Effective field theory methods for light nuclei∗

Saori Pastore @ ODU, Norfolk VA - March 2015

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PRC**78**, 064002 (2008) - PRC**80**, 034004 (2009) - PRC**81**, 034005 (2010)
PRL**105**, 232502, (2010) - PRC**84**, 024001 (2011) - PRC**87**, 014006, (2013)

- Nuclear χEFT approach
- EM charge and current operators up to one loop from χEFT
- $A = 2$ and 3 nuclei: elastic form factors
- Summary and outlook

The Basic Model

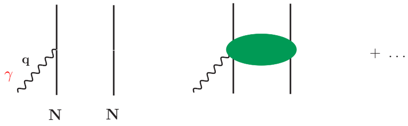
- The nucleus is a system made of A interacting nucleons, its energy is given by

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

where υ_{ij} and V_{ijk} are 2- and 3-nucleon interaction operators

- Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of $1-$, $2-$, ... nucleon operators:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots \,, \qquad \mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$



- Longitudinal EM current operator **j** linked to the nuclear Hamiltonian via

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [T + V, \rho]$$

Nuclear χEFT approach

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

- χEFT exploits the χ symmetry exhibited by QCD at low energy to restrict the form of the interactions of π's with other π's, and with N's, Δ's, …
- The pion couples by powers of its momentum $Q \rightarrow \mathscr{L}_{eff}$ can be systematically expanded in powers n of Q/Λ_χ

  $$\mathscr{L}_{eff} = \mathscr{L}^{(0)} + \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \ldots + \mathscr{L}^{(n)} + \ldots$$

  $\Lambda_\chi \sim 1$ GeV is the hard scale where χEFT breaks down and characterizes the convergence of the expansion $\rightarrow$ we are limited to kinematic regions where $Q \ll \Lambda_\chi$
- The coefficients of the expansion, Low Energy Constants (LECs) are unknown and need to be fixed by comparison with exp data
- The systematic expansion in Q naturally has the feature

  $$\langle \mathscr{O} \rangle_{1-\text{body}} > \langle \mathscr{O} \rangle_{2-\text{body}} > \langle \mathscr{O} \rangle_{3-\text{body}}$$

- A theoretical error due to the truncation of the expansion can be assigned

Transition amplitude in time-ordered perturbation theory

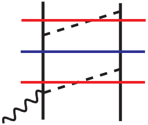
- We use non relativistic N's and π's as mediators of the nuclear interaction at large interparticle distances
- We can construct multiple pion-exchange operators or multiple nucleon contact-terms encoding intermediate– and short–range dynamics

$$T_{\rm fi} = \langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle$$

$$= \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \dots$$

- A contribution with N interaction vertices and L loops scales as

$$\underbrace{e \left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1\,\text{scaling}} \times \underbrace{Q^{-(N-N_K-1)}\, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$



α_i = number of derivatives in H_1 and β_i = number of π's at each vertex

N_K = number of pure nucleonic intermediate states

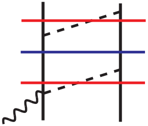
Power counting

- Due to the chiral expansion, the transition amplitude T_{fi} can be expanded as

$$T_{\text{fi}} = T^{\text{LO}} + T^{\text{NLO}} + T^{\text{N2LO}} + \dots \quad \text{and} \quad T^{\text{N}n\text{LO}} \sim (Q/\Lambda_\chi)^n T^{\text{LO}}$$

- N_K energy denominators scale as Q^{-2}

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N}|I\rangle \sim Q^{-2}|I\rangle$$

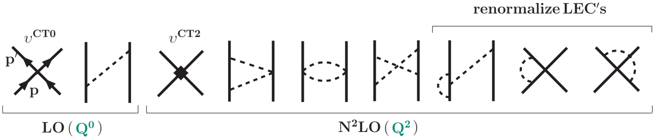


- $(N - N_K - 1)$ energy denominators scale Q^{-1} in the static limit; they can be further expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N - \omega_\pi}|I\rangle \sim -\Big[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \dots\Big]|I\rangle$$

- Relativistic effects taken in to account as $(Q/m_N)^2$ corrections to non-relativistic operators

NN Potential at N2LO (or $Q^{n=2}$)



- Contact potential at LO (or $Q^{n=0}$) depends on 2 LECs
- Contact potential at N2LO (or $Q^{n=2}$) depends on 7 additional LECs

NN potentials with π's and *N*'s

* van Kolck *et al.* (1994–96)
* Kaiser, Weise *et al.* (1997–98)
* Epelbaum, Glöckle, Meissner (1998–2015)
* Entem and Machleidt (2002–2015) *
* …

Observations:
1. EM H_1 obtained by minimal substitution in the π- and N-derivative couplings involve the same LECs entering H_1 strong

$$\begin{aligned} \nabla \pi_{\mp}(\mathbf{x}) &\rightarrow [\nabla \mp i e \mathbf{A}(\mathbf{x})]\, \pi_{\mp}(\mathbf{x}) \\ \nabla N(\mathbf{x}) &\rightarrow [\nabla - i e\, e_N \mathbf{A}(\mathbf{x})]\, N(\mathbf{x})\,, \qquad e_N = (1+\tau_z)/2 \end{aligned}$$

