Effective Field Theory methods for nuclear interactions

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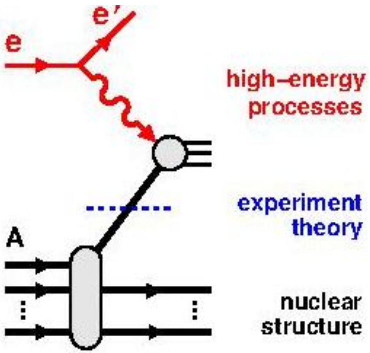

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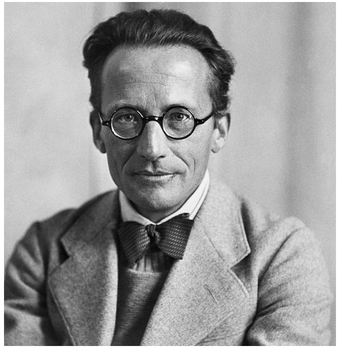
Introduction
The Microscopic Approach
Many-Nucleon Forces and Electroweak Currents
Phenomenological Approach to Many-Nucleon Operators
Quantum Monte Carlo Many-Body Computational Methods
Chiral Effective Field Theory Approach to Many-Nucleon Operators
Quantum Monte Carlo Many-Body Computational Methods
Applications to Electroweak Observables and Relevance of
Many-Nucleon Correlations

Microscopic (or *ab initio*) Description of Nuclei

Comprehensive theory that describes quantitatively and predictably nuclear structure and reactions

Requirements:

1. Accurate understanding of the interactions/correlations between nucleons in **paris**, **triplets**, ... (**two- and three-nucleon forces**)
2. Accurate understanding of the electroweak interactions of external probes (electrons, neutrinos, photons) with nucleons, correlated nucleon-pairs, … (**one- and two-body electroweak currents**)
3. **Computational methods** to solve the many-body nuclear problem of strongly interacting particles



Erwin Schrödinger

$$H\Psi = E\Psi$$

1. Many-nucleon Interactions

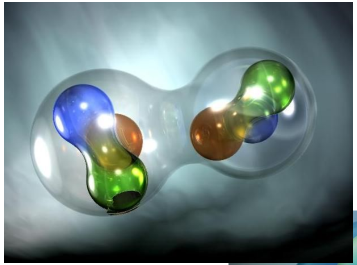
The energy of the nucleus is approximated by the many-body Hamiltonian

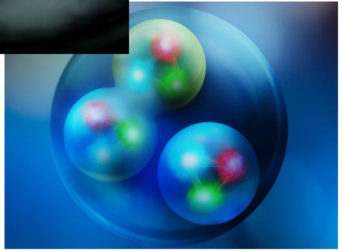
$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

where v_{ij} and V_{ijk} are two- and three-nucleon operators correlating nucleons in pairs and triplets.

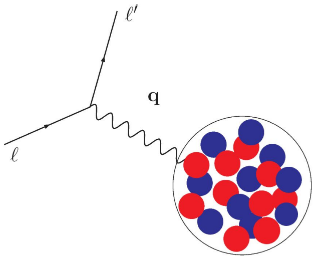
The derivation of these operators is based on experimental data fitting with parameters that subsume underlying QCD.

The non-relativistic description is justified by the observation that nucleon typical velocity inside the nucleus $v^2 \sim 0.05$.

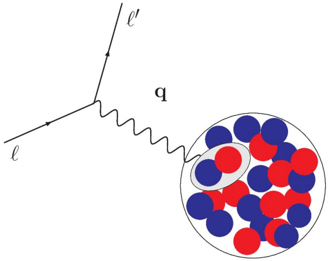




2. Many-nucleon Electroweak Currents



one-body



two-body

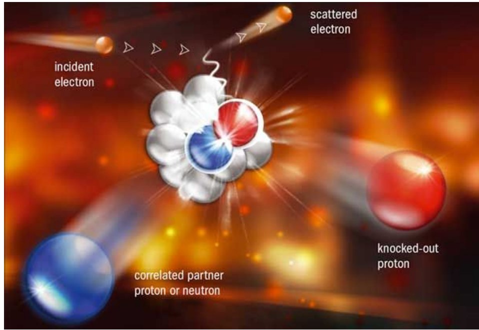
Nuclear Charge Operator

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots$$

Nuclear (Vector) Current Operator

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$

Many-nucleon electroweak currents describe the interaction of external electroweak probes (electrons, neutrinos, photons, …) with single nucleons and pairs of correlated nucleons.



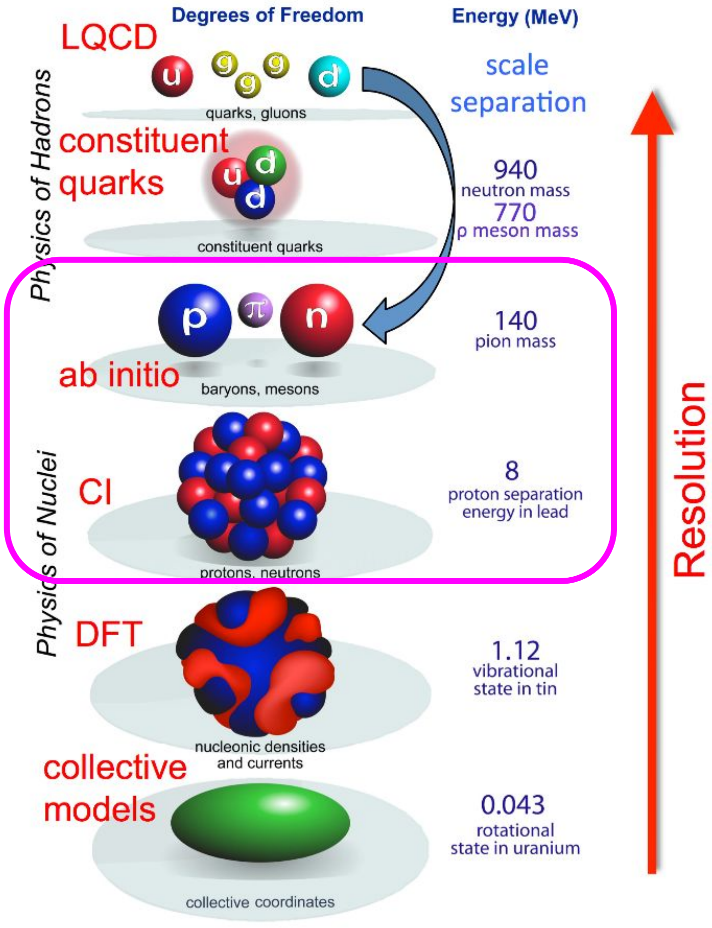
Many-nucleon forces & electroweak currents: history

- 1930s: Yukawa potential
- …
- 1950-1990: Highly sophisticated ``phenomenological” potential based on meson exchange theory
- 1990-: Effective field theory approach

From Quarks to Nuclei

QCD at low-energies is not perturbative. Hard to solve for nuclei with A>3 nucleons.

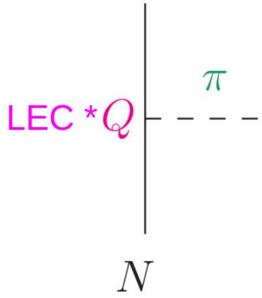
Chiral Effective field theories (XEFT) are low energy approximation of QCD, valid at values of momentum $Q \ll \Lambda_X \sim 1$ GeV (sets the hard scale where the approximation breaks down).



Chiral Effective Field Approach

XEFTs preserve the symmetries exhibited by QCD, especially the approximated chiral symmetry, to constraint the interactions among pions, nucleons, deltas, and external electroweak fields.

The pion couples with nucleons, deltas, … by power of its momentum, with a strength specified by unknown Low Energy Constants (LECs).



