Possible measurement of $\alpha_s(M_{Z_0})$ at EIC with the Bjorken sum rule

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Bjorken sum rule

$$\int g^p_1 - g^n_1 \, dx \triangleq \Gamma_1^{p-n}(Q^2) = \frac{1}{6} g_A \left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 - 175.7\left(\frac{\alpha_s}{\pi}\right)^4 - \sim 893\left(\frac{\alpha_s}{\pi}\right)^5 \ldots\right)$$

$$+ \frac{M^2}{9Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right] + \ldots$$

Nucleon's First spin structure functions

Nucleon axial charge. (Value of Γ_1^{p-n} in the $Q^2 \to \infty$ limit)

pQCD radiative corrections ($\overline{MS}$ Scheme.)

Non-perturbative $1/Q^{2n}$ power corrections. (+rad. corr.)

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$\Rightarrow$ Two possibilities to measure $\alpha_s(M_{Z0})$:

- Do an absolute measurement of $\Gamma_1^{p-n}(Q^2)$ and solve it for $\alpha_s(Q^2)$.
  - One α_s per Γ_1^{p-n} point.
  - Poor systematic accuracy: Such absolute measurements have typically at best a 5% accuracy. Good measurements of α_s should be 2% accurate or better. $\Rightarrow$ Not competitive.

- Measurement of Q^2-dependence of $\Gamma_1^{p-n}(Q^2)$.
  - Need several Γ_1^{p-n} points. Only one value of α_s.
  - Good accuracy: 1990's CERN/SLAC data yielded: $\alpha_s(M_{Z0})=0.120\pm0.009$

Altarelli, Ball, Forte, Ridolfi, Nucl.Phys. B496 337 (1997)

Measurement at 6 GeV in JLab Hall B

EG1dvcs experiment:

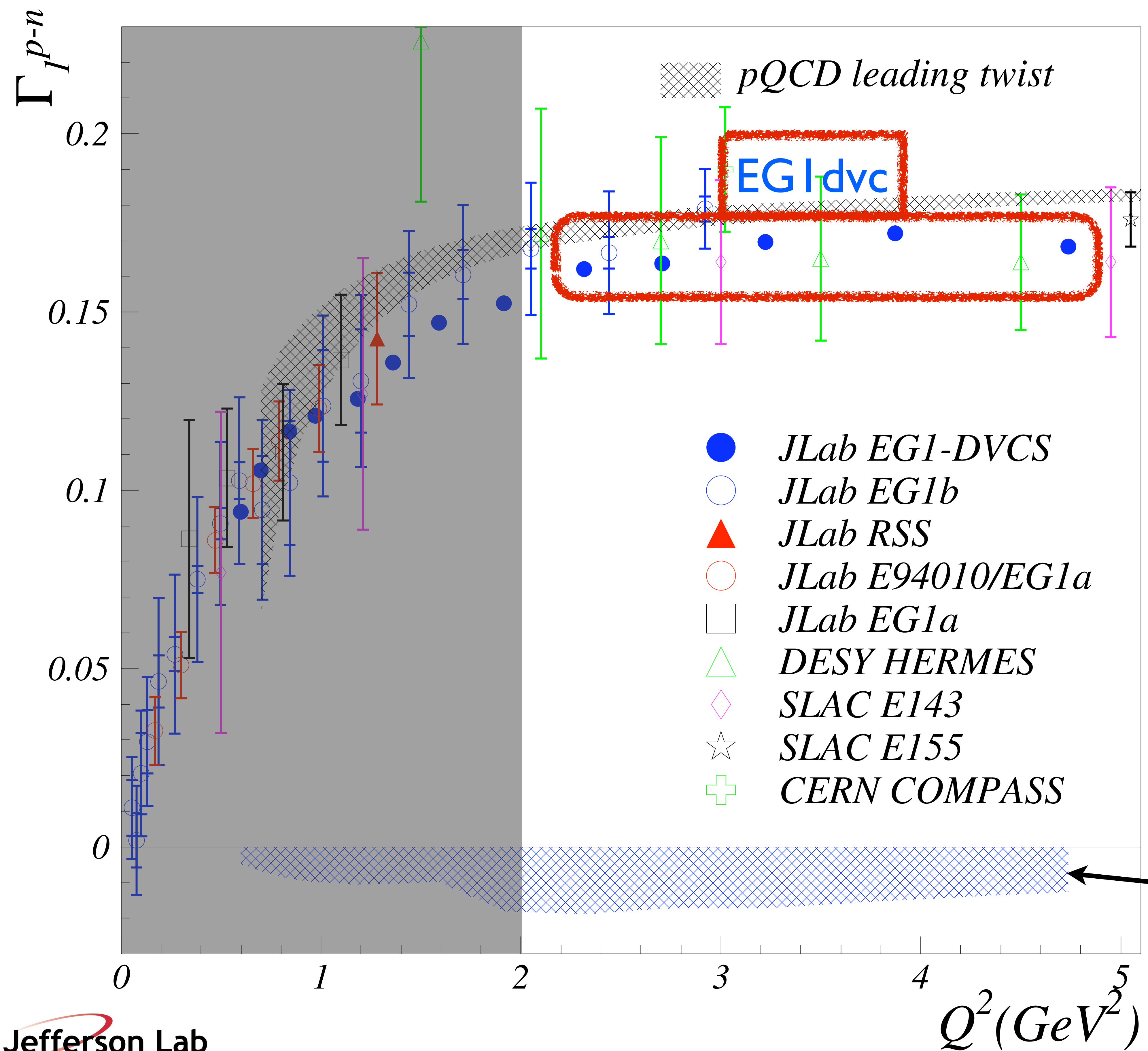
- Cebaf Large Acceptance Spectr. 18-48° polar coverage, ~full azimuthal coverage.
- Polarized NH_3 (50%-64% pol.) and ND_3 (~23% pol.) targets. 0.75 cm eff. length, 1g/cm^2;
- Polarized beam (75%-85%);
- High inclusive statistics (DVCS process meas.): 6 months, 7 nA ⇒ 2×10^{17} e$^-$ on target.

