Effective Field Theory methods for nuclear interactions

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Jun 19–27, 2025 Florida International University, Miami, FL Introduction The Microscopic Approach Many-Nucleon Forces and Electroweak Currents Phenomenological Approach to Many-Nucleon Operators Chiral Effective Field Theory Approach to Many-Nucleon Operators Quantum Monte Carlo Many-Body Computational Methods Applications to Electroweak Observables and Relevance of Many-Nucleon Correlations

Many-body Nuclear Interactions

Many-body Nuclear Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

 v_{ij} and V_{ijk} are two- and three-nucleon operators based on experimental data fitting; fitted parameters subsume underlying QCD dynamics. They correlate nucleons in pairs and triplets.



Contact term: short-range Two-pion range: intermediate-range One-pion range: long-range

- 1. Phenomenological potentials AV18+UIX; AV18+IL7 Wiringa, Schiavilla, Pieper et al.
- 2. Chiral EFT potentials with $\pi N\Delta N3LO + N2LO$ Piarulli *et al.* Norfolk Models

Two-nucleon physics

 $M = \pm 1$

M = 0



Constant density surfaces for a polarized deuteron in the $M=\pm 1$ (left) and M=0 (right) states

Carlson and Schiavilla Rev.Mod.Phys.70(1998)743



Schiavilla Carlson Wiringa Pieper PRL98(2007) & PRC89(2014)

Two-body densities



AV18UX

Piarulli, SP, Wiringa PRC (2023)

<u>data</u>

Many-body Nuclear Electroweak Currents



- 1. Phenomenological meson exchange currents consistent with the AV18 interaction Schiavilla et al.
- 2. Chiral EFT currents with LECs fixed from expt data SP, Schiavilla, Baroni et al.



Electron-Nucleus Scattering Cross Section



Energy and momentum transferred (ω ,q)

Current and planned experimental programs rely on theoretical calculations at different kinematics

Electromagnetic observables at low-energies

Electromagnetic structure of nuclei

Magnetic structure of nuclei

Magnetic moment

$$\mu = \lim_{q \to 0} -i \frac{2m_N}{q} \left\langle JJ | \mathbf{j}_y(q\mathbf{\hat{x}}) | JJ \right\rangle$$

Magnetic moment in the single particle picture

$$\mu_{i} = \mu_{N} \left[(\boldsymbol{L}_{i} + g_{p} \boldsymbol{S}_{i}) \frac{1 + \tau_{i,z}}{2} + g_{n} \boldsymbol{S}_{i} \frac{1 - \tau_{i,z}}{2} \right]$$

$$\vec{\boldsymbol{L}}_{p}$$

$$\vec{\boldsymbol{S}}_{p}$$

$$\vec{\boldsymbol{S}}_{n}$$

Magnetic Moments of Light Nuclei



Hybrid approach: AV18+IL7 and chiEFT currents; predictions are for A>3 nuclei

One-body magnetic density



$$\mu^{1b} \propto \int \rho_M^{1b}(r) dr$$

r single particle coordinate from the c.m.

$$\mu 1b = \mu_N \sum_i \left[(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i(1 - \tau_{i,z})/2 \right]$$

Magnetic moments in light nuclei



Chambers-Wall, King, Gnech et al. 2407.03487 2024

Based on Norfolk interactions and one- plus two-body currents

N

Two-body magnetic densities



$$\mu^{2\mathrm{b}} = \int dr_{ij} 4\pi r_{ij}^2 \rho_M^{2\mathrm{b}}(r_{ij})$$

Cluster effects suppress the two-body contribution for A=9,T=1/2



Elastic scattering

Cross section

$$\frac{d\sigma}{d\Omega} = 4\pi\sigma_M f_{\rm rec}^{-1} \left[\frac{Q^4}{q^4} F_L^2(q) + \left(\frac{Q^2}{2q^2} + \tan^2\theta_e/2 \right) F_T^2(q) \right]$$

$$F_T^2(q) = F_M^2(q) = \frac{1}{2J_i + 1} \sum_{L=1}^{\infty} |\langle J_f || M_L(q) || J_i \rangle|^2$$
$$F_L^2(q) = \frac{1}{2J_i + 1} \sum_{L=0}^{\infty} |\langle J_f || C_L(q) || J_i \rangle|^2$$

Magnetic and Charge Form Factors $\langle JJ | \mathbf{j}_y(q \hat{\boldsymbol{x}}) | JJ \rangle \ \langle J_f M | \rho^{\dagger}(q) | J_i M \rangle$

Magnetic form factors: comparison with the data





First QMC results for form factors in A>6 systems.