S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

Chiral Effective Field Approach

XEFTs provide Lagrangians π's, N's, Δ's, … interactions that are expanded in powers n of the perturbative (small) parameter Q/Λχ where the LECs are the coefficients of the expansion:

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} = & \boxed{\mathcal{L}_{\pi}^{(2)}\left(M_{\pi}, F_{\pi}\right) + \mathcal{L}_{\pi}^{(4)}\left(l_{1,\ldots,7}\right)} \\
& + \boxed{\mathcal{L}_{\pi N}^{(1)}\left(g_{A}\right) + \mathcal{L}_{\pi N}^{(2)}\left(m, c_{1,\ldots,7}\right) + \mathcal{L}_{\pi N}^{(3)}\left(d_{1,\ldots,23}\right) + \mathcal{L}_{\pi N}^{(4)}\left(e_{1,\ldots,118}\right)} \\
& + \boxed{\mathcal{L}_{\text{NN}}^{(0)}\left(C_{S}, C_{T}\right) + \mathcal{L}_{\text{NN}}^{(2)}\left(C_{1,\ldots,7}\right) + \mathcal{L}_{\text{NN}}^{(4)}\left(D_{1,\ldots,12}\right) + \mathcal{L}_{\pi NN}^{(1)}\left(D\right) + \ldots} \\
& + \boxed{\mathcal{L}_{\text{NNN}}^{(0)}\left(E\right) + \mathcal{L}_{\text{NNN}}^{(2)}\left(E_{1,\ldots,10}\right),}
\end{aligned}
$$

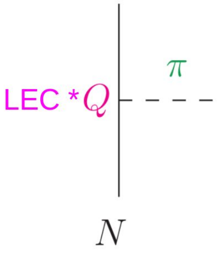
Λχ ~ 1 GeV sets the scale where the low-energy approximation breaks down.

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

Nuclear force at the lowest order

The nuclear force in the lowest orders in the chiral expansion is based on the effective Lagrangian

$$\mathcal{L}^{(0)} = \frac{1}{2}\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - \frac{1}{2}M^2\boldsymbol{\pi}^2 + N^\dagger\left[i\partial_0 + \frac{g_A}{2F}\boldsymbol{\tau}\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi} - \frac{1}{4F^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})\right]N$$
$$- \frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger\vec{\sigma}N)(N^\dagger\vec{\sigma}N) + \ldots\,,$$



Chiral Effective Field Approach

Due to the chiral expansion, many-nucleon operators can in principle be derived at the desired degree of accuracy in perturbative expansion parameter $(Q/\Lambda\chi)^n$ with a theoretical error set by the terms left out with with k>n. This is called truncation error.

$$\frac{1}{1-x} = \sum_{n=0} x^n = 1 + x + x^2 + x^3 \ldots$$

Perturbative techniques are then restored in the low-energy regime.

ChiEFT : $x \sim Q/\Lambda\chi$
Large N_C : $x \sim 1/N_C$
…

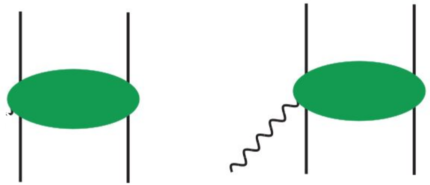
S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

Chiral Effective Field Approach

The systematic expansion in the perturbative (small) parameter Q/Λχ naturally gives the observed scaling of the many-body operators:

$$\langle \mathscr{O} \rangle_{1-\text{body}} > \langle \mathscr{O} \rangle_{2-\text{body}} > \langle \mathscr{O} \rangle_{3-\text{body}}$$

Many-nucleon interactions and currents derived consistently within the same theoretical framework.



The LECs are unknown and increase with increasing n. Need a strategy to determine the LECs without impinging on the predictive power of the theory.

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

1. Many-nucleon interactions

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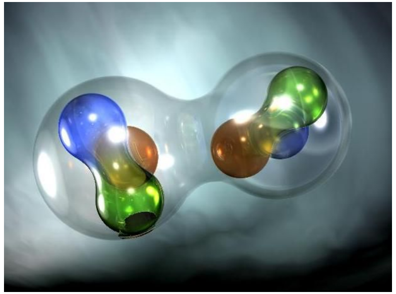


Strong vertices implied by Chiral EFT: Q^1

$$H_{\pi NN} = \frac{g_A}{F_\pi}\int \mathrm{d}\mathbf{x}\, N^\dagger(\mathbf{x})\, [\boldsymbol{\sigma}\cdot\nabla\pi_a(\mathbf{x})]\, \tau_a\, N(\mathbf{x}) \longrightarrow V_{\pi NN} = -i\frac{g_A}{F_\pi}\frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{\sqrt{2\,\omega_k}}\tau_a \sim Q^1 \times Q^{-1/2}$$

$$H_{\pi N\Delta} = \frac{h_A}{F_\pi}\int \mathrm{d}\mathbf{x}\, \Delta^\dagger(\mathbf{x})\, [\mathbf{S}\cdot\nabla\pi_a(\mathbf{x})]\, T_a\, N(\mathbf{x}) \longrightarrow V_{\pi N\Delta} = -i\frac{h_A}{F_\pi}\frac{\mathbf{S}\cdot\mathbf{k}}{\sqrt{2\,\omega_k}}T_a \sim Q^1 \times Q^{-1/2}$$

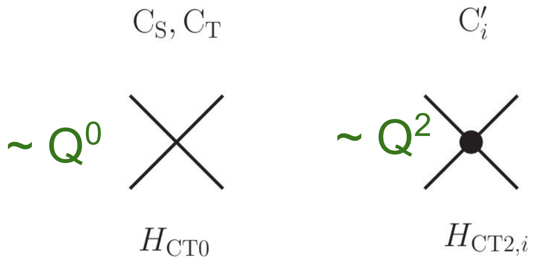
Known LECs

$$\pi_a(\mathbf{x}) = \sum_{\mathbf{p}} \frac{1}{\sqrt{2\,\omega_p}}\left[c_{\mathbf{p},a}\, \mathrm{e}^{i\mathbf{p}\cdot\mathbf{x}} + \mathrm{h.c.}\right]$$ Pion field

$$N(\mathbf{x}) = \sum_{\mathbf{p},\sigma\tau} b_{\mathbf{p},\sigma\tau}\, \mathrm{e}^{i\mathbf{p}\cdot\mathbf{x}}\chi_{\sigma\tau}\ ,$$ Nucleon field

$$\Delta(\mathbf{x}) = \sum_{\mathbf{p},\sigma_\Delta\tau_\Delta} d_{\mathbf{p},\sigma_\Delta\tau_\Delta}\, \mathrm{e}^{i\mathbf{p}\cdot\mathbf{x}}\chi_{\sigma_\Delta\tau_\Delta}$$ Delta field

Strong vertices implied by Chiral EFT: Q^0 and Q^2



$$H_{CT0} = \sum_{\alpha=S,T} \frac{C_\alpha}{2} \int d\mathbf{x} \, [N^\dagger(\mathbf{x}) \, \boldsymbol{\Gamma}_\alpha \, N(\mathbf{x})] \cdot [N^\dagger(\mathbf{x}) \boldsymbol{\Gamma}_\alpha N(\mathbf{x})] \quad \longrightarrow \quad V_{NN,0} = C_S + C_T \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \sim Q^0$$

$$\boldsymbol{\Gamma}_S = 1 \, , \quad \boldsymbol{\Gamma}_T = \boldsymbol{\sigma}$$

Unknown LECs

$$H_{CT2,1} = C'_1 \int d\mathbf{x} \left[\left[N^\dagger \nabla N \right]^2 + \left[(\nabla N)^\dagger \, N \right]^2 \right] \quad \longrightarrow \quad V_{NN,2} = -2 \, C'_1 (\mathbf{p}_1 \cdot \mathbf{p}_2 + \mathbf{p}'_1 \cdot \mathbf{p}'_2) \sim Q^2$$

...

