Effective Field Theory methods for nuclear interactions

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Jun 19–27, 2025 Florida International University, Miami, FL Introduction The Microscopic Approach Many-Nucleon Forces and Electroweak Currents Phenomenological Approach to Many-Nucleon Operators Chiral Effective Field Theory Approach to Many-Nucleon Operators Quantum Monte Carlo Many-Body Computational Methods Applications to Electroweak Observables and Relevance of Many-Nucleon Correlations and Currents



Recitation Session

Anatomy one-pion-exchange potential Phase Shifts and Cross Sections Continuity Equation Non-Relativistic Reduction of the One-Body Electromagnetic Current Derivation of the OPEP

Nuclear Dynamics











electron

Incident



Electron-Nucleus Scattering Cross Section



Energy and momentum transferred (ω ,q)

Current and planned experimental programs rely on theoretical calculations at different kinematics

Strategy

Validate the Nuclear Model against available data for strong and electroweak observables

- Energy Spectra, Electromagnetic Form Factors, Electromagnetic Moments, ...
- Electromagnetic and Beta decay rates, ...
- Muon Capture Rates, ...
- Electron-Nucleus Scattering Cross Sections, ...

Use attained information to make (accurate) predictions for BSM searches and precision tests

- EDMs, Hadronic PV, ...
- BSM searches with beta decay, ...
- Neutrinoless double beta decay, ...
- Neutrino-Nucleus Scattering Cross Sections, ...
- ...

From Quarks to Nuclei

Nuclei are complex systems made of interacting protons and neutrons, which in turns are composite objects made of interacting constituent quarks.

All fundamental forces are at play in nuclei.

The underlying theory of the strong interaction is **Quantum Chromodynamics** (QCD).

Microscopic approaches to the nucleus use bound states of QCD, i.e. protons, neutrons, pions, ..., as fundamental degrees of freedom.

Effective theories of QCD at low-energy are used to construct effective many-nucleon potential and currents.



LQCD for single- and few-nucleon properties

Microscopic approaches rely on accurate inputs at the single- and few-nucleon level from experimental data (where available) and non-perturbative Lattice QCD theoretical calculations.

. . .



Snowmass WP: Theoretical tools for neutrino scattering: interplay between lattice QCD, EFTs, nuclear physics, phenomenology, and neutrino event generators; arXiv:2203.09030, Meyer, Walker-Loud, Wilkinson (2022)

Building blocks of ab initio nuclear approaches:

Nucleonic form factors Transition form factors Pion production amplitudes Two-nucleon couplings (strong and EW)



Microscopic (or ab initio) Description of Nuclei

Comprehensive theory that describes quantitatively and predictably nuclear structure and reactions

Requirements:

- 1. Accurate understanding of the interactions/correlations between nucleons in **paris**, **triplets**, ... (two- and three-nucleon forces)
- Accurate understanding of the electroweak interactions of external probes (electrons, neutrinos, photons) with nucleons, correlated nucleon-pairs, ... (one- and two-body electroweak currents)
- 3. **Computational methods** to solve the many-body nuclear problem of strongly interacting particles



Erwin Schrödinger

 $H\Psi = E\Psi$

1. Many-nucleon Interactions

The energy of the nucleus is approximated by the many-body Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

where v_{ij} and V_{ijk} are two- and three-nucleon operators correlating nucleons in pairs and triplets.

The derivation of these operators is based on experimental data fitting with parameters that subsume underlying QCD.

The non-relativistic description is justified by the observation that nucleon typical velocity inside the nucleus $v^2 \sim 0.05$.



2. Many-nucleon Electroweak Currents



Many-nucleon electroweak currents describe the interaction of external electroweak probes (electrons, neutrinos, photons, ...) with single nucleons and pairs of correlated nucleons.

Nuclear Charge Operator

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i < j} \rho_{ij} + \dots$$

Nuclear (Vector) Current Operator

 $\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i < j} \mathbf{j}_{ij} + \dots$



3. Many-body Nuclear Problem

Nuclear Many-body Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A, \mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_A, \mathbf{t}_1, \mathbf{t}_2, ..., \mathbf{t}_A)$$



$$\Psi$$
 are spin-isospin vectors in 3A dimensions with $2^A \times \frac{A!}{Z!(A-Z)!}$ components

Develop Computational Methods to solve (numerically) exactly or within approximations that are under control the many-body nuclear problem ⁴He : 96 ⁶Li : 1280 ⁸Li : 14336 ¹²C : 540572

Current Status



1. Many-nucleon interactions

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$



Nuclear force: empirical evidences



Nuclear force is short ranged: The binding energy per nucleon increases rapidly for A>4, and saturates at B.E./A~8 MeV, which gives a range ~ 1-2 fm.

