Spin asymmetries and their radiative corrections in low-energy elastic positron scattering

Doris Jakubaßa-Amundsen
University of Munich (LMU)

JLab, March 2025

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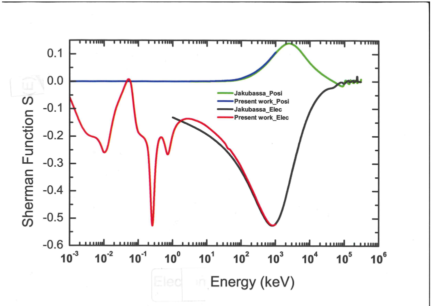
1. Definition of the spin asymmetry

Polarized positron beam $\quad e^+ \overset{\zeta}{\text{—}\uparrow\text{———}\rightarrow} \bullet\, Z_T$

$$\frac{d\sigma}{d\Omega}(\zeta) = (|A|^2 + |B|^2)\,(1 + S\,\boldsymbol{n}\cdot\zeta), \qquad \boldsymbol{n} \sim \boldsymbol{k}_i \times \boldsymbol{k}_f$$

Sherman function

$$S = \frac{d\sigma/d\Omega(\uparrow) - d\sigma/d\Omega(\downarrow)}{d\sigma/d\Omega(\uparrow) + d\sigma/d\Omega(\downarrow)} = \frac{2\,\mathrm{Re}\,\{AB^*\}}{|A|^2 + |B|^2}$$



^{208}Pb, 150°

with A.K.F.Haque, B.C.Saha

2. Motivation for positron scattering

Low energy (< 1 MeV): beam undisturbed by shell electrons

High energy (> 50 MeV): diffraction effects $\Longrightarrow$ ground-state charge distribution

(deformed nuclei, proton skin, particle emission threshold)

Spin asymmetry: additional sensitivity to phase differences

$\Longrightarrow$ better probe of nuclear models

$\Longrightarrow$ effects of two-photon exchange mechanisms

Positrons versus electrons: quantum interference studies between such mechanisms

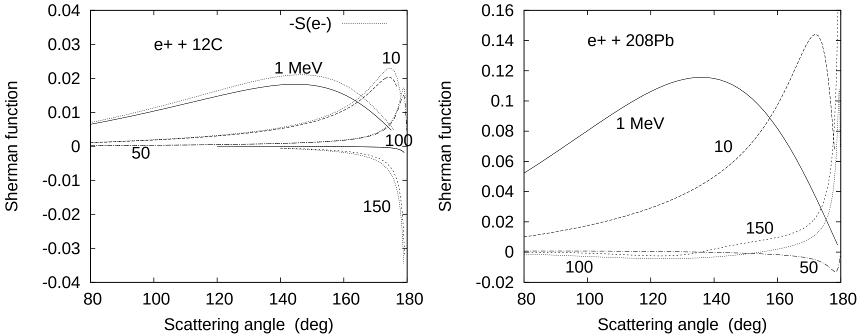
3. Results for ^{12}C and ^{208}Pb (between 1 – 150 MeV)

$E > 1$ MeV, Z_T small: $S \sim Z_T$ (relativistic effect)

$S(e^+) \approx -S(e^-)$

Z_T large: mostly $|S(e^+)| \ll |S(e^-)|$

Angular distribution:



Energy distribution at 170°:

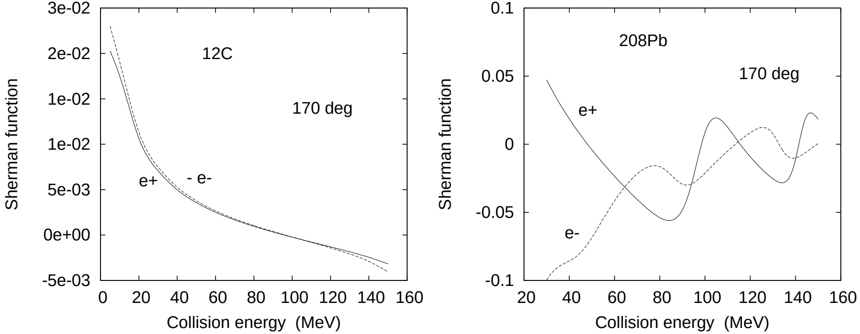
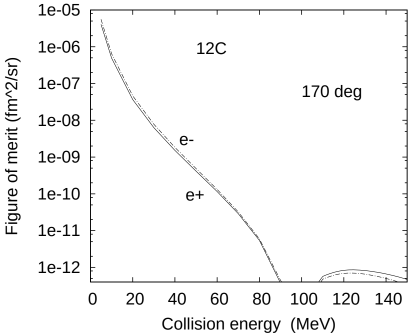
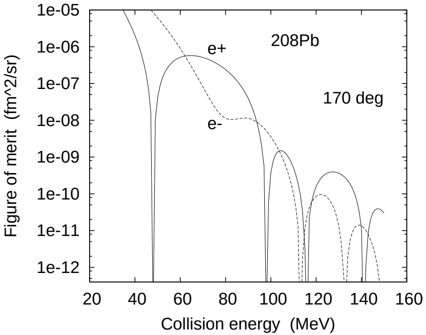


