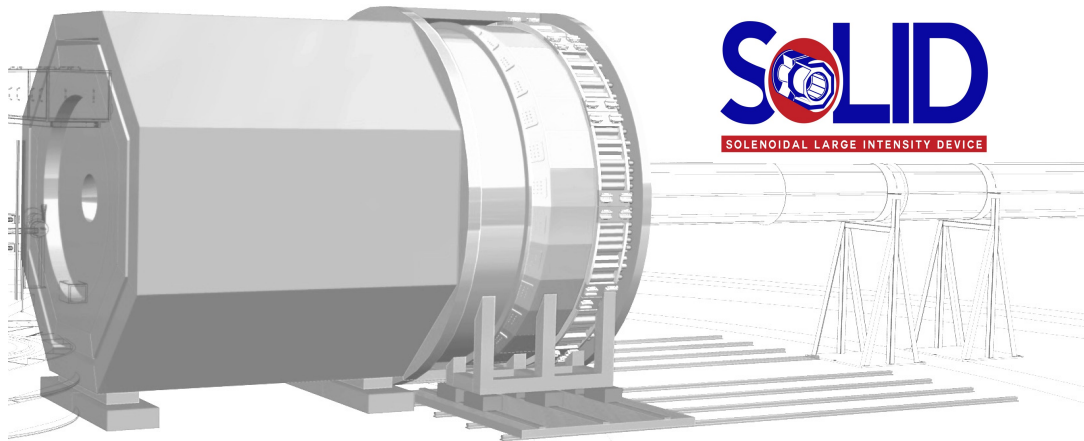


Measurement of the Unpolarized SIDIS Cross Section from a ^3He Target with SoLID



**Run-Group
Proposal Defense**

Ye Tian Syracuse University

On behalf of the spokespersons

Umberto D'Alesio	Università di Cagliari & INFN Sezione di Cagliari
Matteo Cerutti	Christopher Newport University & Jefferson Lab
Shuo Jia	Duke University
Vlad Khachatryan	Indiana University & Duke University

*SoLID Collaboration Meeting, Jefferson Lab, Newport News, VA
January 9-10, 2025*

Outline

- Experimental setup and motivation for our proposed experiment
 - *SoLID ^3He setup and experimental details*
 - *Motivation for unpolarized cross-section measurements*
- SIDIS process and differential cross section
- Estimated systematic uncertainties for the experiment
- Some more physics results in addition to the SoLID impact results
- Summary and outlook

Experimental Setup

➤ Our run group experiment parasitic to SoLID SIDIS experiments of

E12-10-006: Single Spin Asymmetries on Transversely Polarized ^3He (neutron): Rating A

➤ Approved number of days:

- 48 days (11 GeV) & 21 day (8.8 GeV)

➤ 10 days requested for study of x-z factorization with Hydrogen/Deuterium gas using reference target cell

➤ 3 days of reference cell runs for optics and detector check

➤ 5 days of target overhead: spin rotation, polarization measurement

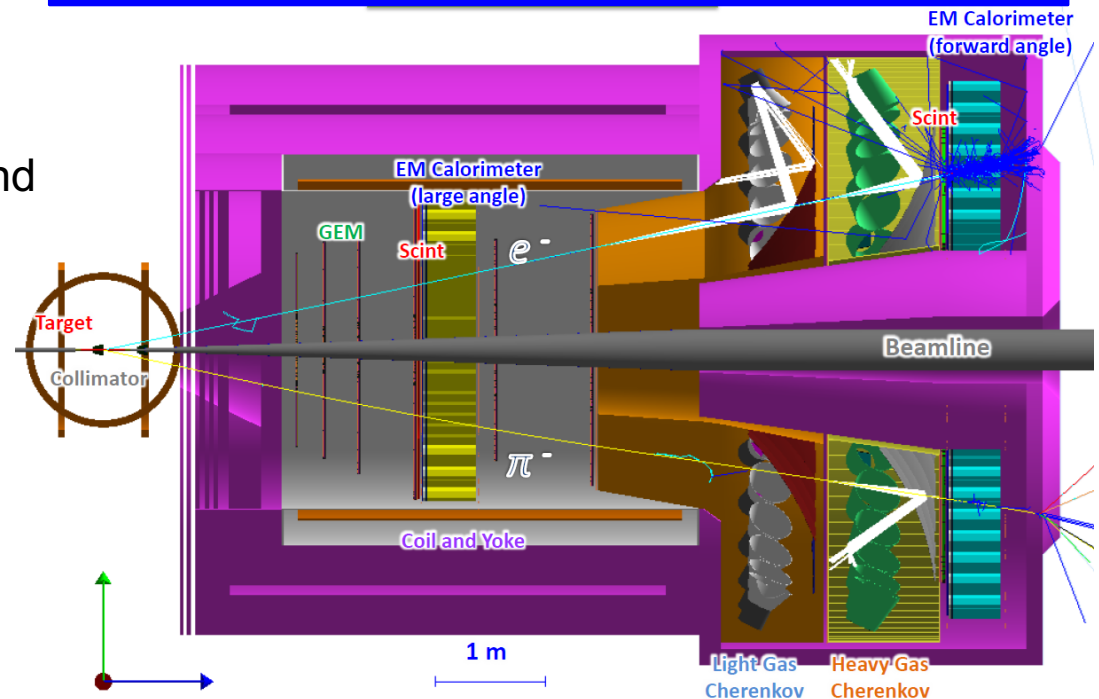
➤ 3 days requested with longitudinal target polarization to study systematics of potential A_{UL} contamination

E12-11-007: Single and Double Spin Asymmetries on Longitudinally Polarized ^3He (neutron): Rating A

➤ Approved number of days:

- 22.5 days (11 GeV) & 9.5 day (8.8 GeV)

SoLID (SIDIS ^3He): 11 GeV & 8.8 GeV beam energies



Data Status

➤ Hall A data

- E06-010: SIDIS π^\pm productions from a **transversely polarized ^3He target** with **5.9 GeV** beam ($0.12 < x_{bj} < 0.45$, $1 < Q^2 < 4 \text{ GeV}^2$, $0.45 < z_h < 0.65$, $0.05 < P_{hT} < 0.55 \text{ GeV}$)

X. Yan, et al., Phys. Rev. C 95, no.3, 035209 (2017)

➤ Hall B data

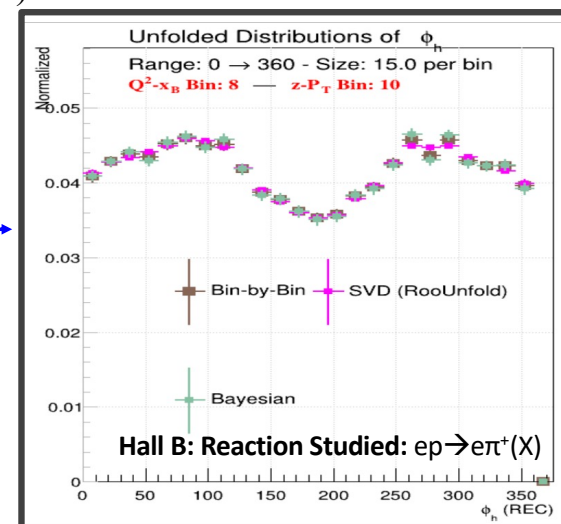
- RG-A: Measurements of the $\cos\phi_h$ and $\cos 2\phi_h$ Moments of the Unpolarized SIDIS π^+ Cross-section with **10.6 GeV** beam and hydrogen target

➤ Hall C data

- E00-108: SIDIS π^\pm productions from **hydrogen** and **deuterium** targets with **5.5 GeV** beam ($0.2 < x_{bj} < 0.6$, $2 < Q^2 < 4 \text{ GeV}^2$, $0.3 < z_h < 1$, and $P_{hT}^2 < 0.2 \text{ GeV}^2$)

R. Asaturyan and et.al Phys. Rev. C 85, 015202 (2012)

- E12-09-017: Transverse momentum (P_{hT}) dependence of SIDIS π^\pm and K^\pm productions from **hydrogen** and **deuterium** targets with **8.8 GeV** and **11 GeV** beam ($0.2 < x_{bj} < 0.5$, $2 < Q^2 < 5 \text{ GeV}^2$, $0.3 < z_h < 0.5$, and $P_{hT} < 0.5 \text{ GeV}$)



R. Capobianco

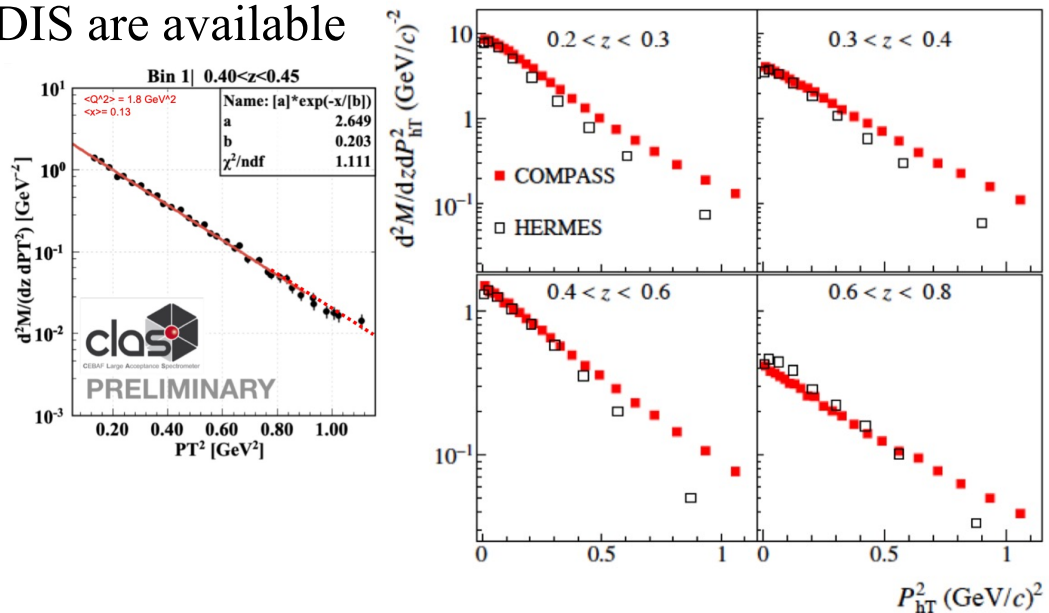
This SoLID proposal: SIDIS π^\pm and K^\pm

$$0 < x_{bj} < 0.7, 1 < Q^2 < 10 \text{ GeV}^2, 0.3 < z_h < 0.7, 0 < P_{hT} < 1.6 \text{ GeV}, -\pi < \phi_h < \pi$$

Motivation

Studies on Multiplicities of hadrons in SIDIS are available

$$\frac{d^2 M^h(x_{bj}, Q^2, z_h, P_{hT}^2)}{dz dP_{hT}^2} = \left(\frac{d^4 \sigma^h}{dx dQ^2 dz dP_{hT}^2} \right) / \left(\frac{d^2 \sigma^{DIS}}{dx dQ^2} \right)$$



Lack of data on SIDIS unpolarized absolute cross sections

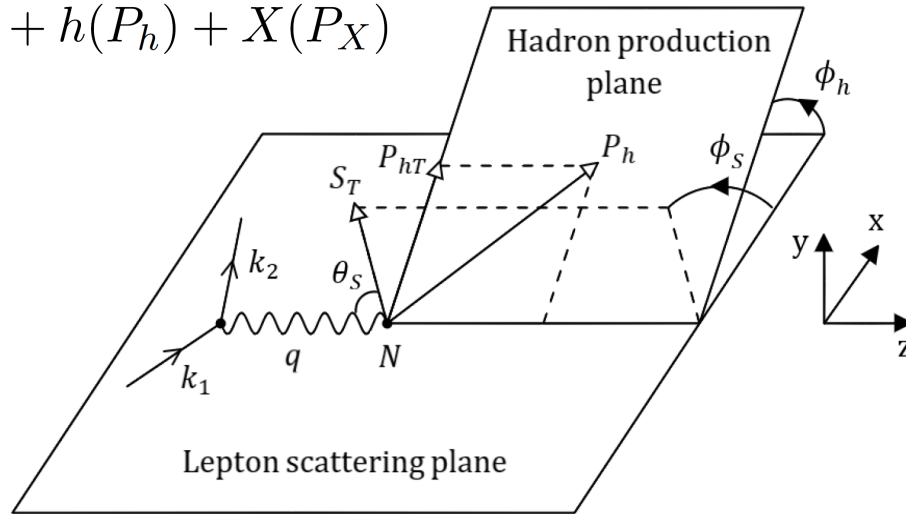
- Study both the shape and the normalization of the SIDIS cross sections
- Ascertain the validity of the factorization theorems
- Nuclear corrections: EMC effect, nuclear binding, Fermi motion, and off-shell effects
- higher-twist effects on azimuthal angular modulations
- TMD flavor dependence