Amplitude in time-ordered perturbation theory

The many-nucleon operators are obtained by direct evaluation of the transition amplitude.

$$v_{ij} \propto \langle N'N' \mid T \mid NN \rangle = \langle N'N' \mid \boxed{H_1} \sum_{n=1}^{\infty} \left(\frac{1}{E_i - \boxed{H_0} + i\,\eta} H_1 \right)^{n-1} \mid NN \rangle$$

Interacting π , N, and Δ, Hamiltonians implied by the chiral Lagrangians

Free π , N, and Δ Hamiltonians

After insertion of complete sets of eigenstates of H_0, the transition amplitude reads

$$\langle f \mid T \mid i \rangle = \langle f \,|H_1|\, i \rangle + \sum_{I_1} \langle f \,|H_1|\, I_1 \rangle \frac{1}{E_i - E_1 + i\,\eta} \langle I_1 \,|H_1|\, i \rangle$$
$$+ \sum_{I_1, I_2} \langle f \,|H_1|\, I_2 \rangle \frac{1}{E_i - E_2 + i\,\eta} \langle I_2 \,|H_1|\, I_1 \rangle \frac{1}{E_i - E_1 + i\,\eta} \langle I_1 \,|H_1|\, i \rangle + \ldots$$

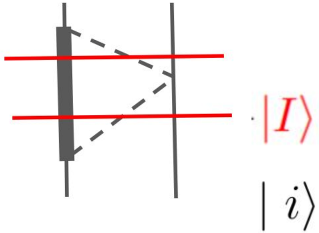
$$T^{(n)} \sim Q^n$$

Power counting

The power counting is a naive way to estimate the size of each contribution to the transition amplitude

$$\underbrace{\left(\prod_{i=1}^{N} Q^{\alpha_i-\beta_i/2}\right)}_{H_1\text{scaling}} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$

$\alpha_i = \#$ of derivatives (momenta) in H_1;
$\beta_i = \#$ of π's;
$N = \#$ of vertices; $N-1 = \#$ of intermediate states;
$L = \#$ of loops



$$H_1 \text{ scaling} \sim \underbrace{Q^1}_{H_{\pi N\Delta}} \times \underbrace{Q^1}_{H_{\pi\pi NN}} \times \underbrace{Q^1}_{H_{\pi N\Delta}} \times Q^{-2} \sim Q^1$$

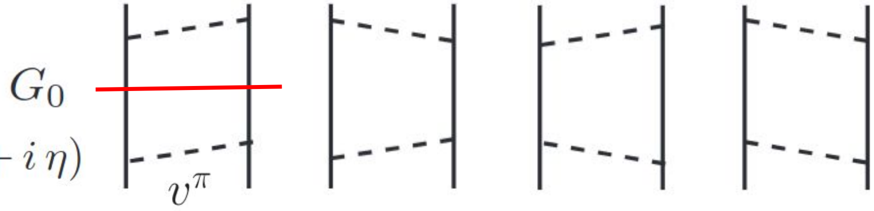
$$\text{denominators} \sim \frac{1}{E_i - H_0}|I\rangle \sim \frac{1}{2\,m_N - (m_\Delta + m_N + \omega_\pi)}|I\rangle = -\frac{1}{m_\Delta - m_N + \omega_\pi}|I\rangle \sim \frac{1}{Q}|I\rangle$$

neglecting nucleons' kinetic energies; *static limit.*

$$Q^2 = Q^1 \times Q^{-2} \times Q^3$$

Technicalities I: reducible diagrams

$$G_0 = 1/(E_i - E_I + i\,\eta)$$



$$|\Psi\rangle \simeq |\phi\rangle + \frac{1}{E_i - H_0} v^\pi |\phi\rangle + \ldots$$

Reducible diagrams are accounted for in the iterated solution of the Schrodinger Eq.

$$v + v\,G_0\,v + v\,G_0\,v\,G_0\,v + \ldots$$

Two strategies have been developed to avoid double counting of these terms:

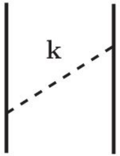
1. The method of the unitary transformation (Bochum-Bonn group) Epelbaum at al. Nucl. Phys. A 671, 295 (2000); Nucl. Phys. A 714,535 (2003); Nucl. Phys. A 747, 362 (2005); Epelbaum, Krebs, Patrick, Frontiers in Physics, 8 (2020); Kölling *et al.* PRC80(2009)045502 & PRC84(2011)054008
2. Order by order subtraction of the Lippmann-Schwinger terms from the transition amplitude (JLab-Pisa group) SP *et al.* PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001, PRC87(2013)014006

From amplitudes to potentials

$$
\begin{aligned}
&Q^0 \qquad v^{(0)} = T^{(0)}\,, \\
&Q^1 \qquad v^{(1)} = T^{(1)} - \left[v^{(0)}\,G_0\,v^{(0)}\right]\,, \\
&Q^2 \qquad v^{(2)} = T^{(2)} - \left[v^{(0)}\,G_0\,v^{(0)}\,G_0\,v^{(0)}\right] \\
&\qquad\qquad\qquad\quad - \left[v^{(1)}\,G_0\,v^{(0)} + v^{(0)}\,G_0\,v^{(1)}\right] \\
&\qquad \ldots
\end{aligned}
$$

Lippmann-Schwinger terms

Two-nucleon potential at leading order: Q^0



$$v^{(0)} = \langle N'N' \mid H_{\mathrm{CT},0} \mid NN\rangle + \sum_{|I\rangle} \langle N'N' \mid H_{\pi NN} | \, I\rangle \frac{1}{E_i - E_I} \langle I \, | H_{\pi NN} \mid NN\rangle$$

$$v^{(0)} = v^{\mathrm{S}} + v^{\pi} = C_S + C_T \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{g_A^2}{F_\pi^2} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k} \, \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

We can recast the LO potential as we did for AV18.

$$v_{12} = \sum_p v_{12}^p(r) \, O_{12}^p$$

$$\boxed{O_{12} = 1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, S_{12} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}$$

Operators at LO

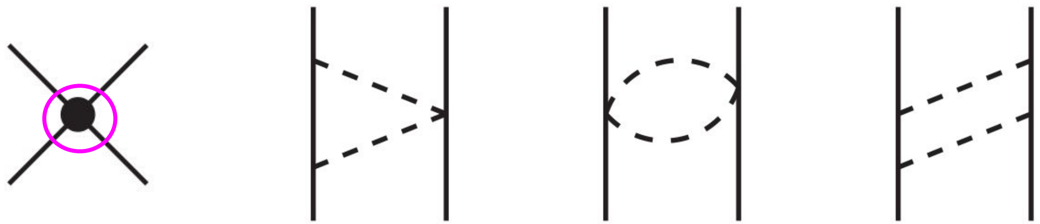
$$\boxed{\begin{array}{l} v_{12}^{p,\pi}(r) \propto Y_\pi(r); T_\pi(r) \\ v_{12}^{p,S}(r) \propto \delta(r) \end{array}}$$

Radial functions at LO

Long range: need to remove singularities at r=0

Short range: need to pick a representation of the delta fnc

Two-nucleon potential at NLO (no Delta's): Q^2



7 new unknown LECs that multiply contact terms with 2 derivatives acting on the nucleon fields (∇N).

Operators at NLO

$$v_{12} = \sum_p v_{12}^p(r)\, O_{12}^p$$

$$\boxed{O_{12} = [1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, S_{12}, \mathbf{L} \cdot \mathbf{S}] \otimes [1, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]}$$

LECs are determined through fits to the NN scattering data, for different values of cutoff used to remove singularities at small r (or large q), and for different representations of the contact interactions.

Technicalities II: Cutoff

Operators in ChiEFT have a power law behaviour in Q.

Cutoff are introduced to remove divergences at large Q.

For example, one could use:

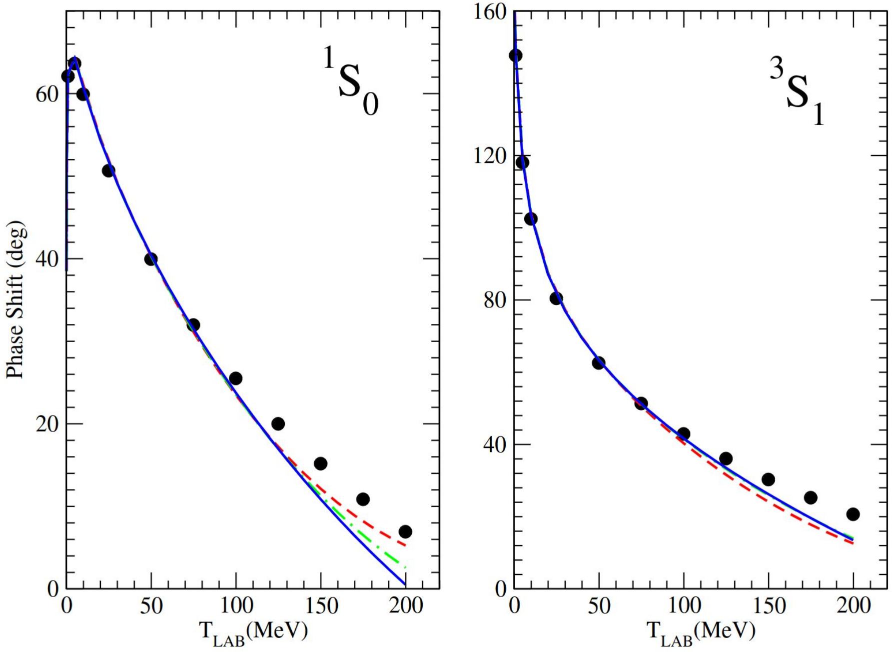
$$C_\Lambda = e^{-(Q/\Lambda)^n}$$

However, n should be large enough so as to not generate spurious contributions:

$$C_\Lambda \sim 1 - \left(\frac{Q}{\Lambda}\right)^n + \dots$$

For each value of the cutoff lambda, one should re-fit the LECs to the data.