Nuclear force has an attractive component: Nuclei exist.

The nuclear force is strongly repulsive at short interparticle distances: The density inside nuclei is constant and independent of the number of nucleons A.



Nuclear force: empirical evidences

The nucleon-nucleon force exhibits charge independence



⁷Li and ⁷Be spectra $(3p, 4n) \rightarrow (4p, 3n)$

The nucleon-nucleon force depends on the spin and isospin of a nucleon pair: Only the S=1 and T=0 combination presents a weakly bound state of np, i.e., the deuteron (with S total spin and T total isospin of the 2N system).



B = -2.2224575(9) MeV J⁺ = 1⁺ L = 0, 2 Q = 0.2859(3) fm²

Nuclear force: empirical evidences

The nucleon-nucleon force has a tensor component: The deuteron presents a quadrupole moment implying that it's not spherical. The tensor operator mixes states with different orbital angular momentum $L = J \pm 1$. The deuteron is predominantly in the ${}^{3}S_{1}$ wave with a small ${}^{3}D_{1}$ component.



$$S_{12} = 3\,\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}}\,\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

The tensor force is non-spherical and correlates spin and spatial orientations.

The Yukawa potential

Yukawa potential (1930s): Yukawa potential is due to the coupling of the interacting nucleons to a massive field.

$$v_Y \sim -\frac{e^{-\mu r_{ij}}}{r_{ij}} \qquad \text{range} \propto \frac{1}{\mu}$$

The pion is observed in 1947: With $\mu = m_{\pi} \sim 140$ MeV implying a range ~ 1.4 fm.



The One-pion Exchange (OPE) potential

Momentum space

$$\widetilde{v}_{12}^{\pi}(\mathbf{q}) = -\frac{f_{\pi NN}^2}{m_{\pi}^2} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q} \, \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{q^2 + m_{\pi}^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Configuration space

$$v_{12}^{\pi}(\mathbf{r}) = \frac{f_{\pi NN}^2}{4\pi} \frac{m_{\pi}}{3} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left\{ T_{\pi}(r) S_{12} + \left[Y_{\pi}(r) - \frac{4\pi}{m_{\pi}^3} \delta(\mathbf{r}) \right] \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right\}$$

Pion-nucleon coupling constant

Tensor operator

 $Y_{\pi}(r) = \frac{e^{-m_{\pi}r}}{m_{\pi}r}, \rightarrow \text{Yukawa function}$ $T_{\pi}(r) = \left(1 + \frac{3}{m_{\pi}r} + \frac{3}{m_{\pi}^2r^2}\right)Y_{\pi}(r) \rightarrow \text{Tensor function}$



m_π q

N

Two-nucleon potential: general features



The NN interaction can be schematically decomposed in short-, intermediate- and long range terms

 $v_{NN} = v^S + v^I + v^{\pi}$



Many-nucleon forces & electroweak currents: history

- 1930s: Yukawa potential
- ...
- 1950-1990: Highly sophisticated ``phenomenological" potential based on meson exchange theory
- 1990-: Effective field theory approach

Phenomenological nucleon-nucleon potentials

Use the most generic form of two-nucleon operators compatible with the allowed symmetries (translation galilean, rotational, time reversal invariance, space reflection, invariance under interchange of particle 1 and 2, isospin symmetry, hermiticity, ...) Okubo and Marshak, Ann. Phys. 4, 166 (1958); with radial functions and strengths determined through fits to NN scattering data.

$$\begin{split} V_{NN} &= V_0(r) + V_{\sigma} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + V_{\tau} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + V_{\sigma\tau} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \text{ central} \\ &+ V_T(r) S_{12} + V_{T\tau}(r) S_{12} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \text{ tensor} \\ &+ V_{LS}(\mathbf{L} \cdot \mathbf{S}) + V_{LS\tau}(\mathbf{L} \cdot \mathbf{S}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \text{ spin-orbit} \\ &+ V_Q Q_{12} + V_{Q\tau} Q_{12} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \text{ quadratic spin-orbit} \\ &+ V_{PP}(r) (\boldsymbol{\sigma}_1 \cdot \mathbf{p}) (\boldsymbol{\sigma}_2 \cdot \mathbf{p}) + V_{PP\tau}(r) (\boldsymbol{\sigma}_1 \cdot \boldsymbol{p}) (\boldsymbol{\sigma}_2 \cdot \mathbf{p}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \text{ p-dependent} \end{split}$$

The AV18 NN potential

The Argonne v18 potential is given in r-space as the sum of OPE and EM long-rage terms, an intermediate TPE term, and a phenomenological short-range term.