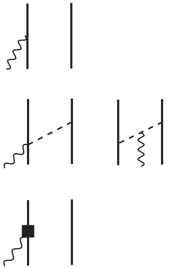
2. LECs entering EM H_1 from the $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$ are not constrained by the strong interaction

χEFT EM current up to $n = 1$ (or up to N3LO)

LO : $\mathbf{j}^{(-2)} \sim e\mathbf{Q}^{-2}$

NLO : $\mathbf{j}^{(-1)} \sim e\mathbf{Q}^{-1}$

N^{2}LO : $\mathbf{j}^{(-0)} \sim e\mathbf{Q}^{0}$

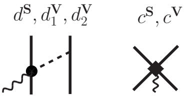


- $n = -2, -1, 0$, and 1-(loops only): depend on known LECs (g_A, F_π, and $\mu_{p/n}$)
- $n = 0$: $(Q/m_N)^2$ relativistic correction to $\mathbf{j}^{(-2)}$ at LO
- Strong contact 'minimal' LECs at $n = 1$ fixed from fits to *np* phases shifts—Q^4 NN potential of Entem&Machleidt
- Unknown 'non-minimal' EM LECs enter the $n = 1$ contact and tree-level currents
- No three-body EM currents at this order !!!
- NLO and N3LO loop-contributions lead to purely isovector operators
- $\mathbf{j}^{(n \leq 1)}$ satisfies the continuity eq. with χEFT two-nucleon potential $\upsilon^{(n \leq 2)}$

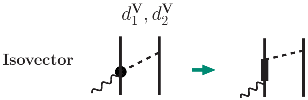
N^{3}LO: $\mathbf{j}^{(1)} \sim e\mathbf{Q}$

unknown LEC's →

χEFT EM currents at N3LO: fixing the EM LECs – Piarulli *et al.*



Five LECs: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



d_2^V and d_1^V are known assuming Δ-resonance saturation

Left with 3 LECs: Fixed in the $A = 2-3$ nucleons' sector

- Isoscalar sector:
  - d^S and c^S from EXPT μ_d and $\mu_S(^3\text{H}/^3\text{He})$
- Isovector sector:
  - model I = c^V from EXPT $npd\gamma$ xsec.

    or

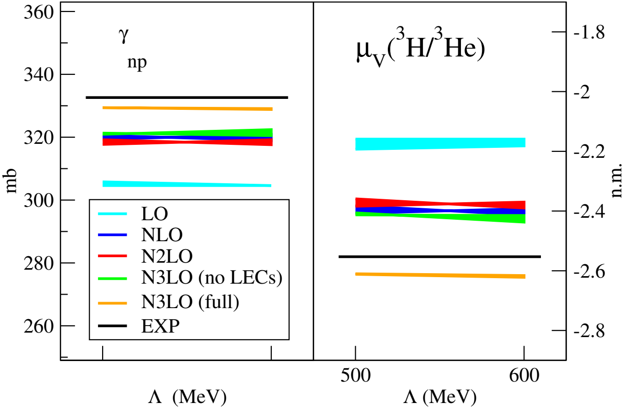
  - model II = c^V from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m. $\leftarrow$ our choice

Note that:
χEFT operators have a power law behavior $\rightarrow$ introduce a regulator to kill divergencies at large Q, *e.g.*, $C_\Lambda = e^{-(Q/\Lambda)^n}$, ...and also, pick n large enough so as to not generate spurious contributions

$$C_\Lambda \sim 1 - \left(\frac{Q}{\Lambda}\right)^n + \dots$$

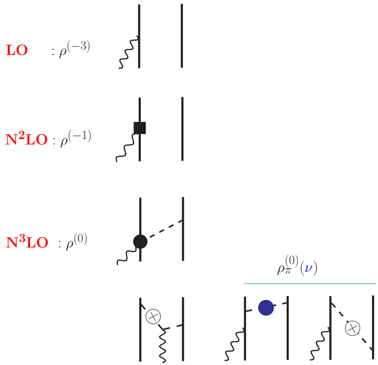
Predictions with χEFT EM currents for $A = 2$–3 systems – Piarulli *et al.*

np capture xsec. (using model II) / μ_V of $A = 3$ nuclei (using model I)
bands represent nuclear model dependence (N3LO/N2LO – AV18/UIX)



- $npd\gamma$ xsec. and $\mu_V(^3\text{H}/^3\text{He})$ m.m. are within 1% and 3% of EXPT
- Two-body currents important to reach agreement with exp data
- Negligible dependence on the cutoff entering the regulator $exp(-(k/\Lambda)^4)$

EM charge up to $n = 0$ (or up to N3LO)



- $n = -3$

  $\rho^{(-3)}(\mathbf{q}) = e\,(2\pi)^3 \delta(\mathbf{p}_1 + \mathbf{q} - \mathbf{p}_1')\,(1 + \tau_{1,z})/2 + 1 \rightleftharpoons 2$