SIDIS Process

- The SIDIS process represented as (four-momenta given in parentheses)

$$l(k_1) + N(P) \rightarrow l'(k_2) + h(P_h) + X(P_X)$$

- l - lepton beam
- N - nucleon target
- h - produced hadron
- X - undetected hadron
- q – virtual photon momentum



Azimuthal angle between hadron production and lepton scattering planes designated as ϕ_h

**Kinematics of the SIDIS process:
assume one-photon exchange approximation**

$$q \equiv l - l'$$

$$Q^2 \equiv -q^2$$

$$x_{bj} = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot k_1}, \quad z_h = \frac{P \cdot P_h}{P \cdot q}, \quad \gamma = \frac{2M_N x_{bj}}{Q}$$

- Project unpolarized cross-section pseudo-data in **5-D binning**

$$x_{bj}, P_{hT}, z_h, Q^2, \phi_h$$

SIDIS Unpolarized Cross Section

We have some updates for the formula

$$\frac{d\sigma}{dx_{bj} dy dz_h dP_{hT}^2 d\phi_h} = 2\pi \frac{\alpha^2}{x_{bj} y Q^2} \left(1 + \frac{\gamma^2}{2x_{bj}}\right) \times$$
$$\times \left[c_1 F_{UU} + c_2 \cos(\phi_h) F_{UU}^{\cos(\phi_h)} + c_3 \cos(2\phi_h) F_{UU}^{\cos(2\phi_h)} \right]$$

Matteo showed SoLID impact-study results with this first term at NNLL

We also have SoLID projection results at LO parton model

There are no ϕ_h -dependent terms computed within TMD factorization (obtained within LO parton model)

NNLL means next-to-next-to-next-to-leading-log

SIDIS Unpolarized Cross Section

$$\frac{d\sigma}{dx_{bj} dy dz_h dP_{hT}^2 d\phi_h} = 2\pi \frac{\alpha^2}{x_{bj} y Q^2} \left(1 + \frac{\gamma^2}{2x_{bj}}\right) \times \left[c_1 F_{UU} + c_2 \cos(\phi_h) F_{UU}^{\cos(\phi_h)} + c_3 \cos(2\phi_h) F_{UU}^{\cos(2\phi_h)} \right]$$

➤ Use the following Gaussian parameterizations for the TMD PDF and TMD FF

$$F_{UU} = \sum_q e_q^2 x_{bj} f_q(x_{bj}) D_q(z_h) \frac{e^{-P_{hT}^2 / \langle P_{hT}^2 \rangle}}{\pi \langle P_{hT}^2 \rangle}$$

where $\langle P_{hT}^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle$

Twist 2 effect

$$F_{UU}^{\cos(\phi_h)} = F_{UU}^{\cos(\phi_h)} \Big|_{\text{Cahn}} + F_{UU}^{\cos(\phi_h)} \Big|_{\text{BM}}$$
Twist 3 effect: $\cos \phi_h$ dependence

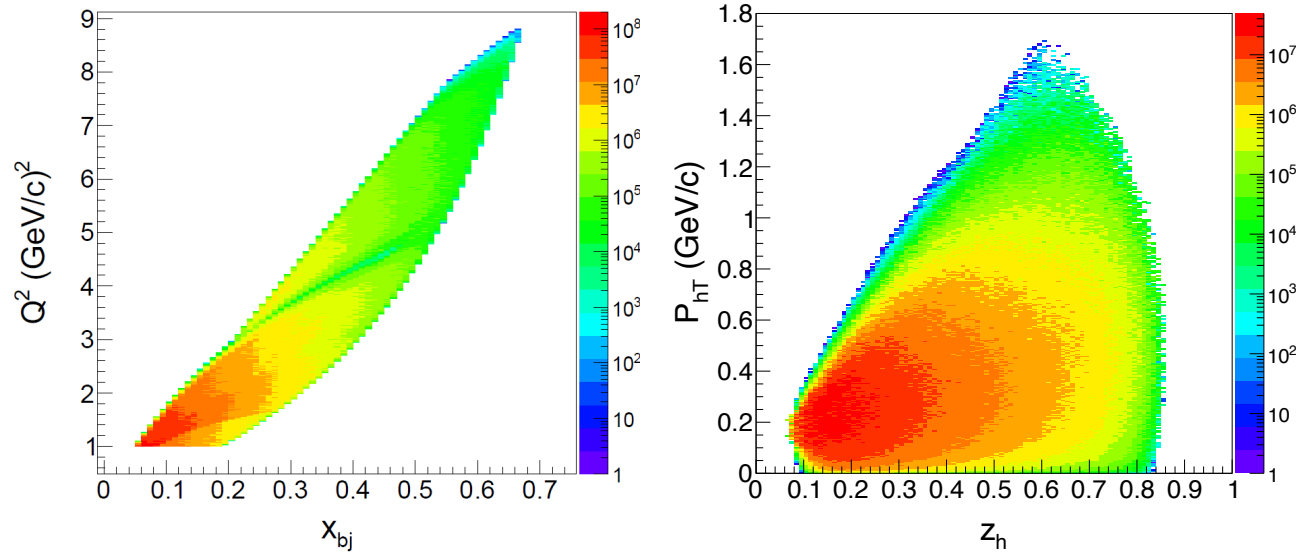
- Cahn effect $\propto f_1 \otimes D_1$
 - Non-zero Cahn effect solely require **non-zero quark transverse momentum**
 - Related to quarks' **intrinsic transverse momentum distribution**

$$F_{UU}^{\cos(2\phi_h)} \approx F_{UU}^{\cos(2\phi_h)} \Big|_{\text{Cahn}} + F_{UU}^{\cos(2\phi_h)} \Big|_{\text{BM}}$$
Twist-4 Cahn & twist-2 Boer-Mulders: $\cos(2\phi_h)$ dependence

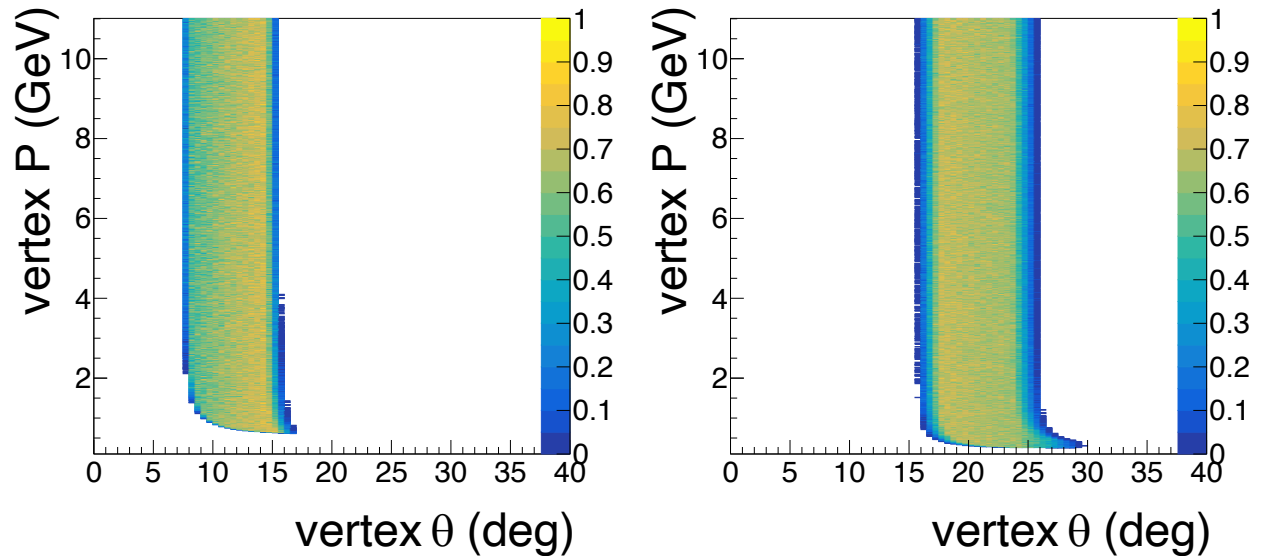
- Boer-Mulders effect $\propto h_{\perp 1} \otimes H_{\perp 1}$
 - Boer-Mulders TMD PDF: transversely polarized quarks in unpolarized nucleon
 - **Twist-4 Cahn effect** could have similar size of contribution to $\cos(2\phi_h)$ as Boer-Mulders [Phys. Rev. D. 81:114026 (2010) based on HERMES/COMPASS results]

Systematic Uncertainties

- Kinematic coverage examples of produced π^+ particles
 - 11 GeV and 8.8 GeV combined
- Phase-space correlation between Q^2 and x_{bj} (top-left)
- Phase-space correlation between x_{bj} and z_h (top-right)



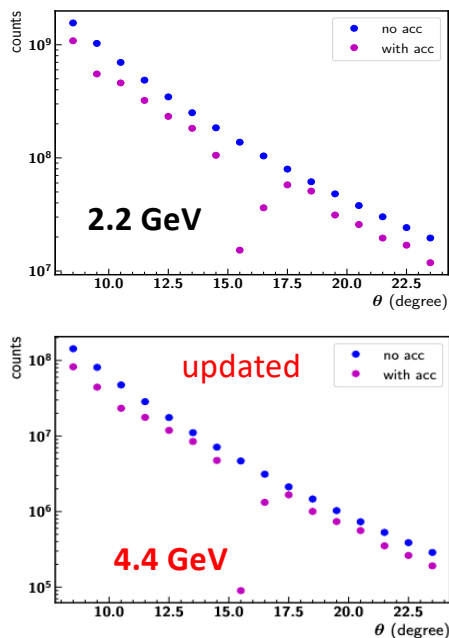
- **Electron acceptance**
 - as a function of polar angle and momentum forward angle (bottom left)
 - as a function of polar angle and momentum large angle (bottom right)



Systematic Uncertainties of Unpolarized Cross Section: Acceptance Uncertainty from Elastic Process

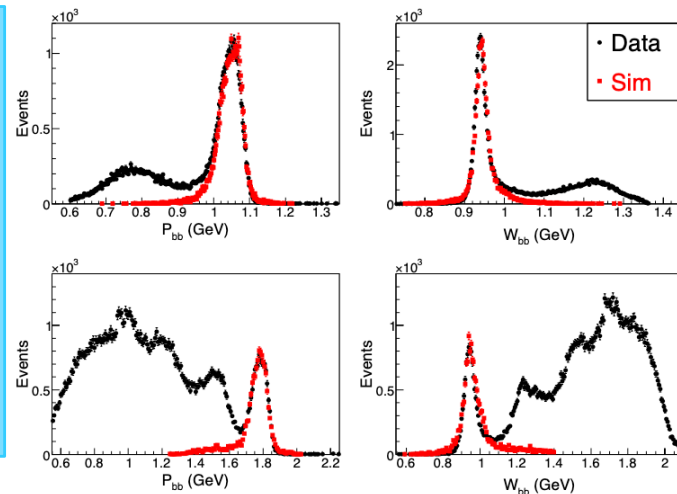
Elastic process:
e – p scattering
With **2.2 GeV** beam energy, 1h could get enough counts

