

# Light Front Structure of Deuterium From NN Interactions

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## Ch. Weiss assignments (=outline):

- formulate the NN interaction on the light front
- solve the Weinberg equation for the deuteron
- include polarization effects
- prescription from FS 81 review constructs LF wave function from NR wf:
- how good is this approx at recoil momenta few hundred MeV?
- can we get the LF wf from NN potentials?
- polarization/ spin

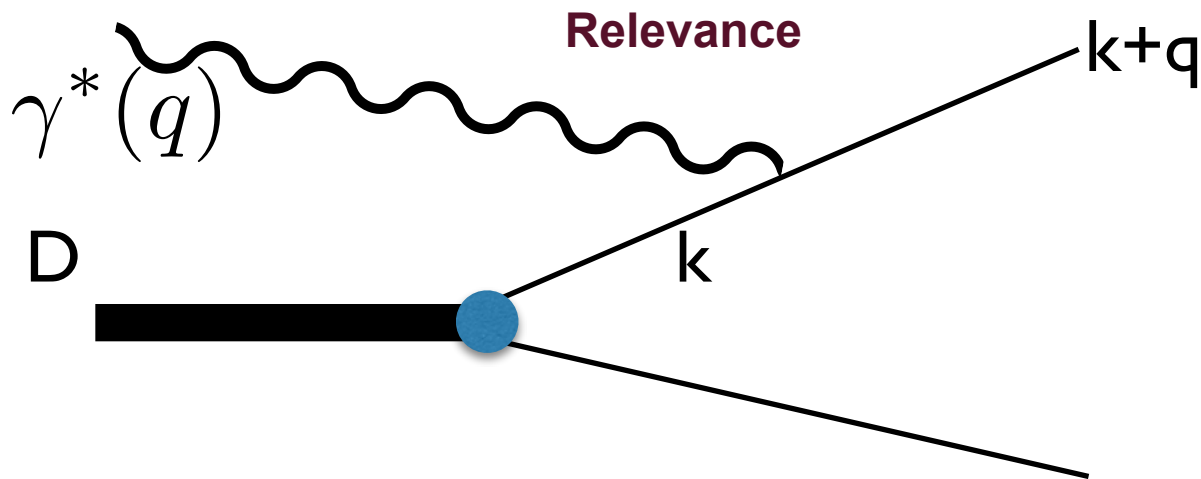
J R Cooke nucl-th/0112029 , Cooke & Miller PRC66, 034002  
Miller & Machleidt PRC60, 035202 Miller, Prog. Nuc. Part. Phys. 45, 83  
Tiburzi & Miller PRC81,035201

# Light front quantization, Infinite momentum frame

“Time”,  $x^+ = x^0 + x^3$ , “Evolve”,  $p^- = p^0 - p^3$

“Space”,  $x^- = x^0 - x^3$ , “Momentum”,  $p^+$  (Bjorken)

Transverse position, momentum  $\mathbf{b}, \mathbf{p}$   
**transverse boosts in kinematic subgroup**



$$p^- = \frac{\mathbf{p}^2 + m^2}{p^+}$$

If Photon energy  $\gg m$  and struck nucleon  $\approx$  on mass – shell :

$$(k + q)^2 = m^2 \rightarrow k^+ q^- \approx Q^2, \quad \frac{Q^2}{\nu^2} \ll 1$$

Integrate over  $k^-$

$$d\sigma \sim \Psi_D^2(\mathbf{k}, \frac{k^+}{P_D^+} \equiv \alpha)$$

# Light front quantization, Infinite momentum frame

$P^-$  is LF Hamiltonian, get from Lagrangian.

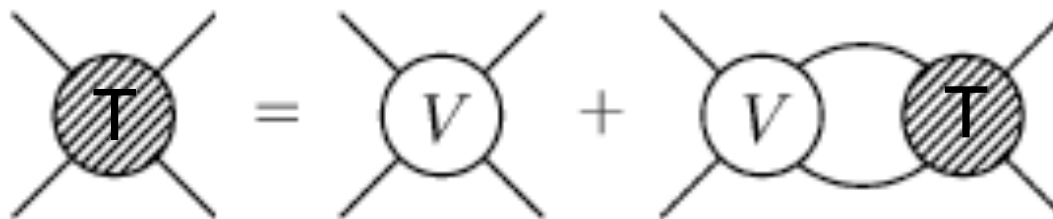
LF Schroedinger eq.  $P^- |\Psi_D\rangle = M_D |\Psi_D\rangle$  Rest frame

One boson exchange

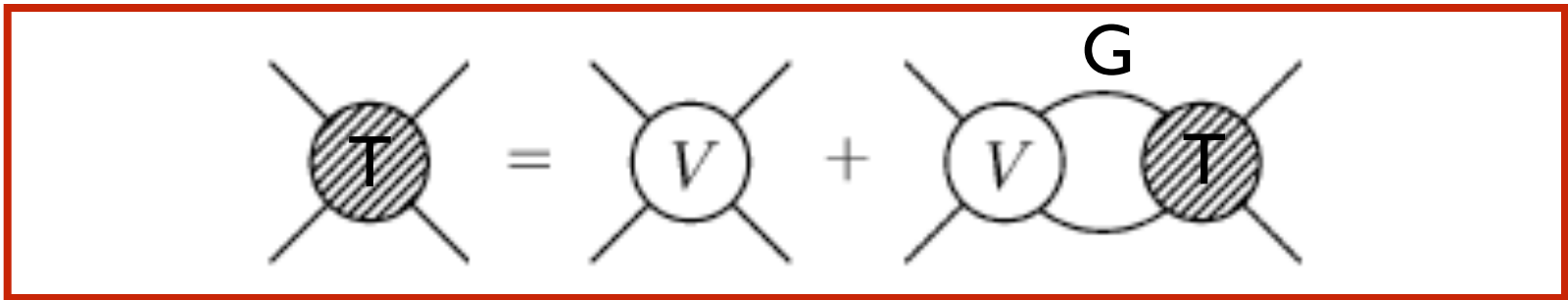
$$\begin{array}{c} 3 \\ | \\ \text{---} \\ | \\ 1 \end{array}
 \begin{array}{c} 4 \\ | \\ \text{---} \\ | \\ 2 \end{array}
 = \frac{g^2}{k^+ (k_1^- + k_2^- - k_3^- - k_2^- - k^-)} = \frac{g^2}{k^2 - \mu^2} \text{ Yukawa}$$

