Tagging with Polarized Electron and Polarized Nuclei

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HENPH with ST workshop

First: **Theoretical framework for Polarized electron DIS off Polarized Light Nuclei with Tagging** **Cosyn & Weiss**

Light-Cone PWIA Approximation

Second: **Final state interaction studies in tagged-DIS (**Wim Cosyn's talk**)**

Generalized Eikonal Approximation

High and intermediate x region

Third: **Extracting "Virtually Free" neutron's structure function**

Pole Extrapolation Method

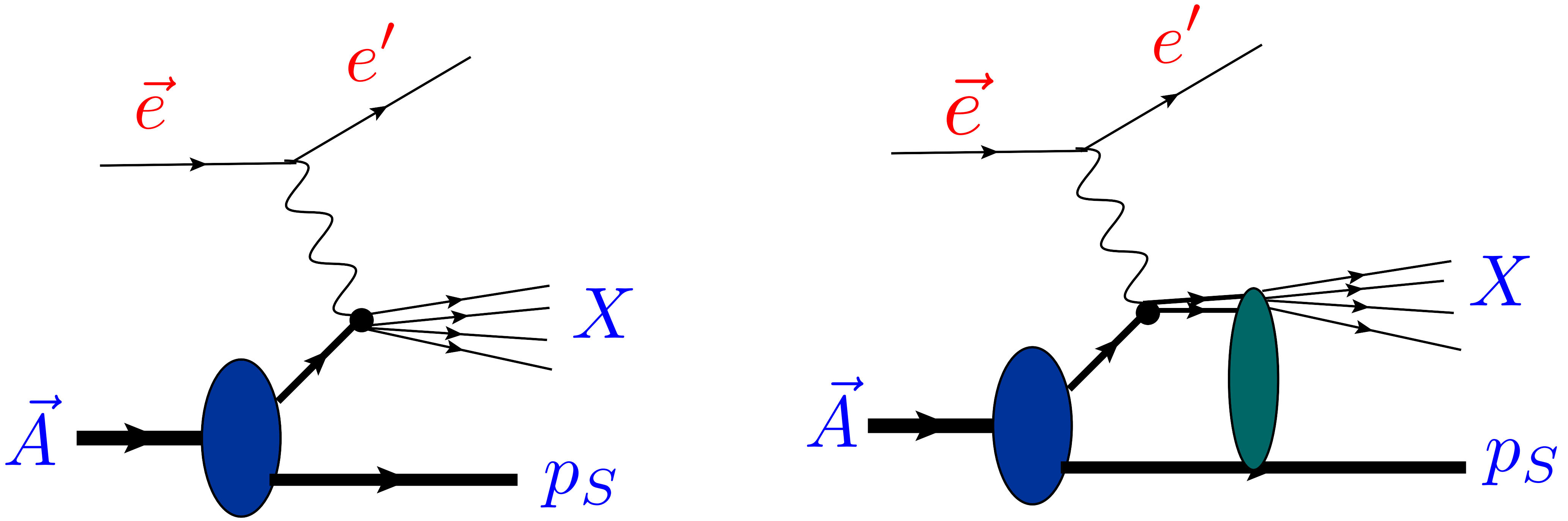
Cancellation of Singularities in the loop

Extraction Procedure

First attempt using Bonus Data Wim Cosyn's talk

Forth: **Conclusion and Outlook**

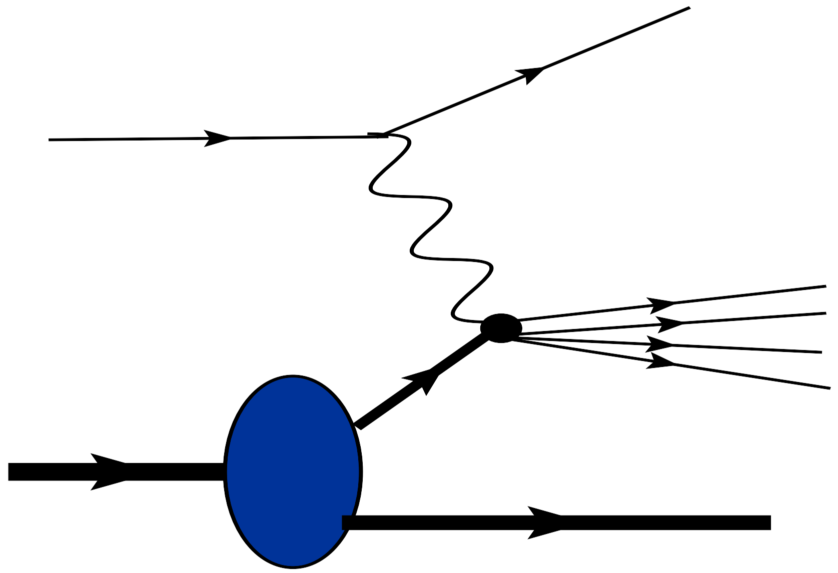
First: Theoretical framework for Polarized electron DIS off Polarized Light Nuclei with Tagging



$$d\sigma = \frac{\alpha^2}{Q^4}\frac{M_A}{kp_A}L_{\mu\nu}W_A^{\mu\nu}\frac{y_A}{2x_A}dQ^2dx_A d\phi$$

$$L_{\mu\nu} = 4\left[(k_\mu - \frac{q_\mu}{2})(k_\nu - \frac{q_\nu}{2})\right] + Q^2\left[-g_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2}\right] + 2im_e\epsilon_{\mu\nu\rho\sigma}q^\rho a^\sigma$$

Plane Wave Impulse Approximation



$$M = \bar{u}(k', s'_e)\gamma^\mu \frac{(1+\gamma_5 \not{s}_e)}{2} u(k, s_e) \frac{e^2}{q^2} J_A^\mu$$

$$J_A^\mu = -\psi_X^\dagger(p_x, s_x)\Gamma^\mu \left[\frac{\sum_{s_N} u(p_N, s_N)\bar{u}(p_N, s_N)}{p_{N-}(p_{A+}-p_{S+}-p_{N+}+i\epsilon}\right] \psi_s^\dagger(p_s, s_s)\Gamma^{A\to NS}(s_A)\chi^{s_A}$$

$$p_+ = \frac{M^2+p_\perp^2}{p_-}$$

$$p_{A+} - p_{S+} - p_{N+} = \frac{1}{p_{A-}}\left(M_A^2 + p_{A,\perp}^2 - A\frac{m_S^2+p_{S\perp}^2}{\alpha_S} - A\frac{m_N^2+p_{N\perp}^2}{A-\alpha_S}\right)$$

$$\psi^{s_A}(\alpha, p_{N,\perp}, s_N, s_S) = -\frac{\psi_s^\dagger \bar{u}(p_N, s_N)\Gamma^{A\to NS}(s_A)\chi^{s_A}}{\frac{1}{A}[M_A^2+p_{A,\perp}^2 - A\frac{m_S^2+p_{S\perp}^2}{\alpha_S} - A\frac{m_N^2+p_{N\perp}^2}{A-\alpha_S}]\sqrt{A}\sqrt{2(2\pi)^3}}$$

$$J_A^{\mu}(p_A, s_A, p_X, s_X, p_S, s_S) = \sum_{s_N} J_N^{\mu}(p_X, s_X, p_N, s_N) \frac{\psi^{s_A}(\alpha_N, p_{N,\perp}, s_N, s_S)}{\alpha_N} \sqrt{A}\sqrt{2(2\pi)^3}$$

$$4\pi M_A W_A^{\mu\nu}(S_A) = \sum_X \sum_{s_S, s_X} \sum_{s_N, s'_N} J_N^{\mu,\dagger}(p_X, s_X, p_N, s_N) J_N^{\nu}(p_A, s_A, p_X, s_X, p_N, s'_N)\Phi_x \frac{\psi^{s_A,\dagger}(\alpha_N, p_{N,\perp}, s_N, s_S)\psi^{s_A}(\alpha_N, p_{N,\perp}, s'_N, s_S)}{\alpha_N^2} 2A(2\pi)^3\Phi_S. \quad (1)$$

$$\begin{aligned}
4\pi M_A W_A^{\mu\nu}(S_A) &= \sum_{X,s_S} J_N^{\mu,\dagger}(p_N, s_{N,z'}) J_N^{\nu}(p_N, s_{N,z'})\Phi_x \frac{\psi^{s_A,\dagger}(p_N, s_{N,z'})\psi^{s_A}(p_N, s_{N,z'})}{\alpha_N^2}\tilde{\Phi}_S \\
&+ \sum_{X,s_S} J_N^{\mu,\dagger}(p_N, -s_{N,z'}) J_N^{\nu}(p_N, -s_{N,z'})\Phi_x \frac{\psi^{s_A,\dagger}(p_N, -s_{N,z'})\psi^{s_A}(p_N, -s_{N,z'})}{\alpha_N^2}\tilde{\Phi}_S \\
&+ \sum_{X,s_S} J_N^{\mu,\dagger}(p_N, s_{N,z'}) J_N^{\nu}(p_N, -s_{N,z'})\Phi_x \frac{\psi^{s_A,\dagger}(p_N, s_{N,z'})\psi^{s_A}(p_N, -s_{N,z'})}{\alpha_N^2}\tilde{\Phi}_S \\
&+ \sum_{X,s_S} J_N^{\mu,\dagger}(p_N, -s_{N,z'}) J_N^{\nu}(p_N, s_{N,z'})\Phi_x \frac{\psi^{s_A,\dagger}(p_N, -s_{N,z'})\psi^{s_A}(p_N, s_{N,z'})}{\alpha_N^2}\tilde{\Phi}_S.
\end{aligned} \quad (1)$$

Diagonal Terms

$$4\pi m_N W_N^{\mu\nu}(s_N)^{dg} = \sum_{X,s_S} J_N^{\mu,\dagger}(p_N, \pm s_{N,z'}) J_N^{\nu}(p_N, \pm s_{N,z'}) \Phi_x$$

$$= \left[-g^{\mu\nu} - \frac{q^\mu q^\nu}{Q^2}\right] W_{1N}(x_N, Q^2, p_N) + [p^\mu + \frac{pq}{Q^2} q^\mu][p^\nu + \frac{pq}{Q^2} q^\nu] \frac{W_{2N}(x_N, Q^2, p_N)}{m_N^2}$$

$$-i\epsilon^{\mu\nu\rho\sigma} q_\rho \left[\zeta_{\pm z',\sigma} \frac{G_1(x_N, Q^2, p_N)}{m_N} + ((p_N q)\zeta_{\pm z',\sigma} - (\zeta_{\pm z'} q) p_{N,\sigma}) \frac{G_2(x_N, Q^2, p_N)}{m_N^3}\right]$$

$$\zeta^\mu_{\pm z'}(p) = \left(\pm \frac{\vec{p} \cdot \hat{z}'}{m_N}, \pm(\hat{z}' + \frac{(\hat{z}' \cdot \vec{p})\vec{p}}{m_N(E + m_N)})\right)$$

$$\hat{z}' = \vec{S}_A = (\sin(\theta_{SA})\cos(\phi_{SA}), \quad \sin(\theta_{SA})\sin(\phi_{SA}), \quad \cos(\theta_{SA}))$$

$$\hat{y}' = \vec{n}_y^{S_A} \equiv \frac{\vec{P}_A \times \vec{S}_A}{|P_A \times S_A|} = (-\sin(\phi_{SA}), \quad \cos(\phi_{SA}), \quad 0)$$

$$\hat{x}' = \vec{n}_x^{S_A} \equiv \vec{n}_y^{S_A} \times \vec{S}_A = (\cos(\theta_{SA})\cos(\phi_{SA}), \quad \cos(\theta_{SA})\sin(\phi_{SA}), \quad -\sin(\theta_{SA}))$$

Off-Diagonal Terms

Diagonal Term

$$\sum_{X,s_S} J_N^{\mu,\dagger}(p_N,\pm s_{N,z'})J_N^{\nu}(p_N,\pm s_{N,z'})\Phi_x = \sum_{X,s_S} \bar{u}(p_N,\pm s_{N,z'})\Gamma^{\dagger\mu}\psi_X(p_X,S_X)\psi_X^{\dagger}(p_X,S_X)\Gamma^{\nu}u(p_N,\pm s_{N,z'})\Phi$$

$$= \sum_{X,s_S}\Gamma^{\dagger\mu}\psi_X(p_X,S_X)\psi_X^{\dagger}(p_X,S_X)\Gamma^{\nu}(\not{p}_N+m)\frac{(1+\gamma_5\not{s}_{\pm z'})}{2}\Phi_x$$

where we used the relation

$$u(p_N,\pm s_{N,z'})\bar{u}(p_N,\pm s_{N,z'}) = (\not{p}_N+m)\frac{(1+\gamma_5\not{s}_{\pm z'}(p))}{2}$$

Off-Diagonal Term

$$u(p_N,\mp s_{N,z'})\bar{u}(p_N,\pm s_{N,z'}) = (\not{p}_N+m)\frac{\gamma_5\zeta_{\mp}}{2}$$

where

$$\zeta_{\mp} = \zeta_{x'}(p) \mp i\zeta_{y'}(p)$$

with

$$\zeta^{\mu}_{x'}(p) = \left(\frac{\vec{p}\cdot\hat{x}'}{m_N}, \hat{x}' + \frac{(\hat{x}'\cdot\vec{p})\vec{p}}{m_N(E+m_N)}\right)$$

$$\zeta^{\mu}_{y'}(p) = \left(\frac{\vec{p}\cdot\hat{y}'}{m_N}, \hat{y}' + \frac{(\hat{y}'\cdot\vec{p})\vec{p}}{m_N(E+m_N)}\right)$$

