} \left( U(k)^2 + W(k)^2 + \frac{k_{\perp}^2}{k^2} P^2(k) \right)$$

$$P(k) = \sqrt{4\pi} \frac{\Gamma_5(k) \sqrt{E_k}}{\sqrt{3}} \frac{k^3}{m_N^3}$$

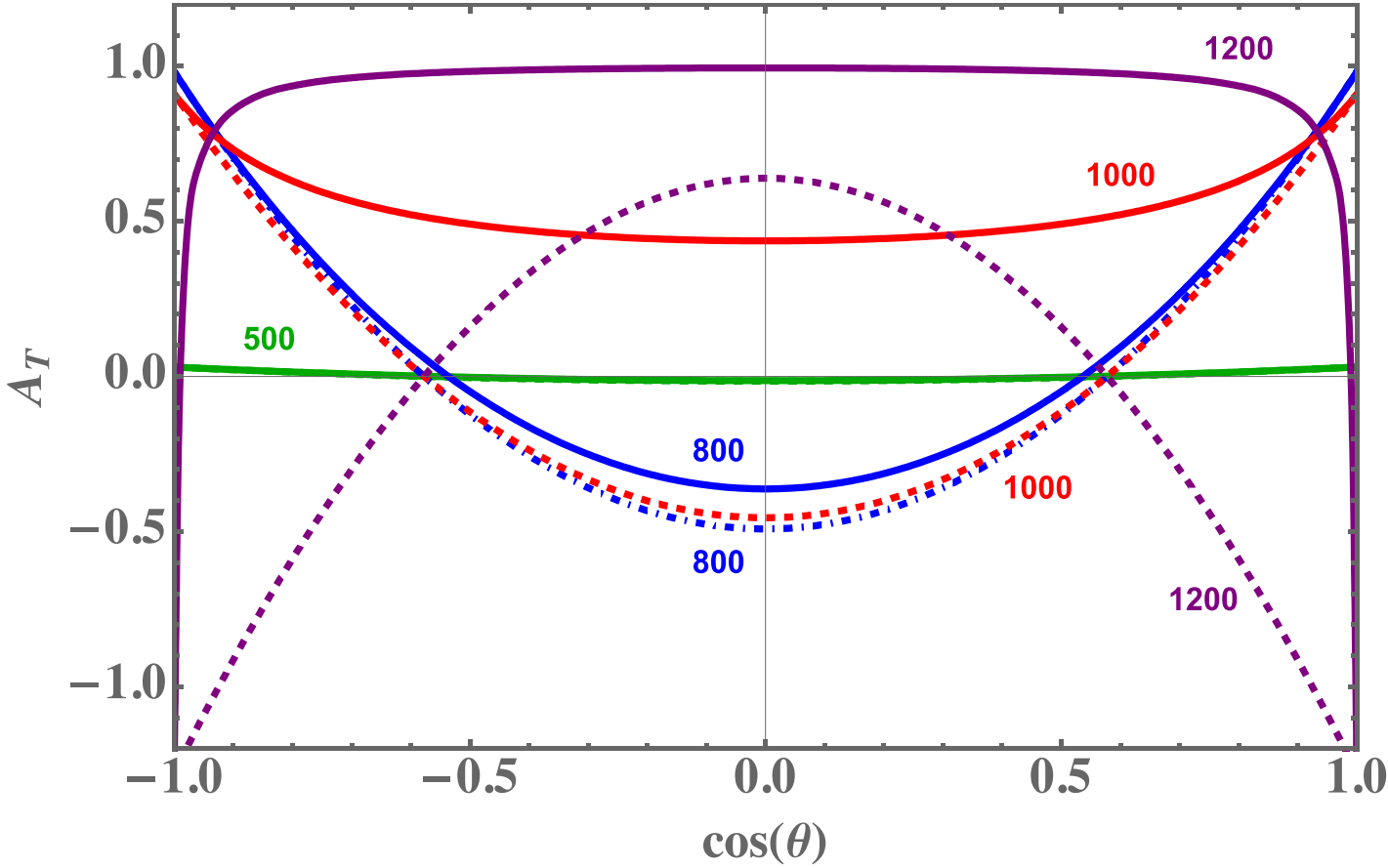
$$\Gamma_5(k) = \frac{A}{(1+\frac{k^2}{0.71})^2}$$

$A$  is estimated by assuming 1% contribution to the total normalization.



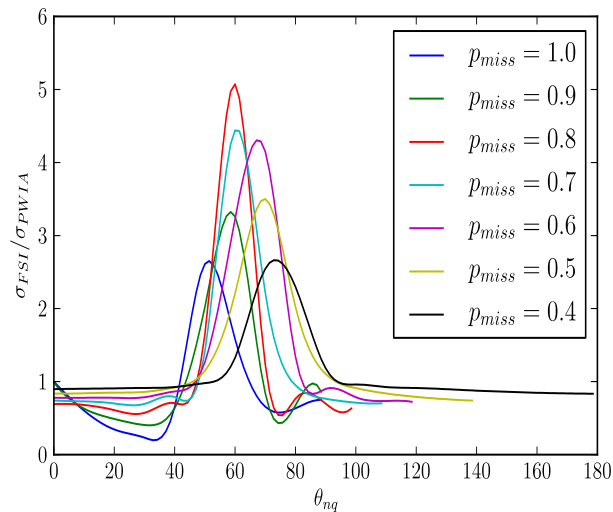
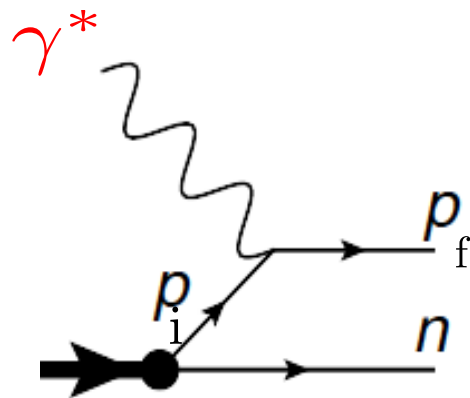
Estimate of the effect

$$A_T = \frac{n_d^{\lambda_d=1}(k,k_\perp) + n_d^{\lambda_d=-1}(k,k_\perp) - 2n_d^{\lambda_d=0}(k,k_\perp)}{n_d(k,k_\perp)}$$



$$\cos \theta = \frac{(\alpha-1)E_k}{k}$$

## Possibility of Experimental Verification



Considering reaction:  $e + d \rightarrow e' + p_f + n$

$$|p_i| = |p_f - q| \gtrsim 800 \text{ MeV}/c$$

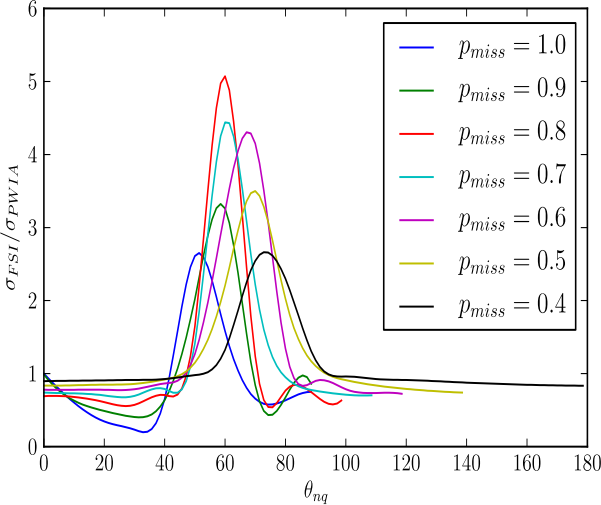
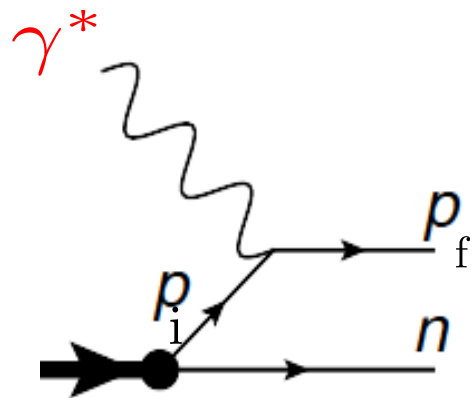
PAC-36, 2010

■ E12-10-003 ( $p_m \square 300 \text{ MeV}$ ): “Deuteron Electro-Disintegration at Very High Missing Momentum”

Rating: B+

data are essential to constrain further theory developments. Overall the experiment was viewed very highly; the lower rating simply reflects the likelihood that the data will not reveal any particular surprise and that their impact may thus be limited to experts in the field.

Possibility of Experimental Verification

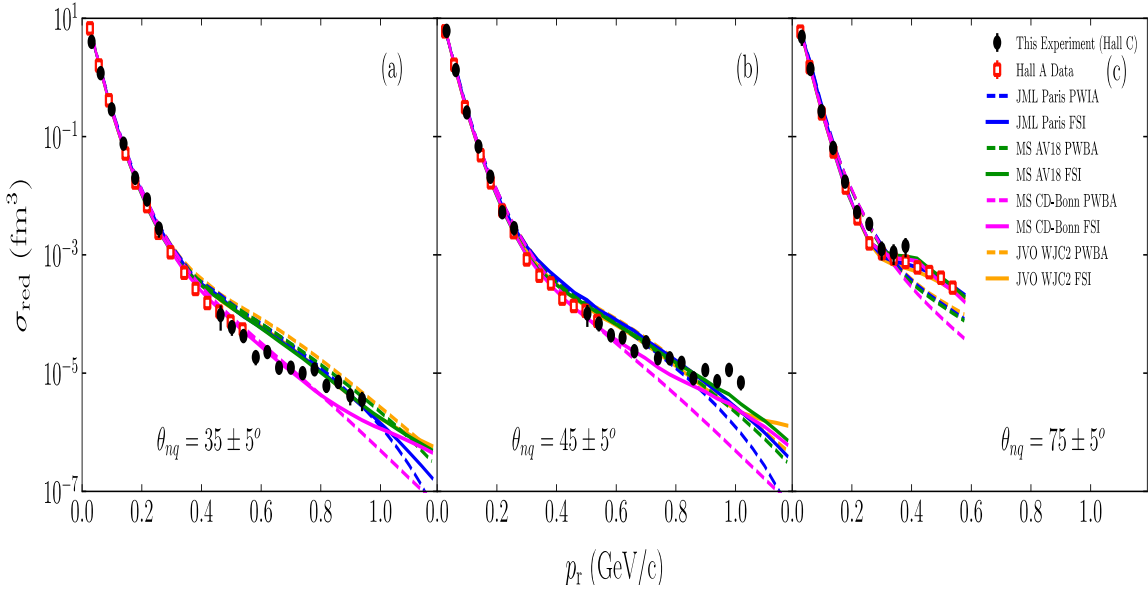


Considering reaction:  $e + d \rightarrow e' + p_f + n$

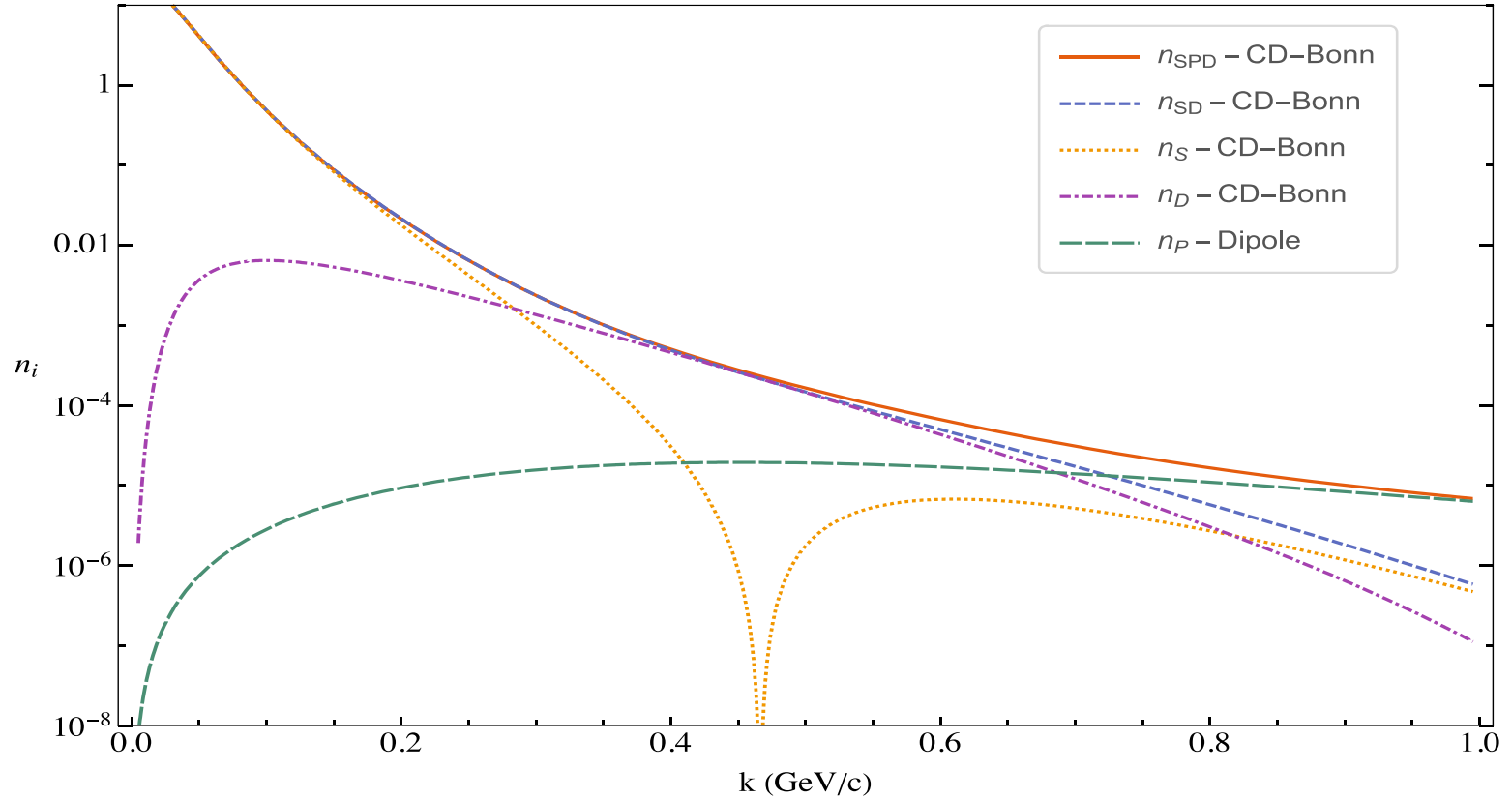
$$|p_i| = |p_f - q| \gtrsim 800 \text{ MeV/c}$$

3-days of commissioning measurement,

JLab experiment  $Q^2 = 4 \text{ GeV}^2$



1-Body Momentum Distribution for Deuteron's <pn> component – Includes: S, D, and P waves



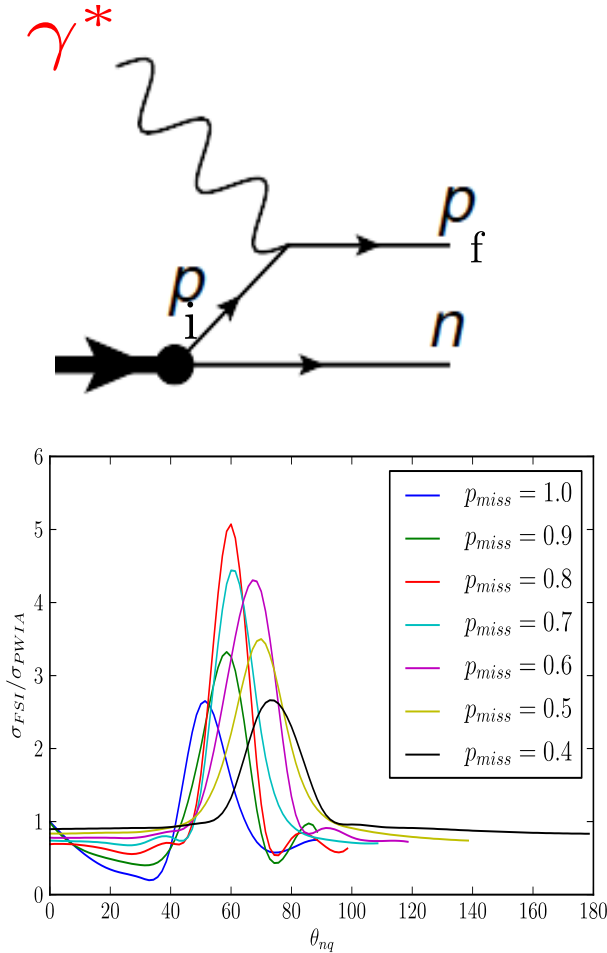


Possibility of Experimental Verification

Considering reaction:  $e + d \rightarrow e' + p_f + n$

$$|p_i| = |p_f - q| \gtrsim 800\text{MeV}/c$$

PAC-49, 2021



PAC 49 SUMMARY OF JEOPARDY RECOMMENDATIONS							
Number	Contact Person	Title	Hall	Previously Approved Days	Days Already Rec'd	Days Awarded	PAC Decision
<a href="#">E12-09-011</a>	Tanja Horn	Studies of the L-T Separated Kaon Electroproduction Cross Section from 5-11 GeV	C	40	32	8	Remain active
<a href="#">E12-10-003</a>	W. Boeglin	Deuteron Electro-Disintegration at Very High Missing Momentum	C	21	3	18	Upgrade Rating to A-

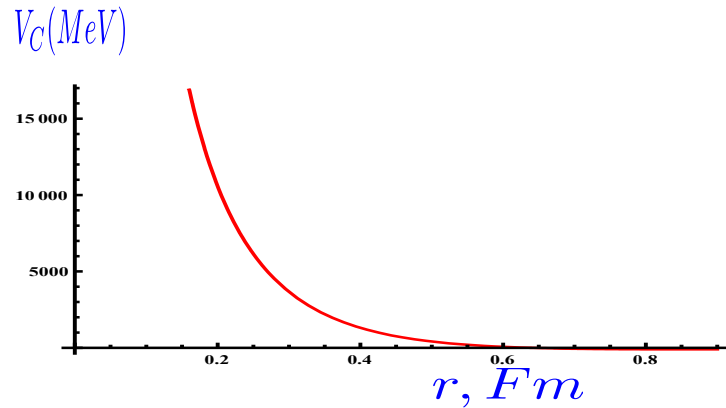
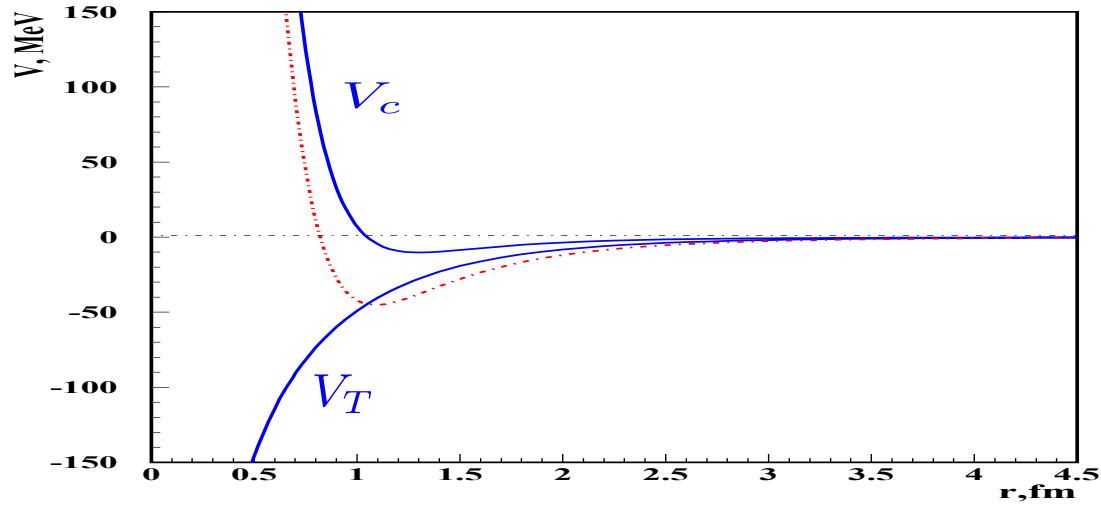
1) Is there any new information that would affect the scientific importance or impact of the Experiment since it was originally proposed?