Used "only" EG1dvcs data to avoid uncorrelated systematics between experiments.

- Point-to-point correlated systematics (e.g. polarimetries, nuclear corrections) have minimal impact on uncertainties.
- EG1dvcs data largely dominates world data for statistics.
- Restricted Q^2 range: $2.32<Q^2<4.74$ GeV^2 rather than $2.<Q^2<10$ GeV^2.
- "only" not accurate: important missing low-x contribution from models fitting world data.

Q^2 (GeV^2)	x−range (p)	x−range (d)	$\Gamma^{p-n}_{1,meas}$	$\Gamma^{p-n}_{1,meas+hi.x}$	σ^{syst}_{meas}	$\sigma^{syst}_{hi.x}$	$\Gamma^{p-n}_{1,tot}$	σ^{syst}	σ^{stat}	$\Gamma^{p-n}_{1,meas+hi.x}/\Gamma^{p-n}_{1.tot}$
2.316	0.263-0.864	0.271-0.798	0.0523	0.0515	0.0177	0.0001	0.1621	0.0188	0.0008	0.317
2.707	0.304-0.825	0.326-0.769	0.0398	0.0388	0.0157	0.0008	0.1636	0.0173	0.0006	0.237
3.223	0.362-0.901	0.385-0.799	0.0322	0.0311	0.0152	0.0000	0.1697	0.0171	0.0005	0.183
3.871	0.438-0.893	0.463-0.762	0.0227	0.0206	0.0121	0.0002	0.1721	0.0150	0.0004	0.120
4.739	0.531-0.909	0.663-0.738	0.0145	0.0113	0.0081	0.0002	0.1684	0.0126	0.0002	0.067

Measurement at 6 GeV in JLab Hall B

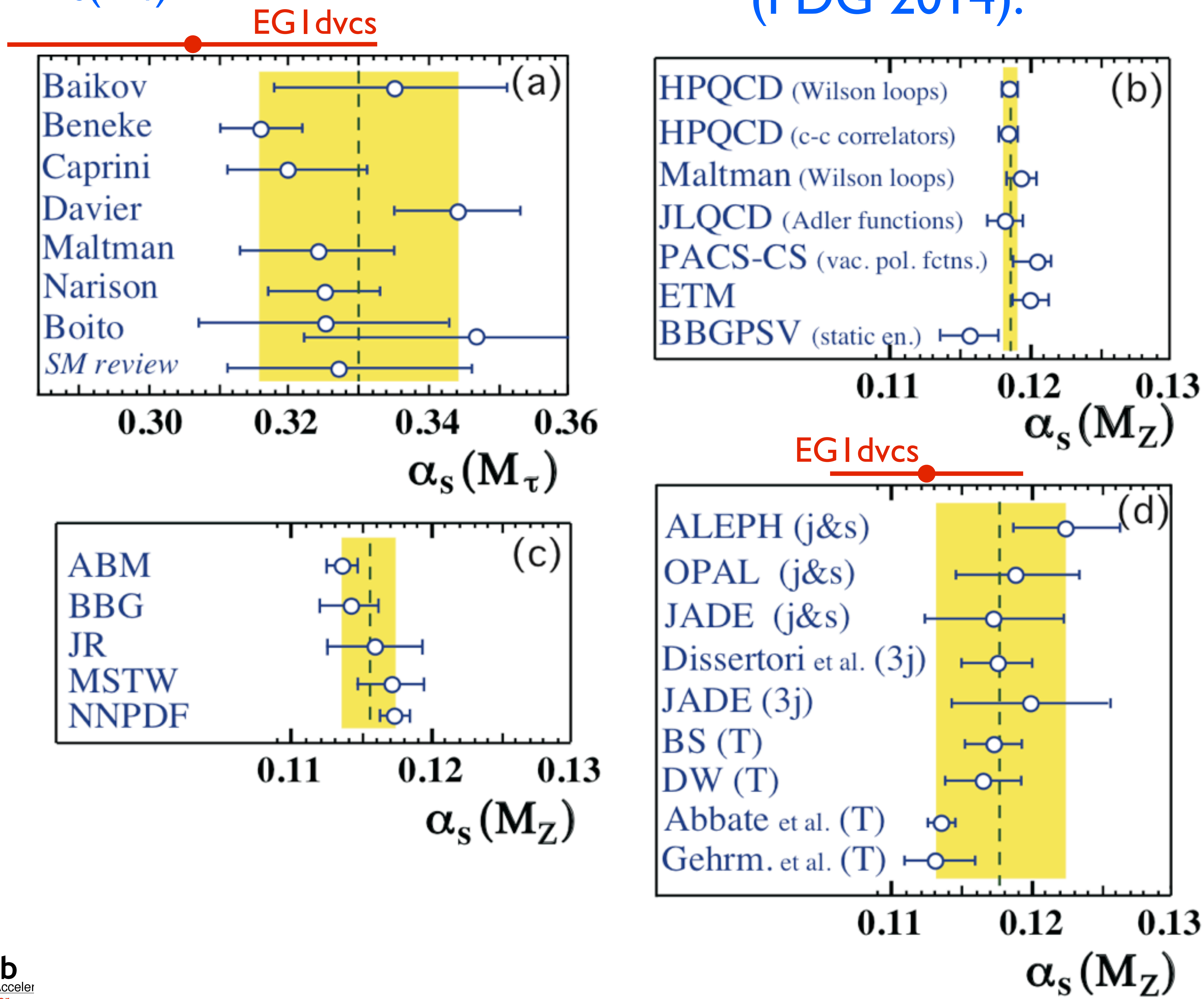


Statistical uncertainties smaller than symbols

Systematic uncertainties (Polarimetries, low-x,...)

