## A Light-Front approach to the 3He Spectral Function

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## This is an exciting Workshop for me...

We studied the process $A\left(e, e^{\prime}(A-1)\right) X$ many years ago


An old idea (Claudio among the first): in this process, in IA,

$$
d^{2} \sigma_{A} \propto F_{2}^{N}(x)
$$

there is no convolution! (nucl-th/9609062)
Example: through ${ }^{3} \mathrm{He}\left(e, e^{\prime} \mathrm{d}\right) \mathrm{X}$, check of the reaction mechanism (EMC effect); measuring ${ }^{3} \mathrm{H}(\mathrm{e}, \mathrm{e}$ 'd $) \mathrm{X}$, direct access to the neutron!
new perspectives
(loi to the JLab PAC,
already in November 2010)

Eur. Phys. J. A 5, 191-207 (1999)
The European Physical Journal a © Springer-Verlog 1999

## Semi-inclusive deep inelastic lepton scattering off complex nuclei

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Abstract. It is shown that in semi-inclusive deep inelostic scottering (DIS) of electrons off complex mucle The detection, in coincidence with the scottered delectran, of an nuclevs $(A-1)$ in the ground state, os wed
of of nucleon ond $a$ nucleus ( $A-2)$, olso in the ground state, may provide unique information on sover
 Ong standing problems, such os: i) the noture ond the relevance of the final state interoction in DIS; $i$ ii) the
volidity of the spectotor mech monism in DIS; $i$ ii) the medium induced modificotions of the nucleon structur function; iv) the origin of the $E M C$ effect.
PACS. 13.40 -f Electrom onnetic processes snd properties - 21.60 -n Nude or-structure rodele snd methods ${ }_{\text {distritutions }}^{24.25 .+p}$

Semi-inclusive Deep Inelastic Scattering from Light Nuclei by Tagging Low Momentum Spectators
R. Dupré, K. Hafidi, and X. Zhan

Argonne National Laboratory, Argonne, Mlinois 60439, USA
S. Dhamija.

Indiana University, Bloomington, Indiana 47405, USA
S. Stepanyan

Thomas Jefferson National Accelerator Faciity,
Newport News, Virginia Q3606, USA
(Dated: November 24, 2010 Abstract
We prop ose to measure the semi-inclusive deep inelastic scattering from light nucle ( $\mathrm{D},{ }^{3} \mathrm{He},{ }^{4} \mathrm{He}$ ).
The detection of the low energy recoil nucleus in the final state will provide unique information bout the nature of nuclear EMC effect and will permit to investigate the modifications of the
nucleon structure functions in the nucleus. We propose to measure a set of observable by using the future 11 GeV electron beam in Hall B CLAS12. The baseline CLAS12 detector is suitable to detect electrons in the valence region, and a new low energy recoil detector with good performance

[^0]March $9^{t h}, 2015$

This is an exciting Workshop for me...
Later, I studied nuclear GPDs...

Which of these pictures is more similar to a nuclear section?


We should perform a tomography It is possible!
Coherent dvcs \& GPDs


Nucleon tomography
Slow nuclear recoil detection is necessary...

## Outline

- Importance of the ${ }^{3} \mathrm{He}$ nucleus for fundamental studies in Hadronic Physics. In particular: the neutron information from ${ }^{3} \mathrm{He}$.
- Crucial quantity: the (distorted) spectral function Recent theoretical developments in DVCS
(M. Rinaldi, S.S. PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)) and SiDIS studies
(L. Kaptari, A. Del Dotto, E. Pace, G. Salmè, S.S., PRC 89 (2014) 035206)

Importance of a relativistic treatment for the description
of the JLab program @ 12 GeV

- The LF spectral function of ${ }^{3} \mathrm{He}$
( E. Pace, A. Del Dotto, M. Rinaldi, G. Salmè, S.S., Few Body Syst. 54 (2013) 1079)
(work in progress): preliminary results
- Conclusions


## Importance of ${ }^{3} \mathrm{He}$ for DIS structure studies

- ${ }^{3} \mathrm{He}$ is theoretically well known. Even a relativistic treatment may be implemented.
- ${ }^{3} \mathrm{He}$ has been used extensively as an effective neutron target, especially to unveil the spin content of the free neutron, due to its peculiar spin structure:

$\boldsymbol{S}$ (~90\%)
${ }^{3} \mathrm{He}$ always promising when the neutron polarization properties have to be studied.
- To this aim, ${ }^{3} \mathrm{He}$ is unique and its spectral function arises in * DIS, together with ${ }^{3} \mathbf{H}$, for the extraction of $F_{2}^{n}$ (Marathon experiment, JLab); * polarized DIS, for the extration of the SSF $g_{1}^{n}$;
* polarized SiDIS, for the extraction of neutron transversity and related observables;
* 

DVCS, for the extraction of neutron GPDs

## Example 1: DVCS off ${ }^{3} \mathrm{He}$

For a $J=\frac{1}{2}$ target, in a hard-exclusive process, $\left(Q^{2}, \nu \rightarrow \infty\right)$
such as (coherent) DVCS
(Definition of GPDs from X. Ji PRL 78 (97) 610):


- $\Delta=P^{\prime}-P, q^{\mu}=\left(q_{0}, \vec{q}\right)$, and $\bar{P}=\left(P+P^{\prime}\right)^{\mu} / 2$
? $x=k^{+} / P^{+} ; \quad \xi=$ "skewness" $=-\Delta^{+} /\left(2 \bar{P}^{+}\right)$
〇 $x \leq-\xi \longrightarrow$ GPDs describe antiquarks;
$-\xi \leq x \leq \xi \longrightarrow$ GPDs describe $q \bar{q}$ pairs; $x \geq \xi \longrightarrow$ GPDs describe quarks
the GPDs $H_{q}\left(x, \xi, \Delta^{2}\right)$ and $E_{q}\left(x, \xi, \Delta^{2}\right)$ are introduced:

$$
\begin{aligned}
\int \frac{d \lambda}{2 \pi} e^{i \lambda x}\left\langle P^{\prime}\right| \bar{\psi}_{q}(-\lambda n / 2) & \gamma^{\mu} \\
& +\quad \psi_{q}(\lambda n / 2)|P\rangle=H_{q}\left(x, \xi, \Delta^{2}\right) \bar{U}\left(P^{\prime}\right) \gamma^{\mu} U(P) \\
& +\quad E_{q}\left(x, \xi, \Delta^{2}\right) \bar{U}\left(P^{\prime}\right) \frac{i \sigma^{\mu \nu} \Delta_{\nu}}{2 M} U(P)+\ldots
\end{aligned}
$$

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- $x=k^{+} / P^{+} ; \quad \xi=$ "skewness" $=-\Delta^{+} /\left(2 \bar{P}^{+}\right)$
- $x \leq-\xi \longrightarrow$ GPDs describe antiquarks;
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and the helicity dependent ones, $\tilde{H}_{q}\left(x, \xi, \Delta^{2}\right)$ and $\tilde{E}_{q}\left(x, \xi, \Delta^{2}\right)$, obtained as follows:

$$
\begin{array}{rll}
\int \frac{d \lambda}{2 \pi} e^{i \lambda x}\left\langle P^{\prime}\right| \bar{\psi}_{q}(-\lambda n / 2) & \gamma^{\mu} \gamma_{5} & \psi_{q}(\lambda n / 2)|P\rangle=\tilde{H}_{q}\left(x, \xi, \Delta^{2}\right) \bar{U}\left(P^{\prime}\right) \gamma^{\mu} U(P) \\
& +\quad \tilde{E}_{q}\left(x, \xi, \Delta^{2}\right) \bar{U}\left(P^{\prime}\right) \frac{\gamma_{5} \Delta^{\mu}}{2 M} U(P)+\ldots
\end{array}
$$

March $9^{\text {th }}, 2015$

## GPDs of ${ }^{3} \mathrm{He}$ : the Impulse Approximation

## coherent DVCS in I.A.

$\left({ }^{3} \mathrm{He}\right.$ does not break-up, $\Delta^{2} \ll M^{2}, \xi^{2} \ll 1$, ):

In a symmetric frame $\left(\bar{p}=\left(p+p^{\prime}\right) / 2\right)$ :


$$
\begin{aligned}
k^{+} & =(x+\xi) \bar{P}^{+}=\left(x^{\prime}+\xi^{\prime}\right) \bar{p}^{+} \\
(k+\Delta)^{+} & =(x-\xi) \bar{P}^{+}=\left(x^{\prime}-\xi^{\prime}\right) \bar{p}^{+}
\end{aligned}
$$

one has, for a given GPD, $H_{q}, \tilde{G}_{M}^{q}=H_{q}+E_{q}$, or $\tilde{H}_{q}$

$$
\left.G P D_{q}\left(x, \xi, \Delta^{2}\right) \simeq \sum_{N} \int \frac{d z^{-}}{4 \pi} e^{i x \bar{P}^{+} z^{-}}{ }_{A}\left\langle P^{\prime} S^{\prime}\right| \hat{O}_{q}^{\mu, N}|P S\rangle_{A}\right|_{z+=0, z_{\perp}=0}
$$

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$$

By properly inserting a tensor product complete basis for the nucleon (PW) and the fully interacting recoiling system :