- $n = -1$:

  $(Q/m_N)^2$ relativistic correction to $\rho^{(-3)}$

- $n = 0$:

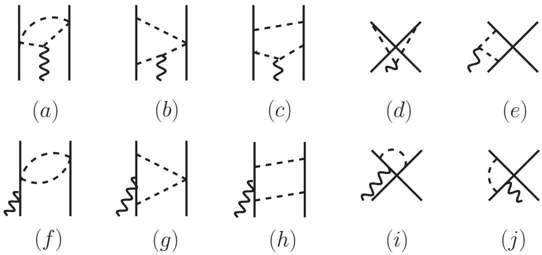
  i) 'static' tree-level current (originates from a $\gamma\pi N$ vertex of order $e\,Q$)

  ii) 'non-static' OPE charge operators, $\rho_\pi^{(0)}(\nu)$ depends on $\upsilon_\pi^{(2)}(\nu)$

- No unknown LECs up to N4LO or $n = 1$ (g_A, F_π) !!

EM charge @ $n = 1$ (or N4LO)

N^{4}LO : $\rho^{(1)}$



- Charge operators up to $n = 1$ satisfy the condition

$$\rho^{(n>-3)}(\mathbf{q}=0) = 0$$

which follows from charge conservation

$$\rho(\mathbf{q}=0) = \int \mathrm{d}\mathbf{x}\, \rho(\mathbf{x}) = e\, \frac{(1+\tau_{1,z})}{2} + 1 \rightleftharpoons 2 = \rho^{(-3)}(\mathbf{q}=0)$$

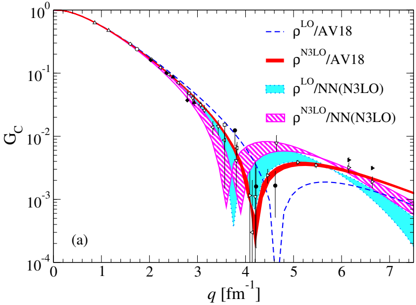
- Divergencies associated with (b) + (g), (c) + (h), and (e) + (j) cancel out—as they must since there are no counter-terms at N4LO
- $\rho^{(1)}$ does not depend on unknown LECs and it is purely isovector

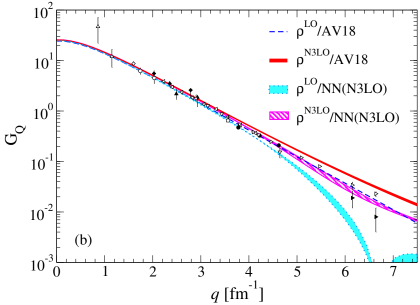
Previous work using χEFT with pions and nucleons

Electroweak currents with π's and N's

* Park, Rho *et al.* (1996–2009);
  hybrid studies in A=2–4 by Song *at al.* (2009-2011)
* Meissner *et al.* (2001), Kölling *et al.* (2009–2011);
  applications to d and ^{3}He photodisintegration by Rozpedzik *et al.* (2011);
  applications to d magnetic f.f. by Kölling, Epelbaum, Phillips (2012)
* Phillips (2003-2007);
  applications to deuteron static properties and f.f.'s

Deuteron Charge and Quadrupole f.f.'s – Piarulli *et al.*



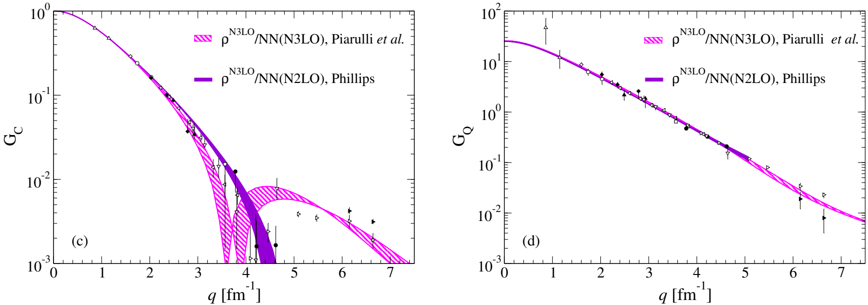


- N4LO contributions do not enter as they lead to isovector operators
- Calculations include nucleonic f.f.'s taken from exp data
- Q_d within 1% (2%) of the exp data with N3LO(AV18)
- Bands' thickness gives cutoff dependence $\Lambda = 500 - 600$ MeV
- G_Q agrees with exp up to $q \simeq 6$ fm^{-1}, *i.e.*, beyond the $3-4\ m_\pi$ range

Λ MeV	$< r_d >$ (fm)	$< r_d >$ EXP	Q_d (fm^2)	Q_d (fm^2) EXP
500	1.976 (1.969)	1.9734(44)	0.285 (0.281)	0.2859(3)
600	1.968 (1.969)		0.282 (0.280)	