Based on Norfolk interactions and one- and two-body currents.

Error band = truncation error in the ChiEFT expansion.

 $q \, [\mathrm{fm}^{-1}]$ Chambers-Wall, King, Gnech et al. <u>2407.03487</u> 2024

Magnetic form factors: comparison with the data





Nice agreement with the data beyond the expected validity of ChiEFT.

M5 explains the dip in ¹⁰B at high q.

 $q \text{ [fm}^{-1}\text{]}$ Chambers-Wall, King, Gnech et al. <u>2407.03487</u> 2024

Magnetic form factors: predictions





Two-body currents provide 40-60%.

Note the swapping of M1 and M3 in mirror nuclei. Also observed in A=7 nuclei.

It would be interesting to have data for mirror nuclei.

Maybe ⁷Be?

Chambers-Wall, King, Gnech et al. 2407.03487 2024

A comment on the data

Nucleus	Reference	Data type	ratio/method
$^{3}\mathrm{H}$	Sick 2001 89	Ν	1
³ He	Sick 2001 89	Ν	1
⁶ Li	Peterson 1962 90	Ν	Eq. (C2)
	Goldemberg 1963 91	N	Eq. $(C2)$
	Rand 1966 92	Ν	Eq. $(C1)$
	Lapikas 1978 93	D	$1/4\pi$
	Bergstrom 1982 94	Ν	$Z^{2}/4\pi$
⁷ Li	Peterson 1962 90	Ν	Eq. (C2)
	Goldemberg 1963 91	Ν	Eq. (C2)
	Van Niftrik 1971 95	D	Eq. (C1)
	Lichtenstadt 1983 96	Ν	$Z^2/4\pi$
$^{9}\mathrm{Be}$	Goldemberg 1963 91	Ν	Eq. (C2)
	Vanpraet 1965 98	N	Eq. (C1)
	Rand 1966 92	Ν	Eq. (C1)
	Lapikas 1975 <u>97</u>	Ν	Eq. (C2)
^{10}B	Goldemberg 1963 91	Ν	Eq. (C2)
	Goldemberg 1965 100	Ν	Eq. (C2)
	Vanpraet 1965 98	Ν	Eq. (C1)
	Rand 1966 92	Ν	Eq. (C1)
	Lapikas 1978 93	D	$1/4\pi$

Available data to date.



Magnetic form factors: comparison with the data





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 $q \, [\text{fm}^{-1}]$ Chambers-Wall, King, Gnech et al. PRL 2024 2407.03487

Magnetic form factors: comparison with the data



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Chambers-Wall, King, Gnech et al. PRL 2024 2407.03487



Charge radii

Extracted from low-momentum transfer behavior of form factor.

$$\frac{1}{Z} \langle JJ | \rho(q\hat{\mathbf{z}}) | JJ \rangle \approx 1 - \frac{1}{6} r_E^2 q^2 + \mathcal{O}(q^4)$$

Accounts for two-body correlations, finite size/nucleon level corrections via nucleonic form factors.



Agreement of $\sim 5\%$ or better.

King et al. submitted to PRC 2025

Magnetic radii

Extracted from low-momentum transfer behavior of form factor.

$$-i\frac{2m}{q\mu}\left\langle JJ|\mathbf{j}_{y}(q\hat{\mathbf{x}})|JJ\right\rangle \approx 1-\frac{1}{6}r_{M}^{2}q^{2}+\mathcal{O}(q^{4})$$

Accounts for two-body currents, finite size/nucleon level corrections via nucleonic form factors.



Limited data, predictions available for A up to 10.

King et al. submitted to PRC 2025

Electromagnetic transitions

Two-body electromagnetic currents bring the theory in agreement with the data

~ 60 – 70% of total two-body current is due to one-pion-exchange currents



Neutrino cross section anatomy



The neutrino program is in need of accurate determinations of neutrino nucleus cross sections in a wide range of energies.

