From nuclear structure to high-energy processes

C. Weiss (JLab), Summer School “Light ion physics in the EIC era”,
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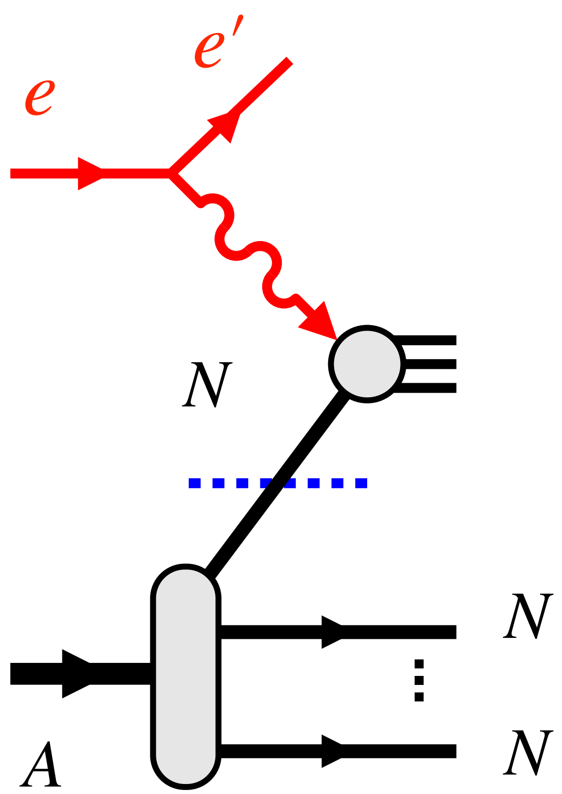
Jefferson Lab



High-energy scattering on nuclei

How to justify/implement a composite description in terms of nucleons?

How to separate/combine structure of nucleus and nucleon?

How to account for nuclear interactions - non-nucleonic DoF?

→ Relativity

→ Light-front methods

Outline

Basic considerations

High-energy scattering on nuclei

Challenges of composite description

Quantum mechanics and relativity

Essential techniques

Non-covariant representation of interactions

Light-front form of relativistic dynamics

High-energy scattering on nuclei

Energy nonconservation in intermediate states

Need for light-front form

Separating nucleus and nucleon structure

Light-front nuclear structrure

Dynamical equation

Rotationally symmetric representation
in 2-body sector (k-vector)

Deuteron and nonrelativistic approximation

Spin degrees of freedom

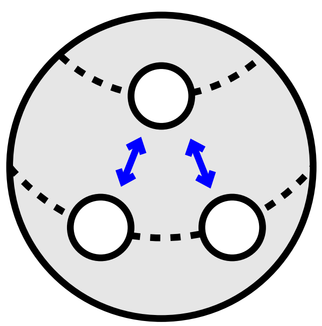
Current operators, good/bad components

Deep-inelastic scattering on nuclei

DIS on deuteron, tagged and inclusive

Structure functions

A > 2 nuclei

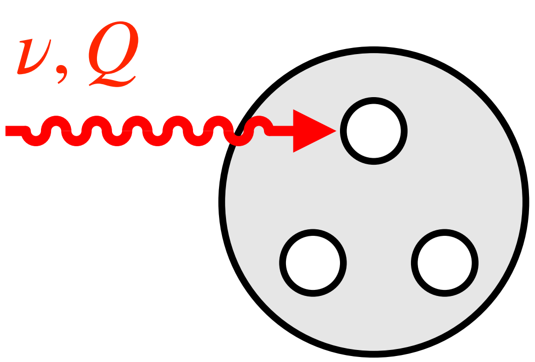


Low-energy structure and processes

Nucleus described in nucleon DoF: Motion, interactions

Other hadrons (π, vectors, Δ, …) “integrated out” $\rightarrow$ NN interactions

Current operators describe low-energy processes $Q \sim k_{\text{bind}} \sim$ few 10 MeV



High-energy processes

Scattering processes with energy/momentum transfer $\nu, Q \gg 1$ GeV:
Various probes, final states

How to obtain composite description in terms of nucleons?
Use nucleon-level process as input, combine with nuclear structure?

Nucleon motion?

Nucleon interactions? Non-nucleonic DoF?

This lecture: Use hadronic picture. Consider general high-energy process and focus on combining nuclear and nucleon structure. Connection with QCD (factorization, partonic structure) later.

Basics: Quantum mechanics and relativity

Quantum mechanics $\times$ Relativity

Superposition of configurations,
wave function

Transitions to intermediate states
lifetime $\Delta t \sim 1/\Delta E$

$\vdots$

Scattering kinematics

Boost invariance,
light-front form of dynamics

Hadron creation/annihilation
in intermediate states

$\vdots$

Consequential application of these concepts can lead to surprising conclusions

You know the basic concepts, but may not have seen them applied in this combination