Diagonal

$$4\pi m_N W_N^{\mu\nu}(s_N)^{dg} = \sum_{X,s_S} J_N^{\mu,\dagger}(p_N, \pm s_{N,z'}) J_N^{\nu}(p_N, \pm s_{N,z'}) \Phi_x$$

$$= \sum_{X,s_S} \Gamma^{\dagger\mu} \psi_X(p_X, S_X) \psi_X^{\dagger}(p_X, S_X) \Gamma^{\nu} (\not{p}_N + m) \frac{(1 + \gamma_5 \not{\zeta}_{\pm z'})}{2} \Phi_x$$

$$= \left[-g^{\mu\nu} - \frac{q^\mu q^\nu}{Q^2} \right] W_{1N}(x_N, Q^2, p_N) + [p^\mu + \frac{pq}{Q^2} q^\mu][p^\nu + \frac{pq}{Q^2} q^\nu] \frac{W_{2N}(x_N, Q^2, p_N)}{m_N^2}$$

$$-i\epsilon^{\mu\nu\rho\sigma} q_\rho \left[\zeta_{\pm z',\sigma} \frac{G_1(x_N, Q^2, p_N)}{m_N} + ((p_N q)\zeta_{\pm z',\sigma} - (\zeta_{\pm z'} q) p_{N,\sigma}) \frac{G_2(x_N, Q^2, p_N)}{m_N^3} \right],$$

OffDiagonal

$$4\pi m_N W_N^{\mu\nu}(s_N)^{ndg} = \sum_{X,s_S} J_N^{\mu,\dagger}(p_N, \pm s_{N,z'}) J_N^{\nu}(p_N, \mp s_{N,z'}) \Phi_x$$

$$= \sum_{X,s_S} \Gamma^{\dagger\mu} \psi_X(p_X, S_X) \psi_X^{\dagger}(p_X, S_X) \Gamma^{\nu} (\not{p}_N + m) \frac{\gamma_5 \not{\zeta}_{\mp}}{2} \Phi_x$$

$$-i\epsilon^{\mu\nu\rho\sigma} q_\rho \left[\zeta_{\mp,\sigma} \frac{G_1(x_N, Q^2, p_N)}{m_N} + [(p_N q)\zeta_{\mp,\sigma} - (\zeta_{\mp} q) p_{N,\sigma}] \frac{G_2(x_N, Q^2, p_N)}{m_N^3} \right],$$

Introduce

$$H_0^{\mu\nu} = \left[-g^{\mu\nu} - \frac{q^\mu q^\nu}{Q^2}\right] W_{1N}(x_N, Q^2, p_N) + [p^\mu + \frac{pq}{Q^2} q^\mu][p^\nu + \frac{pq}{Q^2} q^\nu] \frac{W_{2N}(x_N, Q^2, p_N)}{m_N^2}$$

$$H_j^{\mu\nu} = -i\epsilon^{\mu\nu\rho\sigma} q_\rho \left[\zeta_{j',\sigma} \frac{G_1(x_N, Q^2, p_N)}{m_N} + [(p_N q)\zeta_{j',\sigma} - (\zeta_{j'} q) p_{N,\sigma}] \frac{G_2(x_N, Q^2, p_N)}{m_N^3}\right],$$

where $j' = x', y', z'$

$$\zeta^\sigma_{(x')} = \left(\frac{\vec{p}_N \vec{n}_x^{S_A}}{m_N}, \ \vec{n}_x^{S_A} + \frac{(\vec{n}_x^{S_A} \vec{p}_N)\vec{p}_N}{m_N(E_N + m_N)}\right)$$

$$\zeta^\sigma_{(y')} = \left(\frac{\vec{p}_N \vec{n}_y^{S_A}}{m_N}, \ \vec{n}_y^{S_A} + \frac{(\vec{n}_y^{S_A} \vec{p}_N)\vec{p}_N}{m_N(E_N + m_N)}\right)$$

$$\zeta^\sigma_{(z')} = \left(\frac{\vec{p}_N \vec{S}_A}{m_N}, \ \vec{S}_A + \frac{(\vec{S}_A \vec{p}_N)\vec{p}_N}{m_N(E_N + m_N)}\right)$$

$$\begin{aligned} 4\pi M_A W_A^{\mu\nu}(S_A) = \ & 4\pi m_N \Big\{ \\ & (H_0^{\mu\nu} + H_{z'}^{\mu\nu}) \frac{|\psi^{S_A}(z')|^2}{\alpha_N^2} \\ + \ & (H_0^{\mu\nu} - H_{z'}^{\mu\nu}) \frac{|\psi^{S_A}(-z')|^2}{\alpha_N^2} \\ + \ & (H_{x'}^{\mu\nu} - H_{y'}^{\mu\nu}) \frac{\psi^{S_A,\dagger}(z')\psi^{S_A}(-z')}{\alpha_N^2} \\ + \ & (H_{x'}^{\mu\nu} + H_{y'}^{\mu\nu}) \frac{\psi^{S_A,\dagger}(-z')\psi^{S_A}(z')}{\alpha_N^2} \Big\} \tilde{\Phi}_S \end{aligned}$$

$$W_A^{\mu\nu}(S_A) = \frac{Am_N}{M_A}\left(H_0^{\mu\nu}\rho_0^{S_A}(\alpha_N, p_{N,t}) + \sum_{j=1}^{3} H_j^{\mu\nu}\rho_j^{S_A}(\alpha_N, p_t)\right)\frac{d\alpha_S}{(A-\alpha_S)^2}d^2p_{S,t} \quad (1)$$

where

$$\rho_0^{S_A}(\alpha_N, p_{N,t}) = \frac{|\psi^{S_A}(z')|^2 + |\psi^{S_A}(-z')|^2}{\alpha_S}$$

$$\rho_z^{S_A}(\alpha_N, p_{N,t}) = \frac{|\psi^{S_A}(z')|^2 - |\psi^{S_A}(-z')|^2}{\alpha_S}$$

$$\rho_x^{S_A}(\alpha_N, p_{N,t}) = \frac{2Re\left[\psi^{S_A}(z')\psi^{S_A}(-z')\right]}{\alpha_S}$$

$$\rho_y^{S_A}(\alpha_N, p_{N,t}) = \frac{2Im\left[\psi^{S_A}(z')\psi^{S_A}(-z')\right]}{\alpha_S}$$

$$d\sigma = \frac{\alpha^2}{Q^4}\frac{M_A}{kp_A}L_{\mu\nu}W_A^{\mu\nu}\frac{y_A}{2x_A}dQ^2dx_Ad\phi$$

$$\frac{d\sigma^{\hat{S_A}}}{dQ^2 dx_A d\alpha_S d^2 p_{St}} =$$

$$\frac{2\pi\alpha^2}{Q^4} y_A^2 \left\{ \left(2F_{1N}(x_N, Q^2) + \frac{1}{2x_N y_N^2} \left[(2 - y_N)^2 - y_N^2 \left(1 + \frac{4p_N^2 x_N^2}{Q^2} \right) \right] F_2(x_N, Q^2) \right) \frac{\rho_0^{S_A}(\alpha, p_t)}{(2-\alpha_s)^2} \right.$$

$$\left. + \quad 2m_N h_e \sum_{j=1}^{3} \left[(\zeta_j q - 2\zeta_j k) \frac{g_{1N}(x_N, Q^2)}{(p_N q)} + 2 \left(\frac{\zeta_j q}{y_N} - \zeta_j k \right) \frac{g_{2N}(x_N, Q^2)}{(p_N q)} \right] \frac{\rho_j^{S_A}(\alpha, p_t)}{(2-\alpha_s)^2} \right\}$$

$$\zeta^{\sigma}_{(x')} = \left(\frac{\vec{p}_N \vec{n}_x^{S_A}}{m_N}, \ \vec{n}_x^{S_A} + \frac{(\vec{n}_x^{S_A} \vec{p}_N)\vec{p}_N}{m_N(E_N + m_N)} \right)$$

$$\zeta^{\sigma}_{(y')} = \left(\frac{\vec{p}_N \vec{n}_y^{S_A}}{m_N}, \ \vec{n}_y^{S_A} + \frac{(\vec{n}_y^{S_A} \vec{p}_N)\vec{p}_N}{m_N(E_N + m_N)} \right)$$

$$\zeta^{\sigma}_{(z')} = \left(\frac{\vec{p}_N \vec{S}_A}{m_N}, \ \vec{S}_A + \frac{(\vec{S}_A \vec{p}_N)\vec{p}_N}{m_N(E_N + m_N)} \right)$$

$$\rho_0^{S_A}(\alpha_N, p_{N,t}) = \frac{|\psi^{S_A}(z')|^2 + |\psi^{S_A}(-z')|^2}{\alpha_S}$$

$$\rho_z^{S_A}(\alpha_N, p_{N,t}) = \frac{|\psi^{S_A}(z')|^2 - |\psi^{S_A}(-z')|^2}{\alpha_S}$$

$$\rho_x^{S_A}(\alpha_N, p_{N,t}) = \frac{2Re\left[\psi^{S_A}(z')\psi^{S_A}(-z')\right]}{\alpha_S}$$

$$\rho_y^{S_A}(\alpha_N, p_{N,t}) = \frac{2Im\left[\psi^{S_A}(z')\psi^{S_A}(-z')\right]}{\alpha_S}$$

Fixed-Target

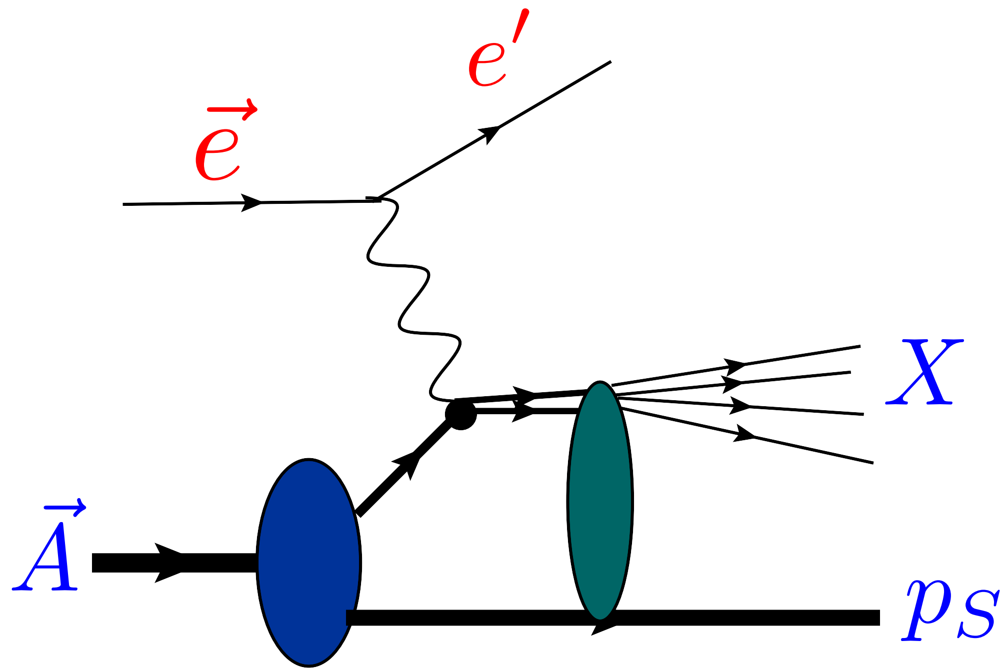
3He: For Inclusive Quasi-Elastic Non-Relativistic Approximation: Blankleider & Woloshin, PRC 1984

3He: Inclusive QE and DIS: Non-Relativistic Approximation: Ciofi, Salme, Pace, Scopetta, PRC

3He DIS: Non-Relativistic Approximation: Scopetta talk

d: Light-Cone with polarized Deuteron: Frankfrut, Strikman, Nucl. Phys. A 1983

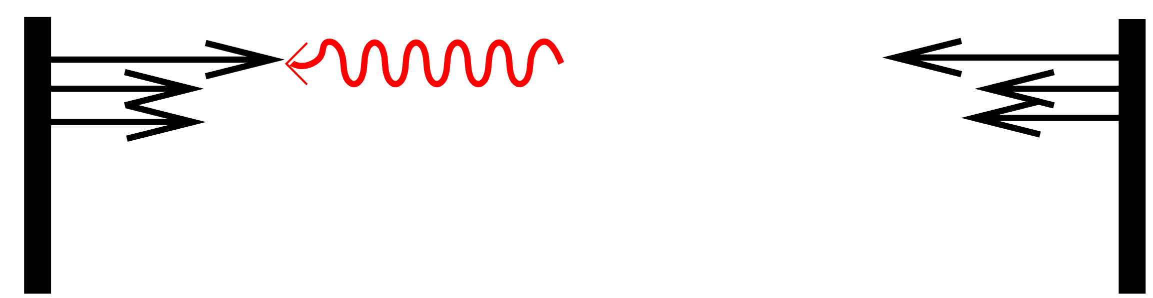
Second: Final state interaction studies in Tagged-DIS