## GPDs of ${ }^{3} \mathrm{He}$ : the Impulse Approximation

coherent DVCS in I.A.
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$$
\begin{aligned}
& G P D_{q}\left(x, \xi, \Delta^{2}\right) \simeq \sum_{N} \int \frac{d z^{-}}{4 \pi} e^{i x^{\prime} \bar{p}^{+} z^{-}}\left\langle P^{\prime} S^{\prime}\right| \sum_{\vec{P}_{R}^{\prime}, f_{A-1}^{\prime}, \vec{p}^{\prime}, s^{\prime}}\left\{\left|P_{R}^{\prime}, \Phi_{A-1}^{f^{\prime}}\right\rangle \otimes\left|p^{\prime} s^{\prime}\right\rangle\right\} \\
& \left\langle P_{R}^{\prime}, \Phi_{A-1}^{f^{\prime}}\right| \otimes\left\langle p^{\prime} s^{\prime}\right| \hat{O}_{q}^{\mu, N} \sum_{\vec{P}_{R}, f_{A-1}, \vec{p}, s}\left\{\left|P_{R}, \Phi_{A-1}^{f}\right\rangle \otimes|p s\rangle\right\}\left\{\left\langle P_{R}, \Phi_{A-1}^{f}\right| \otimes\langle p s|\right\}|P S\rangle,
\end{aligned}
$$

and, since

$$
\left\{\left\langle P_{R}, \Phi_{A-1}^{f}\right| \otimes\langle p s|\right\}|P S\rangle=(2 \pi)^{3} \delta^{3}\left(\vec{P}-\vec{P}_{R}-\vec{p}\right)\left\langle\Phi_{A-1}^{f}, p s \mid P S\right\rangle
$$

## (NR! Separation of the global motion from the intrinsic one!)

March $9^{t h}, 2015$

## GPDs of ${ }^{3} \mathrm{He}$ in IA: the spectral function

$H_{q}^{A}$ can be obtained in terms of $H_{q}^{N}$ (S.S. PRC 70, 015205 (2004), PRC 79, 025207 (2009)):

$$
H_{q}^{A}\left(x, \xi, \Delta^{2}\right)=\sum_{N} \int d E \int d \vec{p} \overline{\sum_{\mathcal{M}}} \sum_{s} P_{\mathcal{M} \mathcal{M}, s s}^{N}\left(\vec{p}, \vec{p}^{\prime}, E\right) \frac{\xi^{\prime}}{\xi} H_{q}^{N}\left(x^{\prime}, \Delta^{2}, \xi^{\prime}\right),
$$

$\tilde{G}_{M}^{3, q}$ in terms of $\tilde{G}_{M}^{N, q}$ (M. Rinaldi, S.S. PRC 85, 062201(R)(2012); PRC 87, 035208 (2013)):

$$
\tilde{G}_{M}^{3, q}\left(x, \Delta^{2}, \xi\right)=\sum_{N} \int d E \int d \vec{p}\left[P_{+-,+-}^{N}-P_{+-,-+}^{N}\right]\left(\vec{p}, \vec{p}^{\prime}, E\right) \frac{\xi^{\prime}}{\xi} \tilde{G}_{M}^{N, q}\left(x^{\prime}, \Delta^{2}, \xi^{\prime}\right)
$$

and $\tilde{H}_{q}^{A}$ can be obtained in terms of $\tilde{H}_{q}^{N}$ :

$$
\tilde{H}_{q}^{A}\left(x, \xi, \Delta^{2}\right)=\sum_{N} \int d E \int d \vec{p}\left[P_{++,++}^{N}-P_{++,--}^{N}\right]\left(\vec{p}, \vec{p}^{\prime}, E\right) \frac{\xi^{\prime}}{\xi} \tilde{H}_{q}^{N}\left(x^{\prime}, \Delta^{2}, \xi^{\prime}\right),
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H_{q}^{A}\left(x, \xi, \Delta^{2}\right)=\sum_{N} \int d E \int d \vec{p} \sum_{\mathcal{M}} \sum_{s} P_{\mathcal{M} \mathcal{M}, s s}^{N}\left(\vec{p}, \vec{p}^{\prime}, E\right) \frac{\xi^{\prime}}{\xi} H_{q}^{N}\left(x^{\prime}, \Delta^{2}, \xi^{\prime}\right),
$$

$\tilde{G}_{M}^{3, q}$ in terms of $\tilde{G}_{M}^{N, q}$ (M. Rinaldi, S.S. PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)):

$$
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$$

where $P_{\mathcal{M}^{\prime} \mathcal{M}, s^{\prime} s}^{N}\left(\vec{p}, \vec{p}^{\prime}, E\right)$ is the one-body, spin-dependent, off-diagonal spectral function for the nucleon $N$ in the nucleus,

$$
\begin{gathered}
\mathbf{P}_{\mathcal{M}^{\prime} \mathcal{M} \sigma^{\prime} \sigma}^{N}\left(\vec{p}, \vec{p}^{\prime}, E\right)=\sum_{f_{\text {intrinsic overlaps }} \nearrow}^{\sum_{f_{A-1}} \delta\left(E-E_{A-1}+E_{A}\right)} \\
\underbrace{S_{A}\left\langle\Psi_{A} ; J_{A} \mathcal{M} \pi_{A} \mid \vec{p}, \sigma ; \phi_{f_{A-1}}\right\rangle} \underbrace{\left\langle\phi_{f_{A-1}} ; \sigma^{\prime} \vec{p}^{\prime} \mid \pi_{A} J_{A} \mathcal{M}^{\prime} ; \Psi_{A}\right\rangle_{S_{A}}}
\end{gathered}
$$

## The spectral function: a few words more

$$
\begin{aligned}
& \mathbf{P}_{\mathcal{M}^{\prime} \mathcal{M} \sigma^{\prime} \sigma}^{N}\left(\vec{p}, \vec{p}^{\prime}, E\right)=\sum_{f} \delta\left(E-E_{\min }-E_{f}^{*}\right) \\
& S_{A}\left\langle\Psi_{A} ; J_{A} \mathcal{M} \pi_{A} \mid \vec{p}, \sigma ; \phi_{f}\left(E_{f}^{*}\right)\right\rangle\left\langle\stackrel{f}{\phi_{f}}\left(E_{f}^{*}\right) ; \sigma^{\prime} \vec{p}^{\prime} \mid \pi_{A} J_{A} \mathcal{M}^{\prime} ; \Psi_{A}\right\rangle_{S_{A}} \\
& { }^{3} \mathrm{He} \underbrace{\overrightarrow{\mathrm{P}}, \mathrm{E}_{\mathrm{f}}^{*}}_{\overrightarrow{\mathrm{P}}}
\end{aligned}
$$

- the two-body recoiling system can be either the deuteron or a scattering state;
- when a deeply bound nucleon, with high removal energy $E=E_{\text {min }}+E_{f}^{*}$, leaves the nucleus, the recoling system is left with high excitation energy $E_{f}^{*}$;
- the three-body bound state and the two-body bound or scattering state are evaluated within the same interaction (in our case, Av18, from the Pisa group (Kievsky, Viviani): the extension of the treatment to heavier nuclei would be very difficult


## GPDs of ${ }^{3} \mathrm{He}$ : importance of relativity

- What we have:
* An instant form, I.A. calculation of $H^{3}, \tilde{G}_{M}^{3}, \tilde{H}^{3}$, within AV18;
* the neutron contribution dominates $\tilde{G}_{M}^{3}$ and $\tilde{H}^{3}$ at low $\Delta^{2}$;
* an extraction procedure of the neutron information, able to take into account all the nuclear effects encoded in an IA analysis;
- What we can do now: to estimate $X$-sections (DVCS, BH, Interference)
$\longrightarrow$ a proposal of coherent DVCS off ${ }^{3} \mathrm{He}$ at JLab@12 GeV?


## BUT

- In case experiments are performed at higher $\Delta^{2}$ :
* a RELATIVISTIC TREATMENT is mandatory:
a sizable difference in momentum between the initial and final states requires proper boosting
* The fulfillment of polinomiality requires covariance; In NR calculations, number of particle sum rule, momentum sum rule, (slightly) violated.
- A relativistic extension of the ${ }^{3} \mathrm{He}$ spectral function definition is necessary


## Example II: Single Spin Asymmetries (SSAs)

$$
\vec{A}\left(e, e^{\prime} h\right) X \text { : Unpolarized beam and T-polarized target } \rightarrow \sigma_{U T}
$$

$$
x=\frac{Q^{2}}{2 P \cdot q} \quad y=\frac{P \cdot q}{P \cdot l} \quad z=\frac{P \cdot h}{P \cdot q} \quad \hat{q}=-\hat{e}_{z}
$$

The number of emitted hadrons at a given $\phi_{h}$ depends on the orientation of $\vec{S}_{\perp}$ ! In SSAs 2 different mechanisms can be experimentally distinguished

$$
A_{U T}^{\text {Sivers }(\text { Collins })}=\frac{\int d \phi_{S} d^{2} P_{h \perp} \sin \left(\phi_{h}-(+) \phi_{S}\right) d^{6} \sigma_{U T}}{\int d \phi_{S} d^{2} P_{h \perp} d^{6} \sigma_{U U}}
$$

with

$$
d^{6} \sigma_{U T}=\frac{1}{2}\left(d^{6} \sigma_{U \uparrow}-d^{6} \sigma_{U \downarrow}\right) \quad d^{6} \sigma_{U U}=\frac{1}{2}\left(d^{6} \sigma_{U \uparrow}+d^{6} \sigma_{U \downarrow}\right)
$$