Figure of merit: $\frac{d\sigma}{d\Omega} \cdot S^2$ (for count-rate estimate)

Example: Energy distribution at 170°





4. Radiative corrections: (a) QED effects

Vacuum polarization: Uehling potential $V_{\rm vac} = V_{\rm vac}(\rho_0)$
Vertex + self-energy correction: V_{vs}

Nonperturbative approach: solve Dirac equation

$$[-ic\boldsymbol{\alpha}\boldsymbol{\nabla} + V_T(r) + V_{\rm vac}(r) + V_{vs}(r)]\,\psi(\boldsymbol{r}) = E\,\psi(\boldsymbol{r})$$

Construction of V_{vs}:

Relation between potential V_T and the corresponding transition amplitude in 1.Born

$$A_{V_T}^{B1}(q) = A_0 \int d\boldsymbol{r}\, e^{i\boldsymbol{q}\boldsymbol{r}}\, V_T(r), \qquad A_0 = \frac{\sqrt{E_i E_f}}{2\pi c^2}\,(u_{k_f}^{+(\sigma_f)}\, u_{k_i}^{(\sigma_i)})$$

$$\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_f$$

Inverse Fourier transform

$$V_T(r) \;=\; \frac{1}{(2\pi)^3}\int d\boldsymbol{q}\; e^{-i\boldsymbol{q}\boldsymbol{r}}\; A^{B1}_{V_T}(q)/A_0$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$V_{vs}(r) \qquad\qquad\qquad A^{B1}_{vs}(q) \;\approx\; F^{vs}_1(q)\, A^{B1}_{V_T}(q)$$

$$F^{vs}_1 \;=\; \text{electric form factor} \qquad \text{(Tsai 1961)}$$

$$V_{vs}(r) \;=\; -\,\frac{2Z}{\pi}\int_0^\infty dq\; \frac{\sin(qr)}{qr}\; F_L(q)\, F^{vs}_1(q)$$

$$F_L \;=\; \text{charge form factor}$$

Sherman function

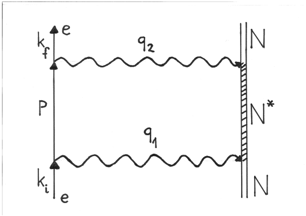
$$S_{QED} \;=\; \frac{2\,\mathrm{Re}\,\{A_{QED}B^*_{QED}\}}{|A_{QED}|^2 \;+\; |B_{QED}|^2}$$

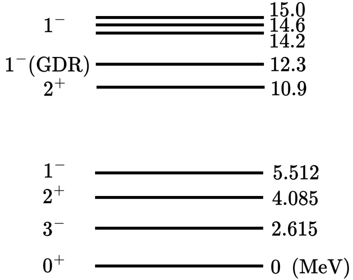
QED modification: $dS_{QED} = S_{QED} - S$
Relative QED change: $\Delta S_{QED} = dS_{QED}/S$

(b) Dispersion: Transient nuclear excitation during scattering (Born approximation)

$$\frac{d\sigma_{\mathrm{box}}}{d\Omega}(\zeta) = |A_{fi}(\zeta) + A_{fi}^{\mathrm{box}}(\zeta)|^2 \approx \frac{d\sigma}{d\Omega}(\zeta) + 2\,\mathrm{Re}\,\{A_{fi}^*(\zeta)\,A_{fi}^{\mathrm{box}}(\zeta)\}$$