PAC 36 graded the proposal with B+ because, even though the physics motivation was viewed highly, the foreseen impact of the result was judged to be limited. The results of the three days commissioning in April 2018, published in Physical Review Letters 125, 262501 (2020), exhibit an unexpected behavior when compared with theoretical calculations. Therefore, the expected impact of future data has increased.

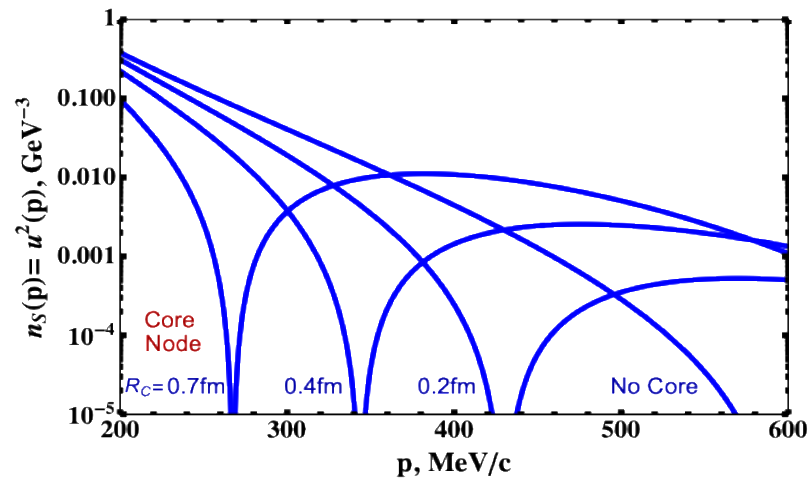
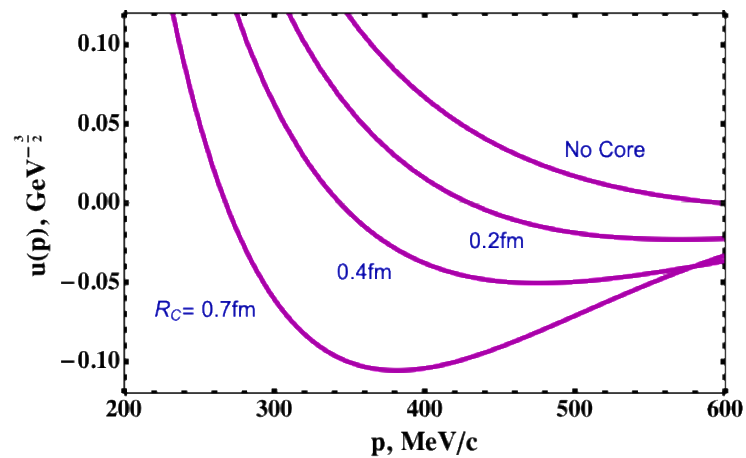
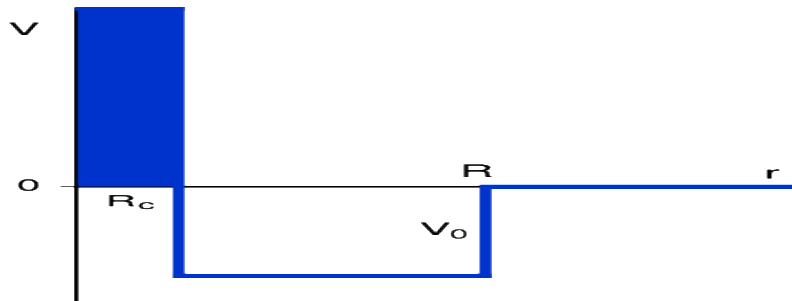
## Outlook on Experimental Verification of the Effect

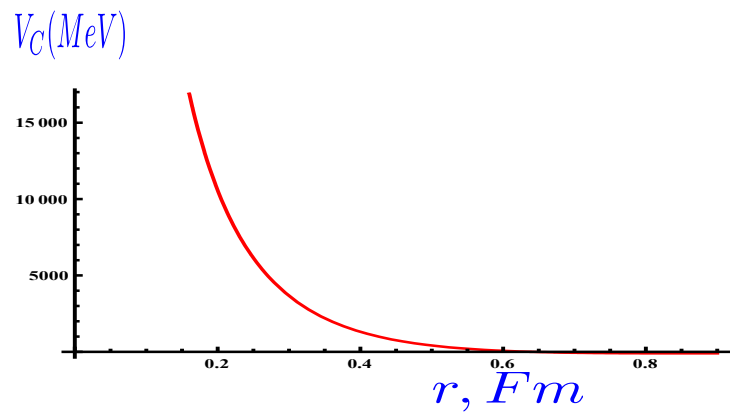
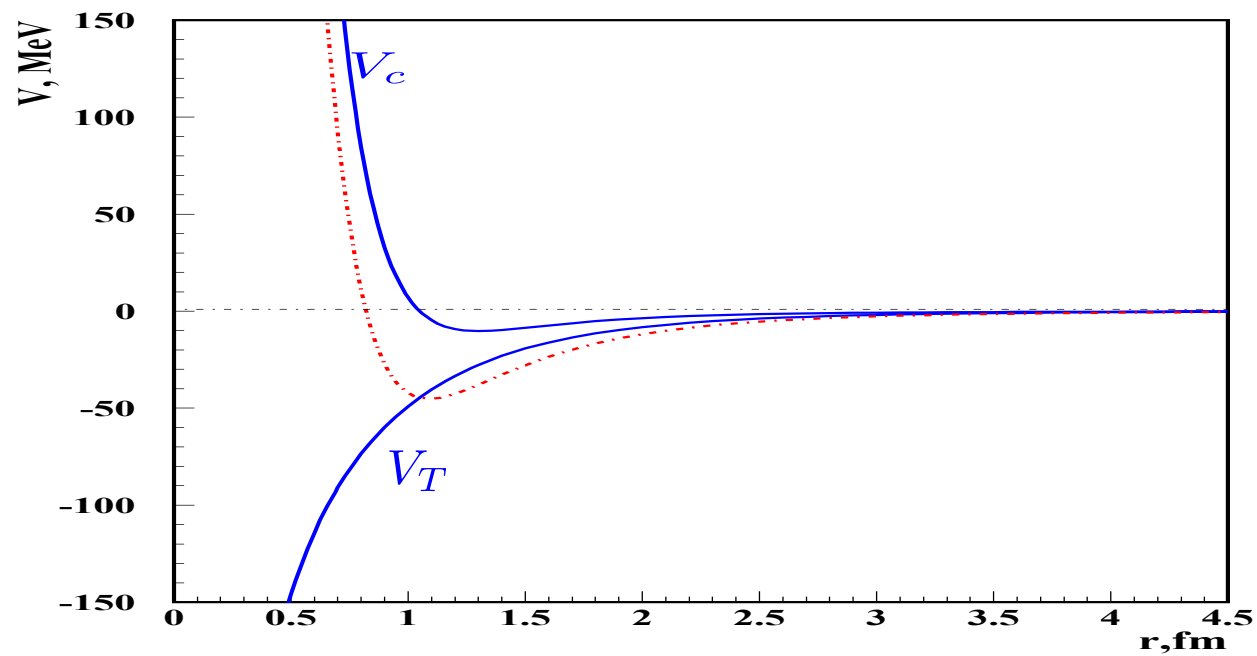
- analysis of the experiment will require careful account for competing nuclear effects most importantly final state interactions
- If angular dependence is found it will motivate new area of research
  - a: modeling non-nucleonic components in the deuteron,
  - b: understanding their origin and nature
  - c: evaluating parameters that can be used for Equation of State of high density Nuclear Matter
- If no angular dependence is found,
  - a: nucleonic degrees persist at very high density fluctuations
  - b: non-nucleonic components conspire to preserve angular condition
  - c: theory was wrong

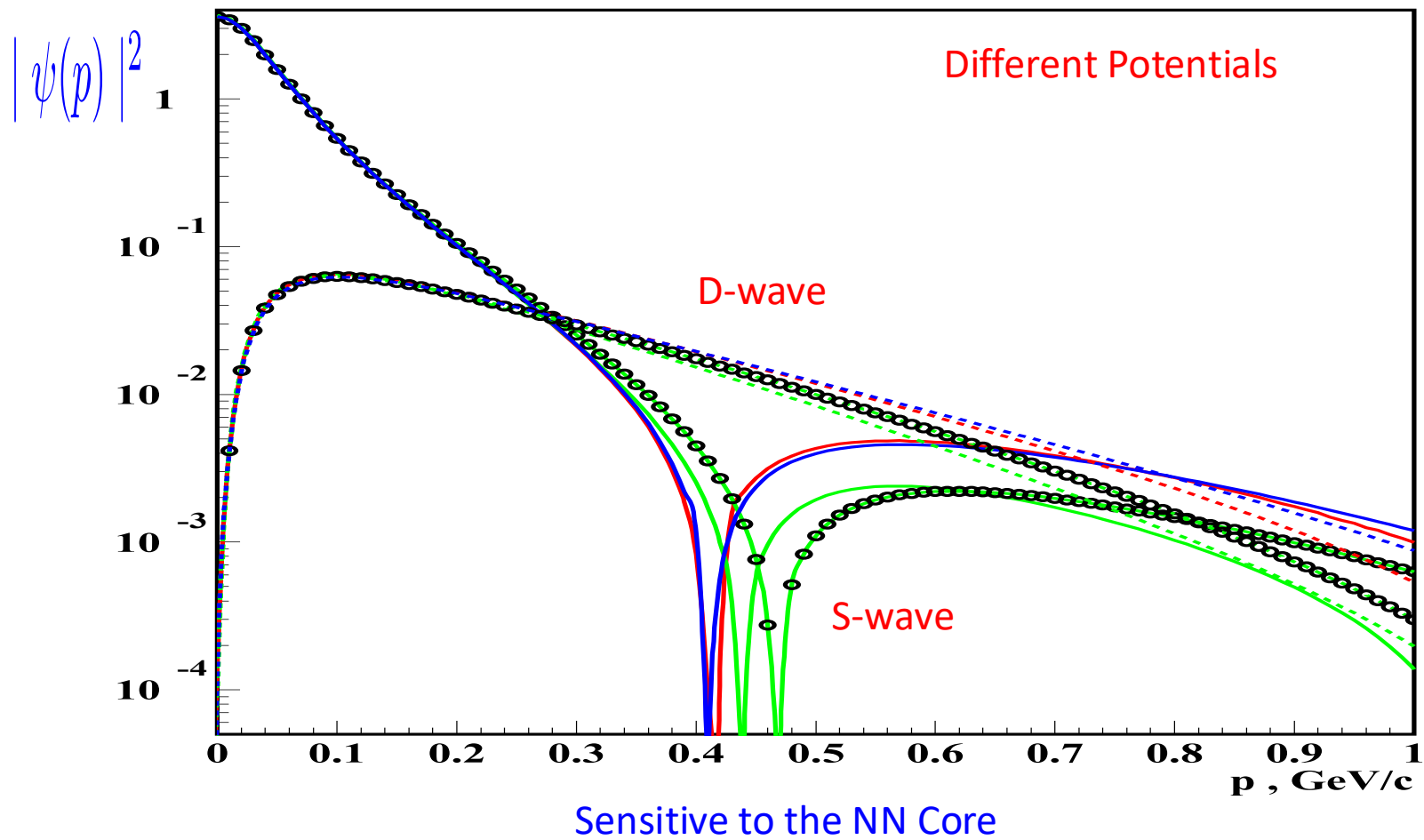
# Node of the Core:



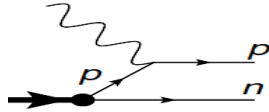
# Core Effects:







PWIA

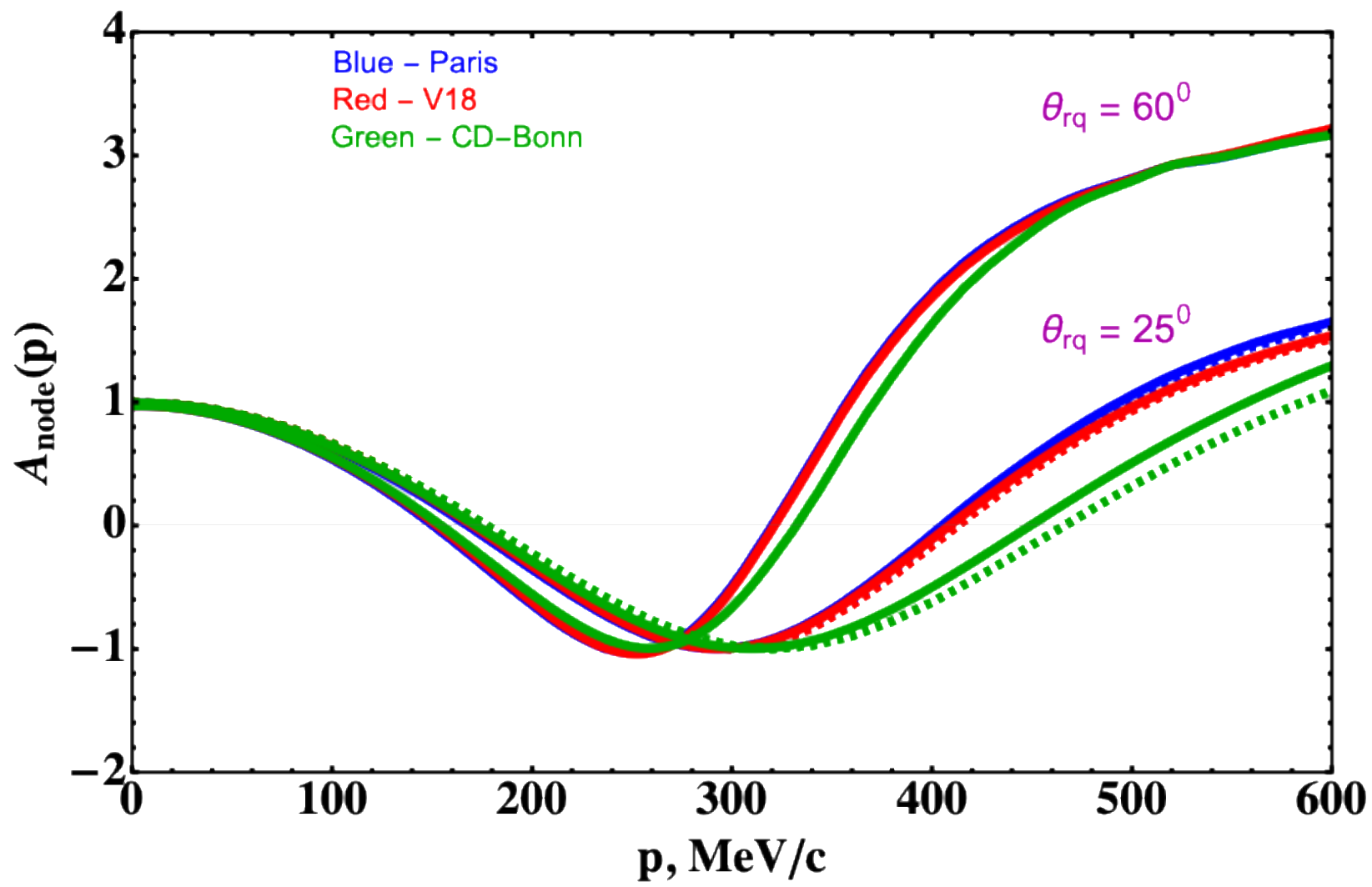


$$\rho_{20}(p, \theta_N) \equiv \frac{|\psi_d^1|^2 + |\psi_d^{-1}|^2 - 2|\psi_d^0|^2}{3} = \frac{3 \cos^2(\theta_N) - 1}{2} \left[ 2\sqrt{2}u(p)w(p) - w^2(p) \right]$$

$$\rho_{node}(p) = \rho_{unp}(p) + \frac{2\rho_{20}}{3 \cos^2(\theta_N) - 1} = u^2(p) + 2\sqrt{2}u(p)w(p)$$

$$A_{node}(p) \equiv \frac{\rho_{node}(p)}{\rho_{unp}(p)} = 1 + \frac{2A_{zz}(p, \theta_N)}{3 \cos^2(\theta_N) - 1} \frac{u^2(p) + 2\sqrt{2}u(p)w(p)}{u(p)^2 + w(p)^2}$$

$$A_{zz} = \frac{\rho_{20}}{\rho_{unp}} \longrightarrow T_{20} = \frac{\sigma_{tensor}}{\sigma_{unp}}$$





## V. Emergence of small distance nuclear dynamics

-start with A-body Lipman Schwinger/Weinberg type equation for nuclear wave function interacting through NN -potential

$$\phi_A(k_1, \dots, k_n, \dots, k_A) = \frac{-\frac{1}{2} \int \sum_{i \neq j} V_{ij}(q) \phi_A(k_1, \dots, k_i - q, \dots, k_j + q, \dots, k_A) \frac{d^3 q}{(2\pi)^3}}{\sum_{i=1}^A \frac{k_i^2}{2m_N} - E_B}$$

$$k = p, \quad \frac{p^2}{2m} \gg E_B, \quad k_i - q = p - q \approx 0 \rightarrow q \approx p, \quad k_j + q \approx k_j + p \approx 0 \rightarrow k_i \approx -k_j \approx p$$

$$\phi_A^{(1)}(k_1, \dots, k_i = p, \dots, k_j \approx -p, \dots, k_A) \sim \frac{V_{NN}(p)}{p^2} f(k_1, \dots, \dots)$$

$$\phi_A^{(2)}(\dots p, \dots) \sim \frac{1}{p^2} \int \frac{V_{NN}(q)V_{NN}(p)}{(p-q)^2} d^3 q$$

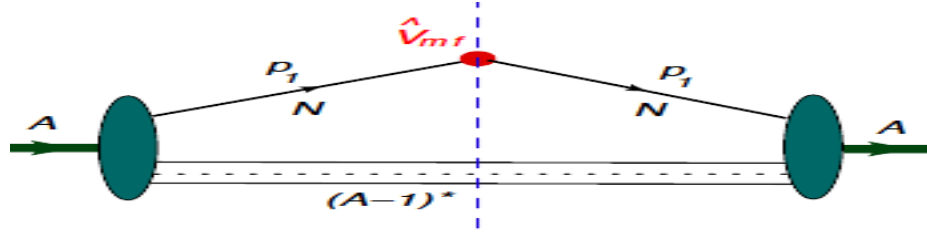
- IF:  $V_{NN} = q^{-n}$  with  $n > 1$  and is finite range  $q_{min}$

$$\phi_A^{(2)}(\dots p, \dots) \sim \frac{V(p)}{p^2} \int_{q_{min}}^{\infty} \frac{dq}{q^n}$$

$$\phi_A^{(2)} \ll \phi_A^{(1)}$$

Frankfurt, Strikman 1988  
Frankfurt, MS, Strikman 2008

# Spectral Function Calculations



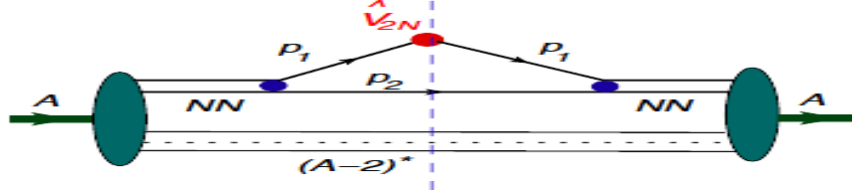
$$S_A^{MF} = -Im \int \chi_A^\dagger \Gamma_{A,N,A-1}^\dagger \frac{p_1 + m}{p_1^2 - m^2 + i\varepsilon} \hat{V}^{MF} \frac{p_1 + m}{p_1^2 - m^2 + i \times \varepsilon} \left[ \frac{G_{A-1}(p_{A-1})}{p_{A-1}^2 - M_{A-1}^2 + i\varepsilon} \right]^{on} \Gamma_{A,N,A-1} \chi_A \frac{d^4 p_{A-1}}{i(2\pi)^4}$$

$$\hat{V}^{MF} = ia^\dagger(p_1, s_1) \delta^3(p_1 + p_{A-1}) \delta(E_m - E_\alpha) a(p_1, s_1)$$

$$\psi_{N/A}(p_1, s_1, s_A, E_\alpha) = \frac{\bar{u}(p_1, s_1) \Psi_{A-1}^\dagger(p_{A-1}, s_{A-1}, E_\alpha) \Gamma_{A,N,A-1} \chi_A}{(M_{A-1}^2 - p_{A-1}^2) \sqrt{(2\pi)^3 2E_{A-1}}}$$

$$S_A^{MF}(p_1, E_m) = \sum_{\alpha} \sum_{s_1, s_{A-1}} | \psi_{N/A}(p_1, s_1, s_A, E_\alpha) |^2 \delta(E_m - E_\alpha)$$