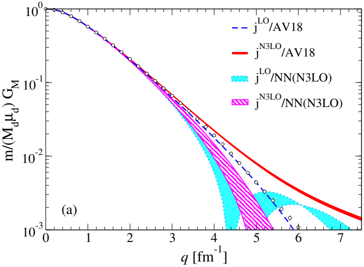
χEFT predictions for the Deuteron Charge and Quadrupole f.f.'s

χEFT calculations from Piarulli *et al.*, Phillips



- Good agreement between theoretical calculations and data for low q-values

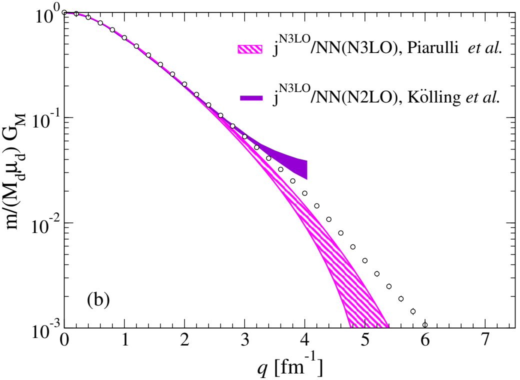
Deuteron Magnetic f.f.'s – Piarulli *et al.*



- μ_d is used to constrain the EM current $\rightarrow$ predictions are for $q > 0$
- Calculations include nucleonic f.f.'s taken from exp data
- Good agreement up to $q \simeq 2$ fm^{-1}
- Exhibited sensitivity to nuclear Hamiltonians due to differences in the S- and D-wave functions
- Cutoff dependence large in the chiral formulation

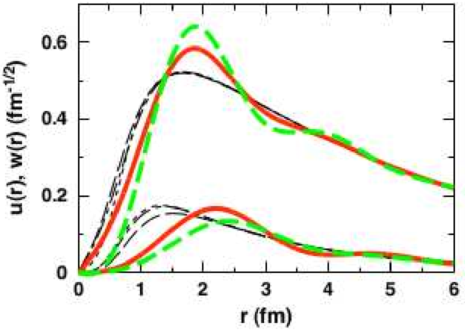
χEFT predictions for the deuteron magnetic f.f.

χEFT calculations from Piarulli *et al.*, Kölling *et al.*



- Good agreement between theoretical calculations and data for low q-values

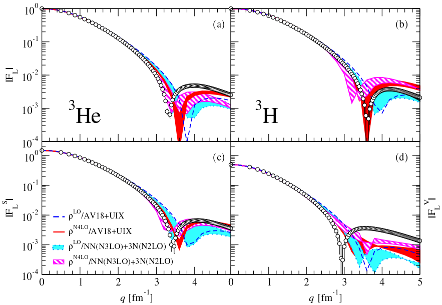
Deuteron wave functions



from Entem&Machleidt 2011 Review

- Entem&Machleidt N3LO
- Epelbaum *et al.* 2005
- black lines = conventional potentials, *i.e.* AV18, CD-Bonn, Nijm-I

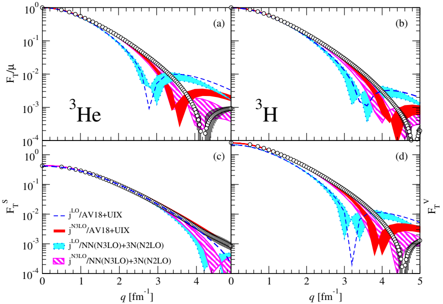
^{3}He and ^{3}H charge f.f.'s – Piarulli *et al.*



- Excellent agreement up to $q \simeq 2$ fm^{-1}
- N3LO and N4LO comparable

	^{3}He $< r >_{EXP} = 1.959 \pm 0.030$ fm		^{3}H $< r >_{EXP} = 1.755 \pm 0.086$	
Λ	500	600	500	600
LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)
N4LO	1.966 (1.950)	1.958 (1.950)	1.762 (1.743)	1.750 (1.743)

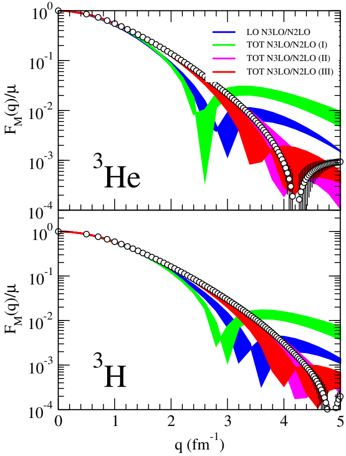
^{3}He and ^{3}H magnetic f.f.'s – Piarulli *et al.*



- ^{3}He and ^{3}H μ's used to fix the EM current; 10% correction from two-body currents
- Two-body current crucial to improve agreement with exp data

	$^3\text{He} < r >_{EXP} = 1.965 \pm 0.153$ fm		$^3\text{H} < r >_{EXP} = 1.840 \pm 0.181$ fm	
Λ	500	600	500	600
LO	2.098 (2.092)	2.090 (2.092)	1.924 (1.918)	1.914 (1.918)
N3LO	1.927 (1.915)	1.913 (1.924)	1.808 (1.792)	1.794 (1.797)