*(Note: The diagram above shows two vertical lines representing particles. The left line has labels 3 at the top and 1 at the bottom. The right line has labels 4 at the top and 2 at the bottom. A dashed line labeled 'k' connects the two lines. A horizontal red line is drawn across the middle of the diagram, labeled 'V=' on the left.)*

Solve



Weinberg equation



$$G \sim \frac{1}{\alpha(1-\alpha)} \frac{d^2 p_{\perp} d\alpha}{P^2 - \frac{p_{\perp}^2 + m^2}{\alpha(1-\alpha)}}$$

define  $p_z : \frac{p_{\perp}^2 + m^2}{4\alpha(1-\alpha)} = p_{\perp}^2 + p_z^2 + m^2 = \vec{p}^2 + m^2$

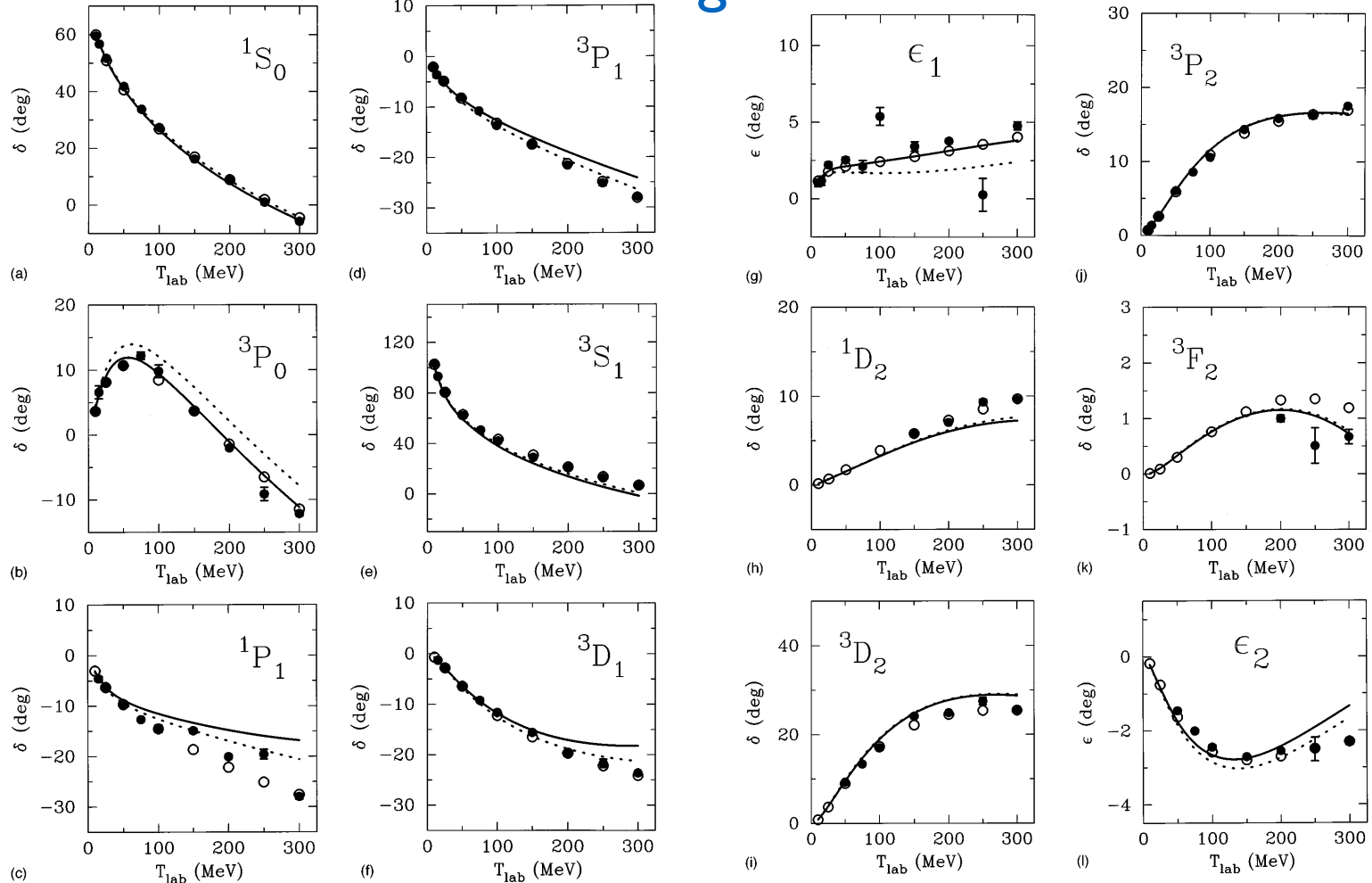
$$G \sim \frac{m^2}{\sqrt{\vec{p}^2 + m^2}} \frac{d^3 p}{p_i^2 - p^2} \text{ Usual propagator with extra factor}$$