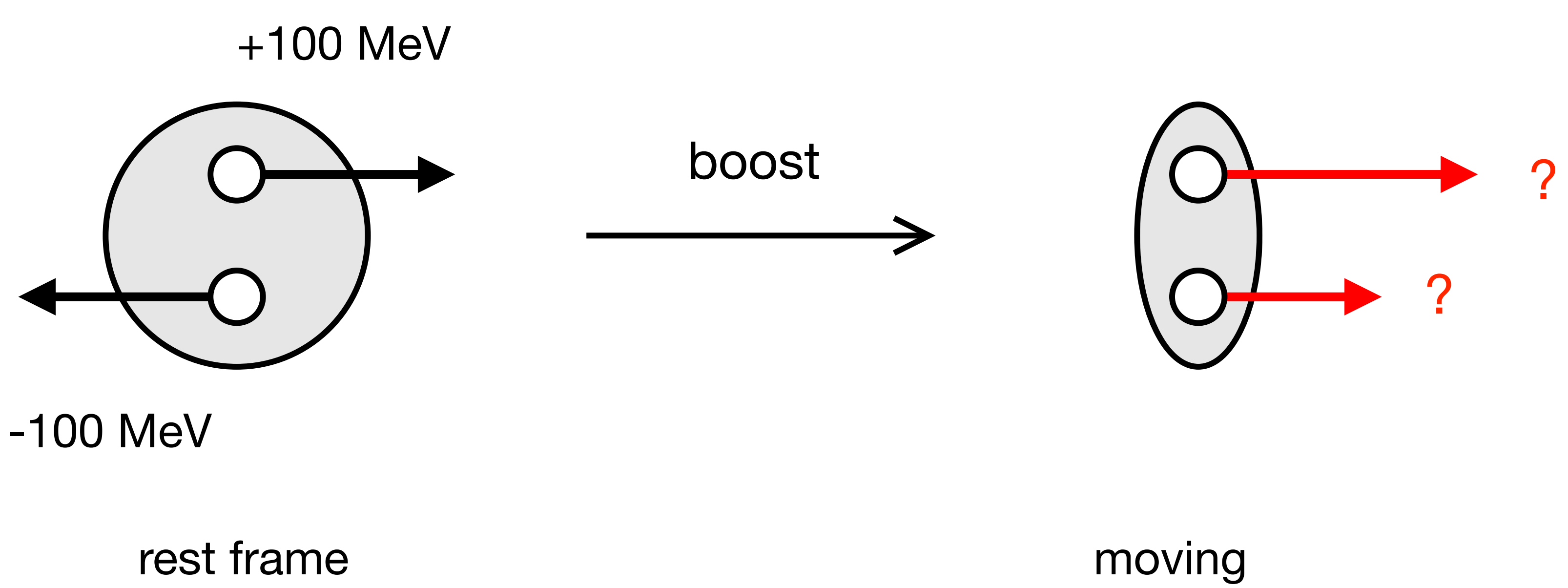
Most/all conclusions can be understood and communicated in simple form

Basics: Nuclear motion

Naive expectation: Motion of nucleons in nucleus with momenta ~ few 10 MeV has little effect on high-energy scattering processes at $\gg$ 1 GeV

Simple example shows that this is not so!

Example: Nucleon momentum distribution in fast-moving deuteron nucleus



Deuteron moving with $E_D = 200$ GeV
Consider nucleon configuration with z-momentum $p_{1,2} = \pm$ 100 MeV in rest frame
What are the nucleon momenta in the moving deuteron?

Basics: Nuclear motion

Exercise: Perform boost of nucleon configuration in deuteron

$$\begin{pmatrix} E \\ p \end{pmatrix}_{\text{moving}} = \begin{pmatrix} \frac{1}{\sqrt{1-v^2}} & \frac{v}{\sqrt{1-v^2}} \\ \frac{v}{\sqrt{1-v^2}} & \frac{1}{\sqrt{1-v^2}} \end{pmatrix} \begin{pmatrix} E \\ p \end{pmatrix}_{\text{rest}}$$

Lorentz boost

Applies to deuteron as a whole and to individual nucleons

Deuteron: $$\left(E_D\right)_{\text{moving}} = \frac{1}{\sqrt{1-v^2}} \left(E_D\right)_{\text{rest}} = \frac{1}{\sqrt{1-v^2}} M_D$$

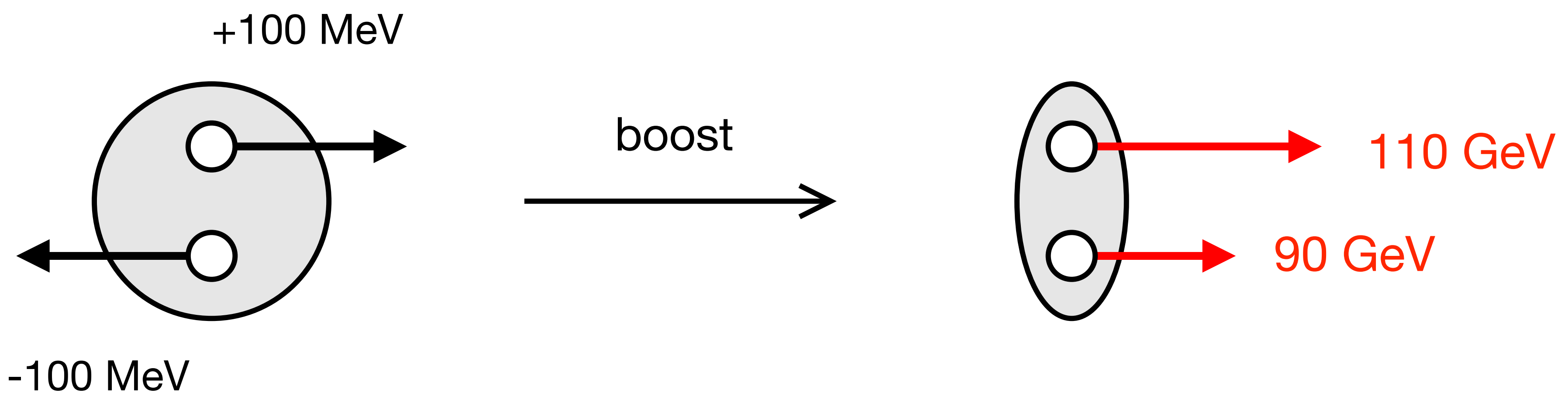
$$\frac{1}{\sqrt{1-v^2}} = \frac{\left(E_D\right)_{\text{moving}}}{M_D} = \frac{200 \text{ GeV}}{2 \text{ GeV}} = 100 \qquad \frac{v}{\sqrt{1-v^2}} \approx \frac{1}{\sqrt{1-v^2}} \qquad (v \approx 1)$$

Nucleon 1, 2: $\left(E_{1,2}\right)_{\text{rest}} \approx 1 \text{ GeV}$ $\quad \left(p_{1,2}\right)_{\text{rest}} = \pm\, 0.1 \text{ GeV}$

$$\left(p_{1,2}\right)_{\text{moving}} = 100 \times (1 \text{ GeV} \pm 0.1 \text{ GeV}) = \begin{cases} 110 \text{ GeV} \\ 90 \text{ GeV} \end{cases}$$

Large momentum difference!