Argonne v_{18} $v_{ij} = v_{ij}^{\gamma} + v_{ij}^{\pi} + v_{ij}^{I} + v_{ij}^{S} = \sum v_p(r_{ij})O_{ij}^p$

18 operatorial structures

$$O_{ij}^{p} = [\mathbf{1}, \sigma_{i} \cdot \sigma_{j}, S_{ij}, \mathbf{L} \cdot \mathbf{S}, \mathbf{L}^{2}, \mathbf{L}^{2}(\sigma_{i} \cdot \sigma_{j}), (\mathbf{L} \cdot \mathbf{S})^{2}] \\ + [\mathbf{1}, \sigma_{i} \cdot \sigma_{j}, S_{ij}, \mathbf{L} \cdot \mathbf{S}, \mathbf{L}^{2}, \mathbf{L}^{2}(\sigma_{i} \cdot \sigma_{j}), (\mathbf{L} \cdot \mathbf{S})^{2}] \otimes \tau_{i} \cdot \tau_{j} \\ + [\mathbf{1}, \sigma_{i} \cdot \sigma_{j}, S_{ij}, \mathbf{L} \cdot \mathbf{S}] \otimes T_{ij} \\ + [\mathbf{1}, \sigma_{i} \cdot \sigma_{j}, S_{ij}, \mathbf{L} \cdot \mathbf{S}] \otimes (\tau_{i} + \tau_{j})_{z}$$

 $S_{ij} = 3\sigma_i \cdot \hat{r}_{ij}\sigma_j \cdot \hat{r}_{ij} - \sigma_i \cdot \sigma_j \qquad T_{ij} = 3\tau_{iz}\tau_{jz} - \tau_i \cdot \tau_j$

42 parameters (I, P, Q, R)

determining the strengths of the TPE and short-range W-S radial functions

 $v_{ij}^{\gamma}: pp, pn \& nn \text{ electromagnetic terms}$ $v_{ij}^{\pi} \sim [Y_{\pi}(r_{ij})\sigma_i \cdot \sigma_j + T_{\pi}(r_{ij})S_{ij}] \otimes \tau_i \cdot \tau_j$ $v_{ij}^I = \sum_p I^p T_{\pi}^2(r_{ij})O_{ij}^p$ $v_{ij}^S = \sum_p [P^p + Q^p r + R^p r^2]W(r)O_{ij}^p$

Wiringa, Stoks, & Schiavilla, PRC 51, 38 (1995)



Nucleon-nucleon scattering data



The parameters entering the NN potential are determined through fits to NN scattering data.

There are more than 7k data available for NN scattering (data collected from 1950-2013).

Database:

Nijmegen SAID Granada Database

Navarro-Perez et al. PRC 89, (2014)

Determining the NN potential through fits to the data

The parameters entering the NN potential are determined from fits to NN scattering data by solving for the scattering states of the Schrodinger equation with the AV18 (for example).



Argonne AV18: phase shifts



The AV18 model uses 42 I^p, P^p, Q^p, R^p parameters, and one cutoff parameter in Yukawa and tensor functions, $Y_{\pi}(r)$, $T_{\pi}(r)$.

Argonne v18 fits Nijmegen PWA93 database of 1787 pp & 2514 np observables for $E_{lab} \leq 350$ MeV with χ^2 /datum = 1.1 plus nn scattering length and ²H binding energy.

Argonne v18 reproduces phase shift beyond $E_{lab} \leq 350$ MeV

$$(1 - e^{-c r^2})^2$$

Argonne AV18

OPE potential is used in all NN models fitted to NN scattering data. Moreover, an indirect evidence of OPE dominance in the long range NN interaction comes from the 1993 Nijmegen phase-shift analysis when m_{π} was left as free-parameter. The best fit was obtained with the physical pion mass.



Deuteron wave functions



Deuteron S and D components



Wiringa, Stocks, Schiavilla PRC51 (1995)

Two-nucleon correlation & the deuteron shape



Constant density surfaces for a polarized deuteron in the $M=\pm 1$ (left) and M=0 (right) states

Carlson and Schiavilla Rev.Mod.Phys.70(1998)743

Two-nucleon correlations & momentum distributions



Tensor correlations lead to large differences in the **np** versus **pp** distributions.