Extra factor is close to unity for D wave function

# Miller & Machleidt PRC 60,035202

$\pi, \eta, \rho, \omega, a_0, \sigma$  exchange with extra factor in G

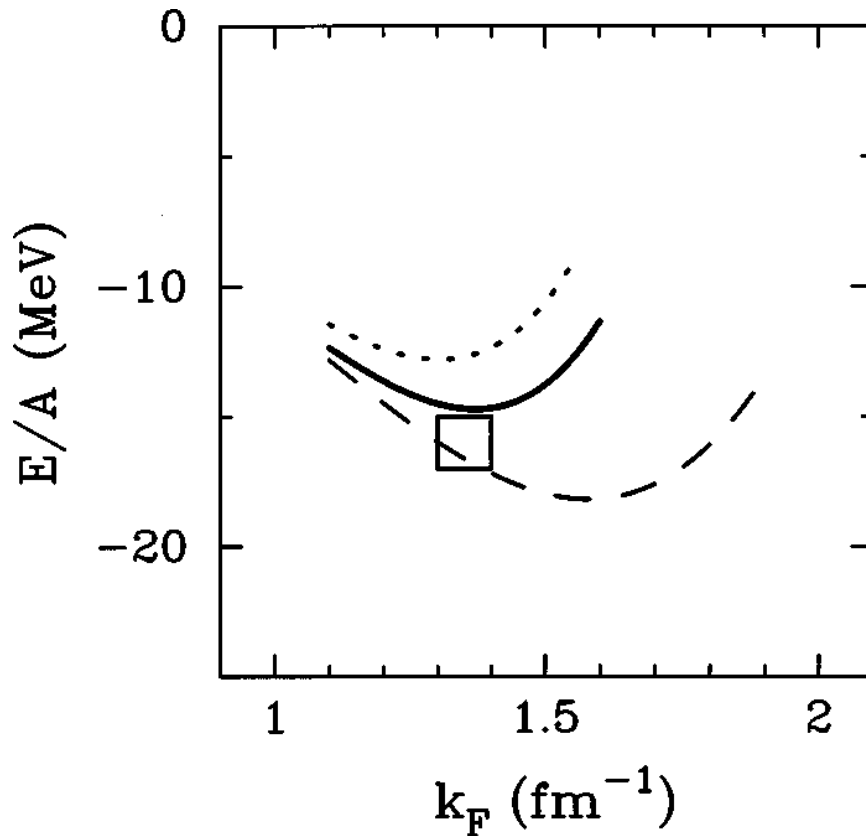
## NN scattering



$$B_D = 2.245 \text{ MeV}$$

# Miller & Machleidt PRC 60,035202

## Nuclear Matter Saturation

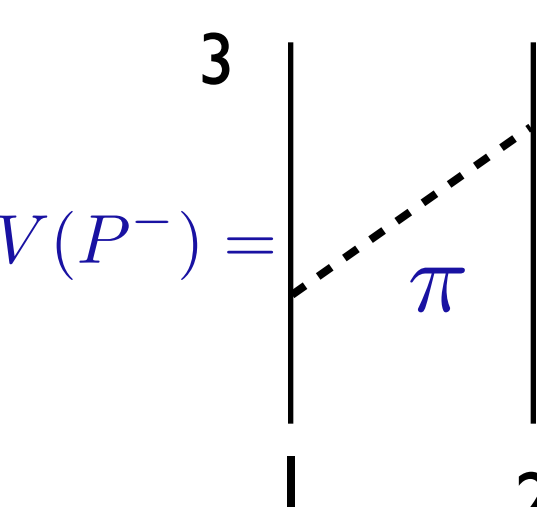


Solid -our light front  
Dashed- ET formalism

# Jason Cooke nucl-th/0112029, Cooke & Miller PRC66, 034002

Solves LF Schroedinger eq (LFSSE)

$$[P_0^- + V(P^-)] |\Psi_D\rangle = P^- |\Psi_D\rangle \quad P^- = 2m - B \quad \text{rest frame}$$



Manifest rotational invar. broken

$$= g^2 \frac{1}{P^- - k_3^- - k_2^- - k_\pi^-}$$

Different meson propagator than Machleidt Miller

2 Solve LSSE using transformation from  $\alpha$  to  $k_z$ :

$$\alpha = \frac{k^+}{P^+} = \frac{k^+}{2M - B} = \frac{1}{2} \frac{\sqrt{\vec{k}^2 + m^2} + k_z}{\sqrt{\vec{k}^2 + m^2}}$$

Solve w. rot. inv. in  $\perp$  plane (polar coords)

Computed B depends on magnetic quantum number!

# Cooke nucl-th/0112029, Cooke & Miller PRC66, 034002

## Dynamics

- Chiral Lagrangian with  $\pi, \eta, \rho, \omega, \delta, \sigma$
- Two meson exchange!
- Explicit  $P^-$  dependence

$$K = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \text{[Diagram 4]} + \text{[Diagram 5]} + \dots$$

FIG. 1. The first several terms of the full kernel for the Bethe-Salpeter equation of the nuclear model with chiral symmetry.



# Cooke nucl-th/0112029, Cooke & Miller PRC66, 034002

## Two Meson Dynamics

### Instantaneous terms

$$(a) V_{\text{TME:EM}} = \left( \begin{array}{c} \text{Diagram 1} \\ q_f = k_{1m} - k_{1f} \\ q_i = k_{1m} - k_{1i} \end{array} + \begin{array}{c} \text{Diagram 2} \\ q_f = k_{2m} - k_{2f} \\ q_i = k_{2m} - k_{2i} \end{array} \right)$$

$$(b) V_{\text{TME:SB}} = \left( \begin{array}{c} \text{Diagram 1} \\ q_f = k_{1f} - k_{1m} \\ q_i = k_{1m} - k_{1i} \end{array} + \begin{array}{c} \text{Diagram 2} \\ q_f = k_{2f} - k_{2m} \\ q_i = k_{2m} - k_{2i} \end{array} \right)$$