Final State Interaction off the Spectator Nucleons
depends on x

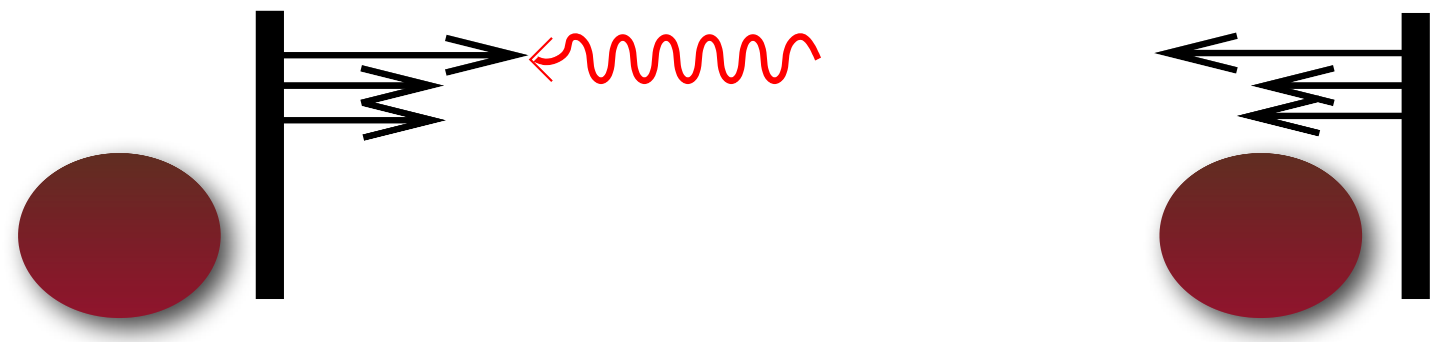
$\gamma^* N$ DIS Interaction

Minimal Fock Component Approximation

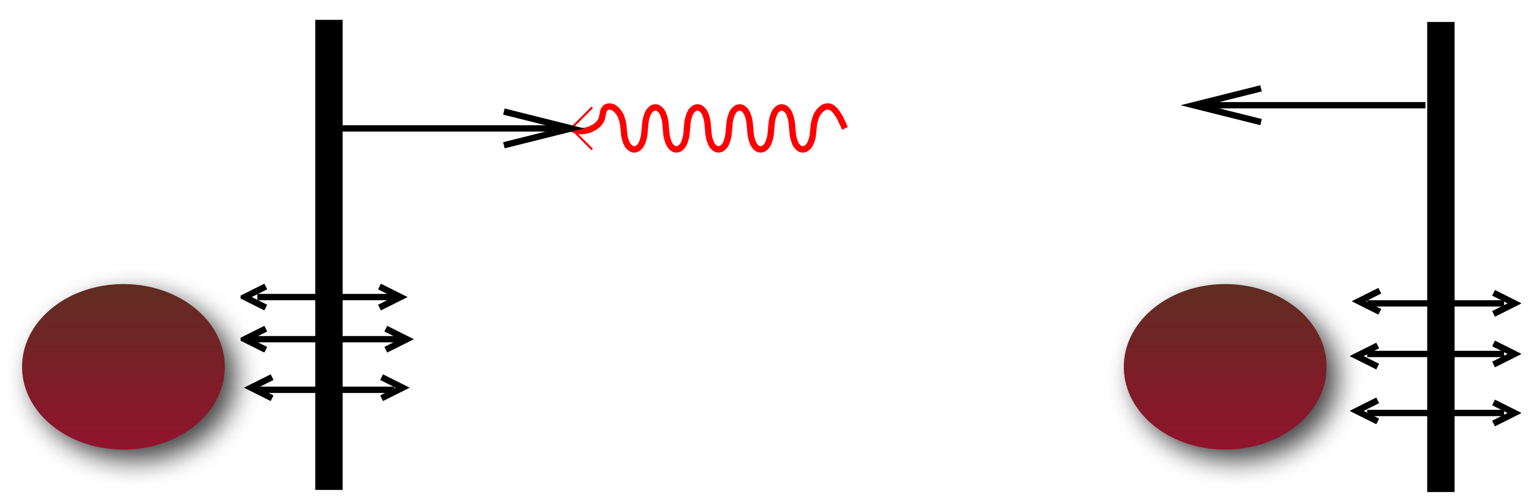


Feynman Mechanism

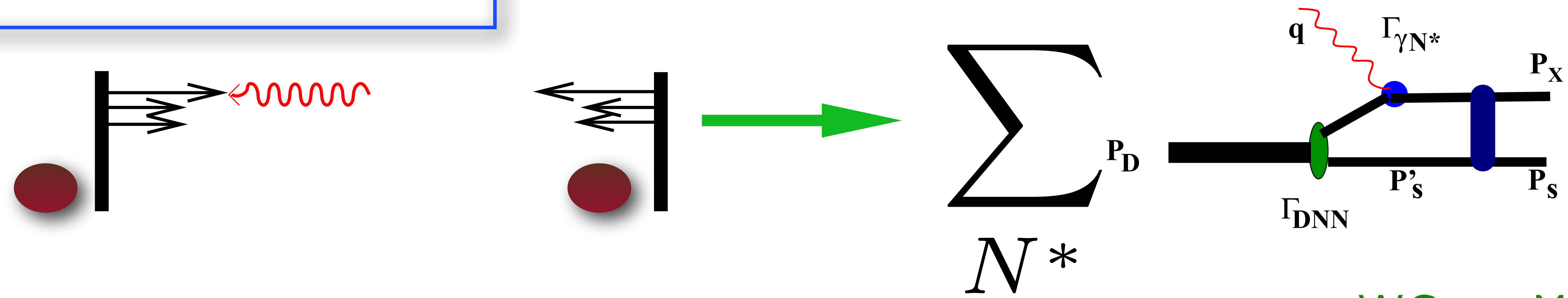
Minimal Fock Component Approximation



Feynman Mechanism



Minimal Fock Component Approximation

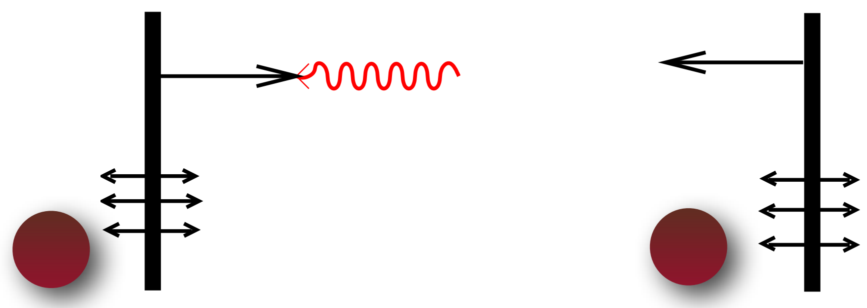


W.Cosyn, MS PRC 2011

$$F_{FSI}(N^*) = -\frac{\sqrt{(2\pi)^3 E_s}}{2}\int \frac{d^3p'_s}{(2\pi)^3}\frac{f^{N^*N}(p_{st}-p'_{st}) < N^*|J(Q^2)|\gamma^*, N >}{[p'_{sz}-p_{sz}+\Delta_{N^*}+i\epsilon]}\psi_D(p'_s)$$

$$\Delta_{N^*} = \frac{M_d + q_0}{q}T_s + \frac{W_N^2 - M_*^2}{2q}$$

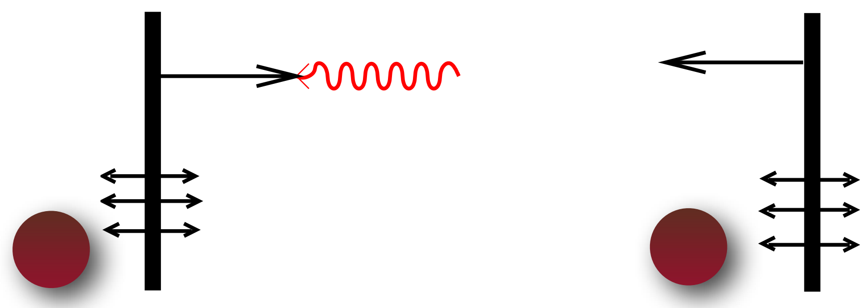
Feynman Mechanism



$$\psi_D(p_s) \rightarrow \int d^3k \psi_{NN}^{p_s \dagger}(k) \cdot \psi_D(k)$$

$$\psi_{\mathbf{NN}}^{\mathbf{p_s}\dagger}(\mathbf{k}) = \delta^{\mathbf{3}}(\mathbf{p_s} - \mathbf{k}) + \frac{\mathbf{1}}{\mathbf{2}\pi^{\mathbf{2}}} \frac{\hat{\mathbf{t}}_{\mathbf{NN}}^{\mathbf{off}\ \mathbf{shell}}(\mathbf{p_s}, \mathbf{k})}{\mathbf{k^2} - \mathbf{p_s^2} - \mathbf{i}\varepsilon}$$

Feynman Mechanism



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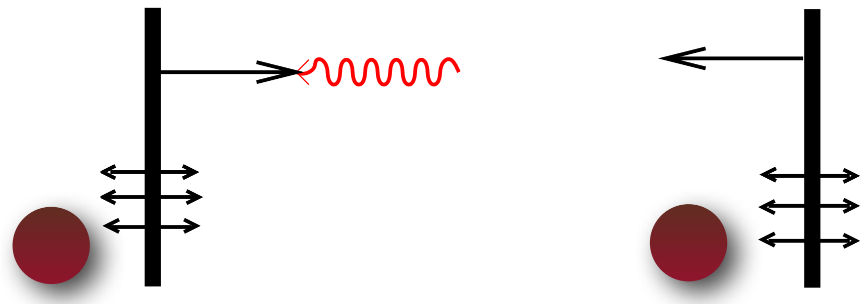
- First term recovers PWIA contribution

Feynman Mechanism



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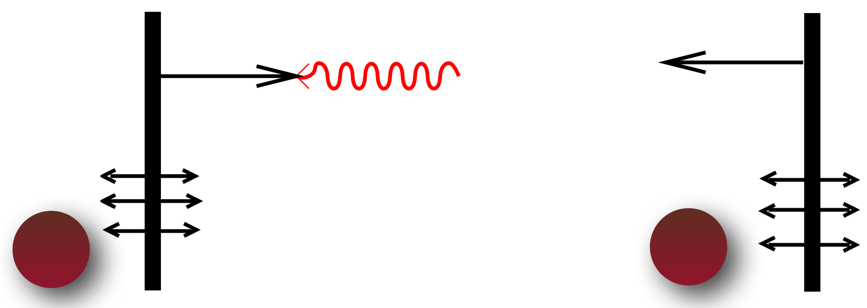
- First term recovers PWIA contribution

- Pole term cancels due to orthogonality

Feynman Mechanism



$$\psi_D(p_s) \to \int d^3k \psi_{NN}^{p_s \dagger}(k) \cdot \psi_D(k)$$

$$\psi_{\mathbf{NN}}^{\mathbf{p_s}\dagger}(\mathbf{k}) = \delta^{\mathbf{3}}(\mathbf{p_s} - \mathbf{k}) + \frac{\mathbf{1}}{\mathbf{2}\pi^{\mathbf{2}}} \frac{\hat{\mathbf{t}}_{\mathbf{NN}}^{\mathbf{off\ shell}}(\mathbf{p_s}, \mathbf{k})}{\mathbf{k^2} - \mathbf{p_s^2} - \mathbf{i}\varepsilon}$$