Result: $\alpha_s(M_{Z_0})=0.1177\pm0.0064$
$\alpha_s(M_\tau)=0.306\pm0.053$

Compared to best world data (PDG 2014):



Uncertainties

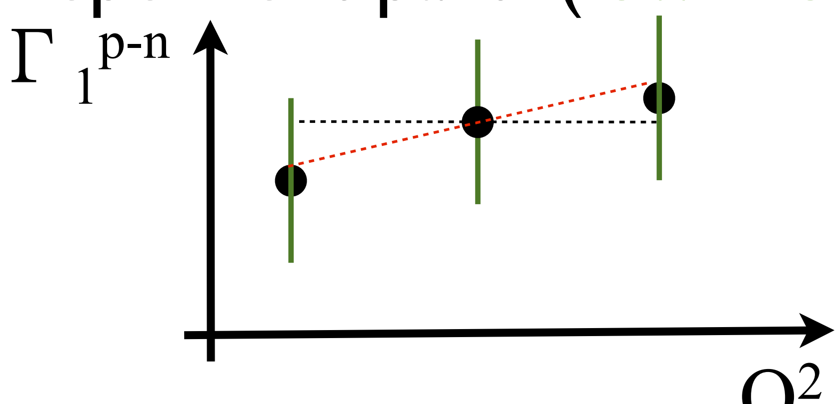
Experimental systematic uncertainty: Separate it between point-to-point correlated and point-to-point uncorrelated parts:

- Fit data with expected pQCD form.
- Force χ^2 to 1 by scaling down uncertainties ("unbiased estimate").
- The scaled downed uncertainties is the point-to-point uncorrelated uncert.
- Point-to-point correlated $\oplus$ uncorrelated = full syst. uncertainties.

Prescription may introduces bias and has assumptions.

Low-x systematic uncertainties: Separate between Q^2-dependent and independent parts.

- Assume Q^2-dependent part=(low-x contribution) $\frac{1}{\Gamma_1^{p-n}} \frac{d\Gamma_1^{p-n}}{dQ^2}$ Q^2-bin size:



Add point-to-point correlated uncertainty to Q^2-dependent low-x part. Use it as uncertainties for χ^2 minimization of the fit.

⇒ large parts of the low-x and exp. syst. uncertainties are suppressed, but prescription makes assumptions.

Uncertainties

Leading uncertainties:

- Point-to-point uncorrelated uncertainties: 4.4%
- Point-to-point correlated uncertainties:3.3%

} 5.5%

Negligible uncertainties:

- Twist-4 contributions: $\frac{M^2}{9Q^2}[a_2(\alpha_s)+4d_2(\alpha_s)+4f_2(\alpha_s)]$

  a_2 (PDF fits) d_2 (meas.) f_2, (Sidorov-Weiss model, with 50% uncertainty). Twist>4 neglected.

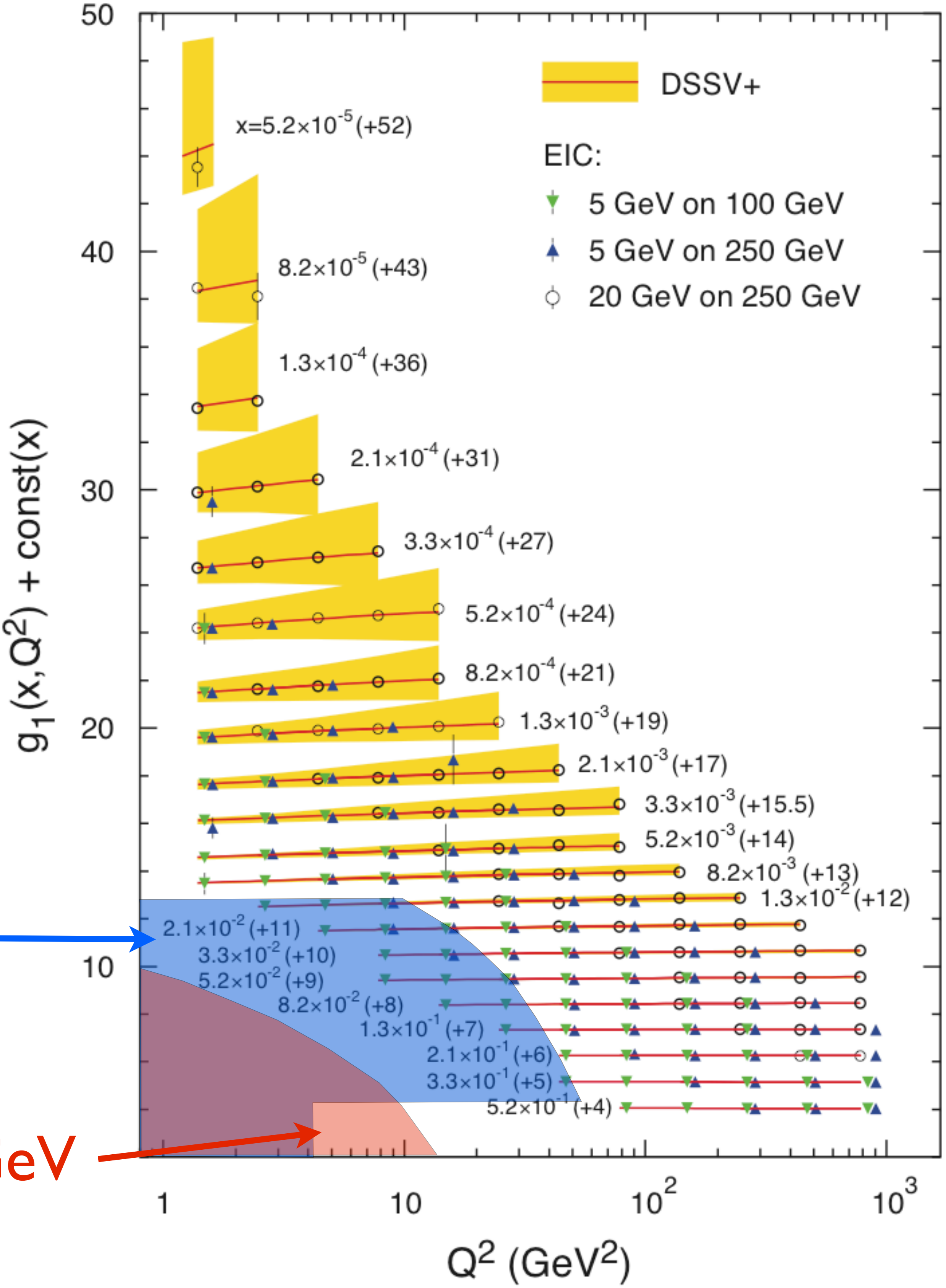
  Uncertainties: <0.1%
- β-series truncation. (Need it to evolve α_s to M_Z.): 0.1%