$$(c) V_{\text{TME:SBII}} = \left( \begin{array}{c} \text{Diagram 1} \\ q_f = k_{1f} - k_{1m} \\ q_i = k_{1m} - k_{1i} \end{array} + \begin{array}{c} \text{Diagram 2} \\ q_f = k_{2f} - k_{2m} \\ q_i = k_{2m} - k_{2i} \end{array} \right) + \left( \begin{array}{c} \text{Diagram 3} \\ q_f = k_{1f} - k_{1m} \\ q_i = k_{1m} - k_{1i} \end{array} + \begin{array}{c} \text{Diagram 4} \\ q_f = k_{2f} - k_{2m} \\ q_i = k_{2m} - k_{2i} \end{array} \right)$$

$$(d) V_{\text{TME:SBII}} = \left( \begin{array}{c} \text{Diagram 1} \\ q_f = k_{1f} - k_{1m} \\ q_i = k_{1m} - k_{1i} \end{array} + \begin{array}{c} \text{Diagram 2} \\ q_f = k_{2f} - k_{2m} \\ q_i = k_{2m} - k_{2i} \end{array} \right)$$

### Chiral contact terms

$$(a) V_{\text{TME:C}} = \left( \begin{array}{c} \text{Diagram 1} \\ q_f = k_{2f} - k_{2m} \\ q_i = k_{2i} - k_{2m} \end{array} + \begin{array}{c} \text{Diagram 2} \\ q_f = k_{1i} - k_{1m} \\ q_i = k_{1i} - k_{1m} \end{array} \right)$$

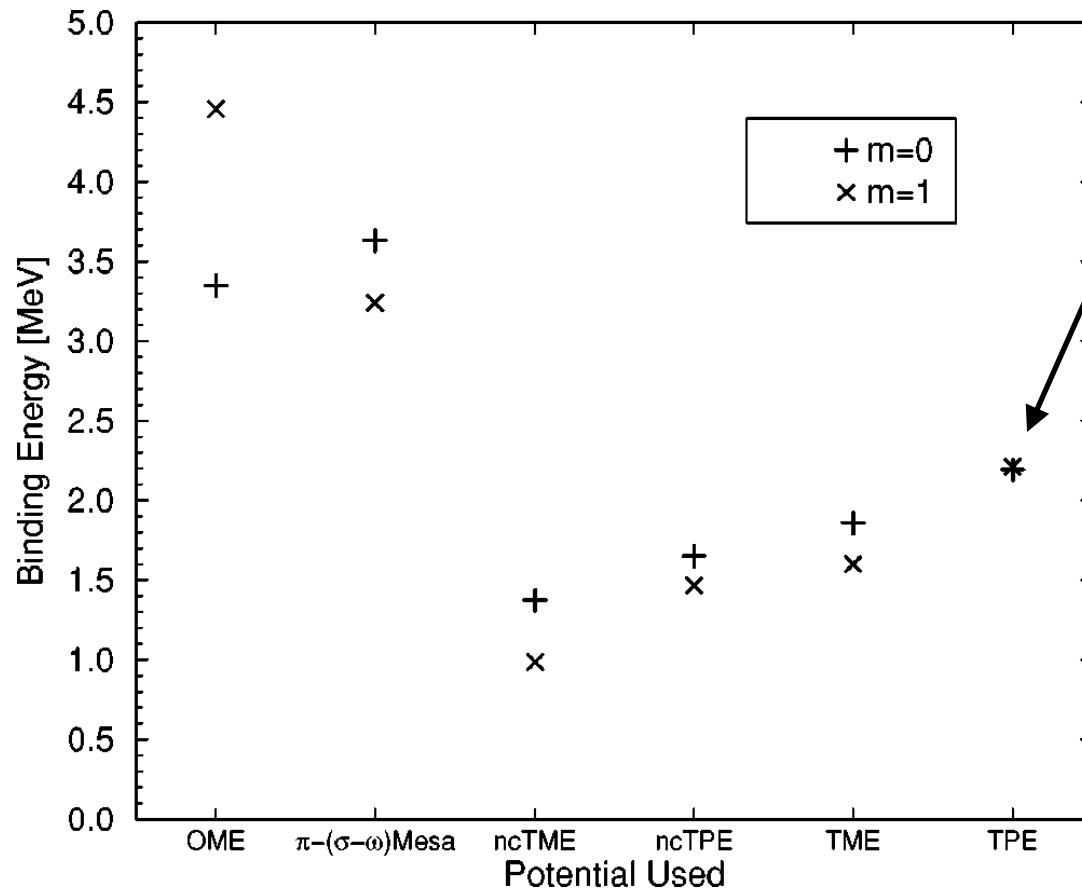
$$(b) V_{\text{TME:SBC}} = \left( \begin{array}{c} \text{Diagram 1} \\ q_f = k_{2m} - k_{2f} \\ q_i = k_{2i} - k_{2m} \end{array} + \begin{array}{c} \text{Diagram 2} \\ q_f = k_{1m} - k_{1f} \\ q_i = k_{1i} - k_{1m} \end{array} \right) + \left( \begin{array}{c} \text{Diagram 3} \\ q_f = k_{1f} - k_{1m} \\ q_i = k_{1m} - k_{1i} \end{array} + \begin{array}{c} \text{Diagram 4} \\ q_f = k_{2f} - k_{2m} \\ q_i = k_{2m} - k_{2i} \end{array} \right)$$

$$(c) V_{\text{TME:SBIC}} = \left( \begin{array}{c} \text{Diagram 1} \\ q_f = k_{1f} - k_{1m} \\ q_i = k_{1m} - k_{1i} \end{array} + \begin{array}{c} \text{Diagram 2} \\ q_f = k_{2f} - k_{2m} \\ q_i = k_{2m} - k_{2i} \end{array} \right) + \left( \begin{array}{c} \text{Diagram 3} \\ q_f = k_{2m} - k_{2f} \\ q_i = k_{2i} - k_{2m} \end{array} + \begin{array}{c} \text{Diagram 4} \\ q_f = k_{1m} - k_{1f} \\ q_i = k_{1i} - k_{1m} \end{array} \right)$$

$$(d) V_{\text{TME:SBCC}} = \left( \begin{array}{c} \text{Diagram 1} \\ q_f \\ q_i \end{array} + \begin{array}{c} \text{Diagram 2} \\ q_f \\ q_i \end{array} \right)$$