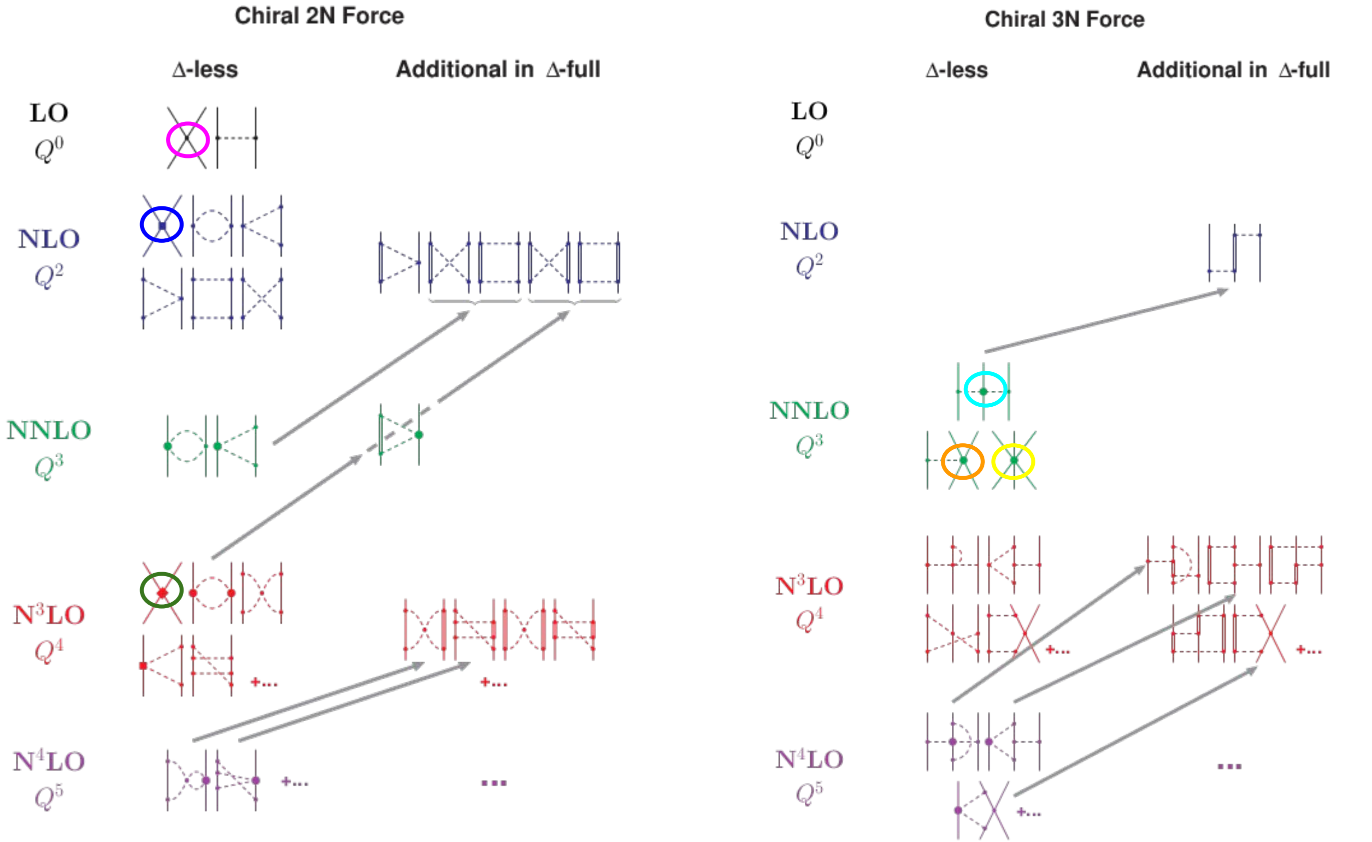
A simple nucleon-nucleon potential at N2LO



Cutoff ~ $\exp(-2\ Q^4/\Lambda^4)$, (cutting off momenta $Q > 3\text{–}4\ m_\pi$), Λ=500, 600, and 700 MeV

SP et al. PRC80(2009)034004

Hierarchy of the chiral many-nucleon interaction



$$m_\Delta - m_N \sim 2m_\pi$$

Including the delta improves the convergence of the chi expansion.

LECs are `saturated by the Delta'.

LECs: 2, 7, 15, 3, 1, 1, …

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ChiEFT interactions: references

ChiEFT many-nucleon interactions have been extensively developed in the past two decades. A very incomplete list of references is provided below:

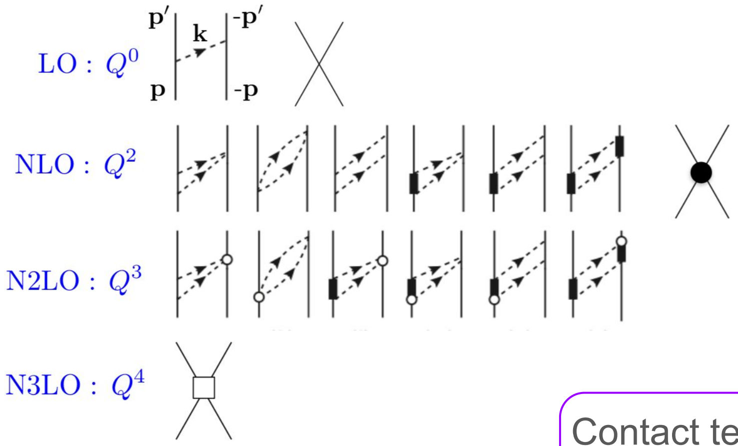
Reinert, Krebs, Epelbaum (Springer Proc. Phys. 238497-501 (2020)
Entem and Machleid (Phys.Rept.503(2011)1)
Ekström et al. (PRL 110, 192502 2013;JPG 42, 034003 2015)
Epelbaum et al. (PRL.112, 102501, 2014; EPJ A 51, 53 2015; PRL. 115, 122301, 2015)
Entem et al. (PRC 91, 014002,2015; PRC 92, 064001 2015, PRC 96, 024004 (2017)

Many-nucleon ChiEFT interactions suitable for Quantum Monte Carlos methods have been developed in:

Gezerlis et al. (PRL 111, 032501 2013, PRC 90, 054323 2014); Lynn et al. (PRL 113,192501, 2014)
Piarulli et al. (PRC 91, 024003 2015,PRC 94, 054007 2016)

Norfolk potentials

Norfolk Two-body Potentials NV2



26 LECs fitted to np and pp Granada database (2700-3700 data points) with a chi-square/datum ~1

NV2-I fits data up to 125 MeV

NV2-II fits data up to 200 MeV

Contact terms functional representation

$$C_{R_S}(r) = \frac{1}{\pi^{3/2} R_S^3} e^{-(r/R_S)^2}$$

Singularities are removed using

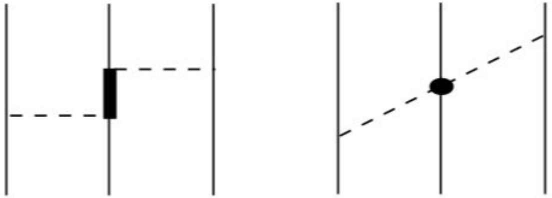
$$C_{R_L}(r) = 1 - \frac{1}{(r/R_L)^6 \, e^{(r-R_L)/a_L} + 1}$$