1h run with **4.4 GeV** beam energy could also reach enough counts



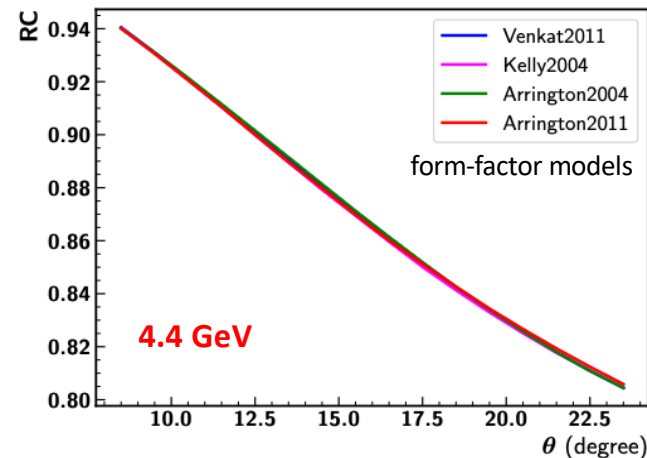
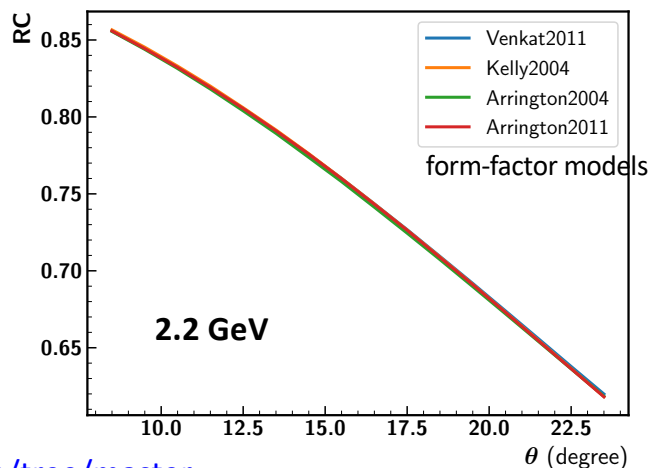
Elastic process
for Big Bite
using 1.2 (top) and
2.4 (bottom) GeV
beam on H₂

Data/Sim from
6 GeV era
experiment



Phys. Rev. C 95, 035209

At low beam energy, cross section difference between different calculations being **negligible**

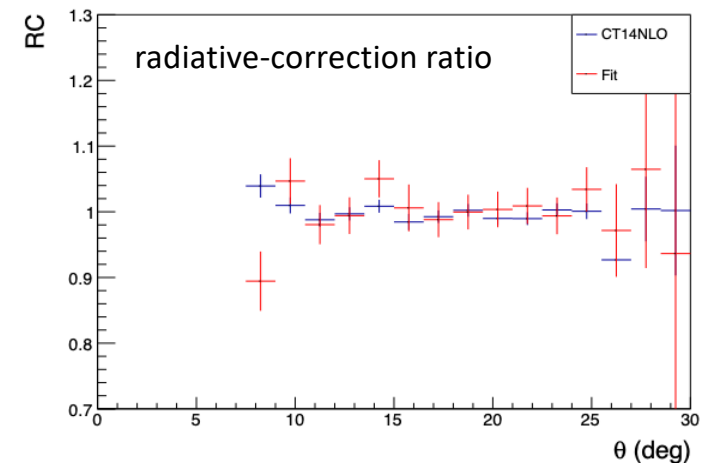
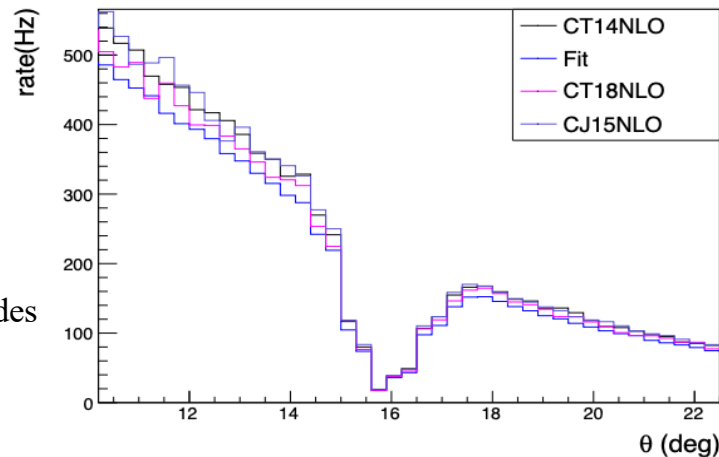


<https://github.com/JeffersonLab/PRadSim/tree/master>

Systematic Uncertainties of Unpolarized Cross Section: Acceptance Uncertainty from DIS

Deep Inelastic Scattering Process

- different global structure functions
- “Fits” (as Christy’s fits includes 12 GeV data)



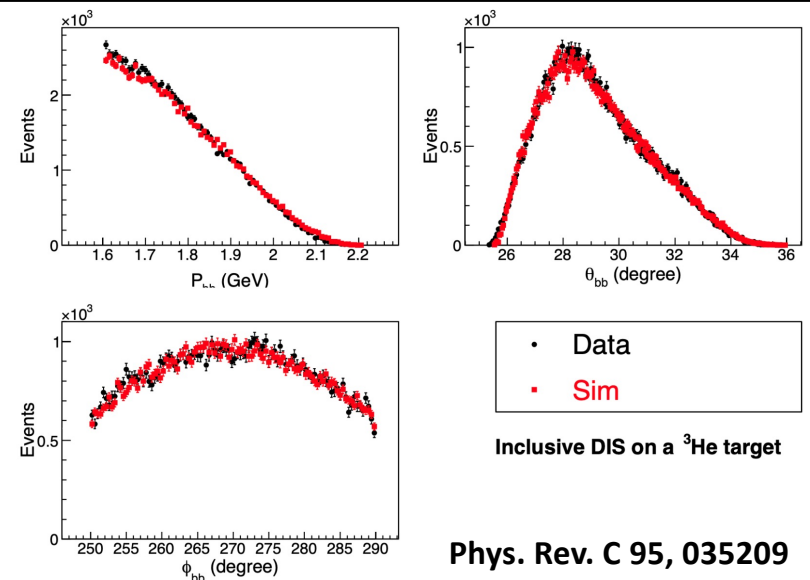
https://github.com/JeffersonLab/evgen_inclusive_e

Uncertainty from different DIS models being 3~4%,
and from RC being 3%

Data and Simulation comparison for DIS process from 6 GeV era,
for Big Bite spectrometer shown on right

Overall normalization can be calibrated by elastic process, then DIS
cross section shape to be assigned uncertainty of 3 ~ 4%

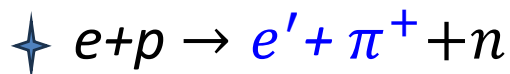
The total uncertainty is around 6%



Phys. Rev. C 95, 035209

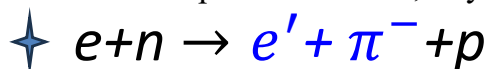
Systematic Uncertainties of Unpolarized Cross Section: Coincidence Acceptance Study Plan

10days of 11 GeV unpolarized hydrogen and deuterium runs (SIDIS transversely polarized ^3He experiment E12-10-006)
above the resonance region $d\sigma/dt(x_{bj}, Q^2)$



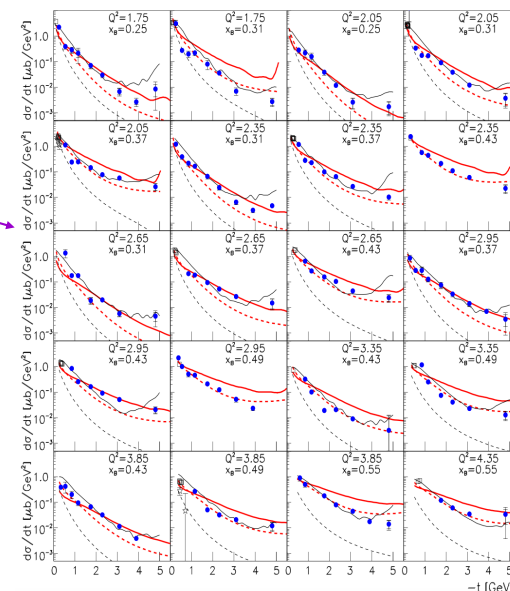
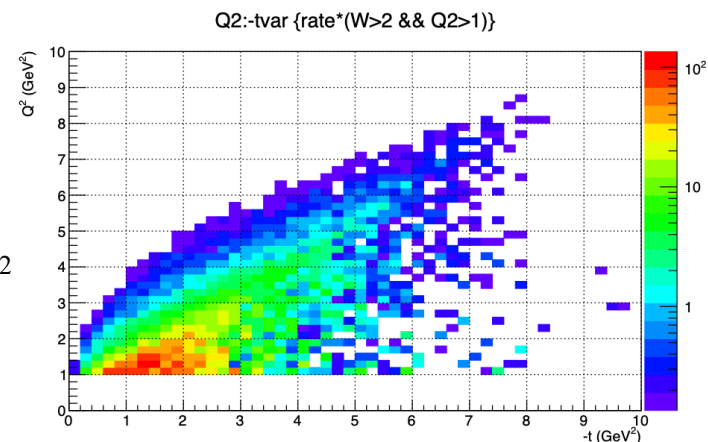
Proton target data

- Hall C
 - $Q^2=0.6-2.45\text{ GeV}^2$, $W=1.9$ and 2.0 GeV , $0.026\text{ GeV}^2 \leq -t \leq 0.365\text{ GeV}^2$
H. P. Blok and et.al., Phys. Rev. C 78, 045202 (2008)
 - $Q^2=2.4\text{ GeV}^2$, $W=2.0\text{ GeV}$, $0.272\text{ GeV}^2 < -t < 2.127\text{ GeV}^2$
S. Basnet and et. al, Phys. Rev. C 100 (2019) 6, 065204
- Hall B
 - $0.16 < x_{bj} < 0.58$, $1.6\text{ GeV}^2 < Q^2 < 4.5\text{ GeV}^2$ and $0.1\text{ GeV}^2 < -t < 5.3\text{ GeV}^2$
K. Park and et al., Phys. J. A 49, 16 (2013)
- HERMES
 - $0.02 < x_{bj} < 0.55$, $1\text{ GeV}^2 < Q^2 < 11\text{ GeV}^2$ and $-t < 2\text{ GeV}^2$
A. Airapetian and et al., Phys. Lett. B. 659, 486 (2008).



Deuterium target data

- Hall C
 - $Q^2=0.6-1.6\text{ GeV}^2$, $W=1.95$, $Q^2=2.45\text{ GeV}^2$, $W=2.2$
G. M. Huber and et al., Phys. Rev. C 91, 015202 (2015)

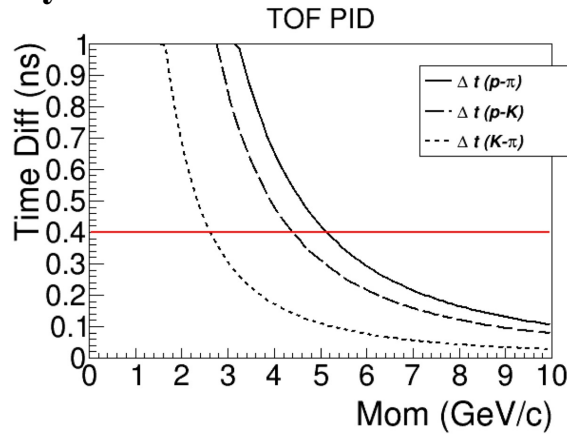


Further comparisons can be made with existing JLab 12 GeV data from Hall B and Hall C after being analyzed

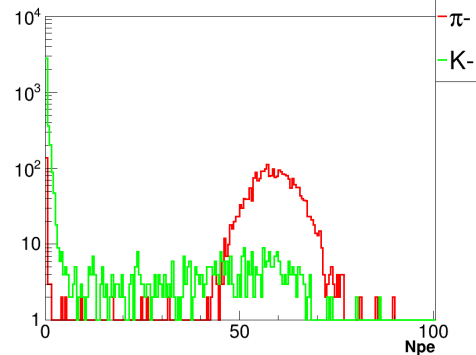
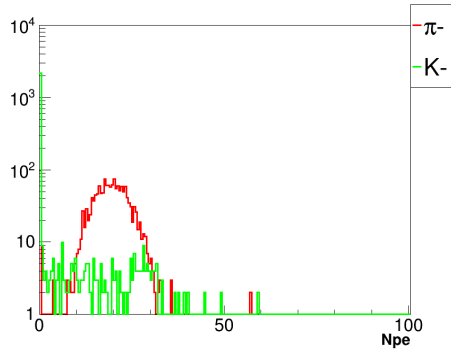
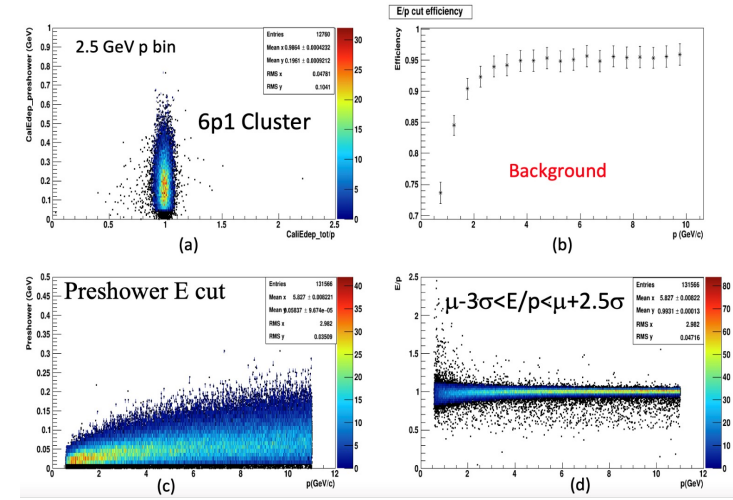
Systematic Uncertainties of Unpolarized Cross Section