Basics: Nuclear motion



Boost conserves nucleon light-cone momentum fractions

$$\frac{(E+p^z)_{1,2}}{(E+p^z)_D} = \text{invariant}$$

Internal motion of nucleons with $\sim$ few 10 MeV momenta has large effect on momentum distribution in nucleus moving with $\gg$ 1 GeV momentum

$\rightarrow$ large effect on high-energy scattering processes on nucleus (in any reference frame)

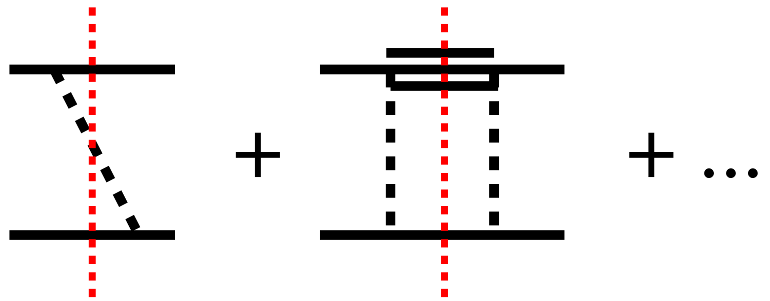
Need boost-invariant description of structure of fast-moving nucleus: Light-front wave function

Basics: Nuclear interactions

Naive expectation: Interactions of nucleons in nucleus
have little effect on high-energy scattering processes at $\gg 1$ GeV

Simple arguments show that this is not necessarily so!

Nucleon interactions involve intermediate states
with additional hadrons: Mesons (π, σ, vector,...),
baryon resonances (Δ, ...), high-mass states



In low-energy structure and reactions $q \sim k_{\text{bind}}$,
the high-mass intermediate states can be "integrated out": EFT approach

High-energy reactions $\omega \gg M_{\text{hadron}}$ can sample intermediate states up to the scale ω:
Cannot a priori assume that they are suppressed!

"Wave function" in relativistic context: Particle number not fixed, depends on scale of probe

Need to organize dynamics such that truncation of nuclear structure to nucleon constituents
becomes possible, and non-nucleonic DoF can be accounted for as corrections

Possible in light-front form of dynamics ($\rightarrow$ will be seen later)

Essential techniques

Interactions: Non-covariant representation

Form of relativistic dynamics: Instant form → light-front form

In describing high-energy scattering on nuclei we use the non-covariant representation of interactions: Wave function, configurations, intermediate states…

Here: Introduce/review basics using example from low-energy interactions → Lectures Pastore, Gnech

$\langle N(p_2')N(p_1') \,|\, \hat{T} \,|\, N(p_2)N(p_1) \rangle$ NN scattering amplitude at nuclear energies

S-matrix element, gives differential cross section

Transition between asymptotic states

$p_2' + p_1' = p_2 + p_1$ 4-momentum conservation

$$\mathbf{p}_2' + \mathbf{p}_1' = \mathbf{p}_2 + \mathbf{p}_1$$
$$E_2' + E_1' = E_2 + E_1$$

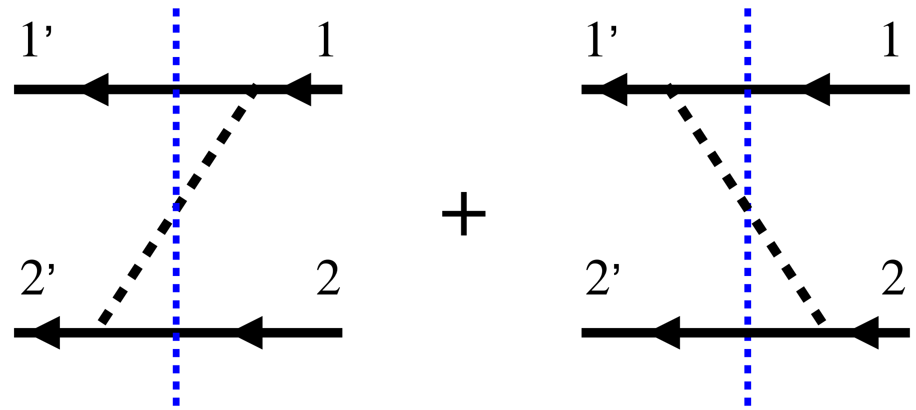
Implies 3-momentum and energy conservation

Consider transition through 1-pion exchange interaction. Apply non-covariant perturbation theory:

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} \qquad \hat{T} = \hat{H}_{\text{int}} + \hat{H}_{\text{int}} \frac{1}{E - \hat{H}_0} H_{\text{int}} + \ldots$$

Interactions: Non-covariant representation

$$\langle N_2' N_1' \,|\, \hat{T} \,|\, N_2 N_1 \rangle \quad = $$



$$\frac{\langle N_2' \,|\, \hat{H}_{\text{int}} \,|\, \pi N_2 \rangle \langle \pi N_1' \,|\, \hat{H}_{\text{int}} \,|\, N_1 \rangle}{(E_\pi + E_1' + E_2) - (E_1 + E_2)} \qquad \frac{\langle N_1' \,|\, \hat{H}_{\text{int}} \,|\, \pi N_1 \rangle \langle \pi N_2' \,|\, \hat{H}_{\text{int}} \,|\, N_2 \rangle}{(E_\pi + E_2' + E_1) - (E_1 + E_2)}$$