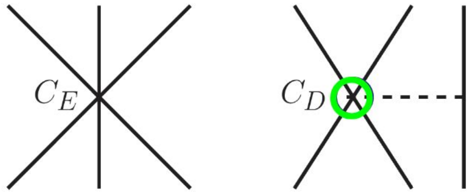
R_L, R_S cutoff parameters

NV2-a: 1.2, 0.8 fm

NV2-b: 1.0, 0.7 fm

Norfolk Three-body Potentials NV3

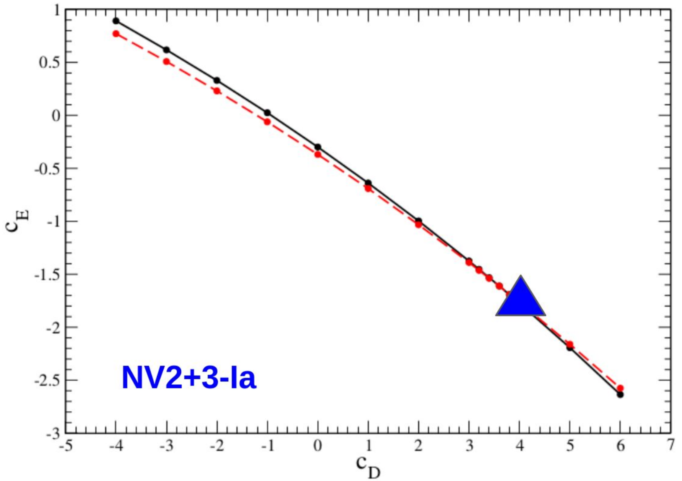




3 LECs fixed on pion-nucleon observables

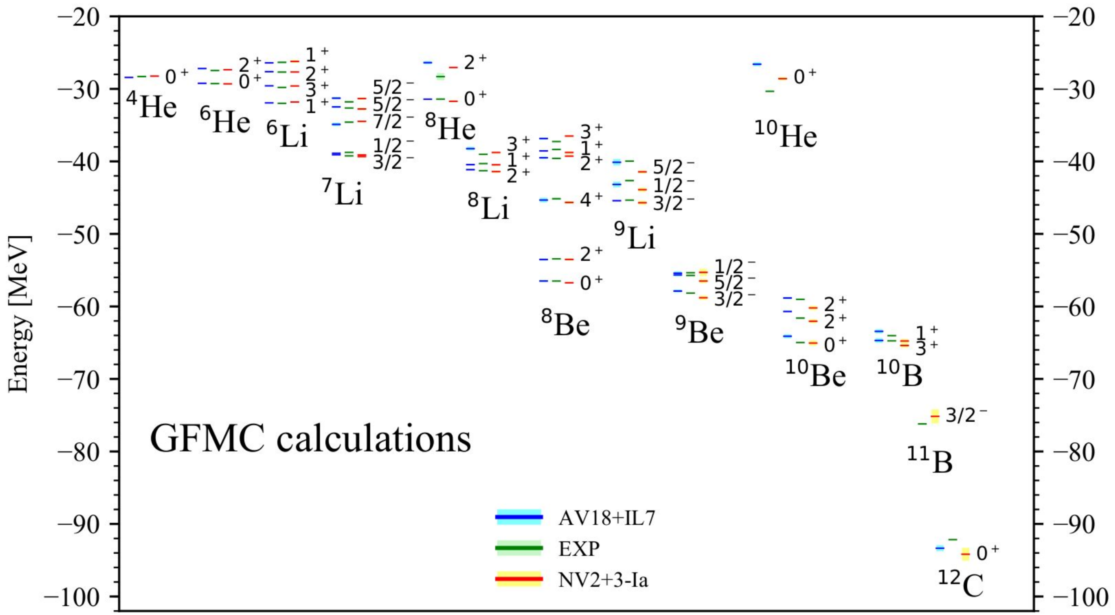
LECs c_D and c_E are fitted to:

trinucleon B.E. and ***nd* doublet scattering length**



Baroni *et al.* PRC98(2018)044003

Energies with Norfolk potentials



Piarulli *et al.* PRL120(2018)052503

Norfolk interactions are determined by NN and 3N data.

ChiEFT many-nucleon interactions

Chiral interactions constrained only in the 2N and 3N sector explain the spectra of A<= 12 nuclei within 1-2% of expt data. This result can be regarded as a validation of the microscopic approach of nuclear properties emerging from nucleonic dynamics.

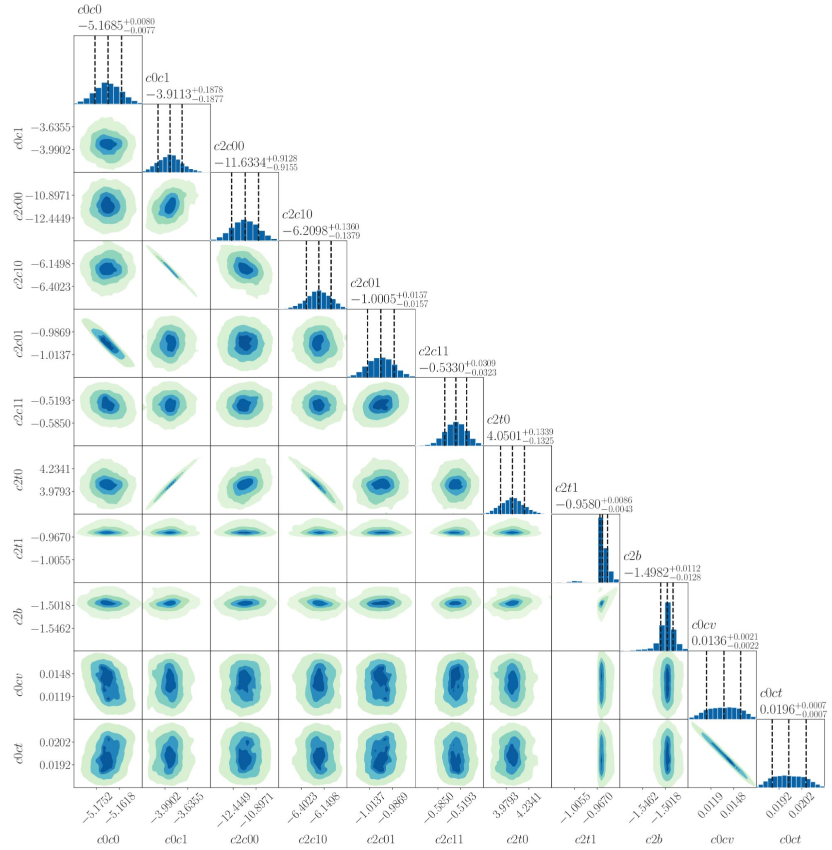
Robust uncertainty quantification of the theoretical error is possible.

The interactions are expressed in terms of OPE, TPE and contact like contributions with strengths specified by LECs.

The determination of LECs require development of strategies to efficiently optimize the interactions.

The presence of regulators and functional representations of the short-range interactions slightly spoils the claimed consistency of chiral interactions and currents.

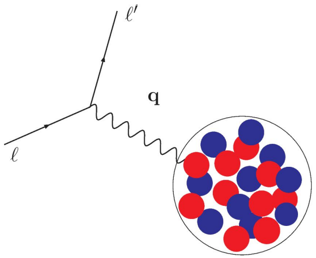
Optimization of Nuclear Two-body Interactions



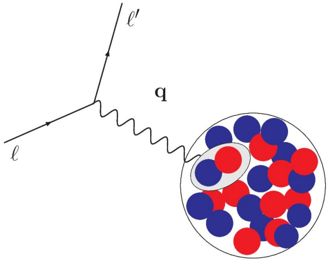
Development and Optimization of two-body interactions based on Bayesian methods

Jason Bub *et al.* arxiv:2408.02480 (2024)

2. Many-nucleon Electroweak Currents



one-body



two-body

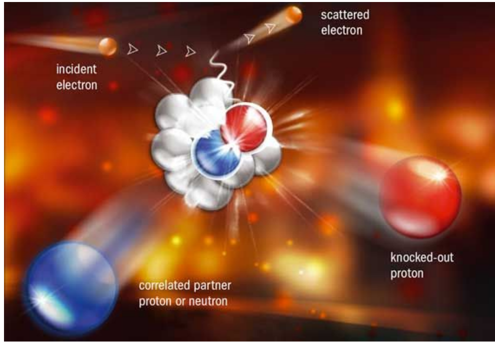
Nuclear Charge Operator

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots$$

Nuclear (Vector) Current Operator

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$

Many-nucleon electroweak currents describe the interaction of external electroweak probes (electrons, neutrinos, photons, …) with single nucleons and pairs of correlated nucleons.