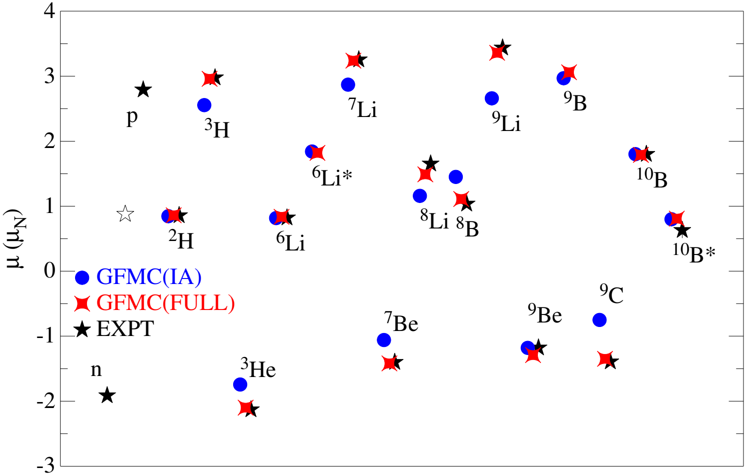
^{3}He and ^{3}H magnetic f.f.'s – Piarulli *et al.*



- If model II is used (with LEC from *npdγ* x-sec.) we get better agreement with exp but larger cutoff dependence and μ_V off by 3%

Magnetic moments in $A \leq 10$ nuclei

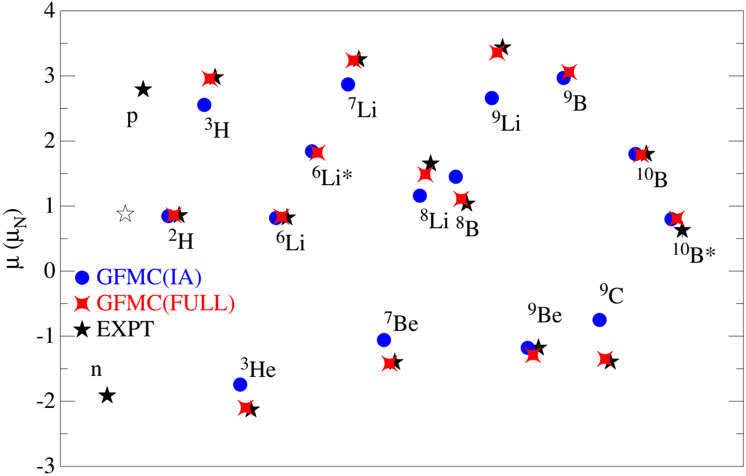
Predictions for $A > 3$ nuclei



$$\mu(\mathrm{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i (1 - \tau_{i,z})/2]$$

Magnetic moments in $A \leq 10$ nuclei - bis

Predictions for $A > 3$ nuclei



- ^{9}C (^{9}Li) dominant spatial symmetry [s.s.] = [432] = $[\alpha, {}^3\text{He}({}^3\text{H}), pp(nn)] \rightarrow$ Large MEC
- ^{9}Be (^{9}B) dominant spatial symmetry [s.s.] = [441] = $[\alpha, \alpha, n(p)]$

Summary

- We derived the EM charge and current operators up to one loop from χEFT with π's and N's
- The charge operator does not involve unknown LECs and two-body corrections (of one-pion range) appear at N3LO
- The current operator depends on 5 LECs which have been fixed to EM data for $A = 2$ ad 3 systems with two-body corrections appearing at NLO (purely isovector)
- Electromagnetic form factors are well reproduced for $q < 3\ \text{fm}^{-1}$
- Sensitivity to cutoff variations and Hamiltonian models is observed at larger values of q
- Convergence pattern of the expansion is spoiled by N3LO EM currents of 'non-minimal' nature

Outlook

* The microscopic description of nuclei successfully reproduces EXPT data provided that many-body effects in nuclear interactions and EM currents are accounted for.

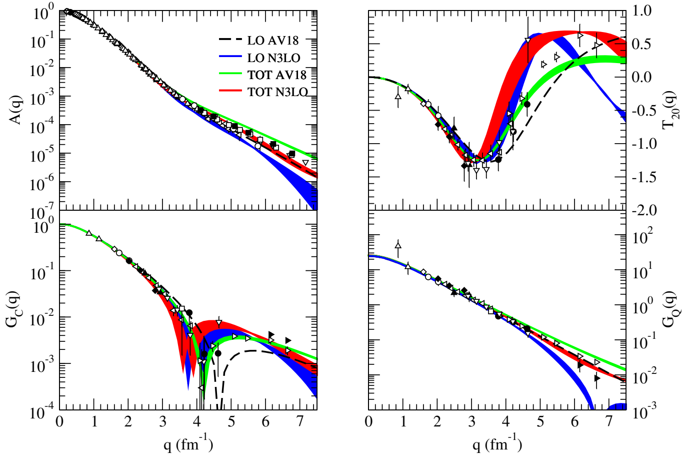
  S. Bacca & S. P. – J. Phys. G: Nucl. Part. Phys. 41, 123002 (2014)

* EM structure and dynamics of light nuclei
  - Charge and magnetic form factors of $A \leq 10$ systems (ongoing)
  - M1/E2 transitions in light nuclei (ongoing)
  - Fully consistent χEFT calculations with 'MEC' for $A > 4$
  - Role of Δ-resonances in 'MEC' !!!