# 2N SRC model



$$\begin{aligned}
 P_{A,2N}^N(\alpha_1, p_{1,\perp}, \tilde{M}_N^2) &= \sum_{s_2, s_{NN}, s_{A-2}} \int \chi_A^\dagger \Gamma_{A \rightarrow NN, A-2}^\dagger \chi_{A-2}(p_{A-2}, s_{A-2}) \\
 &\times \frac{\chi_{NN}(p_{NN}, s_{NN}) \chi_{NN}^\dagger(p_{NN}, s_{NN})}{p_{NN}^2 - M_{NN}^2} \Gamma_{NN \rightarrow NN}^\dagger \frac{u(p_1, s_1) u(p_2, s_2)}{p_1^2 - M_N^2} \\
 &\times \left[ 2\alpha_1^2 \delta(\alpha_1 + \alpha_2 + \alpha_{A-2} - A) \delta^2(p_{1,\perp} + p_{2,\perp} + p_{A-2,\perp}) \delta(\tilde{M}_N^2 - \tilde{M}_N^{(2N),2}) \right] \frac{\bar{u}(p_1, s_1) \bar{u}(p_2, s_2)}{p_1^2 - M_N^2} \\
 &\times \Gamma_{NN \rightarrow NN} \frac{\chi_{NN}(p_{NN}, s_{NN}) \chi_{NN}^\dagger(p_{NN}, s_{NN})}{p_{NN}^2 - M_{NN}^2} \chi_{A-2}^\dagger(p_{A-2}, s_{A-2}) \Gamma_{A, NN, A-2} \chi_A \\
 &\times \frac{d\alpha_2}{\alpha_2} \frac{d^2 p_{2,\perp}}{2(2\pi)^3} \frac{d\alpha_{A-2}}{\alpha_{A-2}} \frac{d^2 p_{A-2,\perp}}{2(2\pi)^3}.
 \end{aligned} \tag{1}$$

O. Artiles & M.S. Phys. Rev. C 2016

$$\rho_A(\alpha_N, p_{N,\perp}) = \int P_A(\alpha_N, p_{N,\perp}, \tilde{M}_N^2) \frac{1}{2} d\tilde{M}_N^2$$

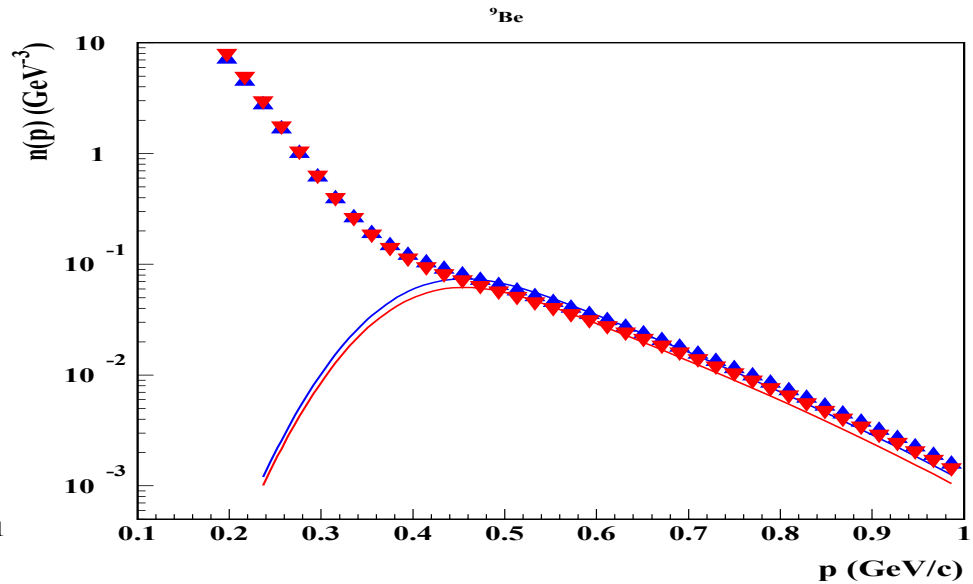
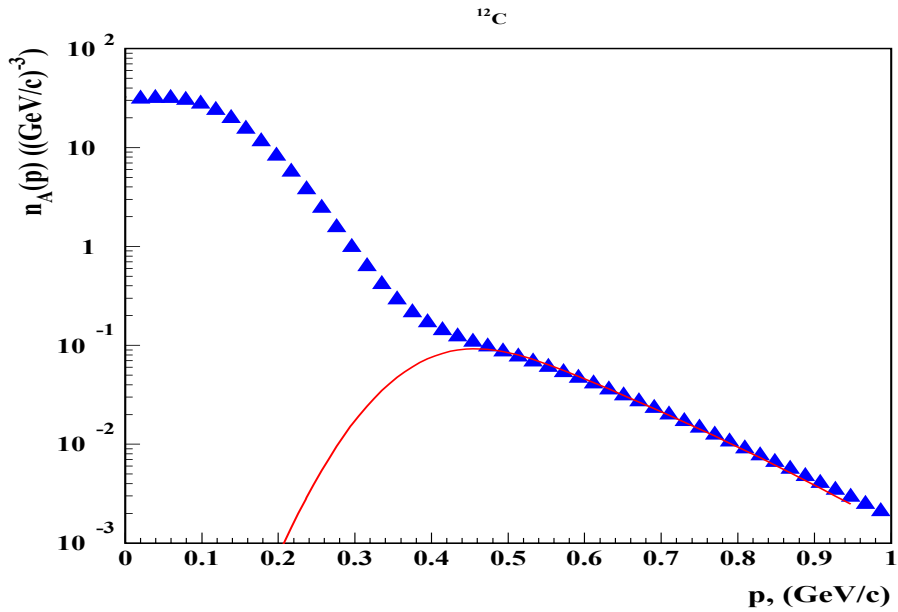
$$\psi_{pn}^{LF}(\alpha, p_\perp) \approx C \psi_d^{LF}(\alpha, p_\perp)$$

$$\psi_{2N}^{s_{NN}}(\beta_1, k_{1,\perp}, s_1, s_2) = -\frac{1}{\sqrt{2(2\pi)^3}} \frac{\bar{u}(p_1, s_1) \bar{u}(p_2, s_2) \Gamma_{NN \rightarrow NN} \chi_{NN}(p_{NN}, s_{NN})}{\frac{1}{2} [M_{NN}^2 - 4(M_N^2 + k_1^2)]}$$

$$\psi_{CM}(\alpha_{NN}, k_{NN,\perp}) = -\frac{1}{\sqrt{\frac{A-2}{2}}} \frac{1}{\sqrt{2(2\pi)^3}} \frac{\chi_{NN}^\dagger(p_{NN}, s_{NN}) \chi_{A-2}^\dagger(p_{A-2}, s_{A-2}) \Gamma_{A \rightarrow NN, A-2} \chi_A^{s_A}}{\frac{2}{A} [M_A^2 - s_{NN, A-2}(k_{CM})]}$$

# 2N SRC model

## Non Relativistic Approximation



# - Nuclear scaling due to 2N SRCs at:

$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$$

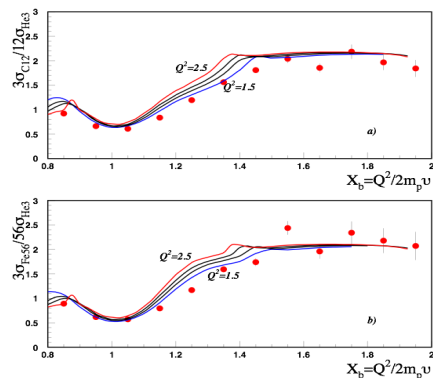
$$\alpha_{2N} = 2 - \frac{q_- + 2m_N}{2m_N} \left( 1 + \frac{\sqrt{W_{2N}^2 - 4m_N^2}}{W_{2N}} \right) \quad 1.3 \leq \alpha_{2N} \leq 1.5$$

Frankfurt, Strikman Phys. Rep, 1988  
Day, Frankfurt, Strikman, MS, Phys.  
Rev. C 1993

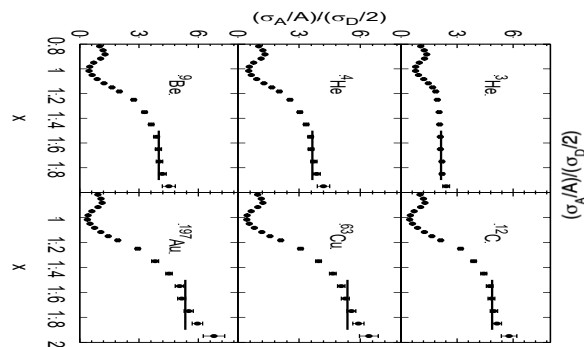
$$\rho_A(\alpha) \approx a_2(A, z) \rho_{NN}(\alpha)$$

$$\frac{2\sigma_{eA}}{A\sigma_{ed}} \approx a_2(A, Z) \quad 1.3 \leq \alpha_{2N} \leq 1.5$$

$A(e, e')$

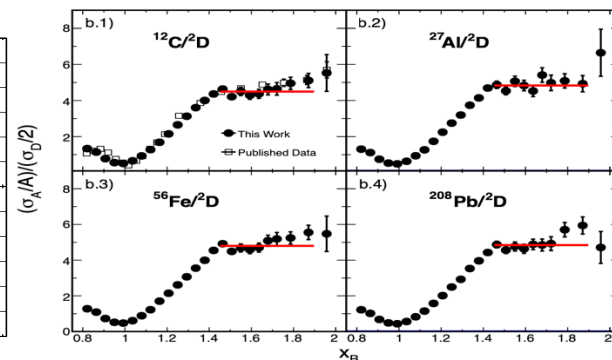


Egiyan et al, PRC, 2002, PRL 2006



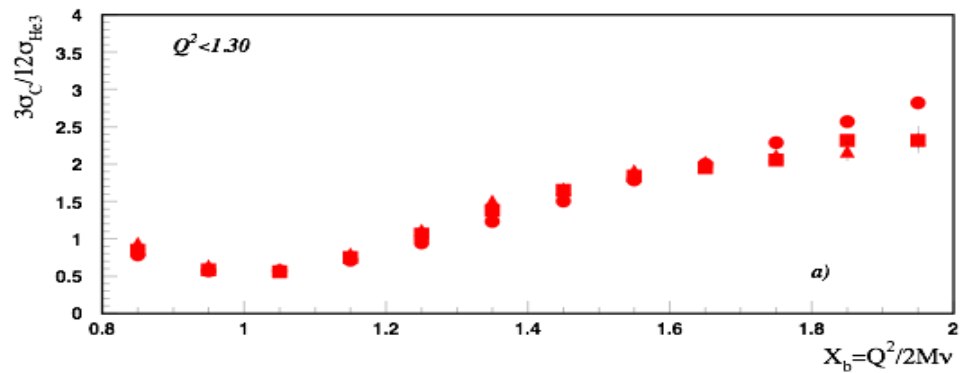
Fomin et al, PRL 2011

## B. Schmookler

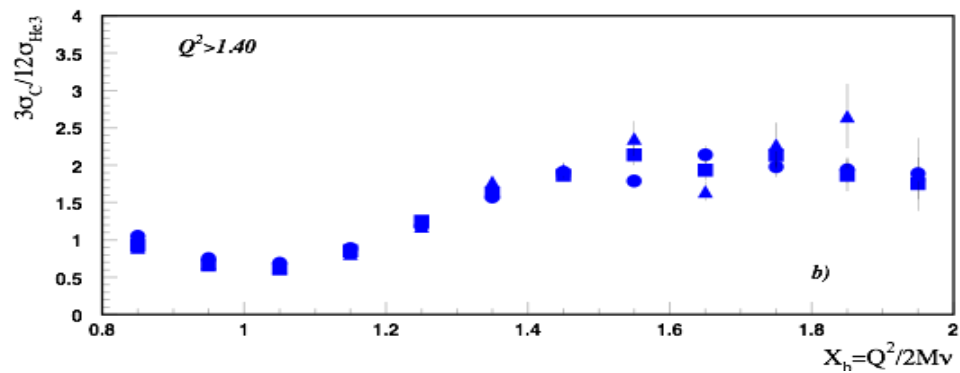


B. Schmookler et al, Nature 2019

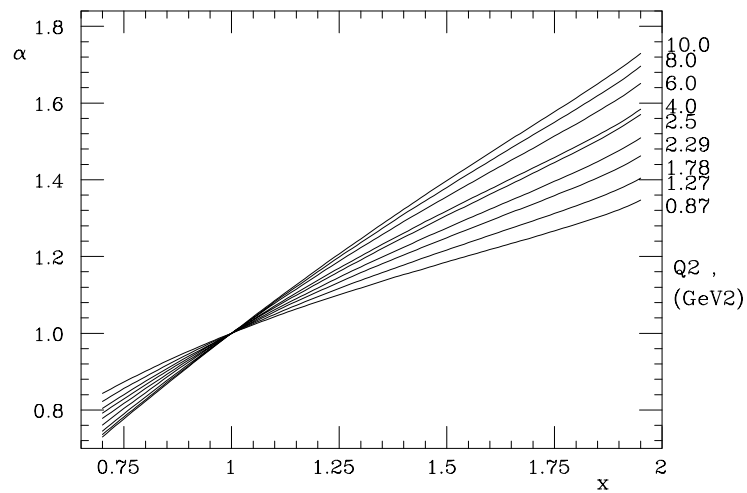
$A(e,e')$



$\alpha_{2N} < 1.3$



$\alpha_{2N} > 1.3$

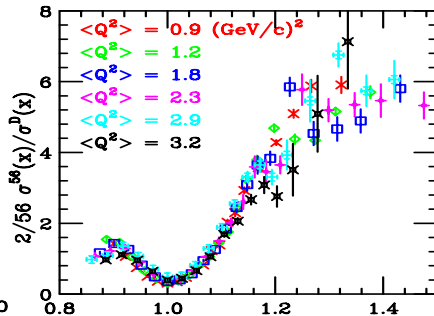
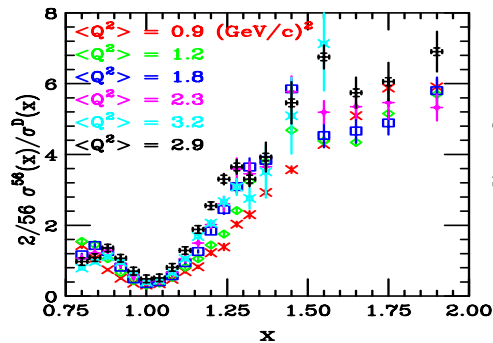


# Back to inclusive A(e,e')X scattering

$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$$

$$1.3 \leq \alpha_{2N} \leq 1.5$$

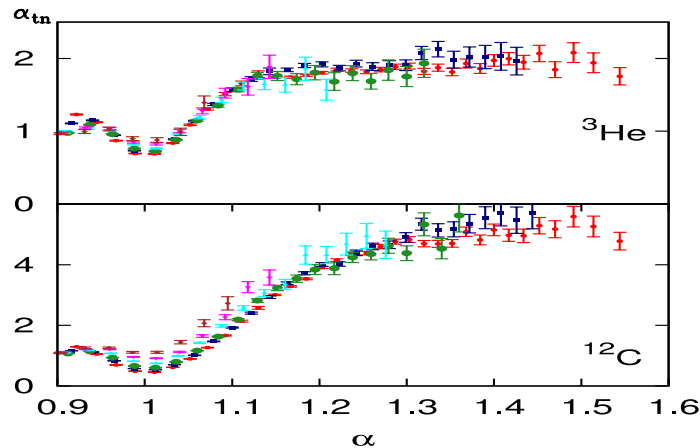
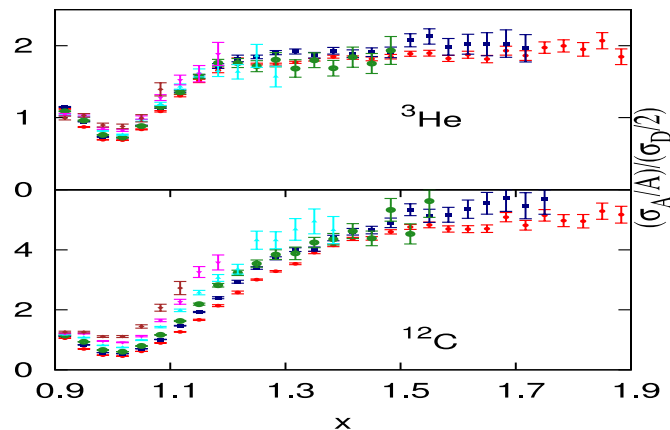
$$\alpha_{2N} = 2 - \frac{q_- + 2m_N}{2m_N} \left( 1 + \frac{\sqrt{W_{2N}^2 - 4m_N^2}}{W_{2N}} \right)$$



$$\alpha \mid Q^2 \rightarrow \infty \rightarrow x$$

$$\alpha \mid x \rightarrow 1 \rightarrow 1$$

J.Arrington, D.Higinbotham  
G.Rosner, M.S. Prog. PNP 2012



N.Fomin, D.Higinbotham  
M.S., P.Sovignon ARNPS, 2017

# The Meaning of scaling values

Day, Frankfurt, MS,  
Strikman, PRC 1993

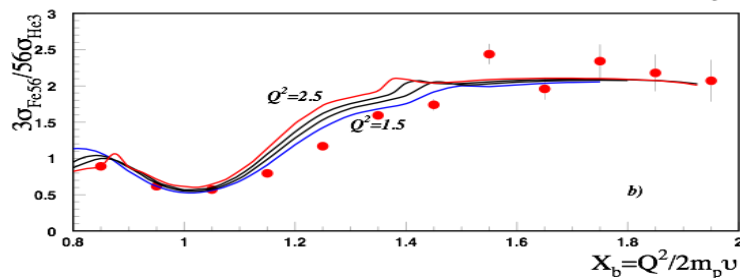
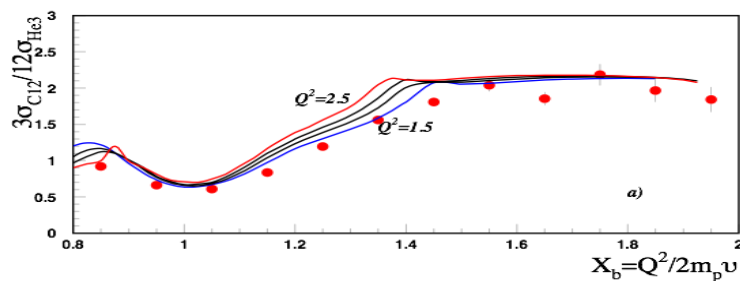
Frankfurt, MS, Strikman,  
IJMP A 2008

Fomin et al PRL 2011

$$R = \frac{A_2 \sigma[A_1(e, e')X]}{A_1 \sigma[A_2(e, e')X]}$$

$$\text{For } 1 < x < 2 \quad R \approx \frac{a_2(A_1)}{a_2(A_2)}$$

$A(e, e')$



Egiyan, et al PRL 2006, PRC 2004



a2's as relative probability of 2N SRCs

Table 1: The results for  $a_2(A, y)$

A	y	This Work	Frankfurt et al	Egiyan et al	Famin et al
$^3\text{He}$	0.33	$2.07\pm0.08$	$1.7\pm0.3$		$2.13\pm0.04$
$^4\text{He}$	0	$3.51\pm0.03$	$3.3\pm0.5$	$3.38\pm0.2$	$3.60\pm0.10$
$^9\text{Be}$	0.11	$3.92\pm0.03$			$3.91\pm0.12$
$^{12}\text{C}$	0	$4.19\pm0.02$	$5.0\pm0.5$	$4.32\pm0.4$	$4.75\pm0.16$
$^{27}\text{Al}$	0.037	$4.50\pm0.12$	$5.3\pm0.6$		
$^{56}\text{Fe}$	0.071	$4.95\pm0.07$	$5.6\pm0.9$	$4.99\pm0.5$	
$^{64}\text{Cu}$	0.094	$5.02\pm0.04$			$5.21\pm0.20$
$^{197}\text{Au}$	0.198	$4.56\pm0.03$	$4.8\pm0.7$		$5.16\pm0.22$

