# Effective Field Theory methods for nuclear interactions

#### Saori Pastore Washington University in St Louis



Jun 19–27, 2025 Florida International University, Miami, FL Introduction The Microscopic Approach Many-Nucleon Forces and Electroweak Currents Phenomenological Approach to Many-Nucleon Operators Chiral Effective Field Theory Approach to Many-Nucleon Operators Quantum Monte Carlo Many-Body Computational Methods Applications to Electroweak Observables and Relevance of Many-Nucleon Correlations

# Many-body Nuclear Interactions

Many-body Nuclear Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

 $v_{ij}$  and  $V_{ijk}$  are two- and three-nucleon operators based on experimental data fitting; fitted parameters subsume underlying QCD dynamics. They correlate nucleons in pairs and triplets.



Contact term: short-range Two-pion range: intermediate-range One-pion range: long-range

- 1. Phenomenological potentials AV18+UIX; AV18+IL7 Wiringa, Schiavilla, Pieper et al.
- 2. Chiral EFT potentials with  $\pi N\Delta N3LO + N2LO$  Piarulli *et al.* Norfolk Models

# Two-nucleon physics

 $M = \pm 1$ 

M = 0



Constant density surfaces for a polarized deuteron in the  $M=\pm 1$  (left) and M=0 (right) states

#### Carlson and Schiavilla Rev.Mod.Phys.70(1998)743



Schiavilla Carlson Wiringa Pieper PRL98(2007) & PRC89(2014)

# Two-body densities



AV18UX

Piarulli, SP, Wiringa PRC (2023)

<u>data</u>

### Many-body Nuclear Electroweak Currents



- 1. Phenomenological meson exchange currents consistent with the AV18 interaction Schiavilla et al.
- 2. Chiral EFT currents with LECs fixed from expt data SP, Schiavilla, Baroni et al.



# **Electron-Nucleus Scattering Cross Section**



Energy and momentum transferred ( $\omega$ ,q)

Current and planned experimental programs rely on theoretical calculations at different kinematics

# Electromagnetic observables at low-energies

Electromagnetic structure of nuclei

## Magnetic structure of nuclei

Magnetic moment

$$\mu = \lim_{q \to 0} -i \frac{2m_N}{q} \left\langle JJ | \mathbf{j}_y(q\mathbf{\hat{x}}) | JJ \right\rangle$$

Magnetic moment in the single particle picture

$$\mu_{i} = \mu_{N} \left[ (\boldsymbol{L}_{i} + g_{p} \boldsymbol{S}_{i}) \frac{1 + \tau_{i,z}}{2} + g_{n} \boldsymbol{S}_{i} \frac{1 - \tau_{i,z}}{2} \right]$$

$$\vec{\boldsymbol{L}}_{p}$$

$$\vec{\boldsymbol{S}}_{p}$$

$$\vec{\boldsymbol{S}}_{n}$$

# Magnetic Moments of Light Nuclei



Hybrid approach: AV18+IL7 and chiEFT currents; predictions are for A>3 nuclei

### One-body magnetic density



$$\mu^{1b} \propto \int \rho_M^{1b}(r) dr$$

r single particle coordinate from the c.m.

$$\mu 1b = \mu_N \sum_i \left[ (L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i(1 - \tau_{i,z})/2 \right]$$

# Magnetic moments in light nuclei



Chambers-Wall, King, Gnech et al. 2407.03487 2024

Based on Norfolk interactions and one- plus two-body currents

N

# Two-body magnetic densities



$$\mu^{2\mathrm{b}} = \int dr_{ij} 4\pi r_{ij}^2 \rho_M^{2\mathrm{b}}(r_{ij})$$

Cluster effects suppress the two-body contribution for A=9,T=1/2



# Elastic scattering

Cross section

$$\frac{d\sigma}{d\Omega} = 4\pi\sigma_M f_{\rm rec}^{-1} \left[ \frac{Q^4}{q^4} F_L^2(q) + \left( \frac{Q^2}{2q^2} + \tan^2\theta_e/2 \right) F_T^2(q) \right]$$

$$F_T^2(q) = F_M^2(q) = \frac{1}{2J_i + 1} \sum_{L=1}^{\infty} |\langle J_f || M_L(q) || J_i \rangle|^2$$
$$F_L^2(q) = \frac{1}{2J_i + 1} \sum_{L=0}^{\infty} |\langle J_f || C_L(q) || J_i \rangle|^2$$

Magnetic and Charge Form Factors  $\langle JJ | \mathbf{j}_y(q \hat{\boldsymbol{x}}) | JJ \rangle \ \langle J_f M | \rho^{\dagger}(q) | J_i M \rangle$ 

#### Magnetic form factors: comparison with the data





First QMC results for form factors in A>6 systems.