Interactions cause transition to intermediate states

Interactions conserve 3-momentum: $\sum \mathbf{p}\ \text{(interm)} = \sum \mathbf{p}\ \text{(initial)} = \sum \mathbf{p}\ \text{(final)}$

All particles are on their mass shell, also in intermediate states: $p_{1,2}^2 = m^2, \;\; E_{1,2} = \sqrt{|\mathbf{p}_{1,2}|^2 + m^2}, \quad p_\pi^2 = M_\pi^2, \quad E_\pi = \sqrt{|\mathbf{p}_\pi|^2 + M_\pi^2}$

Energy not conserved in intermediate states: $\sum E\ \text{(interm)} \neq \sum E\ \text{(initial)} = \sum E\ \text{(final)}$

Transition amplitude $\propto$ energy denominator $1/\Delta E$

Interactions: Non-covariant representation

Properties of non-covariant representation

Particles always on mass shell, even in intermediate states

3-momentum always conserved, even in intermediate states

Energy not conserved in intermediate states, only between asymptotic states

$\rightarrow$ 4-momentum not conserved in intermediate states!

Reasons for use

Intermediate states with well-defined particle content - configurations, constituents

Interpretation of processes as transitions

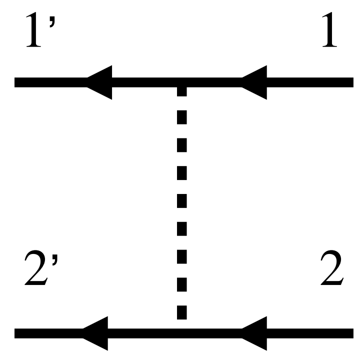
Concept of wave function defined in non-covariant representation

$$\langle N_1, N_2, \ldots N_N | A \rangle \qquad \sum_i^N \mathbf{p}_i = \mathbf{p}_A \qquad \sum_i^N E_i \neq E_A$$

Appropriate for high-energy scattering on composite systems

Interactions: Covariant representation

$\langle N_2' N_1' \,|\, \hat{T} \,|\, N_2 N_1 \rangle$ =



$p_\pi = p_1 - p_1' = p_2' - p_2$ pion 4-momentum

(Feynman diagram)

$$\frac{\text{couplings}}{p_\pi^2 - M_\pi^2}$$

Interactions conserve 4-momentum

Intermediate particles are off mass shell: $p_\pi^2 \neq M_\pi^2$ ("virtual particles")

Used in point-particle field theories (QCD, EFTs): Off-shell behavior of Green functions well-defined

Equivalence of non-covariant and covariant representations can be demonstrated in simple field theories

Some uses with composite systems are possible, but require additional considerations

Relativistic dynamics: Forms

In treatment of high-energy scattering on nuclei we use
the non-covariant representation of interactions

Depends on choice of time $\leftrightarrow$ energy and coordinate $\leftrightarrow$ momentum variables

Introduce/review forms of relativistic dynamics: Instant form, light-front form

Relativistic dynamics: Instant form

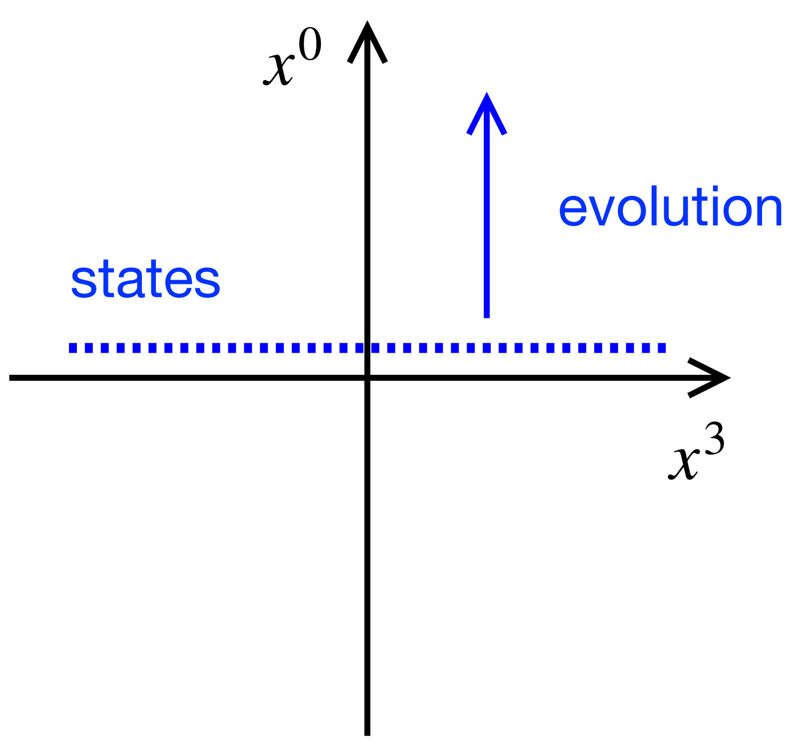
$x^\mu = (x^0, x^1, x^2, x^3)$ 4-dim spacetime

$x^0 = t$ time, $\mathbf{x} \equiv (x^1, x^2, x^3)$ space usual choice of variables

States of system defined at fixed x^0

Evolution in time x^0 described by hamiltonian P^0

Translational invariance in 3-coordinate $\mathbf{x}$ ensures conservation of 3-momentum $\mathbf{P}$



Relativistic dynamics: Instant form

Free particle

$$p^\mu = (E, \mathbf{p}) \qquad p^2 = p^\mu p_\mu = E^2 - |\mathbf{p}|^2 = m^2 \qquad E = +\sqrt{|\mathbf{p}|^2 + m^2}$$

Energy-momentum relation

$|\mathbf{p}\rangle$ Particle states labeled by 3-momentum.
Sometimes write $|p\rangle$, but independent variable is always 3-momentum

$$\langle \mathbf{p}' | \mathbf{p} \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{p}' - \mathbf{p})$$