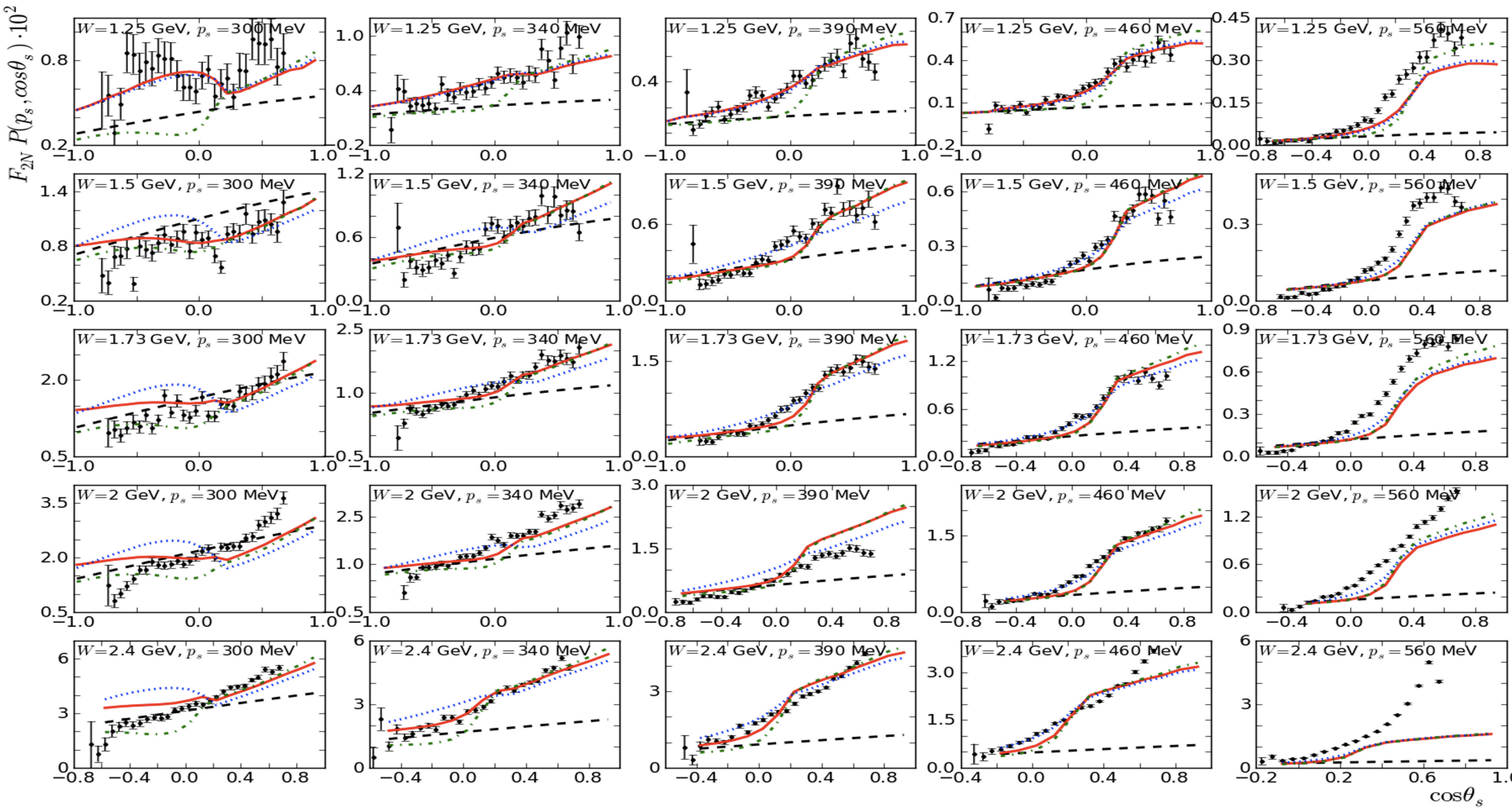
- First term recovers PWIA contribution

- Pole term cancels due to orthogonality

- Non-Pole (off-shell) term accounts for FSI

First data on $e + d \to e'p + X$ at large Bjorken x and moderate Q^2

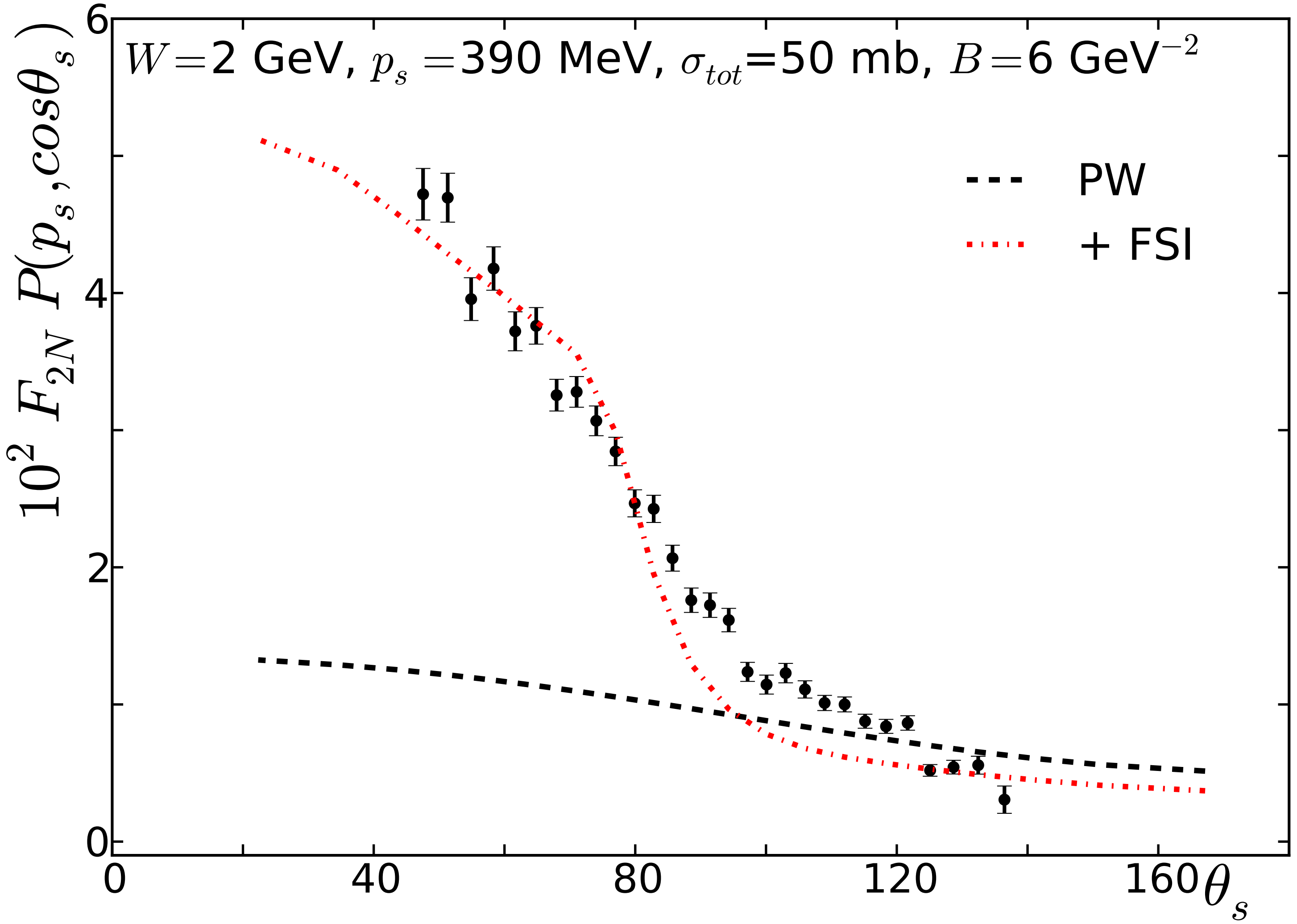
W.Cosyn & M.S., PRC 2011   Data, Klimenko et al, PRC 2006



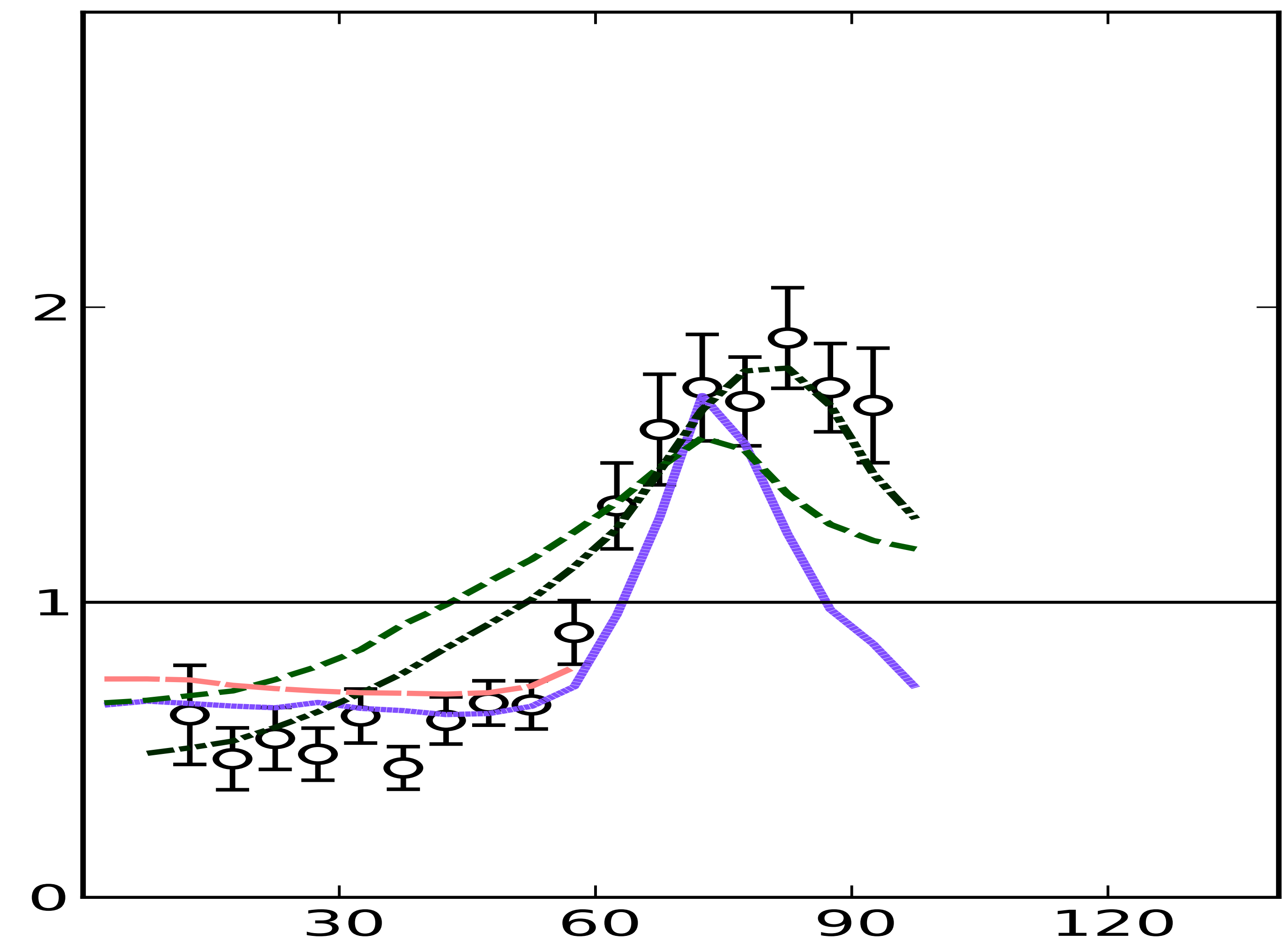
W.Cosyn & M.S., PRC 2011 Data, Klimenko et al, PRC 2006

MS, PRC 2010 Data, Boeglin et al, PRL 2011

For the DIS processes of e+ d → e' + X + p_s



For quasielastic of e+ d → e' + p_f + p_s



Extend the FSI Calculations to Moderate to Small x ~ 0.1

Third: Extracting Virtually "Free" Neutron Structure Function

MS, M.Strikman PLB 2006

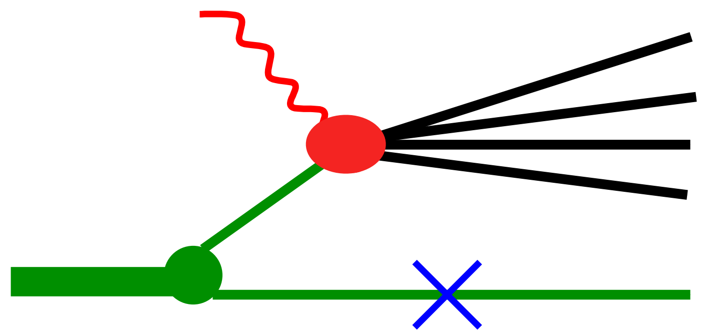
$$e + d \to e' + N_S + X$$

$$\frac{d\sigma}{dxdQ^2d^3p_s/E_s} = \frac{4\pi\alpha_e^2 m}{xQ^4}(1 - y - \frac{x^2y^2m_N^2}{Q^2})\left[F_{2D}^{SI} + 2tan^2(\frac{\theta}{2})\frac{\nu}{m_N}F_{1D}^{SI}\right]$$

where

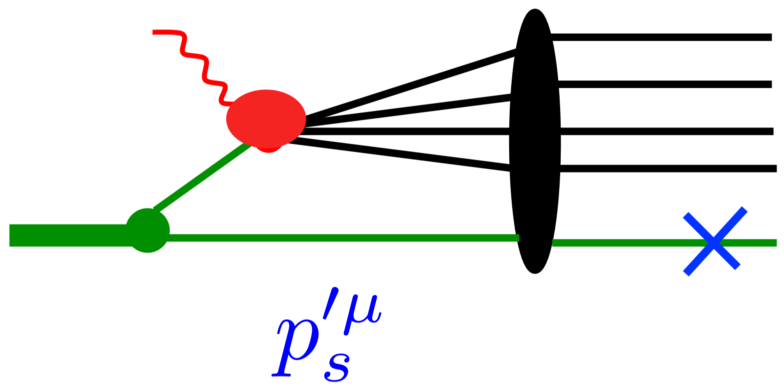
$$F_{2D}^{SI}(x, Q^2, \alpha_s, p_t) = F_L^D + \frac{Q^2}{2q^2}\frac{\nu}{m_N}F_T^D$$

$$F_{1D}^{SI}(x, Q^2, \alpha_s, p_t) = \frac{F_T^D}{2}$$



Plane Wave Impulse Approximation

$$A^{\mu}_{IA} = < X|J^{\mu}_{em}(Q^2,\nu,p_s)\frac{\not{p}_d - \not{p}_s + m}{m_N^2 - (p_d - p_s)^2}\bar{u}(p_s)\Gamma_d = < X|J^{\mu}_{em}(Q^2,\nu,p_s)\frac{\not{p}_d - \not{p}_s + m}{m_N^2 - t}\bar{u}(p_s)\Gamma_d$$