Based on Norfolk interactions and one- and two-body currents.

Error band = truncation error in the ChiEFT expansion.

 $q \, [\mathrm{fm}^{-1}]$  Chambers-Wall, King, Gnech et al. <u>2407.03487</u> 2024

#### Magnetic form factors: comparison with the data





Nice agreement with the data beyond the expected validity of ChiEFT.

M5 explains the dip in <sup>10</sup>B at high q.

 $q \text{ [fm}^{-1}\text{]}$  Chambers-Wall, King, Gnech et al. <u>2407.03487</u> 2024

# Magnetic form factors: predictions





#### Two-body currents provide 40-60%.

Note the swapping of M1 and M3 in mirror nuclei. Also observed in A=7 nuclei.

It would be interesting to have data for mirror nuclei.

Maybe <sup>7</sup>Be?

Chambers-Wall, King, Gnech et al. 2407.03487 2024

# A comment on the data

Nucleus	Reference	Data type	ratio/method
$^{3}\mathrm{H}$	Sick 2001 89	Ν	1
<sup>3</sup> He	Sick 2001 89	Ν	1
<sup>6</sup> Li	Peterson 1962 90	Ν	Eq. (C2)
	Goldemberg 1963 91	N	Eq. $(C2)$
	Rand 1966 92	Ν	Eq. $(C1)$
	Lapikas 1978 93	D	$1/4\pi$
	Bergstrom 1982 94	Ν	$Z^{2}/4\pi$
<sup>7</sup> Li	Peterson 1962 90	Ν	Eq. (C2)
	Goldemberg 1963 91	Ν	Eq. (C2)
	Van Niftrik 1971 95	D	Eq. (C1)
	Lichtenstadt 1983 96	Ν	$Z^2/4\pi$
$^{9}\mathrm{Be}$	Goldemberg 1963 91	Ν	Eq. (C2)
	Vanpraet 1965 98	N	Eq. (C1)
	Rand 1966 92	Ν	Eq. (C1)
	Lapikas 1975 <u>97</u>	Ν	Eq. (C2)
$^{10}B$	Goldemberg 1963 91	Ν	Eq. (C2)
	Goldemberg 1965 100	Ν	Eq. (C2)
	Vanpraet 1965 98	Ν	Eq. (C1)
	Rand 1966 92	Ν	Eq. (C1)
	Lapikas 1978 93	D	$1/4\pi$

Available data to date.



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Chambers-Wall, King, Gnech et al. PRL 2024 2407.03487



# Charge radii

Extracted from low-momentum transfer behavior of form factor.

$$\frac{1}{Z} \langle JJ | \rho(q\hat{\mathbf{z}}) | JJ \rangle \approx 1 - \frac{1}{6} r_E^2 q^2 + \mathcal{O}(q^4)$$

Accounts for two-body correlations, finite size/nucleon level corrections via nucleonic form factors.



Agreement of  $\sim 5\%$  or better.

King et al. submitted to PRC 2025

# Magnetic radii

Extracted from low-momentum transfer behavior of form factor.

$$-i\frac{2m}{q\mu}\left\langle JJ|\mathbf{j}_{y}(q\hat{\mathbf{x}})|JJ\right\rangle \approx 1-\frac{1}{6}r_{M}^{2}q^{2}+\mathcal{O}(q^{4})$$

Accounts for two-body currents, finite size/nucleon level corrections via nucleonic form factors.



Limited data, predictions available for A up to 10.

King et al. submitted to PRC 2025

# **Electromagnetic transitions**

Two-body electromagnetic currents bring the theory in agreement with the data

~ 60 – 70% of total two-body current is due to one-pion-exchange currents



### Neutrino cross section anatomy



The neutrino program is in need of accurate determinations of neutrino nucleus cross sections in a wide range of energies.

Quasi-elastic: dominated by single-nucleon knockout

Resonance: excitation to nucleonic resonant states which decay into mesons

Deep-inelastic scattering: where the neutrino resolves the nucleonic quark content

Each of these regimes requires knowledge of both the **nuclear ground state** and the **electroweak coupling** and **propagation of the struck nucleons, hadrons, or partons** 

#### Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

$$R_{\alpha}(q,\omega) = \sum_{f} \delta\left(\omega + E_0 - E_f\right) |\langle f|O_{\alpha}(\mathbf{q})|0\rangle|^2$$

Longitudinal response induced by the charge operator  $O_L = \rho$ Transverse response induced by the current operator  $O_T = j$ 5 Responses in neutrino-nucleus scattering

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[ v_L R_L(\mathbf{q}, \omega) + v_T R_T(\mathbf{q}, \omega) \right]$$



For a recent review on QMC, SF methods see Rocco Front. In Phys.8 (2020)116

# **Inclusive Cross Sections with Integral Transforms**

Exploit integral properties of the response functions and closure to avoid explicit calculation of the final states (Lorentz Integral Transform **LIT**, **Euclidean**, ...)