Quasi-elastic: dominated by single-nucleon knockout

Resonance: excitation to nucleonic resonant states which decay into mesons

Deep-inelastic scattering: where the neutrino resolves the nucleonic quark content

Each of these regimes requires knowledge of both the **nuclear ground state** and the **electroweak coupling** and **propagation of the struck nucleons, hadrons, or partons**

Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

$$R_{\alpha}(q,\omega) = \sum_{f} \delta\left(\omega + E_0 - E_f\right) |\langle f|O_{\alpha}(\mathbf{q})|0\rangle|^2$$

Longitudinal response induced by the charge operator $O_L = \rho$ Transverse response induced by the current operator $O_T = j$ 5 Responses in neutrino-nucleus scattering

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[v_L R_L(\mathbf{q}, \omega) + v_T R_T(\mathbf{q}, \omega) \right]$$



For a recent review on QMC, SF methods see Rocco Front. In Phys.8 (2020)116

Inclusive Cross Sections with Integral Transforms

Exploit integral properties of the response functions and closure to avoid explicit calculation of the final states (Lorentz Integral Transform **LIT**, **Euclidean**, ...)



Sobczyk et al, PRL127 (2021)



Lovato et al. PRX10 (2020)

Lepton-Nucleus scattering: Data

5

Transverse Sum Rule

 $S_T(q) \propto \langle 0 | \mathbf{j}^{\dagger} \mathbf{j} | 0 \rangle \propto \langle 0 | \mathbf{j}_{1b}^{\dagger} \mathbf{j}_{1b} | 0 \rangle + \langle 0 | \mathbf{j}_{1b}^{\dagger} \mathbf{j}_{2b} | 0 \rangle + \dots$



Observed transverse enhancement explained by the combined effect of two-body correlations and currents in the interference term

$$\langle \mathbf{j}_{1b}^{\dagger} \ \mathbf{j}_{1b} \rangle > 0$$

Leading one-body term

$$\langle \mathbf{j}_{1b}^{\dagger} \; \mathbf{j}_{2b} \; v_{\pi} \rangle \propto \langle v_{\pi}^2 \rangle > 0$$

Interference term



Transverse/Longitudinal Sum Rule Carlson *et al.* PRC65(2002)024002

Beyond Inclusive: Short-Time-Approximation

Short-Time-Approximation Goals:

- Describe electroweak scattering from A
 > 12 without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects



Subedi et al. Science320(2008)1475



Stanford Lab article



Short-Time-Approximation

Short-Time-Approximation:

- Based on Factorization
- Retains two-body physics
- Correctly accounts for interference



$$R(q,\boldsymbol{\omega}) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} \,\mathrm{e}^{i(\boldsymbol{\omega}+E_0)t} \,\langle 0|O^{\dagger}\,\mathrm{e}^{-iHt}\,O|0\rangle$$

$$O_i^{\dagger} e^{-iHt} O_i + O_i^{\dagger} e^{-iHt} O_j + O_i^{\dagger} e^{-iHt} O_{ij} + O_{ij}^{\dagger} e^{-iHt} O_{ij}$$

$$H \sim \sum_i t_i + \sum_{i < j} v_{ij}$$

STA: regime of validity

The typical (conservative estimate) energy (time) scale in a nucleus with A correlated nucleons in pairs is

$$\epsilon_{pair} \sim 20 \text{ MeV}$$
 (t ~ 1/ ϵ_{pair})

This sets a natural expansion parameter in the QE region characterized by ω_{QE}

 ϵ_{pair} / ω_{QE}

The STA neglects terms of order $O((\epsilon_{pair} / \omega_{QE})^2)$



Short-Time-Approximation

Short-Time-Approximation:

- Based on Factorization
- Retains two-body physics
- Response functions are given by the scattering from pairs of fully interacting nucleons that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides "more" exclusive information in terms of nucleon-pair kinematics via the Response Densities

Response Functions ∞ Cross Sections

$$R_{\alpha}(q,\omega) = \sum_{f} \delta\left(\omega + E_0 - E_f\right) \left|\langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle\right|^2$$

Response *Densities*

$$R(q,\omega) \sim \int \delta \left(\omega + E_0 - E_f\right) dP' dp' \mathcal{D}(p',P';q)$$

P' and *p*' are the CM and relative momenta of the struck nucleon pair

Transverse Response Density: *e*-⁴He scattering

Transverse Density q = 500 MeV/c



SP et al. PRC101(2020)044612

Transverse Response Density: two-body physics



q=500

e-⁴He scattering in the back-to-back kinematic



SP et al. PRC101(2020)044612

Helium-4: data & model dependence





Benchmark in ⁴He

SP et al. PRC101(2020)044612

¹²C Response Densities



Andreoli et al. Phys. Rev. C 110 (2024) 6, 064004 arXiv:2407.06986

¹²C response functions

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$

Andreoli et al. Phys. Rev. C 110 (2024) 6, 064004 arXiv:2407.06986





Andreoli *et al. Phys.Rev.C* 110 (2024) 6, 064004 <u>arXiv:2407.06986</u> Data From https://discovery.phys.virginia.edu/research/groups/qes-archive/index.html