➤ Pion PID uncertainty 4%

Below Cherenkov threshold, MRPC can separate particles using time of flight



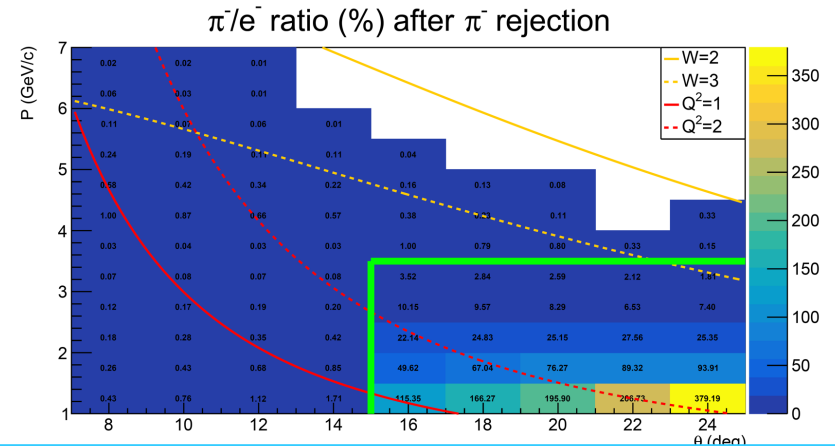
➤ Electron PID uncertainty < 2%



$2.5 < P < 3.0$ GeV
 $8^\circ < \theta < 9^\circ$

$7.0 < P < 7.5$ GeV
 $14^\circ < \theta < 15^\circ$

Pion results in red color, kaon results in green color



- (i) in FA ECal region, π^-/e^- ratio is $< 1.0\%$ for $P > 1.5$ GeV/c, while it is $< 2\%$ for $1.0 < P < 2.0$ GeV/c
- (ii) in LA ECal region, π^-/e^- ratio is $< 1.0\%$ for $P > 3.5$ GeV/c (the electron threshold in LA region)

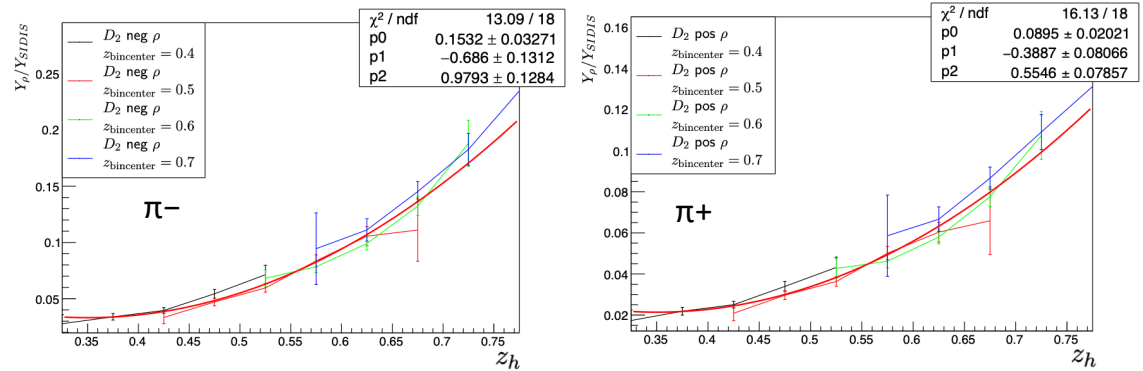
Systematic Uncertainties of Unpolarized Cross Section

➤ Other systematic uncertainty sources

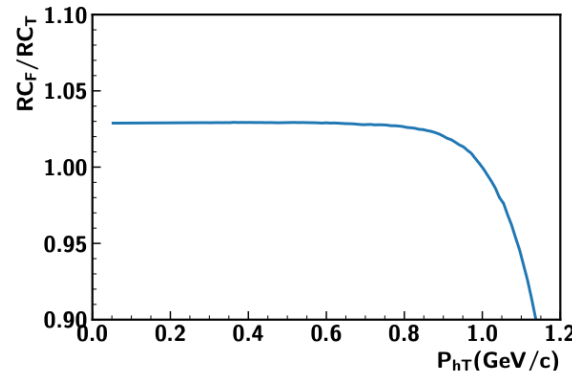
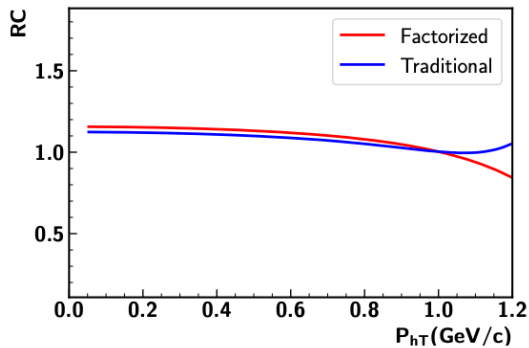
Diffractive ρ fraction to SIDIS for $x_{bj} = 0.35$, $Q^2 = 4$ (GeV/c)² for D_2

Discrepancy between models agreed to 10% according to 6 GeV era study

Multiplied by ρ yield ratio; **uncertainty is < 1%**



Obtained from SIMC HallC Simulation Package



x	z	Q^2 (GeV/c) ²	π_H^+ (%)	π_H^- (%)	π_D^+ (%)	π_D^- (%)
0.22	0.55	1.59	6.1 ± 0.2	–	3.7 ± 0.1	5.1 ± 0.2
0.26	0.55	1.88	5.2 ± 0.1	–	3.5 ± 0.1	5.1 ± 0.1
0.30	0.55	2.17	4.6 ± 0.1	–	3.4 ± 0.1	5.3 ± 0.1
0.34	0.55	2.46	4.6 ± 0.1	–	3.3 ± 0.1	5.1 ± 0.1
0.38	0.55	2.75	4.2 ± 0.1	–	2.9 ± 0.1	4.8 ± 0.1
0.42	0.55	3.04	3.8 ± 0.1	–	2.7 ± 0.1	4.9 ± 0.1
0.46	0.55	3.32	3.7 ± 0.1	–	2.6 ± 0.1	4.2 ± 0.1
0.50	0.55	3.61	3.1 ± 0.1	–	2.3 ± 0.1	3.6 ± 0.1
0.54	0.55	3.90	3.2 ± 0.1	–	1.9 ± 0.1	3.1 ± 0.1
0.58	0.55	4.19	2.5 ± 0.1	–	1.5 ± 0.1	2.5 ± 0.1

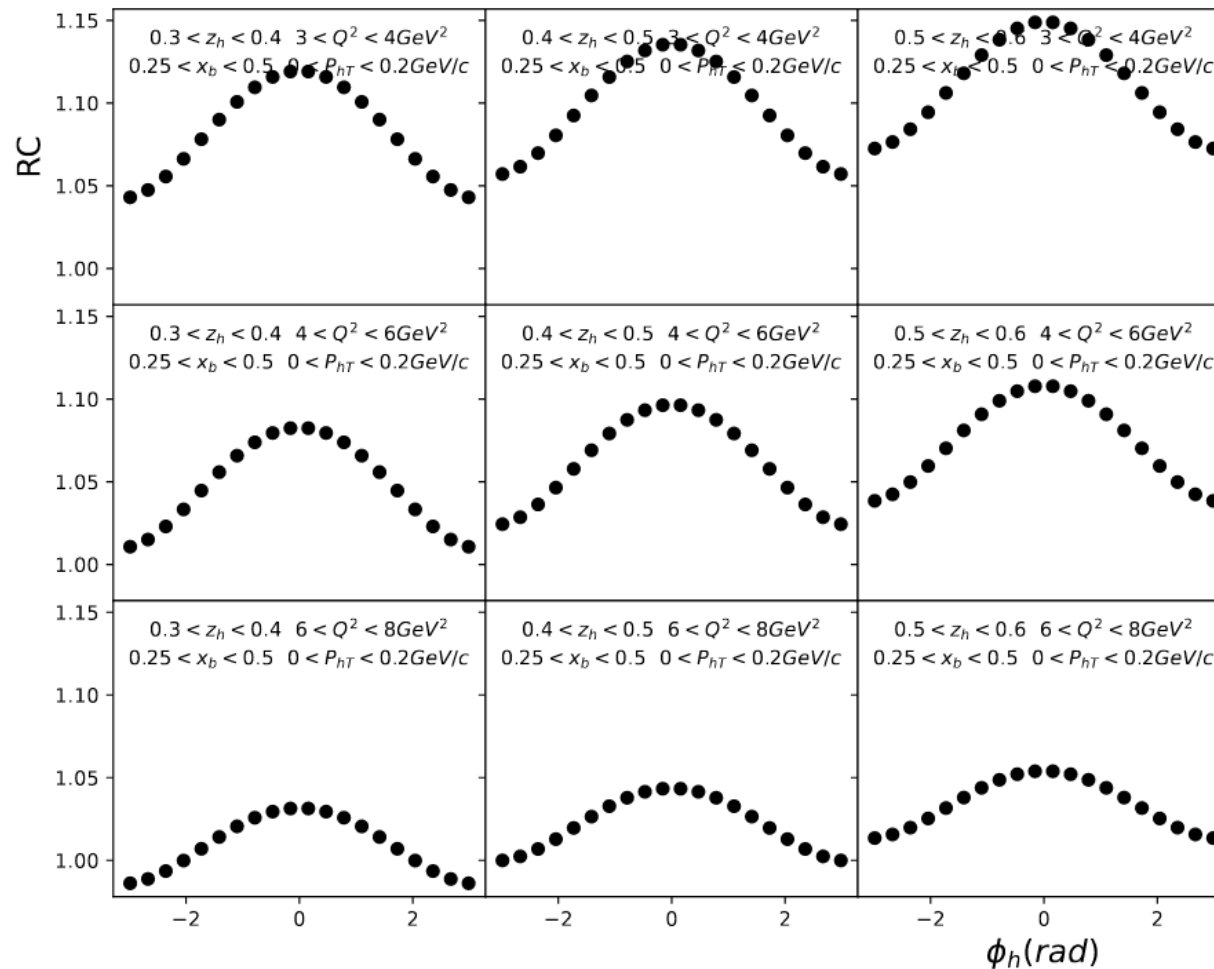
Radiative correction factor for typical JLab kinematic setting at $\sqrt{s} = 4.90$ GeV, $Q^2 = 8$ (GeV/c)², $z_h = 0.375$, $x_{bj} = 0.48$
Discrepancy between two methods is **around 2.5%**

https://indico.bnl.gov/event/18419/contributions/80386/attachments/49832/85265/Jia_Khachatryan_SIDIS-RC.pdf

Exclusive radiative tail yield to SIDIS yield ratio from 6 GeV era; decreasing with increasing Q^2
Discrepancy between models agreed to 10-15%; **uncertainty to be < 0.6%**

Systematic Uncertainties of Unpolarized Cross Section

Radiative corrections introduce an additional phi dependence



RC factors calculated based on using the traditional method (Lowest order QED radiative effects in polarized SIDIS) for several kinematic bins.

