DOE contracts:
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CHRISTOPHER NEWPORT UNIVERSITY

Jefferson Lab

Measurement of the Unpolarized SIDIS Cross Section with SoLID:

advanced MAP framework and physics impact results

Matteo Cerutti

on behalf of the spokespersons

U. D'Alesio, S. Jia, V. Khachatryan,Y. Tian

Transverse-Momentum Distributions (TMDs)

3-***dimensional*** ***map*** of the internal structure of the nucleon

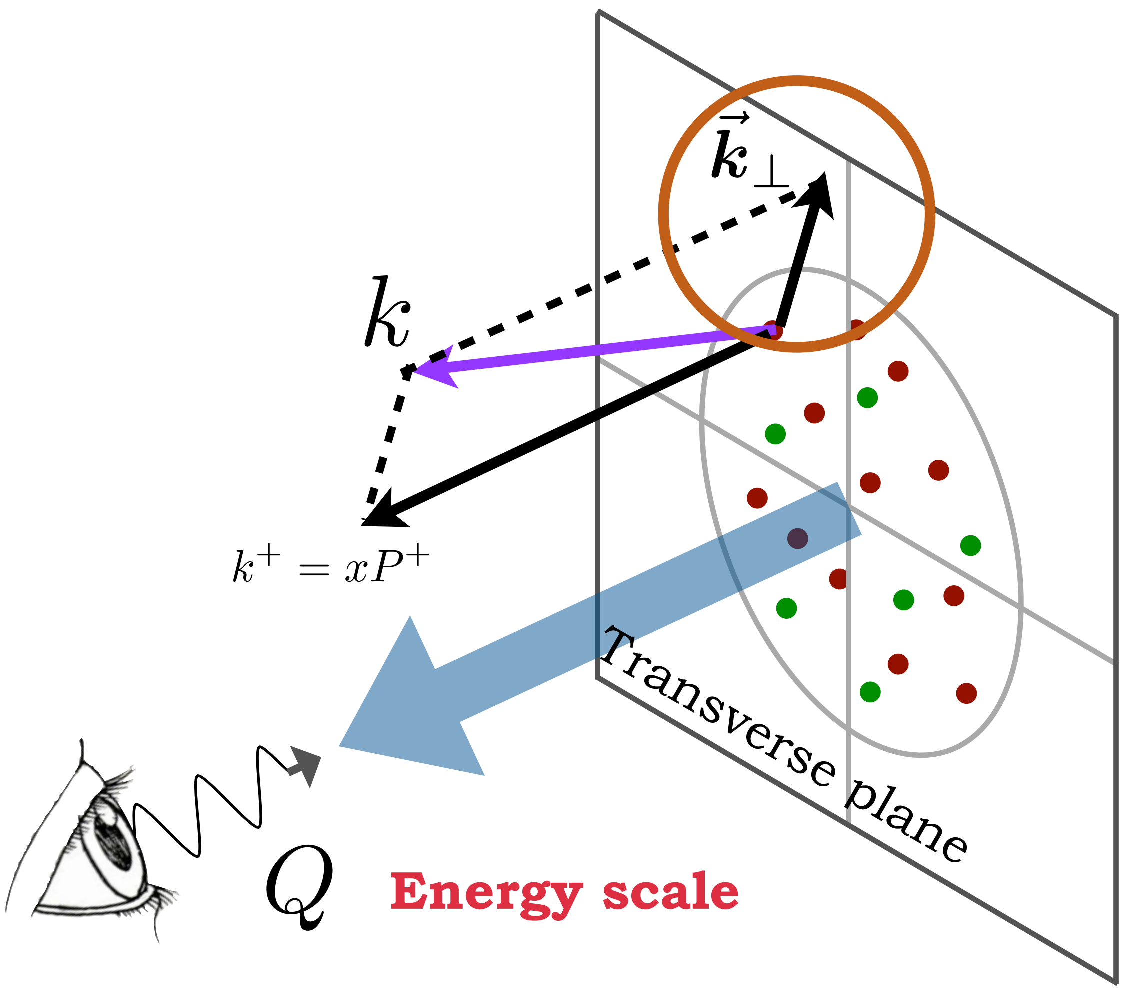
Non-collinear framework

Quark Polarization

Nucleon Pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 \, h_{1T}^\perp$

Time-reversal odd   Time-reversal even



TMD PDFs

$$F(x, \boldsymbol{k}_\perp^2, \mu, \zeta)$$

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3-***dimensional map*** of the internal structure of the nucleon

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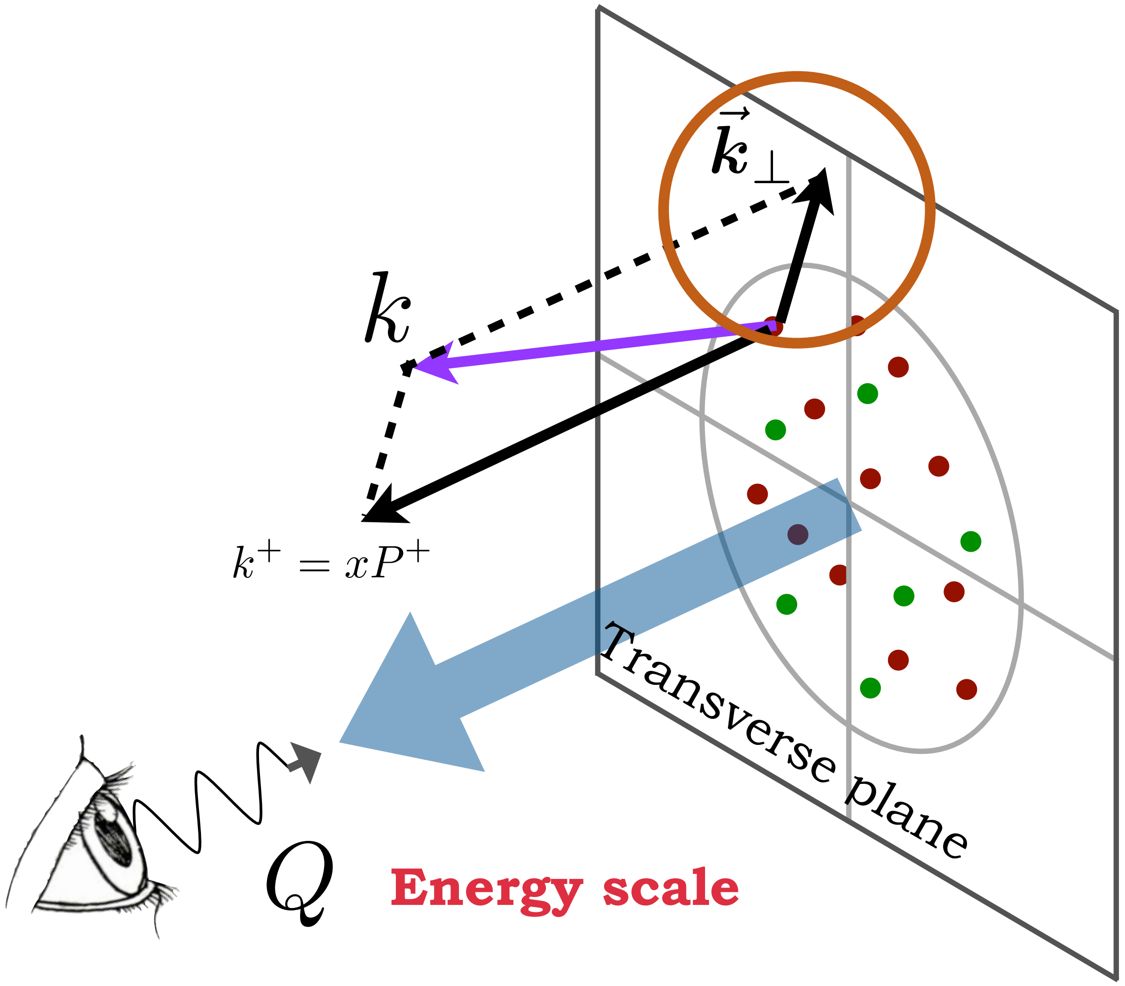


Quark Polarization

Nucleon Pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Time-reversal odd   Time-reversal even

TMD PDFs

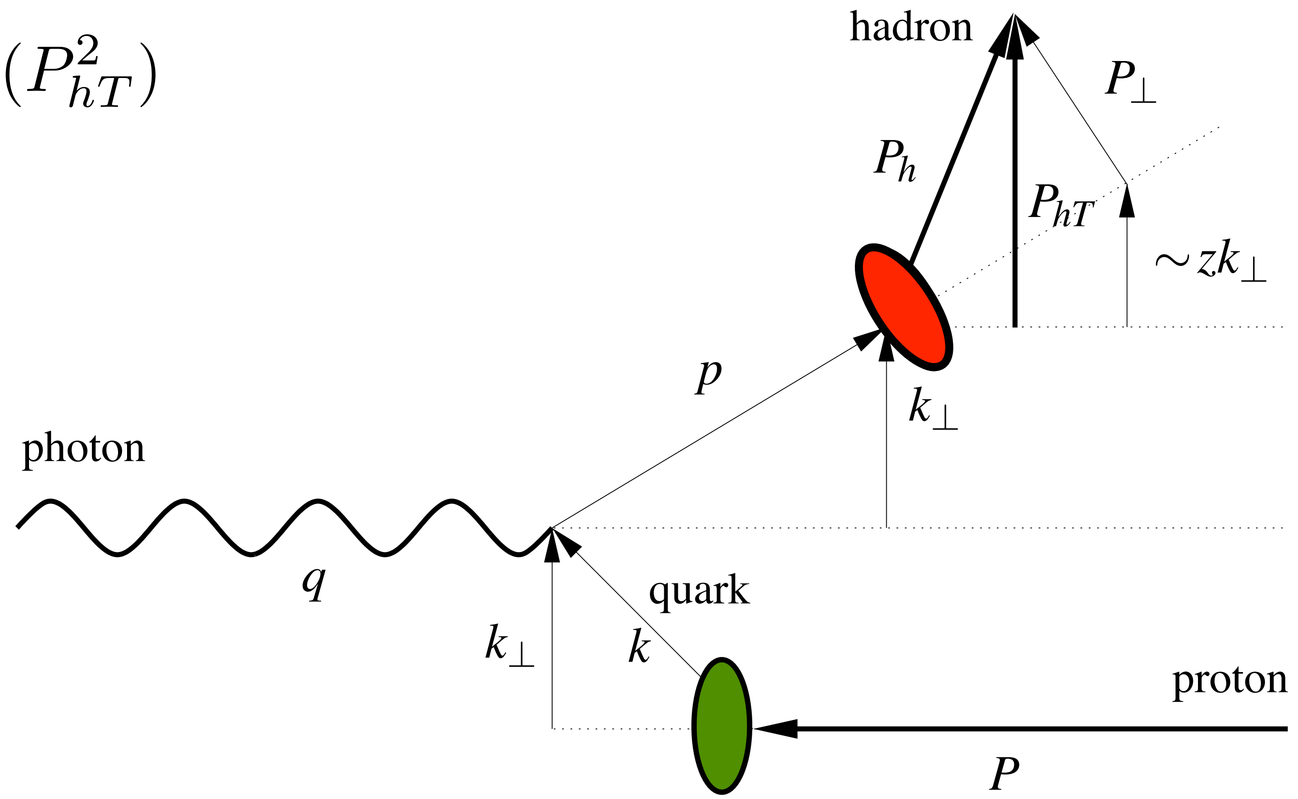
$$F(x, \boldsymbol{k}_\perp^2, \mu, \zeta)$$

SIDIS structure function (TMD factorization)

If $Q^2 \gg M^2$ and $Q^2 \gg q_T^2 (P_{hT}^2)$

TMD FF

TMD PDF



$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2)$$
$$= x \sum_q \mathcal{H}_{UU,T}^q (Q^2, \mu^2) \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x, \boldsymbol{k}_\perp^2; \mu^2) \, D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; \mu^2) \, \delta(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$
$$= x \sum_a \mathcal{H}_{UU,T}^q (Q^2, \mu^2) \int db_T b_T J_0(b_T |\boldsymbol{P}_{h\perp}|) \hat{f}_1^q(x, z^2 b_\perp^2; \mu^2) \, \hat{D}_1^{a\to h}(z, b_\perp^2; \mu^2)$$

- The **W term** dominates in the region where $q_T \ll Q$

Bacchetta, Diehl, et al., JHEP 02 (2007)
Collins, "Foundations of Perturbative QCD"

Expression of a TMD (CSS formalism)

TMD in Fourier space

$$\hat{F}(x, b_T^2; \mu, \zeta) = \int \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp} F(x, k_\perp^2; \mu, \zeta)$$

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$$\hat{f}_1^q(x, b_T^2; \mu, \zeta) = \sum_j C_{q/j}(x, b_*; \mu_{b_*}, \mu_{b_*}^2) \otimes f_1^j(x, \mu_{b_*}) \qquad : \mathrm{A}$$

Perturbative TMD at the initial scale

J. Collins, “Foundation of Perturbative QCD”

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Perturbative TMD at the initial scale

$$\times \exp\left\{ K(b_*; \mu_{b_*}) \ln \frac{\sqrt{\zeta}}{\mu_{b_*}} + \int_{\mu_{b_*}}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta}}{\mu'} \right] \right\} \quad : \mathrm{B}$$

Evolution to final scale (of the process)

J. Collins, “Foundation of Perturbative QCD”

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Evolution to final scale (of the process)

$$\times f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \quad : \text{C}$$

Non-perturbative part of the TMD

J. Collins, “Foundation of Perturbative QCD”

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Perturbative

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Collinear extractions

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Non-perturbative part of the TMD

Parameterization

J. Collins, "Foundation of Perturbative QCD"

TMDs: logarithmic accuracy

Resummation of large logs : B

-

TMDs: logarithmic accuracy

Resummation of large logs : B

$$S_{\text{pert}}(\mu_b, \mu) = 1 + \sum_{k=0}^{\infty} \boxed{R_{\text{N}^k\text{LL}}}$$

$$S_{\text{pert}}(\mu_b, \mu) = 1 + \sum_{k=0}^{\infty} \sum_{n=1+[k/2]}^{\infty} \left(\frac{\alpha_S(\mu)}{4\pi}\right)^n \sum_{k=1}^{2n} L^{2n-k} R^{(n,2n-k)}$$

-

TMDs: logarithmic accuracy

Resummation of large logs : B

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TMDs: logarithmic accuracy

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: B

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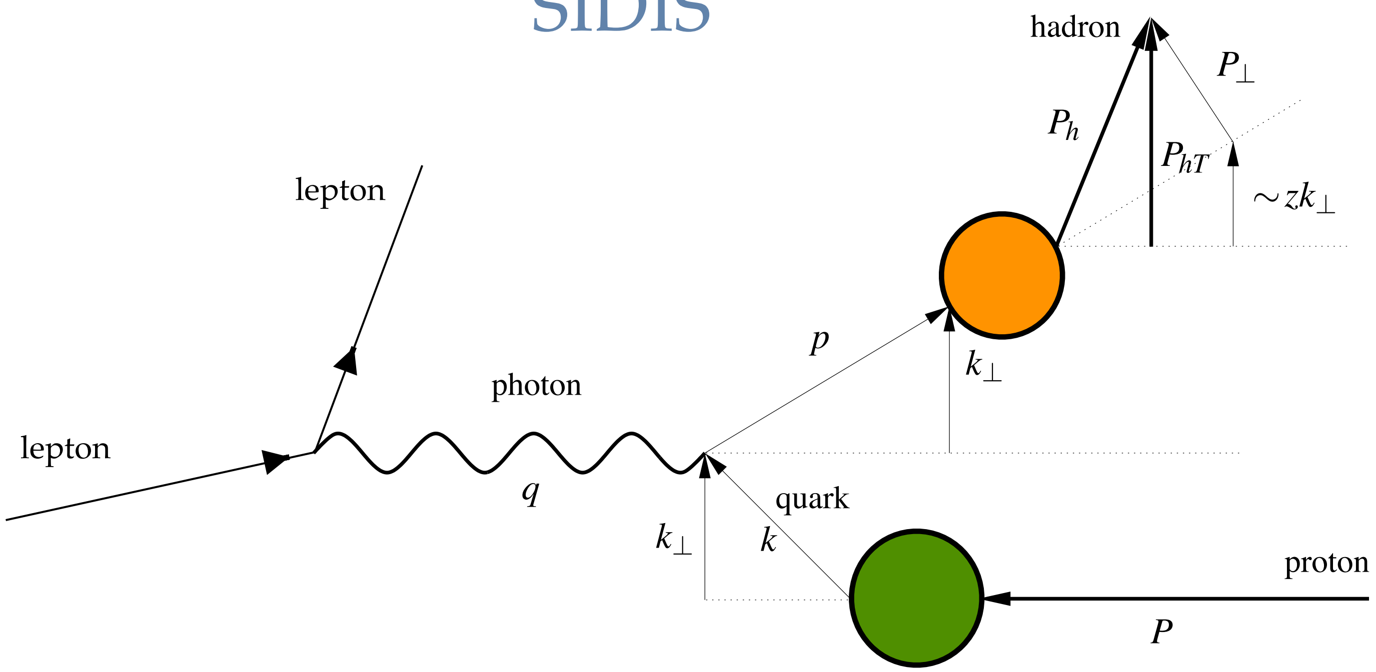
Accuracy	H and C	K and γ_F	γ_K	PDF/FF and α_S evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL^-	2	3	4	NNLO/NLO
N^3LL	**2**	**3**	**4**	**NNLO**
$\text{N}^3\text{LL}'$	3	3	4	N^3LO