# Restoring Rot. Inv.

PRC66, 034002



Uses only  
2 pion exch  
in 2BE

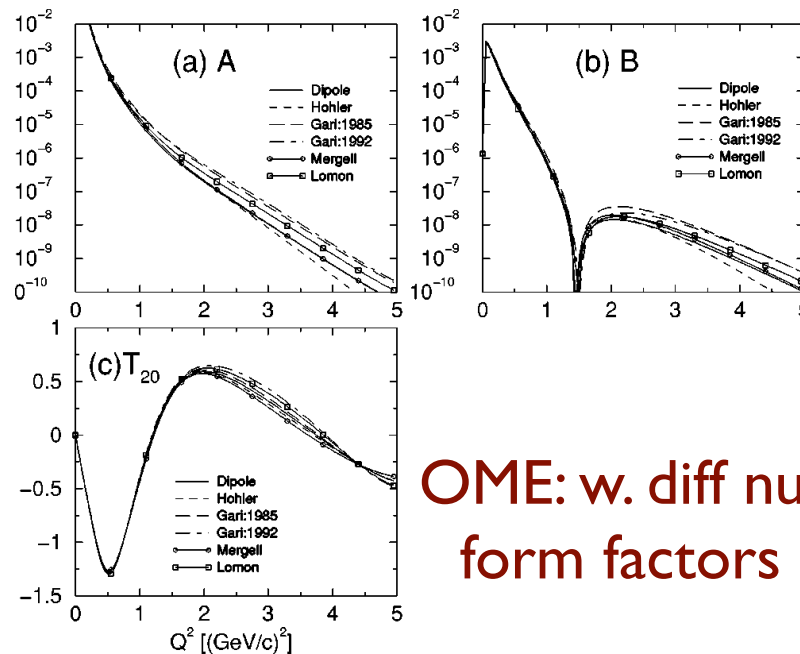
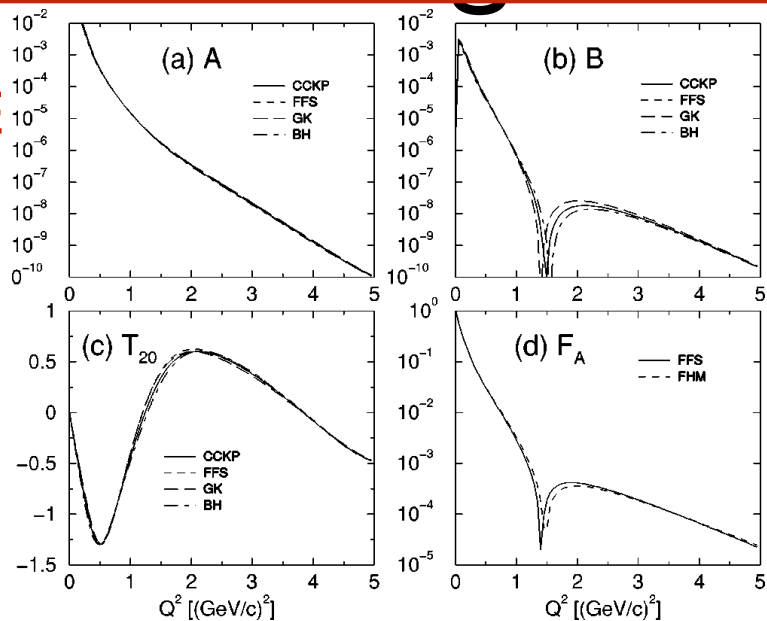
FIG. 9. The values of the binding energy for the  $m=0$  and  $m=1$  states for different nucleon-nucleon light-front potentials. The  $\sigma$

# Restoring RI in form factors

- Rotational invariance gives angular condition FS
- Angular condition is upheld better when Deut is computed using only one meson exchange OME potentials than two meson exchange TME
- However, form factors do not depend much on choice of bad currents

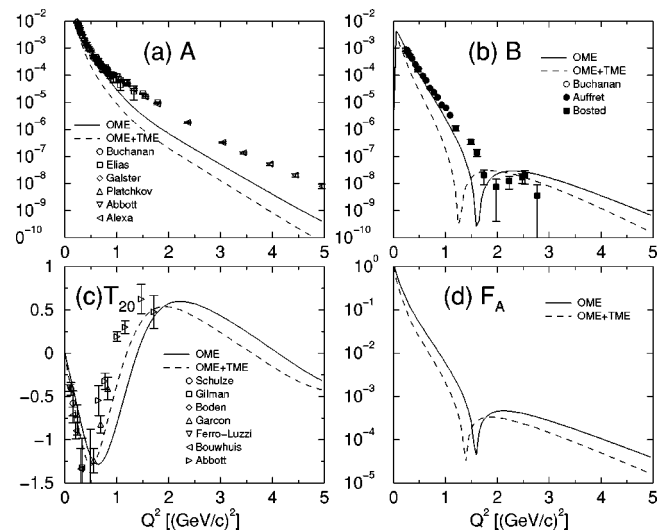
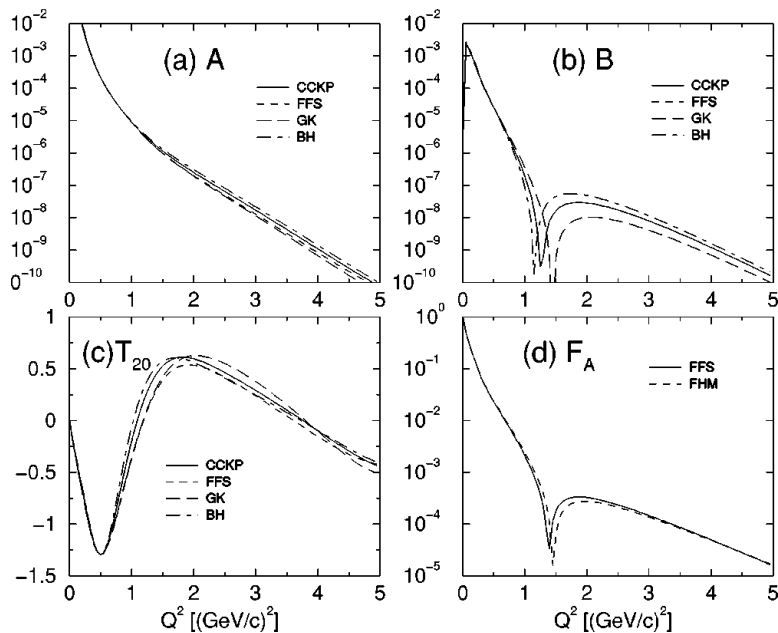
# Restoring? RI in form factors