Systematic Uncertainty Budget for Unpolarized Cross Section

Charged pions

Sources	Uncertainty
Acceptance correction	6%
Experimental resolution	3.5%
Pion detection efficiency	4%
Electron detection efficiency	< 2%
Radiative corrections	2.5%
Radiative backgrounds	0.6%
Vector meson production	1%
Luminosity determination	2.5%
Total	< 10%

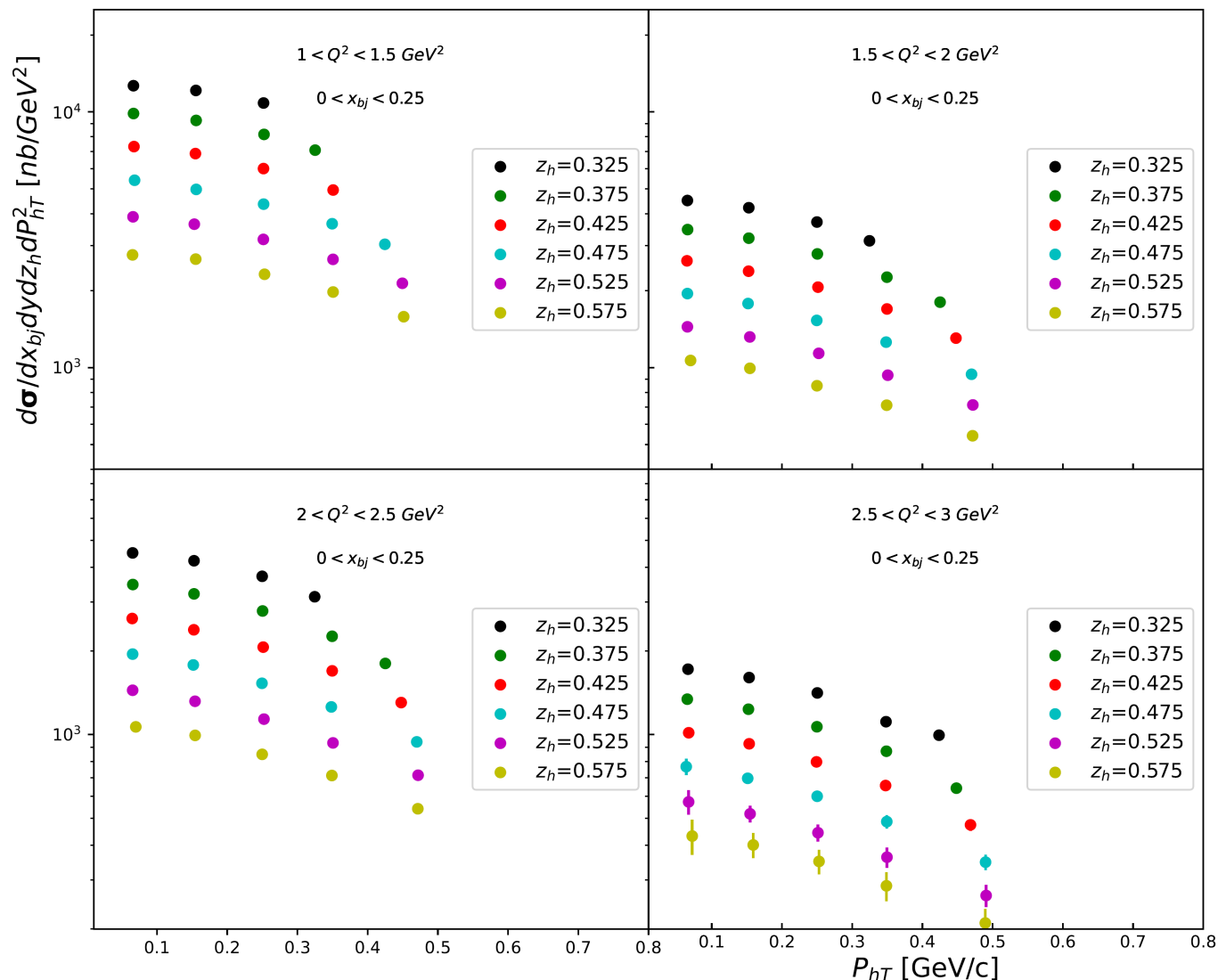
Charged kaons

Sources	Uncertainty
Acceptance correction	6%
Experimental resolution	3.5%
Kaon detection efficiency	12%
Electron detection efficiency	< 2%
Radiative corrections	2.5%
Radiative backgrounds	0.6%
Vector meson production	1%
Luminosity determination	2.5%
Total	< 15%

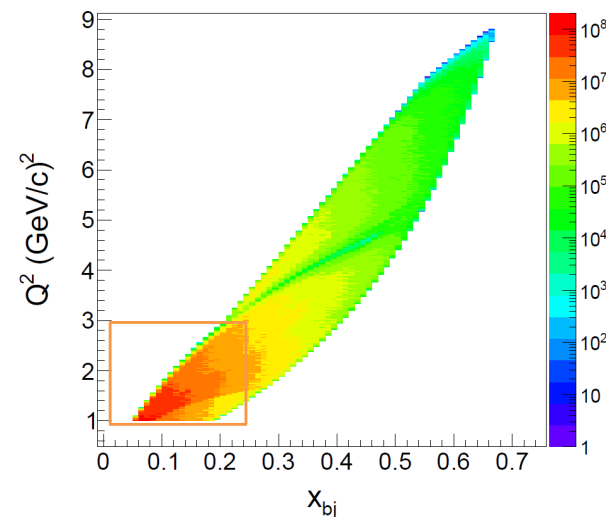
Total uncertainty calculated by rounding off the quadrature sum of separate contributions

More Physics Projections

➤ Produced π^+ unpolarized cross section at 11 GeV beam energy



SoLID low- Q^2 region



SoLID pseudo-data

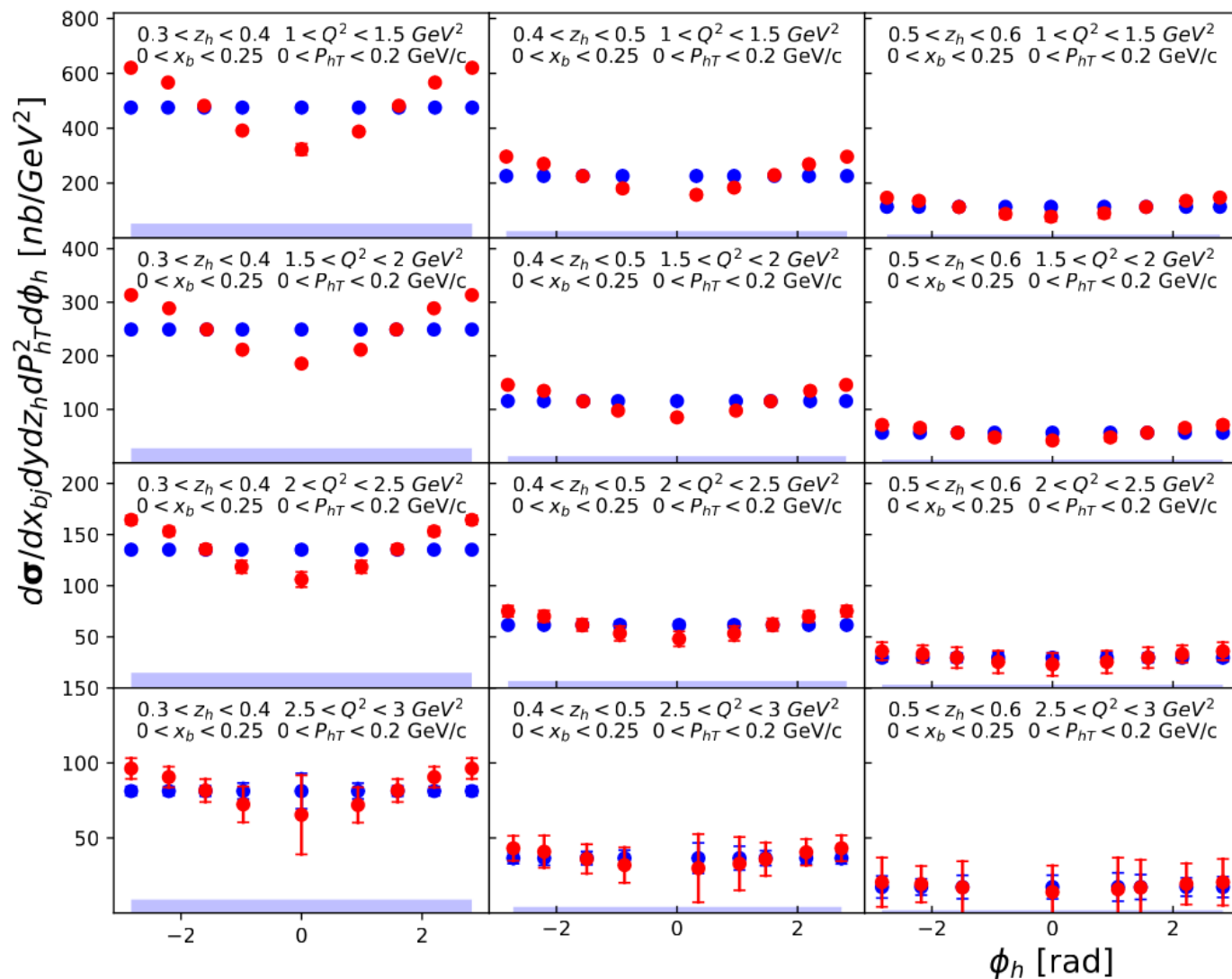
*Integrated cross section
shown with
MAP central points*