Bacchetta, et al., JHEP 07 (2020)
TMD handbook, Boussarie, et al., 2023

TMD factorization — Universality

TMD factorization — Universality

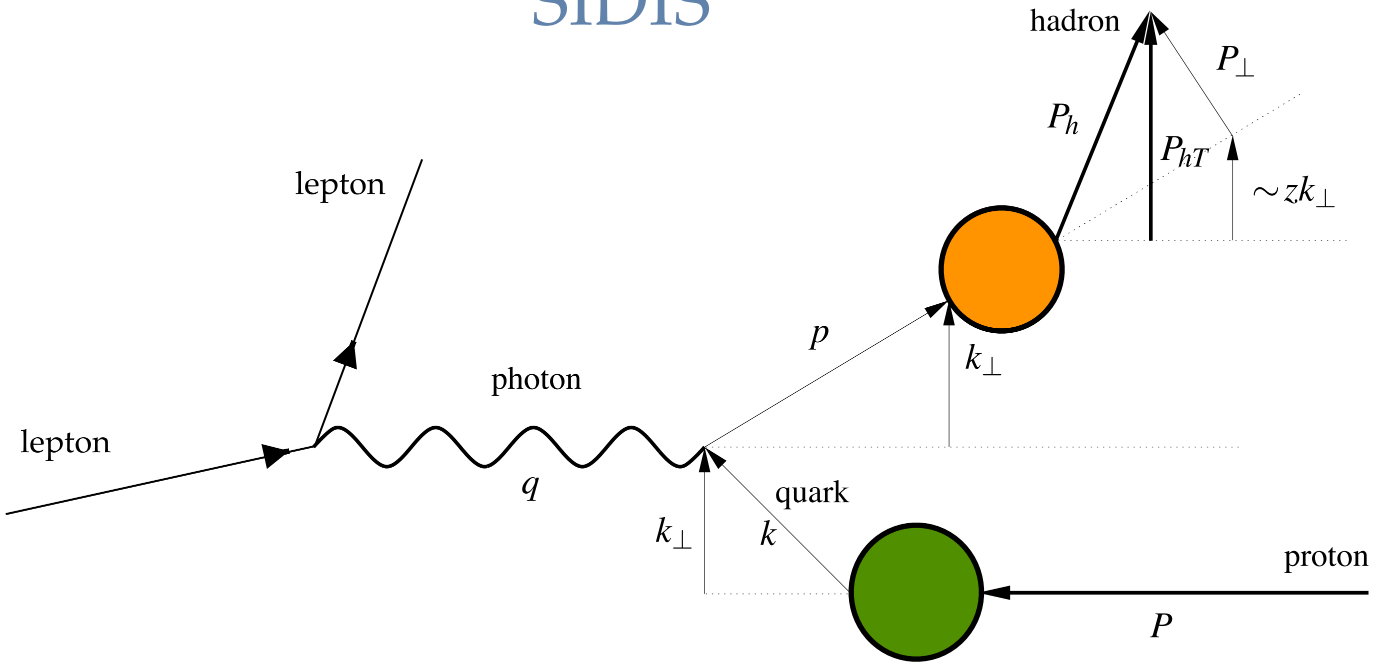
SIDIS



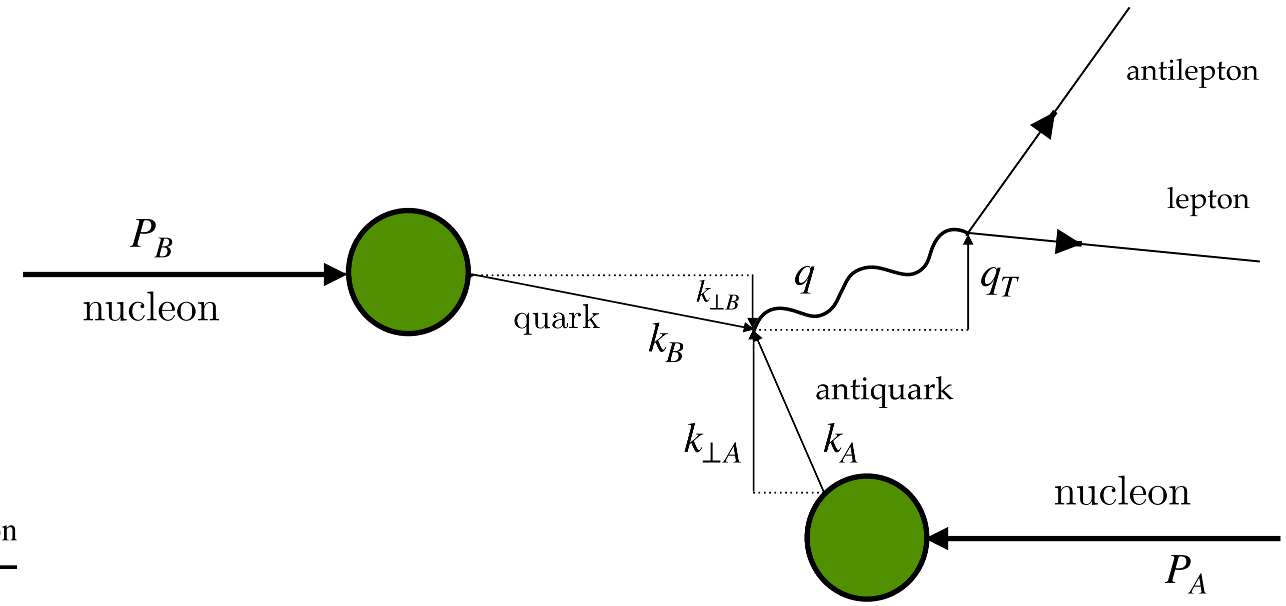
$$F_{UU,T}\big(x,z,|\boldsymbol{q}_T|,Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T|J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big)\,\hat{f}_1^a(x,b_T^2;\mu,\zeta_A)\,\hat{D}_1^{a\to h}(z,b_T^2;\mu,\zeta_B)$$

TMD factorization — Universality

SIDIS



Drell-Yan



$$F_{UU,T}\big(x, z, |\boldsymbol{q}_T|, Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big)\, \hat{f}_1^a(x, b_T^2; \mu, \zeta_A)\, \hat{D}_1^{a\to h}(z, b_T^2; \mu, \zeta_B)$$

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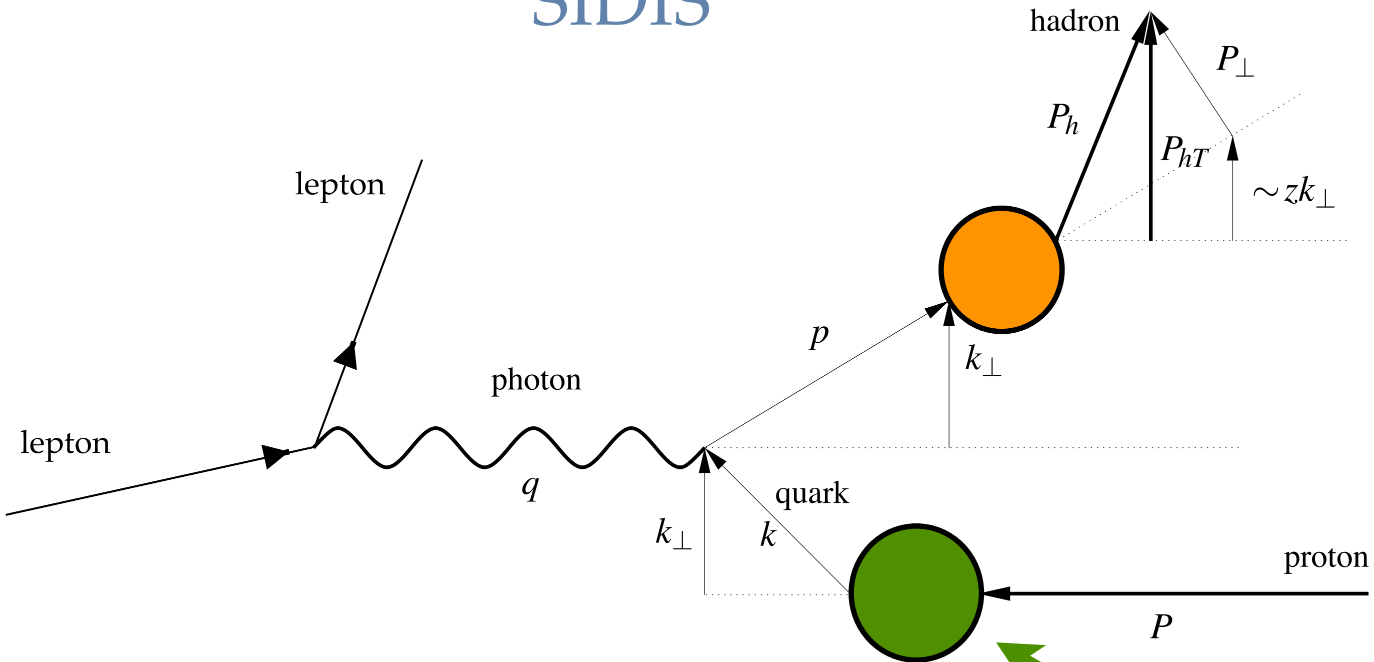
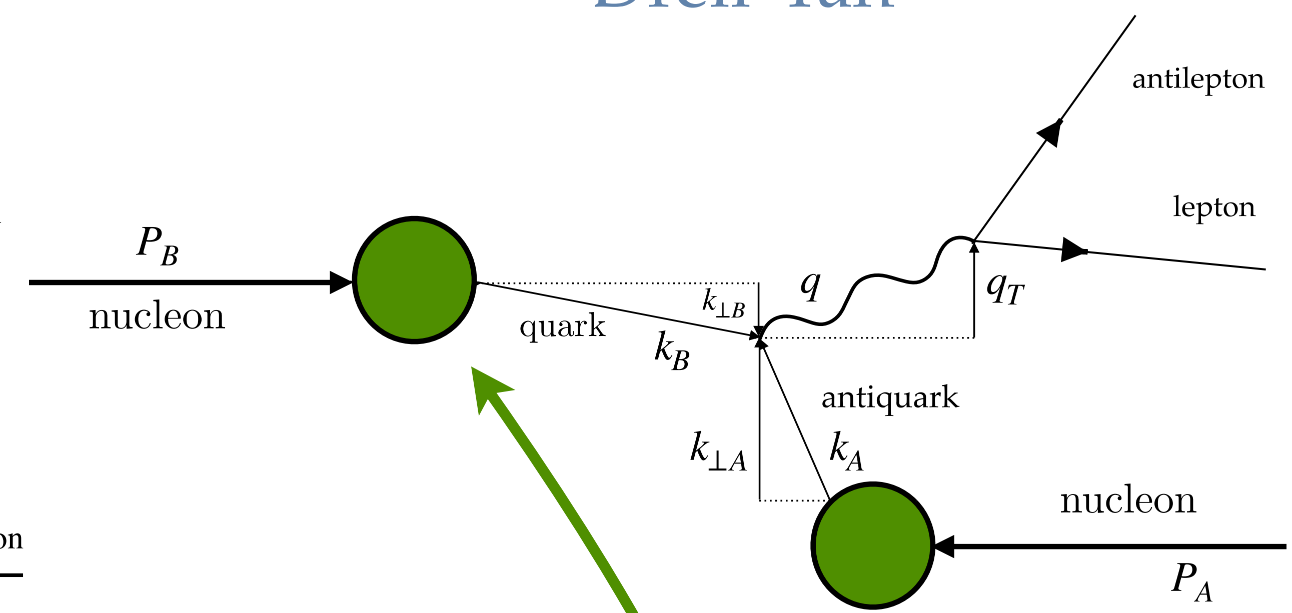
TMD factorization — Universality

SIDIS



Drell-Yan



Same functions

$$F_{UU,T}\big(x, z, |\boldsymbol{q}_T|, Q\big) \sim \int_0^{+\infty} d|\boldsymbol{b}_T||\boldsymbol{b}_T| J_0\big(|\boldsymbol{b}_T||\boldsymbol{q}_T|\big) \hat{f}_1^a(x, b_T^2; \mu, \zeta_A) \hat{D}_1^{a\to h}(z, b_T^2; \mu, \zeta_B)$$

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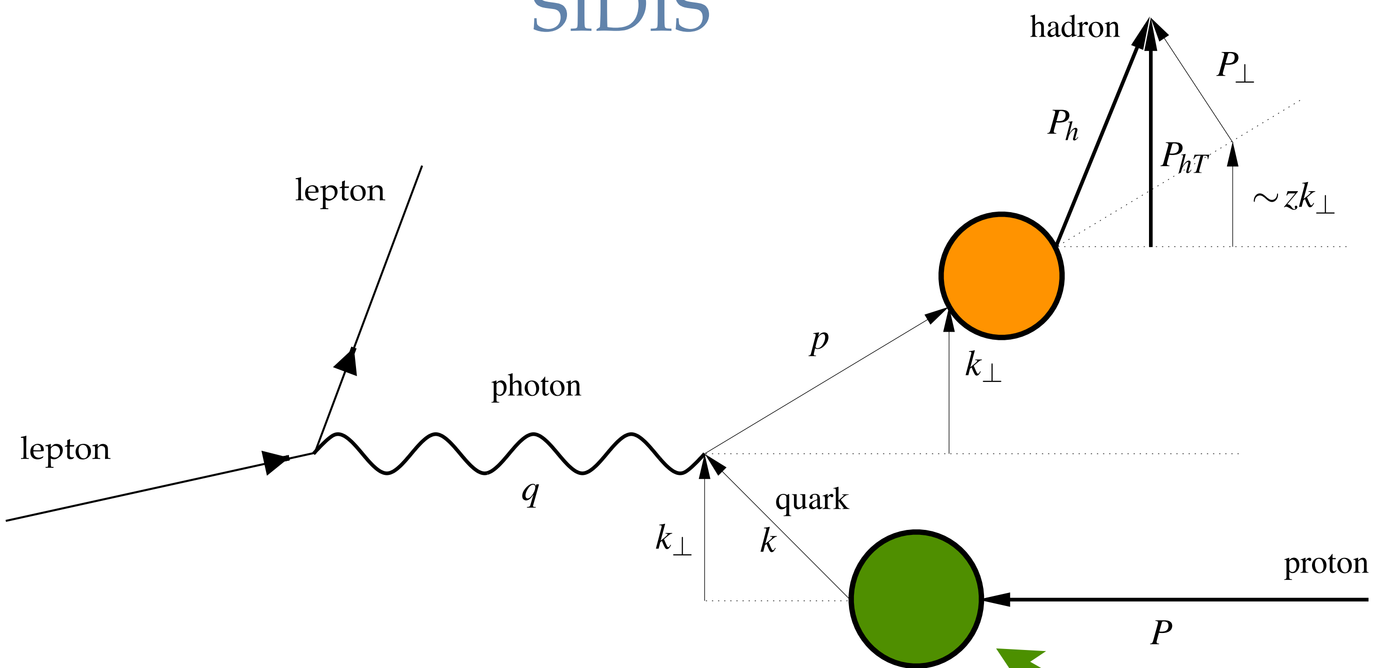
TMD factorization — Universality

SIDIS



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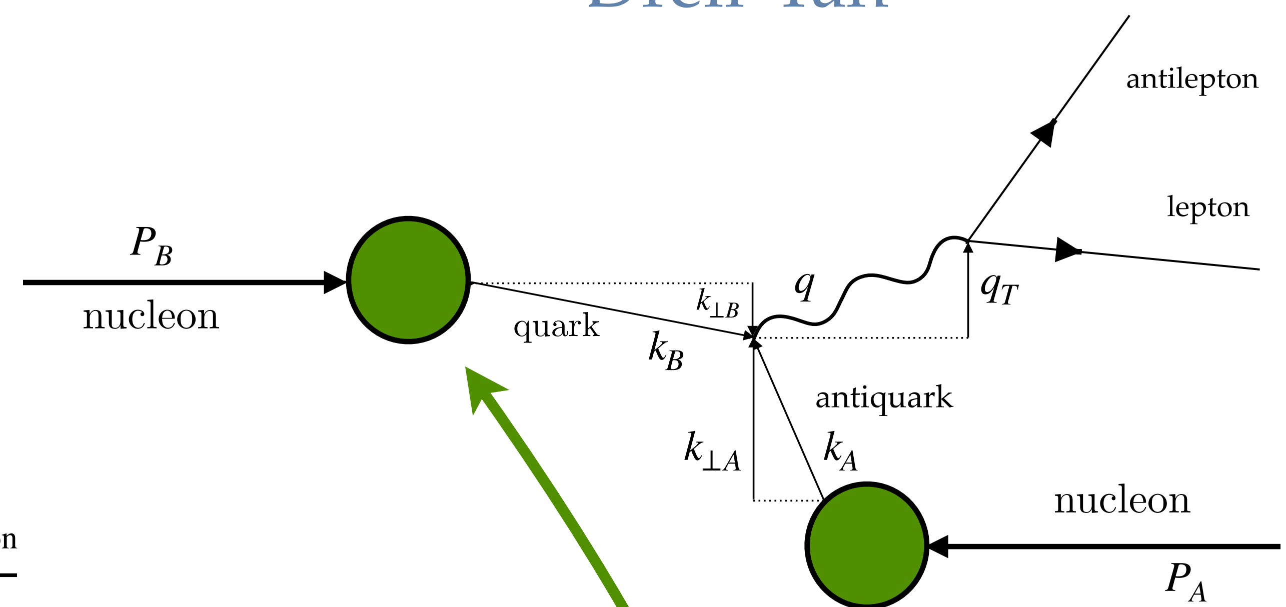

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GLOBAL FITs

Available GLOBAL fits

	Accuracy	SIDIS	DY	Z production	Flav. Dependence	N of points	χ^2/N_{data}
Pavia 2017 arXiv:1703.10157	NLL	✔	✔	✔	✘	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✔	✔	✔	✘	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✔	✔	✔	✘	2031	1.06
MAP24 arXiv:2405.13833	***N^3LL***	✔	✔	✔	✔	***2031***	***1.08***

MAP Collaboration — Fitting framework

https://github.com/MapCollaboration/NangaParbat

README.md

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

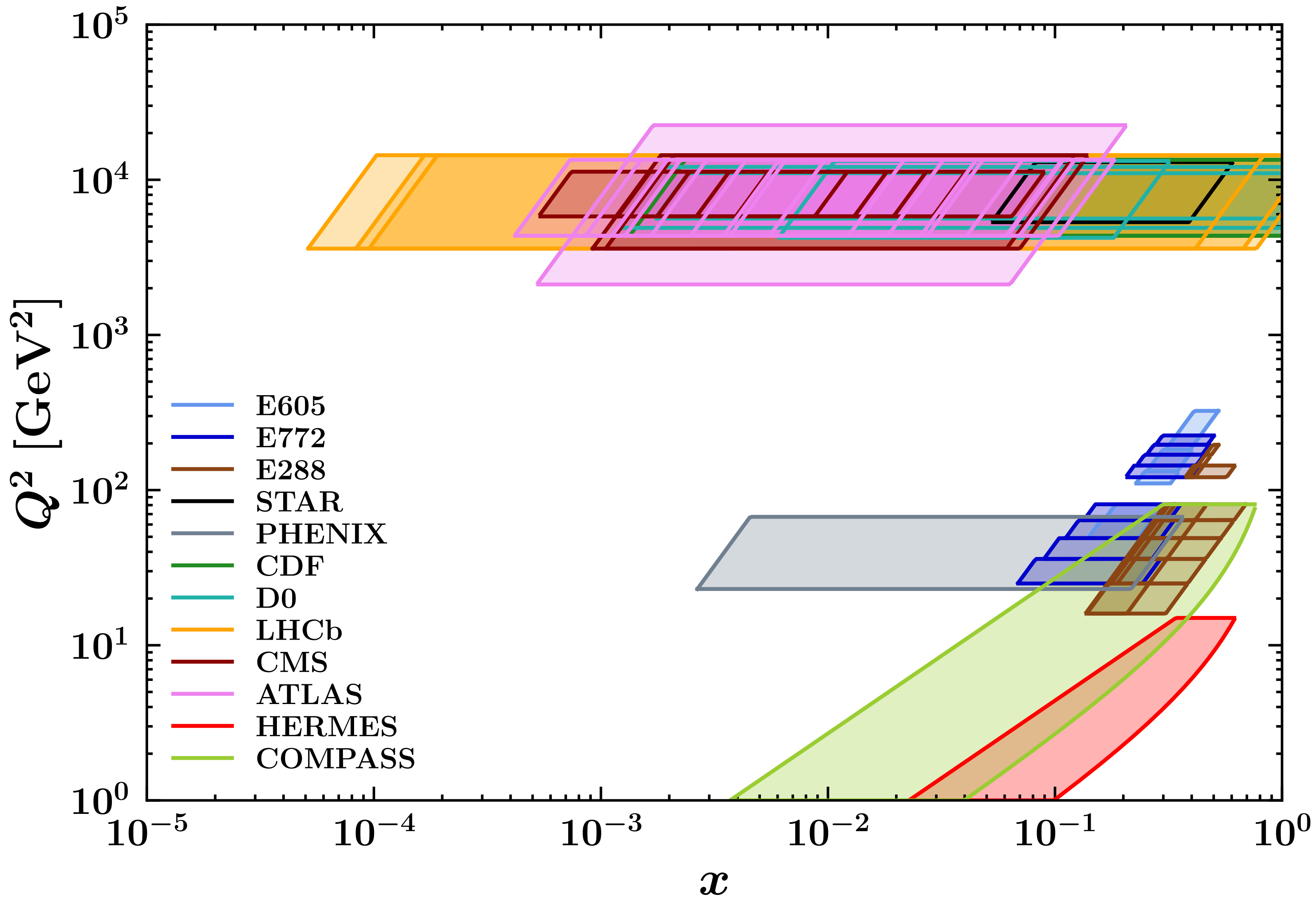
You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