OME



OME: w. diff nucleon form factors

TME



# The real problem- Bethe Salpeter Eq. (BSE)

$$T = \text{diagram} = \text{diagram } K + \text{diagram } K \overset{G}{\circlearrowleft} K$$

K is sum of irred. diagrams

$G$  is 4 dimensional -product of two Feynman propagators.

Intermediate state 4 dimensional integral  $d^4k = dk^0 d^3k = dk^+ dk^- d^2k_{\perp}$

Reduce to 3 dimensions:

ET: integrate over  $k^0$ . Ignore  $k^0$  except in  $G$ . Sets relative time to 0.

LF: Integrate over  $k^-$ . Ignore  $k^-$  except in  $G$ . Sets relative  $\tau = 0$

3 dimensional version of  $G$  is  $g_{ET}$  (Blankenbecler Sugar) or  $g_{LF}$  (Weinberg)

Puts particles on mass shell

$$T = \text{diagram} = \text{diagram } V + \text{diagram } V \overset{g}{\circlearrowleft} V$$

Either  $g: V = K + K(G - g)V$ . Same on-shell  $T$ ,  $V$ 's and wave fcn's differ

No relation between wave functions in principle

## Relation between equal-time and light-front wave functions

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The relation between equal-time and light-front wave functions is studied using models for which the four-dimensional solution of the Bethe-Salpeter wave function can be obtained. The popular prescription of defining the longitudinal momentum fraction using the instant-form free kinetic energy and third component of momentum is found to be incorrect except in the nonrelativistic limit. One may obtain light-front wave functions from rest-frame, instant-form wave functions by boosting the latter wave functions to the infinite momentum frame.

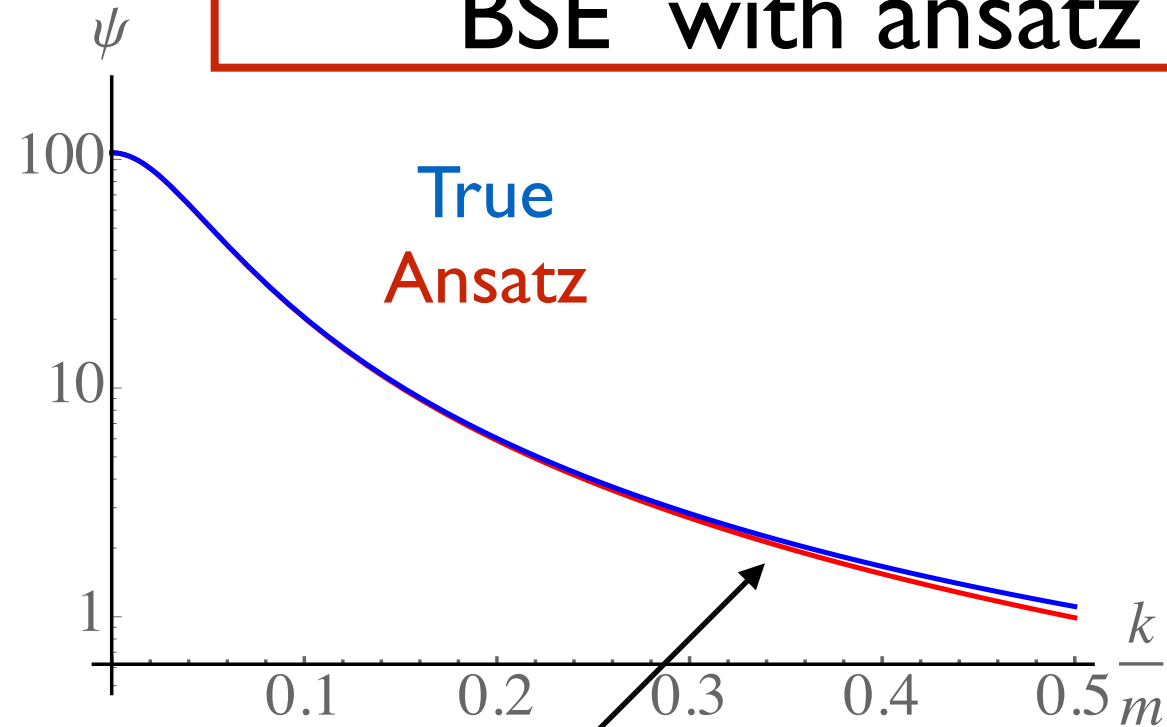
- How bad is the problem?
- Is  $D$  non-relativistic?
- Is  ${}^3\text{He}$  non-relativistic?
- Answer by using solutions of Bethe-S eqn.