*Errors are
SoLID uncertainties*

More Physics Projections

➤ Produced π^+ unpolarized cross section at 11 GeV beam energy

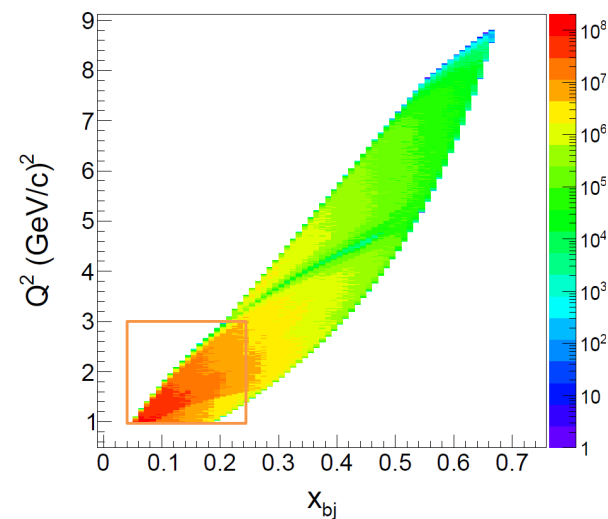
SoLID low- Q^2 region



SoLID pseudo-data
Central points from simple TMD
model

Blue points: Integrated cross
section

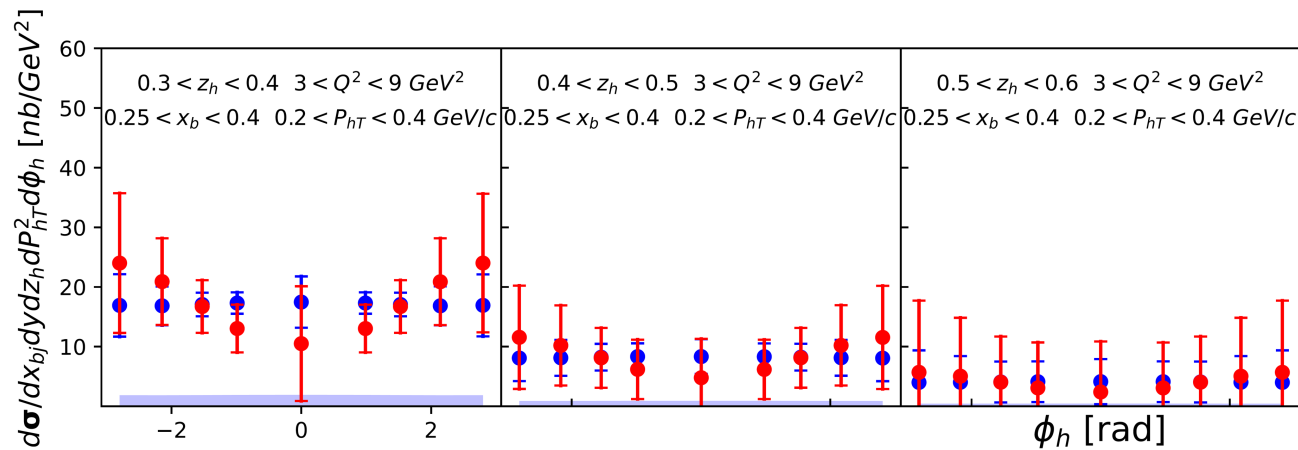
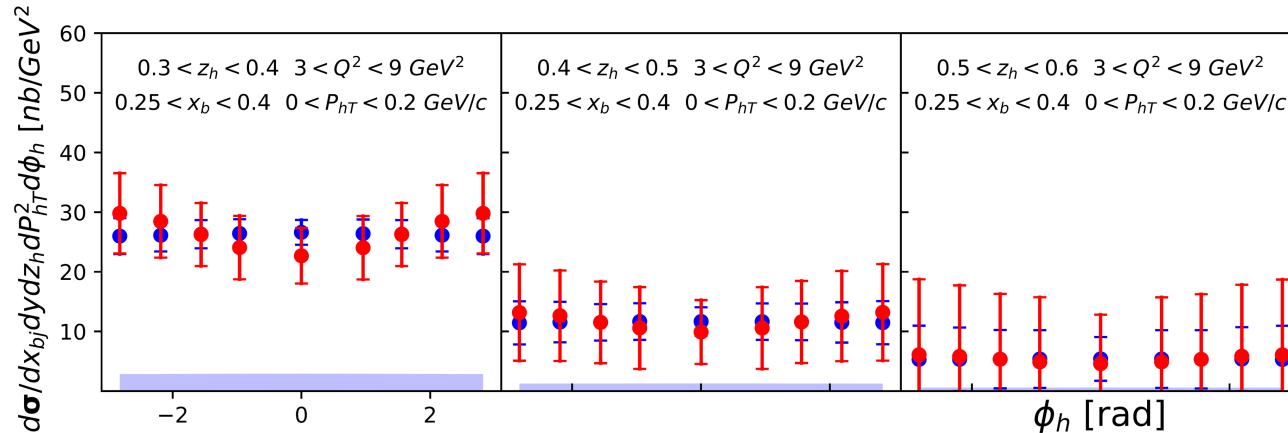
Red points: Cross section
including azimuthal modulations



More Physics Projections

➤ Produced π^+ unpolarized cross section at 11 GeV beam energy

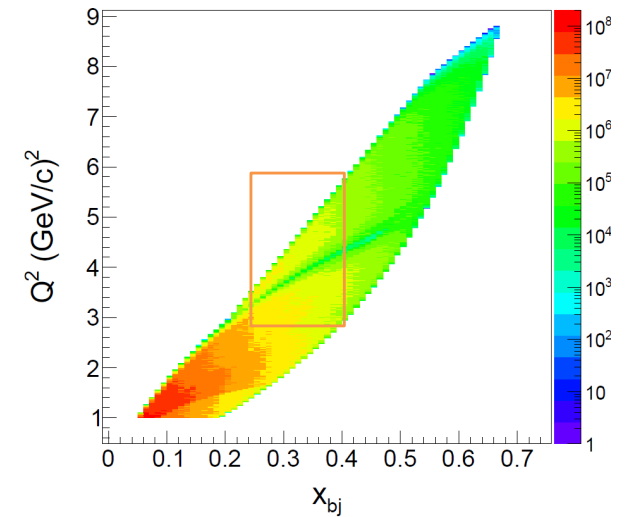
SoLID high- Q^2 region



SoLID pseudo-data
Central points from simple TMD model

Blue points: Integrated cross section

Red points: Cross section including azimuthal modulations

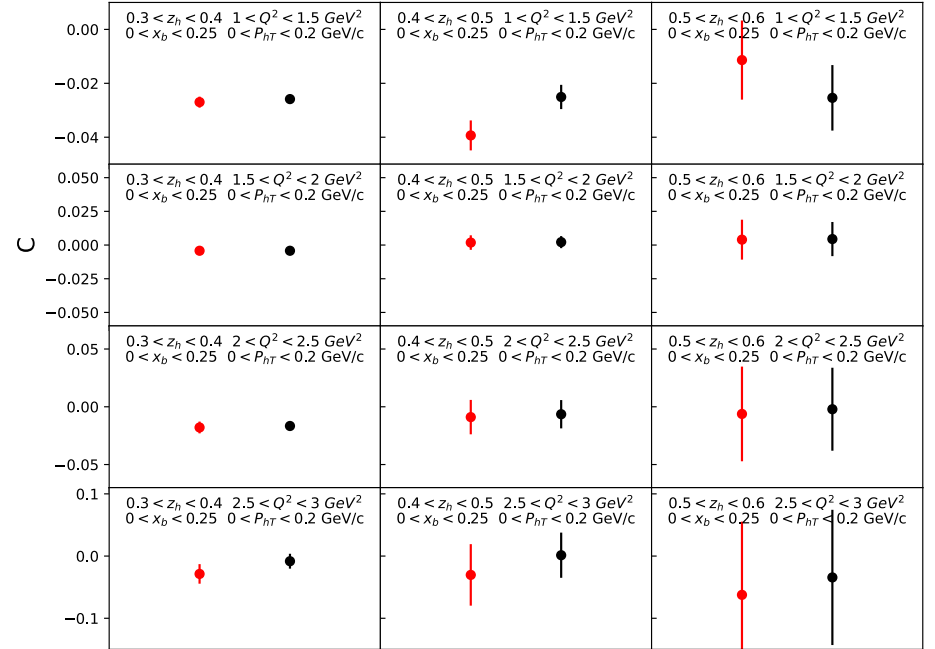
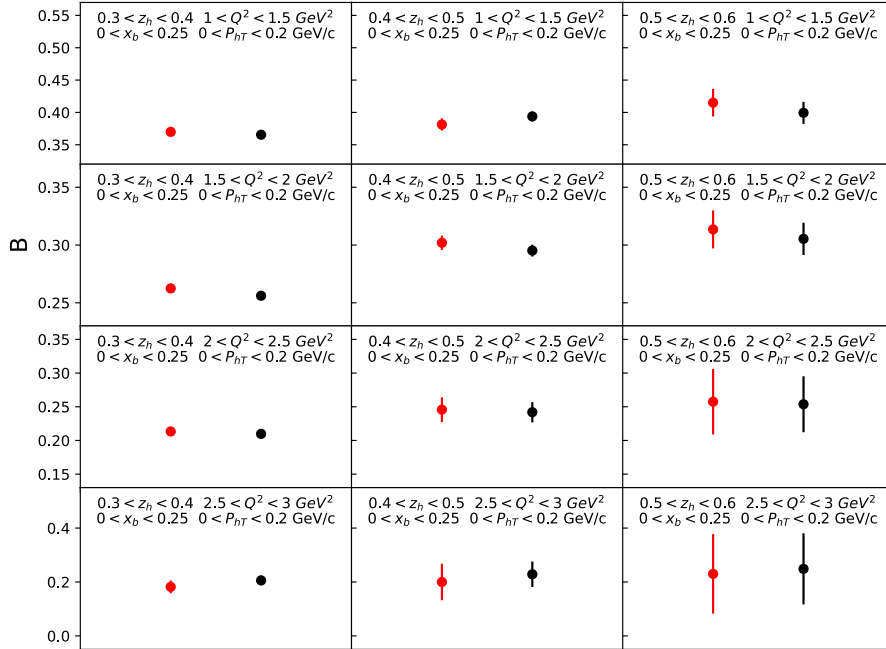


More Physics Projections

➤ Azimuthal modulation effect

$$\frac{d\sigma}{dx_{bj}dydz_h dP_{hT}^2 d\phi_h} \equiv \mathcal{F}_{\mathcal{U}} = \mathcal{F}_{\mathcal{U},A} \cos 0 + \mathcal{F}_{\mathcal{U},B} \cos(\phi_h) + \mathcal{F}_{\mathcal{U},C} \cos(2\phi_h)$$

Fitting ϕ_h distribution with a simple function: $A(1 - B \cdot \cos(\phi_h) - C \cdot \cos(2\phi_h))$



Red points for π^+ , black points for π^-

More Physics Projections

➤ Transverse momentum widths

$$F_{UU} = \sum_q e_q^2 x_{bj} f_q(x_{bj}) D_q(z_h) \frac{e^{-P_{hT}^2 / \langle P_{hT}^2 \rangle}}{\pi \langle P_{hT}^2 \rangle}$$

$$F_{UU}^{\cos(\phi_h)} = F_{UU}^{\cos(\phi_h)} \Big|_{\text{Cahn}} + F_{UU}^{\cos(\phi_h)} \Big|_{\text{BM}}$$

$$F_{UU}^{\cos(2\phi_h)} \approx F_{UU}^{\cos(2\phi_h)} \Big|_{\text{Cahn}} + F_{UU}^{\cos(2\phi_h)} \Big|_{\text{BM}}$$

where $\langle P_{hT}^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle$

In model, we have (in GeV²)

$$\langle k_{\perp}^2 \rangle = 0.604, \langle p_{\perp}^2 \rangle = 0.114$$

$$\text{Least_Square} = \sum (\text{pseudodata} - \text{Model})^2 / (\text{stat} + \text{sys})^2$$

The fitting results shows (in GeV²):

$$\langle k_{\perp}^2 \rangle = 0.5871 \pm 0.0002 \text{ (GeV/c)}^2$$

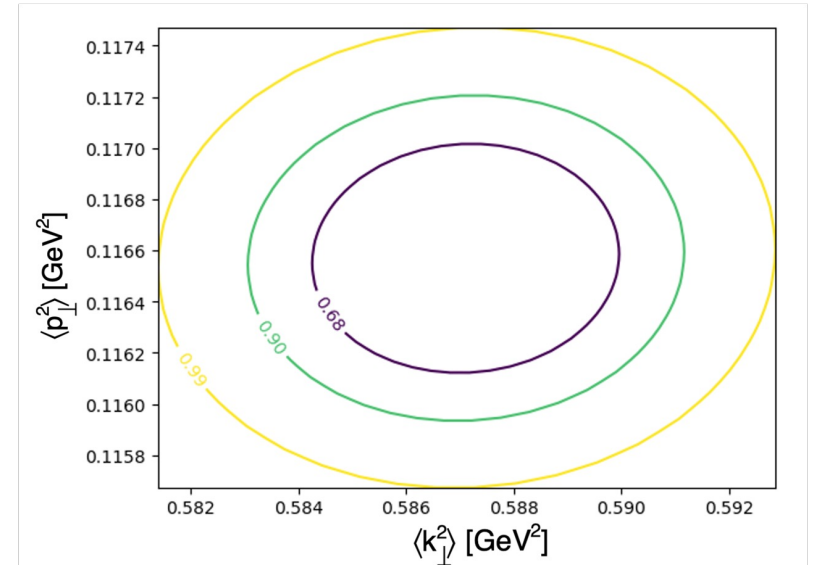
$$\langle p_{\perp}^2 \rangle = 0.1165 \pm 0.0003 \text{ (GeV/c)}^2$$

Three contours corresponding to confidence levels of 68%, 90% and 99%

Both Cahn and Boer-Mulders contributions included

All data from positive and negative polarities are considered

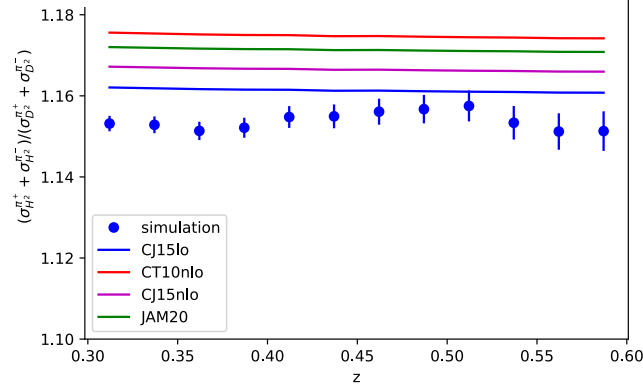
The fitting results differs from the model by 4%