# - pn dominance in 2N SRCs:

for large  $k > k_{Fermi}$

$$P_{pn/pX} = 0.92^{+0.08}_{-0.18}$$

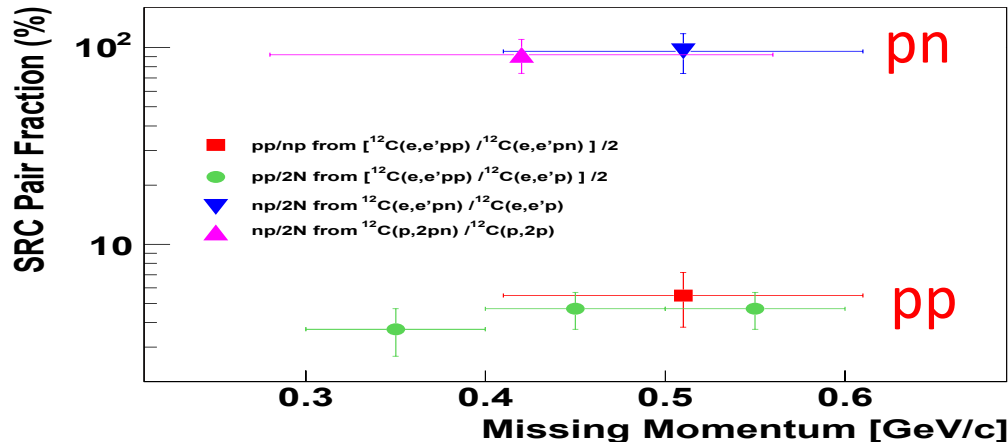
$$\frac{P_{pp}}{P_{pn}} \leq \frac{1}{2}(1 - P_{pn/pX}) = 0.04^{+0.09}_{-0.04}$$

Theoretical analysis of BNL Data  $A(p, 2p)X$  reaction  
E. Piasetzky, MS, L. Frankfurt,  
M. Strikman, J. Watson PRL, 2006

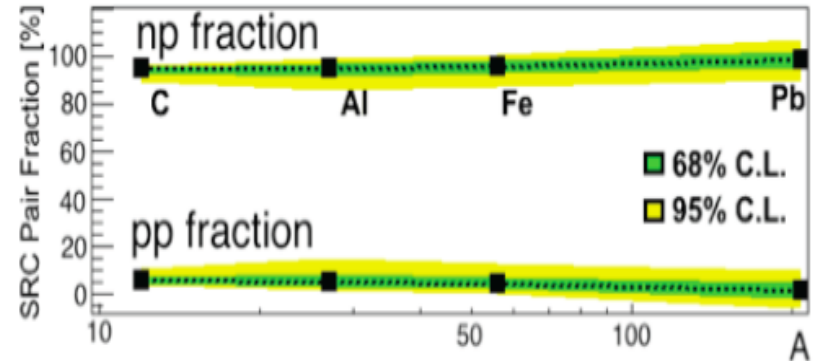
Factor of 20  
Expected 4  
(Wigner counting)

Direct Measurement at JLab R. Subedi, et al Science, 2008

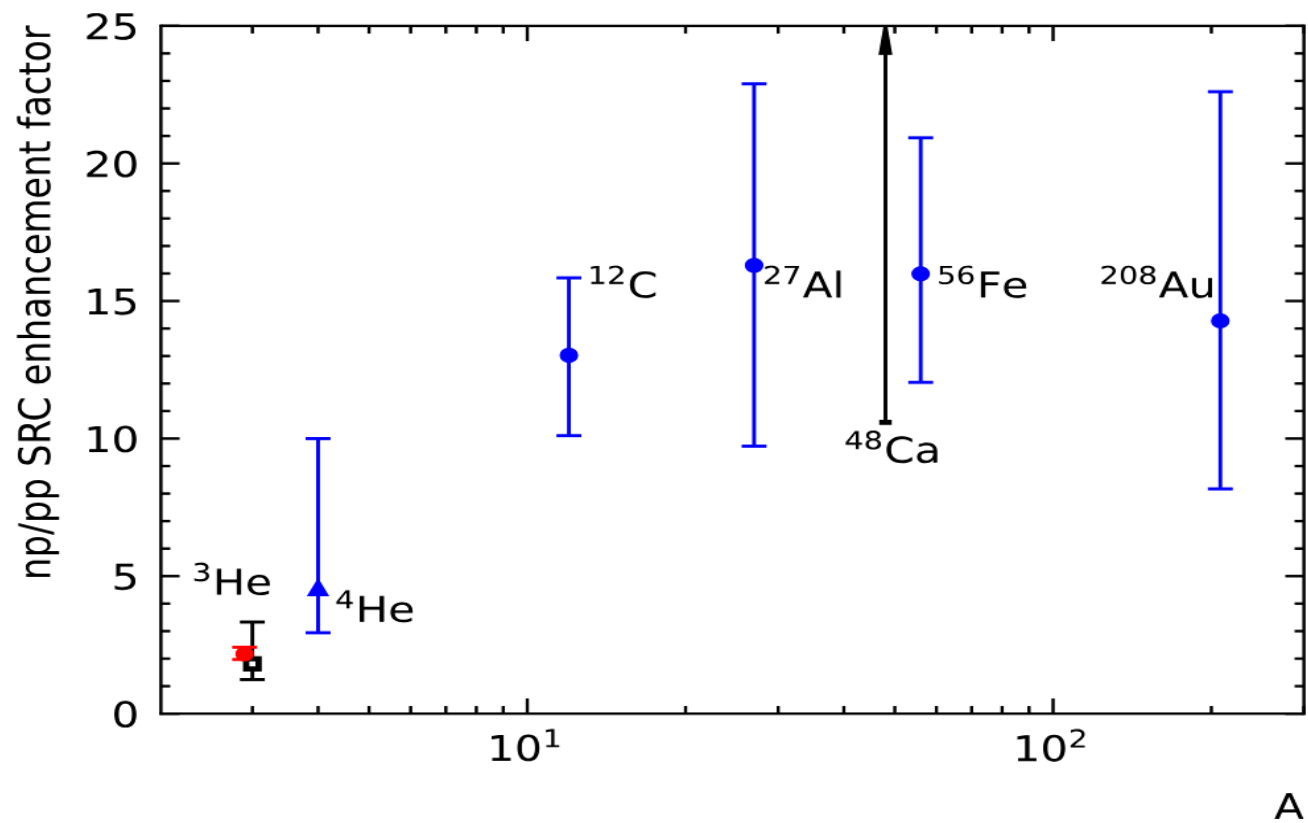
$$P_{pp/ppn} = 0.056 \pm 0.018$$



O. Hen, MS, L. Weinstein et.al. Science, 2014



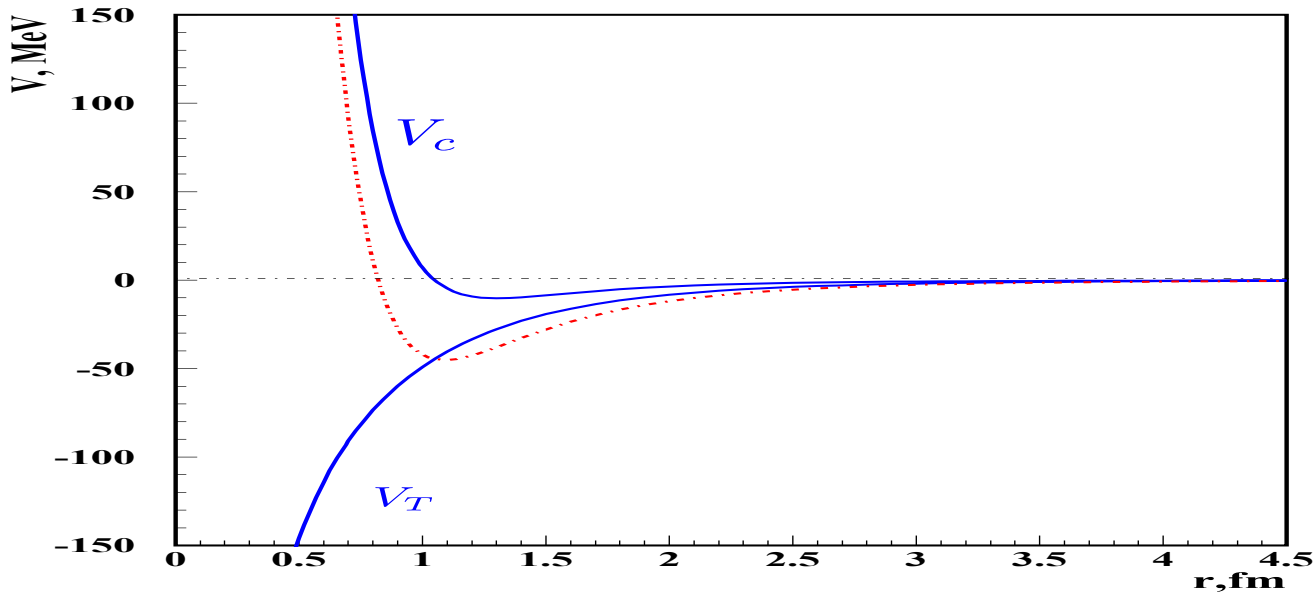
S.Li et al. Nature 2022



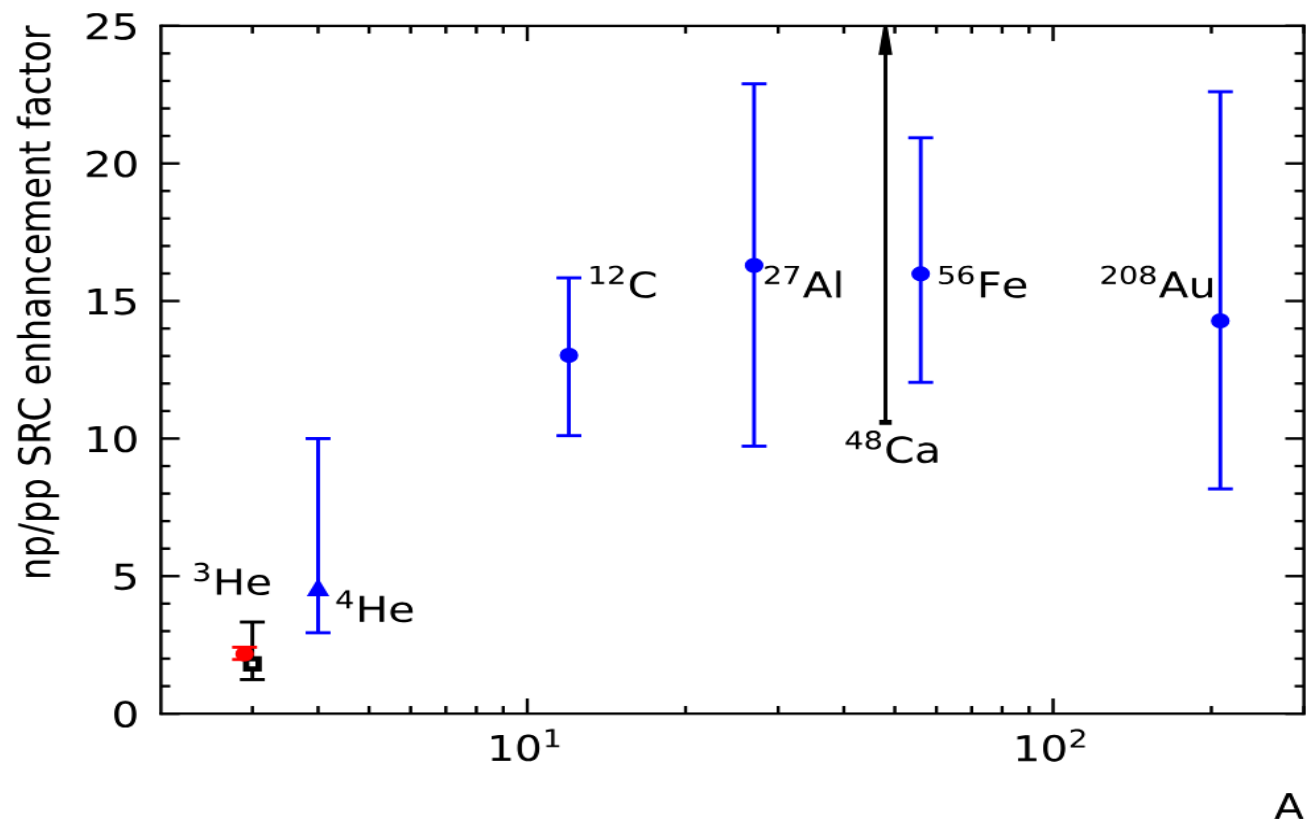
# Theoretical Interpretation

$$\phi_A^{(1)}(k_1, \dots, k_i = p, \dots, k_j \approx -p, \dots, k_A) \sim \frac{V_{NN}(p)}{p^2} f(k_1, \dots, \dots)$$

$$n_A(k) \approx a_{NN}(A) n_{NN}(k)$$



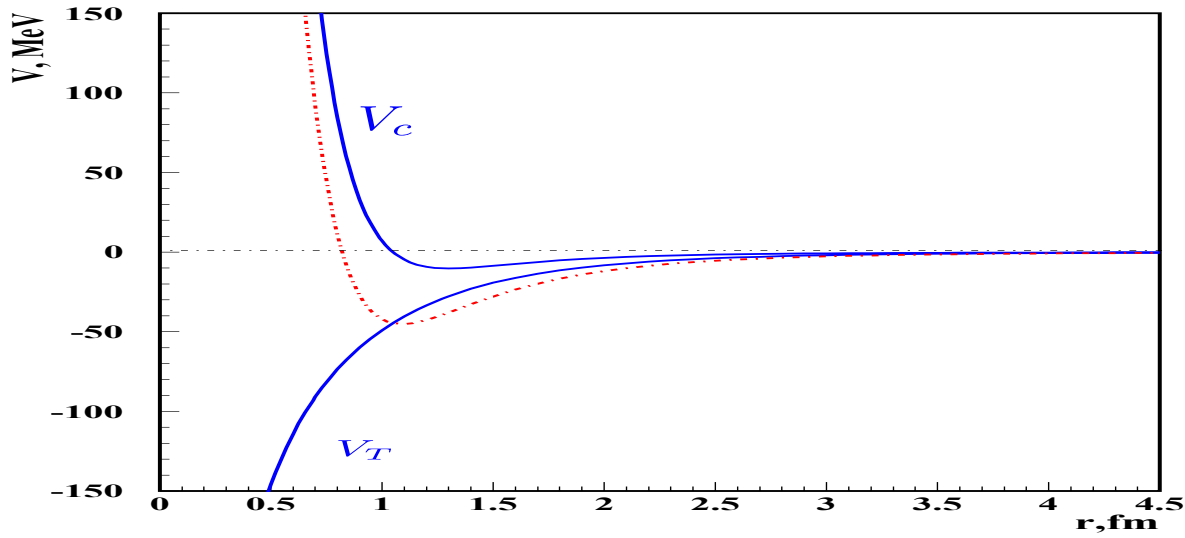
S.Li et al. Nature 2022



# Theoretical Interpretation

$$\phi_A^{(1)}(k_1, \dots, k_i = p, \dots, k_j \approx -p, \dots, k_A) \sim \frac{V_{NN}(p)}{p^2} f(k_1, \dots, \dots) \quad )$$

$$n_A(k) \approx a_{NN}(A) n_{NN}(k)$$



*Explanation lies in the dominance of the tensor part in the NN interaction*

$$V_{NN}(r) \approx V_c(r) + V_t(r) \cdot S_{12}(r) + V_{LS} \cdot \vec{L} \vec{S}$$

$$S_{12} = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \sigma_2$$

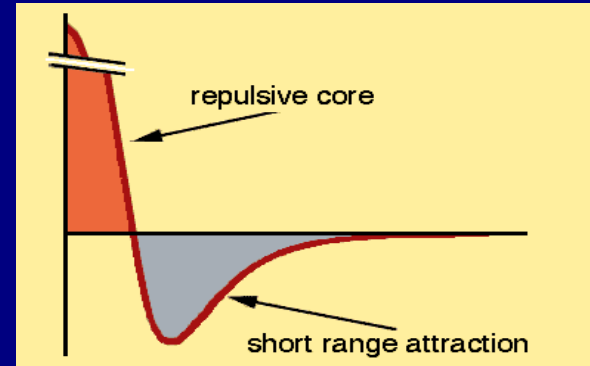
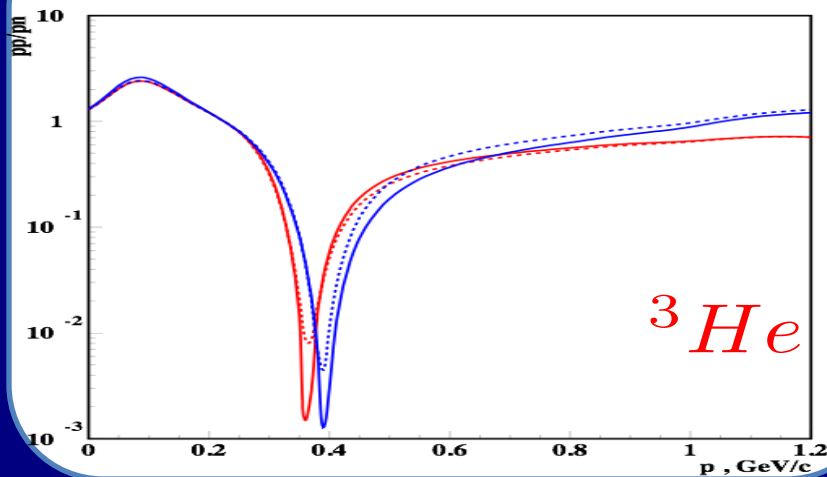
$$S_{12}|pp\rangle = 0$$

$$S_{12}|nn\rangle = 0 \quad \text{Isospin 1 states}$$

$$S_{12}|pn\rangle = 0$$

$$S_{12}|pn\rangle \neq 0 \quad \text{Isospin 0 states}$$

M.S. Abrahamyan, Frankfurt, Strikman PRC, 2005

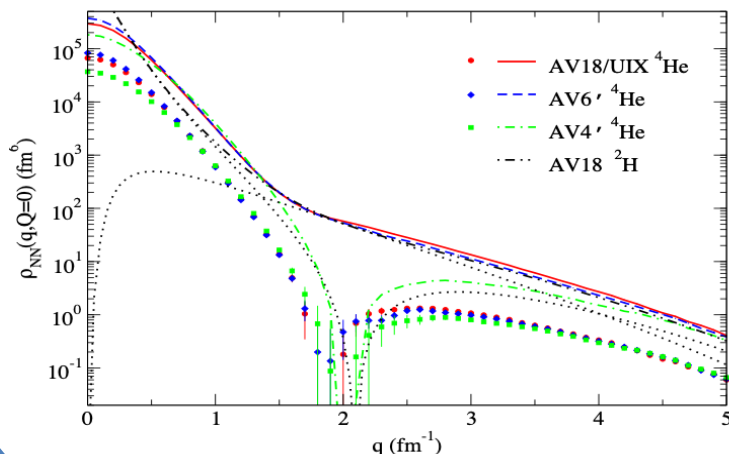


**Explanation lies in the dominance of the tensor part in the NN interaction**

$$V_{NN}(r) \approx V_c(r) + V_t(r) \cdot S_{12}(r) + V_{LS} \cdot \vec{L} \vec{S}$$

$$S_{12} = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \sigma_2$$

Sciavilla, Wiringa, Pieper, Carlson PRL,2007

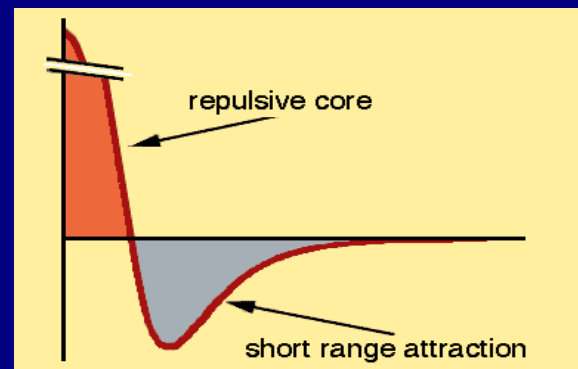


$$S_{12}|pp\rangle = 0$$

$$S_{12}|nn\rangle = 0 \quad \text{Isospin 1 states}$$

$$S_{12}|pn\rangle = 0$$

$$S_{12}|pn\rangle \neq 0 \quad \text{Isospin 0 states}$$





# - High momentum sharing in asymmetric nuclei

- Dominance of pn Correlations  
(neglecting pp and nn SRCs)

Two properties were predicted

MS, arXiv:1210.3280. 2012  
Phys. Rev. C 2014

First Property: Approximate Scaling Relation

$$x_p \cdot n_p^A(p) \approx x_n \cdot n_n^A(p) = \frac{a_{pn}}{2}(A, Z)n_d(p)$$

where  $x_p = \frac{Z}{A}$  and  $x_n = \frac{A-Z}{A}$ .