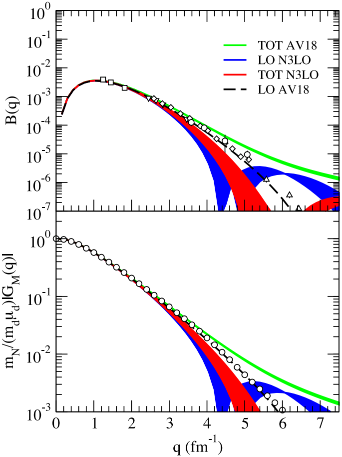
* Electroweak structure and dynamics of light nuclei
  - Text axial currents (chiral and conventional) in light nuclei
  - ν-d pion-production at threshold from HBχPT (ongoing)

EXTRA SLIDES

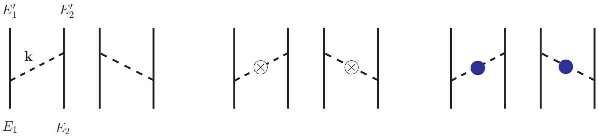
Deuteron $A(q)$ structure function and tensor polarization $T_{20}(q)$ – Piarulli *et al.*



Deuteron $B(q)$ structure function – Piarulli *et al.*



OPEP beyond the static limit



$v_\pi^{(0)} \sim Q^0$ $\qquad$ $v_\pi^{(1)} \sim Q^1$ $\qquad$ $v_\pi^{(2)} \sim Q^2$

On-the-energy-shell, non-static OPEP at N2LO (Q^2) can be equivalently written as

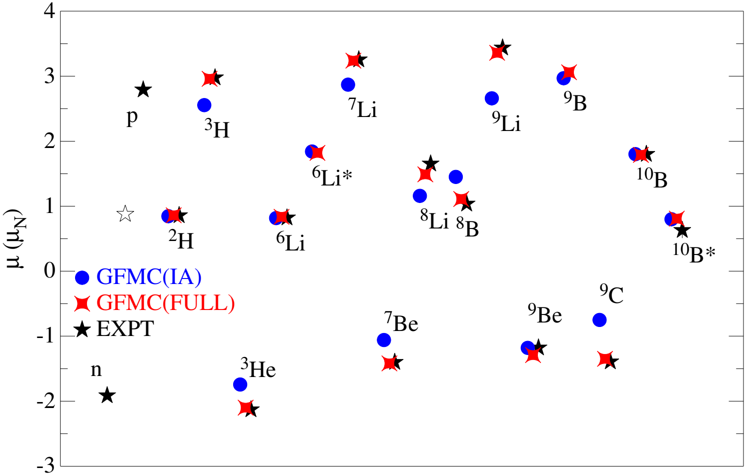
$$
\begin{aligned}
\upsilon_\pi^{(2)}(\nu = 0) &= \upsilon_\pi^{(0)}(\mathbf{k}) \, \frac{(E_1' - E_1)^2 + (E_2' - E_2)^2}{2\,\omega_k^2} \\
\upsilon_\pi^{(2)}(\nu = 1) &= -\upsilon_\pi^{(0)}(\mathbf{k}) \, \frac{(E_1' - E_1)\,(E_2' - E_2)}{\omega_k^2} \\
\upsilon_\pi^{(0)}(\mathbf{k}) &= -\frac{g_A^2}{F_\pi^2} \, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k} \, \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2}
\end{aligned}
$$

$\upsilon_\pi^{(2)}(\nu)$ corrections are different off-the-energy-shell $(E_1 + E_2 \neq E_1' + E_2')$

- TPE contributions are affected by the choice made for the parameter ν

Magnetic moments in $A \leq 10$ nuclei

Predictions for $A > 3$ nuclei



$$\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

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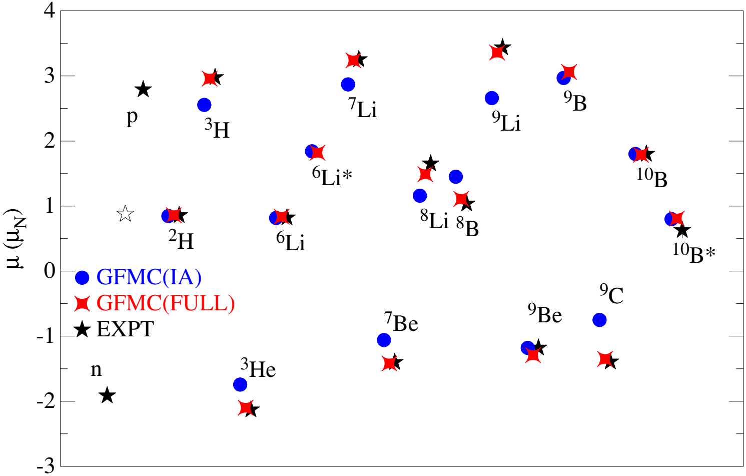