More Physics Projections

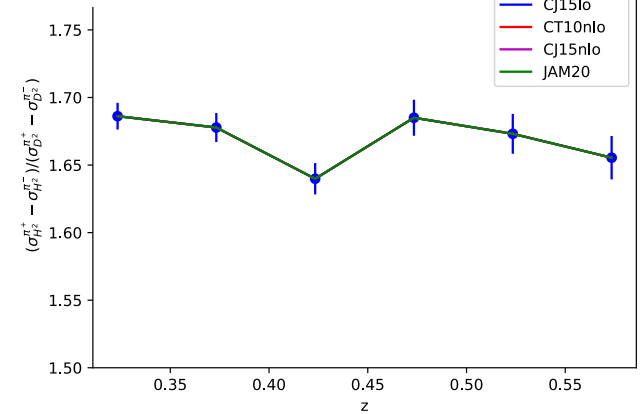
➤ Test of factorization

$$\frac{Y_p^{\pi^+} + Y_p^{\pi^-}}{Y_d^{\pi^+} + Y_d^{\pi^-}} = \frac{4u + 4\bar{u} + d + \bar{d}}{5(u + \bar{u} + d + \bar{d})}$$

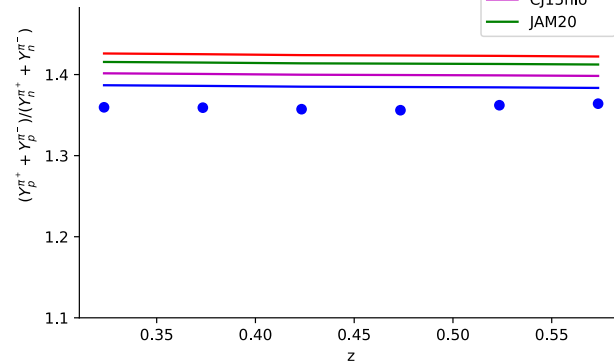


Assume no P_{hT} dependence and ignore heavy quark contributions

$$\frac{Y_p^{\pi^+} - Y_p^{\pi^-}}{Y_d^{\pi^+} - Y_d^{\pi^-}} = \frac{4u_v - d_v}{3(u_v + d_v)}$$

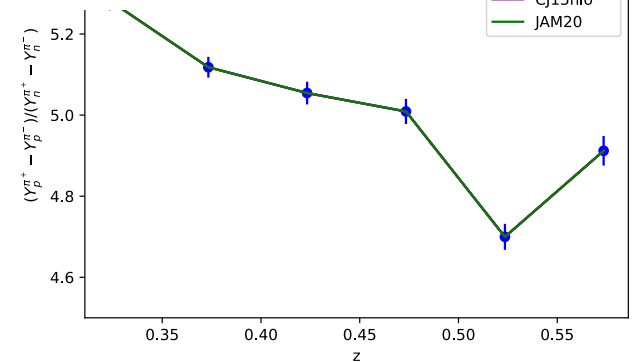


$$\frac{Y_p^{\pi^+} + Y_p^{\pi^-}}{Y_n^{\pi^+} + Y_n^{\pi^-}} = \frac{4u + 4\bar{u} + d + \bar{d}}{u + \bar{u} + 4d + 4\bar{d}}$$



Write two types of simple ratios

$$\frac{Y_p^{\pi^+} - Y_p^{\pi^-}}{Y_n^{\pi^+} - Y_n^{\pi^-}} = \frac{4u_v - d_v}{4d_v - uv}$$



Summary and Outlook

- With high luminosity and large acceptance, SoLID could provide **high-precision** SIDIS unpolarized cross-section data with **full azimuthal angular coverage**

- The updated run-group proposal includes
 - Detailed systematic uncertainty studies
 - Physics impact results for unpolarized TMD using state-of-the-art (MAP framework)
 - SoLID projections for unpolarized **integrated** cross section using MAP and TMD factorization
 - SoLID projections for unpolarized cross section with **azimuthal modulations** using TMD factorization
 - Some results on **azimuthal modulation effects and Gaussian width parameters**
 - Some results on **test of factorization**

- Calibration planned for unpolarized cross-section measurement

Thank You !

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Supported in part by U.S. Department of Energy under contract numbers:
DE-FG02-03ER41231 and DE-FG02-84ER40146

Backups

Calibration Plan

- Calibration studies for unpolarized cross-section measurement of this proposal

Target	Beam energy (GeV)	Field	Time (hour)	Purpose
H ₂ reference cell	2.2	Normal	1	Acceptance study
Empty reference cell	2.2	Normal	1	Backgrounds subtraction
Carbon	2.2	Normal	1	Acceptance study
H ₂ reference cell	4.4	Normal	1	Acceptance study
Empty reference cell	4.4	Normal	1	Backgrounds subtraction
Carbon	4.4	Normal	1	Acceptance study

We are not asking for
new beam time !!!

Reference cell runs, optics and detector check : 3 Days
(using approved calibration time)

Calibration Plan

Details of the beam time request for the transversely polarized ^3He experiment E12-10-006 (and for the longitudinally polarized ^3He experiment E12-11-007 shown in parentheses).

	Time (Hour)	Time (Day)
Production on Pol. ^3He at 11 GeV	1152	48
Production on Pol. ^3He at 8.8 GeV	504	21
Longitudinal on Pol. ^3He at 11 GeV	38(538)	2(22.5)
Longitudinal on Pol. ^3He at 8.8 GeV	24(228)	1(9.5)
Dedicated Hydrogen run at 11 GeV	84	3.5
Dedicated Deuterium run at 11 GeV	84	3.5
Dedicated Hydrogen run at 8.8 GeV	36	1.5
Dedicated Deuterium run at 8.8 GeV	36	1.5
Reference cell runs, optics and detector check	72	3
Target Overhead: spin rotation, polarization measurement	120(60)	5(2.5)
Total Time Request	2160(826)	90(34.5) days

Target	Beam energy (GeV)	Field	Time (hour)
Carbon	8.8	Normal	1
Carbon	8.8	50%	1
Carbon	8.8	0%	1
Carbon	11	Normal	1
Carbon	11	50%	1
Carbon	11	0%	1

Calibration arrangement for related detector alignment and particle tracking, for unpolarized cross-section measurement

- Momentum coverage: 1.0 - 7.0 GeV/c; Polar angular coverage: 8.0° - 14.8°
(for hadron & electron ID)
- Momentum coverage: 3.5 - 6.0 GeV/c; Polar angular coverage: 15.7° - 24.0°
(for electron ID)
- Momentum resolution: ~ 2%; Polar angular resolution: 2 mrad
- Azimuthal angular coverage: 2π ; Azimuthal angular resolution: 6 mrad
- PID (electron): detection efficiency $\geq 90\%$; pion contamination $< 1\%$
- PID (pion): detection efficiency $\geq 90\%$; kaon contamination $< 1\%$
- Total luminosity: $3.74 \cdot 10^{36} \text{ cm}^{-2} \text{ sec}^{-1}$
- Beam polarimetry: $< 3\%$; Beam current: 15 μA
- Many other details in **SoLID (Solenoidal Large Intensity Device) Updated Preliminary Conceptual Design Report**, <https://solid.jlab.org/>

$$F_{UU,B} = 2\pi \frac{\alpha^2}{x_{bj} y Q^2} \left(1 + \frac{\gamma^2}{2x_{bj}} \right) c_2 F_{UU}^{\cos(\phi_h)},$$

- The second structure function $F_{UU}^{\cos(\phi_h)}$, associated to the $\cos(\phi_h)$ modulation of the cross section, is a twist-3 quantity of the order of $1/Q$

$$F_{UU}^{\cos(\phi_h)} = F_{UU}^{\cos(\phi_h)} \Big|_{\text{Cahn}} + F_{UU}^{\cos(\phi_h)} \Big|_{\text{BM}}$$

where

$$F_{UU}^{\cos(\phi_h)} \Big|_{\text{Cahn}} = -2 \sum_q e_q^2 x \int d^2 \mathbf{k}_\perp \frac{(\mathbf{k}_\perp \cdot \mathbf{h})}{Q} f_q(x, k_\perp) D_q(z, p_\perp)$$

as the Cahn convolution of unpolarized **TMD PDF** and **TMD FF**

$$F_{UU}^{\cos(\phi_h)} \Big|_{\text{BM}} = \sum_q e_q^2 x \int d^2 \mathbf{k}_\perp \frac{k_\perp}{Q} \frac{P_{hT} - z (\mathbf{k}_\perp \cdot \mathbf{h})}{k_\perp} \Delta f_{q\uparrow/p}(x, k_\perp) \Delta D_{h/q\uparrow}(z, p_\perp)$$

as the Boer-Mulders convolution of **Boer-Mulders TMD PDF** and **Collins TMD FF**

➤ Analytical forms of the Cahn and Boer-Mulders azimuthal modulation given by

$$\begin{aligned}
 F_{UU} &= \sum_q e_q^2 x_{bj} f_q(x_{bj}) D_q(z_h) \frac{e^{-P_{hT}^2/\langle P_{hT}^2 \rangle}}{\pi \langle P_{hT}^2 \rangle}, \\
 F_{UU}^{\cos(\phi_h)} \Big|_{\text{Cahn}} &= -2 \frac{P_{hT}}{Q} \sum_q e_q^2 x_{bj} f_q(x_{bj}) D_q(z_h) \frac{z_h \langle k_{\perp}^2 \rangle}{\langle P_{hT}^2 \rangle} \frac{e^{-P_{hT}^2/\langle P_{hT}^2 \rangle}}{\pi \langle P_{hT}^2 \rangle}, \\
 F_{UU}^{\cos(\phi_h)} \Big|_{\text{BM}} &= 2e \frac{P_{hT}}{Q} \sum_q e_q^2 x_{bj} \frac{\Delta f_{q\uparrow/p}(x_{bj})}{M_{\text{BM}}} \frac{\Delta D_{h/q\uparrow}(z_h)}{M_{\text{C}}} \frac{e^{-P_{hT}^2/\langle P_{hT}^2 \rangle_{\text{BM}}}}{\pi \langle P_{hT}^2 \rangle_{\text{BM}}^4} \\
 &\quad \times \frac{\langle k_{\perp}^2 \rangle_{\text{BM}}^2 \langle p_{\perp}^2 \rangle_{\text{C}}^2}{\langle k_{\perp}^2 \rangle \langle p_{\perp}^2 \rangle} \left[z_h^2 \langle k_{\perp}^2 \rangle_{\text{BM}} (P_{hT}^2 - \langle P_{hT}^2 \rangle_{\text{BM}}) + \langle p_{\perp}^2 \rangle_{\text{C}} \langle P_{hT}^2 \rangle_{\text{BM}} \right], \\
 F_{UU}^{\cos(2\phi_h)} \Big|_{\text{Cahn}} &= 2 \frac{P_{hT}^2}{Q^2} \sum_q e_q^2 x_{bj} f_q(x_{bj}) D_q(z_h) \frac{z_h^2 \langle k_{\perp}^2 \rangle^2}{\langle P_{hT}^2 \rangle^2} \frac{e^{-P_{hT}^2/\langle P_{hT}^2 \rangle}}{\pi \langle P_{hT}^2 \rangle}, \\
 F_{UU}^{\cos(2\phi_h)} \Big|_{\text{BM}} &= -e P_{hT}^2 \sum_q e_q^2 x_{bj} \frac{\Delta f_{q\uparrow/p}(x_{bj})}{M_{\text{BM}}} \frac{\Delta D_{h/q\uparrow}(z_h)}{M_{\text{C}}} \frac{e^{-P_{hT}^2/\langle P_{hT}^2 \rangle_{\text{BM}}}}{\pi \langle P_{hT}^2 \rangle_{\text{BM}}^3} \\
 &\quad \times \frac{z_h \langle k_{\perp}^2 \rangle_{\text{BM}}^2 \langle p_{\perp}^2 \rangle_{\text{C}}^2}{\langle k_{\perp}^2 \rangle \langle p_{\perp}^2 \rangle},
 \end{aligned}$$

where

$$\langle P_{hT}^2 \rangle_{\text{BM}} = \langle p_{\perp}^2 \rangle_{\text{C}} + z_h^2 \langle k_{\perp}^2 \rangle_{\text{BM}}$$

$$\langle p_{\perp}^2 \rangle_{\text{C}} = \frac{\langle p_{\perp}^2 \rangle M_{\text{C}}^2}{\langle p_{\perp}^2 \rangle + M_{\text{C}}^2}$$

$$\langle k_{\perp}^2 \rangle_{\text{BM}} = \frac{\langle k_{\perp}^2 \rangle M_{\text{BM}}^2}{\langle k_{\perp}^2 \rangle + M_{\text{BM}}^2}$$