Sobczyk et al, PRL127 (2021)



Lovato et al. PRX10 (2020)

# Lepton-Nucleus scattering: Data

5

Transverse Sum Rule

 $S_T(q) \propto \langle 0 | \mathbf{j}^{\dagger} \mathbf{j} | 0 \rangle \propto \langle 0 | \mathbf{j}_{1b}^{\dagger} \mathbf{j}_{1b} | 0 \rangle + \langle 0 | \mathbf{j}_{1b}^{\dagger} \mathbf{j}_{2b} | 0 \rangle + \dots$ 



Observed transverse enhancement explained by the combined effect of two-body correlations and currents in the interference term

$$\langle \mathbf{j}_{1b}^{\dagger} \ \mathbf{j}_{1b} \rangle > 0$$

Leading one-body term

$$\langle \mathbf{j}_{1b}^{\dagger} \; \mathbf{j}_{2b} \; v_{\pi} \rangle \propto \langle v_{\pi}^2 \rangle > 0$$
  
Interference term



Transverse/Longitudinal Sum Rule Carlson *et al.* PRC65(2002)024002

# **Beyond Inclusive: Short-Time-Approximation**

Short-Time-Approximation Goals:

- Describe electroweak scattering from A
   > 12 without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects



Subedi et al. Science320(2008)1475



#### Stanford Lab article



# Short-Time-Approximation

Short-Time-Approximation:

- Based on Factorization
- Retains two-body physics
- Correctly accounts for interference



$$R(q,\boldsymbol{\omega}) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} \,\mathrm{e}^{i(\boldsymbol{\omega}+E_0)t} \,\langle 0|O^{\dagger}\,\mathrm{e}^{-iHt}\,O|0\rangle$$

$$O_i^{\dagger} e^{-iHt} O_i + O_i^{\dagger} e^{-iHt} O_j + O_i^{\dagger} e^{-iHt} O_{ij} + O_{ij}^{\dagger} e^{-iHt} O_{ij}$$

$$H \sim \sum_i t_i + \sum_{i < j} v_{ij}$$

# STA: regime of validity

The typical (conservative estimate) energy (time) scale in a nucleus with A correlated nucleons in pairs is

$$\epsilon_{pair} \sim 20 \text{ MeV}$$
 (t ~ 1/ $\epsilon_{pair}$ )

This sets a natural expansion parameter in the QE region characterized by  $\omega_{QE}$ 

 $\epsilon_{\text{pair}}$  /  $\omega_{\text{QE}}$ 

The STA neglects terms of order  $O((\epsilon_{pair} / \omega_{QE})^2)$ 



# **Short-Time-Approximation**

Short-Time-Approximation:

- Based on Factorization
- Retains two-body physics
- Response functions are given by the scattering from pairs of fully interacting nucleons that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides "more" exclusive information in terms of nucleon-pair kinematics via the Response Densities

Response Functions  $\infty$  Cross Sections

$$R_{\alpha}(q,\omega) = \sum_{f} \delta\left(\omega + E_0 - E_f\right) \left|\langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle\right|^2$$

Response *Densities* 

$$R(q,\omega) \sim \int \delta \left(\omega + E_0 - E_f\right) dP' dp' \mathcal{D}(p',P';q)$$

*P*' and *p*' are the CM and relative momenta of the struck nucleon pair

# Transverse Response Density: *e*-<sup>4</sup>He scattering

Transverse Density q = 500 MeV/c



SP et al. PRC101(2020)044612

### Transverse Response Density: two-body physics



q=500

# *e*-<sup>4</sup>He scattering in the back-to-back kinematic



SP et al. PRC101(2020)044612

#### Helium-4: data & model dependence





Benchmark in <sup>4</sup>He

SP et al. PRC101(2020)044612

<sup>12</sup>C Response Densities



Andreoli et al. Phys. Rev. C 110 (2024) 6, 064004 arXiv:2407.06986

# <sup>12</sup>C response functions

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[ v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$

Andreoli et al. Phys. Rev. C 110 (2024) 6, 064004 arXiv:2407.06986





Andreoli *et al. Phys.Rev.C* 110 (2024) 6, 064004 <u>arXiv:2407.06986</u> Data From https://discovery.phys.virginia.edu/research/groups/qes-archive/index.html

# Relativistic effects in e-<sup>3</sup>H scattering



Andreoli et al. Phys. Rev. C 105 (2022) 1, 014002



# Relativistic effects in e-<sup>12</sup>C scattering



Andreoli et al. Phys. Rev. C 110 (2024) 6, 064004 arXiv:2407.06986

# **Relativistic corrections**

Traditional non relativistic expansion of the covariant single nucleon electromagnetic current assumes initial and final nucleon momentum small.