## SSAs $\rightarrow$ the neutron $\rightarrow{ }^{3} \mathrm{He}$

SSAs in terms of parton distributions and fragmentation functions:
$\bigcirc \quad A_{U T}^{\text {Sivers }}=N^{\text {Sivers }} / D \quad A_{U T}^{\text {Collins }}=N^{\text {Collins }} / D$
$N^{\text {Sivers }} \propto \sum_{q} e_{q}^{2} \int d^{2} \kappa_{\mathbf{T}} d^{2} \mathbf{k}_{\mathbf{T}} \delta^{2}\left(\mathbf{k}_{\mathbf{T}}+\mathbf{q}_{\mathbf{T}}-\kappa_{\mathbf{T}}\right) \frac{\hat{\mathbf{P}_{\mathbf{h} \perp}} \cdot \mathbf{k}_{\mathbf{T}}}{\mathbf{M}} f_{1 T}^{\perp q}\left(x, \mathbf{k}_{\mathbf{T}}{ }^{2}\right) D_{1}^{q, h}\left(z,\left(z \kappa_{\mathbf{T}}\right)^{2}\right)$
$N^{\text {Collins }} \propto \sum_{q} e_{q}^{2} \int d^{2} \kappa_{\mathbf{T}} d^{2} \mathbf{k}_{\mathbf{T}} \delta^{2}\left(\mathbf{k}_{\mathbf{T}}+\mathbf{q}_{\mathbf{T}}-\kappa_{\mathbf{T}}\right) \frac{\hat{\mathbf{P}_{\mathbf{h} \perp}} \cdot \kappa_{\mathbf{T}}}{\mathbf{M}_{\mathbf{h}}} h_{1}^{q}\left(x, \mathbf{k}_{\mathbf{T}}{ }^{2}\right) H_{1}^{\perp q, h}\left(z,\left(z \kappa_{\mathbf{T}}\right)^{2}\right)$
$D \propto \sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1}^{q, h}(z)$

- LARGE $A_{U T}^{S i v e r s}$ measured in $\vec{p}\left(e, e^{\prime} \pi\right) x$ HERMES PRL 94, 012002 (2005)
- SMALL $A_{U T}^{\text {Sivers }}$ measured in $\vec{D}\left(e, e^{\prime} \pi\right) x$; COMPASS PRL 94, 202002 (2005)

A strong flavor dependence confirmed by recent data Importance of the neutron for flavor decomposition!

March $9^{t h}, 2015$

## The neutron information from ${ }^{3} \mathrm{He}$

As we have seen already ${ }^{3} \mathrm{He}$ is the ideal target to study the polarized neutron:


$$
\begin{gathered}
\text { In } S \text {-wave } \\
{ }^{3} \vec{H} e=\vec{n}!
\end{gathered}
$$

... But the bound nucleons in ${ }^{3} \mathrm{He}$ are moving!
Dynamical nuclear effects in inclusive DIS ( $\left.{ }^{3} \vec{H} e\left(e, e^{\prime}\right) X\right)$ were evaluated with a realistic spin-dependent spectral function for ${ }^{3} \vec{H} e, P_{\sigma, \sigma} \mathcal{M}(\vec{p}, E)$. It was found that the formula

$$
\begin{array}{r}
A_{n} \simeq \frac{1}{p_{n} f_{n}}\left(A_{3}^{e x p}-2 p_{p} f_{p} A_{p}^{e x p}\right), \quad(\text { Ciofi degli Atti et al., } P R C 48(1993) R 968) \\
\left(f_{p}, f_{n} \text { dilutionfactors }\right)
\end{array}
$$

can be safely used $\longrightarrow$ widely used by experimental collaborations.
The nuclear effects are hidden in the "effective polarizations"

$$
p_{p}=-0.023 \quad(A v 18) \quad p_{n}=0.878 \quad(A v 18)
$$

## $\vec{n}$ from ${ }^{3} \vec{H} e$ : SiDIS case

Can one use the same formula to extract the SSAs ? in SiDIS also the fragmentation functions can be modified by the nuclear environment !

The process ${ }^{3} \vec{H} e\left(e, e^{\prime} \pi\right) X$ has been evaluated :
 in IA $\rightarrow$ no FSI between the measured fast, ultrarelativistic $\pi$ the remnant and the two nucleon recoiling system

$$
E_{\pi} \simeq 2.4 \mathrm{GeV} \text { in JLAB exp at } 6 \mathrm{GeV} \text { - Qian et al., PRL } 107 \text { (2011) } 072003
$$

SSAs involve convolutions of the transverse spin-dependent nuclear spectral function, $P_{\perp}(\vec{p}, E)$, with parton distributions AND fragmentation functions [S.Scopetta, PRD 75 (2007) 054005] :

$$
A \simeq \int d \vec{p} d E \ldots P_{\perp}(\vec{p}, E) f_{1 T}^{\perp q}\left(\frac{Q^{2}}{2 p \cdot q}, \mathbf{k}_{\mathbf{T}}^{2}\right) D_{1}^{q, h}\left(\frac{p \cdot h}{p \cdot q},\left(\frac{p \cdot h}{p \cdot q} \kappa_{\mathbf{T}}\right)^{2}\right)
$$

The nuclear effects on fragmentation functions are new with respect to the DIS case and have been studied carefully, using models for $f_{1 T}^{\perp q}, D_{1}^{q, h} \ldots$ and the $A v 18$ (Pisa group w.f.) spectralfunction.
March $9^{\text {th }}, 2015$

## Results: $\vec{n}$ from ${ }^{3} \vec{H} e: A_{U T}^{\text {Sivers }}$, @ JLab




FULL: Neutron asymmetry (model)
DOTS: Neutron asymmetry extracted from ${ }^{3} \mathrm{He}$ (calculation) neglecting the contribution of the proton polarization

$$
\bar{A}_{n} \simeq \frac{1}{f_{n}} A_{3}^{\text {calc }}
$$

DASHED : Neutron asymmetry extracted from ${ }^{3} \mathrm{He}$ (calculation) taking into account nuclear structure effects through the formula:

$$
A_{n} \simeq \frac{1}{p_{n} f_{n}}\left(A_{3}^{\text {calc }}-2 p_{p} f_{p} A_{p}^{\text {model }}\right)
$$

## Results: $\vec{n}$ from ${ }^{3} \vec{H} e: A_{U T}^{\text {Collins }}$, @ JLab



The extraction procedure successful in DIS works also in SiDIS, for both the Collins and the Sivers SSAs !

1 - What about FSI effects? A pion is detected, now...
2 - What about relativistic effects @ 12 GeV JLab?
E12-09-018 experiment, approved with rate A, G. Cates et al.

## FSI: distorted spin-dependent spectral function of ${ }^{3} \mathrm{He}$

L. Kaptari, A. Del Dotto, E. Pace, G. Salmè, S.S., PRC 89 (2014) 035206


Relative energy between $A-1$ and the remnants: a few GeV
$\longrightarrow$ eikonal approximation.
Relevant part of the (distorted ) spin dependent spectral function:

$$
\begin{aligned}
& \mathcal{P}_{\|}^{I A(F S I)}=\mathcal{O}_{\frac{1}{2} \frac{1}{2}}^{I A(F S I)}-\mathcal{O}_{-\frac{1}{2}-\frac{1}{2}}^{I A(F S I)} ; \quad \text { with: } \\
& \mathcal{O}_{\lambda \lambda^{\prime}}^{I A(F S I)}\left(p_{N}, E\right)=\sum_{\epsilon_{A-1}^{*}} \rho\left(\epsilon_{A-1}^{*}\right)\left\langle S_{A}, \mathbf{P}_{\mathbf{A}} \mid\left(\hat{S}_{G l}\right)\left\{\Phi_{\epsilon_{A-1}^{*}}, \lambda^{\prime}, \mathbf{p}_{N}\right\}\right\rangle \times \\
& \quad\left\langle\left(\hat{S}_{G l}\right)\left\{\Phi_{\epsilon_{A-1}^{*}}, \lambda, \mathbf{p}_{N}\right\} \mid S_{A}, \mathbf{P}_{\mathbf{A}}\right\rangle \delta\left(E-B_{A}-\epsilon_{A-1}^{*}\right) .
\end{aligned}
$$

Glauber operator: $\hat{S}_{G l}\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \mathbf{r}_{3}\right)=\prod_{i=2,3}\left[1-\theta\left(z_{i}-z_{1}\right) \Gamma\left(\mathbf{b}_{1}-\mathbf{b}_{i}, z_{1}-z_{i}\right)\right]$ (generalized) profile function: $\quad \Gamma\left(\mathbf{b}_{1 i}, z_{1 i}\right)=\frac{(1-i \alpha) \sigma_{e f f}\left(z_{1 i}\right)}{4 \pi b_{0}^{2}} \exp \left[-\frac{\mathbf{b}_{1 i}^{2}}{2 b_{0}^{2}}\right]$, (hadronization model: Kopeliovich et al., NPA 2004; $\sigma_{e f f}$ model: Ciofi \& Kopeliovich, EPJA 2003)

## GEA = Generalized Eikonal Approximation

(succesfull application to unpolarized ${ }^{2} H\left(e, e^{\prime} p\right) X$ : Ciofi \& Kaptari PRC 2011)

## Recoil detection in ${ }^{3} H e\left(e, e^{\prime} D\right) X,{ }^{3} \vec{H} e\left(\vec{e}, e^{\prime} D\right) X$

FSI have been included into the scheme in the unpolarized case;
(Ciofi and Kaptari, PRC 83 (2011) 044602)
Everything has been extended to the spin-dependent case
(Kaptari, Del Dotto, Pace, Salmè, S.S.
PRC 89 (2014) - 035206)

Properly defined observables can distiguish among different descriptions of the EMC effect.