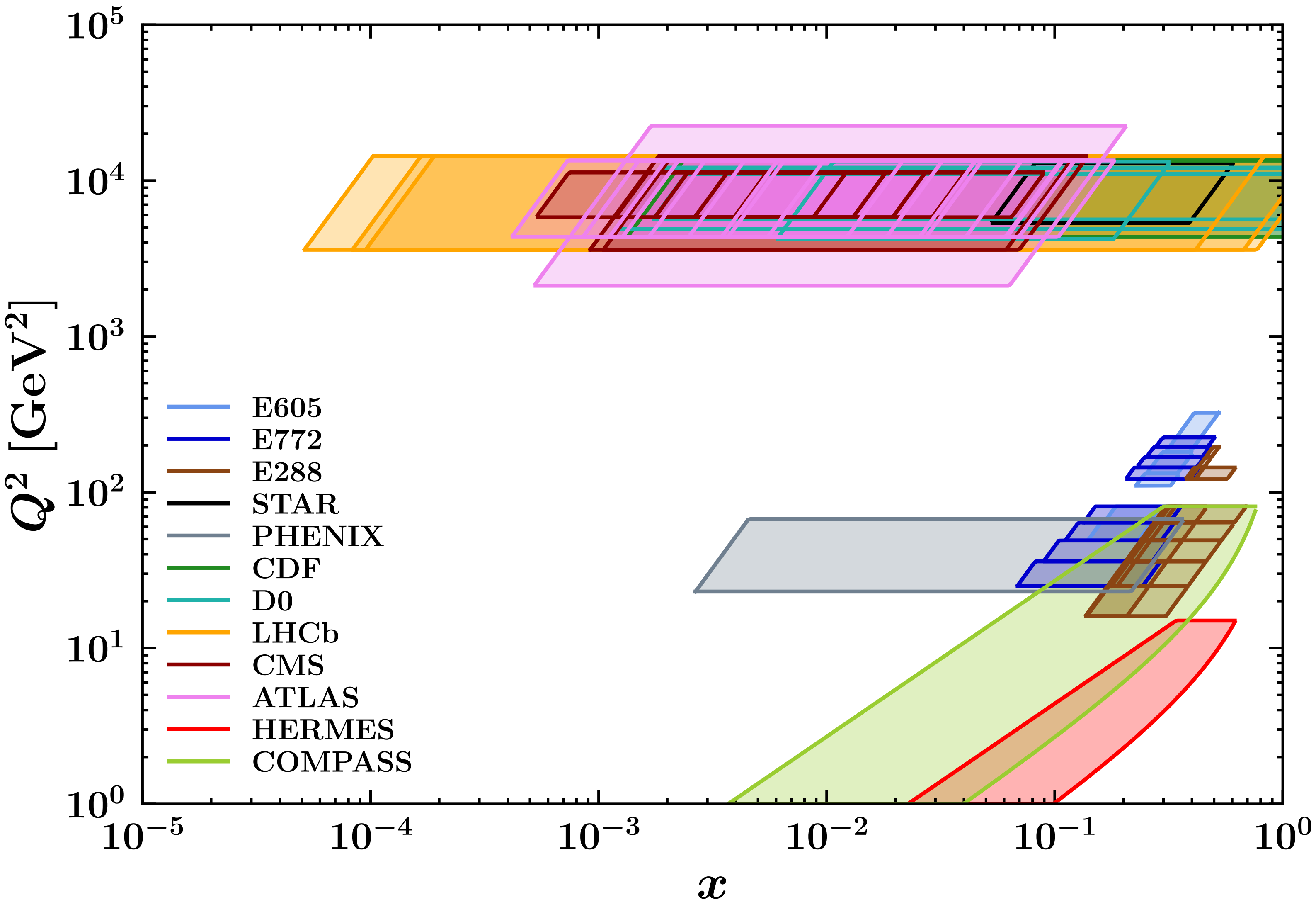
```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

MAP global fits: dataset included



MAP global fits: dataset included

Drell-Yan data 484



MAP global fits: dataset included

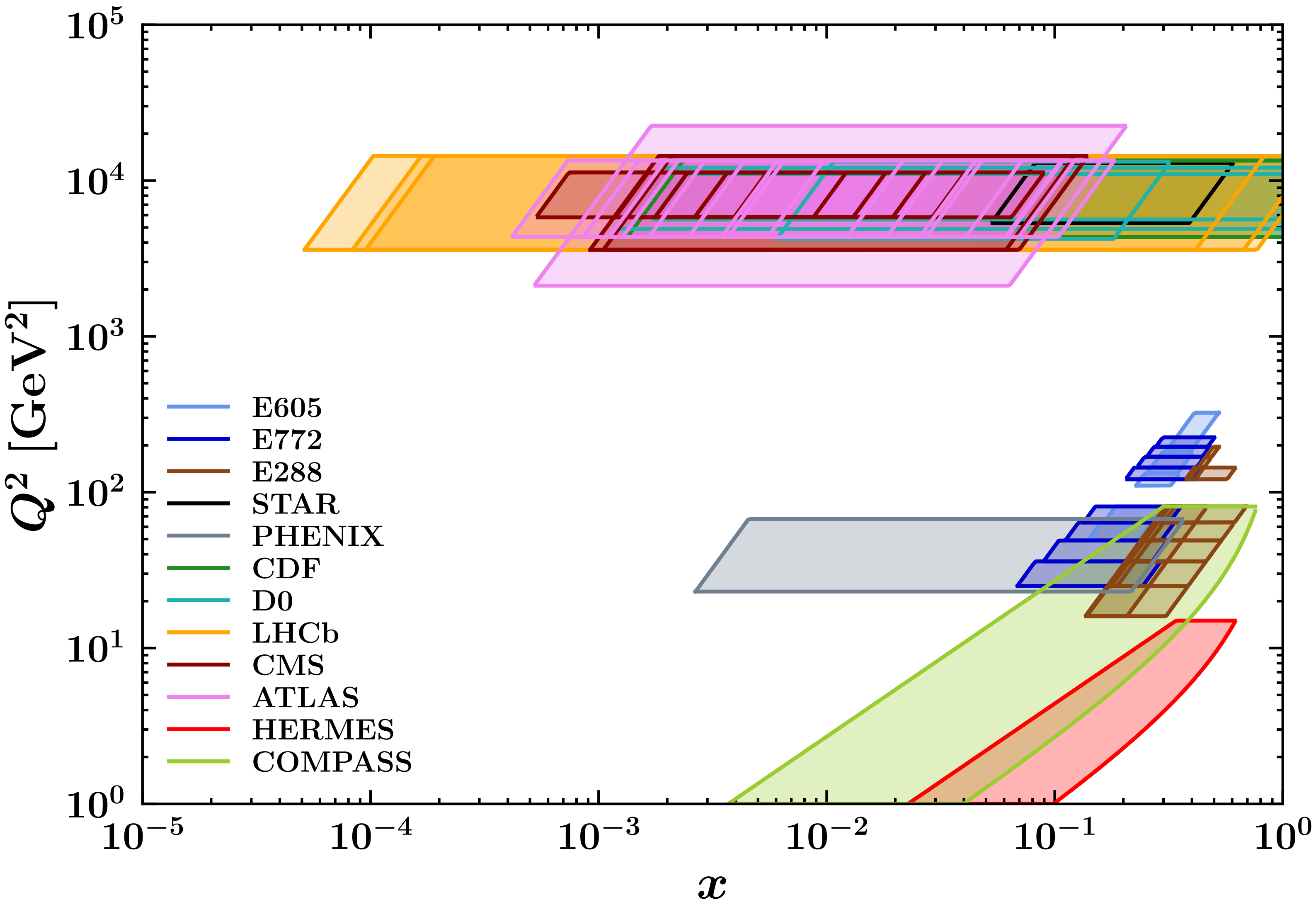
Drell-Yan data 484

Fixed-target:

E288, E605, E772

Collider mode:

RHIC, Tevatron, LHC



MAP global fits: dataset included

Drell-Yan data 484

Fixed-target:
E288, E605, E772

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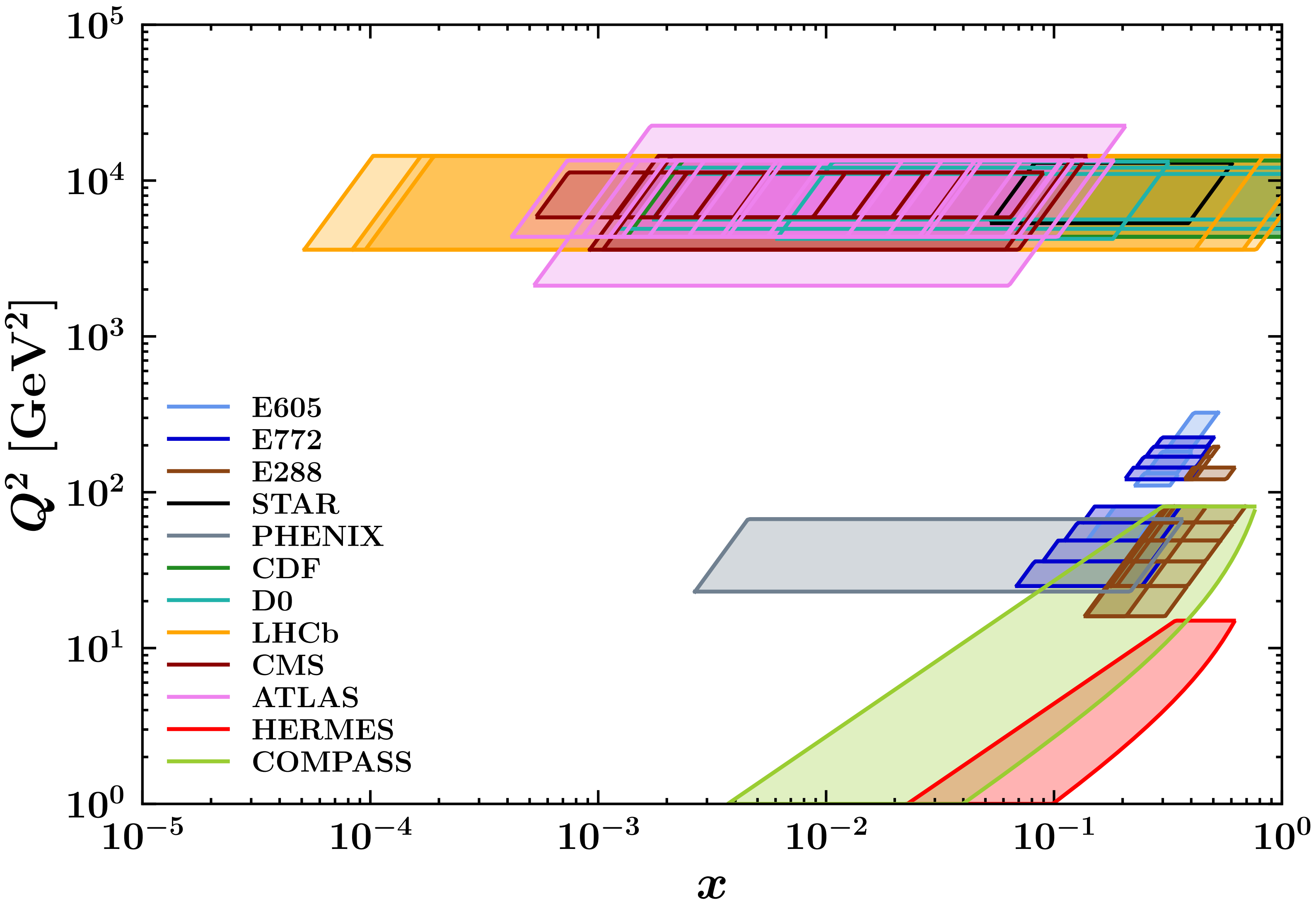
SIDIS data 1547



MAP global fits: dataset included

Drell-Yan data 484

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E288, E605, E772

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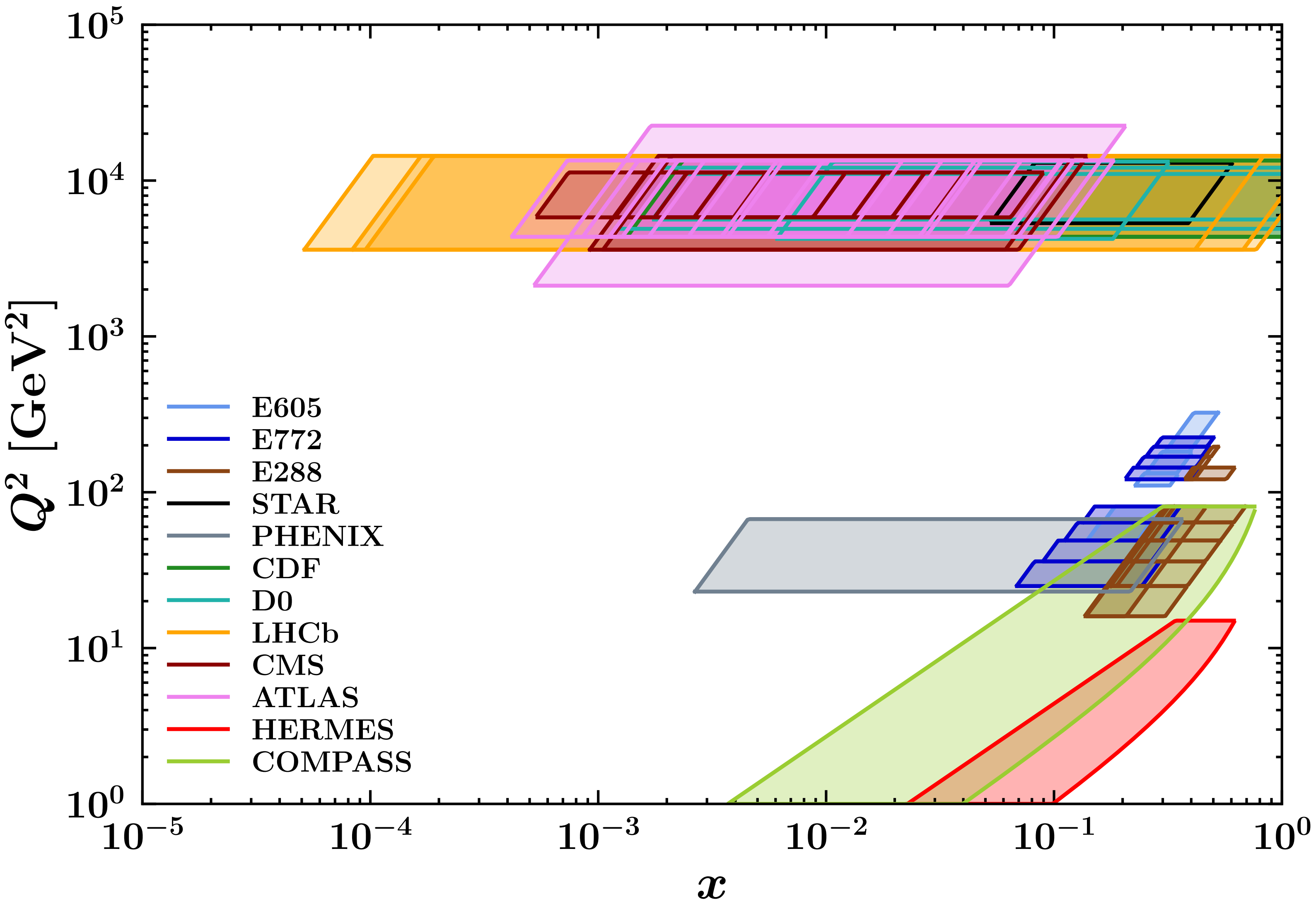
HERMES, COMPASS



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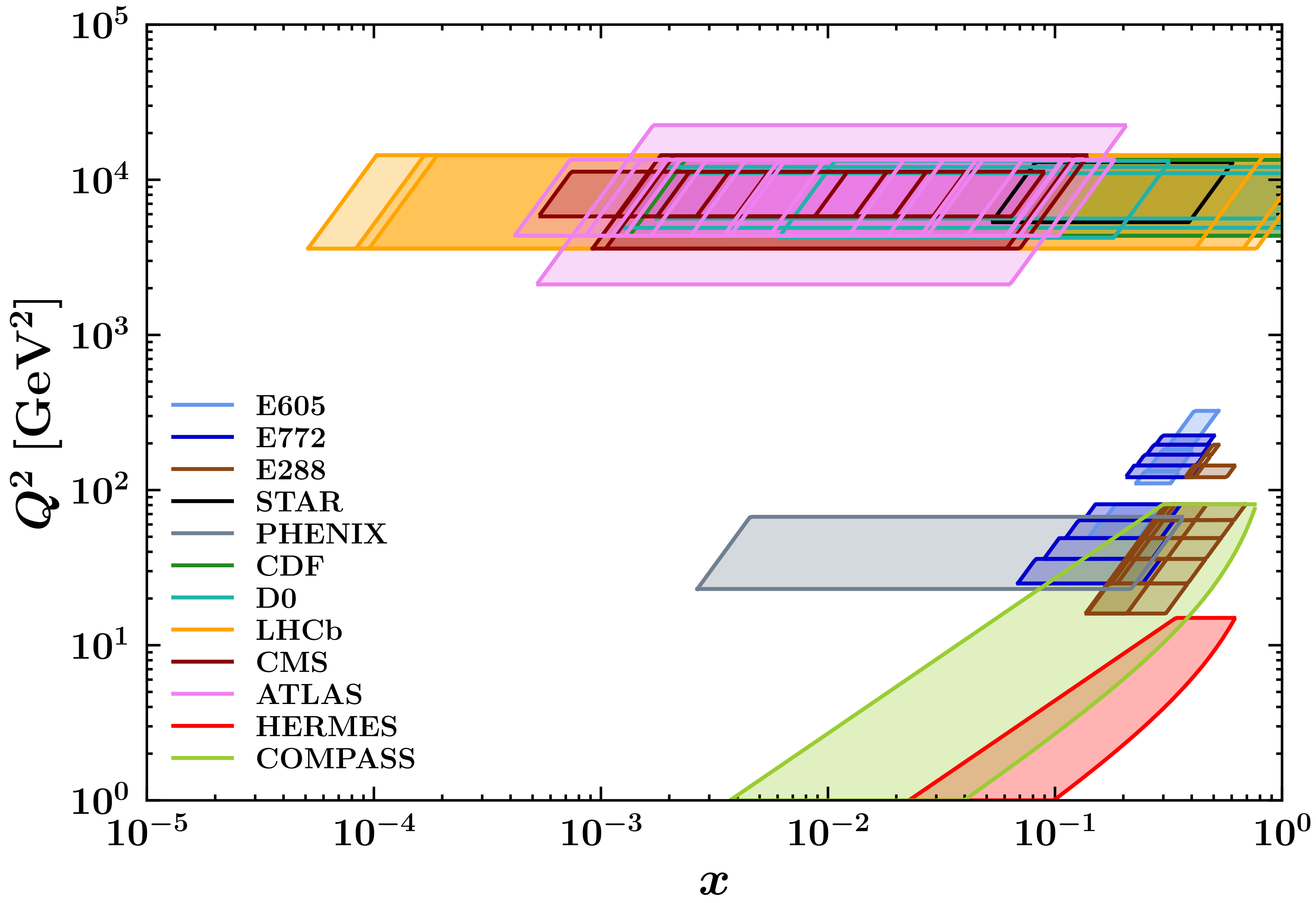

Total number of data: 2031

MAP global fits: TMD parameterization

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

Bacchetta, Conti, Radici, PRD 78 (2008)
Pasquini, Cazzaniga, Boffi, PRD 78 (2008)
Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)
Burkardt, Pasquini, EPJA (2016)
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$$f_{1\mathrm{NP}}(x, b_T^2) \propto \mathrm{F.T.\ of} \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

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$$g_1(x) = N_1 \frac{(1-x)^\alpha \; x^\sigma}{(1-\hat{x})^\alpha \; \hat{x}^\sigma}$$

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$$D_{1\mathrm{NP}}(x, b_T^2) \propto \mathrm{F.T.\ of} \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

$$g_3(z) = N_3 \frac{(z^\beta + \delta)(1-z)^\gamma}{(\hat{z}^\beta + \delta)(1-\hat{z})^\gamma}$$

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$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}}\right)$$

$$g_1(x) = N_1\,\frac{(1-x)^\alpha\; x^\sigma}{(1-\hat{x})^\alpha\;\hat{x}^\sigma}$$

$$D_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of } \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}}\right)$$

$$g_3(z) = N_3\,\frac{(z^\beta+\delta)(1-z)^\gamma}{(\hat{z}^\beta+\delta)(1-\hat{z})^\gamma}$$

$$g_K(b_T^2) = -g_2^2\,\frac{b_T^2}{4}$$

MAP global fits: TMD parameterization

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\}$$