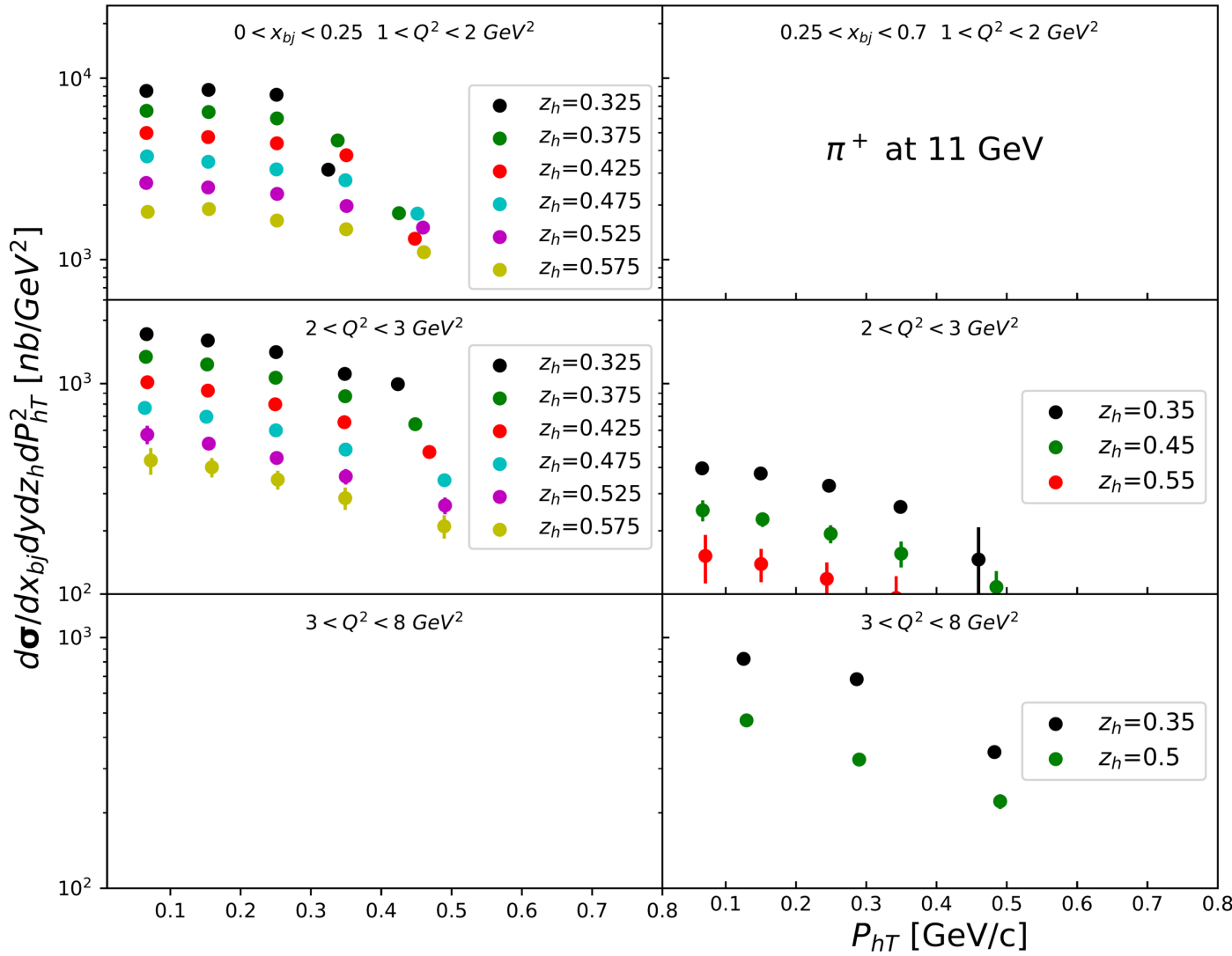
M_{C}^2 and M_{BM}^2 and all the other functional forms to be found in

JHEP 06, 007 (2019)

and

<https://github.com/TianboLiu/LiuSIDIS>

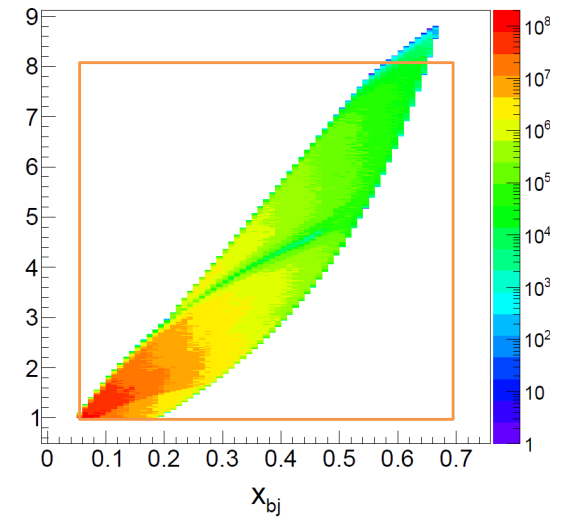
➤ Produced π^+ unpolarized cross section at 11 GeV beam energy



SoLID low- Q^2 and high- Q^2 regions

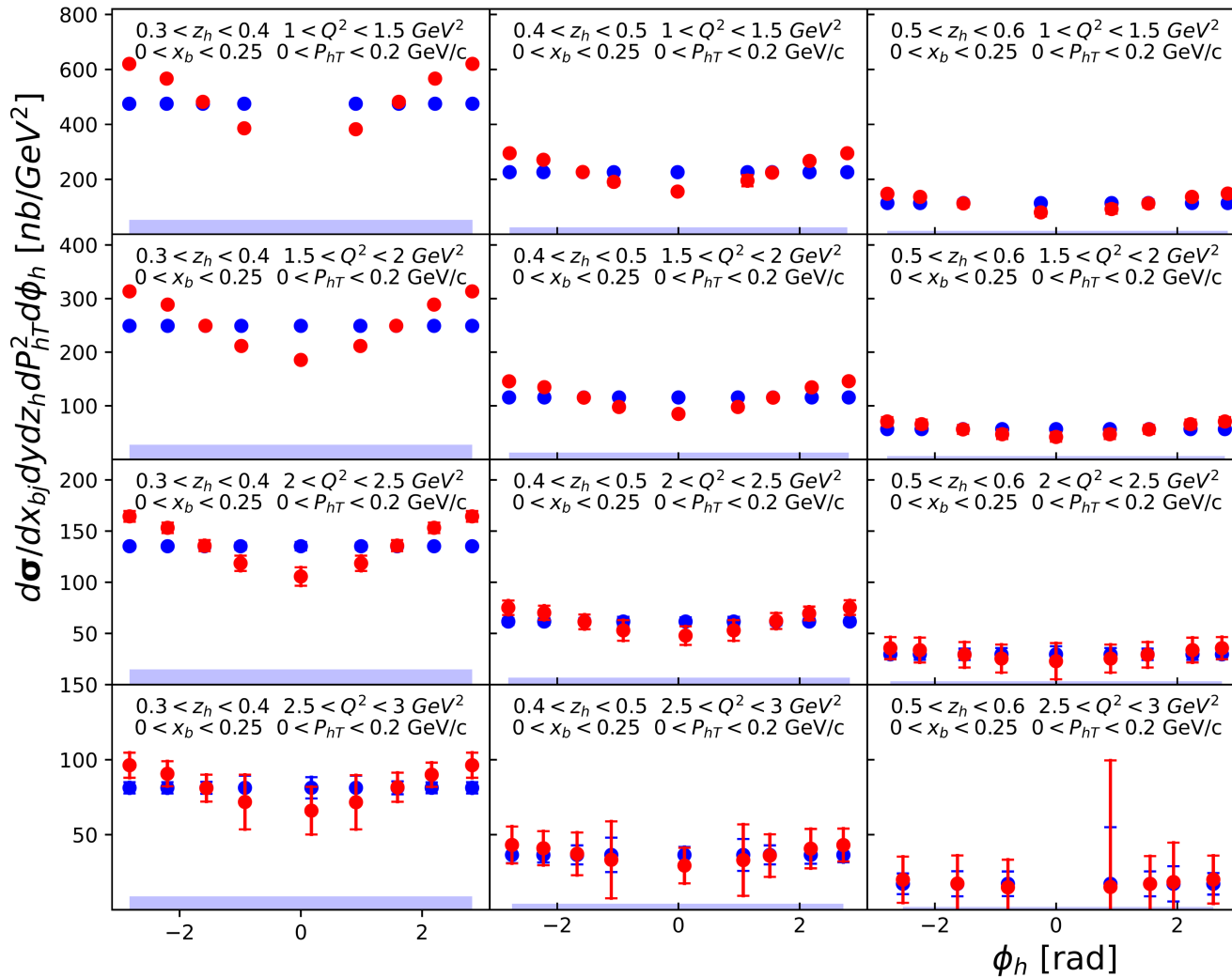
SoLID pseudo-data
Integrated cross section shown with MAP central points

Errors are SoLID uncertainties

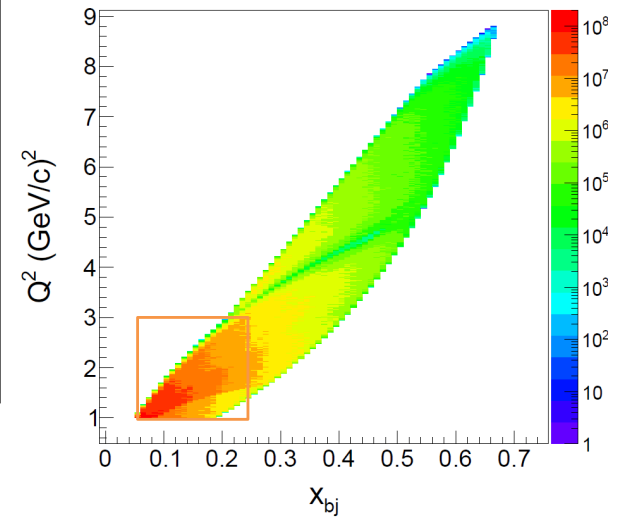


➤ Produced $\underline{\pi}^-$ unpolarized cross section at 11 GeV beam energy

SoLID low- Q^2 region

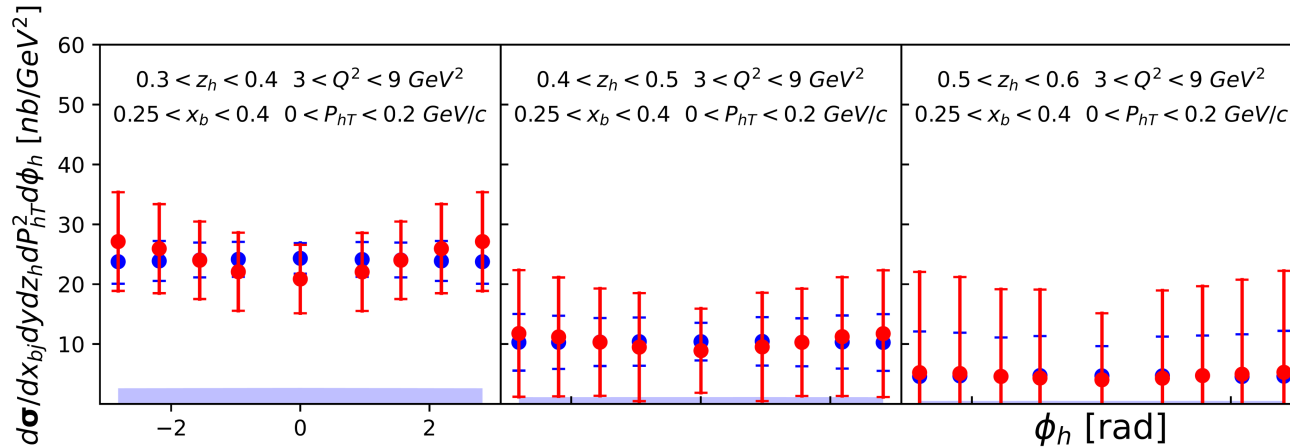


SoLID pseudo-data
 Integrated cross section shown with **blue central points** from simple TMD model
 Cross section including azimuthal modulations shown with **red central points** from simple TMD model



➤ Produced $\underline{\pi}^-$ unpolarized cross section at 11 GeV beam energy

SoLID high- Q^2 region



SoLID pseudo-data
 Integrated cross section shown with **blue central points** from simple TMD model
 Cross section including azimuthal modulations shown with **red central points** from simple TMD model