In the polarized case, convenient kinematical regions have been addreesed to access either the structure of the bound proton (tagged EMC effect)
or details of the hadronization mechanism



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## (preliminary) FSI effects on ${ }^{3} H e \uparrow\left(\vec{e}, e^{\prime} h\right) X$

> FSI in the Parallel (along the target polarization) Spin Dependent Spectral Function Pı ( $\mathrm{p}_{\text {mis }}, \theta_{\text {mis }}$ )
> $\mathcal{F}^{A}(x,\{\alpha\}) \simeq \int_{x}^{A} \mathcal{F}^{N}(\xi / x,\{\alpha\}) f^{A}(\xi) d \xi ; \quad f^{A}(\xi)=\int d E \int_{\mathbb{P}_{\text {min }}}^{p_{\text {max }}} P^{A}(\mathbf{p}, E) \delta\left(\xi-\frac{p q}{m \nu}\right) d^{3} \mathbf{p}$
> $\mathbf{p}=\mathbf{p}_{\text {min }} \rightarrow\left|\cos \theta_{\mathbf{p}}\right|=\mathbf{1}($ FSI minimized, Spectral Function maximized !!! )

$$
\begin{aligned}
& \mathbf{p}_{\mathrm{N}}^{\mathrm{FSI}}=\int \mathbf{P}_{\|}^{N}\left(\vec{p}_{m i s}, E\right) d E d^{3} \mathrm{p}_{\text {mis }}=\mathbf{p}_{\mathrm{N}}^{\mathrm{PWIA}}-\delta \mathbf{p}_{\mathrm{N}}^{\mathrm{FSI}}\left(\mathbf{Q}^{2}, \mathrm{X}_{\mathrm{Bj}}\right) \quad \text { Preliminary } \delta \mathrm{p}_{\mathrm{N}}^{\mathrm{FSI}}\left(Q^{2}, \mathrm{X}_{\mathrm{Bj}}\right) \sim 10-15 \%
\end{aligned}
$$

$\mathcal{P}_{\|}^{P W I A}$ and $\mathcal{P}_{\|}^{F S I}$ can be very different: but observables evaluated in IA (GEA) are obtained through their integrals, dominated by the low momentum region, where they are rather close - not dramatic differences: e.g., $p_{p(n)}$ differ by 10-15 \%.

Is this the effect in the extraction of the neutron information?

## Actually, one should also consider the effect on dilution factors...

## DILUTION FACTORS

$$
A_{3}^{\text {exp }} \simeq \frac{\Delta \vec{\sigma}_{3}^{\text {exp. }}}{\sigma_{\text {unpol. }}^{\text {exp. }}} \Longrightarrow \frac{\left\langle\overrightarrow{\mathbf{s}}_{\mathbf{n}}\right\rangle \Delta \vec{\sigma}(\mathbf{n})+2\left\langle\overrightarrow{\mathbf{s}}_{\mathbf{p}}\right\rangle \Delta \vec{\sigma}(\mathbf{p})}{\left\langle\mathbf{N}_{\mathbf{n}}\right\rangle \sigma_{\text {unpol. }}(\mathbf{n})+2\left\langle\mathbf{N}_{\mathbf{p}}\right\rangle \sigma_{\text {unpol. }}(\mathbf{p})}=\left\langle\overrightarrow{\mathbf{s}}_{\mathbf{n}}\right\rangle \mathbf{f}_{\mathbf{n}} \mathbf{A}_{\mathbf{n}}+2\left\langle\overrightarrow{\mathbf{s}}_{\mathbf{p}}\right\rangle \mathbf{f}_{\mathbf{p}} \mathbf{A}_{\mathbf{p}}
$$

PWIA: $\begin{aligned} & \begin{array}{l}\left\langle\vec{s}_{n(p)}\right\rangle=\int d E \int d^{3} p P_{\|}(E, \mathbf{p})=\mathbf{p}_{\mathbf{n}(\mathbf{p})} ; \\ \langle N\rangle=\int d E \int d^{3} p P_{\text {unpol }}(E, \mathbf{p})=1 .\end{array}\end{aligned}$

FSI: $\begin{aligned} & \left\langle\vec{s}_{n(p)}\right\rangle=\int d E \int d^{3} p P_{\|}^{F S I}(E, \mathbf{p})=\mathbf{p}_{\mathbf{n}(\mathbf{p})}^{\mathbf{F S I}} ; \\ & \langle N\rangle=\int d E \int d^{3} p P_{\text {unpol. }}^{F S I}(E, \mathbf{p})<\mathbf{1} .\end{aligned} \Longrightarrow \mathbf{f}_{\mathbf{n},(\mathbf{p})}^{\mathbf{F S I}}(\mathbf{x}, \mathbf{z})=\frac{\sum_{\mathbf{q}} \mathbf{e}_{\mathbf{q}}^{2} \mathbf{f}_{1}^{\mathbf{q}, \mathbf{n}(\mathbf{p})}(\mathbf{x}) \mathbf{D}_{1}^{\mathbf{q}, \mathbf{h}}(\mathbf{z})}{\sum_{\mathbf{N}}\langle\mathbf{N}\rangle \sum_{\mathbf{q}} \mathbf{e}_{\mathbf{q}}^{\mathbf{2}} \mathbf{f}_{\mathbf{1}}^{\mathbf{q}, \mathbf{N}}(\mathbf{x}) \mathbf{D}_{1}^{\mathbf{q}, \mathbf{h}}(\mathbf{z})}$

## Good news from preliminary GEA studies of FSI!

## Results (Preliminary)

1) PWIA: $\left\langle p_{n}\right\rangle=0.876,\left\langle p_{p}\right\rangle=-0.0237, \theta_{e}=30^{\circ}, \theta_{\pi}=14^{\circ}$

| $E_{\text {beam }}$, <br> GeV | $x_{B j}$ | $\nu$ <br> GeV | $p_{\pi}$ <br> $\mathrm{GeV} / \mathrm{c}$ | $f_{n}(x, z)$ | $\left\langle p_{n}\right\rangle f_{n}$ | $f_{p}(x, z)$ | $\left\langle p_{p}\right\rangle f_{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8.8 | 0.21 | $\mathbf{7 . 5 5}$ | $\mathbf{3 . 4 0}$ | $\mathbf{0 . 3 0 4}$ | 0.266 | $\mathbf{0 . 3 4 8}$ | $-8.410^{-3}$ |
| 8.8 | 0.29 | $\mathbf{7 . 1 5}$ | $\mathbf{3 . 1 9}$ | $\mathbf{0 . 2 8 6}$ | 0.251 | $\mathbf{0 . 3 5 7}$ | $-8.510^{-3}$ |
| 8.8 | 0.48 | $\mathbf{6 . 3 6}$ | $\mathbf{2 . 7 7}$ | $\mathbf{0 . 2 5 7}$ | 0.225 | $\mathbf{0 . 3 7 2}$ | $-8.910^{-3}$ |
| 11 | 0.21 | $\mathbf{9 . 6 8}$ | 4.29 | $\mathbf{0 . 3 0 2}$ | 0.265 | $\mathbf{0 . 3 4 9}$ | $-8.310^{-3}$ |
| 11 | 0.29 | $\mathbf{9 . 2 8}$ | $\mathbf{4 . 1 1}$ | $\mathbf{0 . 2 8 5}$ | 0.25 | $\mathbf{0 . 3 5 7}$ | $-8.510^{-3}$ |

2) FSI: $\left\langle p_{n}\right\rangle=\mathbf{0 . 7 5 6},\left\langle p_{p}\right\rangle=\mathbf{- 0 . 0 2 6 5},\left\langle N_{n}\right\rangle=0.85,\left\langle N_{p}\right\rangle=0.87,\left\langle\sigma_{e f f .}\right\rangle=71 \mathrm{mb}$

| $E_{\text {beam }}$, <br> GeV | $x_{B j}$ | $\nu$ <br> GeV | $p_{\pi}$ <br> $\mathrm{GeV} / \mathbf{c}$ | $f_{n}(x, z)$ | $\left\langle p_{n}\right\rangle f_{n}$ | $f_{p}(x, z)$ | $\left\langle p_{p}\right\rangle f_{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8.8 | 0.21 | $\mathbf{7 . 5 5}$ | $\mathbf{3 . 4 0}$ | $\mathbf{0 . 3 5 3}$ | 0.267 | $\mathbf{0 . 4 0 5}$ | $-1.110^{-2}$ |
| 8.8 | 0.29 | $\mathbf{7 . 1 5}$ | $\mathbf{3 . 1 9}$ | $\mathbf{0 . 3 3 2}$ | 0.251 | $\mathbf{0 . 4 1 5}$ | $-1.110^{-2}$ |
| 8.8 | 0.48 | $\mathbf{6 . 3 6}$ | $\mathbf{2 . 7 7}$ | $\mathbf{0 . 2 9 8}$ | 0.225 | $\mathbf{0 . 4 3 2}$ | $-1.210^{-2}$ |
| 11 | 0.21 | $\mathbf{9 . 6 8}$ | $\mathbf{4 . 2 9}$ | $\mathbf{0 . 3 5 1}$ | 0.266 | $\mathbf{0 . 4 0 5}$ | $-1.10^{-2}$ |
| 11 | 0.29 | $\mathbf{9 . 2 8}$ | $\mathbf{4 . 1 1}$ | $\mathbf{0 . 3 3 1}$ | 0.250 | $\mathbf{0 . 4 1 5}$ | $-1.110^{-2}$ |

$$
A_{\mathrm{n}} \simeq \frac{1}{p_{\mathrm{n}}^{\mathrm{FSI}} \mathbf{f}_{\mathrm{n}}^{\mathrm{FSI}}}\left(\mathrm{~A}_{3}^{\exp }-2 \mathbf{p}_{\mathrm{p}}^{\mathrm{FSI}} \mathbf{f}_{\mathrm{p}}^{\mathrm{FSI}} \mathrm{~A}_{\mathrm{p}}^{\exp }\right) \simeq \frac{1}{\mathbf{p}_{\mathrm{n}} f_{\mathrm{n}}}\left(\mathrm{~A}_{3}^{\exp }-2 \mathbf{p}_{\mathrm{p}} f_{\mathrm{p}} A_{\mathrm{p}}^{\exp }\right)
$$

The effects of FSI in the dilution factors and in the effective polarizations are found to compensate each other to a large extent: the usual extraction seems to be safe!

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## Relativity for Few Nucleon Systems: many efforts

O Relativistic Mean Field Theory for Few-Nucleon Systems? NO!