Bacchetta, Conti, Radici, PRD 78 (2008)
Pasquini, Cazzaniga, Boffi, PRD 78 (2008)
Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)
Burkardt, Pasquini, EPJA (2016)
Grewal, Kang, Qiu, Signori, PRD 101 (2020)

$$\boxed{f_{1\mathrm{NP}}(x, b_T^2)} \propto \text{F.T. of } \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha \; x^\sigma}{(1-\hat{x})^\alpha \; \hat{x}^\sigma}$$

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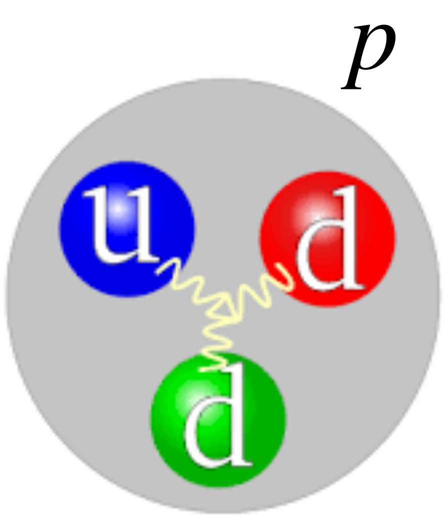
$$\boxed{g_K(b_T^2)} = -g_2^2 \frac{b_T^2}{4}$$

MAP24: flavor dependence

Flavor dependence

Flavor dependence



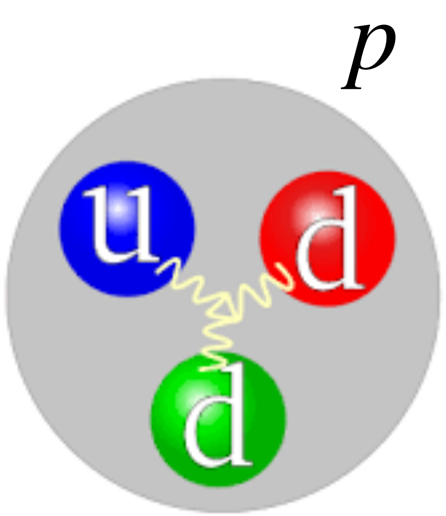
u, d

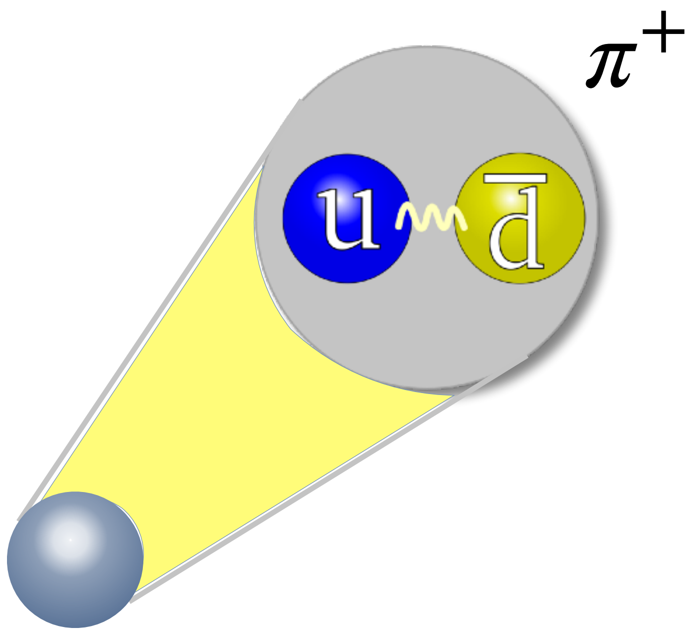
$\bar{u}, \bar{d}$

$s \ (sea)$

MAP24: flavor dependence

Flavor dependence





u, d

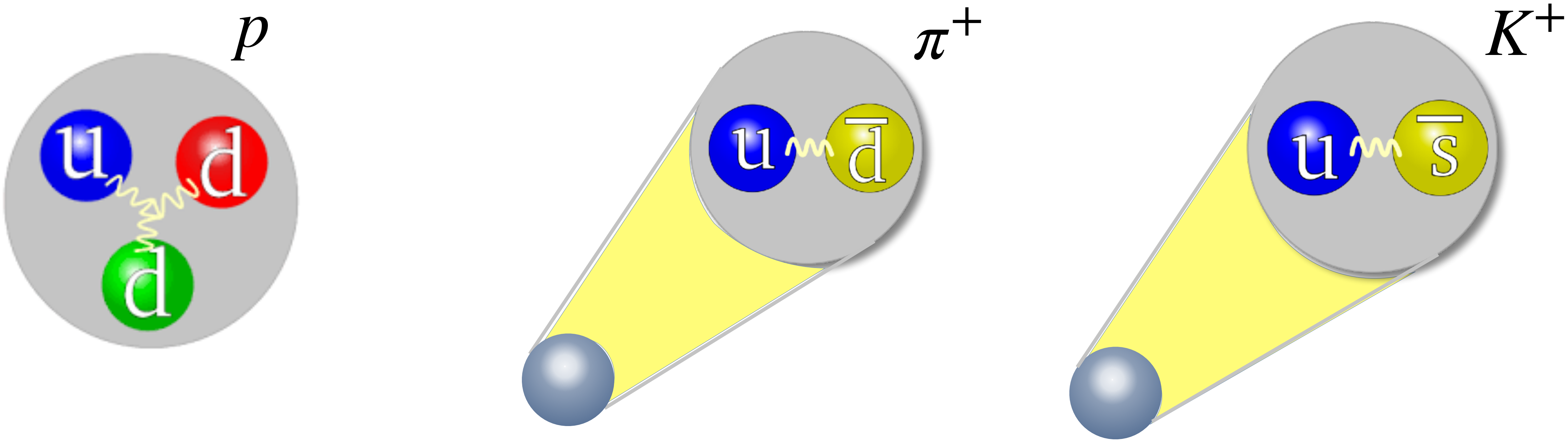
$\bar{u}, \bar{d}$

$s\ (sea)$

$u \rightarrow \pi^+, \ldots$

$d \rightarrow \pi^+, \ldots$

Flavor dependence



u, d	$u \to \pi^+, \ldots$	$u \to K^+, \ldots$
$\bar{u}, \bar{d}$	$d \to \pi^+, \ldots$	$\bar{s} \to K^+, \ldots$
$s\ (sea)$		$d \to K^+, \ldots$

MAP24: flavor dependence

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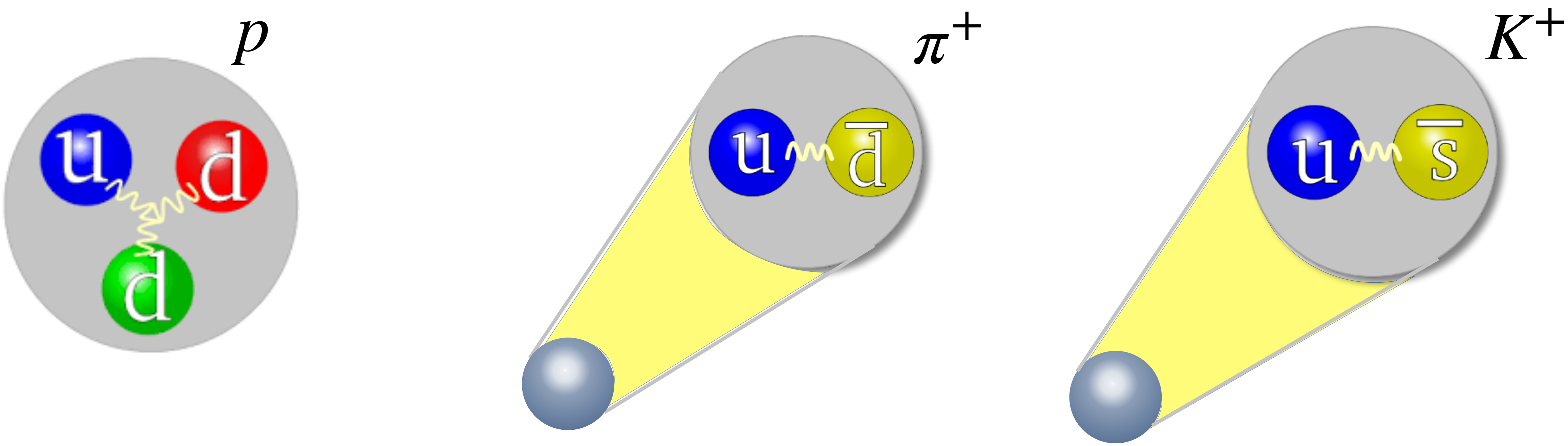

u, d

$\bar{u}, \bar{d}$

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$u \rightarrow \pi^+, \ldots$

$d \rightarrow \pi^+, \ldots$

$u \rightarrow K^+, \ldots$

$\bar{s} \rightarrow K^+, \ldots$

$d \rightarrow K^+, \ldots$

charge conjugation

MAP24: error propagation

PDF set for $f_1(x, Q^2)$ *NNPDF3.1nnlo*

Ball et al. (NNPDF), EPJ C 77 (2017)

$D_1(z, Q^2)$ *MAPFFnnlo*

Khalek et al. (MAP), PLB 834 (2022)

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100 MC replicas of unpolarized PDFs

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100 MC replicas of unpolarized PDFs

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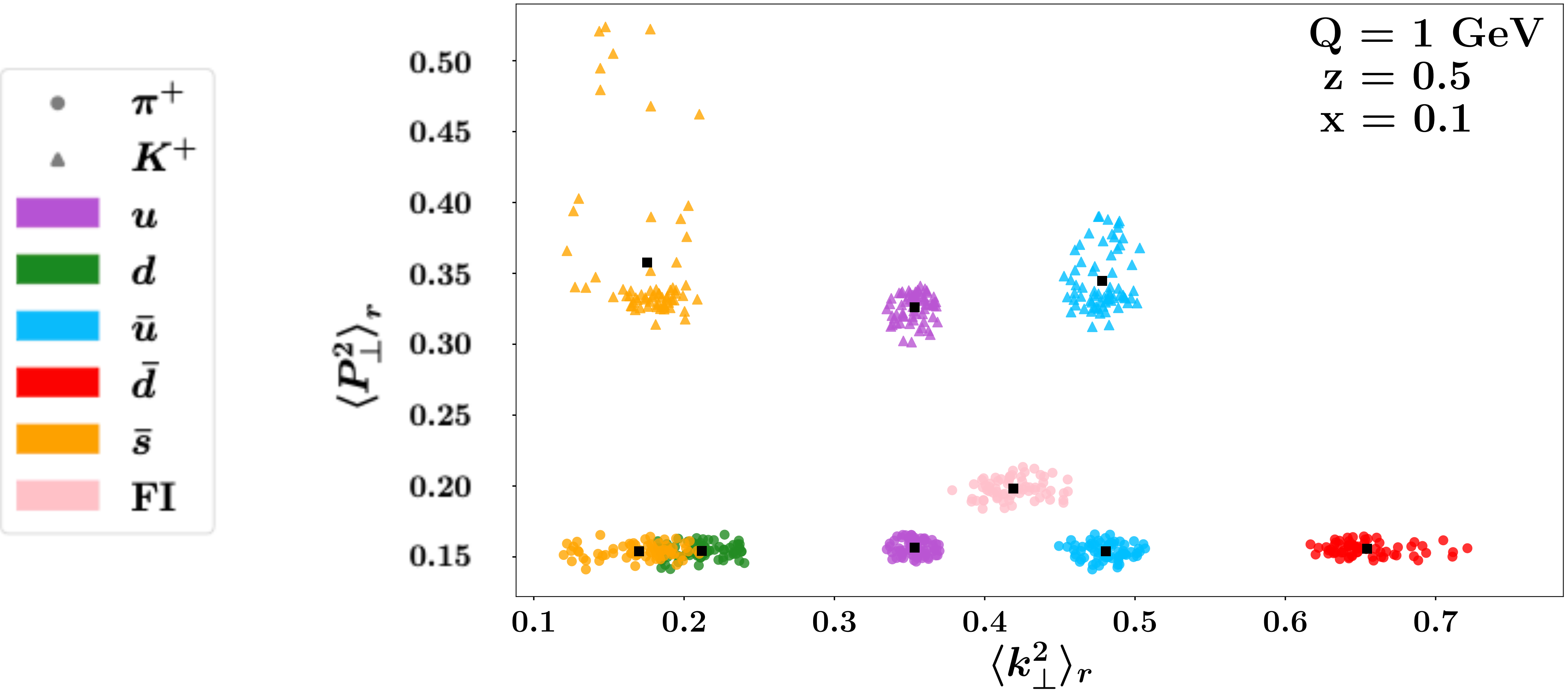
MAP24: main results

TMD's "effective width"



Evidence of different behaviors for different flavors

Evidence of different behaviors for different measured hadrons

SoLID impact studies with MAP24

Impact study of SoLID pseudodata

Kinematics

See Ye's talk

	$\sqrt{s}$	x	Q2	z
π^+	4.0631 or 4.7	[0,0.7] (3 bins)	[1,8] (5 bins)	[0.3,0.6] (6 bins)
π^-	4.0631 or 4.7	[0,0.7] (3 bins)	[1,8] (5 bins)	[0.3,0.6] (6 bins)