Uncertainties not accounted for:

- Bjorken twist-2 series truncation: 2.3%

EIC

Use the 5 GeV on 100 GeV, 5 GeV on 250 GeV and 20 GeV on 250 GeV kinematic coverage:



Present world data

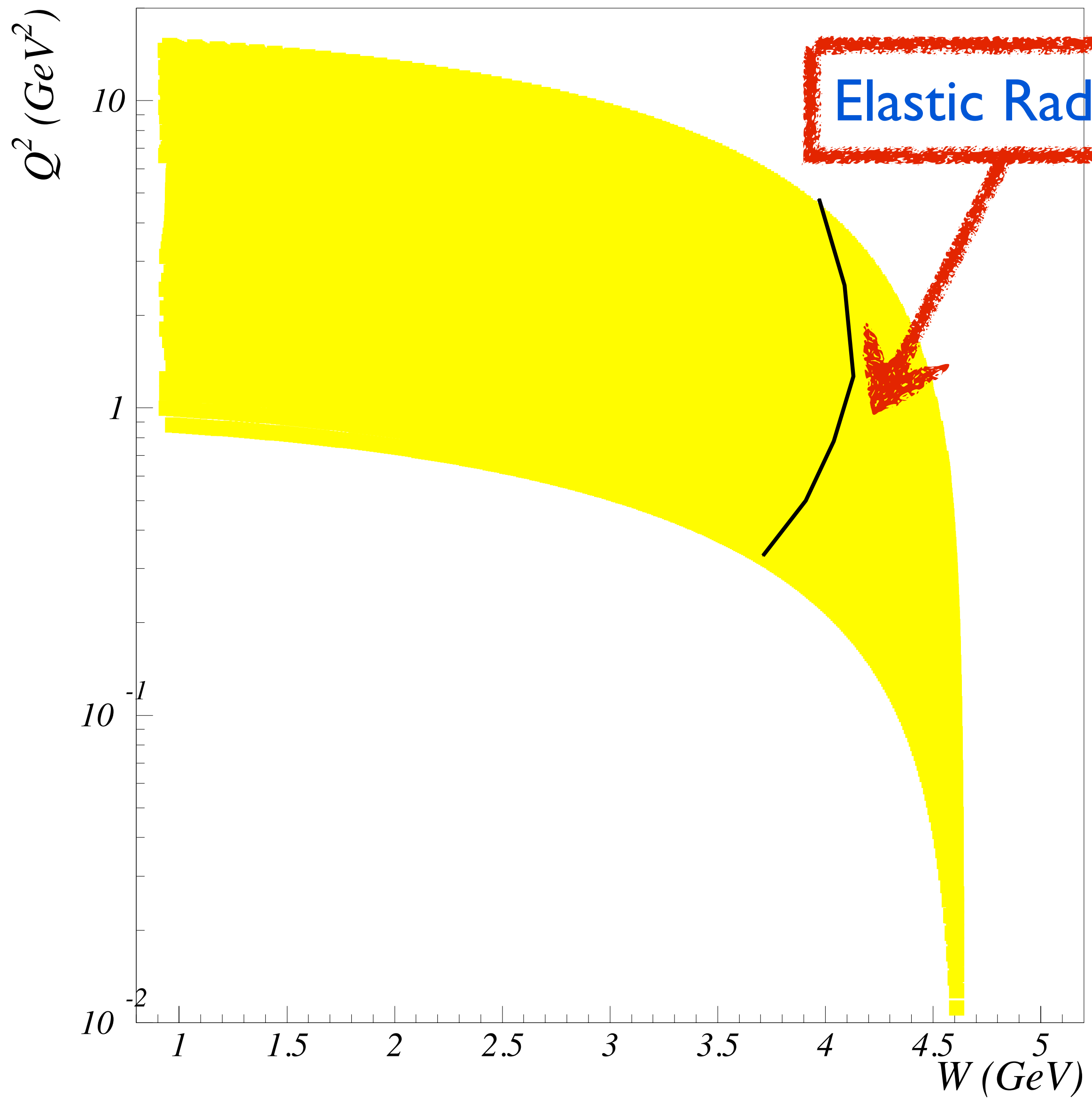
Jlab 12 GeV

From the EIC white paper, arXiv:1212.1701

EIC

Fixed target experiment limitation: Elastic tails

Ex: CLAS12 at 11 GeV:

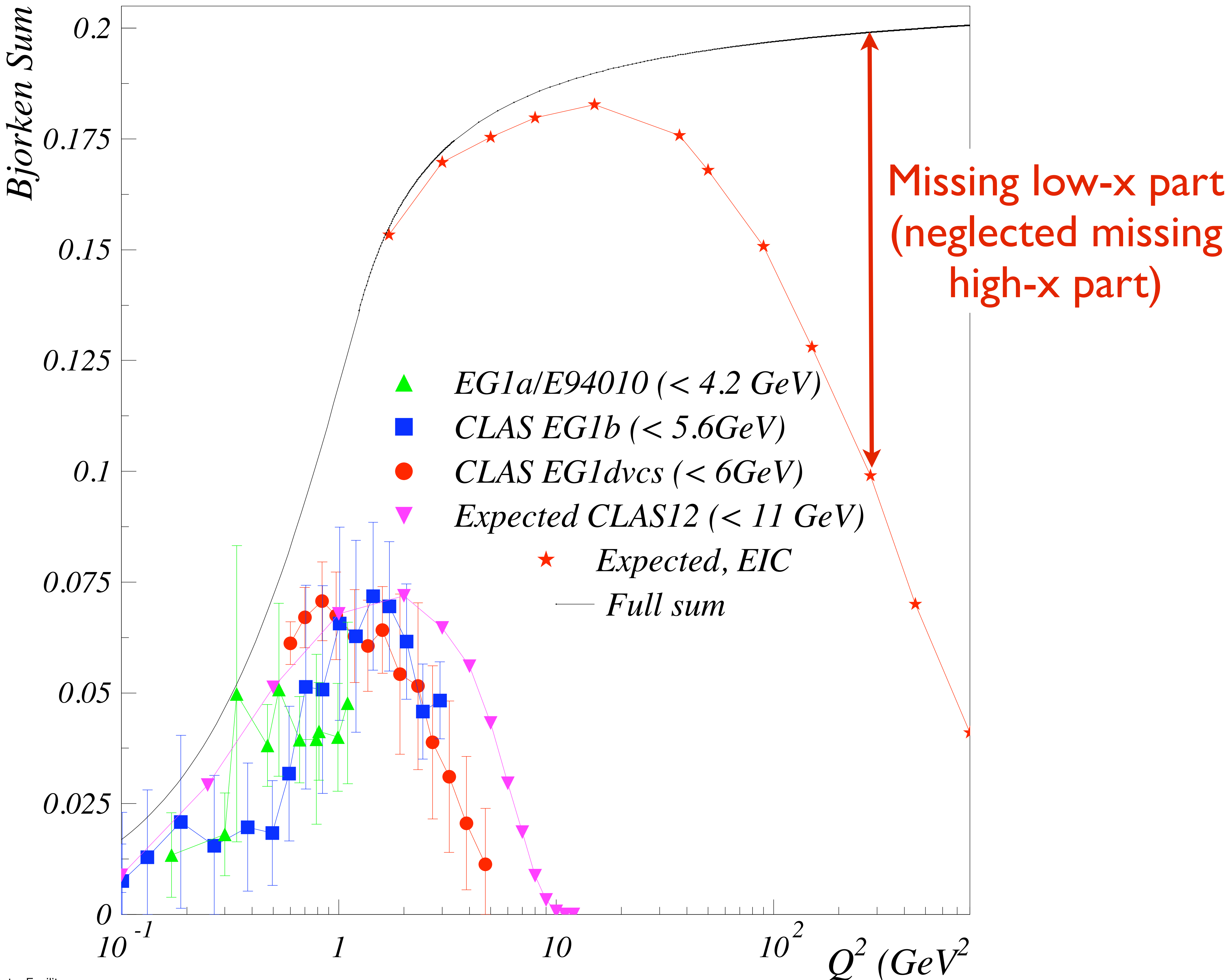


CLAS12 at 6 GeV: tails becomes equal to inelastic cross-section at 2.4<W<4 GeV.

Colliders: no external bremsstrahlung on incoming e^-.