Electromagnetic currents in ChiEFT

The construction of the current operators follows the prescription outlined for the strong interactions.

$$v_\gamma = A^0\,\rho - \mathbf{A}\cdot\mathbf{j} \propto \langle N'N' \mid T \mid NN;\gamma\rangle = \langle N'N' \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\,\eta} H_1 \right)^{n-1} \mid NN;\gamma\rangle$$

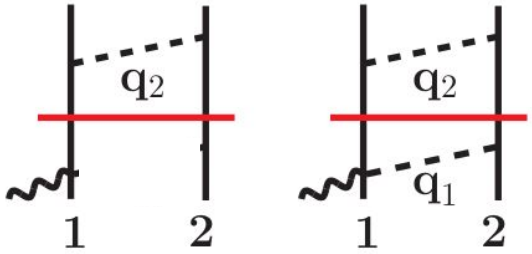
Terms generated in the Lippmann-Schwinger equation are subtracted from the transition amplitude:

Q^{-3} $\quad v_\gamma^{(-3)} = T_\gamma^{(-3)}$

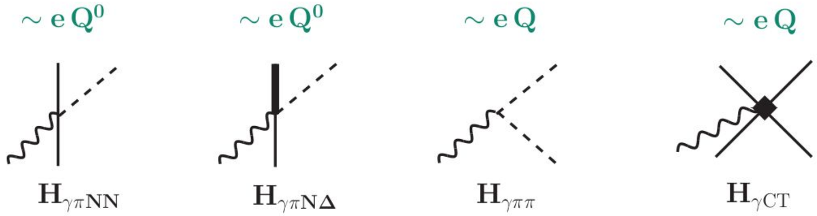
Q^{-2} $\quad v_\gamma^{(-2)} = T_\gamma^{(-2)} - \left[v_\gamma^{(-3)}\, G_0\, v^{(0)} + v^{(0)}\, G_0\, v_\gamma^{(-3)} \right] ,$

Q^{-1} $\quad v_\gamma^{(-1)} = T_\gamma^{(-1)} - \left[v_\gamma^{(-3)}\, G_0\, v^{(0)}\, G_0\, v^{(0)} + \text{permutations} \right] - \left[v_\gamma^{(-2)}\, G_0\, v^{(0)} + v^{(0)}\, G_0\, v_\gamma^{(-2)} \right] ,$

…



Electromagnetic vertices implied by Chiral EFT



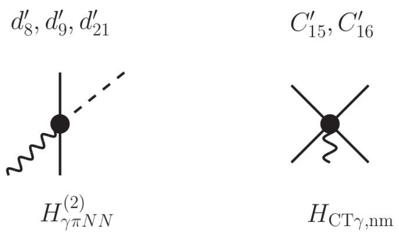
``Minimal” Hamiltonians

Minimal Hamiltonians are obtained by minimal substitution in the π- and N-derivative couplings (p → p + e A).

$$
\begin{aligned}
\nabla \pi_{\mp}(\mathbf{x}) &\quad\rightarrow\quad [\nabla \mp i e \mathbf{A}(\mathbf{x})]\, \pi_{\mp}(\mathbf{x}) \\
\nabla N(\mathbf{x}) &\quad\rightarrow\quad [\nabla - i e\, e_N \mathbf{A}(\mathbf{x})]\, N(\mathbf{x}) \,, \qquad e_N = (1+\tau_z)/2
\end{aligned}
$$

This ensures consistency between the strong and electromagnetic sectors. Moreover, the strong and the corresponding minimal electromagnetic vertices share the same LECs.

Electromagnetic vertices implied by Chiral EFT



``Non-minimal” Hamiltonians

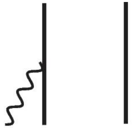
Non-minimal Hamiltonians are generated by electromagnetic tensor interactions with the hadrons.

$$F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$$

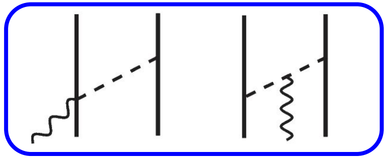
They involve 5 new unconstrained LECs.

Vector electromagnetic current at N3LO

LO : $\mathbf{j}^{(-2)} \sim \mathbf{eQ^{-2}}$

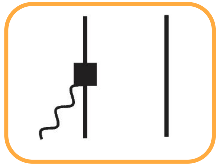


NLO : $\mathbf{j}^{(-1)} \sim \mathbf{eQ^{-1}}$



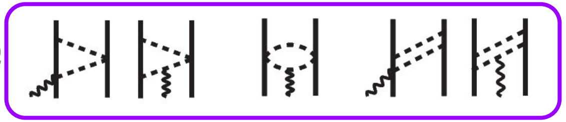
OPE long range

N²LO : $\mathbf{j}^{(-0)} \sim \mathbf{eQ^{0}}$



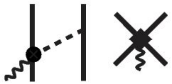
One-body relativistic corrections

N³LO: $\mathbf{j}^{(1)} \sim \mathbf{eQ}$



TPE intermediate range

unknown LEC's

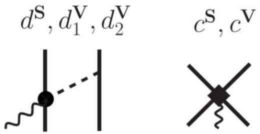


There are no three-body currents at this order.

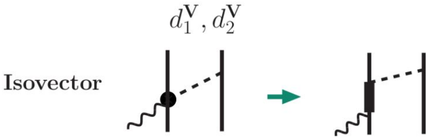
5 LEcs need to be determined in order to constrain the current operator.

Analogous expansions have been derived for the electromagnetic charge operator. There, two-nucleon contributions appear at N3LO.

A fitting strategy



d^S, d_1^V, and d_2^V could be determined by $\pi\gamma$-production data on the nucleon



$d_2^V = 4\mu^* h_A / 9 m_N (m_\Delta - m_N)$ and
$d_1^V = 0.25 \times d_2^V$
assuming Δ-resonance saturation

Left with 3 LECs: Fixed in the $A = 2-3$ nucleons' sector

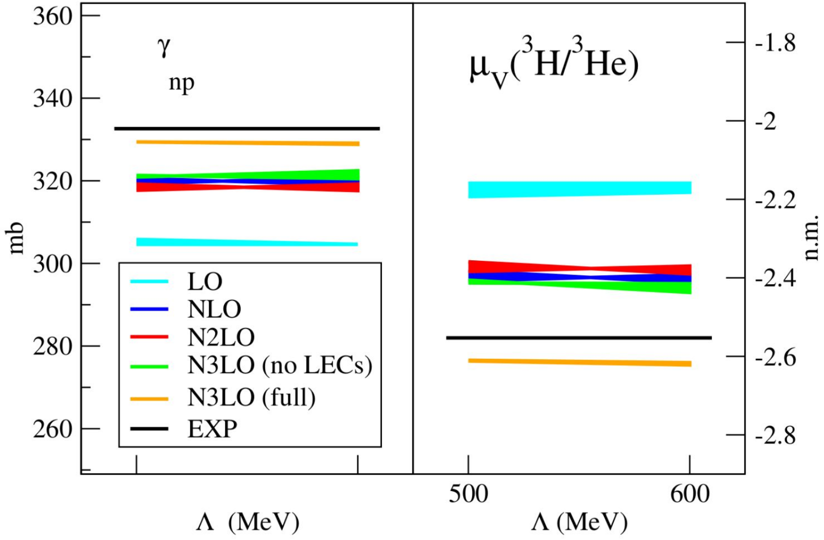
- Isoscalar sector:
  - d^S and c^S from EXPT μ_d and $\mu_S(^3\mathrm{H}/^3\mathrm{He})$
- Isovector sector:
  - c^V from EXPT $npd\gamma$ xsec.

    or
  - c^V from EXPT $\mu_V(^3\mathrm{H}/^3\mathrm{He})$ m.m.