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- ^{9}Be (^{9}B) dominant spatial symmetry [s.s.] = [441] = $[\alpha, \alpha, n(p)]$

From amplitudes to potentials

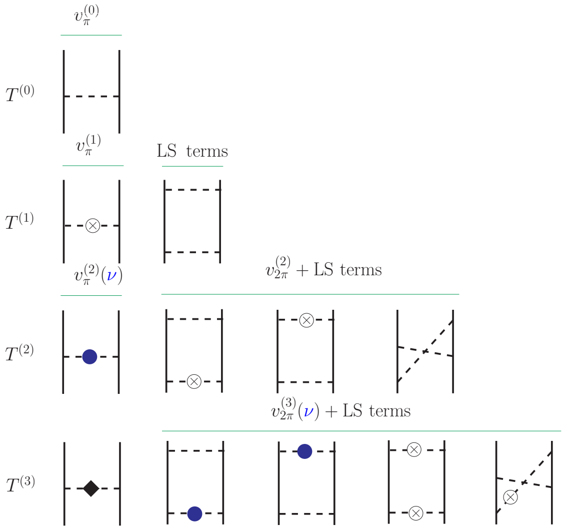
The two-nucleon potential $\upsilon = \upsilon^{(0)} + \upsilon^{(1)} + \upsilon^{(2)} + \dots$ (with $\upsilon^{(n)} \sim Q^n$) is iterated into the Lippmann-Schwinger (LS) equation *i.e.*

$$\upsilon + \upsilon\, G_0\, \upsilon + \upsilon\, G_0\, \upsilon\, G_0\, \upsilon + \dots\,, \qquad G_0 = 1/(E_i - E_I + i\,\eta)$$

$\upsilon^{(n)}$ is obtained subtracting from the transition amplitude $T_{\rm fi}^{(n)}$ terms already accounted for into the LS equation

$$\begin{aligned}
\upsilon^{(0)} &= T^{(0)}\,, \\
\upsilon^{(1)} &= T^{(1)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right]\,, \\
\upsilon^{(2)} &= T^{(2)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(1)}\right]\,, \\
\upsilon^{(3)}(\nu) &= T^{(3)} \underbrace{- \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)} + \text{permutations}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(1)}\right] - \left[\upsilon^{(2)}(\nu)\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(2)}(\nu)\right]}_{\text{LS terms}}
\end{aligned}$$

From amplitudes to potentials: an example with OPE and TPE only



- To each $\upsilon_\pi^{(2)}(\nu)$ corresponds a $\upsilon_{2\pi}^{(3)}(\nu)$

Unitary equivalence of $\upsilon_\pi^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$

- Different off-the-energy-shell parameterizations lead to unitarily equivalent two-nucleon Hamiltonians

$$H(\nu) = t^{(-1)} + \upsilon_\pi^{(0)} + \upsilon_{2\pi}^{(2)} + \upsilon_\pi^{(2)}(\nu) + \upsilon_{2\pi}^{(3)}(\nu)$$

  $t^{(-1)}$ is the kinetic energy, $\upsilon_\pi^{(0)}$ and $\upsilon_{2\pi}^{(2)}$ are the static OPEP and TPEP

- The Hamiltonians are related to each other via

$$H(\nu) = \mathrm{e}^{-iU(\nu)} H(\nu=0)\, \mathrm{e}^{+iU(\nu)} \,, \qquad i\,U(\nu) \simeq i\,U^{(0)}(\nu) + i\,U^{(1)}(\nu)$$

  from which it follows

$$H(\nu) = H(\nu=0) + \left[t^{(-1)} + \upsilon_\pi^{(0)}\,,\, i\,U^{(0)}(\nu)\right] + \left[t^{(-1)}, i\,U^{(1)}(\nu)\right]$$

- Predictions for physical observables are unaffected by off-the-energy-shell effects

From amplitudes to EM charge and current operators

- In presence of EM interaction the transition amplitude T_γ is expanded as

$$T_\gamma = T_\gamma^{(-3)} + T_\gamma^{(-2)} + T_\gamma^{(-1)} + \dots, \qquad T_\gamma^{(n)} \sim e\,Q^n$$

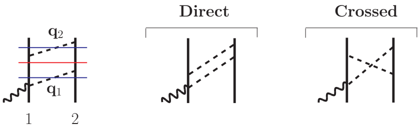
and the charge and current operators are related to $T_\gamma^{(n)}$ via

$$\upsilon_\gamma^{(n)} = A^0\,\rho^{(n)} - \mathbf{A}\cdot\mathbf{j}^{(n)} = T_\gamma^{(n)} - \text{ LS terms}$$

that is

Technical issue II - Recoil corrections at N^{3}LO

$$\mathbf{j}^{\mathrm{N^3LO}} =$$



- Reducible contributions

$$\mathbf{j}_{\mathrm{red}} \sim \int \upsilon^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{NLO}}(\mathbf{q}_1)$$
$$- \int 2\,\frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