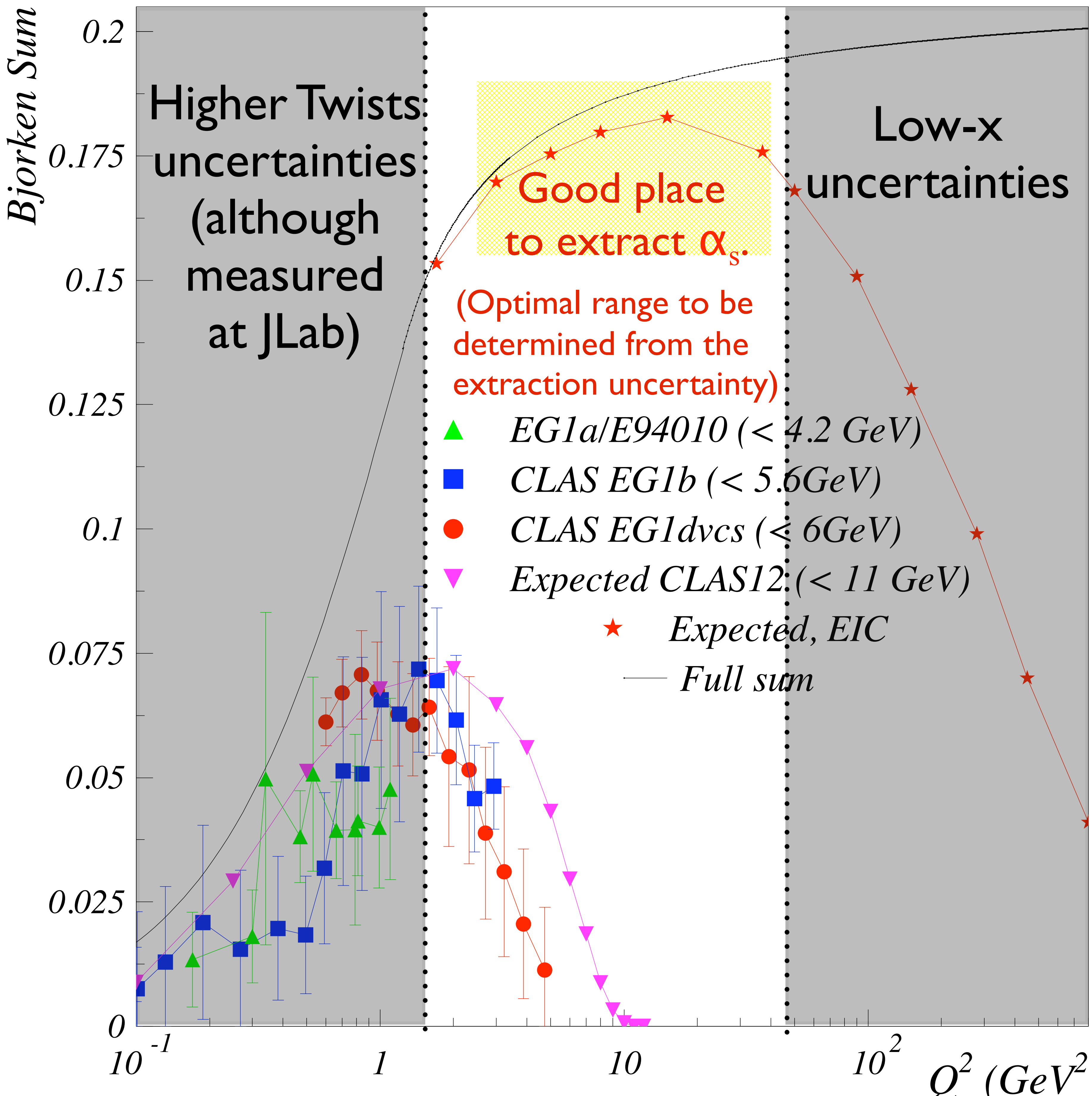
⇒ Radiative tails are suppressed.

Measured fraction of the Bjorken sum



Jefferson Lab
Thomas Jefferson National Accelerator Facility
Exploring the Nature of Matter

Measured fraction of the Bjorken sum



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Uncertainty budget

Statistics:

- Assume $\Delta\Gamma_1^{p-n}$=0.5% (Q^2=3 GeV^2) to $\Delta\Gamma_1^{p-n}$=0.05% (Q^2=15 GeV^2), not counting other world data (JLab@6&12 GeV, SLAC,CERN, DESY)

Statistics assumed twice better than those of CLAS EG1b experiment:

- Luminosity: 10^{34}/s,
- PbPt:0.2-0.6
- Dilution factor: ~80%
- Duration: Analyzed data gathered in a few months.
- Q^2 range for α_s fit: $1<Q^2<3$ GeV^2

With a collider:

- Luminosity: 10^{34}/s,
- P_{e}-P_N: 0.5-0.6
- Dilution factor: 0%
- Duration: a few months.
- Q^2 range for α_s fit: $1.5<Q^2<15$ GeV^2

Uncertainty budget

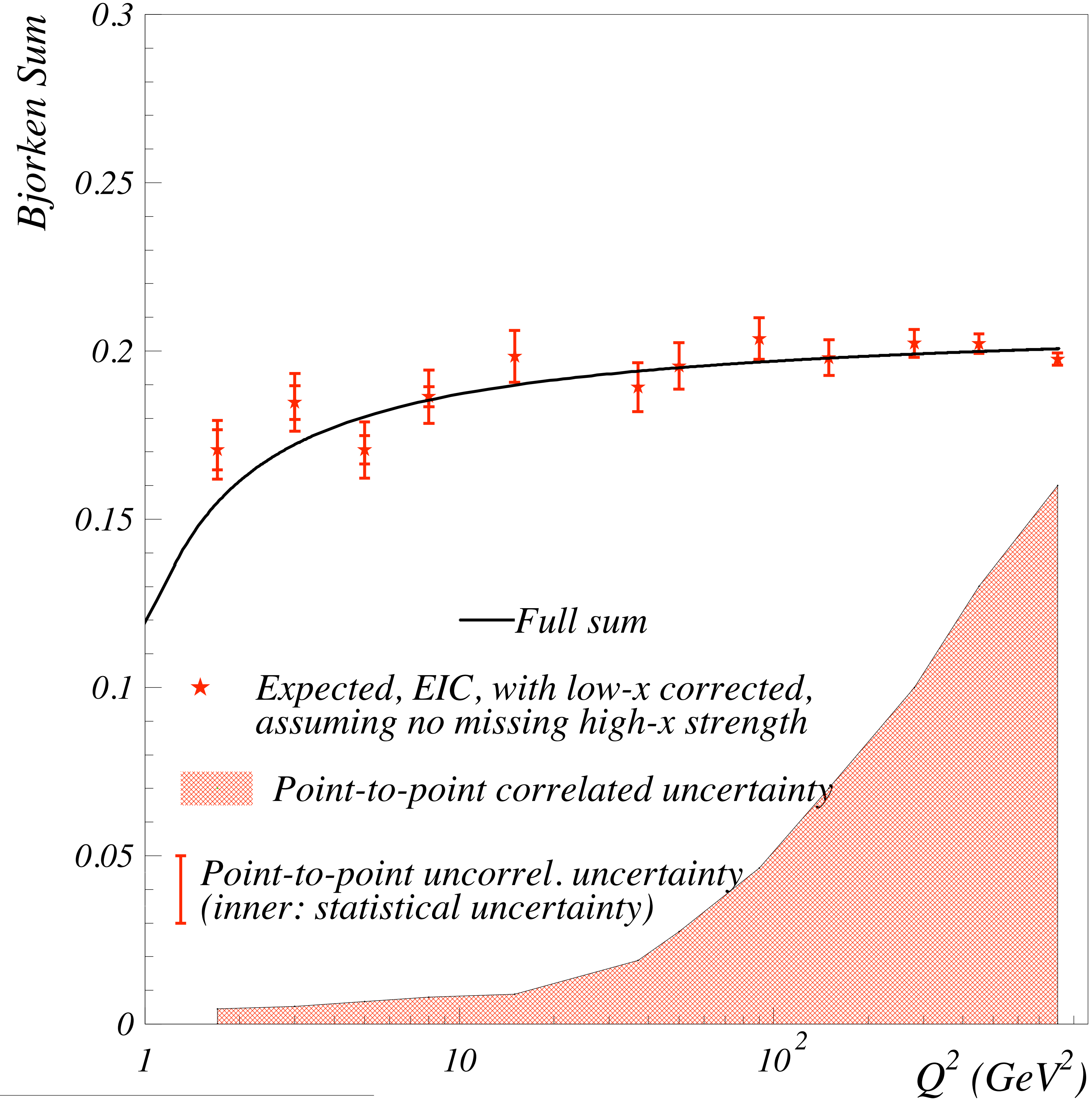
Systematics:

- Nuclear corrections (^{3}He, D): Neglected for tagged program. 4% otherwise.
- Missing low-x part: Assume 100% uncertainty on it.
- Polarimetries: A_1 data overlap with 12 GeV program⇒Normalize to 12 GeV polarimetry performance. Assume ΔP_e-ΔP_N=5%.
- Radiative corrections: Mostly internal RC on e^- line. Lower energy data exist. Assume 4%.
- F_1 to form g_1 from A_1: 2.5% (assumed F_2: 2% for proton and neutron. R: 10%.)
- Dilution/purity: 0
- Miss-PID contamination: Assumed negligible.
- g_2 contribution: Measured with transversally pol. ion beam.
- Kinematic corrections for Q^2: Assumed negligible.
- Detector/trigger efficiencies, acceptance, beam currents: Neglected (asym).

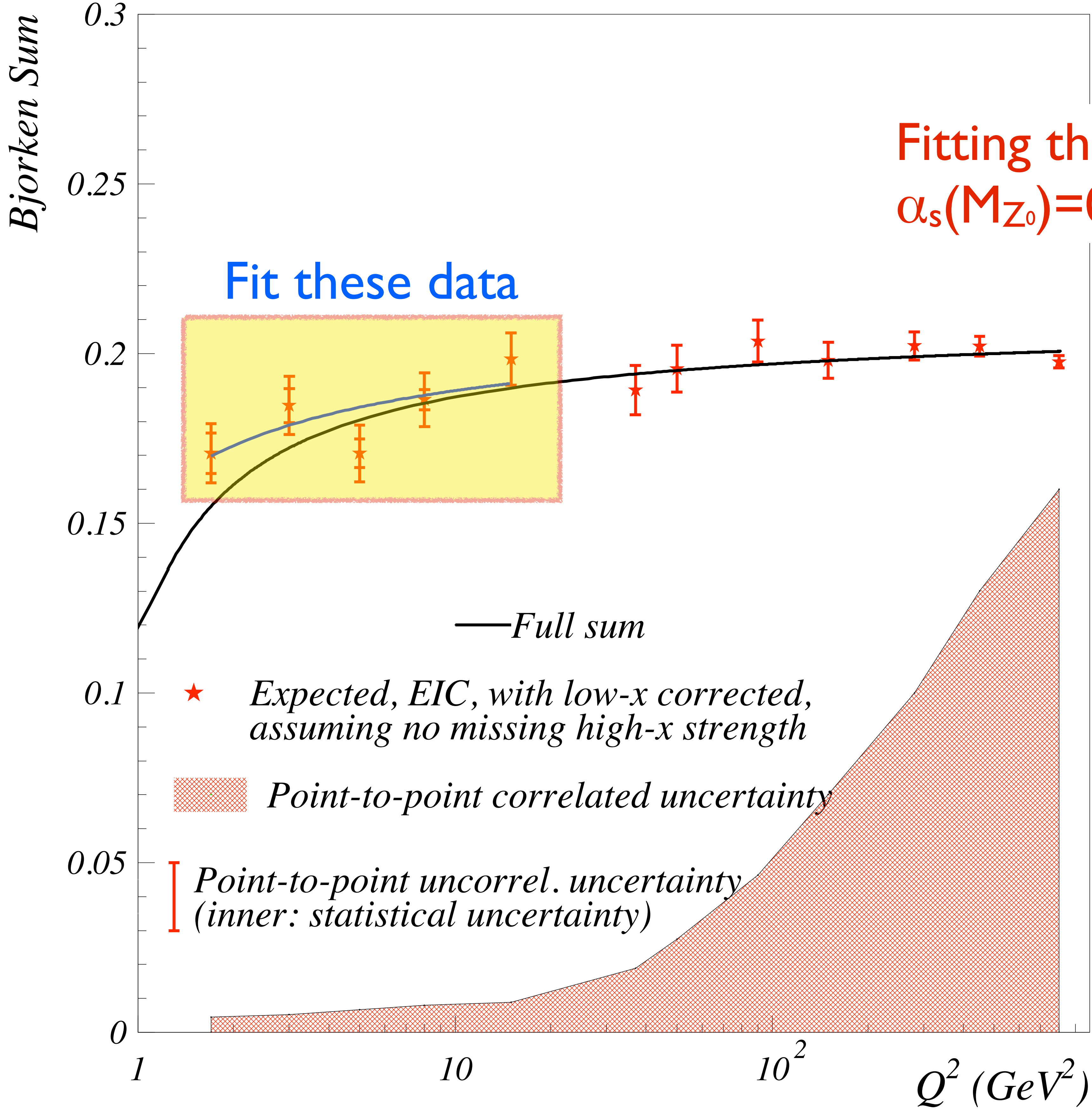
Extraction of $\alpha_s(M_{Z_0})$

For CLAS EG1dvcs, 60% of syst. is point-to-point uncorrelated (excluding the low-x error) ⇒ add to stat. uncert.