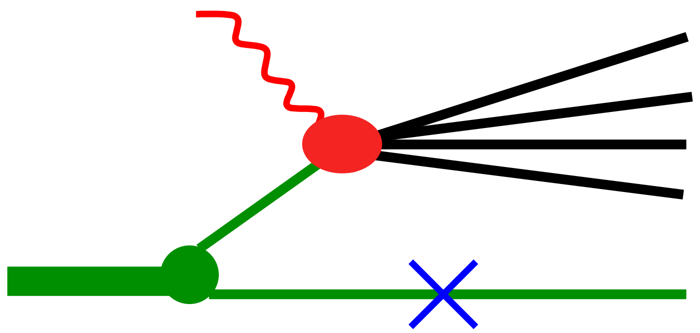
Final State Interactions

$$A_{FSI} = \sum_{X'}\int \frac{d^4 p_{s'}}{i(2\pi)^4}\langle X,s|\hat{A}_{FSI}\cdot G(X')\cdot \hat{J}^{em}(Q^2,x)\frac{\not{p}_d - \not{p}_{s'} + m_N}{(p_d - p_{s'})^2 - m_N^2 + i\epsilon}\frac{\not{p}_{s'} + m_{N_2}}{p_{s'}^2 - m_{N_1}^2 + i\epsilon}\Gamma_d$$

MS, M.Strikman, PLB 2006

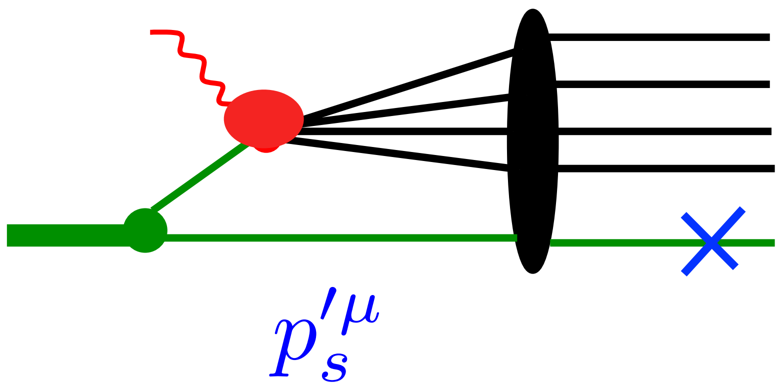
$$F_{2D}^{SI}(x,Q^2,\alpha_s,p_t) = S^{DWIA}(\alpha_s,p_t)\frac{m_N\nu}{pq} \times \left[(1+cos\delta)^2(\alpha+\frac{pq}{Q^2}\alpha_q)^2+\frac{1}{2}sin^2\delta\frac{p_t^2}{m_N^2}\right]F_{2N}^{eff}(\tilde{x},Q^2,\alpha,p_t),$$

$$F_{1D}^{SI}(x,Q^2,\alpha_s,p_t) = S^{DWIA}(\alpha_s,p_t)\left[F_{1N}^{eff}(\tilde{x},Q^2,\alpha,p_t)+\frac{p_t^2}{2pq}F_{2N}^{eff}(\tilde{x},Q^2,\alpha,p_t)\right].$$



Plane Wave Impulse Approximation

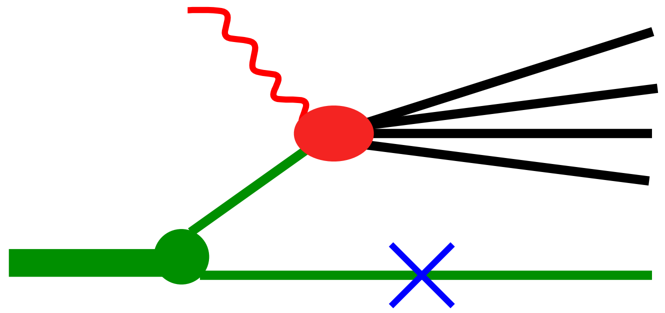
$$A^{\mu}_{IA} =< X|J^{\mu}_{em}(Q^2,\nu,p_s)\frac{\not{p}_d - \not{p}_s + m}{m_N^2 - (p_d - p_s)^2}\bar{u}(p_s)\Gamma_d =< X|J^{\mu}_{em}(Q^2,\nu,p_s)\frac{\not{p}_d - \not{p}_s + m}{m_N^2 - t}\bar{u}(p_s)\Gamma_d$$



Final State Interactions

$$A_{FSI} = \sum_{X'}\int \frac{d^4 p_{s'}}{i(2\pi)^4}\langle X, s|\hat{A}_{FSI}\cdot G(X')\cdot \hat{J}^{em}(Q^2,x)\frac{\not{p}_d - \not{p}_{s'} + m_N}{(p_d - p_{s'})^2 - m_N^2 + i\epsilon}\frac{\not{p}_{s'} + m_{N_2}}{p_{s'}^2 - m_{N_1}^2 + i\epsilon}\Gamma_d$$

MS, M.Strikman, PLB 2006



Plane Wave Impulse Approximation

$$A_0 = \langle X|J^{em}(Q^2,x)|n\rangle \bar{u}(p_d - p_s)\bar{u}(s)\frac{\Gamma_d}{m_n^2 - (p_d - p_s)^2}$$

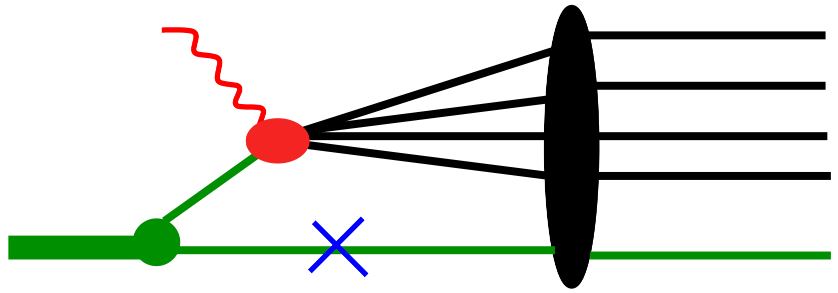
$t = (p_s - p_d) \to m_N^2$

Pole

$$A_0 = \langle X|J^{em}(Q^2,x)|n\rangle \bar{u}(p_d - p_s)\bar{u}(s)\frac{\Gamma_d}{|\epsilon_B|(M_d + m_n - m_p) + 2M_d T_s}$$

$$T_s^{pole} = -\frac{|\epsilon_B|}{2}(1 + \frac{m_n - m_p}{M_d}) \approx -\frac{|\epsilon_B|}{2}$$

Chew and Low, 1959, for n+d ⟹ *p+n+n*



Final State Interaction

Loop Theorem

$$\mathbf{A_{FSI}} = \int \frac{\mathbf{d^4 p_{s'}}}{\mathbf{i(2\pi)^4}} \frac{\langle \mathbf{X, s} | \mathbf{A_{FSI}} | \mathbf{X' s'} \rangle \langle \mathbf{X'} | \mathbf{J^{em}(Q^2, x)} | \mathbf{N} \rangle \bar{\mathbf{u}}(\mathbf{p_d} - \mathbf{p_{s'}}) \bar{\mathbf{u}}(\mathbf{p_{s'}})}{\mathbf{p^2_{s'}} - \mathbf{m^2_N} + \epsilon} \frac{\mathbf{\Gamma_d}}{(\mathbf{p_d} - \mathbf{p_{s'}})^2 - \mathbf{m^2_N}}$$

$$\int dp^0_{s'} \frac{1}{p^2_{s'} - m^2_N + \epsilon} = -\frac{2\pi}{2E_{s'}}$$

$$A_{FSI} = \int \frac{d^3 p_{s'}}{(2\pi)^3} \frac{\langle X, s | A_{FSI} | X' s' \rangle \langle X' | J^{em}(Q^2, x) | N \rangle \bar{u}(p_d - p_{s'}) \bar{u}(p_{s'})}{2E_{s'}} \frac{\Gamma_d}{m^2_N - (p_d - p_{s'})^2}$$

Introducing $k = p_s - p_{s'}$

$$A_{FSI} = \int \frac{d^3 k}{(2\pi)^3} \frac{\langle X, s | A_{FSI} | X' s' \rangle \langle X' | J^{em}(Q^2, x) | N \rangle \bar{u}(p_d - p_s - k) \bar{u}(p_s - k)}{2(m_N + T_s - k_0)} \frac{\Gamma_d}{-M^2_d + 2M_d(m + T_s - k0)}$$

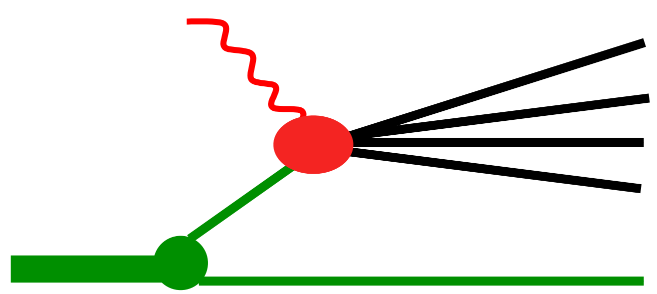
$$\int \frac{d^3k}{M_d|\epsilon_b| + 2M_dT_s - 2M_dk_0}$$

$$k_0 = \sqrt{m_N^2 + p_s^2} - \sqrt{m_N^2 + (p_s - k)^2} = \frac{\vec{p}_s\vec{k}}{m_N} - \frac{k^2}{2m_N}$$

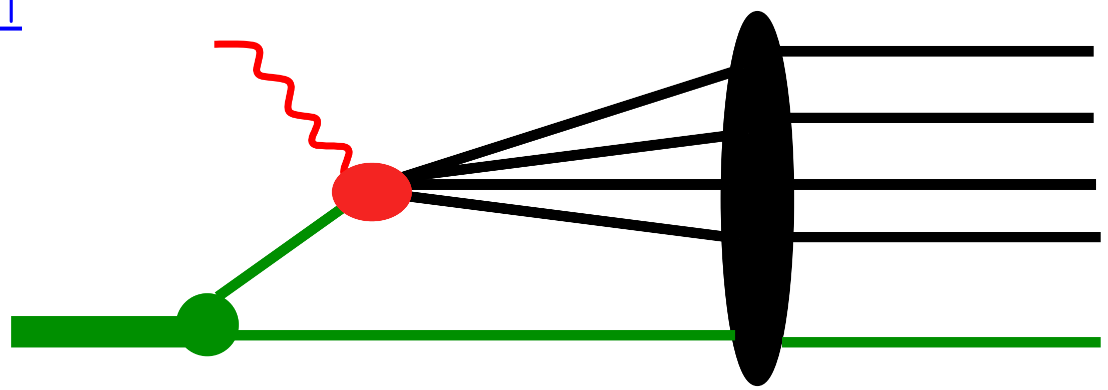
analytic continuation $T_s \to -\frac{|\epsilon_b|}{2}$

$$\frac{k^2dk}{2M_d(\frac{k^2}{2m_N})} \to \frac{1}{2}$$

at $T_s \to -\frac{|\epsilon_B|}{2}$



$\sim \frac{const}{T_s + \frac{|\epsilon_B|}{2}}$



$const_2$

Extracting "Free" Neutron Structure Function:

Extraction Procedure

$$F_{2D}^{SI}(x,Q^2,\alpha_s,p_t) = S^{DWIA}(\alpha_s,p_t)\frac{m_N\nu}{pq} \times \left[(1+cos\delta)^2(\alpha+\frac{pq}{Q^2}\alpha_q)^2+\frac{1}{2}sin^2\delta\frac{p_t^2}{m_N^2}\right]F_{2N}^{eff}(\tilde{x},Q^2,\alpha,p_t),$$

$$F_{1D}^{SI}(x,Q^2,\alpha_s,p_t) = S^{DWIA}(\alpha_s,p_t)\left[F_{1N}^{eff}(\tilde{x},Q^2,\alpha,p_t)+\frac{p_t^2}{2pq}F_{2N}^{eff}(\tilde{x},Q^2,\alpha,p_t)\right].$$

Analytic Property of Distorted Spectral Function

$$S^{DWIA} \sim \left|\frac{const_1}{t-m_n^2} - const_2\right|^2$$

Model Independent Model For FSI

$$S_{VN}^{DWIA} = E_s \frac{M_d}{2(M_d - E_s)} \left[\Psi_d^2(p_s) - \frac{1}{2}\sqrt{\frac{M_d - E_s}{E_s}} Im \int \frac{d^2 k_t}{(2\pi)^2} f(k_t) \left[\Psi_d^\dagger(p_s)\Psi_d(\tilde{p}_s) - i\Psi_d^\dagger(p_s)\Psi_d'(\tilde{p}_s) \right] + \left(\frac{1}{4}\sqrt{\frac{M_d - E_s}{E_s}} \right)^2 \left| \int \frac{d^2 k_t}{(2\pi)^2} f(k_t) [\psi_d(\tilde{p}_s) - i\psi'(\tilde{p}_s)] \right|^2 \right],$$

where $\tilde{p}_s \equiv (\tilde{p}_{sz}, \tilde{p}_{st}) = (p_{sz} - \Delta_{VN}, p_{st} - k_t)$

$$\Delta_{VN} = \frac{(M_d + \nu)}{\mathbf{q}}(E_s - m) + \frac{W^2 - W_0^2}{2\mathbf{q}}$$

where $W^2 = (q + M_d - p_s)^2$ and $W_0^2 = (q + m)^2$.