- Field theoretical approaches for two-body system (Bethe-Salpeter Equation, primarily): very important, difficult to be numerically implemented
- However, for $A \geq 2$, Relativistic Hamiltonian Dynamics (RHD), devised by Dirac in 1949 (RMP 21 (1949) 392), allow one to fulfill the Poincaré covariance, with finite dof, and therefore they fall in between the NR framework and the field theory, in its full glory.

The Few-Nucleon system has to be described through a Poincaré covariant formalism, where both wave functions and operators transform according to the extended Poincaré group $\mathcal{G}_{P}$ (4D translations + Lorentz group + parity and time reversal)

This represents a reasonable compromise: i) fulfilling Poincaré covariance in a non perturbative way; ii) embedding the whole successful non relativistic phenomenology; iii) affordable numerical calculations; iv) fixed number of constituents; v) large class of allowed interactions.

## Choosing a Relativistic Hamiltonian Dynamics

- DIRAC: A quantum states evolves in time under the action of Hamiltonian operators that contain the Dynamics. The initial state lives onto a given hyper-surface in the Minkowski space, with its-own symmmetries wrt $\mathcal{G}_{P}$. In the non relativistic framework, since any value for the velocity is possible, one has only one choice for the initial hyper-surface : $t=0$ and any $\{x, y, z\}$,

In a relativistic framework, given the existence of a limiting velocity (the speed of the light), one has a set of possibilities: Forms of Dynamics:

- Instant Form; familiar surface $t=0$, invariant wrt to $\vec{P}$ and $\vec{J}$.
- Front Form or Light-Front Form, initial surface i) fully "illuminated", at a given time $_{L F}=c t+z$, by an electromagnetic wave and ii) tangent to the light-cone, natural for DIS and SIDIS
$\square$ Point Form, $t^{2}-x^{2}-y^{2}-z^{2}$ invariant for Lorentz transformations.
The symmetry properties of the initial surface distinguish the generators of $\mathcal{G}_{P}$ :
$\square$ The ones that leave the initial hypersurface invariant are called kinematical, since are untouched by the interactions.
- The remaining generators are dynamical: they push the system away from the inital hypersurface, and therefore they contain the interaction, that governs the evolution. They are also called Hamiltonians.


## 4D surfaces with maximal symmetry under $\mathcal{G}_{P}$



The instant form

$$
\begin{aligned}
& \tilde{\mathrm{x}}^{0}=\mathrm{ct} \\
& \tilde{\mathrm{x}}^{1}=\mathrm{x} \\
& \tilde{\mathrm{x}}^{2}=\mathrm{y} \\
& \tilde{\mathrm{x}}^{3}=\mathrm{z}
\end{aligned}
$$

$$
\tilde{\mathrm{g}}_{\mu \nu}=\left(\begin{array}{rrrr}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{array}\right)
$$



The front form

$$
\begin{aligned}
\tilde{\mathrm{x}}^{0} & =\mathrm{ct}+\mathrm{z} \\
\tilde{\mathrm{x}}^{1} & =\mathrm{x} \\
\tilde{\mathrm{x}}^{2} & =\mathrm{y} \\
\tilde{\mathrm{x}}^{3} & =\mathrm{ct}-\mathrm{z}
\end{aligned}
$$



The point form

$$
\widetilde{\mathrm{x}}^{0}=\tau \quad, \quad \mathrm{ct}=\tau \cosh \omega
$$

$\tilde{\mathrm{x}}^{1}=\omega, \quad \mathrm{x}=\tau \sinh \omega \sin \theta \cos \phi$
$\tilde{x}^{2}=\theta \quad, \quad y=\tau \sinh \omega \sin \theta \sin \phi$
$\tilde{\mathrm{x}}^{3}=\phi \quad, \quad \mathrm{x}=\tau \sinh \omega \cos \theta$

After S.J. Brodsky, H.C. Pauli and S.S Pinsky, Phys. Rep. 301, 299 (1998).
The thick arrows indicate the flow of the time variable, that labels the states reached by the interacting system under the action of the generators containing the Dynamics.

Summarizing: different forms of HD $\rightarrow$ different form of the variable "time".

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## Choosing LFHD: benefits and problems

+ In LFHD, one has the maximal set of kinematical generators. They are 7 $P^{+}=P^{0}+P_{z}, \vec{P}_{\perp}, J_{z}, K_{z}, \vec{E}_{\perp}$. The two generators $\left\{E_{x},=K_{x}+J_{y}, E_{y}=K_{y}-J_{x}\right\}$ are the transverse LF boosts.
+ The LF boosts: $K z, \vec{E}_{\perp}$, given their kinematical nature, produce trivial transformation rules for boosting quantum states, and allows one to separate the intrinsic motion from the global motion, in complete analagy with the non relativistic case.
$+P^{+} \geq 0$ leads to a meaningful Fock expansion.
+ The IMF description of DIS is easily included.
+ The dynamical set is composed by only 3 generators: $P^{-}=P^{0}-P_{z}$ and $F_{x}=K_{x}-J_{y}, F_{y}=K_{y}+J_{x}$. The last two generators are the transverse LF rotations.
- Although one can define a kinematical, intrinsic angular momentum in a particular construction of the generators, as discussed below, the transverse LF-rotations are dynamical.

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## Poincaré generators for an interacting system

Finite number of dof: an explicit construction of the 10 Poincaré generators given by Bakamjian and Thomas (BT) (PR 92 (1953) 1300). Essential feature of the BT construction: i) the dynamical generators of $\mathcal{G}_{P}$ are expressed in terms of the mass operator of the interacting system, and ii) only the latter contains the interaction (remember that the mass operator is one of the Casimir of $\mathcal{G}_{P}$ ) For the LFHD, the BT construction is implemented through the following steps (see Keister and Polyzou Adv. NP 20 (1991))

- First step: construct the 10 generators, $\left\{P_{0}^{-}, J_{3}, \vec{F}_{0 \perp}, P^{+}, \vec{P}_{\perp}, K_{3}, \vec{E}_{\perp}\right\}$ for the non interacting system
Second step: choose 10 auxiliary operators, $\left\{M_{0}, \vec{j}_{0 L F}, P^{+}, \vec{P}_{\perp}, K_{3}, \vec{E}_{\perp},\right\}$. The non interacting mass, $M_{0}$, and the angular momentum, $\vec{j}_{0 L F}$ in the LF intrinsic frame, are given by

$$
M_{0}^{2}=P_{0}{ }^{-} P^{+}-\left|\vec{P}_{\perp}\right|^{2} \quad\left(0, \vec{j}_{0 L F}\right)=\left[B_{L F}^{-1}\left(\frac{P_{0}}{M_{0}}\right)\right]_{\nu}^{\mu} \frac{W_{0}^{\nu}}{M_{0}}
$$

$\left[B_{L F}^{-1}\right]_{\nu}^{\mu}$ is a LF boost, and $W_{0}^{\nu}$ is the Pauli-Lubanski 4-vector $\left(W_{0}^{2}=M_{0}^{2}\left|\vec{j}_{0 L F}\right|^{2}\right)$
NB the commutation rules of the Poincaré generators imply the ones of the auxiliary operators (and viceversa)
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## Poincaré generators for an interacting system

- Third step: add to $M_{0}$ an interaction $V$ that commutes with $\left\{P^{+}, \vec{P}_{\perp}, K_{3}, \vec{E}_{\perp}, \vec{j}_{0 L F}\right\}$. Then, the set $\left\{M=M_{0}+V, P^{+}, \vec{P}_{\perp}, K_{3}, \vec{E}_{\perp}, \vec{j}_{0 L F}\right\}$ have the same commutation rules of the non interacting set (i.e. the one with $M_{0}$ ).
- Fourth step: invert the second step, starting from $\left\{M=M_{0}+V, P^{+}, \vec{P}_{\perp}, K_{3}, \vec{E}_{\perp}, \vec{j}_{0 L F}\right\}$ and obtaining 10 Poincaré generators, that fulfill the correct commutation rules, and contain the interaction.

A first lesson:

- The key ingredient is the mass operator, Casimir of $\mathcal{G}_{P}$, that contains the interaction, and generates the dependence upon the interaction of the dynamical generators, $P^{-}$and the LF transverse rotations $\vec{F}_{\perp}$, in LFHD.
- The interaction, $V$, must commute with all the kinematical generators, and in addition with the non interacting spin. These constraints lead to the independence upon the global (CM) motion, as in the non relativistic case and the property to conserve the BT angular momentum. $\longrightarrow \mathrm{NB}\left|\vec{j}_{0 L F}\right|^{2}$ and the third component of $\vec{j}_{0 L F}$ can be used for labelling the states !!!
- NB NB the BT construction holds for an interacting system with a finite number of dof and it is not unique.
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## The BT Mass operator for $\mathrm{A}=3$ nuclei - I

$$
M_{B T}(123)=M_{0}(123)+V_{12,3}^{B T}+V_{23,1}^{B T}+V_{31,2}^{B T}+V_{123}^{B T}
$$

where
$\bigcirc M_{0}(123)=\sqrt{m^{2}+k_{1}^{2}}+\sqrt{m^{2}+k_{2}^{2}}+\sqrt{m^{2}+k_{3}^{2}}=\sqrt{M_{0}^{2}(i j)+p_{\ell}^{2}}+\sqrt{m^{2}+p_{\ell}^{2}}$
is the free mass operator, with i) $\vec{k}_{1}+\vec{k}_{2}+\vec{k}_{3}=0$, and ii) $\vec{p}_{\ell}$ the Jacobi momentum with respect to the CM of the free pair (ij).