⇒ data may look like (assume no tagging, i.e. include nuclear correction uncertainty):



Extraction of $\alpha_s(M_{Z_0})$

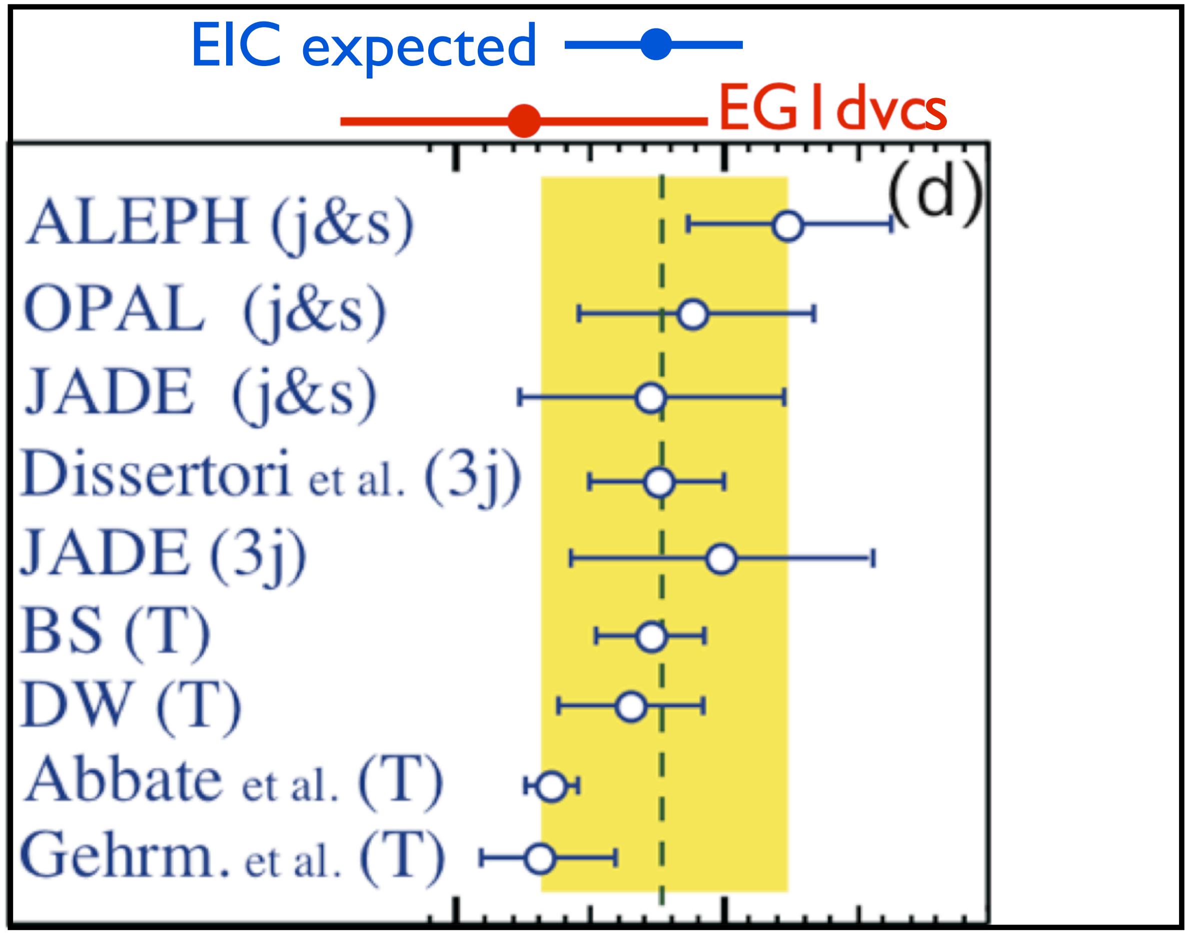


Fitting the data [yellow box] yields:
$\alpha_s(M_{Z_0})$=0.1175±0.0033±0.0005

(Adding next data point yields:
$\alpha_s(M_{Z_0})$=0.1162±0.0030±0.0021
⇒ Not worth including more point).

Other processes:
τ-decay: $\Delta\alpha_s(M_{Z_0})$=0.015
DIS fits: $\Delta\alpha_s(M_{Z_0})$=0.019
Z-pole fits: $\Delta\alpha_s(M_{Z_0})$=0.025
e^+e^- annihil.: $\Delta\alpha_s(M_{Z_0})$=0.042

Compared to EG1dvcs and best world data (PDG 2014):



Conclusion:

- Reasonable assumptions for EIC yield a very accurate measurement of acceptable precision. Tagging not necessary as long as we are statistics (really stat+point-to-point uncor.) dominated.
- Assumed statistics similar to a typical CLAS experiment aiming at measuring inclusive spin structure functions.
- Increasing statistics by factor10 would yield: $\Delta\alpha_s(M_{Z_0})=\pm0.0021\pm0.0003$.
- Then, adding tagging would yield: $\Delta\alpha_s(M_{Z_0})=\pm0.0016\pm0.0003$. A very competitive measurement.