Considered excited states: $E_x < 30$ MeV, $L \leq 3$





^{12}C: $E_x = 17.7,\ 23.5$ MeV (1^-), $4.439,\ 9.84$ MeV (2^+)
$9.64,\ 14.8$ MeV (3^-)

^{208}Pb: right-hand figure plus 21.6 MeV (2^+), 28.94 MeV (3^-)

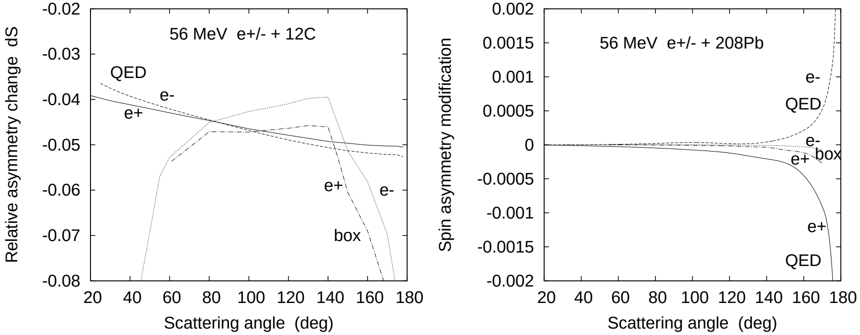
Results for ^{12}C and ^{208}Pb: Angular distribution at 56 MeV

Spin asymmetry modification $\quad dS_{\text{box}} = S_{\text{box}} - S$

Relative spin asymmetry change $\quad \Delta S_{\text{box}} = S_{\text{box}}/S - 1$

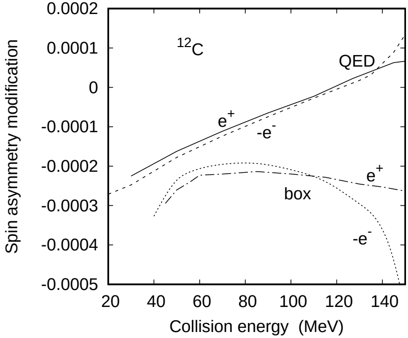
$$S_{\text{box}} = \frac{d\sigma_{\text{box}}/d\Omega(\uparrow) - d\sigma_{\text{box}}/d\Omega(\downarrow)}{d\sigma_{\text{box}}/d\Omega(\uparrow) + d\sigma_{\text{box}}/d\Omega(\downarrow)}$$

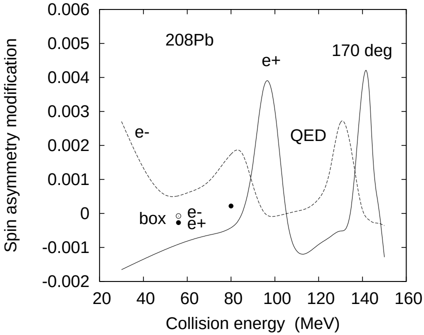
$\Delta S_{QED} = S_{QED}/S - 1$ $\qquad\qquad$ $dS_{QED} = S_{QED} - S$



Spin asymmetry modification: Energy distribution at 170°

$$dS_{QED} = S_{QED} - S$$
$$dS_{\mathrm{box}} = S_{\mathrm{box}} - S$$





Summary for positrons

Sherman function S:

Increase with Z_T
Decrease with E beyond 5 MeV at large angles
Existence of diffraction zeros:
^{12}C: $E = 96$ MeV $\Leftrightarrow$ $R_n \sim 1$ fm
^{208}Pb: $E = 48$ MeV $\Leftrightarrow$ $R_n \sim 2$ fm (170°)

QED corrections:

Average $3 - 5\%$ beyond 30 MeV; diffraction effects

Dispersion corrections:

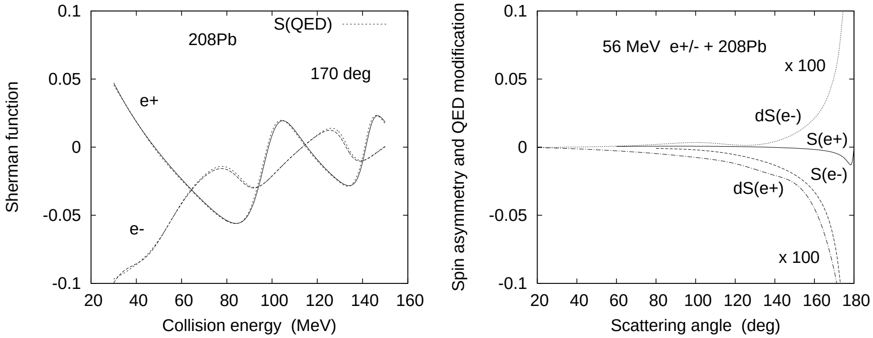
Important for ^{12}C, particularly at foremost and backmost angles ($\sim 10\%$ beyond 50 MeV)
Very small for ^{208}Pb ($\lesssim 1\%$)

Thank you!