Extraction Factor

$$I(p_s,t) = \frac{1}{E_s}\frac{(m_N^2 - t)^2}{[Res(\Psi_d(T_{pole}))]^2} \cdot \frac{1}{\frac{m_N \nu}{pq}\left[(1+cos\delta)^2(\alpha + \frac{pq}{Q^2}\alpha_q)^2 + \frac{1}{2}sin^2\delta\frac{p_t^2}{m_N^2}\right]},$$

"Free" Nucleon Structure Function

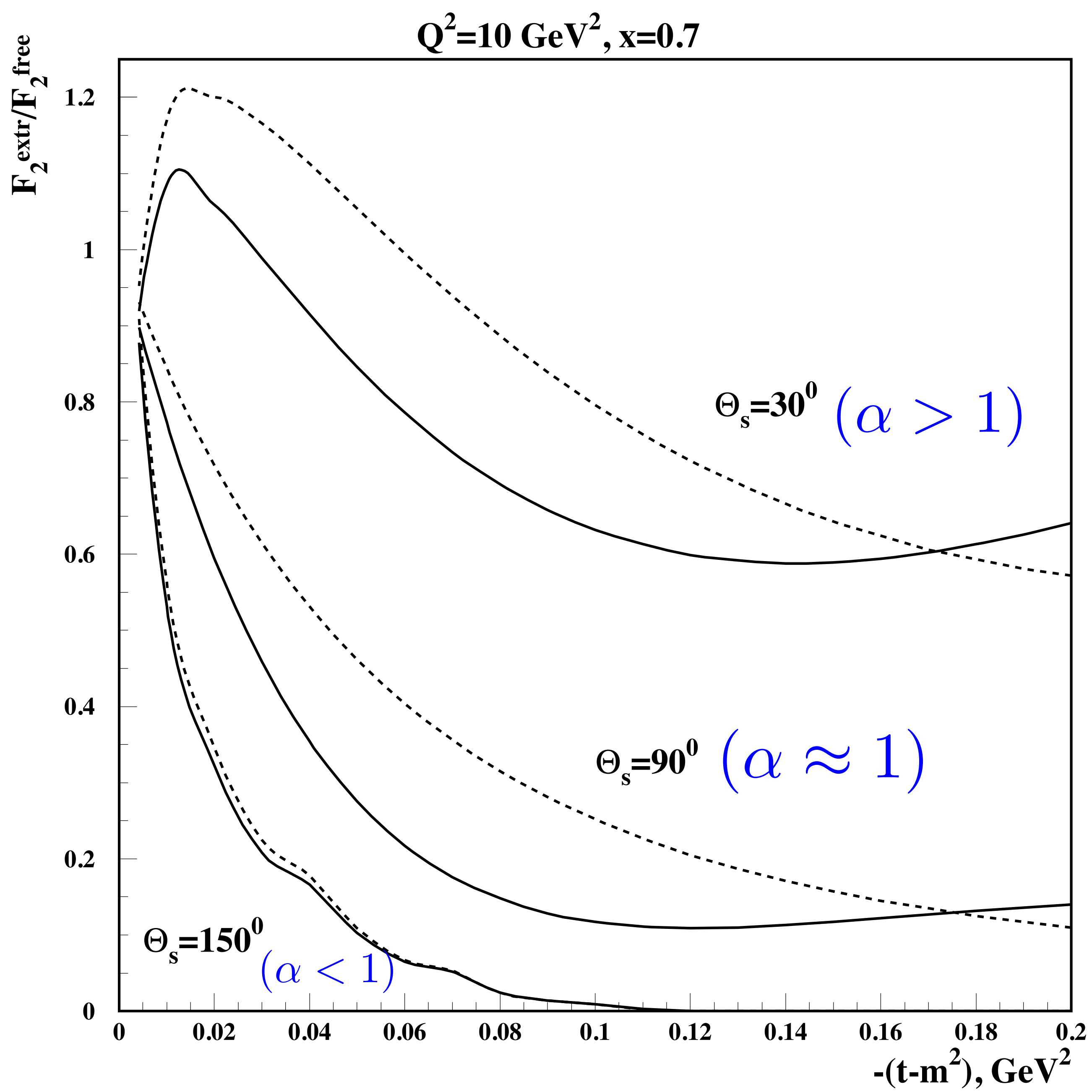
$$F_{2N}^{extr}(Q^2, x, t) = I(p_s, t) \cdot F_{2D}^{SI,EXP}(x, q^2, \alpha_s, p_t),$$

$$t \rightarrow m_n^2 \quad \text{or} \quad T_s \rightarrow -\frac{|\epsilon_B}{2}$$

F_{2N}^{extr} is a quadratic function of t

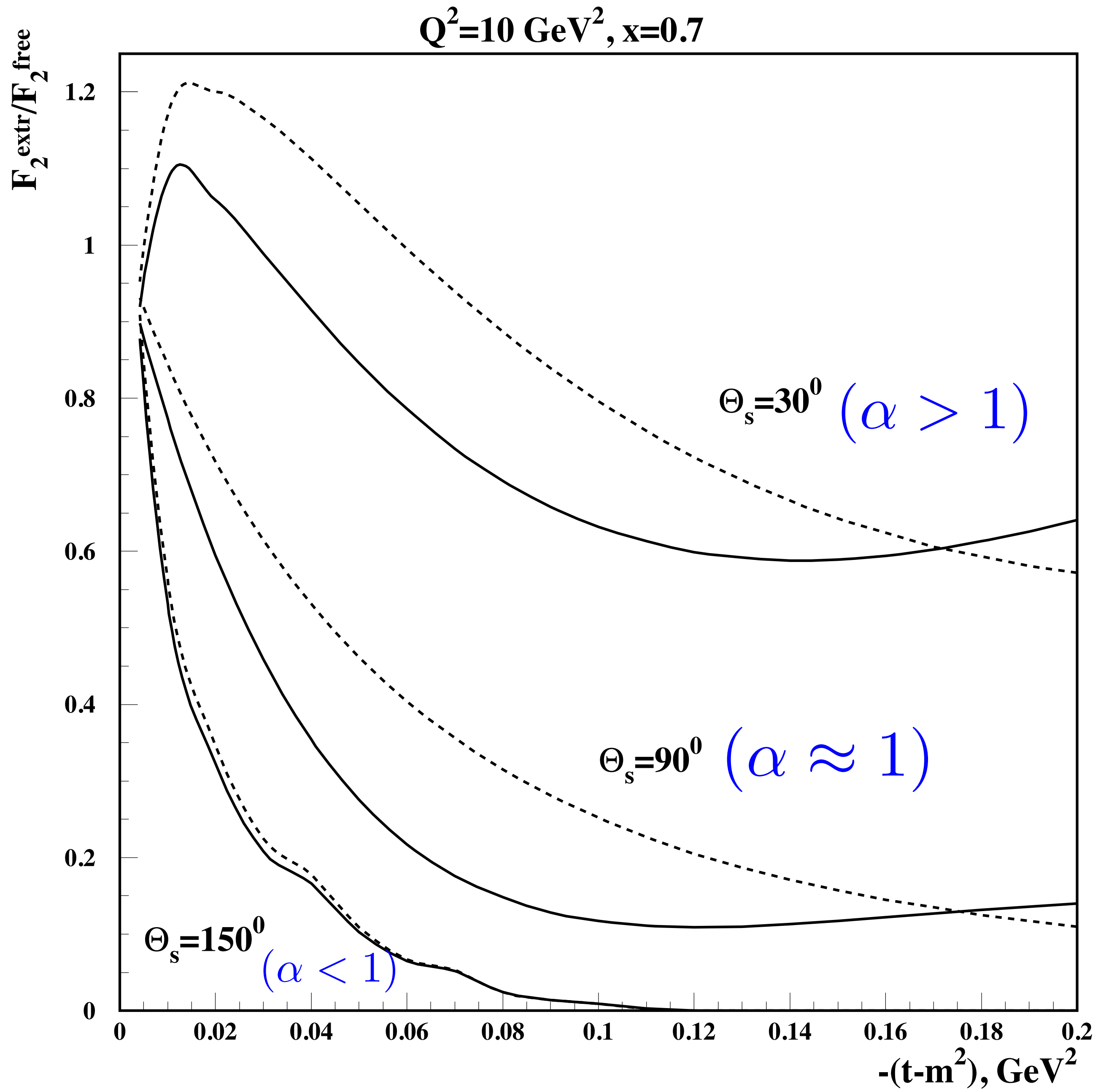
Second: Extracting "Free" Neutron Structure Function: *Estimations*

$$R = \frac{F_{2N}^{extr}(Q^2, x, t)}{F_{2N}^{free}(Q^2, x)}$$



$$W_N^2 \approx \alpha W_{N,0}^2 + (\alpha - 1)(Q^2 - m_N^2)$$

Second: Extracting "Free" Neutron Structure Function: *Estimations*

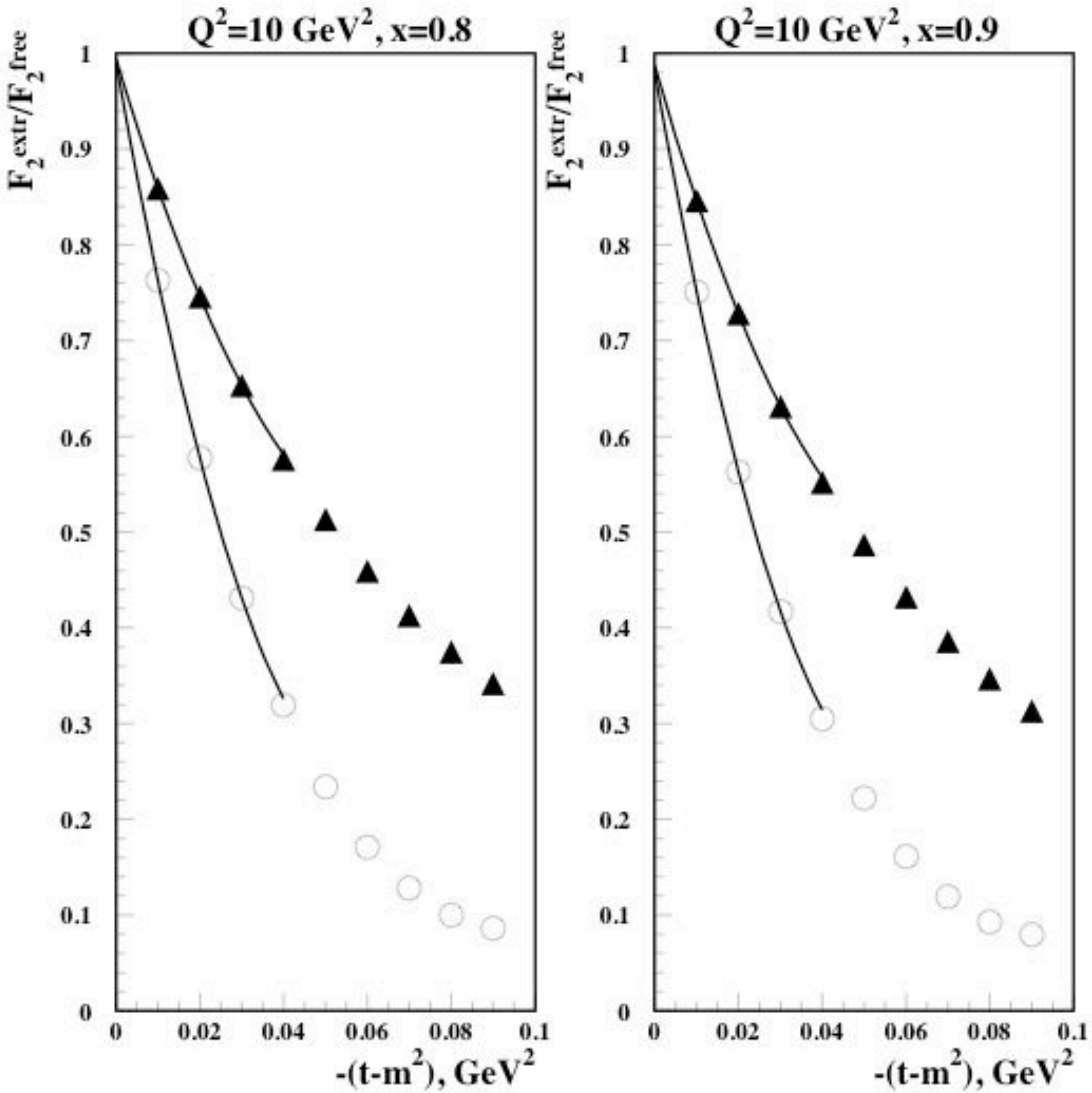


$$R = \frac{F_{2N}^{extr}(Q^2, x, t)}{F_{2N}^{free}(Q^2, x)}$$

$$W_N^2 \approx \alpha W_{N,0}^2 + (\alpha - 1)(Q^2 - m_N^2)$$

Is not ideally quadratic, due to higher mass poles

$\alpha_s = 1$



Extrapolation to the pole

$t = m_n^2$

	PWIA, DWIA	
x=0.8	0.9923, 0.9888	VN
	0.9983, 0.9946	LF
x=0.9	0.9868, 0.9828	VN
	1.0078, 1.0036	LF

Ps Points fitted

54MeV/c, 89MeV/c, 114MeV/c

Forth: Conclusion and Outlook

- Working to develop a theoretical framework for $\vec{e} + \vec{A} \to e' + N_S + X$ reactions

  at collider reference frame & kinematics

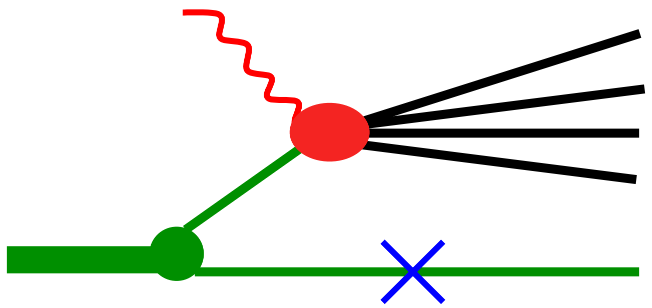
- Studies of DIS FSI for Tagged Reactions