  - Regulator $C(\Lambda) = exp(-(p/\Lambda)^4)$ with $\Lambda = 500 - 600$ MeV

An order by order prediction

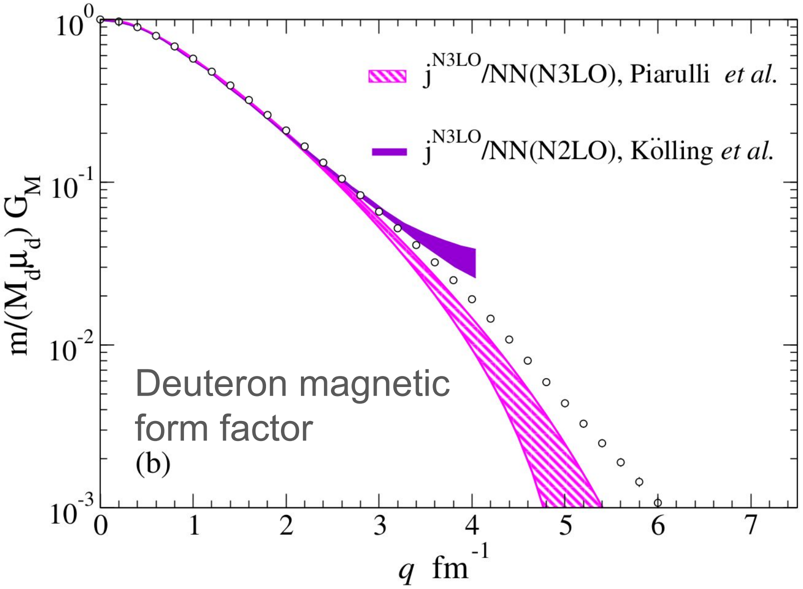
np capture x-section/ μ_V of $A = 3$ nuclei
bands represent nuclear model dependence [NN(N3LO)+3N(N2LO) – AV18+UIX]



* $npd\gamma$ x-section and $\mu_V(^3\mathrm{H}/^3\mathrm{He})$ m.m. are within 1% and 3% of EXPT
* negligible dependence on the cutoff

$$\text{Observables} \propto \langle \Psi_f | \mathbf{j} | \Psi_i \rangle$$

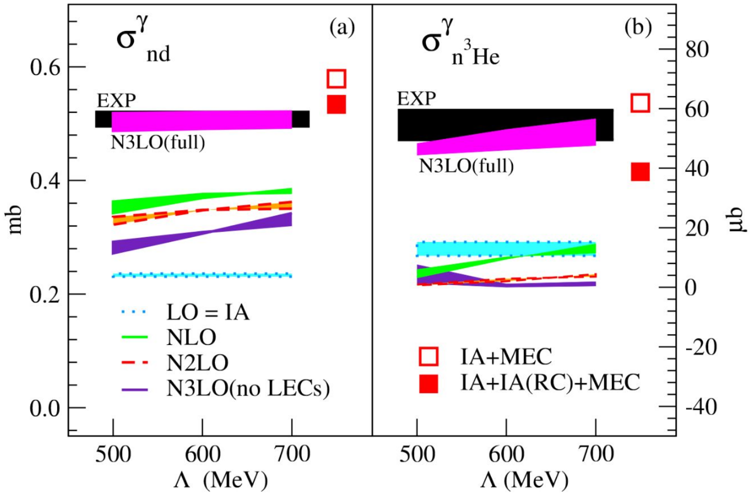
Error bands



$$\text{Observable} \propto \langle \Psi_f | \mathbf{j} | \Psi_i \rangle$$

PRC86(2012)047001 & PRC87(2013)014006

Phenomenological vs Chiral Approach



Girlanda *et al.* PRL105(2010)232502

$$\text{Observables} \propto \langle \Psi_f | \mathbf{j} | \Psi_i \rangle$$

Power Counting doesn't know about suppressions/cancellations at LO

Suppression at LO is a nuclear feature due to "pseudo-orthogonality" of initial and final wave functions

Observable sensitive to many-body components

Electromagnetic currents from ChiEFT

ChiEFT currents consist of OPE, TEP, and contact like contributions. At N3LO, there are 5 LECs to be determined.

OPE EM vector current appears at NLO, OPE charge operators appear at N3LO.

ChiEFT currents are qualitatively in agreement with phenomenological, currents. However, ChiEFT provides a framework to systematically improve them

However there are challenges and open questions:

- numbers of LECs increases as higher orders are accounted for;
- the presence of cutoffs makes is hard to have a consistent description of nuclear interactions and currents;
- Simple power counting not sufficient to predict the correct scaling of the operators; nuclear structure effects can lead to non trivial suppressions and cancellations.

3. Many-body Nuclear Problem

http://exascaleage.org/np/

Nuclear Many-body Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

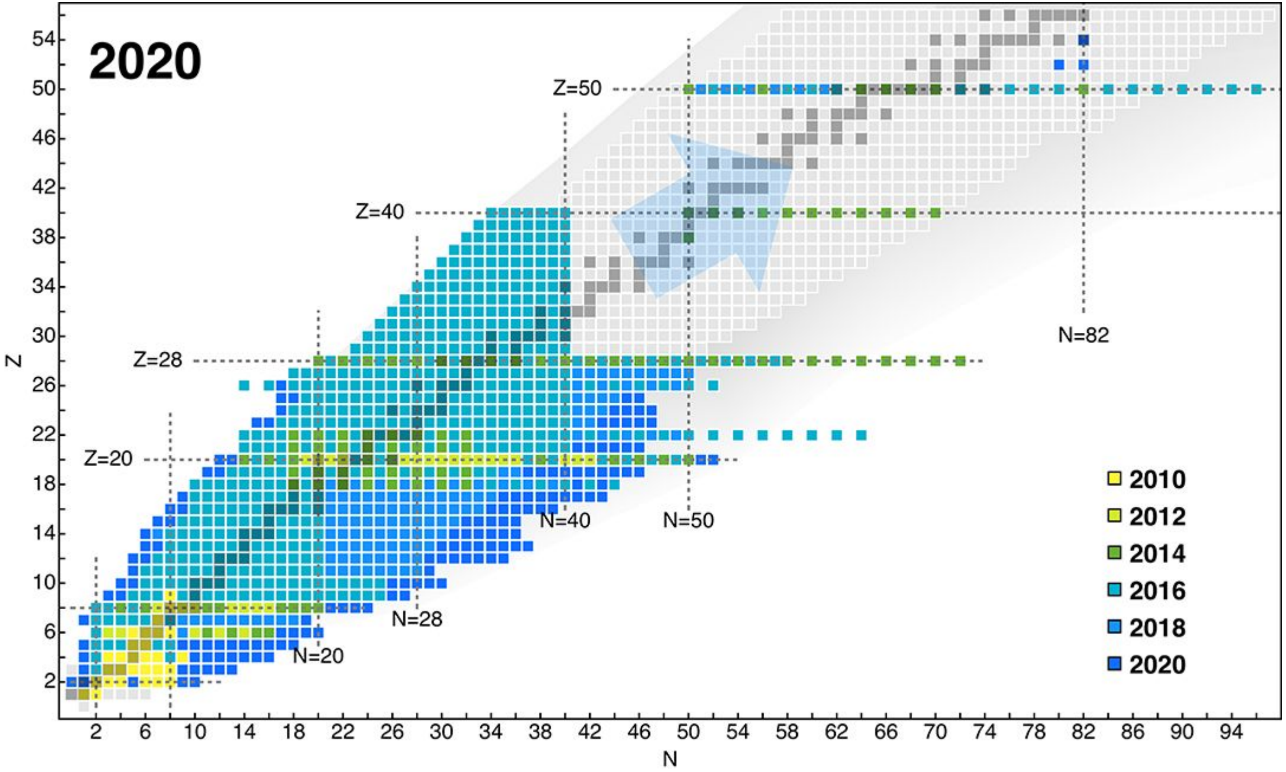
$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A, s_1, s_2, \ldots, s_A, t_1, t_2, \ldots, t_A)$$

Ψ are spin-isospin vectors in 3A dimensions with $2^A \times \frac{A!}{Z!(A-Z)!}$ components