K^+	4.0631 or 4.7	[0,0.7] (3 bins)	[1,8] (5 bins)	[0.3,0.6] (6 bins)
K^-	4.0631 or 4.7	[0,0.7] (3 bins)	[1,8] (5 bins)	[0.3,0.6] (6 bins)

Pseudodata generation

Central value obtained using ***average parameters*** of MAP24 baseline fit

Uncertainties of pseudodata

Stat from simulation

Sys 10% from simulation

Impact study of SoLID pseudodata: complete

Included dataset

$Q^2 > 1.4\ \mathrm{GeV}^2$

$0.2 < z < 0.7$

$x_B < 0.7$

$P_{hT} < \min\left[\min\left[0.2\,Q, 0.5zQ\right] + 0.3\,\mathrm{GeV}, zQ\right]$

Final-state hadrons

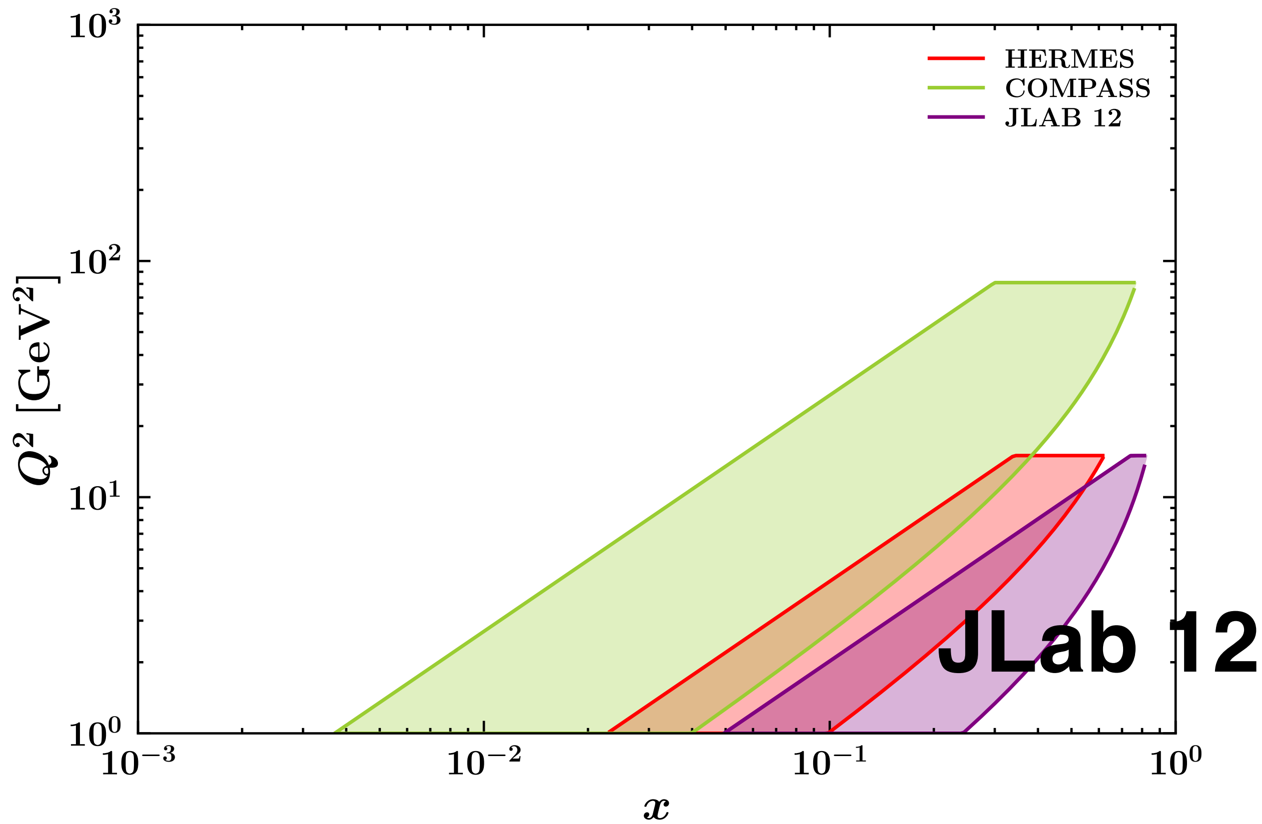
π^+ π^-

K^+ K^-

$\sim\ 2000$ MAP24

+

~ 1600 SoLID pseudodata



$\pi^+ \ \pi^-$
$K^+ \ K^-$

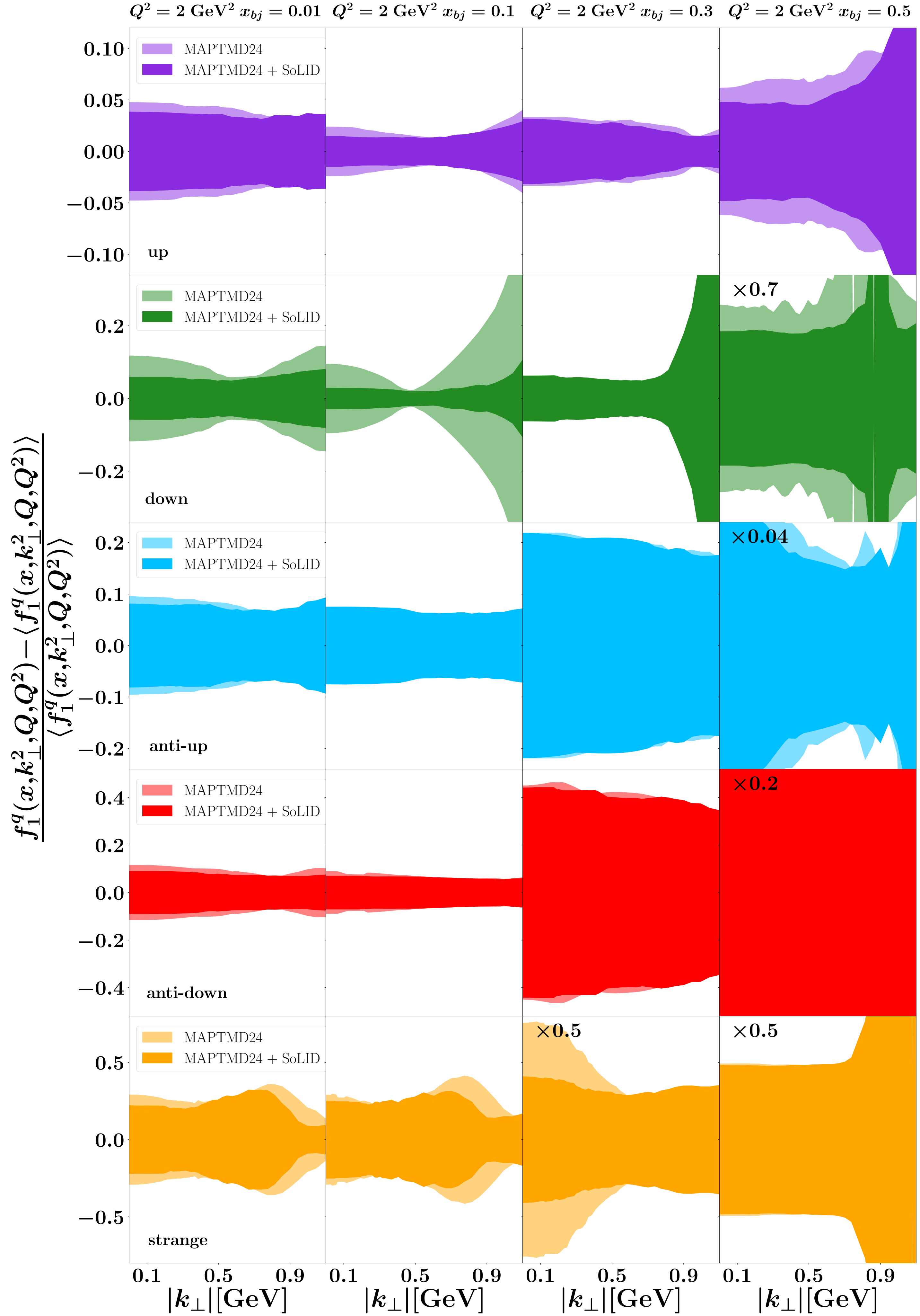
$x = 0.01, 0.1, 0.3, 0.5$

$Q^2 = 2 \text{ GeV}^2$

Plotted quantity

$$\frac{f_1^q(x,k_\perp^2,Q,Q^2) - \langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}{\langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}$$

Uncertainty bands account for 68% CL



$\pi^+ \; \pi^-$

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$x = 0.01, 0.1, 0.3, 0.5$

$Q^2 = 4\ \mathrm{GeV}^2$

Plotted quantity

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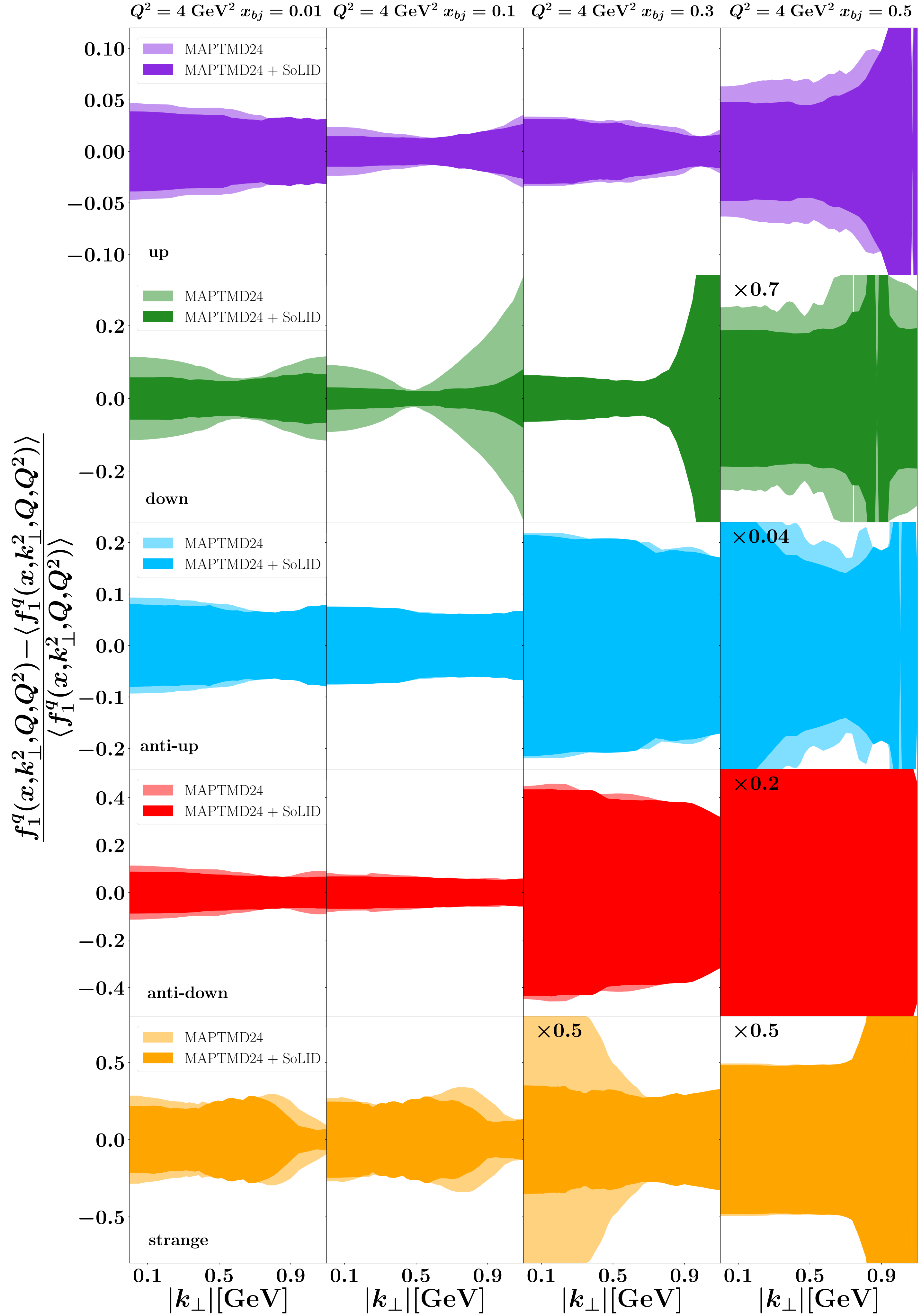

$$\pi^+ \ \pi^- \\ K^+ \ K^-$$

$x = 0.01, 0.1, 0.3, 0.5$

$Q^2 = 6\ \mathrm{GeV}^2$

Plotted quantity

$$\frac{f_1^q(x,k_\perp^2,Q,Q^2) - \langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}{\langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}$$

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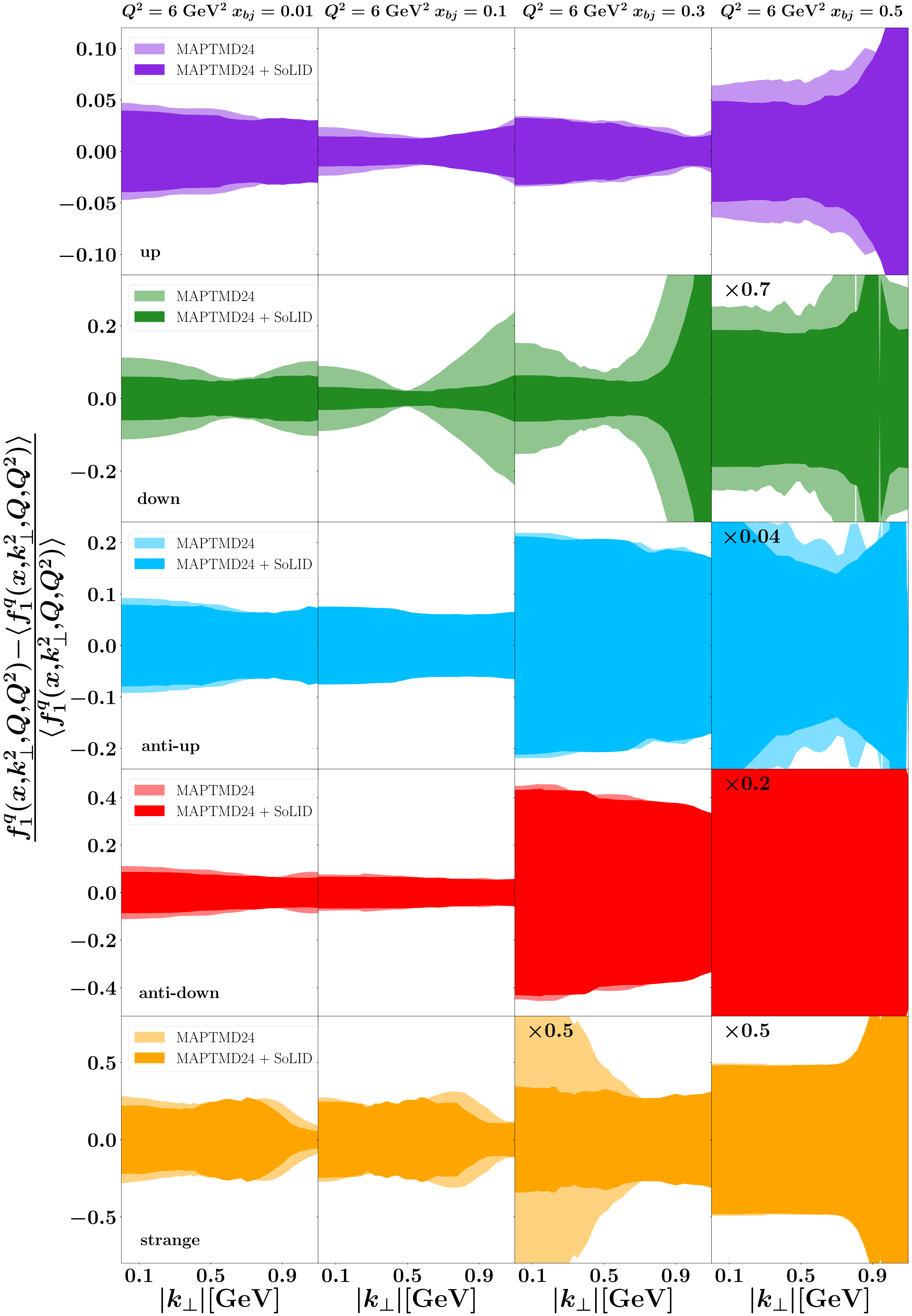

DISCLAIMER for impact studies

It can happen that the error bands after the **impact study are similar or slightly larger** than the baseline ones in certain regions

1- Intrinsic uncertainty from collinear PDF set (unavoidable)

2- Correlations (similar bands for given flavor, but smaller xsec)

3- Specific number of replicas (fixed by MAP24 extraction)

→ Statistical fluctuations

Outlook and Conclusions

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  → no precise exp. data in MAP24 (HERMES)

  → provide a constraint on quark TMDs in the valence region (d, u)

  → useful to understand the role of power corrections in SIDIS

  Hadron-Mass corrections (kinematics) See Accardi's talk in Frascati

  Higher-Twist corrections (dynamics)

  → nuclear (light) corrections never studied in TMD framework