Backup figures

Sherman function including QED corrections

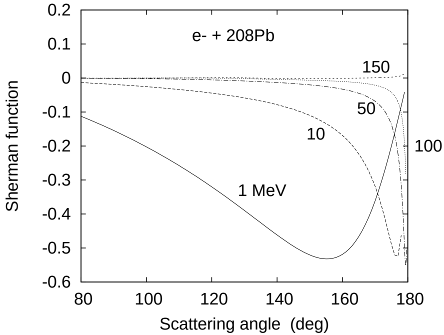
Sherman function and its QED modifications (scaled up by 100)
Note: diffraction leads to a sign change!

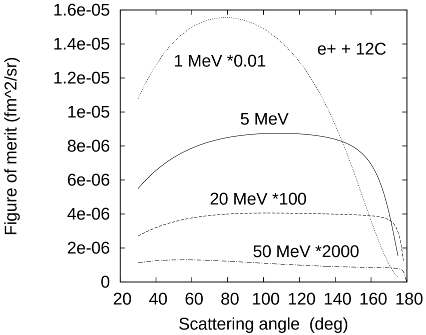


S for electrons, $1 - 150$ MeV

$\frac{d\sigma}{d\Omega} \times S^2$ for e^+, $1 - 50$ MeV

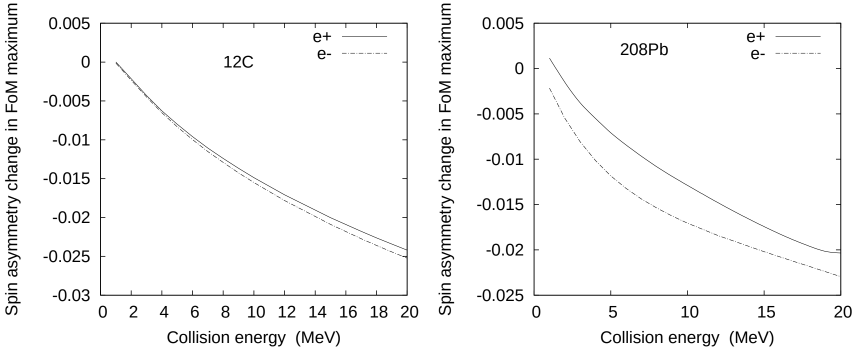
Note the scaling factor!





Relative spin asymmetry change $\Delta S_{\mathrm{QED}} = (S_{\mathrm{QED}} - S)/S$ at the angle where the figure of merit is maximum at the given collision energy:

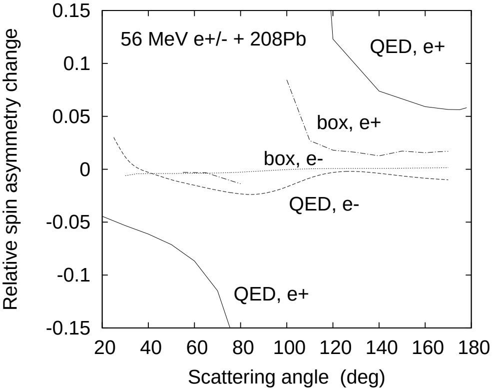
Comparison between positrons (e^+) and electrons (e^-)



Relative spin symmetry change by QED effects and dispersion (box) for 56 MeV collision energy

Due to the zero of the positron Sherman function near 100°, ΔS_{QED} is ill defined between $80^\circ - 120^\circ$.

For electrons, S is nonzero at all angles at 56 MeV.



ΔS_{QED} for positrons
at angles 30° and 170°
Note that $S = 0$ at 96 MeV
for 170°

Modification $S - S_{\mathrm{box}}$ for e^-
Increase $\sim \sin(\theta/2)$ near 0
The result from the 2^+ excitation
at 4.439 MeV is shown by the
lower line (increased by the
factor of 100)