^4He : 96
^6Li : 1280
^8Li : 14336
^{12}C : 540572

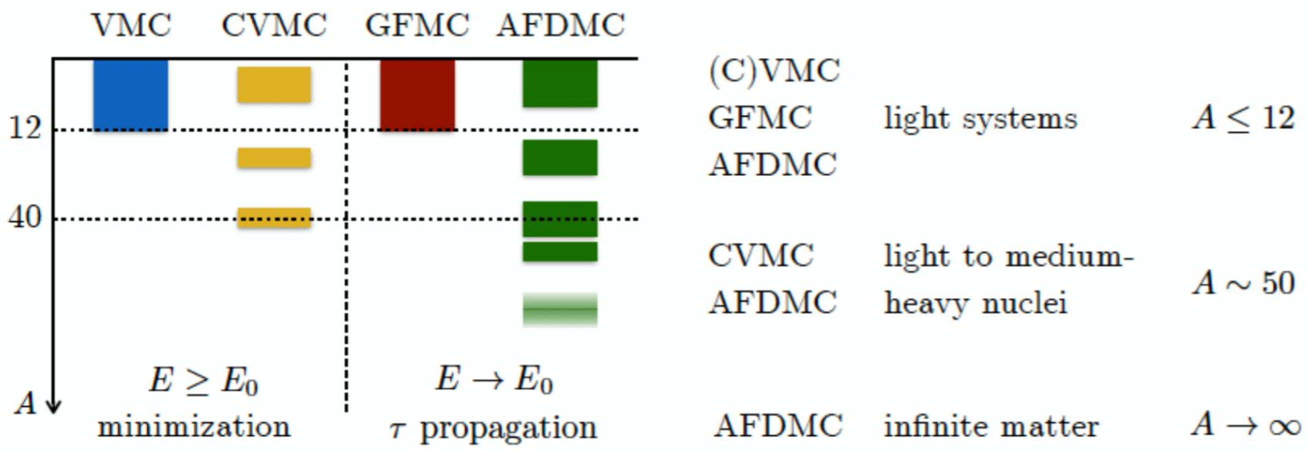
Develop Computational Methods to solve (numerically) exactly or within approximations that are under control the many-body nuclear problem

Current Status



H. Hergert
Front. Phys.
07 October 2020

Quantum Monte Carlo methods in a nutshell



Class of computational methods based on stochastic methods.
Required interactions in r-space.

J. Carlson et al., RMP. 87, 1067 (2015)

Variational Monte Carlo Methods

Minimize the expectation value of the nuclear Hamiltonian: H = T +υ_{ij} + V_{ijk}

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using the trial wave function:

$$|\Psi_V\rangle = \left[\mathcal{S} \prod_{i<j} \left(1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}\right) \right] \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

* single-particle $\Phi_A(JMTT_3)$ is fully antisymmetric and translationally invariant
* central pair correlations $f_c(r)$ keep nucleons at favorable pair separation
* pair correlation operators U_{ij} reflect influence of υ_{ij} (AV18)
* triple correlation operators U_{ijk} reflect the influence of V_{ijk} (IL7)

Lomnitz-Adler, Pandharipande, and Smith NPA361(1981)399

Wiringa, PRC43(1991)1585

Diffusion Quantum Monte Carlo

Diffusion Monte Carlo methods [are used to project the exact ground state of a given Hamiltonian via a propagation in imaginary time.

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = (H - E_T)|\Psi(t)\rangle$$

$$|\Psi(t)\rangle = e^{-i(H-E_T)t}|\Psi_T\rangle$$

Real time

$$-\frac{\partial}{\partial \tau}|\Psi(\tau))\rangle = (H - E_T)|\Psi(\tau)\rangle$$

$$|\Psi(\tau)\rangle = e^{-(H-E_T)\tau}|\Psi_T\rangle$$

Imaginary time

$$|\Psi_T\rangle = \sum_{i=1}^{\infty} c_i|\psi_i\rangle$$

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle \propto c_0\psi_0.$$

Orthonormal basis of H

Green's Function Monte Carlo

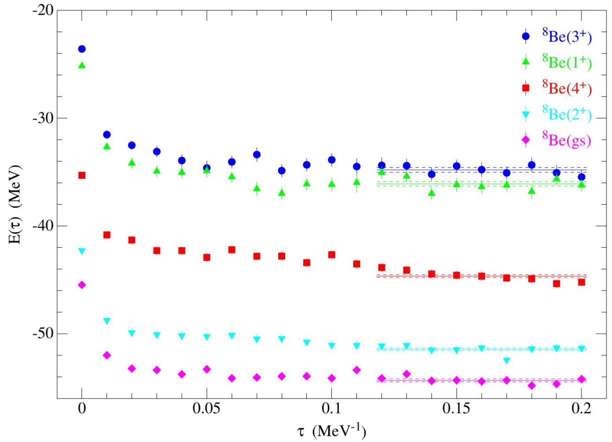
$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau]a_n\psi_n$$

$$\Psi(\tau \to \infty) = a_0\psi_0$$

The time evolution is carried out in small steps in imaginary time

$$|\Psi(\tau)\rangle = \left[e^{-(H-E_0)\Delta\tau}\right]^n |\Psi_T\rangle$$

This procedure allows to further improve the variational wave function by removing excited states contaminations.



Wiringa *et al.* PRC62(2000)014001

Expectation values and mixed estimates

Ideally the evaluation of operators would be done between two propagated waves as

$$\langle O(\tau)\rangle = \frac{\langle\Psi(\tau)|O|\Psi(\tau)\rangle}{\langle\Psi(\tau)|\Psi(\tau)\rangle}$$

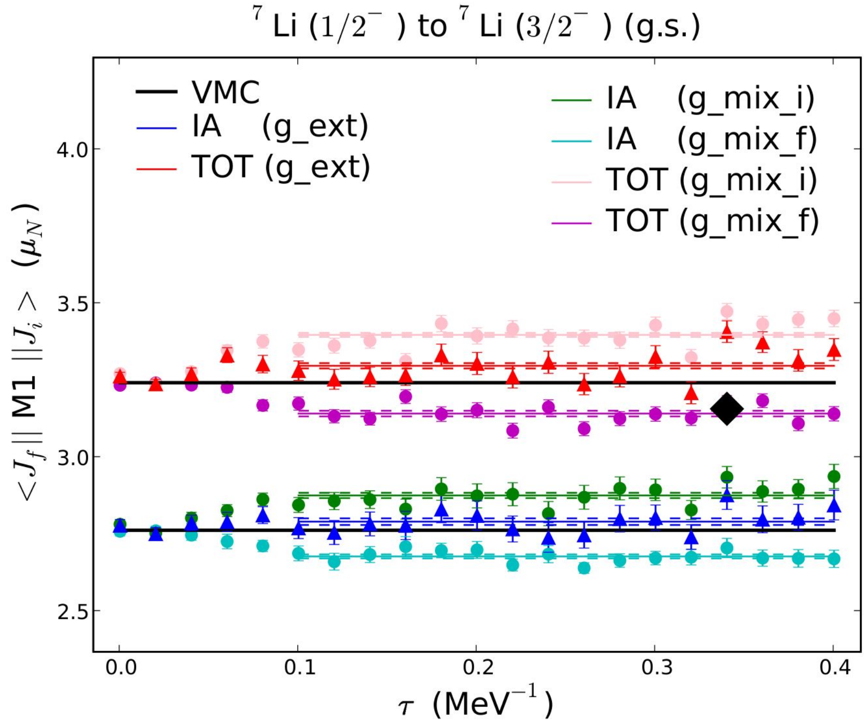
$$\Psi(\tau) = \Psi_V + \delta\Psi(\tau)$$

In practice, we evaluate a "mixed" estimates

$$\langle O(\tau)\rangle = \frac{{}_f\langle\Psi(\tau)|O|\Psi(\tau)\rangle_i}{\langle\Psi(\tau)|\Psi(\tau)\rangle} \approx \langle O(\tau)\rangle^i_{\text{Mixed}} + \langle O(\tau)\rangle^f_{\text{Mixed}} - \langle O\rangle_V$$

$$\langle O(\tau)\rangle^i_{\text{Mixed}} = \frac{{}_f\langle\Psi_V|O|\Psi(\tau)\rangle_i}{{}_f\langle\Psi_V|\Psi(\tau)\rangle_i} \;\; ; \;\; \langle O(\tau)\rangle^f_{\text{Mixed}} = \frac{{}_f\langle\Psi(\tau)|O|\Psi_V\rangle_i}{{}_f\langle\Psi(\tau)|\Psi_V\rangle_i}$$

Green's Function Monte Carlo: M1



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Review Article by Dr. Carlson and Prof. Schiavilla Few-Body

Review Article by Dr. Carlson et al. - QMC methods

Review Article by Epelbaum at al. - ChiEFT