Model

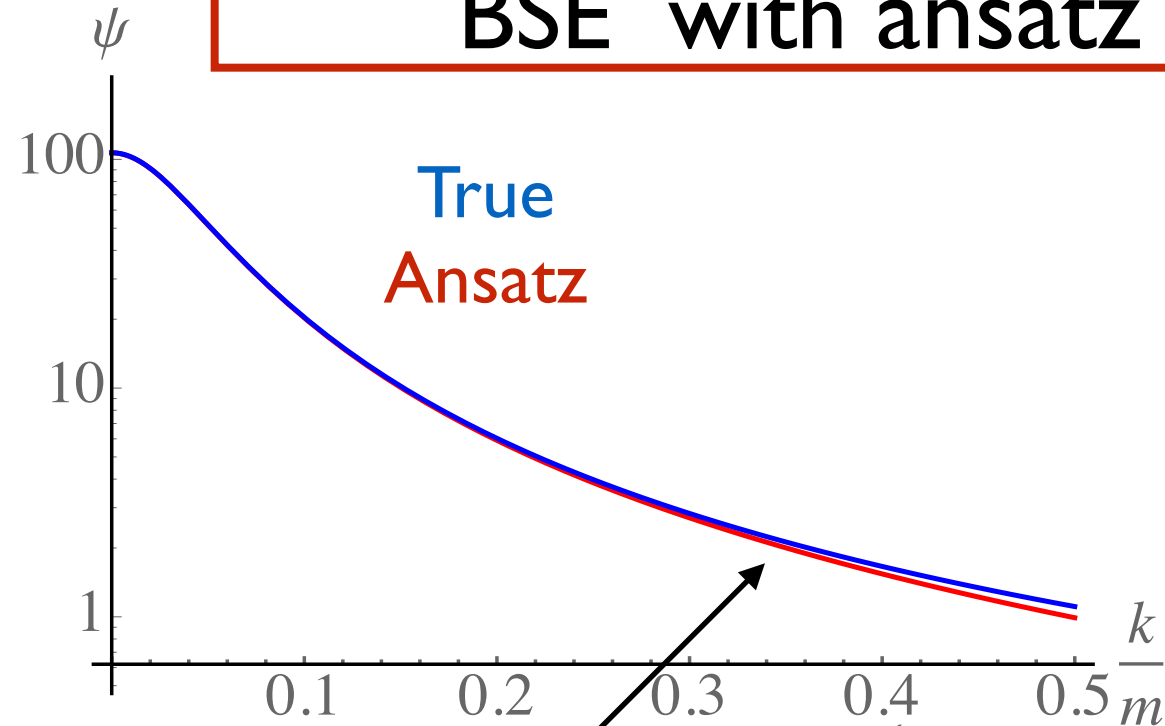
FIG. 2. Bethe-Salpeter equation for a point interaction. The state is bound by the infinite chain of bubbles.