〇 $V_{i j, \ell}^{B T}=\sqrt{M_{0}^{2}(i j)+v_{i j}^{B T}+p_{\ell}^{2}}-\sqrt{M_{0}^{2}(i j)+p_{\ell}^{2}}$ is the two-body interaction in a $\mathrm{A}=3$ system, and $v_{i j}^{B T}$ the two-body interaction in a $\mathrm{A}=2$ system, fullfilling the proper commutation rules.
The structure of $V_{i j, \ell}^{B T}$, is suggested by the analysis of a two-body interacting system + a free third particle. One can naturally write

$$
\begin{gathered}
M_{12,3}=\sqrt{M_{0}^{2}(12)+v_{i j}^{B T}+p_{3}^{2}}+\sqrt{m^{2}+p_{3}^{2}}= \\
=M_{0}(123)+\left[\sqrt{M_{0}^{2}(12)+v_{12}^{B T}+p_{3}^{2}}-\sqrt{M_{0}^{2}(12)+p_{3}^{2}}\right]
\end{gathered}
$$

- $V_{123}^{B T}$ is a short-range three-body forces


## The BT Mass operator for $\mathrm{A}=3$ nuclei - II

Notice that

$$
\begin{gathered}
V_{12,3}^{B T}=\frac{v_{12}^{B T}}{\sqrt{M_{0}^{2}(12)+v_{12}^{B T}+p_{3}^{2}}+\sqrt{M_{0}^{2}(12)+p_{3}^{2}}} \sim \\
\frac{4 m V_{12}^{N R}}{\sqrt{M_{0}^{2}(12)+v_{12}^{B T}+p_{3}^{2}}+\sqrt{M_{0}^{2}(12)+p_{3}^{2}}} \rightarrow V_{12}^{N R}
\end{gathered}
$$

For the two-body case the Schrödinger Eq. can be rewritten as follows

$$
\begin{gathered}
{\left[4 m^{2}+4 k^{2}+4 m V^{N R}\right]\left|\psi_{d}\right\rangle=\left[4 m^{2}-4 m B_{d}\right]\left|\psi_{d}\right\rangle} \\
{\left[M_{0}^{2}(12)+4 m V^{N R}\right]\left|\psi_{d}\right\rangle=\left[M_{d}^{2}+B_{d}^{2}\right]\left|\psi_{d}\right\rangle \sim M_{d}^{2}\left|\psi_{d}\right\rangle}
\end{gathered}
$$

and the identification between $v_{12}^{B T}$ and $4 m V^{N R}$ naturally stems out, disregarding correction of the order $B_{d} / M_{d}$. Final remark: the commutation rules impose to $V^{B T}$ analogous properties as the ones of $V^{N R}$, with respect to the translational invariance, to the total 4-momentum and the total angular momentum

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## The BT Mass operator for A=3 nuclei - III

In the non relativistic framework, it is not taken into account the changes in the two-body interaction when we move from the two-body CM to the three-body CM

The NR mass operator is written as

$$
M^{N R}=3 m+\sum_{i=1,3} \frac{k_{i}^{2}}{2 m}+V_{12}^{N R}+V_{23}^{N R}+V_{31}^{N R}+V_{123}^{N R}
$$

NB The operators describing the two- and three-body forces must obey to the commutation rules proper of the Galilean group, leading to the well-known properties like translational invariance (conservation of total 3-momentum).

Those properties are similar to the ones in the BT construction. This allows us to consider the standard non relativistic mass operators a sensible BT mass operator, and embedding it in a Poincaré covariant approach.

$$
M_{B T}(123)=M_{0}(123)+V_{12,3}^{B T}+V_{23,1}^{B T}+V_{31,2}^{B T}+V_{123}^{B T} \sim M^{N R}
$$

As a consequence, the standard eigensolutions of $M^{N R}$ can be eligible for a Poincaré covariant description of the $A=3$ nuclei.

## The BT Mass operator for A=3 nuclei - IV

To complete the matter:

- within the LFHD, the LF boosts are kinematical and therefore their action result in simple phases
- Coupling intrinsic spins and orbital angular momenta is easily accomplished within the Instant form of RHD: it amounts to the usual non relativistic machinery (Clebsch-Gordan coefficients)
- to embed this machinery in the LFHD one needs unitary operators, the so-called Melosh rotations, that relate the LF spin wave function and the canonical one. For a (1/2)-particle with LF momentum $\tilde{k} \equiv\left\{k^{+}, \vec{k}_{\perp}\right\}$

$$
\left|s, \sigma^{\prime}\right\rangle_{L F}=\sum_{\sigma} D_{\sigma^{\prime}, \sigma}^{1 / 2}\left(R_{M}^{\dagger}(\tilde{k})\right)|s, \sigma\rangle_{c}
$$

where $D_{\sigma^{\prime}, \sigma}^{1 / 2}\left(R_{M}^{\dagger}(\tilde{k})\right)$ is the standard Wigner function for the $J=1 / 2$ case
for the nucleon quantities, like the density distribution or the Spectral Function, the Melosh rotations does not produce an extra algebric burden with respect to the Instant form, viz

$$
O_{\sigma^{\prime \prime \prime}, \sigma}^{L P}=\sum_{\sigma^{\prime \prime}, \sigma^{\prime}} D_{\sigma^{\prime \prime \prime}, \sigma^{\prime \prime}}^{1 / 2}\left(R_{M}^{\dagger}\right) O_{\sigma^{\prime \prime}, \sigma^{\prime}}^{I F} D_{\sigma^{\prime}, \sigma}^{1 / 2}\left(R_{M}\right)
$$

## Second lesson

What has been done till now, within a non relativistic framework, can be re-used in a Poincaré covariant framework NB:

$$
V_{12,3}^{B T}=\frac{v_{12}^{B T}}{\sqrt{M_{0}^{2}(12)+v_{12}^{B T}+p_{3}^{2}}+\sqrt{M_{0}^{2}(12)+p_{3}^{2}}}
$$

and $v_{12}^{B T}$ is the two-body interaction that must describe the whole two-nucleon phenomenology (bound + scattering states), in the $\mathrm{A}=2 \mathrm{CM}$ !

## We are now (almost) ready for phenomenology!

## Example: The SiDIS nuclear hadronic tensor in LF

In Impulse Approximation the LF hadronic tensor for the ${ }^{3} \mathrm{He}$ nucleus is:

$$
\begin{aligned}
\mathcal{W}^{\mu \nu}\left(Q^{2}, x_{B}, z, \tau_{h} f, \hat{\mathbf{h}}, S_{H e}\right) & \propto \sum_{\sigma, \sigma^{\prime}} \sum_{\tau_{h f}} \int_{\epsilon_{S}^{\min }}^{\epsilon_{S}^{\max }} d \epsilon_{S} \int_{M_{N}^{2}}^{\left(M_{X}-M_{S}\right)^{2}} d M_{f}^{2} \\
& \times \int_{\xi_{l o}}^{\xi_{u p}} \frac{d \xi}{\xi^{2}(1-\xi)(2 \pi)^{3}} \int_{P_{\perp}^{\min }}^{P_{\perp}^{\max }} \frac{d P_{\perp}}{\sin \theta}\left(P^{+}+q^{+}-h^{+}\right) \\
& \times w_{\sigma \sigma^{\prime}}^{\mu \nu}\left(\tau_{h f}, \tilde{\mathbf{q}}, \tilde{\mathbf{h}}, \tilde{\mathbf{P}}\right) \mathcal{P}_{\sigma^{\prime}}^{\tau_{h f}}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S_{H e}\right)
\end{aligned}
$$

where $\quad\left(\tilde{\mathbf{v}}=\left\{v^{+}=v^{0}+v^{3}, \mathbf{v}_{\perp}\right\}\right)$
$w_{\sigma \sigma^{\prime}}^{\mu \nu}\left(\tau_{h f}, \tilde{\mathbf{q}}, \tilde{\mathbf{h}}, \tilde{\mathbf{P}}\right)$ is the nucleon hadronic tensor
$\mathcal{P}_{\sigma^{\prime}{ }_{\sigma}}^{\tau_{h}}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S_{H e}\right)$ is the LF nuclear spectral function defined in terms of LF overlaps


## The ${ }^{3} \mathrm{He}$ LF Spectral Function

$$
\mathcal{P}_{\sigma^{\prime} \sigma}^{\tau}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S_{H e}\right) \propto \sum_{\sigma_{1} \sigma_{1}^{\prime}} D^{\frac{1}{2}}\left[\mathcal{R}_{M}^{\dagger}(\tilde{\mathbf{k}})\right]_{\sigma^{\prime} \sigma_{1}^{\prime}} \mathcal{S}_{\sigma_{1}^{\prime} \sigma_{1}}^{\tau}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S_{H e}\right) D^{\frac{1}{2}}\left[\mathcal{R}_{M}(\tilde{\mathbf{k}})\right]_{\sigma_{1} \sigma}
$$

is obtained through the unitary Melosh Rotations: $\quad D^{\frac{1}{2}}\left[\mathcal{R}_{M}(\tilde{\mathbf{k}})\right]=\frac{m+k^{+}-\imath \boldsymbol{\sigma} \cdot\left(\hat{z} \times \mathbf{k}_{\perp}\right)}{\sqrt{\left(m+k^{+}\right)^{2}+\left|\mathbf{k}_{\perp}\right|^{2}}}$ and the instant-form spectral function

$$
\begin{aligned}
\mathcal{S}_{\sigma_{1}^{\prime} \sigma_{1}}^{\tau}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S_{H e}\right) & =\sum_{J_{S} J_{z S} \alpha} \sum_{T_{S} \tau_{S}}\left\langle T_{S}, \tau_{S}, \alpha, \epsilon_{S} J_{S} J_{z S} ; \sigma_{1}^{\prime} ; \tau, \mathbf{k} \mid \Psi_{0} S_{H e}\right\rangle \\
& \times\left\langle S_{H e}, \Psi_{0} \mid \mathbf{k} \sigma_{1} \tau_{;} J_{S} J_{z S} \epsilon_{S}, \alpha, T_{S}, \tau_{S}\right\rangle \\
& =\left[B_{0, S_{H e}}^{\tau}(|\mathbf{k}|, E)+\boldsymbol{\sigma} \cdot \mathbf{f}_{S_{H e}}^{\tau}(\mathbf{k}, E)\right]_{\sigma_{1}^{\prime} \sigma_{1}}
\end{aligned}
$$

with

$$
\mathbf{f}_{S_{H e}}^{\tau}(\mathbf{k}, E)=\mathbf{S}_{A} B_{1, S_{H e}}^{\tau}(|\mathbf{k}|, E)+\hat{\mathbf{k}}\left(\hat{\mathbf{k}} \cdot \mathbf{S}_{A}\right) B_{2, S_{H e}}^{\tau}(|\mathbf{k}|, E)
$$

NOTICE: $\mathcal{S}_{\sigma_{1}^{\prime} \sigma_{1}}^{\tau}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S_{H e}\right)$ is given in terms of THREE independent functions, $B_{0,1,2}$, once parity and t-reversal are imposed. Adding FSI, more terms could be included.