Relativistic effects in e-³H scattering



Andreoli et al. Phys. Rev. C 105 (2022) 1, 014002



Relativistic effects in e-¹²C scattering



Andreoli et al. Phys. Rev. C 110 (2024) 6, 064004 arXiv:2407.06986

Relativistic corrections

Traditional non relativistic expansion of the covariant single nucleon electromagnetic current assumes initial and final nucleon momentum small.

$$egin{aligned} j^{\mu} &= ear{u}ig(m{p}'s'ig)ig(e_N\gamma^{\mu}+rac{i\kappa_N}{2m_N}\sigma^{\mu
u}q_{
u}ig)u(m{p}s)\ m{p}' &=m{p}+m{q} \end{aligned}$$

New paradigme where the relativistic correction is obtained expanding the covariant one-nucleon current for high values of momentum transfer, and small values of initial nucleon momentum p. This changed:

- 1. Expression of the one-body operator
- 2. Energy conserving delta function



With Ronen Weiss and Lorenzo Andreoli

Ronen Weiss Ed Jaynes Fellow at WashU

Implementation single nucleon current

1000

800

600



Response density vs relative and c.m. energy of the struck pair ⁴He Transverse response density at q = 700 MeV/c





Response density vs momenta of individual nucleons

Application to e-³H scattering



Three-body densities







Ronen Weiss, and Stefano Gandolfi *Phys.Rev.C* 108 (2023) 2, L021301

With AFDMC



⁴He Three-Body Momentum Distribution



Summary

Ab initio calculations of light nuclei yield a picture of nuclear structure and dynamics where many-body effects play an essential role to explain available data.



200150100 500 $e \, [\text{MeV}]$ Close collaborations between NP, LQCD, Pheno, Hep, Comp, Expt, ... are required to progress e.g., NP is represented in the Snowmass process

It's a very exciting time!

Transverse Density q = 500 MeV/c

2,000

1.000



Graham Chambers-Wall (WashU GS)



Garrett King (LANL PD)



Lorenzo Andreoli (ODU/JLab PD)

King *et al.* <u>PRC 110</u> (2024) 5, 054325; <u>Ann.Rev.Nucl.Part.Sci. 74</u> (2024) 343 Chambers-Wall, Gnech, King *et al.* <u>PRL 133</u> (2024) 21, 212501; <u>PRC 110</u> (2024) 5, 054316 Andreoli *et al.* <u>PRC 110</u> (2024) 6, 064004

Collaborators

WashU: Bub Chambers-Wall Flores Novario Piarulli Weiss

LANL: Carlson Gandolfi Hayes **King** Mereghetti JLab+ODU: **Andreoli Gnech** Schiavilla ANL: McCoy Lovato Wiringa UW/INT: Cirigliano Dekens Pisa U/INFN: Kievsky Marcucci Viviani Salento U: Girlanda Huzhou U: Dong Wang Fermilab: Gardiner Betancourt Rocco MIT: Barrow









¹²C cross sections: interpolation scheme

We have coarse grid in q for ¹²C. We use an interpolation scheme tested on He4.

$$I_{L/T}(\omega;\mathbf{q}) = rac{\int_0^\omega R_{L/T}(\omega';\mathbf{q})d\omega'}{\int_0^\infty R_{L/T}(\omega';\mathbf{q})d\omega'}$$







Lorenzo Andreoli et al. arXiv:2407.06986

¹²C response functions

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \, \left[v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$

Lorenzo Andreoli et al. arXiv:2407.06986



¹²C cross sections



Lorenzo Andreoli et al. arXiv:2407.06986

GENIE validation using e-scattering

Z = 2, A = 4, Beam Energy = 0.64 GeV, Angle = $60^{\circ} \pm 0.25^{\circ}$



- STA responses used to build the cross sections
- Cross sections are used to generate events in GENIE (a Monte Carlo neutrino event generator)
- Here, we use electromagnetic processes (for which data are available) to validate the generator

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$

Barrow, Gardiner, SP et al. PRD 103 (2021) 5, 052001

Back to back scattering and particle identity





tot

pp nn pp/all % pp/all % from momentum distributions nn/all %

GFMC SF STA: Benchmark & error estimate



Lorenzo Andreoli, et al. PRC 2021