# Deuteron Compares true LFD from BSE with ansatz from ET

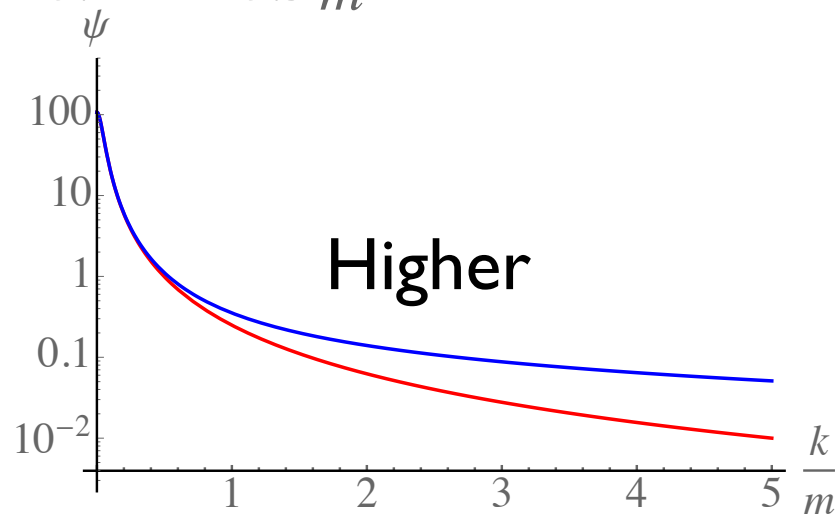


OK up to here

# Deuteron Compares true LFD from BSE with ansatz from ET

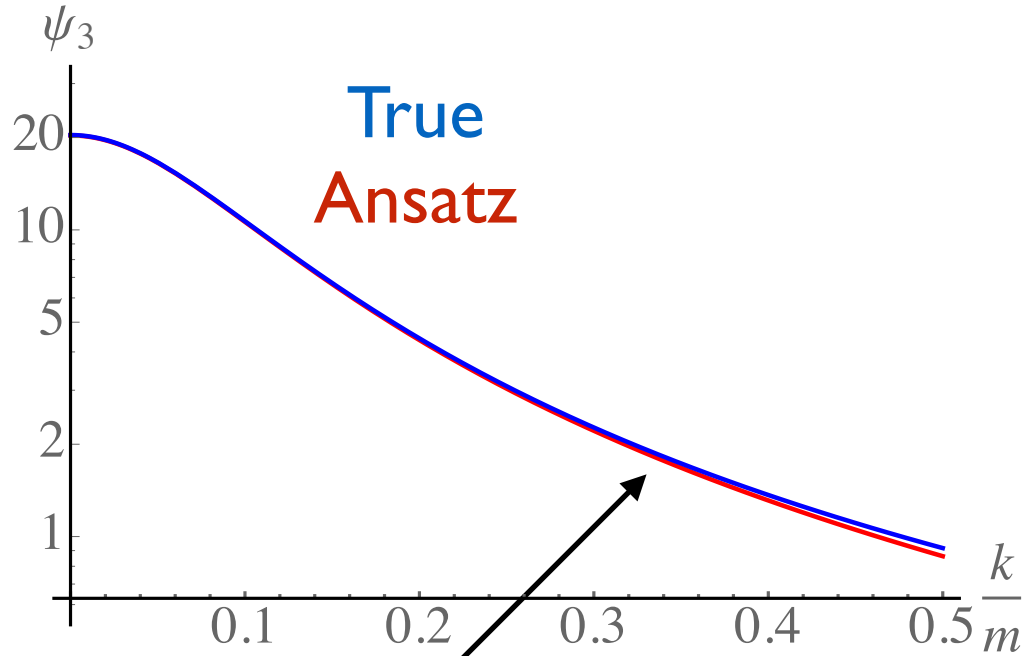


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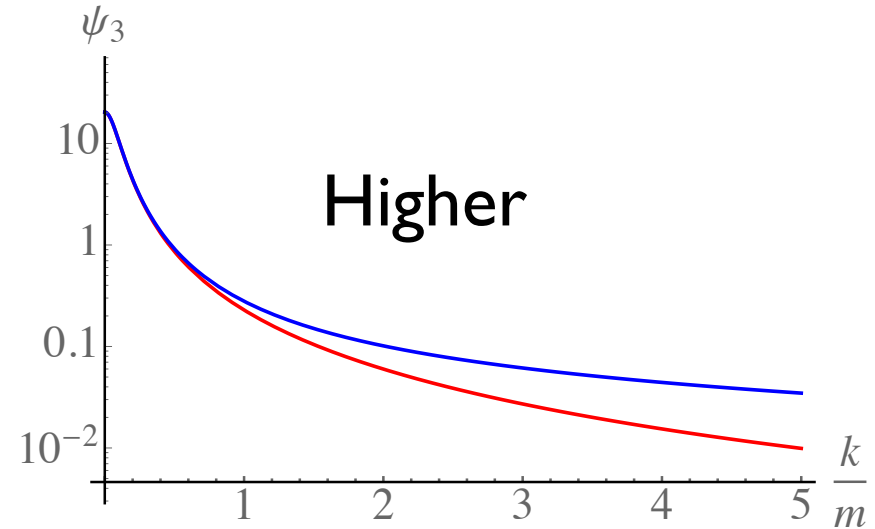
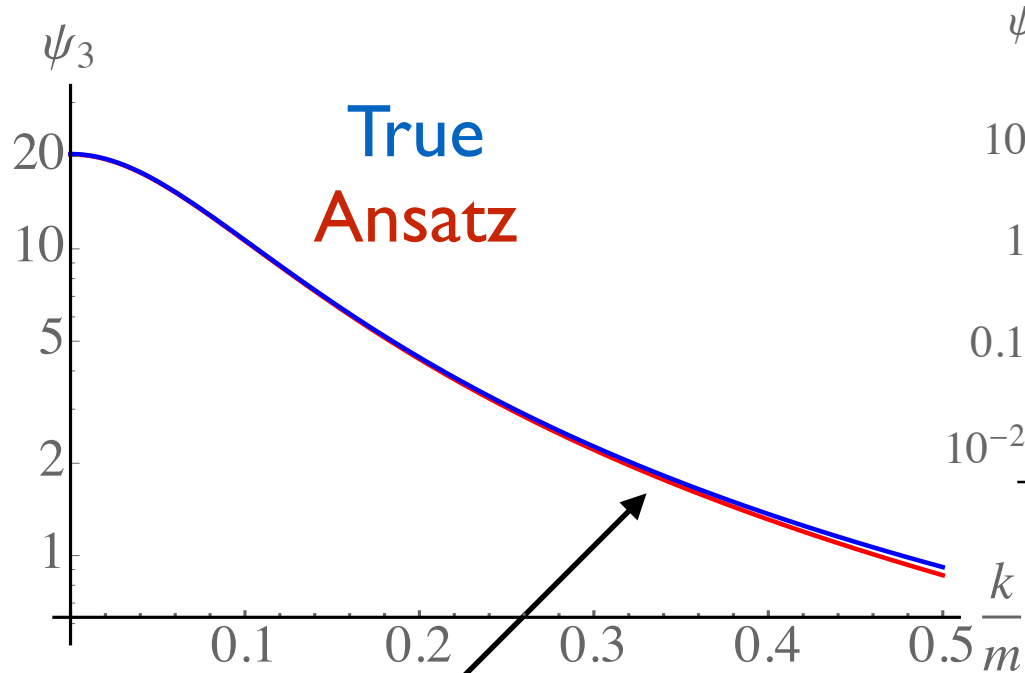


# $^3\text{He}$ Compare true LFD from BSE with ansatz from ET



OK up to here

# $^3\text{He}$ Compare true LFD from BSE with ansatz from ET



OK up to here

# Summary

- formulate the NN interaction on the light front - get from BSE
- solve the Weinberg equation for the deuteron -done only have extra factor in propagator
- include polarization effects -as usual
- prescription from FS 81 review constructs LF wave function from NR wf: study with exact solutions of BSE
- how good is this approx at recoil momenta few hundred MeV?-seems ok up to about 250-300 MeV more study needed
- can we get the LF wf from NN potentials?-seems ok

## Learn more and cite:

J R Cooke nucl-th/0112029 , Cooke & Miller PRC66, 034002  
Miller & Machleidt PRC60, 035202 Miller, Prog. Nuc. Part. Phys. 45, 83  
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**Spares follow**

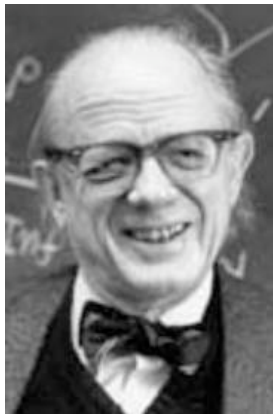
The APS Council and the DNP have endorsed the establishment of the

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If you have any questions, please contact G. A. (Jerry) Miller UW, <millar@uw.edu>.



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