## GOOD preliminary NEWS

We are now evaluating the SSAs using the LF hadronic tensor, to check whether the proposed extraction procedure still holds within the LF approach. We have preliminary encouraging indications:

- LF longitudinal and transverse polarizations change little from the NR ones:

|  | proton NR | proton LF | neutron NR | neutron LF |
| :--- | :---: | :---: | :---: | :---: |
| $\int d E d \vec{p} \frac{1}{2} \operatorname{Tr}\left(\mathcal{P} \sigma_{z}\right)_{\vec{S}_{A}=\widehat{z}}$ | -0.02263 | -0.02231 | 0.87805 | 0.87248 |
| $\int d E d \vec{p} \frac{1}{2} \operatorname{Tr}\left(\mathcal{P} \sigma_{y}\right)_{\vec{S}_{A}=\widehat{y}}$ | -0.02263 | -0.02268 | 0.87805 | 0.87494 |

The difference between the effective longitudinal and transverse polarizations is a measure of the relativistic content of the system (in a proton, it would correspond to the difference between axial and tensor charges).
The extraction procedure works well within the LF approach as it does in the non relativistic case.

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## LF spectral function and LF TMDs - I

The LF spectral function for a $J=1 / 2$ system of three spin $1 / 2$ constituents can be used to find relations among the SIX T-even TMDs, $A_{i}, \widetilde{A}_{i}(i=1,3)$, at the leading twist for the quarks inside a nucleon with momentum $P$ and spin $S$.
In general, the TMDs for a $J=1 / 2$ system are introduced through the q-q correlator

$$
\begin{aligned}
\Phi(k, P, S)= & \int d^{4} z e^{i k \cdot z}\langle P S| \bar{\psi}_{q}(0) \psi_{q}(z)|P S\rangle \\
= & \frac{1}{2}\left\{A_{1} \not P+A_{2} S_{L} \gamma_{5} \not P+A_{3} \not P \gamma_{5} \oiint_{\perp}\right. \\
& \left.+\frac{1}{M} \widetilde{A}_{1} \vec{k}_{\perp} \cdot \vec{S}_{\perp} \gamma_{5} \not P+\widetilde{A}_{2} \frac{S_{L}}{M} \not P \gamma_{5} k_{\perp}+\frac{1}{M^{2}} \widetilde{A}_{3} \vec{k}_{\perp} \cdot \vec{S}_{\perp} \not P \gamma_{5} k_{\perp}\right\},
\end{aligned}
$$

so that particular combinations of the SIX twist-2 TMDs can be obtained by proper traces of $\Phi(k, P, S)$ (instead of $A_{i}, \widetilde{A}_{i}$, the "Amsterdam" notation can be used for the TMDs), :

$$
\begin{aligned}
\frac{1}{2 P^{+}} \operatorname{Tr}\left(\gamma^{+} \Phi\right) & =A_{1} \\
\frac{1}{2 P^{+}} \operatorname{Tr}\left(\gamma^{+} \gamma_{5} \Phi\right) & =S_{L} A_{2}+\frac{1}{M} \vec{k}_{\perp} \cdot \vec{S}_{\perp} \widetilde{A}_{1} \\
\frac{1}{2 P^{+}} \operatorname{Tr}\left(\mathrm{i} \sigma^{\mathrm{i}+} \gamma_{5} \Phi\right) & =S_{\perp}^{i} A_{3}+\frac{S_{L}}{M} k_{\perp}^{i} \widetilde{A}_{2}+\frac{1}{M^{2}} \vec{k}_{\perp} \cdot \vec{S}_{\perp} k_{\perp}^{i} \widetilde{A}_{3}
\end{aligned}
$$

## LF spectral function and LF TMDs - II

Actually, in LF one has on-mass shell quarks. Let us consider therefore the contribution to the correlation function from on-mass-shell fermions

$$
\begin{array}{r}
\Phi_{p}(k, P, S)=\frac{\left(k_{o n}+m\right)}{2 m} \Phi(k, P, S) \frac{\left(k_{o n}+m\right)}{2 m}= \\
=\sum_{\sigma} \sum_{\sigma^{\prime}} u_{L F}\left(\tilde{k}, \sigma^{\prime}\right) \bar{u}_{L F}\left(\tilde{k}, \sigma^{\prime}\right) \Phi(k, P, S) u_{L F}(\tilde{k}, \sigma) \bar{u}_{L F}(\tilde{k}, \sigma)
\end{array}
$$

and let us identify $\bar{u}_{L F}\left(\tilde{k}, \sigma^{\prime}\right) \Phi(k, P, S) u_{L F}(\tilde{k}, \sigma)$ with the LF nucleon spectral function

$$
\bar{u}_{L F}\left(\tilde{k}, \sigma^{\prime}\right) \Phi(k, P, S) u_{L F}(\tilde{k}, \sigma)=\mathcal{P}_{\sigma^{\prime} \sigma}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S\right)
$$

In a reference frame where $\mathbf{P}_{\perp}=0$, the following relation holds between $k^{-}$and the spectator diquark energy $\epsilon_{S}$ :

$$
k^{-}=\frac{M^{2}}{P^{+}}-\frac{\left(\epsilon_{S}+m\right) 4 m+\left|\boldsymbol{k}_{\perp}\right|^{2}}{P^{+}-k^{+}}
$$

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## LF spectral function and LF TMDs - III

The contributions of on-mass-shell fermions to the traces previously introduced in terms of the TMD's, $A_{i}, \widetilde{A}_{i}(i=1,3)$, are

$$
\begin{gathered}
\frac{1}{2 P^{+}} \operatorname{Tr}\left[\gamma^{+} \Phi_{p}(k, P, S)\right]=\frac{k_{o n} \cdot P}{2 m^{2}} \frac{k^{+}}{P^{+}} A_{1} \\
\frac{1}{2 P^{+}} \operatorname{Tr}\left[\gamma^{+} \gamma_{5} \Phi_{p}(k, P, S)\right]=\left(\frac{1}{2}-\frac{k_{o n} \cdot P}{4 m^{2}} \frac{k^{+}}{P^{+}}\right)\left[A_{2} \lambda_{N}+\frac{1}{M} \widetilde{A}_{1} \mathbf{k}_{\perp} \cdot \mathbf{S}_{\perp}\right]+ \\
+\frac{1}{2 m}\left[A_{3} \mathbf{k}_{\perp} \cdot \mathbf{S}_{\perp}+\widetilde{A}_{2} \frac{\lambda_{N}}{M}\left|\mathbf{k}_{\perp}\right|^{2}+\frac{1}{M^{2}} \widetilde{A}_{3} \mathbf{k}_{\perp} \cdot \mathbf{S}_{\perp}\left|\mathbf{k}_{\perp}\right|^{2}\right] \\
\frac{1}{2 P^{+}} \operatorname{Tr}\left[k_{\perp} \gamma^{+} \gamma_{5} \Phi_{p}(k, P, S)\right]=\frac{1}{2 m}\left|\boldsymbol{k}_{\perp}\right|^{2}\left[A_{2} \lambda_{N}+\frac{1}{M} \widetilde{A}_{1} \mathbf{k}_{\perp} \cdot \mathbf{S}_{\perp}\right]+ \\
+\left(\frac{k^{+}}{P^{+}} \frac{k_{o n} \cdot P}{4 m^{2}}-\frac{\left|\boldsymbol{k}_{\perp}\right|^{2}}{4 m^{2}}\right)\left[A_{3} \mathbf{k}_{\perp} \cdot \mathbf{S}_{\perp}+\widetilde{A}_{2} \frac{\lambda_{N}}{M}\left|\mathbf{k}_{\perp}\right|^{2}+\frac{1}{M^{2}} \widetilde{A}_{3} \mathbf{k}_{\perp} \cdot \mathbf{S}_{\perp}\left|\mathbf{k}_{\perp}\right|^{2}\right]
\end{gathered}
$$