Backup

Structure of a TMD: NP content

$$f_{NP}(x, b_T^2) \exp\left\{ g_K(b_T^2) \ln \frac{\sqrt{\zeta}}{\sqrt{\zeta_0}} \right\} \qquad : \mathrm{C}$$

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$$\mu_b > \mu \qquad \infty \quad \xleftarrow{b_T \ll 1} \quad \mu_b = \frac{2e^{-\gamma_E}}{|\boldsymbol{b}_T|} \quad \xrightarrow{b_T \gg 1} \quad 0 \qquad \alpha_S(\mu_b) \to +\infty$$

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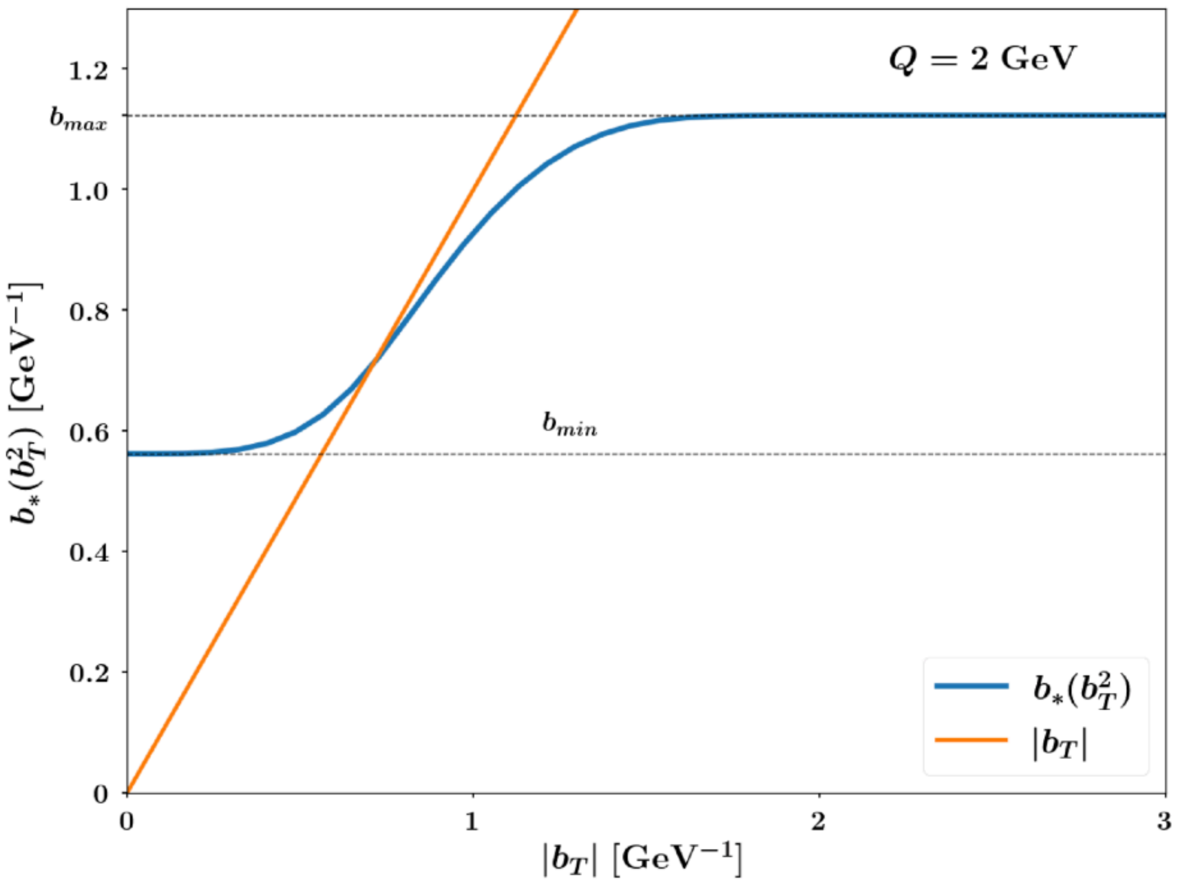
b_*-prescription



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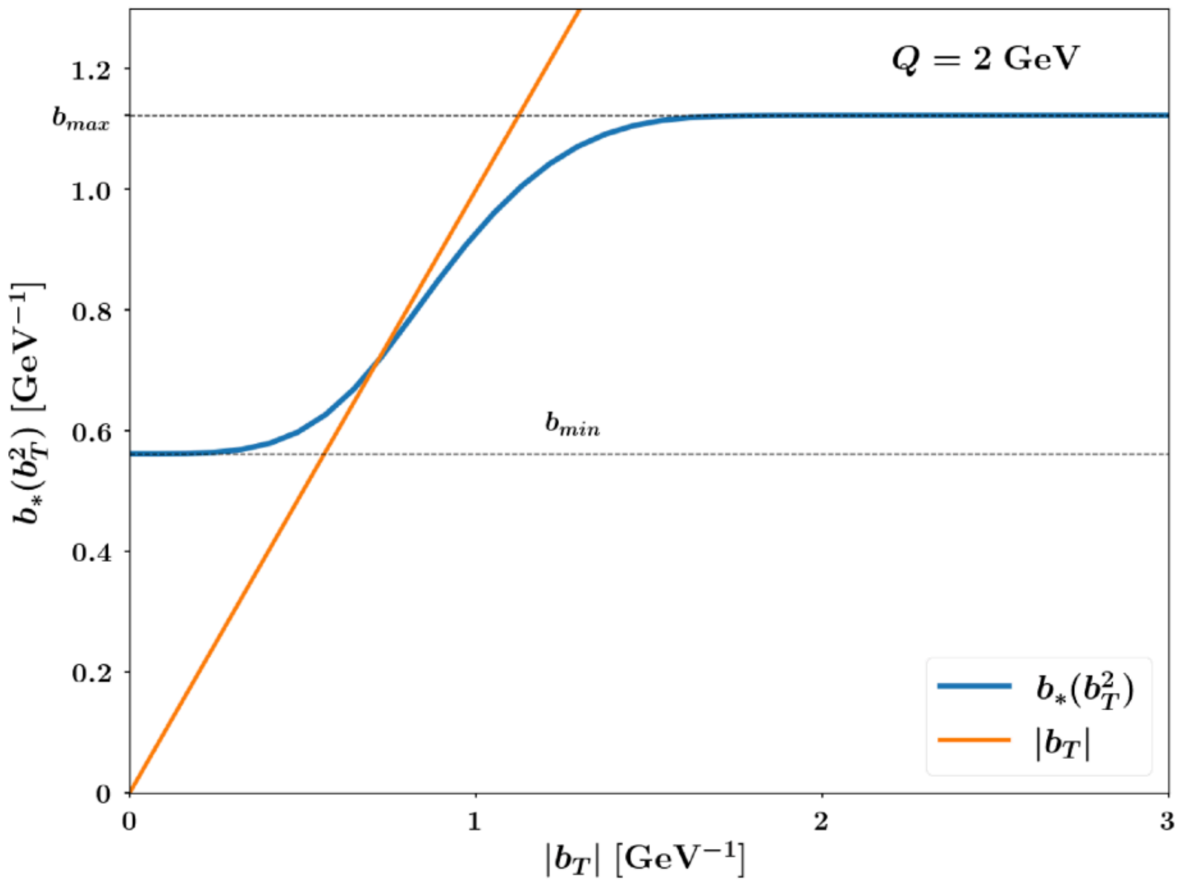
Collins, Soper, Sterman, Nucl. Phys. B250 (1985)

Collins, Gamberg, et al., PRD (2016)

Bacchetta, Echevarria, Mulders, et al., JHEP 11 (2015)



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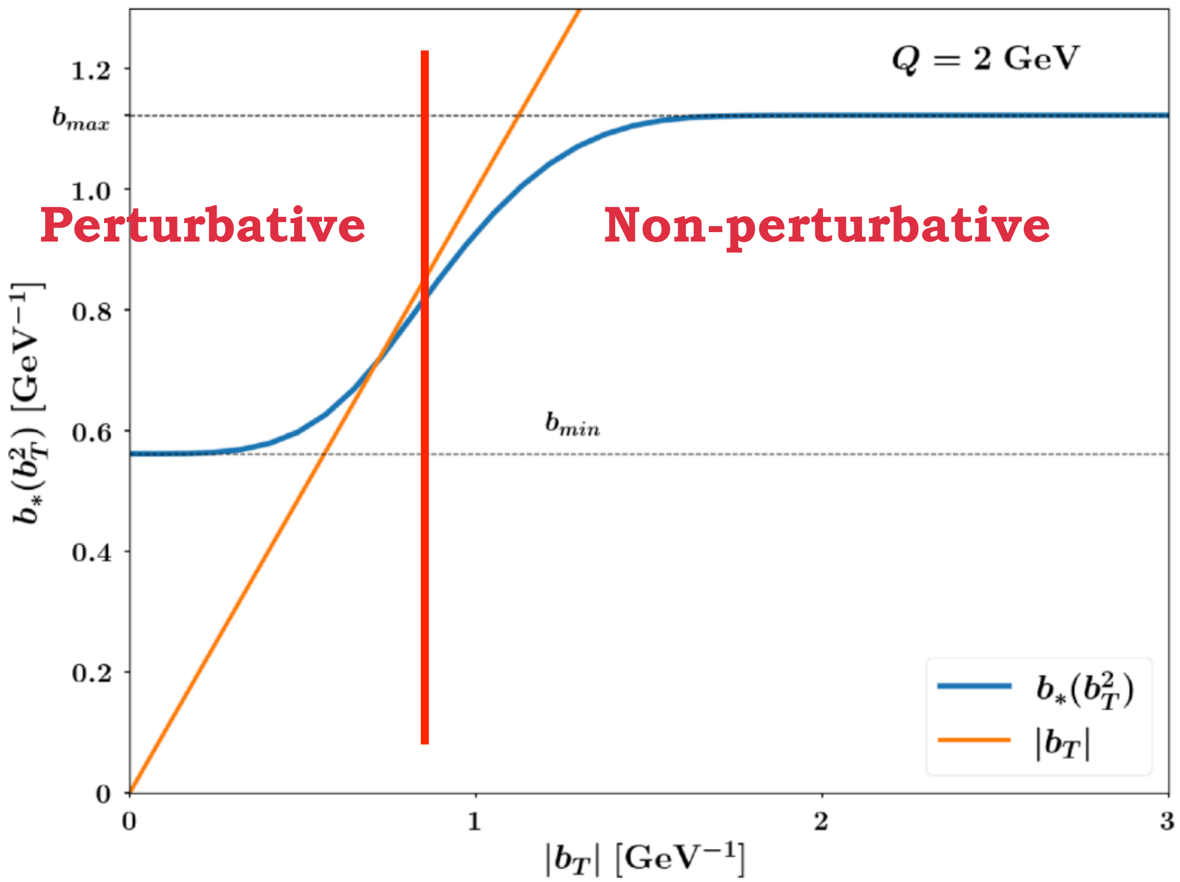

$$\hat{f}_1(x, b_T^2; \mu, \zeta) = \left[\frac{\hat{f}_1(x, b_T^2; \mu, \zeta)}{\hat{f}_1(x, b_*(b_T^2); \mu, \zeta)} \right] \hat{f}_1(x, b_*(b_T^2); \mu, \zeta) \equiv f_{\rm NP}(x, b_T^2; \zeta) \hat{f}_1(x, b_*(b_T^2); \mu, \zeta)$$

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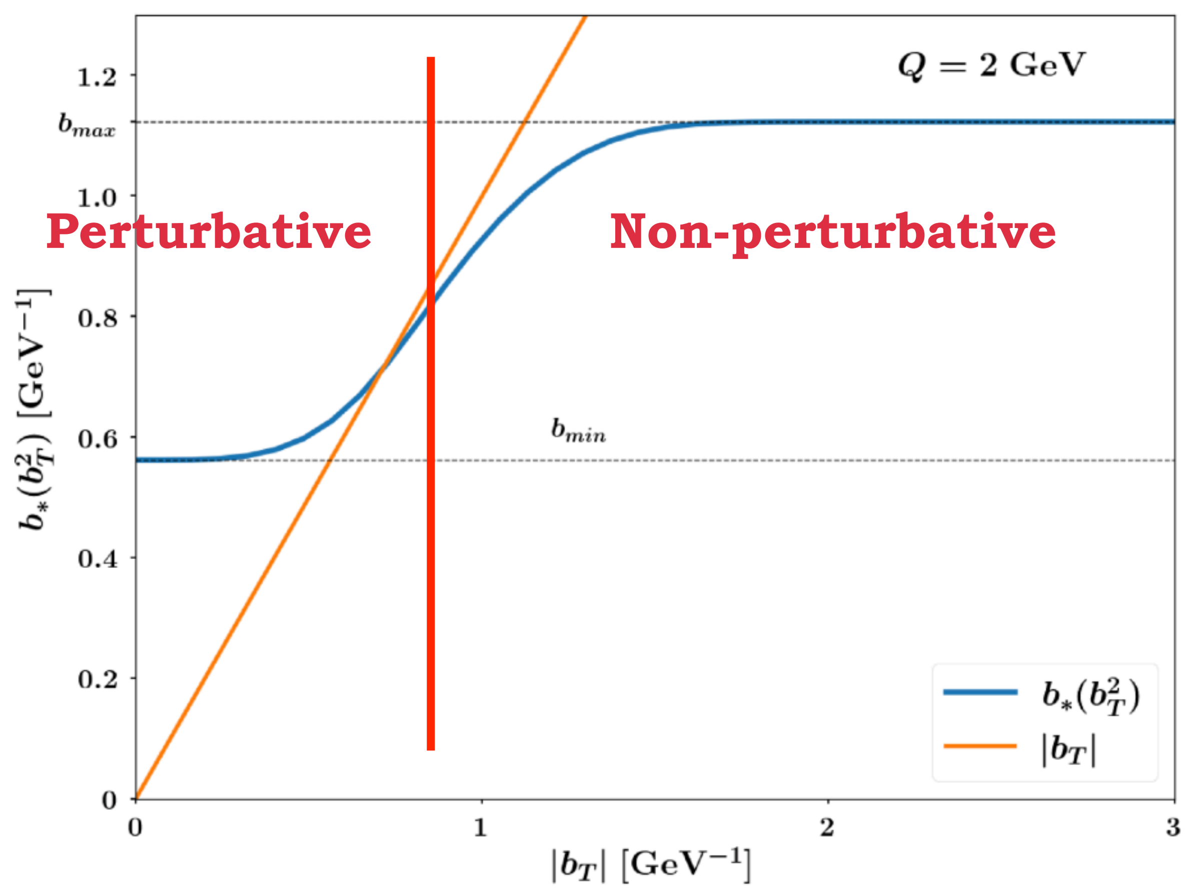

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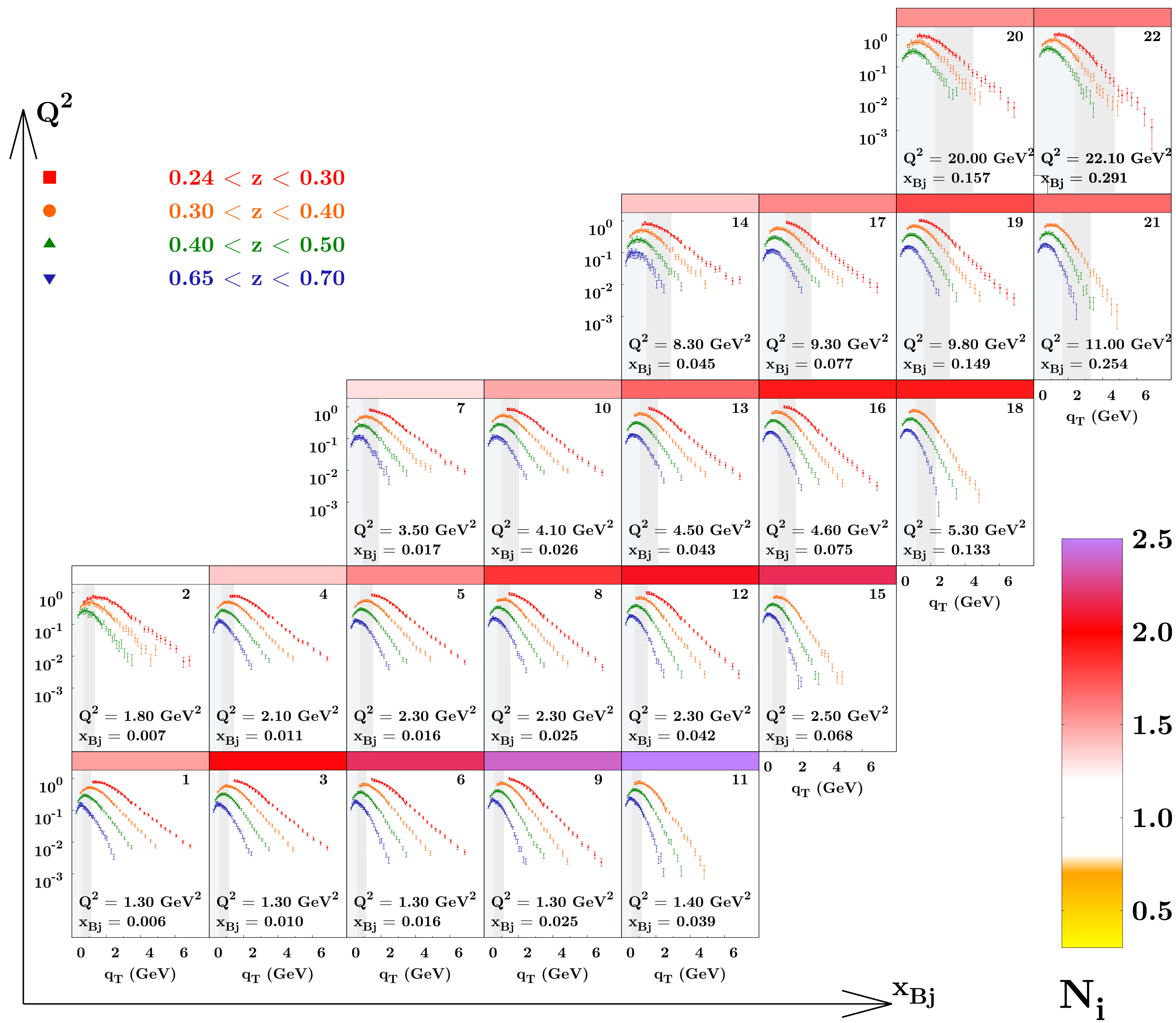
Normalization of SIDIS calculation

Normalization issue confirmed also in other analyses from different collaborations

Vladimirov, JHEP 12 (2023)



Gonzalez-Hernandez, PoS DIS2019 (2019)

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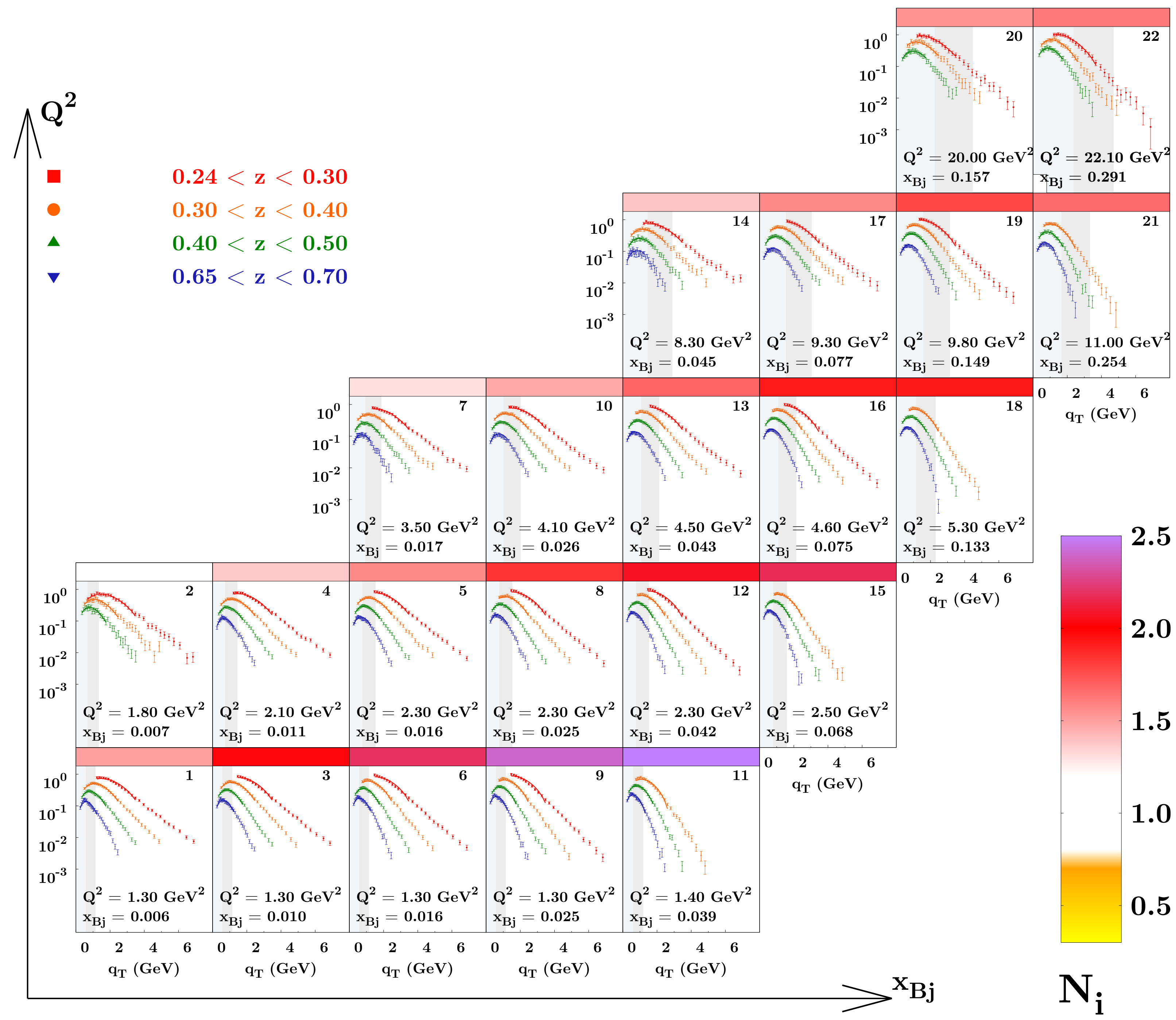

Sun, Isaacson, Yuan, Yuan, IJNP A (2014)

Gonzalez-Hernandez, PoS DIS2019 (2019)

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Normalization of SIDIS calculation

MAP22 work solution

Good agreement for almost all bins

MAP Collaboration, JHEP 10 (2022)

Normalization of SIDIS calculation

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SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dxdQ\,dzdP_{hT}} \Bigg/ \frac{d\sigma}{dxdQ}$$

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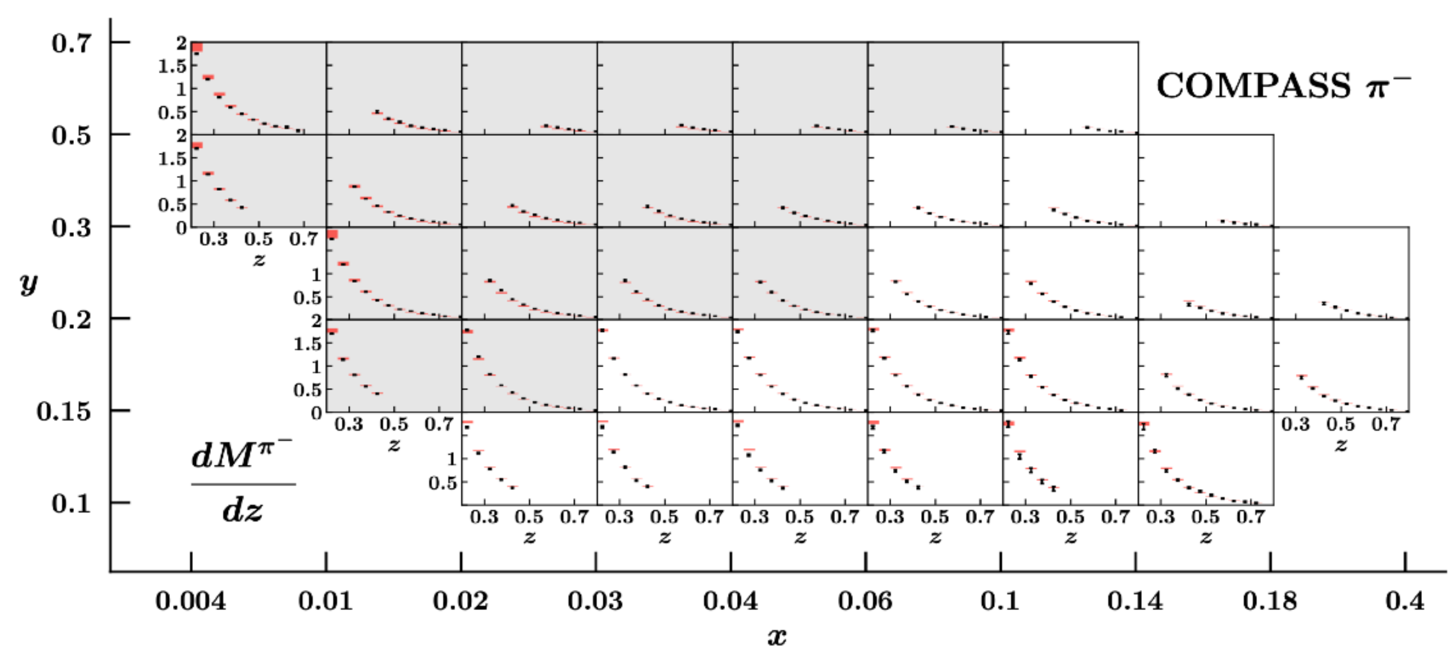

Khalek, Bertone, Nocera, et al., PRD 104 (2021)

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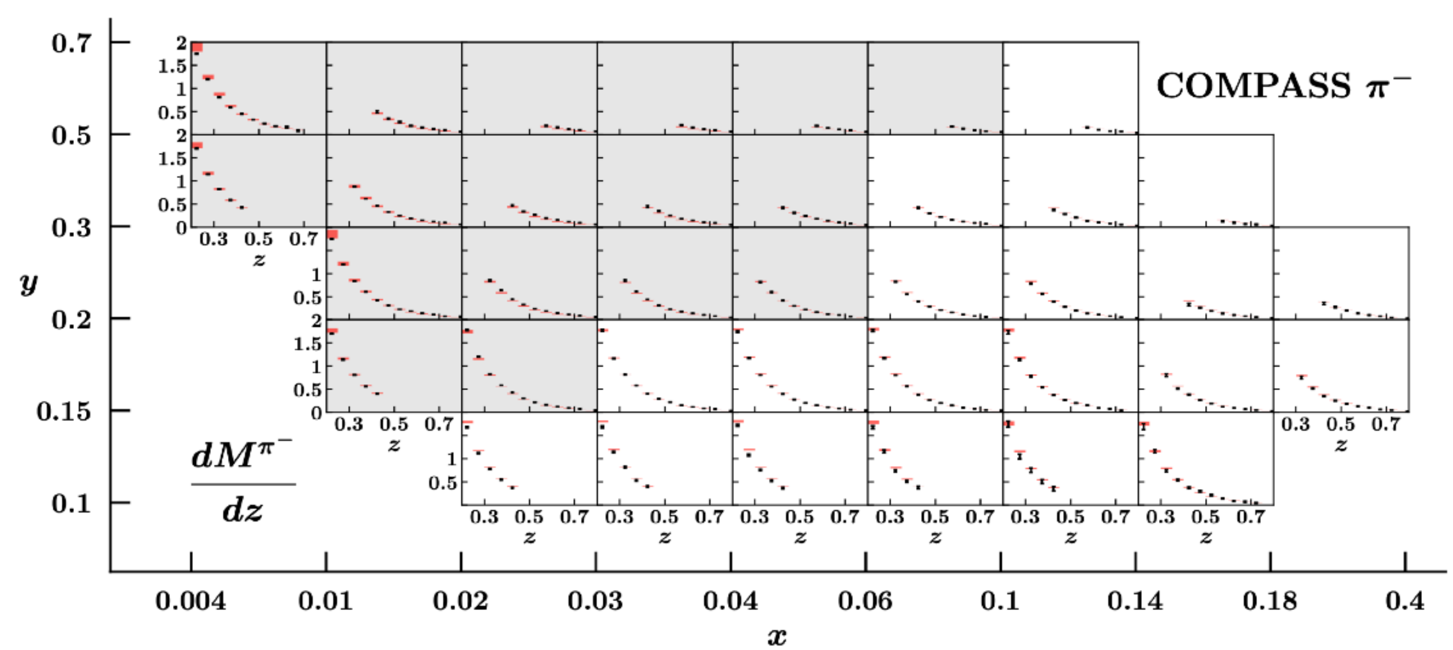
Collinear SIDIS cross section $$\frac{d\sigma}{dx dQ dz}$$