Normalization of states
Factor p^0 included for relativistic covariance

$$\int d\Gamma_p \equiv \int \frac{d^4p}{(2\pi)^4} \, 2\pi\delta(p^2 - m^2)\,\theta(p^0) = \int \frac{d^3p}{(2\pi)^3 2E(\mathbf{p})}$$

Invariant phase space element

Bound state

$$\langle N(\mathbf{p}_1)N(\mathbf{p}_2)\,..\,N(\mathbf{p}_A)\,|\,A(\mathbf{P})\rangle \;=\; 2E_A\,(2\pi)^3\delta^{(3)}(\sum_i^A \mathbf{p}_i - \mathbf{P})\;\Psi(\{\mathbf{p}_i\}\,|\,\mathbf{P})$$

Expand state in products of free-particle states

$$2E_A \int d\Gamma_1\,..\int d\Gamma_A\,(2\pi)^3\delta^{(3)}(\sum_i^A \mathbf{p}_i - \mathbf{P})\;\Psi^*(\{\mathbf{p}_i\}\,|\,\mathbf{P})\,\Psi(\{\mathbf{p}_i\}\,|\,\mathbf{P}) \;=\; 1$$

Normalization of wave function

$$\langle A(\mathbf{P}')\,|\,A(\mathbf{P})\rangle \;=\; 2E_A\,(2\pi)^3\delta^{(3)}(\mathbf{P}' - \mathbf{P})$$

Normalization of state (CM motion)

Wave function describes expansion of bound state in free-particle states

Here: Deal with wave function as abstract object, independent of dynamical equation.
Wave functions for specific interactions can be obtained as solution of dynamical equation.

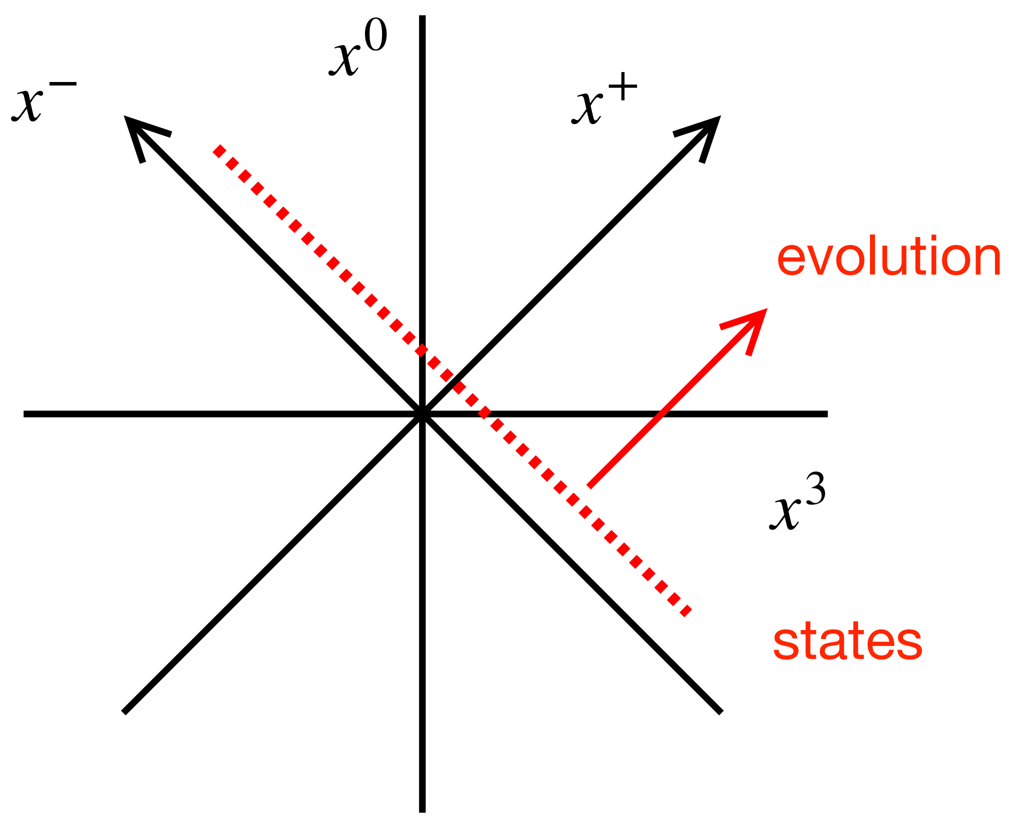
Spin/isospin quantum numbers: Later

Useful to review: Concepts and formulas can be extended to light-front quantization

Light-cone components

$a^{\pm} \equiv a^0 \pm a^3$ $\quad$ $\mathbf{a}_T \equiv (a^1, a^2)$ $\quad$ light-cone components of 4-vector a^{μ}

$ab = \dfrac{a^+b^- + a^-b^+}{2} - \mathbf{a}_T \cdot \mathbf{b}_T$ $\quad$ $a^2 = a^+a^- - |\mathbf{a}_T|^2$ $\quad$ scalar product and square



Light-front form of dynamics

$x^+ = x^0 + x^3 = 0$ $\quad$ hypersurface tangential to light-cone (3D plane)

Wave front of light wave traveling in -3 direction (= surface of constant phase)

Define states of system at light-front time $x^+ = 0$

Evolution in x^+ described by hamiltonian P^-

Relativistic dynamics: Light-front form

Light-front dynamics has many interesting formal properties: Representation of Poincare group, constrained dynamics, …

Here: Interested in applications to high-energy scattering on nuclei processes.
Take practical attitude. Start with basic features, learn about other features as needed

Boosts in light-front form

$p^+ \to e^\eta p^+$ Longitudinal boost (3-direction)