Second Property: Inverse Fractional Dependence of High Momentum Component

$$n_{p/n}^A(p) \approx \frac{1}{2x_{p/n}} a_2(A, y) \cdot n_d(p) \quad n_{p/n}^A(p) \approx \frac{1}{2(x_{p/n})^\gamma} a_2(A, y) \cdot n_d, \quad \gamma \leq 1$$

Minority component has more high momentum fraction

$$\gamma \approx 0.85 \text{ for } {}^3\text{He}$$
$$\gamma \approx 1.00 \text{ for } {}^9\text{Be}, {}^{10}\text{B}$$

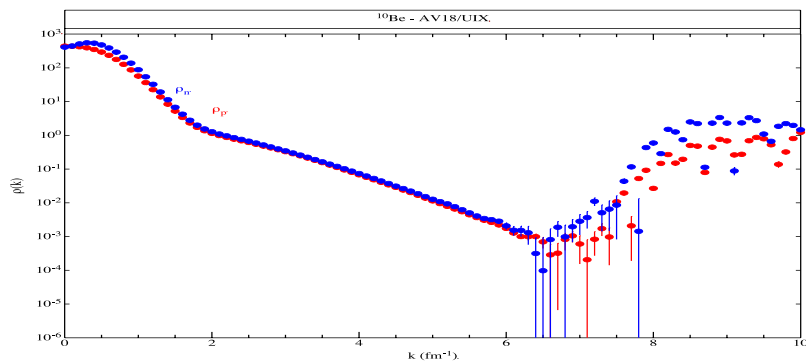
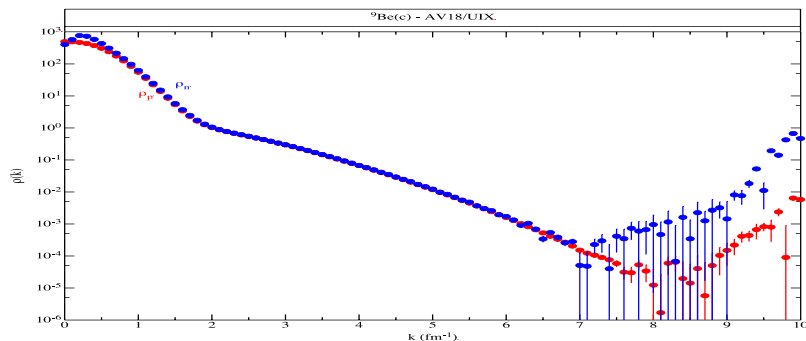
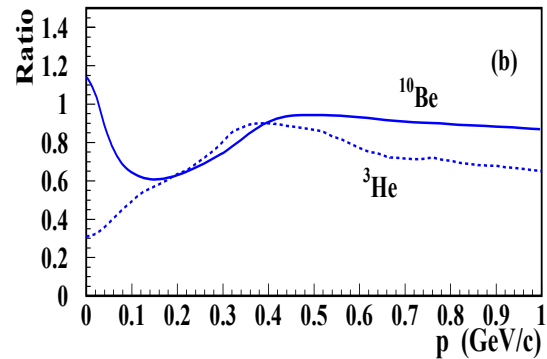
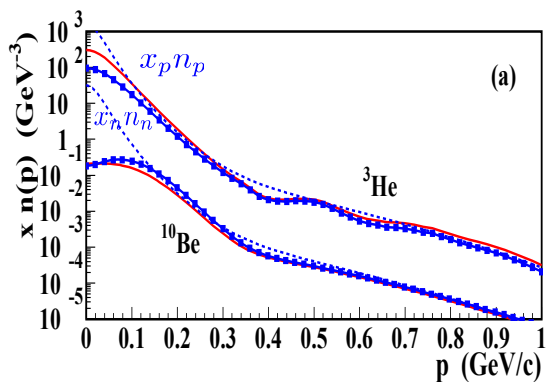
$$P_{p/n}(A, y) = \frac{1}{2x_{p/n}} a_2(A, y) \int_{k_F}^{\infty} n_d(p) d^3p$$

First Property  $x_p \cdot n_p^A(p) \approx x_n \cdot n_n^A(p)$

$^3\text{He}$

Be9,B10 Variational Monte Carlo Calculation: Robert Wiringa 2013

<http://www.phy.anl.gov/theory/research/momenta/>



# Predictions: High Momentum Fractions

MS,arXiv:1210.3280,2012  
Phys. Rev. C 2014

Minority Component has larger high momentum fractions

$$P_{p/n}(A, y) = \frac{1}{2x_{p/n}} a_2(A, y) \int_{k_F}^{\infty} n_d(p) d^3p$$

$y = |x_p - x_n|$

A	P <sub>p</sub> (%)	P <sub>n</sub> (%)
12	20	20
27	23	22
56	27	23
197	31	20

Table 1: Kinetic energies (in MeV) of proton and neutron

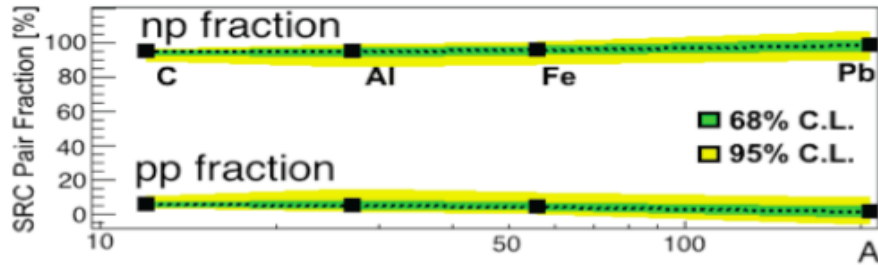
A	y	$E_{kin}^p$	$E_{kin}^n$	$E_{kin}^p - E_{kin}^n$
<sup>8</sup> He	0.50	30.13	18.60	11.53
<sup>6</sup> He	0.33	27.66	19.06	8.60
<sup>9</sup> Li	0.33	31.39	24.91	6.48
<sup>3</sup> He	0.33	14.71	19.35	-4.64
<sup>3</sup> H	0.33	19.61	14.96	4.65
<sup>8</sup> Li	0.25	28.95	23.98	4.97
<sup>10</sup> Be	0.2	30.20	25.95	4.25
<sup>7</sup> Li	0.14	26.88	24.54	2.34
<sup>9</sup> Be	0.11	29.82	27.09	2.73
<sup>11</sup> B	0.09	33.40	31.75	1.65

Variational Monte Carlo Calculation: Robert Wiringa 2013

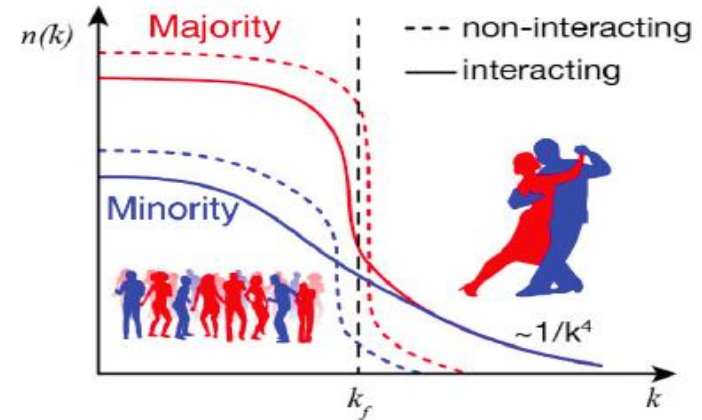
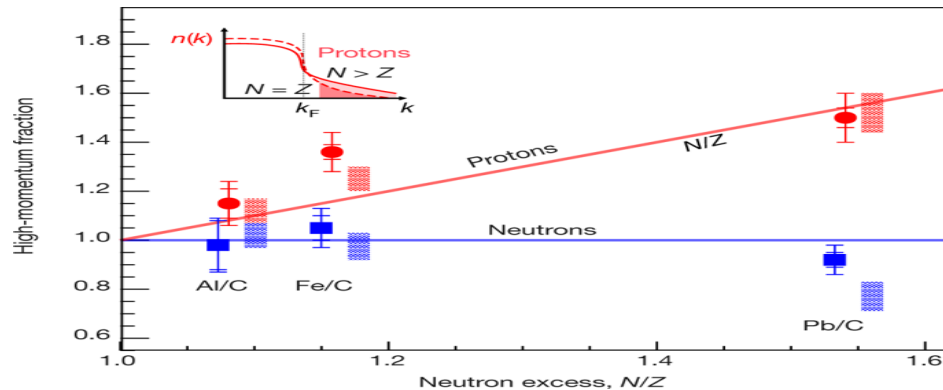
# - Experimental Verification of Momentum Sharing Effects

## - pn dominance persist for heavy nuclei

O. Hen, MS, L, Weinstein et.al. Science, 2014



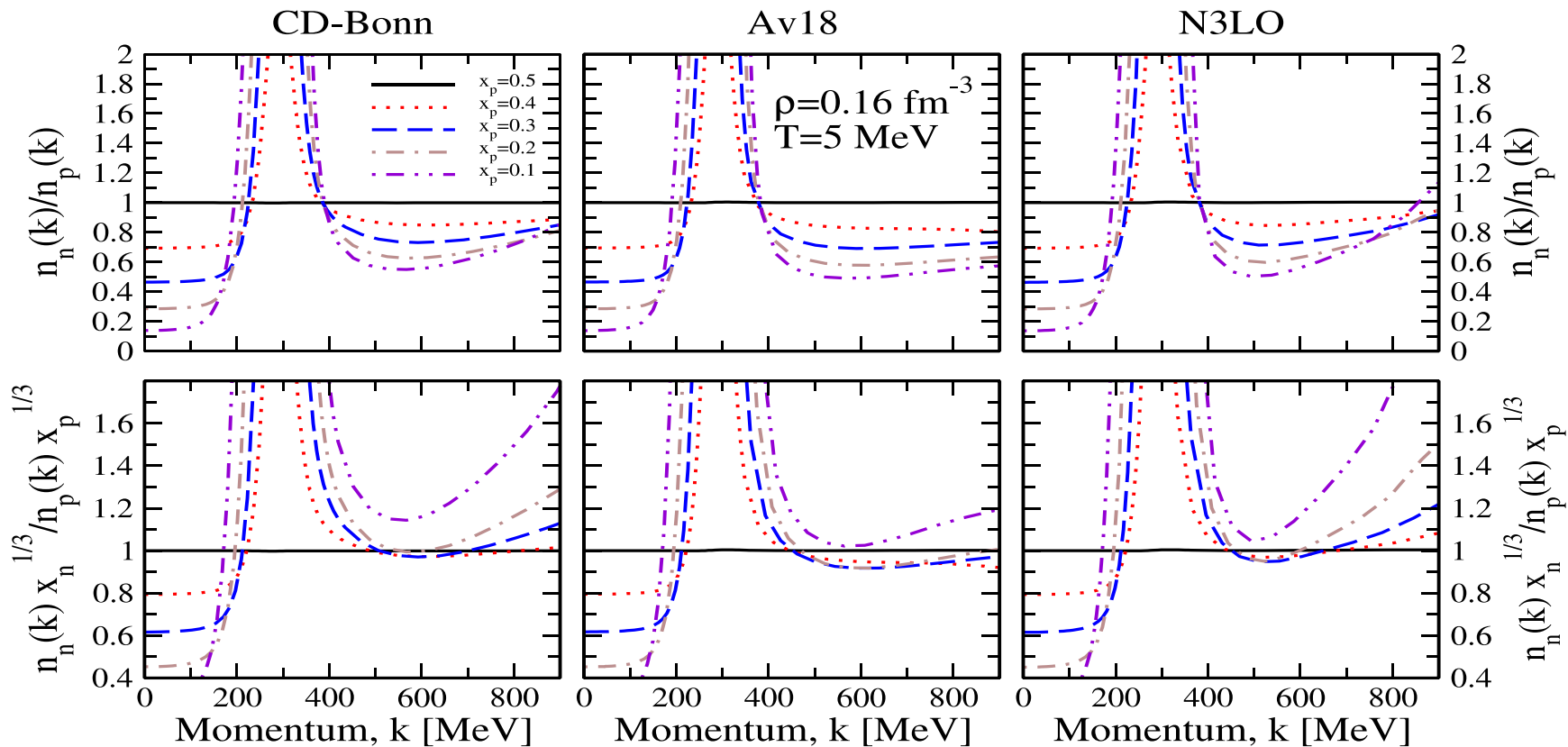
Duer et al, Nature 2018

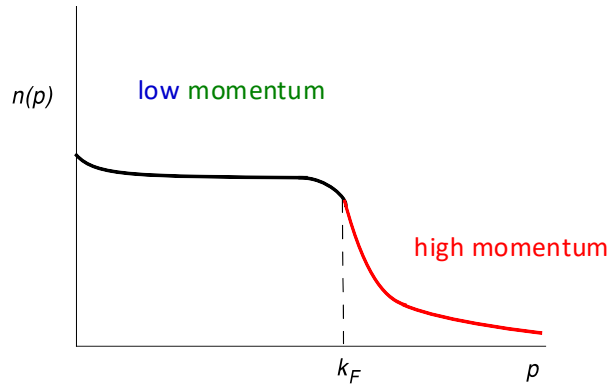


$$n_{p/n}^A(p) \approx \frac{1}{2x_{p/n}} a_2(A, y) \cdot n_d(p)$$

# Asymmetric Nuclear Matter Calculations

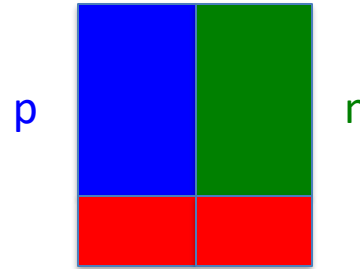
A.Rios, A. Polls and W. H. Dickhoff,  
Phys. Rev. C 2014





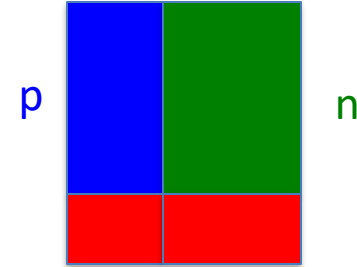
$$k_F = (3\pi^2 \rho_N)^{\frac{1}{3}}$$

### Symmetric Nuclei



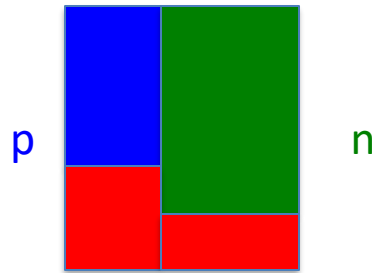
$$K_p = K_n$$

### Asymmetric Nuclei



**Conventional Theory:**  $K_n > K_p$   
(Shell Model, HF, HO Ab Initio)

### Asymmetric Nuclei



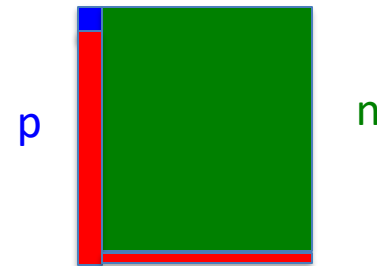
### New Predictions

1. Per nucleon, more protons  
are in high momentum tail

2. Kin Energy Inversion

$$K_p > K_n \quad ?$$

### Neutron Stars



Protons may completely  
populate the high momentum  
tail

- **New Properties of High Momentum**

**Distribution of Nucleons in Asymmetric Nuclei**

MS, arXiv:1210.3280, 2012

Phys. Rev. C 2014

- **Protons are more Energetic in**

**Neutron Rich High Density Nuclear Matter**

M. McGauley, MS

arXiv:1102.3973, 2011

- **First Experimental Indication**

O. Hen, et al.

Science, 2014,

- **Confirmed by VMC calculations for  $A < 12$**

R.B. Wiringa et al,

Phys. Rev. C 2014

- **For Nuclear Matter**

W. Dickhoff et al

Phys. Rev. C 2014

- **For Medium/Heavy Nuclei**

J. Ryckebusch, W. Cosyn

M. Vanhalst., J. Phys 2015

- **In Light-Cone Approximation:**

O. Artiles, M.S.

Phys. Rev. C 2016

## Implications/Predictions for Nuclear Physics and Astrophysics

- more/less protons/neutrons per nucleon in neutron rich nuclei
- protons are extremely energetic in Neutron Stars
- protons are more modified in neutron rich nuclei
- u-quarks are more modified than d-quarks in large A Nuclei

### Experimental Implications:

- Flavor Dependence of EMC effect
- A dependence of NuTeV Anomaly
- u/d modification can be checked in neutrino-nuclei or  $\nu$ DIS processes

## Implication for High Density Asymmetric Nuclear Matter



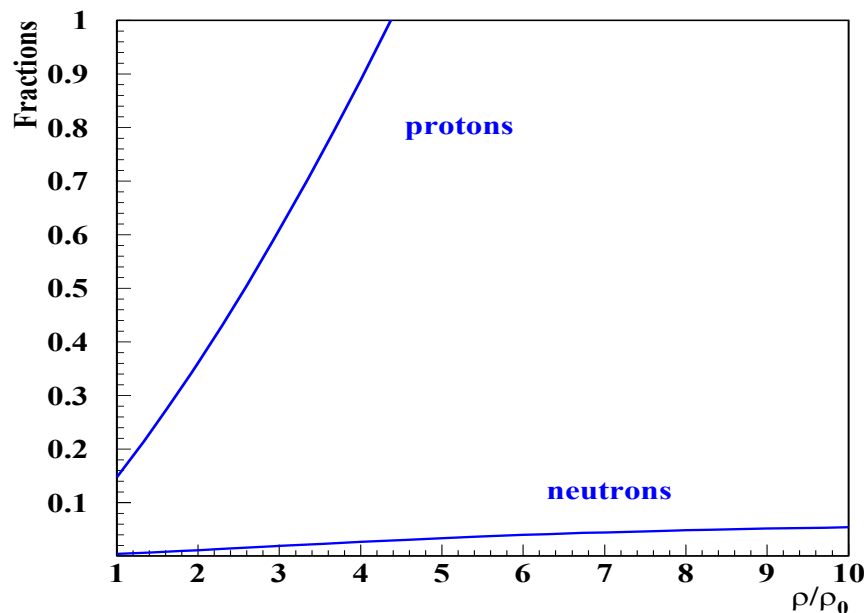
## Implications: For Nuclear Matter

$$P_{p/n}(A, y) = \frac{1}{2x_{p/n}} a_2(A, y) \int_{k_F}^{\infty} n_d(p) d^3p$$

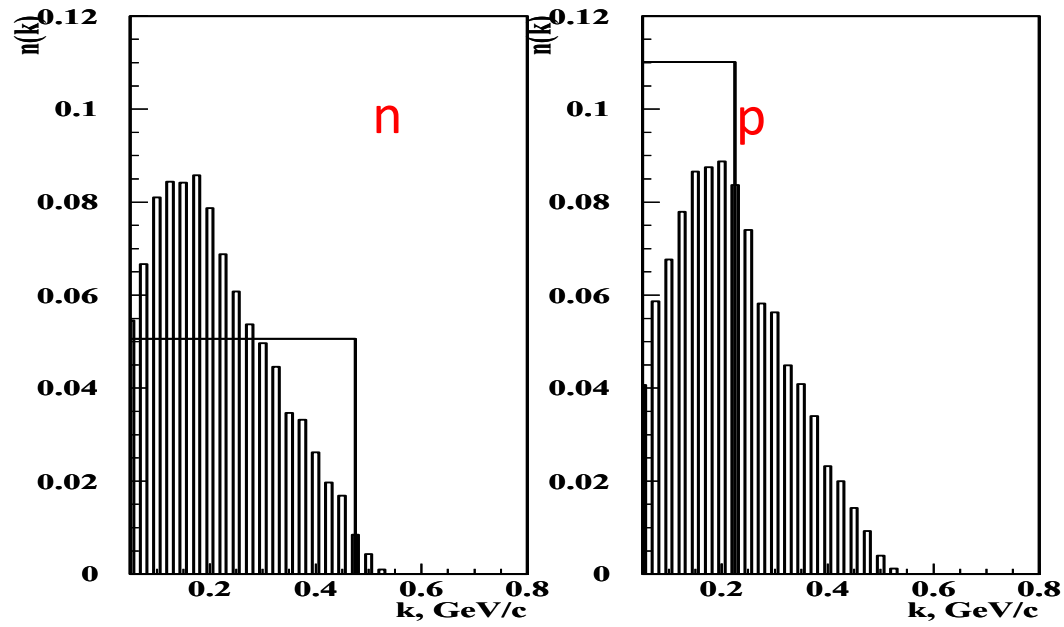
For  $x_p = \frac{1}{9}$  and  $y = \frac{7}{9}$   
and using  $k_{F,N} = (3\pi^2 x_N \rho)^{\frac{1}{3}}$

For  $x_p = \frac{1}{9}$  and  $y = \frac{7}{9}$   
and using  $k_{F,N} = (3\pi^2 x_N \rho)^{\frac{1}{3}}$

$$P_{p/n}(\rho, y) = \frac{a_2(\rho, y)}{2x_{p/n}} \int_{k_F} n_d(k) d^3k$$



## Implication for asymmetric nuclear matter



## Some Possible Implications of our Results

### Cooling of Neutron Star:

Large concentration of the protons above the Fermi momentum will allow the condition for Direct URCA processes  $p_p + p_e > p_n$  to be satisfied even if  $x_p < \frac{1}{9}$ . This will allow a situation in which intensive cooling of the neutron stars will be continued well beyond the critical point  $x_p = \frac{1}{9}$ .