Normalization of prediction such that

$$\int dP_{hT} W(x, z, Q, P_{hT}) = \frac{d\sigma}{dx dQ dz}$$

Piacenza, PhD thesis (2020)

Good agreement theory/data



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Normalization of prediction such that

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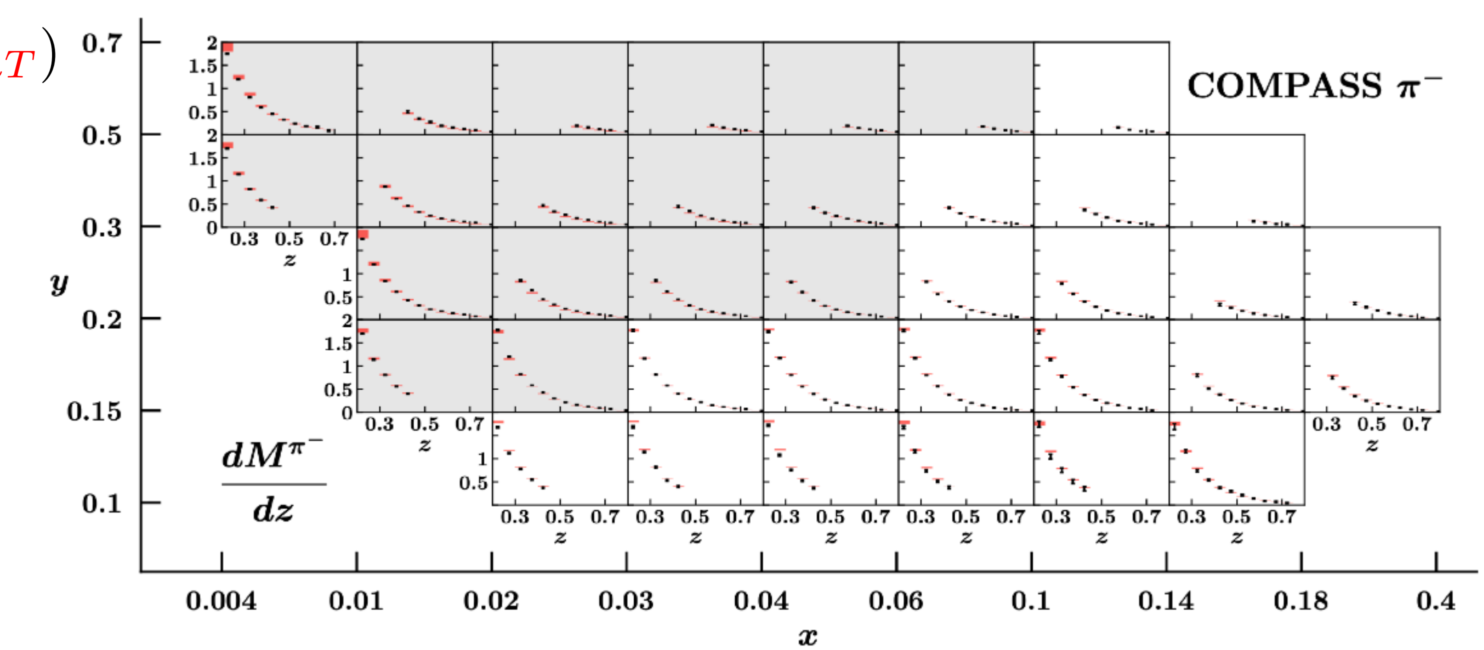

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MAP Collaboration, JHEP 10 (2022)

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SIDIS multiplicity $$M(x,z,P_{hT},Q) = \frac{d\sigma}{dxdQ\color{red}{dzdP_{hT}}} \bigg/ \frac{d\sigma}{dxdQ}$$

Collinear SIDIS cross section $$\frac{d\sigma}{dxdQ\color{red}{dz}}$$

Normalization of prediction such that

$$n(x,z,Q) = \frac{d\sigma}{dxdQdz} \bigg/ \int d\color{red}{P_{hT}}\color{black}{W(x,z,Q,}\color{red}{P_{hT}}\color{black}{)}$$

Piacenza, PhD thesis (2020)

$$M(x,z,P_{hT},Q) = n(x,z,Q)W(x,z,Q,P_{hT}) \bigg/ \frac{d\sigma}{dxdQ}$$

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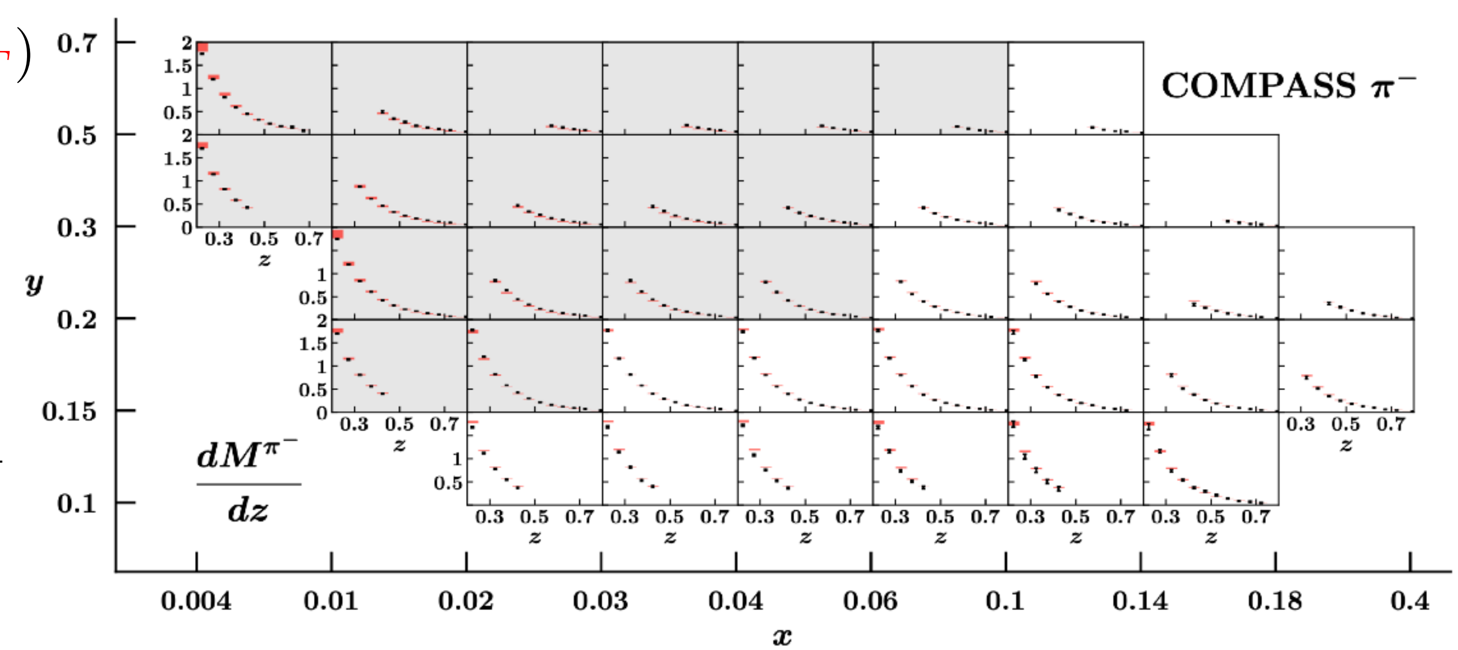

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Normalization of SIDIS calculation

MAP22 work solution

SIDIS multiplicity $$M(x,z,P_{hT},Q) = \frac{d\sigma}{dx\,dQ\,dz\,dP_{hT}} \Big/ \frac{d\sigma}{dx\,dQ}$$

Collinear SIDIS cross section $$\frac{d\sigma}{dx\,dQ\,dz}$$

Normalization of prediction such that

$$n(x,z,Q) = \frac{d\sigma}{dx\,dQ\,dz} \Big/ \int dP_{hT}\, W(x,z,Q,P_{hT})$$

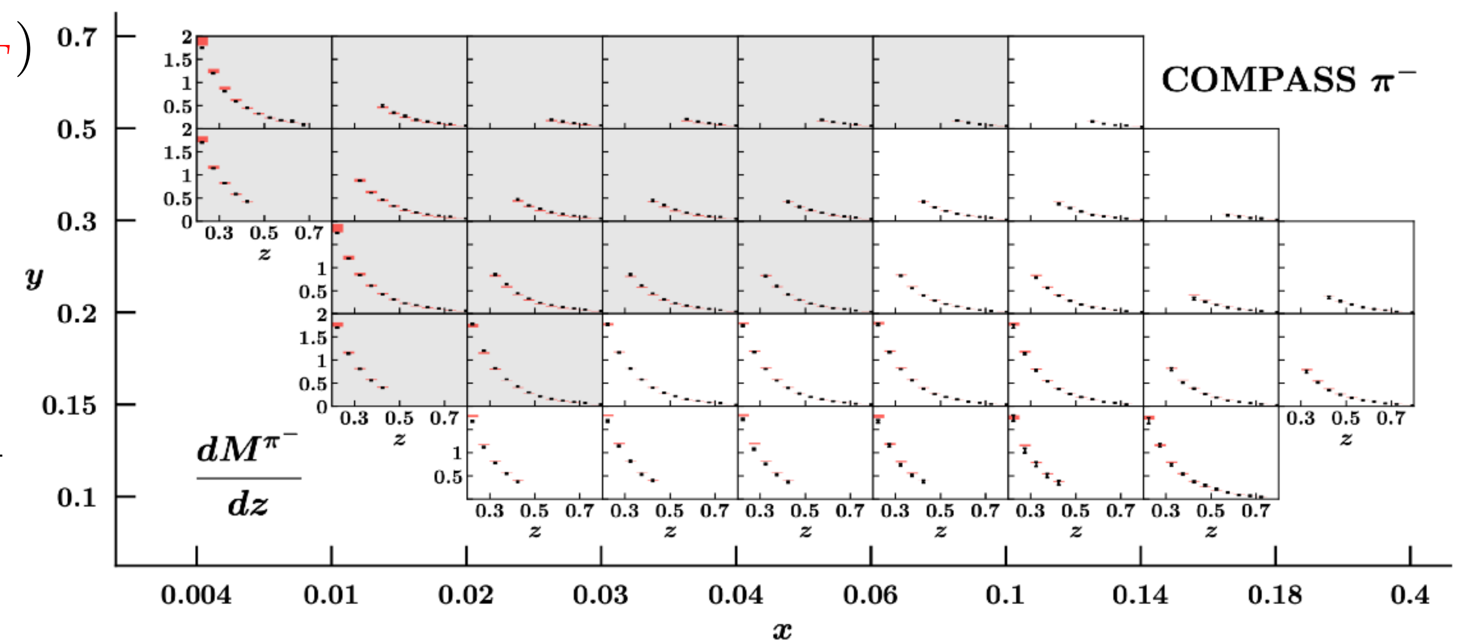
Piacenza, PhD thesis (2020)

$$M(x,z,P_{hT},Q) = n(x,z,Q)\, W(x,z,Q,P_{hT}) \Big/ \frac{d\sigma}{dx\,dQ}$$

Calculable before the fit

Good agreement for almost all bins

Good agreement theory/data



Khalek, Bertone, Nocera, et al., PRD 104 (2021)

MAP Collaboration, JHEP 10 (2022)

MAPTMD24: new approach

high sensitivity to flavor dependence

$$e + p \to e' + \pi^+ + X$$
$$e + p \to e' + \pi^- + X$$
$$e + p \to e' + K^+ + X$$
$$e + p \to e' + K^- + X$$

+ deuteron target

$$e + d \to e' + h^+ + X$$
$$e + d \to e' + h^- + X$$

$$p + p \to \gamma^*/Z \to l^+ + l^-$$

$q\bar{q}$ in the initial state

low sensitivity to flavor dependence

Impact study of SoLID pseudodata: pions

Included dataset

$Q^2 > 1.4\ \text{GeV}^2$

$0.2 < z < 0.7$

$x_B < 0.7$

$P_{hT} < \min\left[\min\left[0.2\,Q, 0.5zQ\right] + 0.3\,\text{GeV}, zQ\right]$