$p^- \to e^{-\eta} p^-$ Light-cone components diagonalize boost, transform multiplicatively

η rapidity = hyperbolic angle, $v = \tanh \eta$

$$\alpha = \frac{p^+}{p^+_{\text{ref}}}$$

Light-cone fractions boost-invariant

Simple technique for performing boosts of kinematic variables:
Compute fraction α in "old" frame, take p^+_{ref} in "new" frame, obtain p^+ in new frame

Exercise: Perform boost of nucleon configuration in deuteron using light-front variables

Boost-invariant momentum variables for wave functions: Light-front wave functions $\to$ following

Free particle

$p^+, \mathbf{p}_T$ momentum $\qquad p^-$ energy

$$p^2 = p^+ p^- - |\mathbf{p}_T|^2 = m^2 \qquad p^- = \frac{m^2 + |\mathbf{p}_T|^2}{p^+}$$

energy p^- fixed by mass shell condition

$p^+ > 0$ for physical particle because $\quad p^+ = p^0 + p^3 = \sqrt{m^2 + |\mathbf{p}_T|^2 + (p^3)^2} + p^3 \; > \; 0$

$p^- > 0$

regardless of sign of p^3

$|p^+, \mathbf{p}_T\rangle$ free particle state

$$\langle p'^+, \mathbf{p}'_T | p^+, \mathbf{p}_T \rangle = 2p^+ (2\pi)^3 \delta(p'^+ - p^+)\, \delta^{(2)}(\mathbf{p}'_T - \mathbf{p_T})$$

normalization of states

$$\int d\Gamma_p \;=\; \int_0^\infty \frac{dp^+}{(2\pi)\, 2p^+} \int \frac{d^2 p_T}{(2\pi)^2}$$

invariant phase space element

Relativistic dynamics: Light-front form

Bound state

$$\langle N(p_1^+, \mathbf{p}_{1T}) \, .. \, N(p_A^+, \mathbf{p}_{AT}) \, | \, A(\mathbf{P}) \rangle \;=\; (2\pi)^3 \, 2P^+ \, \delta(\sum_i^A p_i^+ - P^+) \; \delta^{(2)}(\sum_i^A \mathbf{p}_{iT} - \mathbf{P}_T) \; \Psi(\{p_i^+, \mathbf{p}_{iT}\} \, | \, \mathbf{P})$$

Nucleon light-cone momenta satisfy $\sum_i^A p_i^+ = P^+, \qquad \sum_i^A \mathbf{p}_{iT} = \mathbf{P}_T$

Boost invariance (longitudinal): Wave function depends only on light-cone fractions

$$\alpha_i \equiv \frac{A p_i^+}{P_A^+} \qquad (i = 1,..,A) \qquad\qquad \sum_{i=1}^A \alpha_i = A$$

$\Psi \equiv \Psi(\{\alpha_i, \mathbf{p}_{iT}\} \, | \, \mathbf{P}_T)$ independent of P^+

In many applications we can use nucleus rest frame $\mathbf{P}_T = 0$

Relativistic dynamics: Light-front form

Deuteron

Two nucleons: 1,2 or p, n $\qquad \alpha_1 + \alpha_2 = 2, \qquad \mathbf{p}_{1T} + \mathbf{p}_{2T} = \mathbf{P}_T$

Wave function effectively depends on variables of one nucleon: $\quad \Psi(\alpha_1, \mathbf{p}_{1T} \,|\, \mathbf{P}_T)$

Normalization:

$$\int \frac{d\alpha_1}{\alpha_1(2-\alpha_1)} \int \frac{d^2 p_{1T}}{(2\pi)^2} \, \Psi^*(\alpha_1, \mathbf{p}_{1T} \,|\, \mathbf{P}_T) \Psi(\alpha_1, \mathbf{p}_{1T} \,|\, \mathbf{P}_T) = 1$$

Spin/isospin quantum numbers, dynamical equation, connection with non-relativistic WF: Later

Spin degrees of freedom

Light-front helicity states: Later $\qquad$ → Lecture Cosyn

High-energy scattering on nuclei

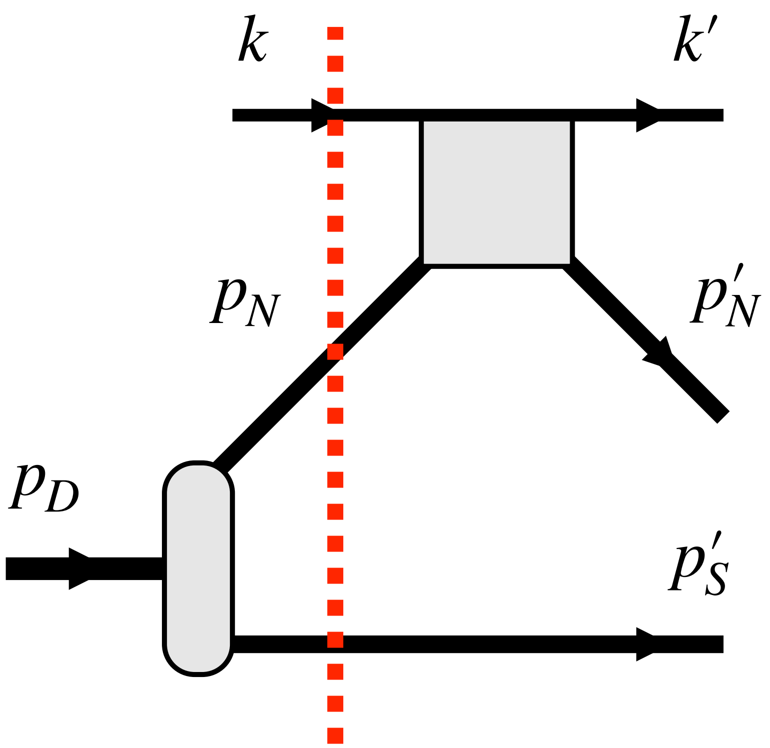
Study scattering processes at multi-GeV energy/momentum transfers