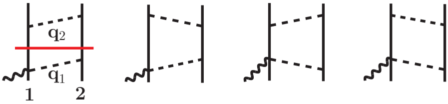
- Irreducible contributions

$$\mathbf{j}_{\mathrm{irr}} = \int 2\,\frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$
$$- \int 2\,\frac{\omega_1^2+\omega_2^2+\omega_1\,\omega_2}{\omega_1\,\omega_2(\omega_1+\omega_2)}[V_{\pi NN}(2,\mathbf{q}_2), V_{\pi NN}(2,\mathbf{q}_1)]_-\,V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

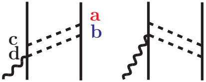
- Observed partial cancellations at N^{3}LO between recoil corrections to reducible diagrams and irreducible contributions

The box diagram: an example at N^{3}LO

Reducible



Irreducible direct



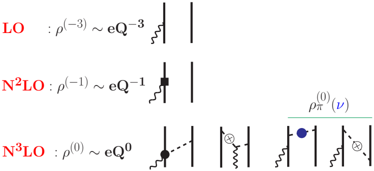
Irreducible crossed

$$\text{direct} = f_d(\omega_1,\omega_2)\,V_a\,V_b\,V_c\,V_d$$

$$\text{crossed} = f_c(\omega_1,\omega_2)\,V_b\,V_a\,V_c\,V_d \qquad V_b V_a = V_a V_b - [V_a, V_b]_-$$

$$\text{irreducible} = [f_d(\omega_1,\omega_2)+f_c(\omega_1,\omega_2)]V_a\,V_b\,V_c\,V_d - f_c(\omega_1,\omega_2)[V_a,V_b]_-\,V_c\,V_d$$

EM charge up to $n = 0$ (or up to N3LO)



- $n = -3$

  $\rho^{(-3)}(\mathbf{q}) = e\,(2\pi)^3\delta(\mathbf{p}_1 + \mathbf{q} - \mathbf{p}_1')\,(1+\tau_{1,z})/2 + 1 \rightleftharpoons 2$

- $n = -1$:

  $(Q/m_N)^2$ relativistic correction to $\rho^{(-3)}$

- $n = 0$:

  i) 'static' tree-level current (originates from a $\gamma\pi N$ vertex of order $e\,Q$)

  ii) 'non-static' OPE charge operators, $\rho_\pi^{(0)}(\nu)$ depends on $\upsilon_\pi^{(2)}(\nu)$

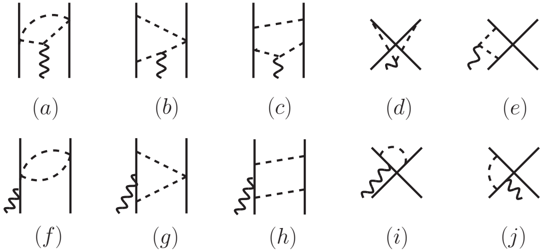
- $\rho_\pi^{(0)}(\nu)$'s are unitarily equivalent

$$\rho_\pi^{(0)}(\nu) \quad = \quad \rho_\pi^{(0)}(\nu = 0) + \left[\rho^{(-3)}\,,\, i\,U^{(0)}(\nu)\right]$$

- No unknown LECs up to this order (g_A, F_π)

EM charge @ $n = 1$ (or N4LO) 1.

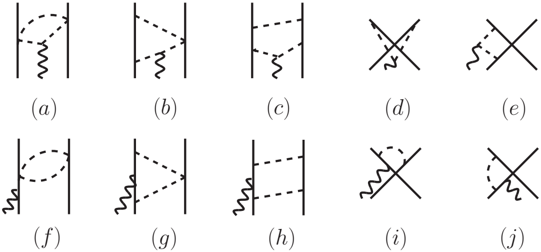
N^{4}LO : $\rho^{(1)}$



- (a), (f), (d), and (i) vanish
- Divergencies associated with (b) + (g), (c) + (h), and (e) + (j) cancel out—as they must since there are no counter-terms at N4LO
- $\rho_{\rm h}^{(1)}(\nu)$ depends on the parametrization adopted for $\upsilon_\pi^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$
- $\rho_{\rm h}^{(1)}(\nu)$'s are unitarily equivalent

$$\rho_{\rm h}^{(1)}(\nu) = \rho_{\rm h}^{(1)}(\nu = 0) + \left[\rho^{(-3)}, i\, U^{(1)}(\nu)\right]$$

N^{4}LO : $\rho^{(1)}$



- Charge operators (ν-dependent included) up to $n=1$ satisfy the condition

$$\rho^{(n>-3)}(\mathbf{q}=0)=0$$

which follows from charge conservation

$$\rho(\mathbf{q}=0)=\int \mathrm{d}\mathbf{x}\,\rho(\mathbf{x})=e\,\frac{(1+\tau_{1,z})}{2}+1\rightleftharpoons 2=\rho^{(-3)}(\mathbf{q}=0)$$

- $\rho^{(1)}$ does not depend on unknown LECs and it is purely isovector