These differences are observed in A(e, e'np) and A(e, e'pp) reactions.

Schiavilla Carlson Wiringa Pieper PRL98(2007) & PRC89(2014)



Two-nucleon correlations & the shape of nuclei



Beyond two-nucleon forces: Three-nucleon forces

The AV18 alone fails to provide the adequate binding in light nuclei, primarily due to a cancellation between kinetic and two-nucleon terms.

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots \qquad \text{Expected to be suppressed by ~ 0.05}$$

W.r.t. 3N force
Ti + Vij = -38.2131 (0.1433) + Vijk = -46.7975 (0.1150)

Ti = 290.3220 (1.2932) Vij =-328.5351 (1.1983) Vijk = -8.5844 (0.0892)

 $V_{ijk} \sim (0.02 \text{ to } 0.07) v_{ij} \sim (0.15 - 0.6) H$

 $v_{\pi} \sim 0.83 v_{ij}$

Three-body force



3N forces are similar in nature to the three-body force between three magnetic dipoles due to an induced moment on one of them.

In the interaction between sun-earth-moon, in addition to the two-body interactions, there is three-body interaction that depends on the position of the three-body. It takes into account the change in the gravitational field of the earth due to the polarization of ocean waters via the tides caused by moon's gravity.

Nucleons' excitation





Due to its large strength and low energy, the Δ -resonance at W = 1.24 GeV plays an important role in pion-exchange interactions between nucleons.

The spin and isospin of this resonance have values of 3/2 each.

The total and elastic $\pi^+ p$ scattering cross sections

Urbana three-nucleon force

Progress of Theoretical Physics, Vol. 17, No. 3, March 1957

Pion Theory of Three-Body Forces

Jun-ichi FUJITA and Hironari MIYAZAWA Department of Physics, University of Tokyo, Tokyo



The Urbana model consists of the attractive Fujita-Miyazawa interactions plus a repulsive contribution required to additional repulsion in nuclear matter. Two parameters regulate the strengths of these terms are are fixed to reproduce the triton binding energy and the saturation density of symmetric nuclear matter.

Carlson, Pandharipande, & Wiringa, NP A401, 59 (1983)

Illinois three-nucleon force



The Illinois model add to the Urbana interaction the two-pion S-wave contribution and three-pion ring diagrams. It has 4 additional parameters determined from fits to 23 states in A \leq 10 nuclei.

Pieper, Pandharipande, Wiringa, & Carlson, PRC 64, 014001 (2001)

Spectra of light nuclei with AV18+IL7





Phenomenological many-nucleon interactions

The microscopic description of nuclei is very successful! Spectra described within 1-2% of expt data.

The NN interaction is dominated by the OPE potential at long interparticle distances, intermediate range forces are described in terms of OPE and TPE mechanisms, short-range terms are given by special functions (e.g. W.S.). AV18 has a fixed cutoff.

The parameters entering the NN and 3N interaction are fixed to reproduce large set of scattering data plus binding energies. 3N forces need to reach good agreement due to cancellations between 2N and K terms in the nuclear Hamiltonian.

AV18 fitted up to 350 MeV, however phase shifts are in good agreement with data up to ~ 1 GeV.

OPE physics dominates and it's crucial to explain the data.

Cons: Phenomenological theories are hard to be systematically improved. Also, it's hard to assign a theoretical error. Model dependencies are provided by comparing results based on different interactions.

2. Many-nucleon Electroweak Currents



Many-nucleon electroweak currents describe the interaction of external electroweak probes (electrons, neutrinos, photons, ...) with single nucleons and pairs of correlated nucleons.

Nuclear Charge Operator

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i < j} \rho_{ij} + \dots$$

Nuclear (Vector) Current Operator

 $\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i < j} \mathbf{j}_{ij} + \dots$



Electromagnetic probes

 $e', p'^{\mu}_{e} \qquad \qquad P^{\mu}_{f}, |\Psi_{f}\rangle$ $e, p^{\mu}_{e} \qquad \qquad \qquad P^{\mu}_{e} - p'^{\mu}_{e}$ $= (\omega, \mathbf{q})$ $e, p^{\mu}_{e} \qquad \qquad \qquad P^{\mu}_{i}, |\Psi_{i}\rangle$

 α ~ 1/137 allows for a perturbative treatment of the EM.

x-sections are factorized into a well-known part specified by the electron kinematics and a part proportional to the matrix element of the EM current.