Exhibit effect of energy non-conservation in intermediate states of scattering amplitude

Compare equal-time and light-front form of dynamics

Arguments based on: Frankfurt, Strikman, Phys. Rept. 76, 215 (1981) [INSPIRE]

High-energy scattering: Example



High-energy electron-deuteron quasi-elastic scattering

N active nucleon, S spectator nucleon

$$e(k) + D(p_D) \to e'(k') + N'(p'_N) + S'(p'_S)$$

$k + p_D = k' + p'_N + p'_S$ 4-momentum conserved in overall process (asymptotic states)

Active nucleon momentum p_N in intermediate state determined by rules of noncovariant interactions

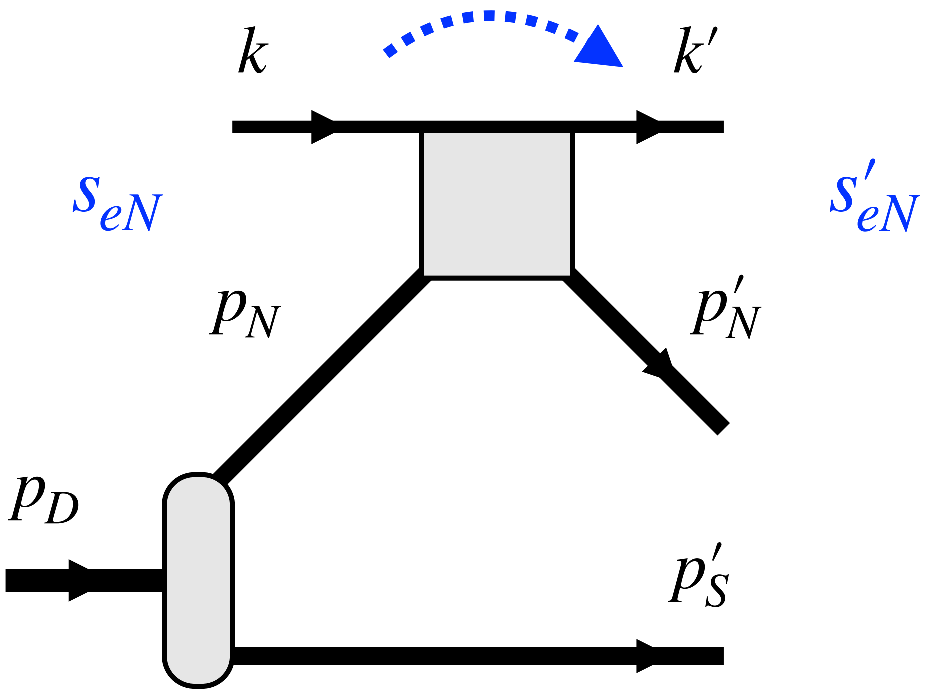
$D \to N + S$ matrix element preserves 3-momentum (wave function)

$p_N^2 = m^2$ mass shell condition fixes nucleon energy

$p_N \neq p_D - p_S \quad \curvearrowright \quad p_N \neq k - k' + p'_N$

$k + p_N \neq k' + p'_N$ 4-momentum not conserved in electron-nucleon subprocess

High-energy scattering: eN subprocess



Quantify effect of 4-momentum non-conservation in electron-nucleon subprocess

$$s_{eN} = (k + p_N)^2$$
$$s'_{eN} = (k' + p'_N)^2$$

Invariant energy before and after eN interaction

$$s'_{eN} = (k' + p'_N)^2 = (k + p_D - p_S)^2$$

using external 4-momentum conservation

Compute difference of subprocess invariant energies

$$\begin{aligned} s'_{eN} - s_{eN} &= (k + p_D - p_S)^2 - (k + p_N)^2 \\ &= \underbrace{2k \cdot (p_D - p_S - p_N)}_{\text{involves projectile 4-momentum } k\text{, large!}} \quad + \quad \underbrace{(p_D - p_S)^2 - p_N^2}_{\text{related to deuteron binding energy, small}} \end{aligned}$$

High-energy scattering: Equal-time dynamics

Use deuteron rest frame: $p_D = (M_D, \mathbf{0}), \qquad k = (\omega, -\omega \mathbf{e}_3)$ initial electron in -3 direction

energy $\omega \gtrsim 1$ GeV (or $\gg$)

Equal-time dynamics

$(p_D - p_S - p_N)^0 \neq 0$ non-conservation in 0-component (conventional energy)

$$p_D^0 = M_D \qquad p_S^0 \approx m + \frac{|\mathbf{p}_S|^2}{2m} \qquad p_N^0 \approx m + \frac{|\mathbf{p}_N|^2}{2m}$$

$\mathbf{p}_N = -\mathbf{p}_S$

$|\mathbf{p}_{S,N}| \sim$ few 100 MeV

$$(p_D - p_S - p_N)^0 \quad = \quad \frac{|\mathbf{p}_S|^2}{m} \;+\; (M_D - 2m)$$

= kinetic + binding energy

$$s'_{eN} - s_{eN} \quad = \quad 2k^0(p_D - p_S - p_N)^0 \quad \approx \quad 2\omega \frac{|\mathbf{p}_S^2|}{m}$$

grows with incident energy!