### Superfluidity of Protons in the Neutron Stars:

Transition of protons to the high momentum spectrum will smear out the energy gap which will remove the superfluidity condition for the protons. This will also result in significant changes in the mechanism of generation of neutron star magnetic fields.

## Protons in the Neutron Star Cores:

The concentration of protons in the high momentum tail will result in proton densities  $\rho_p \sim p_p^3 \gg k_{F,p}^3$ . This will result in an equilibrium condition with "neutron skin" effect in which large concentration of protons will populate the core rather than the crust of the neutron star. This situation may provide very different dynamical conditions for generation of magnetic fields of the stars.

## Isospin Locking and Large Masses of Neutron Stars

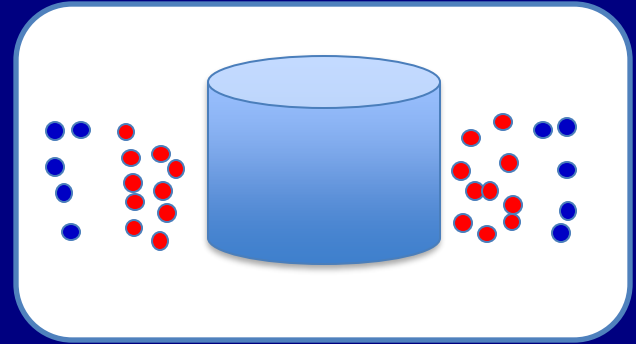
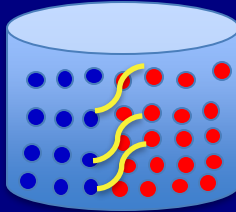
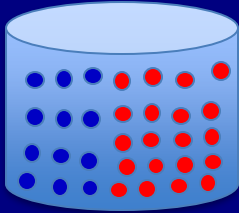
With an increase of the densities more and more protons move to the high momentum tail where they are in short range tensor correlations with neutrons. In this case one will expect that high density nuclear matter will be dominated by configurations with quantum numbers of tensor correlations  $S = 1$  and  $I = 0$ . In such scenario protons and neutrons at large densities will be locked in the NN isosinglet state. Such situation will double the threshold of inelastic excitation from  $NN \rightarrow N\Delta$  to  $NN \rightarrow \Delta\Delta(NN^*)$  transition thereby stiffening the equation of the state. This situation may explain the observed neutron star masses in Ref.[?] which are in agreement with the calculation of equation of state that include only nucleonic degrees of freedom

## Is the Observed Effects Universal for Two Component Asymmetric Fermi Systems?

- *Start with Two Component Asymmetric Degenerate Fermi Gas*
- *Asymmetric:  $\rho_1 \ll \rho_2$*
- *Switch on the short-range interaction between two-component*
- *While interaction between each components is weak*
- *Spectrum of the small component gas will strongly deform*

Cold Atoms

## Possible Experiment with Trapped Atoms

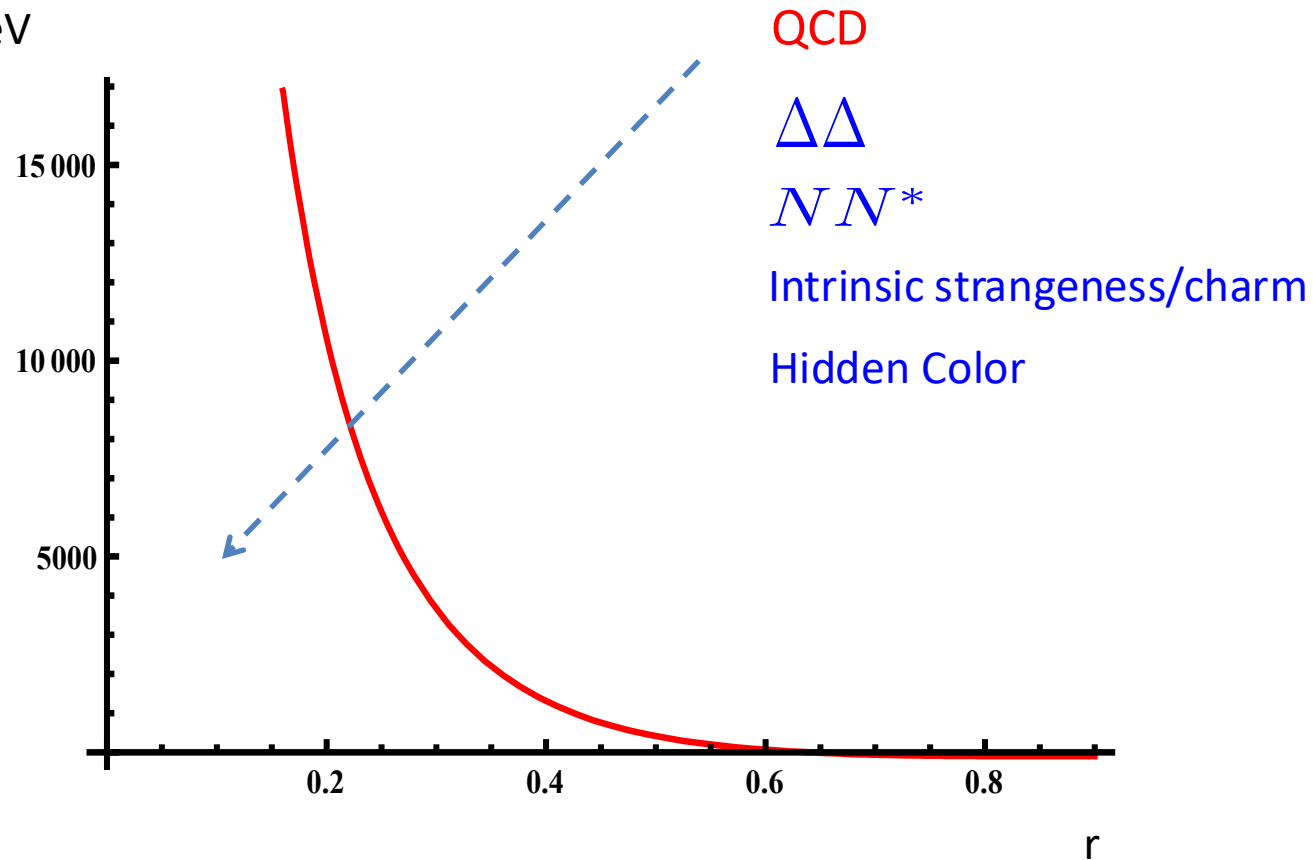


## Conclusions and Outlook

- We observe two new properties of high momentum distribution of proton and neutron in nuclei
- Predicting more energetic/virtual protons in neutron reach matter
- May have strong implication for protons in neutron stars *Cooling & Magnetic Fields*
- Can be a universal effect for any two-component asymmetric fermi system with short-range interaction between unlike components



$V_c$ , MeV

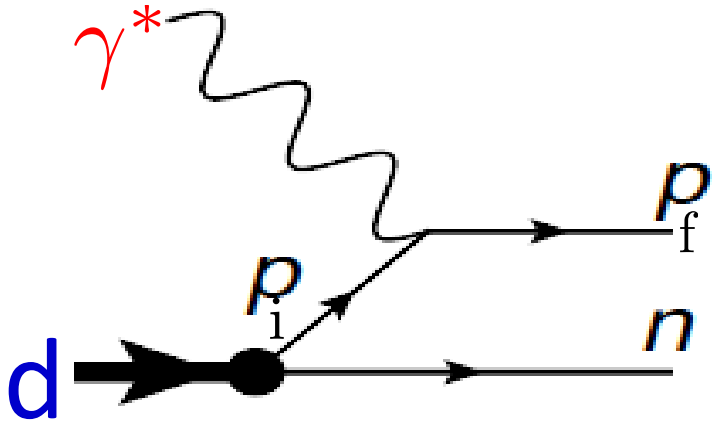


~80% hidden color  
Brodsky, Ji, Lepage, PRL 83

# Probing NN interaction at very short distances

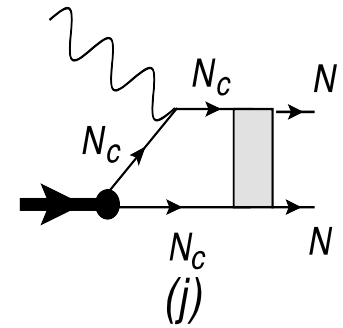
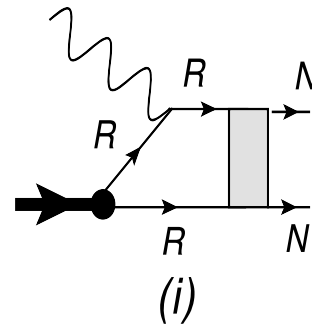
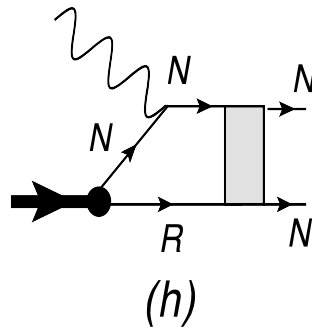
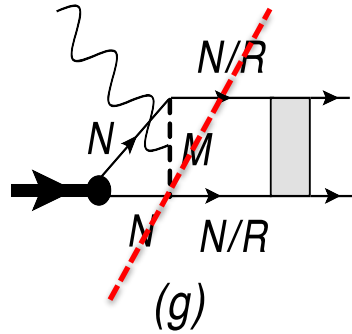
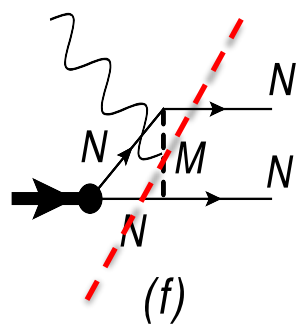
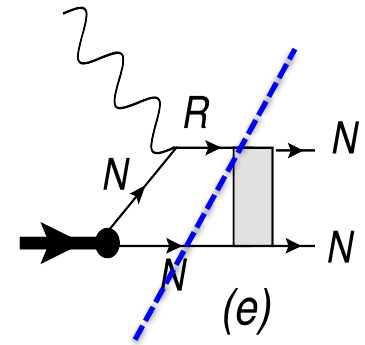
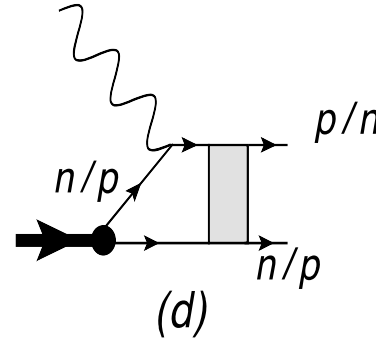
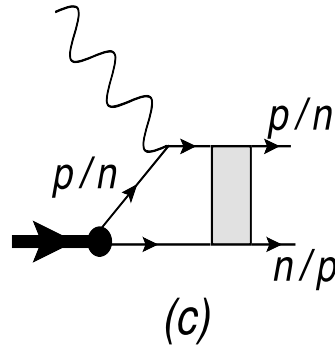
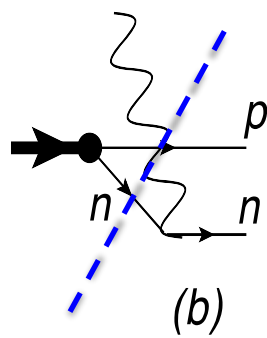
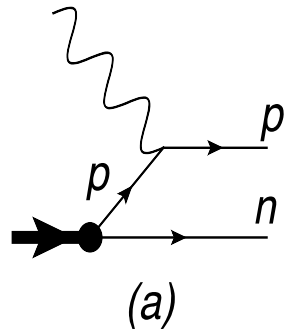
Considering reaction:  $e + d \rightarrow e' + p_f + n$

$$|p_i| = |p_f - q| \leq 550 \text{ MeV}/c$$



$$|p_i| = |p_f - q| > 550 \text{ MeV}/c$$

Considering reaction:  $e + d \rightarrow e' + p_f + n$



For the Deuteron it means, at Short Distances

$$\Psi_d = \Psi_{pn} + \Psi_{\Delta\Delta} + \Psi_{NN^*} + \Psi_{hc} \cdots + \Psi_{p\Lambda^0 K^0} \cdots$$

$$\Psi_{hc} = \Psi_{N_c, N_c}$$

The NN repulsive core can be due to the orthogonality of

$$\langle \Psi_{NonNucleonic} \mid \Psi_{NN} \rangle = 0$$

# Some Paradigm Shift

Our current mindset about deuteron is fully non-relativistic, the observation that it has total spin,  $J=1$  and parity,  $P=+$ , together with the relation that for non-relativistic wave function,  $P=(-1)^l$ , one concludes that the deuteron consists of S- and D- partial waves for proton-neutron system.

**Paradigm Shift:** The above reaction at high  $Q^2$ , measures the probability of observing proton and neutron in the deuteron at very large relative momenta. In such a formulation the deuteron is not a composite system consisting of proton and neutron but it is a composite pseudo - vector ( $J=1, P=+$ ) "particle" from which one extracts proton and neutron.

How such a proton and neutron produced at such extremal conditions is related to the dynamical structure of Light-Front deuteron wave function, which may include internal elastic  $pn \rightarrow pn$  as well as inelastic  $\Delta\Delta \rightarrow pn$ ,  $N^*N \rightarrow pn$  or  $N_C N_C \rightarrow pn$  transitions.

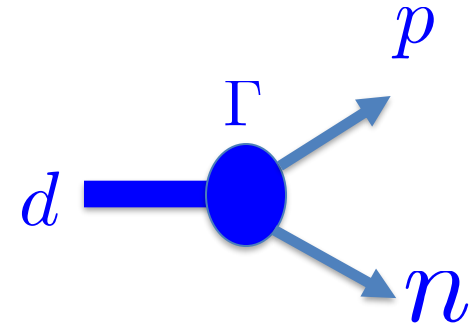
# New Structure in the Deuteron and possible non-nucleonic components

M.S & Frank Vera PRL 2023

## Paradigm shift:

- consider a deuteron not a nucleus that consist of proton and neutron
- but **pseudovector composite particle** from which we **extract** proton and neutron
- Light-Front Deuteron wave function

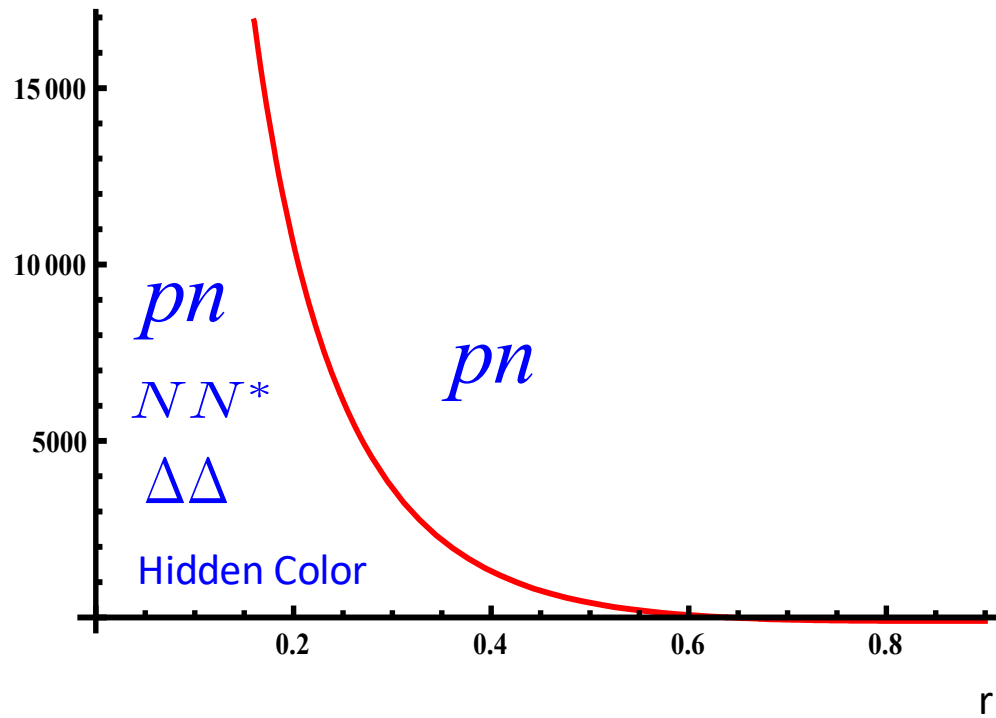
$$\psi_d^{\lambda_d}(\alpha_i, p_{\perp}, \lambda_1 \lambda_2) = - \frac{\bar{u}(p_2, \lambda_2) \bar{u}(p_1, \lambda_1) \Gamma_d \chi^{\lambda_d}}{\frac{1}{2} (m_d^2 - 4 \frac{m_N^2 + p_{\perp}^2}{\alpha_i (2 - \alpha_i)}) \sqrt{2} (2\pi)^3}$$



- Absorbing the energy denominator into the vertex function and using crossing symmetry

$$\psi_d^{\mu}(\alpha_i, p_{\perp}, \lambda_1, \lambda_2) = -\bar{u}(p_2, \lambda_2) \Gamma_d^{\mu}(k) \frac{(i\gamma_2 \gamma_0)}{\sqrt{2}} \bar{u}(p_1, \lambda_1)^T = - \sum_{\lambda'_1} \bar{u}(p_1, \lambda_1) \Gamma_d^{\mu} \gamma_5 \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} u(p_1, \lambda'_1)$$

Vc, MeV



$$\psi_d^\mu(\alpha_i, p_\perp, \lambda_1, \lambda_2) = -\bar{u}(p_2, \lambda_2) \Gamma_d^\mu(k) \frac{(i\gamma_2 \gamma_0)}{\sqrt{2}} \bar{u}(p_1, \lambda_1)^T = - \sum_{\lambda'_1} \bar{u}(p_1, \lambda_1) \Gamma_d^\mu \gamma_5 \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} u(p_1, \lambda'_1)$$