## LF spectral function and LF TMDs - IV

However these same contributions can be also expressed through the LF spectral function

$$
\begin{gathered}
\frac{1}{2 P^{+}} \operatorname{Tr}\left[\gamma^{+} \Phi_{p}(k, P, S)\right]= \\
=\frac{1}{2 P^{+}} \sum_{\sigma \sigma^{\prime}} \bar{u}_{L F}(\tilde{k}, \sigma) \gamma^{+} u_{L F}\left(\tilde{k}, \sigma^{\prime}\right) \bar{u}_{L F}\left(\tilde{k}, \sigma^{\prime}\right) \Phi(k, P, S) u_{L F}(\tilde{k}, \sigma)= \\
=\frac{1}{2 P^{+}} \sum_{\sigma \sigma^{\prime}} \bar{u}_{L F}(\tilde{k}, \sigma) \gamma^{+} u_{L F}\left(\tilde{k}, \sigma^{\prime}\right) \mathcal{P}_{\sigma^{\prime} \sigma}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S\right)=\frac{k^{+}}{2 m P^{+}} \operatorname{Tr}\left[\mathcal{P}_{\sigma^{\prime} \sigma}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S\right)\right] \\
\frac{1}{2 P^{+}} \operatorname{Tr}\left[\gamma^{+} \gamma_{5} \Phi_{p}(k, P, S)\right]=\sum_{\sigma \sigma^{\prime}} \bar{u}_{L F}(\tilde{k}, \sigma) \gamma^{+} \gamma_{5} u_{L F}\left(\tilde{k}, \sigma^{\prime}\right) \mathcal{P}_{\sigma^{\prime} \sigma}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S\right)= \\
=\frac{k^{+}}{2 m P^{+}} \sum_{\sigma \sigma^{\prime}} \chi_{\sigma}^{\dagger} \sigma_{z} \chi_{\sigma^{\prime}} \mathcal{P}_{\sigma^{\prime} \sigma}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S\right)=\frac{k^{+}}{2 m P^{+}} \operatorname{Tr}\left[\sigma_{z} \mathcal{P}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S\right)\right] \\
\frac{1}{2 P^{+}} \operatorname{Tr}\left[k_{\perp} \gamma^{+} \gamma_{5} \Phi_{p}(k, P, S)\right]=\frac{1}{2 P^{+}} \sum_{\sigma \sigma^{\prime}} \bar{u}_{L F}(\tilde{k}, \sigma) k_{\perp} \gamma^{+} \gamma_{5} u_{L F}\left(\tilde{k}, \sigma^{\prime}\right) \mathcal{P}_{\sigma^{\prime} \sigma}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S\right)= \\
=\frac{k^{+}}{2 m P^{+}} \sum_{\sigma \sigma^{\prime}} \chi_{\sigma}^{\dagger} \boldsymbol{k}_{\perp} \cdot \boldsymbol{\sigma} \chi_{\sigma^{\prime}} \mathcal{P}_{\sigma^{\prime} \sigma}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S\right)=\frac{k^{+}}{2 m P^{+}} \operatorname{Tr}\left[\boldsymbol{k}_{\perp} \cdot \boldsymbol{\sigma} \mathcal{P}\left(\tilde{\mathbf{k}}, \epsilon_{S}, S\right)\right]
\end{gathered}
$$

## LF spectral function and LF TMDs - V

$$
\begin{aligned}
\frac{1}{2} \operatorname{Tr}(\mathcal{P} I) & =c B_{0} \\
\frac{1}{2} \operatorname{Tr}\left(\mathcal{P} \sigma_{z}\right) & =S_{A z}\left[a\left(B_{1}+B_{2} \cos ^{2} \theta\right)+b \cos \theta \frac{\left|\mathbf{k}_{\perp}\right|^{2}}{k} B_{2}\right] \\
& +\mathbf{S}_{\mathbf{A} \perp} \cdot \mathbf{k}_{\perp}\left[a B_{2} \frac{\cos \theta}{k}+b\left(B_{1}+B_{2} \sin ^{2} \theta\right)\right] \\
\frac{1}{2} \operatorname{Tr}\left(\mathcal{P} \sigma_{y}\right) & =S_{A y}\left[\left(a+d\left|\mathbf{k}_{\perp}\right|^{2}\right) B_{1}\right] \\
& +S_{A z} k_{y}\left[a \frac{\cos \theta}{k} B_{2}-b\left(B_{1}+B_{2} \cos ^{2} \theta\right)\right] \\
& +k_{y} \mathbf{S}_{\mathbf{A}} \cdot \mathbf{k}_{\perp}\left[\left(\frac{a}{k^{2}}-b \frac{\cos \theta}{k}\right) B_{2}-d B_{1}\right]
\end{aligned}
$$

The SIX TMDs can be expressed in terms of the 3 independent functions $B_{0}, B_{1}, B_{2}$ ! $a, b, c, d$ are kinematical factors, predicted by the LF procedure!

In the LF approach only THREE of the $\operatorname{SIX} A_{i}, \widetilde{A}_{i}(i=1,3)$ distributions are independent!


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## Help from ${ }^{3} \mathrm{He}$ for proton studies?

${ }^{3} \mathrm{He}$ is a spin $1 / 2$ system with 3 spin $1 / 2$ constituents. The same as the proton, in the valence region $\Rightarrow$ they have the same symmetries.

One could investigate the analogy:

```
                        proton \(\Leftrightarrow{ }^{3} \mathrm{He}\)
    Valence quark contribution \(\Leftrightarrow\) nucleon contribution
    twist-2 Approximation \(\Leftrightarrow\) ImpulseApproximation
    SiDIS \(\Leftrightarrow \quad\left(e, e^{\prime} p\right)\) athigh \(Q^{2}\)
    6 independent asymmetries \(\Leftrightarrow 6\) independent Responsefunctions
    6 independent TMDS @twist \(-2 \Leftrightarrow 6\) independent momentum distributions
```

A one-to-one correspondence can be obtained $\Rightarrow$ check of the 3 relations found in RHD among the TMDS looking at ${ }^{3} \mathrm{He}$ data (may be, in part, available)
$\Rightarrow$ test of RHD! Hint for TMDs data analysis?

## Conclusions

We are studying DIS processes off ${ }^{3} \mathrm{He}$ beyond the realistic, NR, IA approach.
We have encouraging results concerning:

- FSI effects evaluated through the GEA:
a distorted spin dependent spectral function is studied
- An analysis of a LF spectral function (in IA); besides, within LF dynamics only 3 of the 6 T-even TMDs are independent. The relations among them are precisely predicted within LF Dynamics, and could be experimentally checked to test the LF description of SiDIS.


## Next steps:

- complete this program!
- apply the LF spectral function to other processes (e.g., DVCS);
- relativistic FS ? $?$
${ }^{3} \mathrm{He}$ : an effective neutron AND a LAB for Light-Front studies


## Nuclear DIS: Workshop at ECT*!!!



## Please come and help us to boost our Physics in Europe!

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## backup: $\vec{n}$ from ${ }^{3} \vec{H} e$ : SiDIS case

Ingredients of the calculations:

- A realistic spin-dependent spectral function of ${ }^{3} \mathrm{He}$ (C. Ciofi degli Atti et al., PRC 46 (1992) R 1591; A. Kievsky et al., PRC 56 (1997) 64) obtained using the AV18 interaction and the wave functions evaluated by the Pisa group [ A. Kievsky et al., NPA 577 (1994) 511 ] (small effects from 3-body interactions)
- Parametrizations of data for pdfs and fragmentation functions whenever available:
$f_{1}^{q}\left(x, \mathbf{k}_{\mathbf{T}}^{2}\right)$, Glueck et al., EPJ C (1998) 461,
$f_{1 T}^{\perp q}\left(x, \mathrm{k}_{\mathrm{T}}{ }^{\mathbf{2}}\right)$, Anselmino et al., PRD 72 (2005) 094007,
$D_{1}^{q, h}\left(z,\left(z \kappa_{\mathbf{T}}\right)^{2}\right)$, Kretzer, PRD 62 (2000) 054001
- Models for the unknown pdfs and fragmentation functions:
$h_{1}^{q}\left(x, \mathrm{k}_{\mathrm{T}}{ }^{2}\right)$, Glueck et al., PRD 63 (2001) 094005,
$H_{1}^{\perp q, h}\left(z,\left(z \kappa_{\mathbf{T}}\right)^{2}\right)$ Amrath et al., PRD 71 (2005) 114018
Results will be model dependent. Anyway, the aim for the moment is to study nuclear effects.

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## backup: the hadronization model

hadronization model: Kopeliovich et al., Predazzi, Hayashigaki, NPA 2004;
$\sigma_{e f f}$ model: Ciofi \& Kopeliovich, EPJA 2003)


At the interaction point, a color string, denoted X1, and a nucleon N1, arising from target fragmentation, are formed; the color string propagates and gluon radiation begins. The first $\pi$ is created at $z_{0}=0.6$ by the breaking of the color string, and pion production continues until it stops at a maximum value of $z=z_{\max }$, when energy conservation does not allow further pions to be created, and the number of pions remains constant. Once the total effective cross section has been obtained, the elastic slope $b_{0}$ and the ratio $\alpha$ of the real to the imaginary parts of the elastic amplitude remain to be determined.

## Macroscopic locality

Macroscopic locality meets our physical intuition
For instance, if the spacelike distance increases, and the interaction dies, one should expect two completely isolated subsystems, for which the Hamiltonian clusterizes as follows

$$
\text { For }\left|r_{12}-r_{3}\right| \gg d \Rightarrow H(123)=H_{12}+H_{3}^{\text {free }}
$$

(the same for the other generators)
NB Spacelike separations are not Lorentz invariant, this leads to a mathematical formulation of the macroscopic locality given for infinitely large spacelike separations: $d \rightarrow \infty$.

Imposing macroscopic locality means that all the properties valid for a system must hold for any subsystem in isolation.

Then, e.g., the two-body interaction extracted from the study of NN systems can be adopted in the description of many-nucleon systems (modulo the presence of many-body interactions).

The macroscopic locality can be easily implemented if one takes as a basis the tensor product of the (A-1)-body interacting states and the single particle states (with the proper symmetrization). Within the BT construction, the macrolocality cannot be implemented,
but there are unitary operators, the packing operators, that relate the states obtained in the BT approach and the one related to the tensor-product approach

The packing operators, fortunately give very small effects, and therefore one can adopt the BT framework safely (Coester-Polyzou PRD 26, 1348 (1982) and Keister-Polyzou PRC 86 (2012))014002.

The macroscopic locality (the only property that can be experimentally tested) can be seen as a weak counterpart of the microscopic locality (or microcausality): one of the basic axioms of the Local Field Theory. In this case the constraint is imposed at arbitrarily short spacelike distances to free fields.


[^0]:    - $a$ le $x$