Final-state hadrons

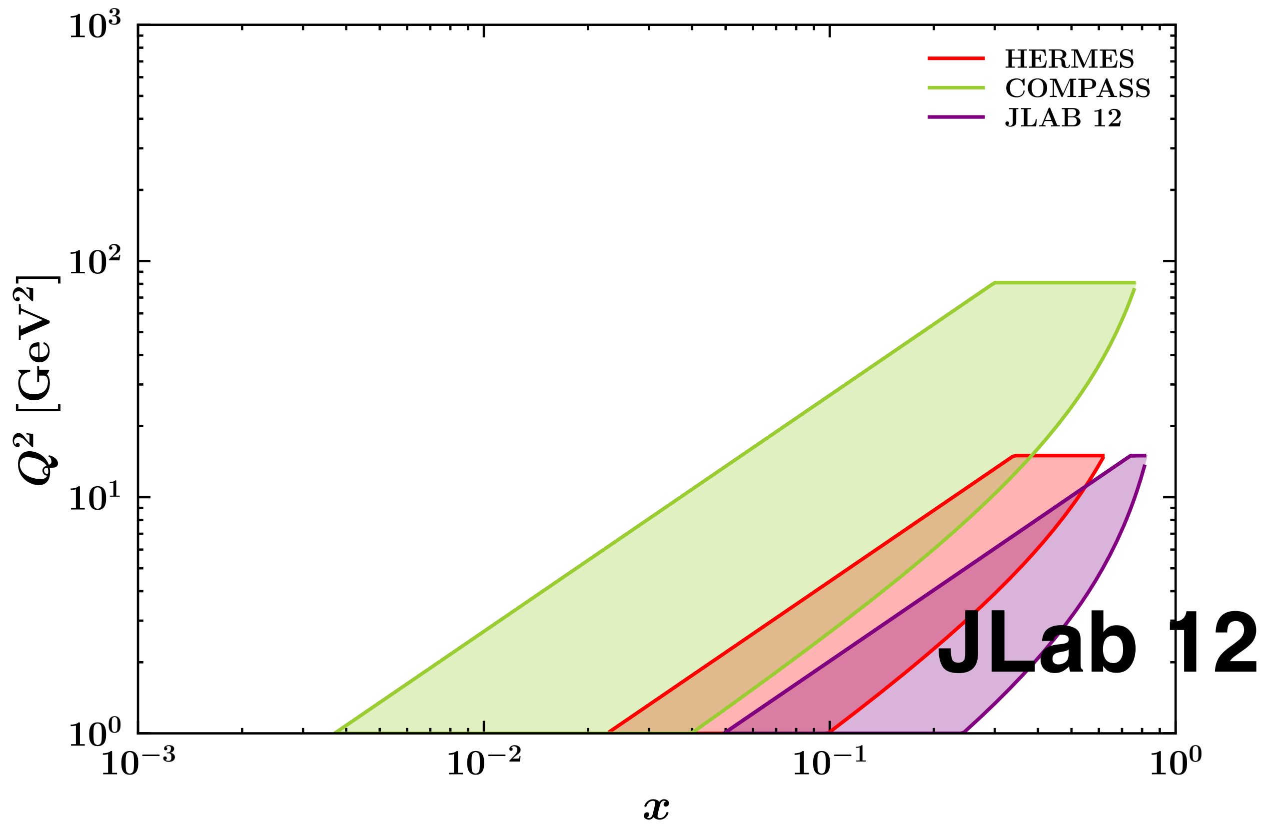
π^+ π^-

K^+ K^-

$\sim$ 2000 MAP24

+

~ 800 SoLID pseudodata



$\pi^+ \pi^-$

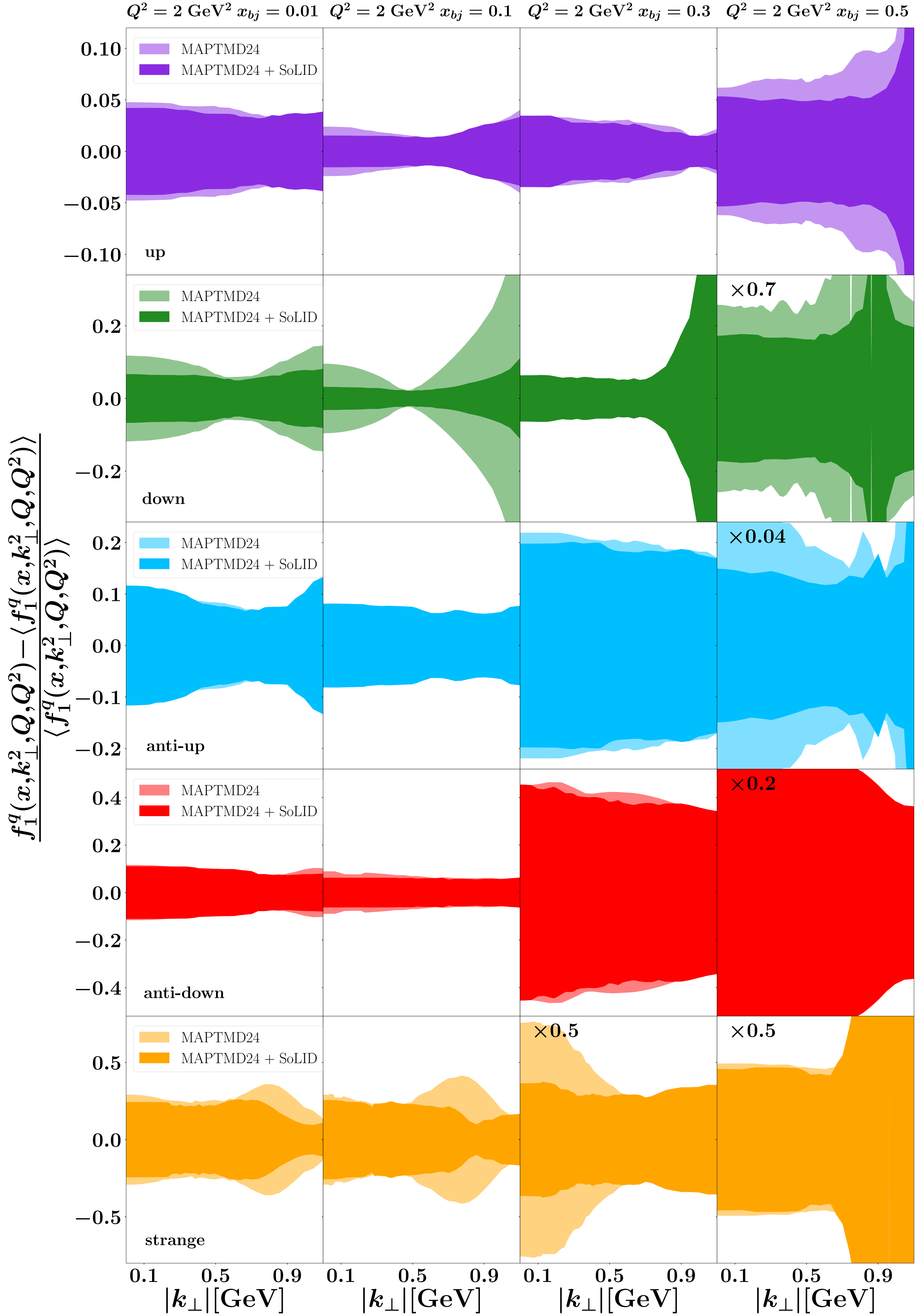
$x = 0.01, 0.1, 0.3, 0.5$

$Q^2 = 2\ \text{GeV}^2$

Plotted quantity

$$\frac{f_1^q(x,k_\perp^2,Q,Q^2) - \langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}{\langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}$$

Uncertainty bands account for 68% CL



$\pi^+ \pi^-$

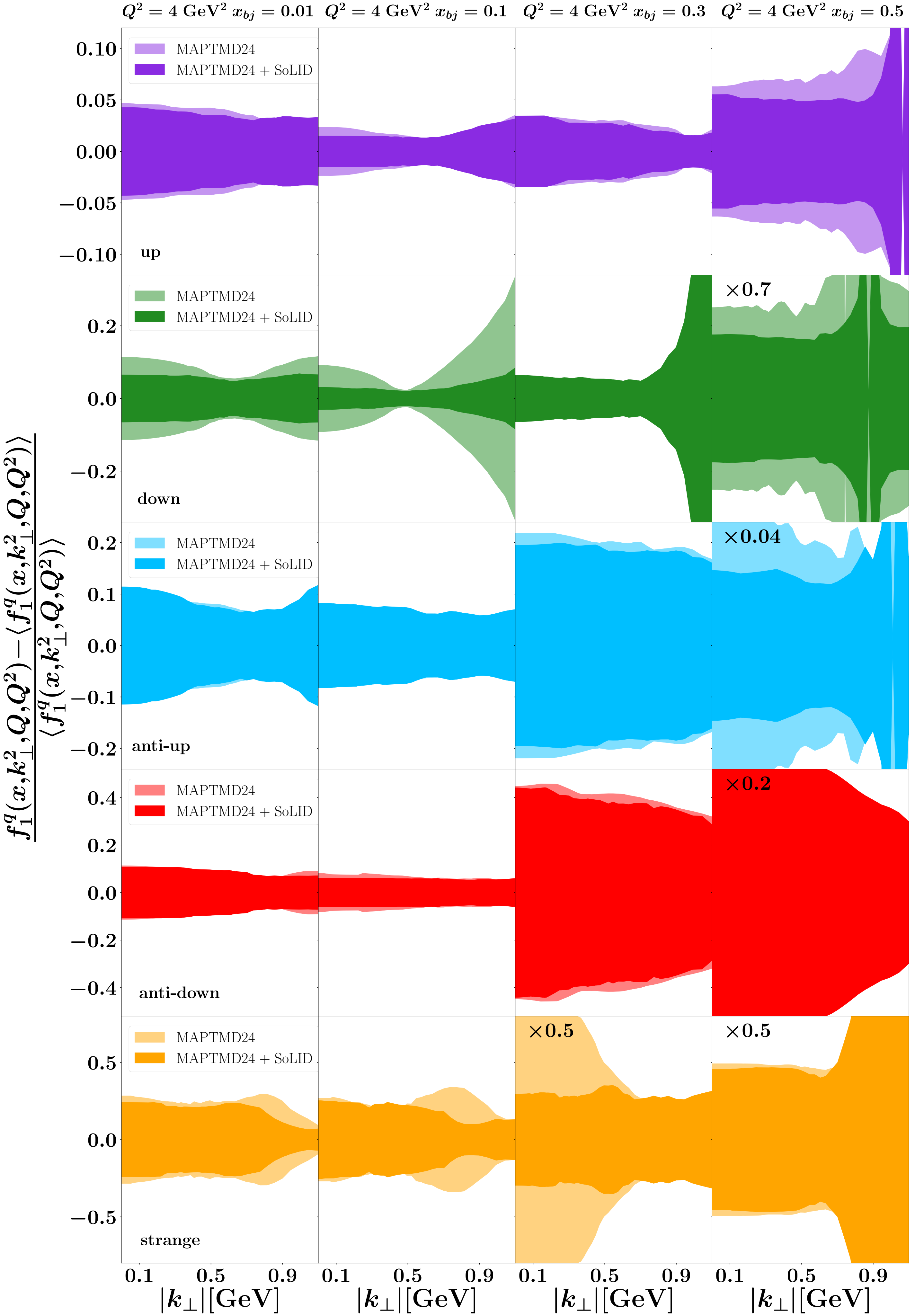
$x = 0.01, 0.1, 0.3, 0.5$

$Q^2 = 4\ \mathrm{GeV}^2$

Plotted quantity

$$\frac{f_1^q(x,k_\perp^2,Q,Q^2) - \langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}{\langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}$$

Uncertainty bands account for 68% CL



$\pi^+ \pi^-$

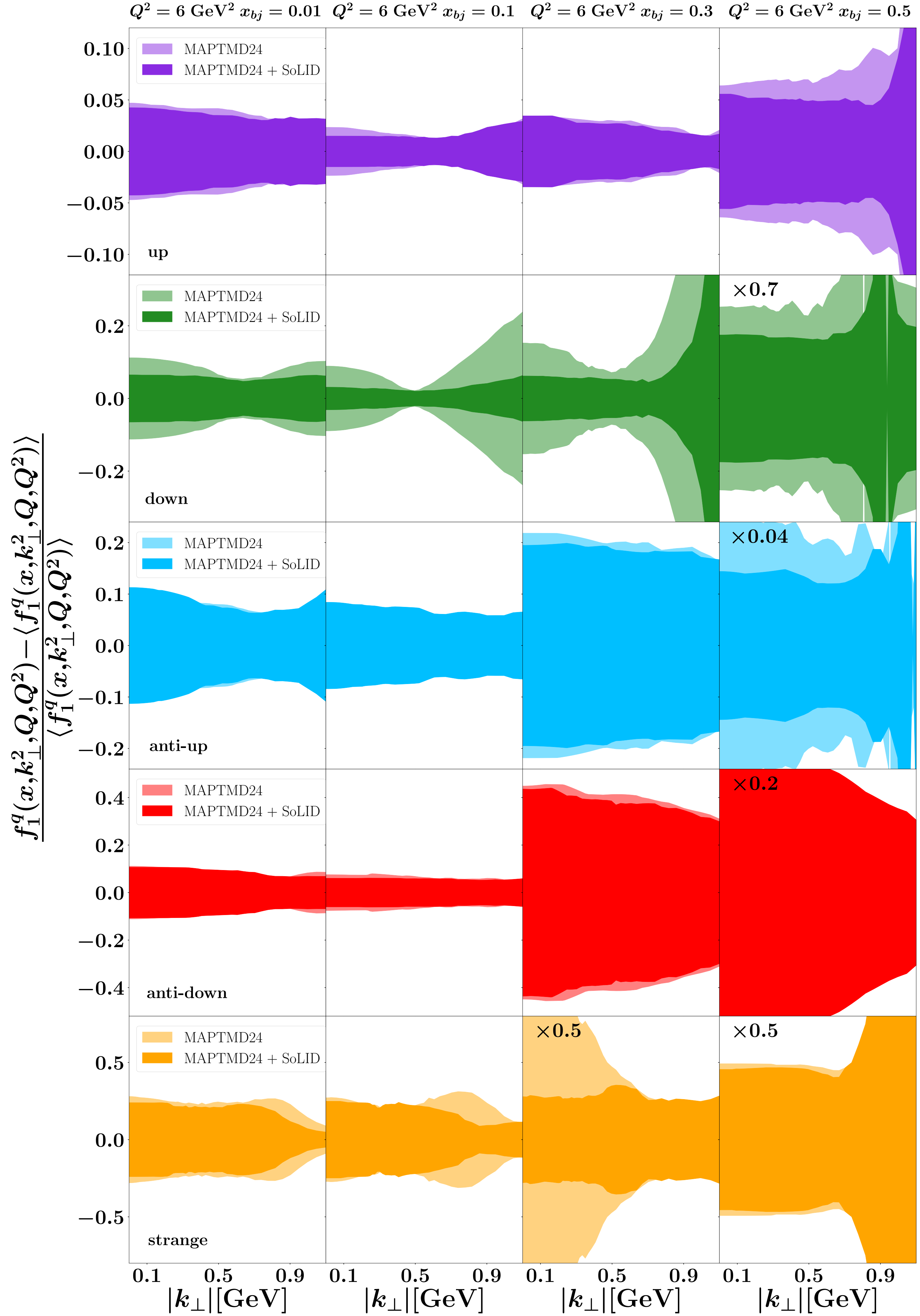
$x = 0.01, 0.1, 0.3, 0.5$

$Q^2 = 6\ \text{GeV}^2$

Plotted quantity

$$\frac{f_1^q(x,k_\perp^2,Q,Q^2) - \langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}{\langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}$$

Uncertainty bands account for 68% CL



Impact study of SoLID pseudodata: kaons

Included dataset

$Q^2 > 1.4\ \mathrm{GeV}^2$

$0.2 < z < 0.7$

$x_B < 0.7$

$P_{hT} < \min\left[\min\left[0.2\,Q, 0.5zQ\right] + 0.3\,\mathrm{GeV}, zQ\right]$

Final-state hadrons

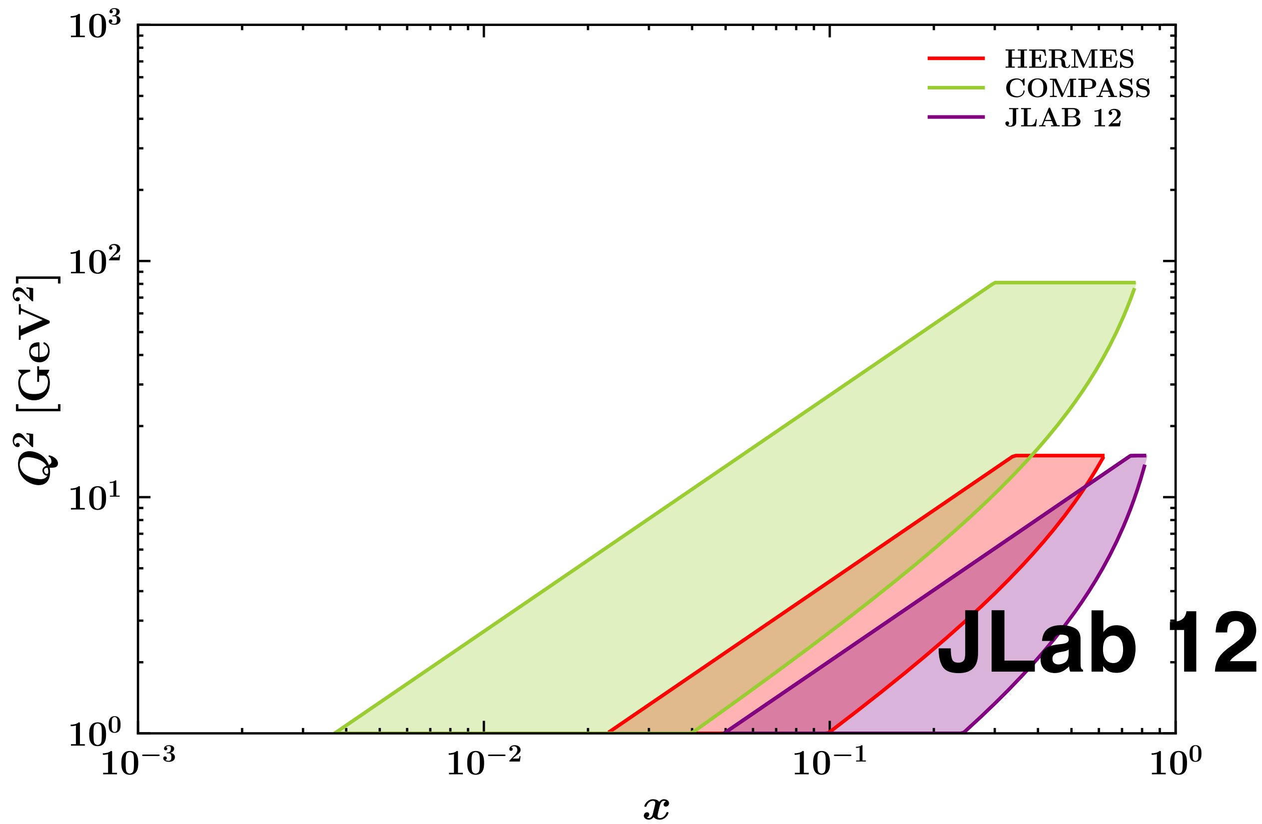
π^+ π^-

K^+ K^-

$\sim\ 2000$ MAP24

+

~ 800 SoLID pseudodata



$K^+ \ K^-$

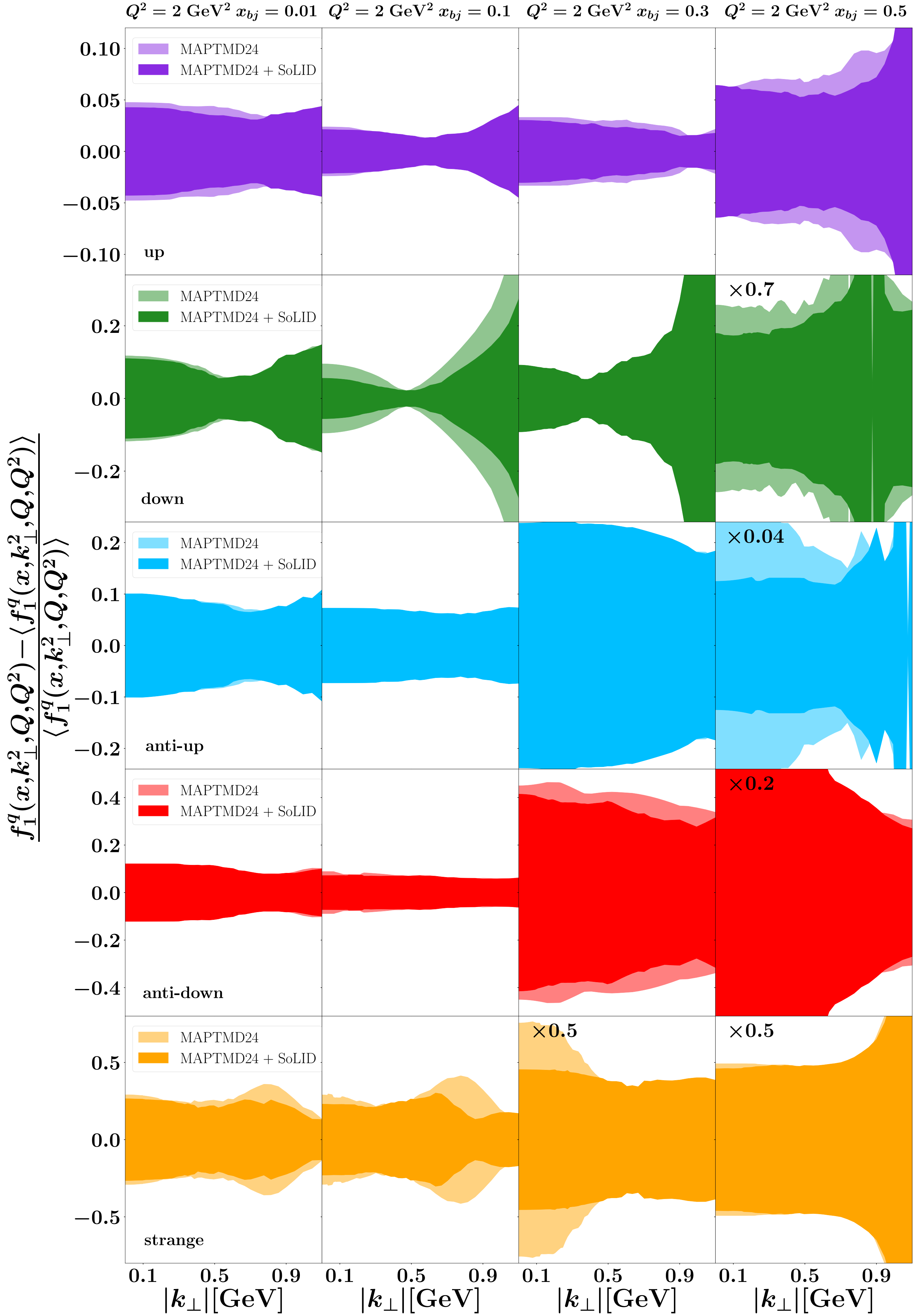
$x = 0.01, 0.1, 0.3, 0.5$

$Q^2 = 2 \ \mathrm{GeV}^2$

Plotted quantity

$$\frac{f_1^q(x,k_\perp^2,Q,Q^2) - \langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}{\langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}$$

Uncertainty bands account for 68% CL



$K^+\ K^-$

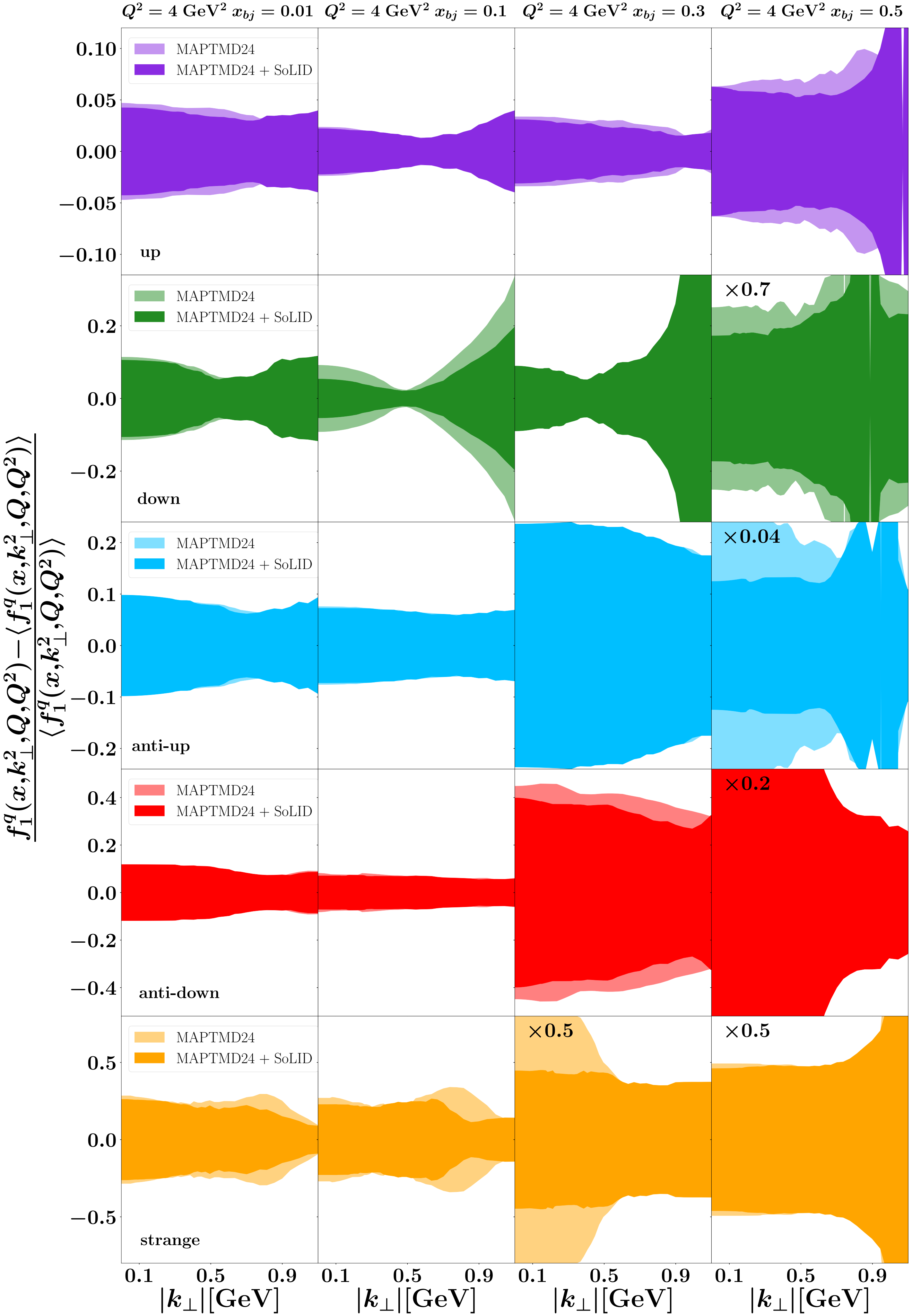
$x = 0.01, 0.1, 0.3, 0.5$

$Q^2 = 4\ \mathrm{GeV}^2$

Plotted quantity

$$\frac{f_1^q(x,k_\perp^2,Q,Q^2) - \langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}{\langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}$$

Uncertainty bands account for 68% CL



$K^+ \; K^-$

$x = 0.01, 0.1, 0.3, 0.5$

$Q^2 = 6 \text{ GeV}^2$

Plotted quantity

$$\frac{f_1^q(x,k_\perp^2,Q,Q^2) - \langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}{\langle f_1^q(x,k_\perp^2,Q,Q^2)\rangle}$$

Uncertainty bands account for 68% CL