Electron-nucleon subprocess amplitude far “off energy shell” in limit of high-energy scattering

Cannot be connected with “on energy shell” amplitude measured in free eN scattering

No composite picture of high-energy scattering

High-energy scattering: Light-front dynamics

Use again deuteron rest frame

$k^+ = 0$ $k^- = 2\omega$ large light-front components of projectile 4-momentum

$(p_D - p_S - p_N)^- \neq 0$ non-conservation in minus component (LF energy)

$$
\begin{aligned}
s'_{eN} - s_{eN} &= 2k \cdot (p_D - p_S - p_N) \\
&= \underbrace{k^+}_{0} \cdot (p_D - p_S - p_N)^- + k^- \cdot (p_D - \underbrace{p_S - p_N)^+}_{0} = 0 + \text{terms independent of energy } \omega
\end{aligned}
$$

Use of light-front dynamics removes the term $\propto \omega$ in $s'_{eN} - s_{eN}$

Energy offshellness of eN subprocess amplitude remains finite in high-energy limit

Subprocess amplitude can be connected with “on energy shell” amplitude measured in eN scattering

Composite picture of high-energy scattering: Compute nuclear high-energy scattering amplitude from on-shell nucleon amplitudes and nuclear structure

High-energy scattering: Light-front dynamics

Light-front dynamics “aligns” the time/energy axis for nuclear dynamics with the direction of the high-energy process, in such a way that the energy nonconservation in intermediate states does not produce large effects

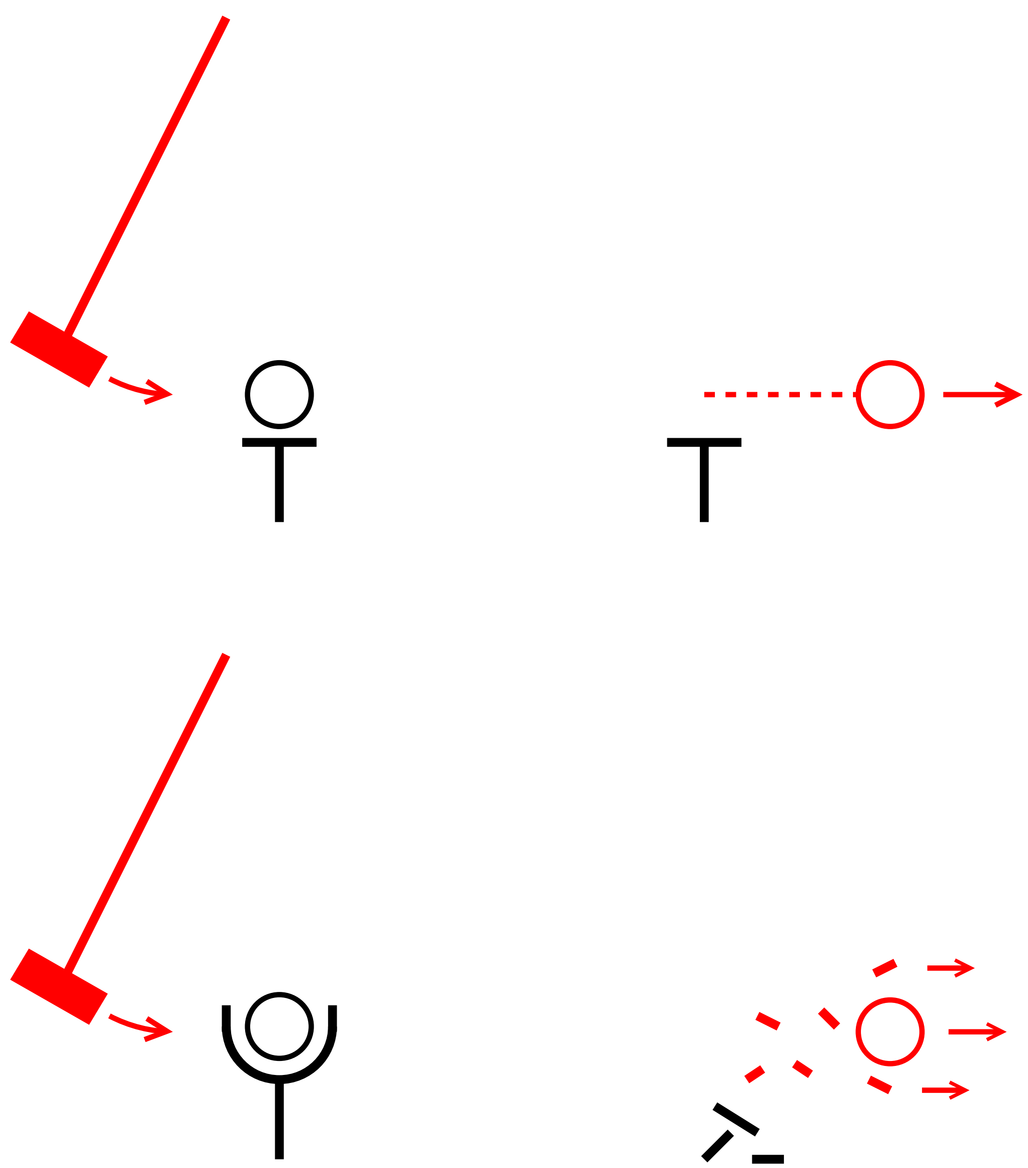
Light-front dynamics is the only scheme that avoids “large” energy offshellness in the nucleon subprocess amplitude and permits a composite description of high-energy scattering.
Its use is necessary, not optional, for a composite description.

In low-energy processes, there is no need to use light-front dynamics

Energy nonconservation in intermediate states is a necessary consequence of interactions and nuclear binding (→ wave function). Its manifestations in high energy scattering are physical effects, not technical artifacts.

Electroproduction and deep-inelastic scattering: Light-front direction usually aligned with momentum transfer 4-vector q. Conclusions re light-front dynamics remain the same as in example here.

High-energy scattering: Analogue



Teeing up a golf ball

Golf club = high energy process

Golf ball = nucleon

Tee = low-energy nuclear structure

Light-front quantization

Low-energy structure aligned with direction of high-energy process

Clean separation of scales

Other quantization schemes

Low-energy structure not aligned with direction of high-energy process

Low-energy structure produces effects of the order of the high collision energy

Supplemental material