-  $\Gamma_d^\mu$  - is a four-vector, which can be constructed in a most general form satisfying time reversal, parity and charge conjugate symmetries

- Because the deuteron is a bound system, in addition to on-shell  $p_1$  and  $p_2$  four momenta one introduces

$$\Delta^\mu \equiv p_1^\mu + p_2^\mu - p_d^\mu \equiv (\Delta^-, \Delta^+, \Delta_\perp) = (\Delta^-, 0, 0)$$

$$\Delta^- = p_1^- + p_2^- - p_d^- = \frac{m_N^2 + k_\perp^2}{p_1^+} + \frac{m_N^2 + k_\perp^2}{p_2^+} - \frac{M_d^2}{p_d^+} = \frac{1}{p_d^+} \left[ \frac{4(m_N^2 + k_\perp^2)}{\alpha_1(2 - \alpha_1)} - M_d^2 \right] = \frac{4}{p_d^+} \left[ m_N^2 - \frac{M_d^2}{4} + k^2 \right]$$

- Constructed vertex:

$$\Gamma_d^\mu = \Gamma_1 \gamma^\mu + \Gamma_2 \frac{(p_1 - p_2)^\mu}{2m_N} + \Gamma_3 \frac{\Delta^\mu}{2m_N} + \Gamma_4 \frac{(p_1 - p_2)^\mu \not{\Delta}}{4m_N^2} + i\Gamma_5 \frac{1}{4m_N^3} \gamma_5 \epsilon^{\mu\nu\rho\gamma} (p_d)_\nu (p_1 - p_2)_\rho (\Delta)_\gamma + \Gamma_6 \frac{\Delta^\mu \not{\Delta}}{4m_N^2}$$



## High Momentum Transfer Kinematics

For large  $Q^2$  limit, Light-Front momenta for the reaction are chosen as follows:

$$p_d^\mu \equiv (p_d^-, p_d^+, p_{d\perp}) = \left( \frac{Q^2}{x\sqrt{s}} \left[ 1 + \frac{x}{\tau} - \sqrt{1 + \frac{x^2}{\tau}} \right], \frac{Q^2}{x\sqrt{s}} \left[ 1 + \frac{x}{\tau} + \sqrt{1 + \frac{x^2}{\tau}} \right], 0_\perp \right)$$
$$q^\mu \equiv (q^-, q^+, q_\perp) = \left( \frac{Q^2}{x\sqrt{s}} \left[ 1 - x + \sqrt{1 + \frac{x^2}{\tau}} \right], \frac{Q^2}{x\sqrt{s}} \left[ 1 - x - \sqrt{1 + \frac{x^2}{\tau}} \right], 0_\perp \right)$$

where  $s = (q + p_d)^2$ ,  $\tau = \frac{Q^2}{M_d^2}$  and  $x = \frac{Q^2}{M_d q_0}$ , with  $q_0$  being virtual photon energy in the deuteron rest frame.

- One observes that for fixed  $x$ ,  $p_d^+ \sim \sqrt{Q^2} \gg m_N$

$$\Delta^\mu \equiv p_1^\mu + p_2^\mu - p_d^\mu \equiv (\Delta^-, \Delta^+, \Delta_\perp) = (\Delta^-, 0, 0),$$

where

$$\begin{aligned} \Delta^- &= p_1^- + p_2^- - p_d^- = \frac{m_N^2 + k_\perp^2}{p_1^+} + \frac{m_N^2 + k_\perp^2}{p_2^+} - \frac{M_d^2}{p_d^+} \\ &= \frac{1}{p_d^+} \left[ \frac{4(m_N^2 + k_\perp^2)}{\alpha_1(2 - \alpha_1)} - M_d^2 \right] = \frac{4}{p_d^+} \left[ m_N^2 - \frac{M_d^2}{4} + k^2 \right]. \end{aligned}$$

In high  $Q^2$  limit  $\frac{\Delta^-}{2m_N} \ll 1$

$$\begin{aligned} \Gamma_d^\mu &= \Gamma_1 \gamma^\mu + \Gamma_2 \frac{(p_1 - p_2)^\mu}{2m_N} + \cancel{\Gamma_3 \frac{\Delta^\mu}{2m_N}} + \Gamma_4 \frac{(p_1 - p_2)^\mu}{4m_N^2} \\ &\quad + i\Gamma_5 \frac{1}{4m_N^3} \gamma_5 \epsilon^{\mu\nu\rho\gamma} (p_d)_\nu (p_1 - p_2)_\rho (\Delta)_\gamma + \cancel{\Gamma_6 \frac{\Delta^\mu}{4m_N^2}} \end{aligned}$$

Consider:  $\epsilon^{\mu,+,\perp,-} p_{d,-} k_{\perp} \Delta_{+}$

Since:  $p_{d,-} = \frac{1}{2} p_d^{+}$  and  $\Delta_{+} = \frac{1}{2} \Delta^{-}$  then  $p_d^{+} \Delta^{-} = p_d^{+} \frac{1}{p_d^{+}} \left[ \frac{4(m_N^2 + k_{\perp}^2)}{\alpha_1(2-\alpha_1)} - M_d^2 \right] = \left[ \frac{4(m_N^2 + k_{\perp}^2)}{\alpha_1(2-\alpha_1)} - M_d^2 \right]$

$\epsilon^{\mu,+,\perp,-} p_{d,-} k_{\perp} \Delta_{+} = \frac{1}{4} \epsilon^{\mu,+,\perp,-} p_d^{+} k_{\perp} \Delta^{-}$  **Leading Order!**

$$\Gamma_d^{\mu} = \Gamma_1 \gamma^{\mu} + \Gamma_2 \frac{(p_1 - p_2)^{\mu}}{2m_N} + \cancel{\Gamma_3 \frac{\Delta^{\mu}}{2m_N}} + \Gamma_4 \frac{(p_1 - p_2)^{\mu} \cancel{\Delta}}{4m_N^2} \\ + i\Gamma_5 \frac{1}{4m_N^3} \gamma_5 \epsilon^{\mu\nu\rho\gamma} (p_d)_{\nu} (p_1 - p_2)_{\rho} (\Delta)_{\gamma} + \Gamma_6 \frac{\cancel{\Delta^{\mu}} \cancel{\Delta}}{4m_N^2}$$

$$\psi_d^{\lambda_d}(\alpha_i, k_{\perp}) = - \sum_{\lambda_2, \lambda_1, \lambda'_1} \bar{u}(-k, \lambda_2) \left\{ \Gamma_1 \gamma^{\mu} + \Gamma_2 \frac{\tilde{k}^{\mu}}{m_N} + \sum_{i=1}^2 i\Gamma_5 \frac{1}{8m_N^3} \epsilon^{\mu+i-} p_d'^{+} k_i \Delta'^{-} \right\} \gamma_5 \frac{\epsilon_{\lambda_1, \lambda'_i}}{\sqrt{2}} u(k, \lambda'_1) s_{\mu}^{\lambda_d}$$

where  $\tilde{k}^{\mu} = (0, k_z, k_{\perp})$

$$\psi_d^{\lambda_d}(\alpha_i, k_{\perp}) = - \sum_{\lambda_2, \lambda_1, \lambda'_1} \bar{u}(-k, \lambda_2) \left\{ \Gamma_1 \gamma^{\mu} + \Gamma_2 \frac{\tilde{k}^{\mu}}{m_N} + \sum_{i=1}^2 i \Gamma_5 \frac{1}{8m_N^3} \epsilon^{\mu+i-} p_d^{\prime+} k_i \Delta^{\prime-} \right\} \gamma_5 \frac{\epsilon_{\lambda_1, \lambda'_i}}{\sqrt{2}} u(k, \lambda'_1) s_{\mu}^{\lambda_d}$$

$$\begin{aligned} \psi_d^{\lambda_d}(\alpha_1, k_t, \lambda_1, \lambda_2) = & \sum_{\lambda'_1} \phi_{\lambda_2}^{\dagger} \sqrt{E_k} \left[ \frac{U(k)}{\sqrt{4\pi}} \sigma \mathbf{s}_{\mathbf{d}}^{\lambda_d} - \frac{W(k)}{\sqrt{4\pi} \sqrt{2}} \left( \frac{3(\sigma \mathbf{k})(\mathbf{k} \mathbf{s}_{\mathbf{d}}^{\lambda})}{k^2} - \sigma \mathbf{s}_{\mathbf{d}}^{\lambda} \right) + \right. \\ & \left. (-1)^{\frac{1+\lambda_d}{2}} P(k) Y_1^{\lambda_d}(\theta, \phi) \delta^{1, |\lambda_d|} \right] \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} \phi_{\lambda'_1} \end{aligned}$$

$$\begin{aligned} U(k) &= \frac{2\sqrt{4\pi}\sqrt{E_k}}{3} \left[ \Gamma_1 \left( 2 + \frac{m_N}{E_k} \right) + \Gamma_2 \frac{k^2}{m_N E_k} \right] \\ W(k) &= \frac{2\sqrt{4\pi}\sqrt{2E_k}}{3} \left[ \Gamma_1 \left( 1 - \frac{m_N}{E_k} \right) - \Gamma_2 \frac{k^2}{m_N E_k} \right] \end{aligned}$$

Where:  $Y_1^{\pm}(\theta, \phi) = \mp i \sqrt{\frac{3}{4\pi}} \sum_{i=1}^2 \frac{(k \times s_d^{\pm 1})_z}{k}$

$$P(k) = \sqrt{4\pi} \frac{\Gamma_5(k) \sqrt{E_k}}{\sqrt{3}} \frac{k^3}{m_N^3}$$

fully relativistic: in addition to

has additional

$\frac{k^{l=1}}{m_N}$  term  
 $\frac{k^2}{m_N^2}$  term

## Light Front Density Matrix and Momentum Distribution

$$\psi_d^{\lambda_d}(\alpha_1, k_t, \lambda_1, \lambda_2) = \sum_{\lambda'_1} \phi_{\lambda'_2}^\dagger \sqrt{E_k} \left[ \frac{U(k)}{\sqrt{4\pi}} \sigma \mathbf{s}_d^{\lambda_d} - \frac{W(k)}{\sqrt{4\pi}\sqrt{2}} \left( \frac{3(\sigma \mathbf{k})(\mathbf{k} \mathbf{s}_d^{\lambda_d})}{k^2} - \sigma \mathbf{s}_d^{\lambda_d} \right) + \right. \\ \left. (-1)^{\frac{1+\lambda_d}{2}} P(k) Y_1^{\lambda_d}(\theta, \phi) \delta^{1,|\lambda_d|} \right] \frac{\epsilon_{\lambda_1, \lambda'_1}}{\sqrt{2}} \phi_{\lambda'_1}$$

$$\rho_d(\alpha, k_\perp) = \frac{n_d(k, k_\perp)}{2-\alpha}$$

$$n_d(k, k_\perp) = \frac{1}{3} \sum_{\lambda_d=-1}^1 | \psi_d^{\lambda_d}(\alpha, k_\perp) |^2 = \frac{1}{4\pi} \left( U(k)^2 + W(k)^2 + \frac{k_\perp^2}{k^2} P^2(k) \right)$$

Baryonic and Momentum Sum Rules  $\int \rho_d(\alpha, k_\perp) \frac{d\alpha}{\alpha} = 1$  and  $\int \alpha \rho_d(\alpha, k_\perp) \frac{d\alpha}{\alpha} = 1$

$$\int \left( U(k)^2 + W(k)^2 + \frac{2}{3} P^2(k) \right) k^2 dk = 1.$$

## Non-Nucleonic Components and the New Structure

$$n_d(k, k_{\perp}) = \frac{1}{3} \sum_{\lambda_d=-1}^1 | \psi_d^{\lambda_d}(\alpha, k_{\perp}) |^2 = \frac{1}{4\pi} \left( U(k)^2 + W(k)^2 + \frac{k_{\perp}^2}{k^2} P^2(k) \right)$$

- Momentum distribution depends on  $k_{\perp}$  separately
- *This is impossible for non-relativistic quantum mechanics of the deuteron since in this case the potential of the interaction is real (no inelasticities) and the solution of Lippmann-Schwinger (or Schroedinger) equation for partial S- and D-waves satisfies "angular condition", according to which the momentum distribution in unpolarized deuteron depends on the magnitude of relative momentum only.*
- On the other hand, in the relativistic domain the definition of the interaction potential is not straightforward to allow to use quantum-mechanical arguments in claiming that momentum distribution should satisfy the angular condition (i.e. depends on magnitude of  $k$  only).

- However, for the Light-Front, there is a remarkable theorem (Frankfurt, Mankiewicz, Sawitzky, Strikman, 1990) which states that if **one considers only pn component in the deuteron**, then for most acceptable forms of NN potential – constructed from elastic pn → pn scattering, the angular condition should be satisfied also for LF momentum distribution.

$$T_{NN}(\alpha_i, k_{i\perp}, \alpha_f, k_{f\perp}) \equiv T_{NN}(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f\perp}) = V(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f\perp}) + \int V(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m\perp}) \times \frac{d^3 k_m}{(2\pi)^3 \sqrt{m^2 + k_m^2}} \frac{T_{NN}(k_{m,z}, k_{m\perp}, k_{f,z}, k_{f\perp})}{4(k_m^2 - k_f^2)}$$

- The realization of the angular condition for relativistic case will require that light-front potential to satisfy a condition

$$V(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m\perp}) = V(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2)$$

- Lorentz invariance for on-shell NN amplitude requires

$$T_{NN}^{on\ shell}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m\perp}) = T_{NN}^{on\ shell}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2)$$

- Existence of the Born term indicates that

$$T_{NN}^{on\ shell}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2) = V_{NN}^{on\ shell}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2) + \int V_{NN}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) \times \\ \frac{d^3 k_m}{(2\pi)^3 \sqrt{m^2 + k_m^2}} \frac{T_{NN}(k_{m,z}, k_{m\perp}, k_{f,z}, k_{f,\perp})}{4(k_m^2 - k_f^2)}$$

- Iterating the equation around the on-shell kinematic point.

$$T_{NN}(\alpha_i, k_{i\perp}, \alpha_f, k_{f,\perp}) \equiv T_{NN}(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f,\perp}) = V(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f,\perp}) + \\ \int V(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) \times \frac{d^3 k_m}{(2\pi)^3 \sqrt{m^2 + k_m^2}} \frac{T_{NN}(k_{m,z}, k_{m\perp}, k_{f,z}, k_{f,\perp})}{4(k_m^2 - k_f^2)}$$

- will result in:

$$T_{NN}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) = T_{NN}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2)$$

$$V_{NN}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) = V_{NN}(\vec{k}_i^2, (\vec{k}_m - \vec{k}_i)^2) \quad \text{for the general case}$$

-  $V_{NN}$  – analytic function of angular momentum and it does not diverge exponentially in the complex-angular momentum space it was shown that also for the off-shell case



- For Non-nucleonic components no such iteration can be done

$$T_{NN}(k_{i,z}, k_{i\perp}, k_{f,z}, k_{f,\perp}) = \int V_{NN^*}(k_{i,z}, k_{i\perp}, k_{m,z}, k_{m,\perp}) \frac{d^3 k_m}{(2\pi)^3 \sqrt{m_m^2 + k_m^2}} \frac{T_{N^*N}(k_{m,z}, k_{m\perp}, k_{f,z}, k_{f,\perp})}{4(k_m^2 - k_f^2 + m_m^2 - m_N^2)}$$

- transition amplitudes such as  $T_{\Delta\Delta\rightarrow NN}$ ,  $T_{N^*,N\rightarrow NN}$  or  $T_{N^c,N^c\rightarrow NN}$  where  $N^cN^c$  represents a hidden color component in the deuteron could not be described with any combination of interaction potentials that satisfies angular condition
- if  $\Gamma_5$  term is not zero then it should originate from non-nucleonic component in the deuteron.
- Our prediction is that the observation of LF momentum distribution depending on the center of mass  $k$  and  $k_\perp$  separately will indicate the presence of non-nucleonic component in the deuteron

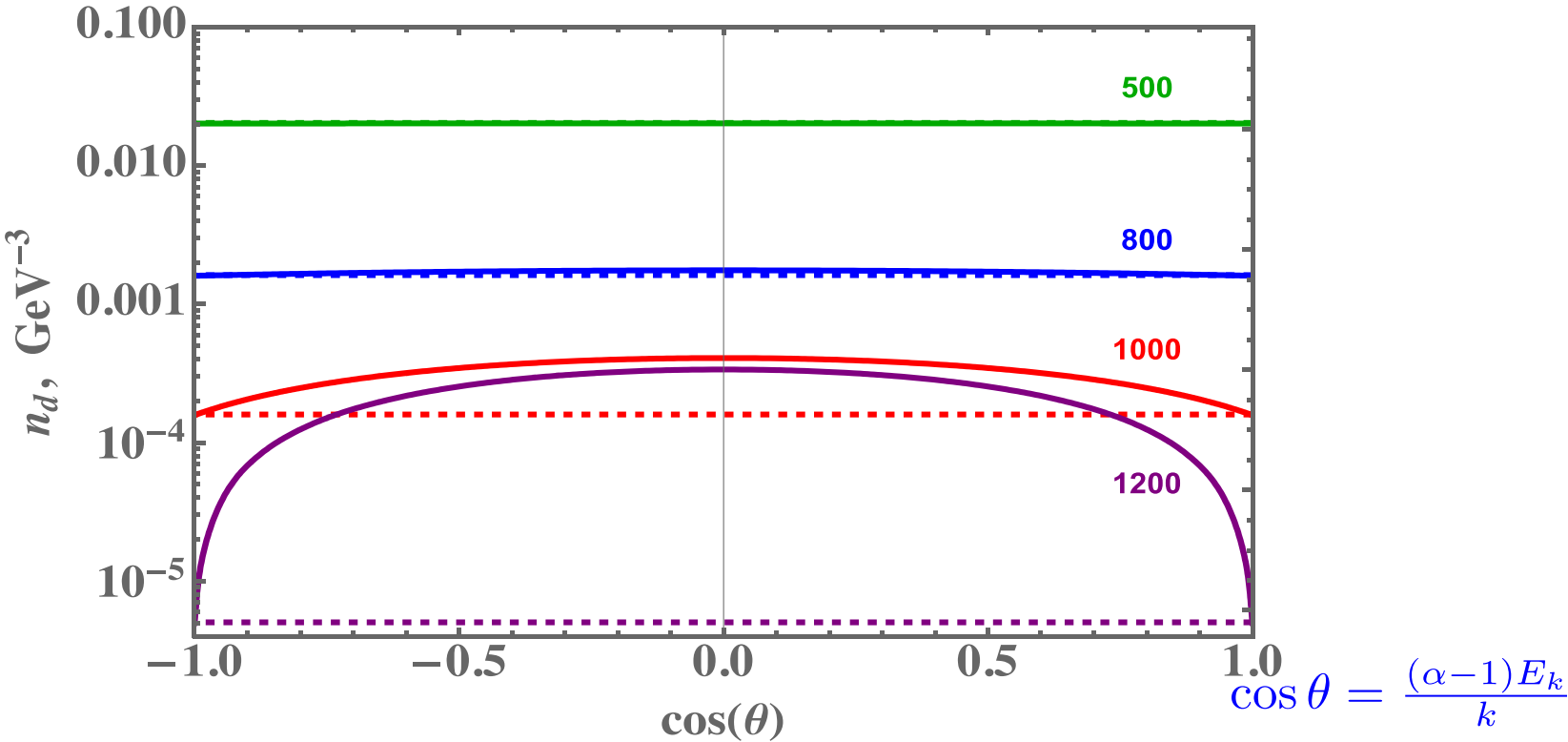
**Estimate of the effect**

$$n_d(k,k_{\perp}) = \frac{1}{3} \sum_{\lambda_d=-1}^1 \left| \psi_d^{\lambda_d}(\alpha,k_{\perp}) \right|^2 = \frac{1}{4\pi} \left( U(k)^2 + W(k)^2 + \frac{k_{\perp}^2}{k^2} P^2(k) \right)$$