$$egin{aligned} j^{\mu} &= ear{u}ig(m{p}'s'ig)ig(e_N\gamma^{\mu}+rac{i\kappa_N}{2m_N}\sigma^{\mu
u}q_{
u}ig)u(m{p}s)\ m{p}' &=m{p}+m{q} \end{aligned}$$

New paradigme where the relativistic correction is obtained expanding the covariant one-nucleon current for high values of momentum transfer, and small values of initial nucleon momentum p. This changed:

- 1. Expression of the one-body operator
- 2. Energy conserving delta function



With Ronen Weiss and Lorenzo Andreoli

Ronen Weiss Ed Jaynes Fellow at WashU

#### Implementation single nucleon current

1000

800

600



Response density vs relative and c.m. energy of the struck pair <sup>4</sup>He Transverse response density at q = 700 MeV/c





Response density vs momenta of individual nucleons

# Application to e-<sup>3</sup>H scattering



#### Three-body densities







Ronen Weiss, and Stefano Gandolfi *Phys.Rev.C* 108 (2023) 2, L021301

With AFDMC



# <sup>4</sup>He Three-Body Momentum Distribution



# Summary

Ab initio calculations of light nuclei yield a picture of nuclear structure and dynamics where many-body effects play an essential role to explain available data.



200150100 500  $e \, [\text{MeV}]$ Close collaborations between NP, LQCD, Pheno, Hep, Comp, Expt, ... are required to progress e.g., NP is represented in the Snowmass process

It's a very exciting time!

Transverse Density q = 500 MeV/c

2,000

1.000



Graham Chambers-Wall (WashU GS)



Garrett King (LANL PD)



Lorenzo Andreoli (ODU/JLab PD)

King *et al.* <u>PRC 110</u> (2024) 5, 054325; <u>Ann.Rev.Nucl.Part.Sci. 74</u> (2024) 343 Chambers-Wall, Gnech, King *et al.* <u>PRL 133</u> (2024) 21, 212501; <u>PRC 110</u> (2024) 5, 054316 Andreoli *et al.* <u>PRC 110</u> (2024) 6, 064004

# Collaborators

#### WashU: Bub Chambers-Wall Flores Novario Piarulli Weiss

LANL: Carlson Gandolfi Hayes **King** Mereghetti JLab+ODU: **Andreoli Gnech** Schiavilla ANL: McCoy Lovato Wiringa UW/INT: Cirigliano Dekens Pisa U/INFN: Kievsky Marcucci Viviani Salento U: Girlanda Huzhou U: Dong Wang Fermilab: Gardiner Betancourt Rocco MIT: Barrow





![](_page_51_Picture_5.jpeg)

![](_page_51_Picture_6.jpeg)

# <sup>12</sup>C cross sections: interpolation scheme

We have coarse grid in q for <sup>12</sup>C. We use an interpolation scheme tested on He4.

$$I_{L/T}(\omega;\mathbf{q}) = rac{\int_0^\omega R_{L/T}(\omega';\mathbf{q})d\omega'}{\int_0^\infty R_{L/T}(\omega';\mathbf{q})d\omega'}$$

![](_page_52_Figure_3.jpeg)

![](_page_52_Figure_4.jpeg)

![](_page_52_Figure_5.jpeg)

Lorenzo Andreoli et al. arXiv:2407.06986

# <sup>12</sup>C response functions

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \, \left[ v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$

Lorenzo Andreoli et al. arXiv:2407.06986

![](_page_53_Figure_3.jpeg)

# <sup>12</sup>C cross sections

![](_page_54_Figure_1.jpeg)

Lorenzo Andreoli et al. arXiv:2407.06986

#### **GENIE** validation using e-scattering

Z = 2, A = 4, Beam Energy = 0.64 GeV, Angle =  $60^{\circ} \pm 0.25^{\circ}$ 

![](_page_55_Figure_2.jpeg)

- STA responses used to build the cross sections
- Cross sections are used to generate events in GENIE (a Monte Carlo neutrino event generator)
- Here, we use electromagnetic processes (for which data are available) to validate the generator

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[ v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$

Barrow, Gardiner, SP et al. PRD 103 (2021) 5, 052001

### Back to back scattering and particle identity

![](_page_56_Figure_1.jpeg)

![](_page_56_Figure_2.jpeg)

tot

pp nn pp/all % pp/all % from momentum distributions nn/all %

#### GFMC SF STA: Benchmark & error estimate

![](_page_57_Figure_1.jpeg)

Lorenzo Andreoli, et al. PRC 2021

![](_page_57_Picture_3.jpeg)