 $|\langle \Psi_f | j^{\mu} | \Psi_i \rangle|^2$

The experimental data are (in most cases) known with great accuracy providing stringent constraints on theories.

For light nuclei, the many-body problem can be solved exactly of within controlled approximations.

EM observables used to validate the microscopic model.

One-body EM currents



One-body EM currents describe the interaction of the external EM probe with a single proton or neutron. They are obtained from the non-relativistic reduction of the covariant single nucleon currents.

$$\begin{split} \rho_{i}(\mathbf{q}) &= e \frac{1}{2} \left[G_{E}^{S}(q_{\mu}^{2}) + G_{E}^{V}(q_{\mu}^{2})\tau_{z,i} \right] e^{i\mathbf{q}\cdot\mathbf{r}_{i}} & \mathsf{EM Charge} \\ \mathbf{j}_{i}(\mathbf{q}) &= \frac{1}{4m} \left[G_{E}^{S}(q_{\mu}^{2}) + G_{E}^{V}(q_{\mu}^{2}) \right]_{z,i} \right] \left\{ \mathbf{p}_{i}, e^{i\mathbf{q}\cdot\mathbf{r}_{i}} \right\} - \frac{i}{4m} \left[G_{M}^{S}(q_{\mu}^{2}) + G_{M}^{V}(q_{\mu}^{2})\tau_{z,i} \right] \mathbf{q} \times \sigma_{i} e^{i\mathbf{q}\cdot\mathbf{r}_{i}} & \mathsf{EM vector current} \\ \mathbf{q} \to \mathbf{0} & \mathsf{Nucleonic form factors} \\ \rho_{i} &= e \frac{1 + \tau_{z,i}}{2} \\ \mu_{i} &= \mu_{N} \left[(L_{i} + g_{p}S_{i}) \frac{1 + \tau_{i,z}}{2} + g_{n}S_{i} \frac{1 - \tau_{i,z}}{2} \right] & \overbrace{S_{p}}^{I} & \overbrace{S_{p}}^{I} & \overbrace{S_{p}}^{I} \\ \end{split}$$

Magnetic moment: single particle picture

EM nucleonic form factors



Gonzalez-Jimenez et al. Phys. Rep. 524, 1 (2013)

Beyond the single particle picture

The single particle picture fails to explain basic EM observables, such as the trinucleon magnetic moments, the np radiative capture, ...

Corrections accounting for the fact that the nucleons are correlated are needed to explain the data.

Nucleons are correlated via OPE mechanisms, EM currents of one-pion range emerge naturally (pions move charge around). They provide the ~10% missing contribution to reach agreement with the data for the magnetic moments of the trinucleons.

Villars, Miyazawa (40s), Chembot, Rho (70s), Riska, Brown, Schiavilla, (80s)

Many-body Nuclear Electroweak Currents



Two-body currents are a manifestation of two-nucleon correlations.

Electromagnetic two-body currents are required to satisfy current conservation.

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + v_{ij} + V_{ijk}], \rho]$$
$$\nabla \cdot \mathbf{j} = -\frac{\partial \rho}{\partial t} \quad \text{classically}$$



Phenomenological Meson-Exchange EM Currents

The continuity equation constraints the longitudinal part of the EM currents. Phenomenological models exploit the continuity equation to construct the two-body longitudinal EM current from the two-nucleon interaction adopted to correlated nucleons in pairs.

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + v_{ij} + V_{ijk}, \rho]$$



Schiavilla et al. PRC 41 (1990); Marcucci et al. PRC72(2005)

EM currents from nuclear many-body interactions

 2 H(p, γ)³He capture



Marcucci et al. PRC72, 014001 (2005)

EM currents from nuclear many-body interactions



Marcucci et al. PRC72, 014001 (2005)

EM currents from many-nucleon interactions

Many-nucleon currents are found to be essential to explain experimental data in a wide range of energy and momentum transfer, including electromagnetic moments, form factors, captures, cross sections.

The regime of validity of the electromagnetic currents constructed from the Argonne v18 goes beyond 350 MeV.

Cons: The phenomenological EM currents have model dependent components and they are hard to be systematically improved making it hard to assign a theoretical error to the calculations.



Lectures by Prof. Rocco Schiavilla

Review Article by Dr. Carlson and Prof. Schiavilla Few-Body

Review Article by Dr. Carlson et al. - QMC methods