- Pole extrapolation procedures for spectator N and 2N

Some additional slides

$\vec{e} + \vec{A} \to e' + N_S + X$ reactions

Plane Wave Impulse Approximation

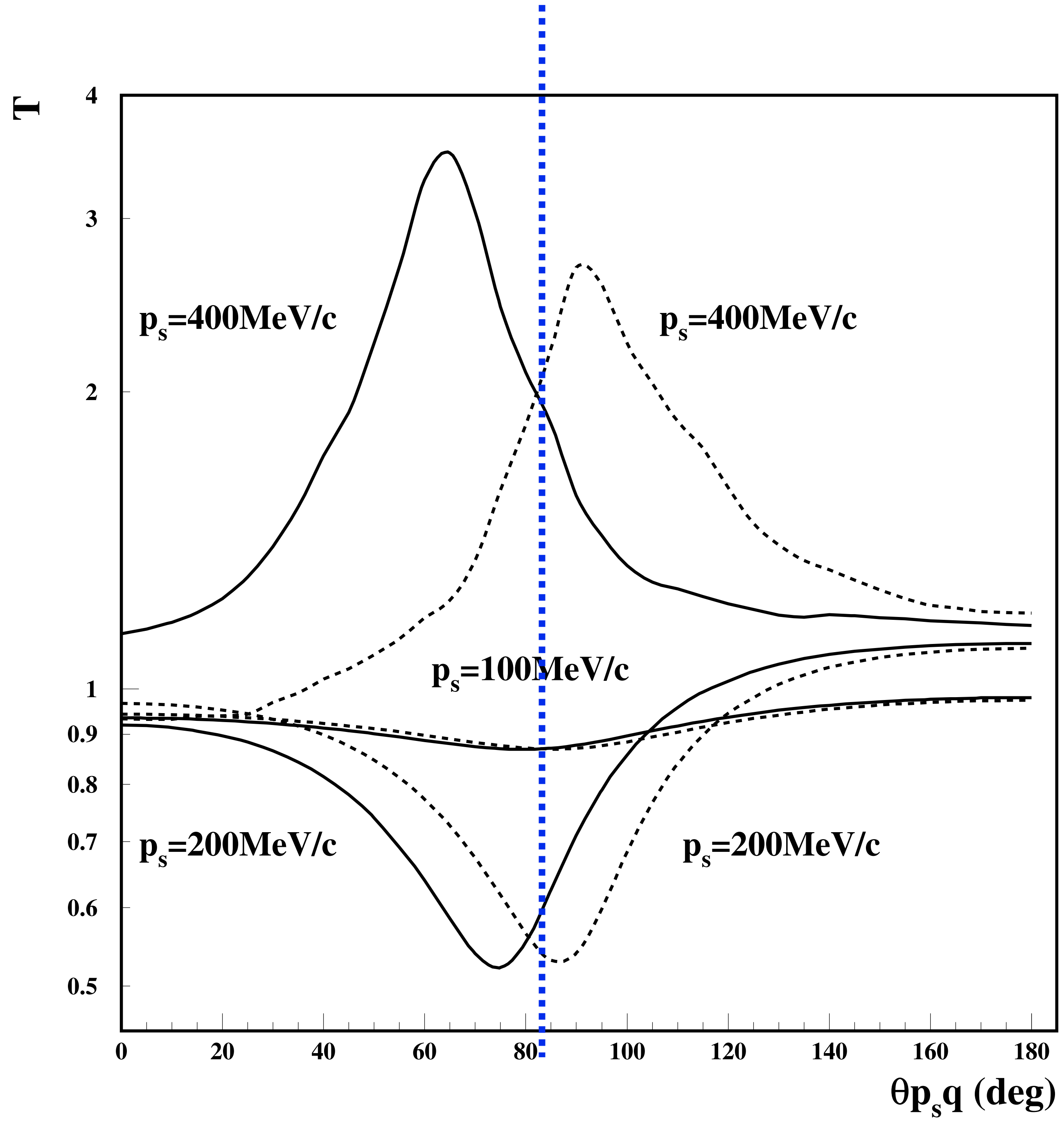


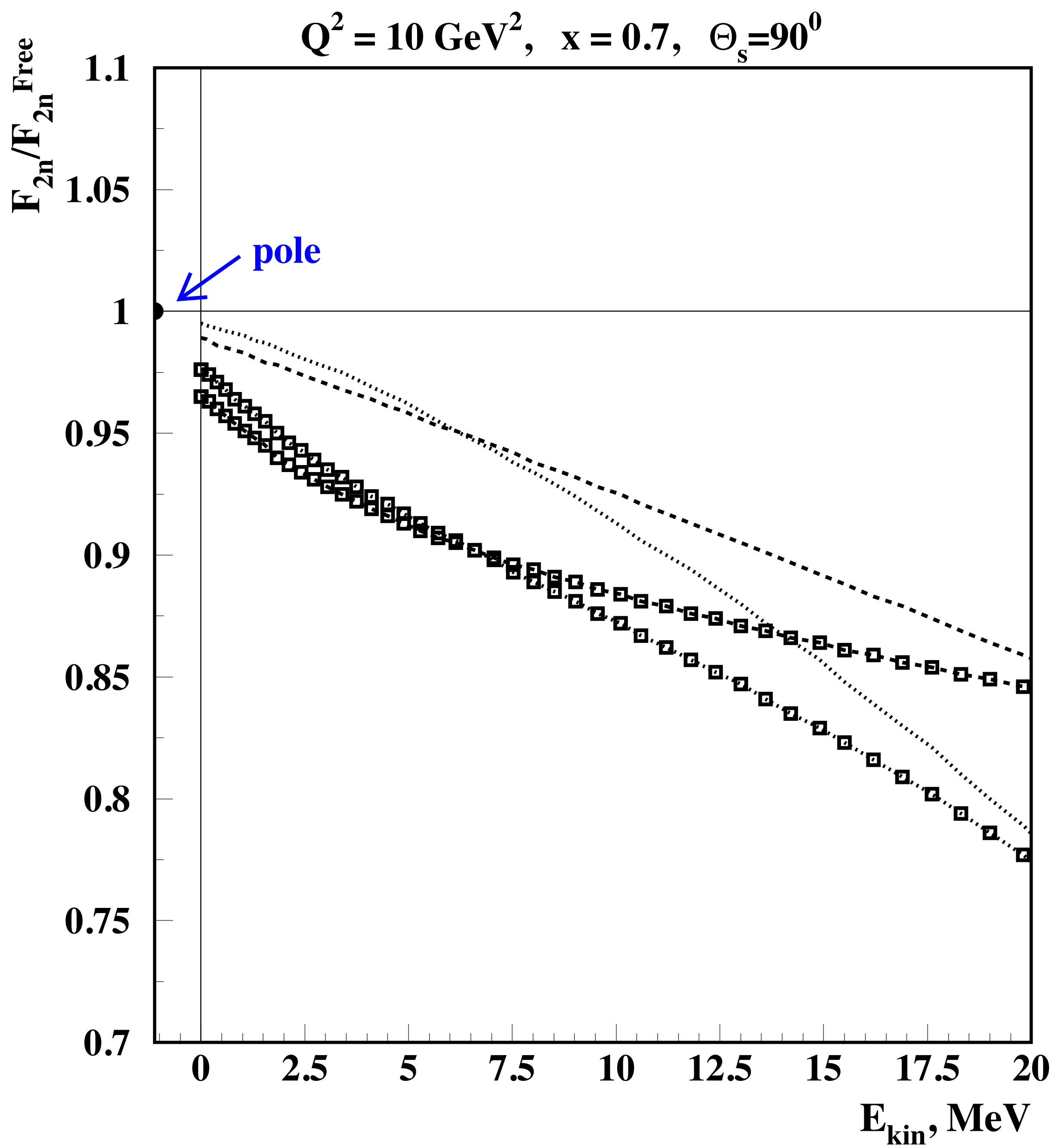
$$A^{\mu}_{IA} = < X|J^{\mu}_{em}(Q^2,\nu,p_s)\frac{\not p_d - \not p_s + m}{m_N^2 - (p_d - p_s)^2}\bar{u}(p_s)\Gamma_d = < X|J^{\mu}_{em}(Q^2,\nu,p_s)\frac{\not p_d - \not p_s + m}{m_N^2 - t}\bar{u}(p_s)\Gamma_d$$

Virtual Nucleon (VN) Approximation

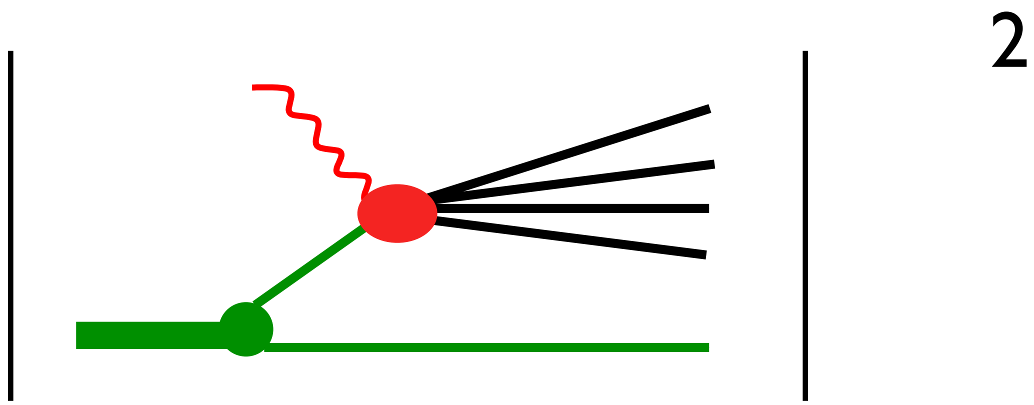
$$\Psi_d^{NR}(p_s) = \frac{\bar{u}(p_s)\bar{u}(p_d - p_s)\Gamma_d}{2\sqrt{(2\pi)^3 E_s}\ \ m_n^2 - (p_d - p_s)^2}$$

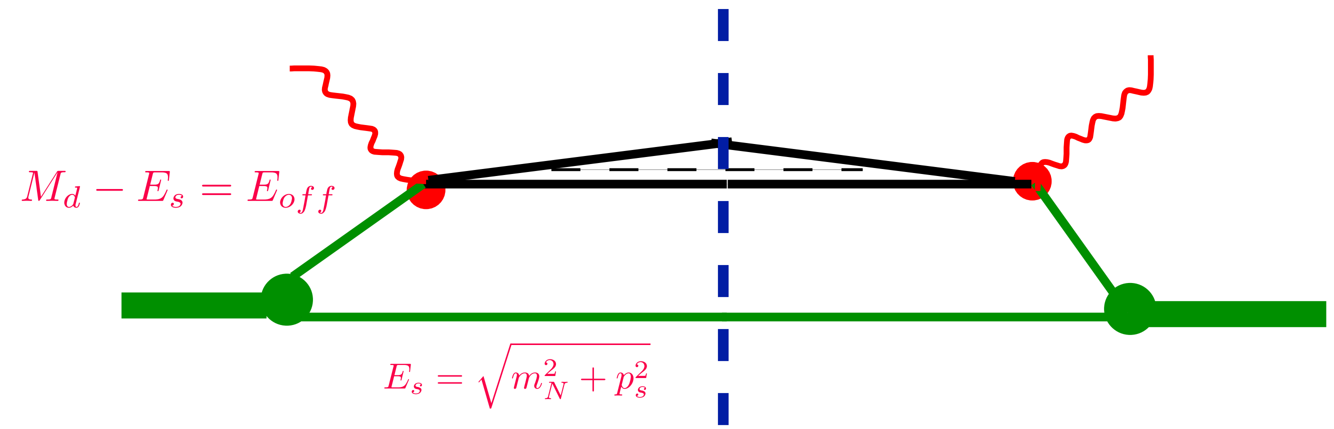
F.Gross
Relativistic Quantum Mechanics
& Field Theory 1993