$$P(k) = \sqrt{4\pi} \frac{\Gamma_5(k) \sqrt{E_k}}{\sqrt{3}} \frac{k^3}{m_N^3}$$

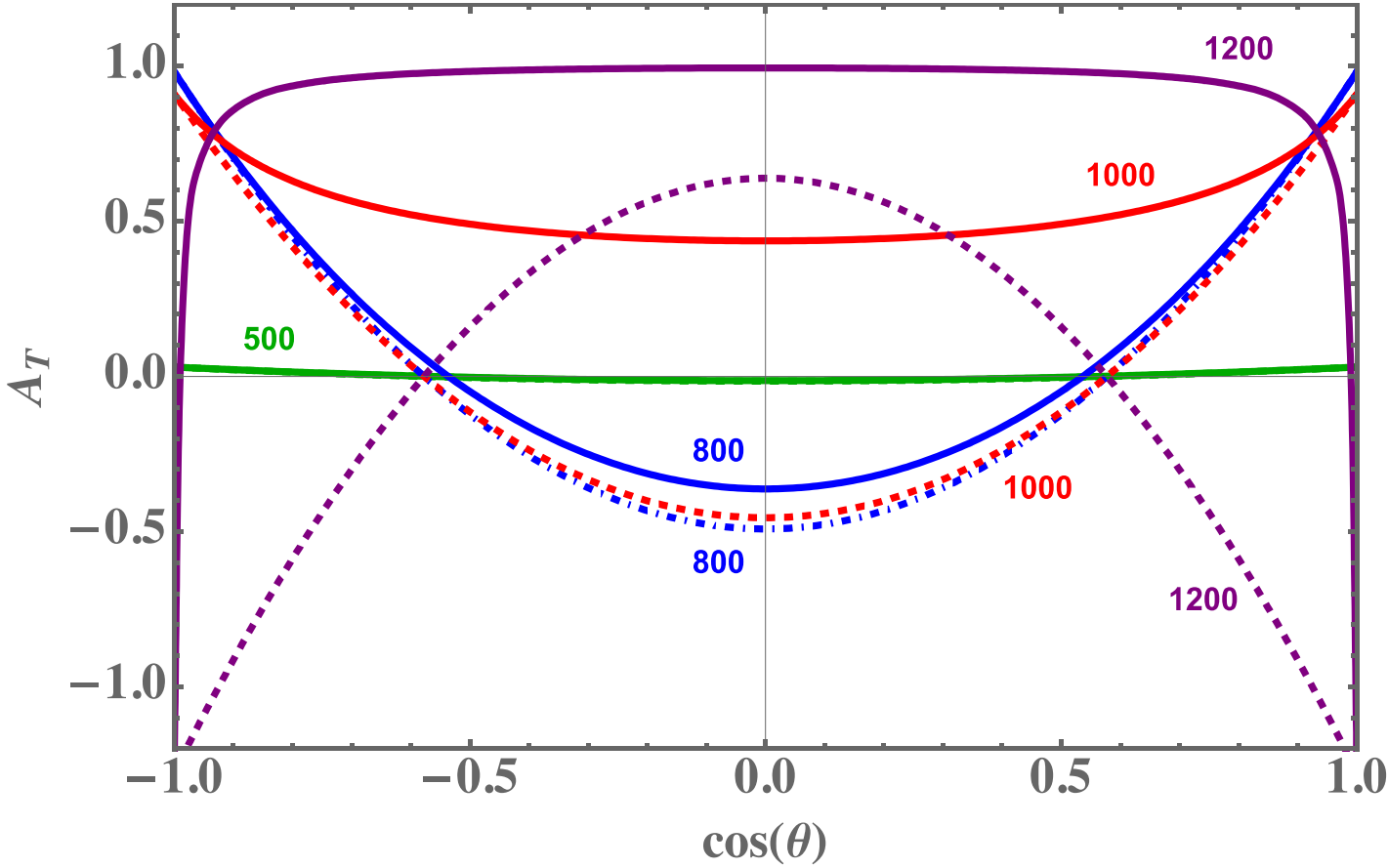
$$\Gamma_5(k) = \frac{A}{(1+\frac{k^2}{0.71})^2}$$

$A$  is estimated by assuming 1% contribution to the total normalization.



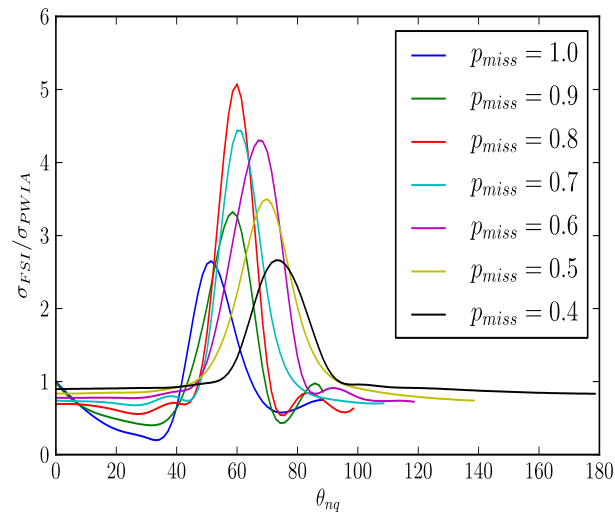
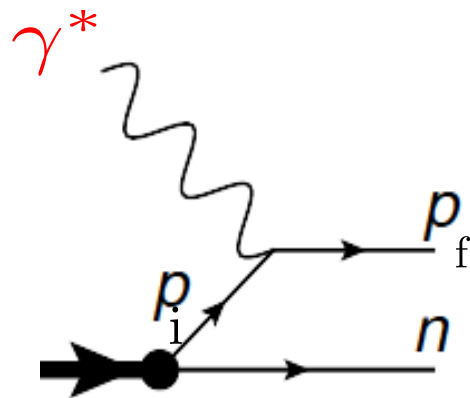
Estimate of the effect

$$A_T = \frac{n_d^{\lambda_d=1}(k,k_\perp) + n_d^{\lambda_d=-1}(k,k_\perp) - 2n_d^{\lambda_d=0}(k,k_\perp)}{n_d(k,k_\perp)}$$



$$\cos \theta = \frac{(\alpha-1)E_k}{k}$$

## Possibility of Experimental Verification



Considering reaction:  $e + d \rightarrow e' + p_f + n$

$$|p_i| = |p_f - q| \gtrsim 800 \text{ MeV}/c$$

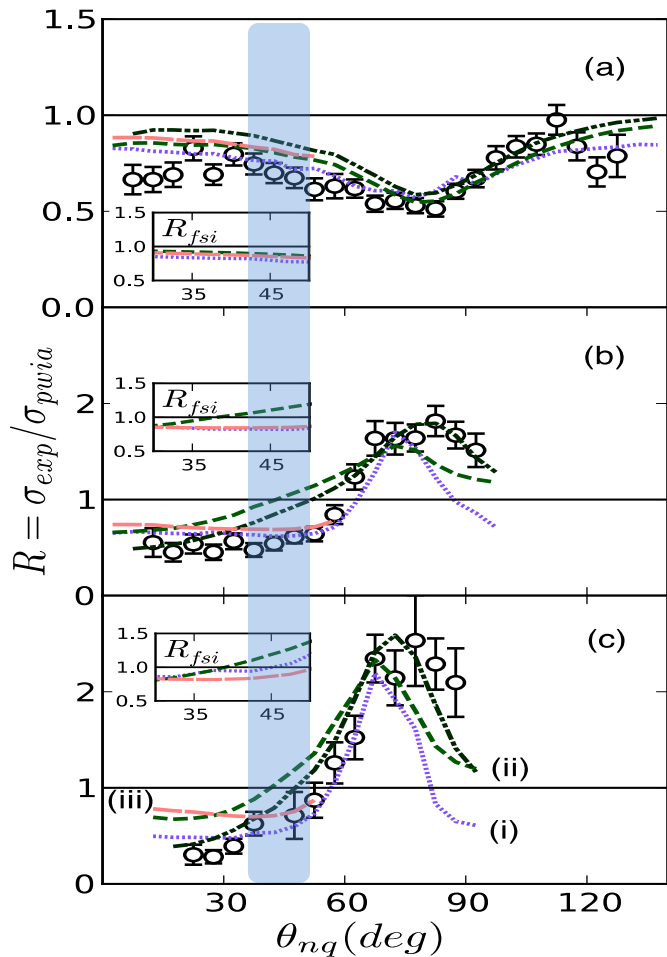
PAC-36, 2010

■ E12-10-003 ( $p_m \square 300 \text{ MeV}$ ): “Deuteron Electro-Disintegration at Very High Missing Momentum”

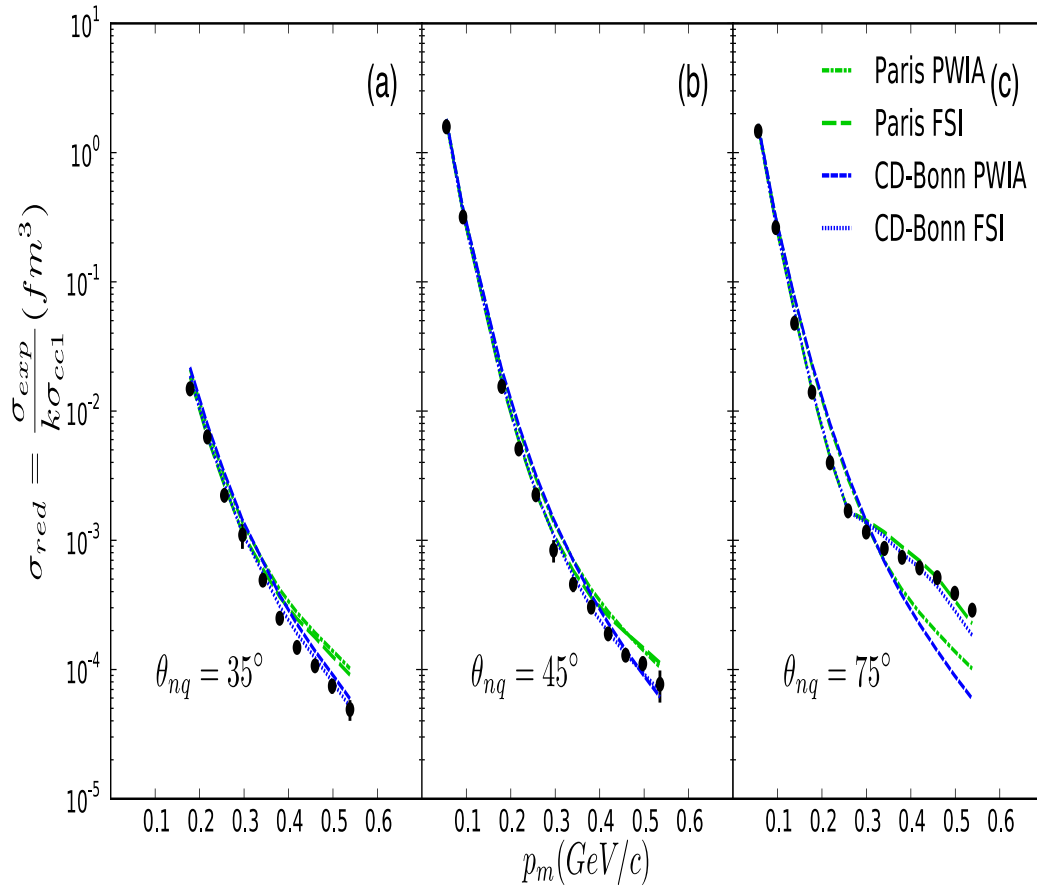
Rating: B+

data are essential to constrain further theory developments. Overall the experiment was viewed very highly; the lower rating simply reflects the likelihood that the data will not reveal any particular surprise and that their impact may thus be limited to experts in the field.

# Probing Deuteron at Small Distances at large $Q^2$

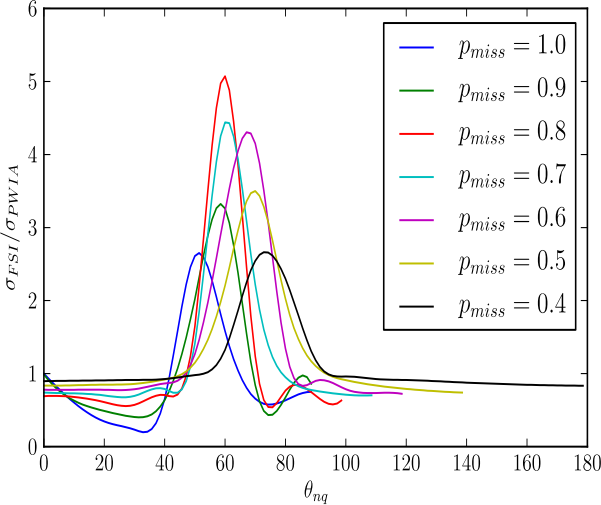
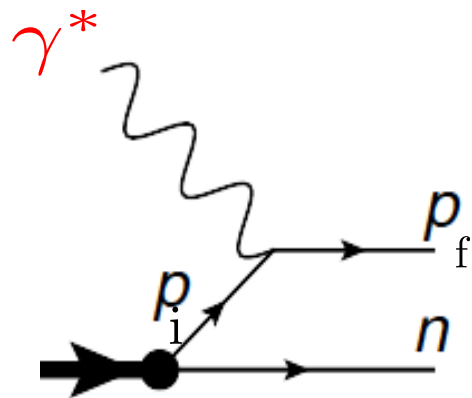


JLab,  $Q^2 = 3.5 \text{ GeV}^2$



Boeglin et al PRL 2011, deuteron probed at up to 550 MeV/c

Possibility of Experimental Verification

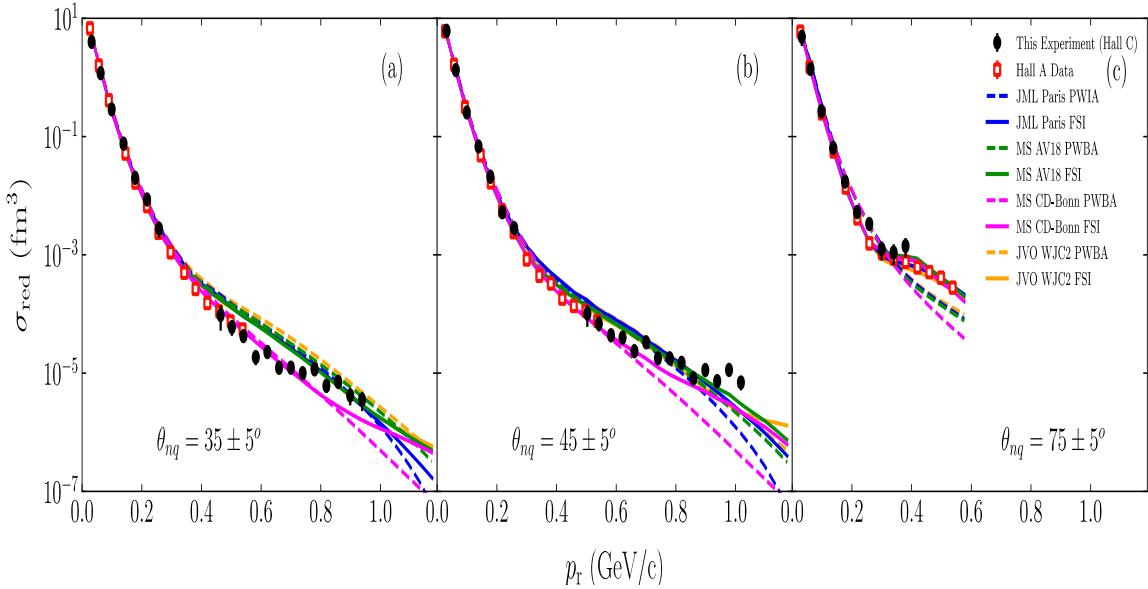


Considering reaction:  $e + d \rightarrow e' + p_f + n$

$$|p_i| = |p_f - q| \gtrsim 800 \text{MeV}/c$$

3-days of commissioning measurement,

JLab experiment  $Q^2 = 4 \text{ GeV}^2$

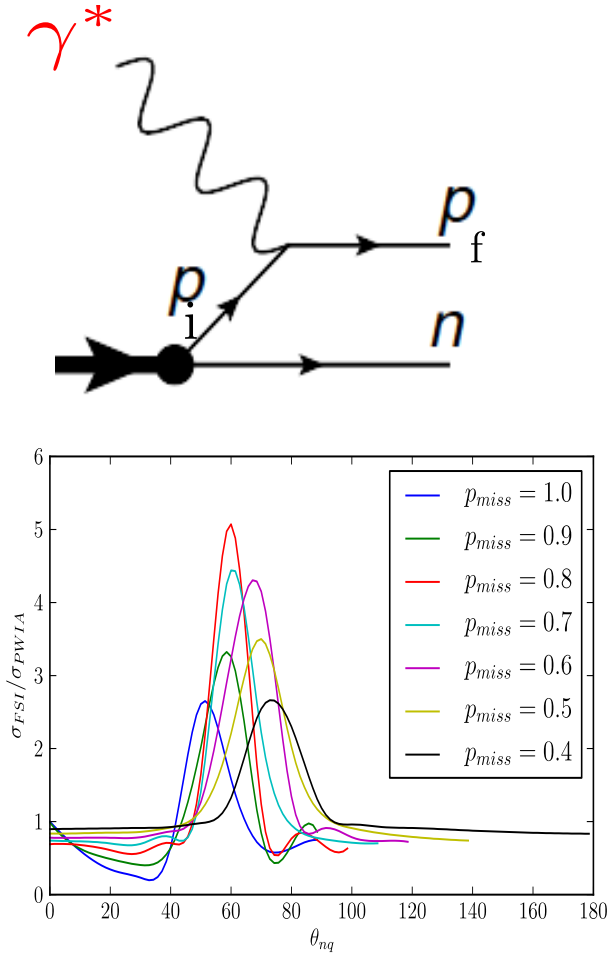


Possibility of Experimental Verification

Considering reaction:  $e + d \rightarrow e' + p_f + n$

$$|p_i| = |p_f - q| \gtrsim 800\text{MeV}/c$$

PAC-49, 2021



PAC 49 SUMMARY OF JEOPARDY RECOMMENDATIONS							
Number	Contact Person	Title	Hall	Previously Approved Days	Days Already Rec'd	Days Awarded	PAC Decision
<a href="#">E12-09-011</a>	Tanja Horn	Studies of the L-T Separated Kaon Electroproduction Cross Section from 5-11 GeV	C	40	32	8	Remain active
<a href="#">E12-10-003</a>	W. Boeglin	Deuteron Electro-Disintegration at Very High Missing Momentum	C	21	3	18	Upgrade Rating to A-

1) Is there any new information that would affect the scientific importance or impact of the Experiment since it was originally proposed?

PAC 36 graded the proposal with B+ because, even though the physics motivation was viewed highly, the foreseen impact of the result was judged to be limited. The results of the three days commissioning in April 2018, published in Physical Review Letters 125, 262501 (2020), exhibit an unexpected behavior when compared with theoretical calculations. Therefore, the expected impact of future data has increased.

## Outlook on Experimental Verification of the Effect

- analysis of the experiment will require careful account for competing nuclear effects most importantly final state interactions
- If angular dependence is found it will motivate new area of research
  - a: modeling non-nucleonic components in the deuteron,
  - b: understanding their origin and nature
  - c: evaluating parameters that can be used for Equation of State of high density Nuclear Matter
- If no angular dependence is found,
  - a: nucleonic degrees persist at very high density fluctuations
  - b: non-nucleonic components conspire to preserve angular condition
  - c: theory was wrong



## Second Property: Inverse Fractional Dependence of High Momentum Component

$$a_{NN}(A, y) \approx a_{NN}(A, 0) \cdot f(y) \quad \text{with } f(0) = 1 \text{ and } f(1) = 0$$

$$f(|x_p - x_n|) = 1 - \sum_{j=1}^n b_j |x_p - x_n|^j \quad \text{with } \sum_{j=1}^n b_j = 0$$

In the limit  $\sum_{j=1}^n b_j |x_p - x_n|^j \ll 1$  Momentum distributions of p & n are inverse proportional to their fractions

$$n_{p/n}^A(p) \approx \frac{1}{2x_{p/n}} a_2(A, y) \cdot n_d(p)$$