T
4
3
2
1
0.9
0.8
0.7
0.6
0.5
p_s=400MeV/c
p_s=400MeV/c
p_s=100MeV/c
p_s=200MeV/c
p_s=200MeV/c
0
20
40
60
80
100
120
140
160
180
$\theta p_s q$ (deg)


$Q^2 = 10\ GeV^2$, x = 0.7, $\Theta_s = 90^0$
F_{2n}/F_{2n}^{Free}
pole
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0
2.5
5
7.5
10
12.5
15
17.5
20
E_{kin}, MeV

Virtual Nucleon (VN) Approximation





$$F^{D,(VN)}_{L,T,TL,TT} = S^{VN}(\alpha_s, p_t)\ F^{N}_{L,T,TL,TT}(x, Q^2, \alpha_s, p_t)$$

where S^{VN} is deuteron spectral function normalized as:

$$\int S^{VN}(\alpha_s, p_t)(2-\alpha_s)d\alpha_s d^2p_t = 1$$

$$\int S^{VN}(\alpha_s, p_t)(2-\alpha_s)^2 d\alpha_s d^2p_t < 1$$

Can be modeled

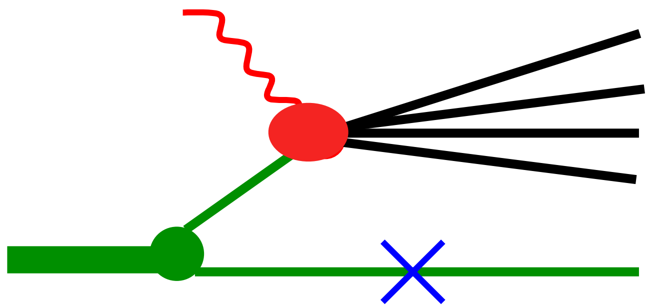
$$S^{VN}(\alpha_s, p_t) = E_s \frac{M_d}{2(M_d - E_s)} \Psi_d^2(p_s)$$

Virtual Nucleon (VN) Approximation

$$F_{2D}^{SI}(x,Q^2,\alpha_s,p_t) = S(\alpha_s,p_t)\frac{m_N\nu}{pq} \times \left[(1+cos\delta)^2(\alpha+\frac{pq}{Q^2}\alpha_q)^2+\frac{1}{2}sin^2\delta\frac{p_t^2}{m_N^2}\right]F_{2N}^{eff}(\tilde{x},Q^2,\alpha,p_t),$$

$$F_{1D}^{SI}(x,Q^2,\alpha_s,p_t) = S(\alpha_s,p_t)\left[F_{1N}^{eff}(\tilde{x},Q^2,\alpha,p_t)+\frac{p_t^2}{2pq}F_{2N}^{eff}(\tilde{x},Q^2,\alpha,p_t)\right].$$

Plane Wave Impulse Approximation

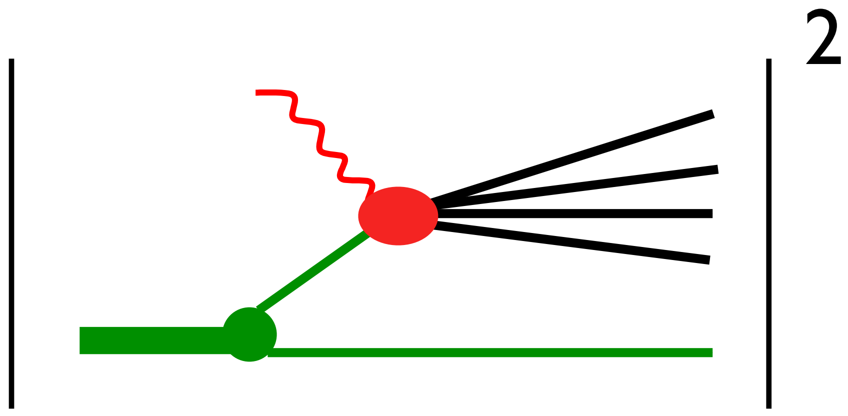


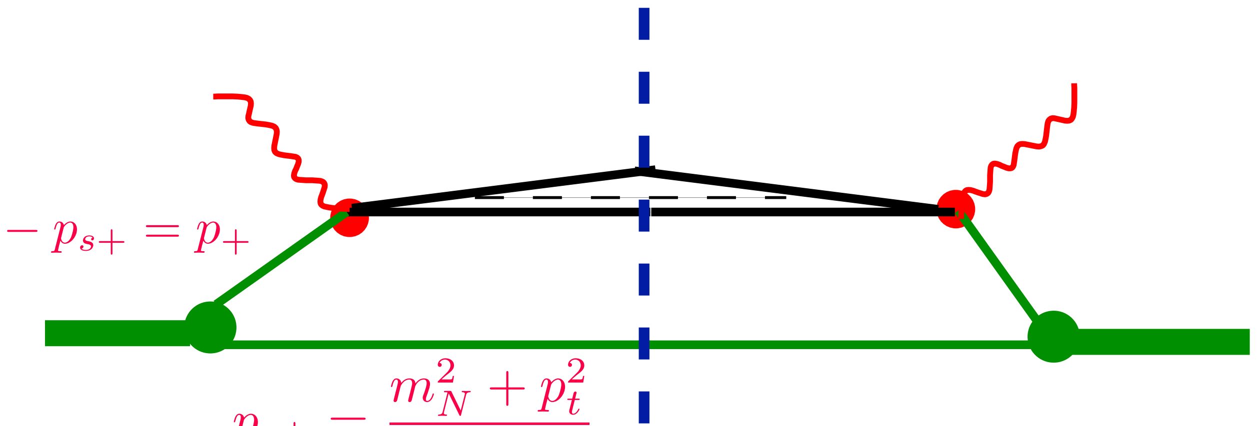
$$A^{\mu}_{IA} = < X|J^{\mu}_{em}(Q^2,\nu,p_s)\frac{\not{p}_d - \not{p}_s + m}{m_N^2 - (p_d - p_s)^2}\bar{u}(p_s)\Gamma_d = < X|J^{\mu}_{em}(Q^2,\nu,p_s)\frac{\not{p}_d - \not{p}_s + m}{m_N^2 - t}\bar{u}(p_s)\Gamma_d$$

Light Front (LF) Approximation

$$\Psi_d^{LC}(\alpha_s, p_{st}) = \frac{\Gamma_d}{2\sqrt{(2\pi)^3}\left(\frac{4(m^2+p_{st}^2)}{\alpha_s(2-\alpha_s)} - M_d^2\right)}$$

L. Frankfurt, M. Strikman
Phys.Rept. 1981





II) Taking the pole at the on-shell $p_+ = (E_s + p_{sz})$ value in the Light Cone Reference Frame

Light Front (LF) Approximation

$$\mathbf{F^{D,LC}_{L,T,TL,TT} = \frac{S^{LC}(\alpha_s, p_t)}{(2-\alpha_s)^2} F^N_{L,T,TL,TT}(x, Q^2, \alpha_s, p_t)}$$

where S^{LC} is the Ligh Cone deuteron spectral function normalized as:

Baryonic Number Conservation

$$\mathbf{\int \frac{S^{LC}(\alpha_s, p_t)}{(2-\alpha_s)} \frac{d\alpha_s}{\alpha_s} d^2p_t = 1}$$

Momentum Sum Rule

$$\int \frac{S^{LC}(\alpha_s, p_t)}{(2-\alpha_s)} d\alpha_s d^2p_t = 1$$

Can be modeled (Frankfurt and Strikman (1976))

$$\mathbf{S^{LC}(\alpha_s, p_t) = E_k \Psi^2_{LC,d}(p_k)}$$

where

$$\mathbf{\alpha_s = \frac{E_k + k_z}{E_k}}$$

Integrated Semi-Inclusive Cross section

$$\frac{\mathbf{d\sigma}}{\mathbf{dxdQ^2d^3p_s/E_s}} = \frac{\mathbf{4\pi\alpha_e^2 m}}{\mathbf{xQ^4}}(\mathbf{1 - y - \frac{x^2y^2m_N^2}{Q^2}})\left[\mathbf{F_{2D}^{SI} + 2tan^2(\frac{\theta}{2})\frac{\nu}{m_N}F_{1D}^{SI}}\right]$$

$$F_{2D}^{SI}(x, Q^2, \alpha_s, p_t) = A \cdot S(\alpha_s, p_t)\frac{m_N q_0}{pq}\left[(1 + cos\delta)^2(\alpha + \frac{pq}{Q^2}\alpha_q)^2 + \frac{1}{2}sin^2\delta\frac{p_t^2}{m_N^2}\right]F_{2N}^{eff}(\tilde{x}, Q^2, \alpha, p_t)$$

$$F_{1D}^{SI}(x, Q^2, \alpha_s, p_t) = A \cdot S(\alpha_s, p_t)\left[F_{1N}^{eff}(\tilde{x}, Q^2, \alpha, p_t) + \frac{p_t^2}{2pq}F_{2N}^{eff}(\tilde{x}, Q^2, \alpha, p_t)\right]$$

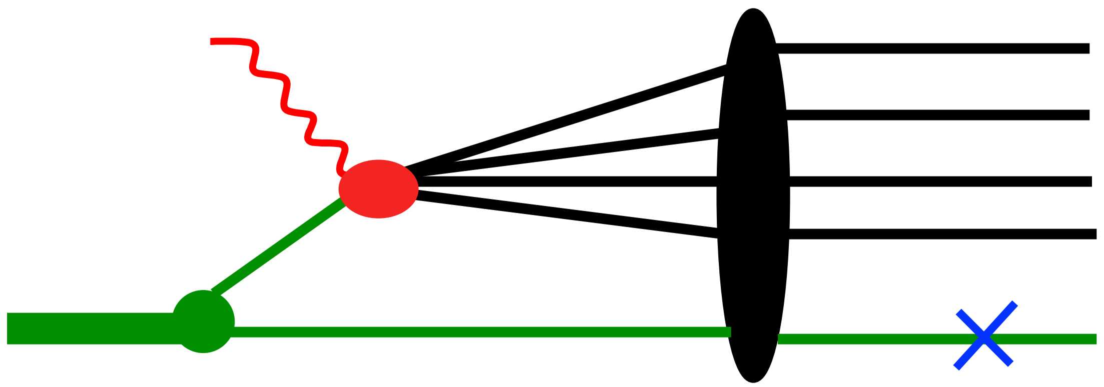
$A^{VN} = 1$ and $A^{LC} = \alpha^{-2}$

in the Bjorken limit $Q^2 \to \infty; \quad q_0 \to \infty; \quad x \ fixed$

$$F_{2D}^{SI}(x, Q^2, \alpha_s, p_t) = A \cdot S(\alpha_s, p_t) \cdot \alpha \cdot F_{2N}^{eff}(\tilde{x}, Q^2, \alpha, p_t)$$

$$F_{1D}^{SI}(x, Q^2, \alpha_s, p_t) = A \cdot S(\alpha_s, p_t) \cdot F_{1N}^{eff}(\tilde{x}, Q^2, \alpha, p_t)$$

Final State Interaction



$$A_{FSI} = \sum_{X'} \int \frac{d^4 p_{s'}}{i(2\pi)^4} \langle X, s | \hat{A}_{FSI} \cdot G(X') \cdot \hat{J}^{em}(Q^2, x) \frac{\not{p}_d - \not{p}_{s'} + m_N}{(p_d - p_{s'})^2 - m_N^2 + i\epsilon} \frac{\not{p}_{s'} + m_{N_2}}{p_{s'}^2 - m_{N_1}^2 + i\epsilon} \Gamma_d$$

Model Independent Model For FSI

$$S_{VN}^{DWIA} = E_s \frac{M_d}{2(M_d - E_s)} \left[\Psi_d^2(p_s) - \frac{1}{2}\sqrt{\frac{M_d - E_s}{E_s}} Im \int \frac{d^2k_t}{(2\pi)^2} f(k_t) \left[\Psi_d^\dagger(p_s)\Psi_d(\tilde{p}_s) - i\Psi_d^\dagger(p_s)\Psi_d'(\tilde{p}_s) \right] + \left(\frac{1}{4}\sqrt{\frac{M_d - E_s}{E_s}} \right)^2 \left| \int \frac{d^2k_t}{(2\pi)^2} f(k_t)[\psi_d(\tilde{p}_s) - i\psi'(\tilde{p}_s)] \right|^2 \right],$$

where $\tilde{p}_s \equiv (\tilde{p}_{sz}, \tilde{p}_{st}) = (p_{sz} - \Delta_{VN}, p_{st} - k_t)$

$$\Delta_{VN} = \frac{(M_d+\nu)}{\mathbf{q}}(E_s - m) + \frac{W^2 - W_0^2}{2\mathbf{q}}$$

where $W^2 = (q + M_d - p_s)^2$ and $W_0^2 = (q + m)^2$.