

EEC and the extraction of Collins-Soper kernel

Alejandro Bris

in collaboration with

Dr. Ignazio Scimemi

Dr. Alexey Vladimirov

Universidad Complutense de Madrid -
Departamento de Física Teórica & IPARCOS



May 20 2025

Outline

1 Introduction

2 ARTEMIDE implementation

3 Data

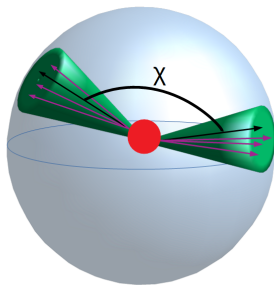
4 Results

- Fits
- SCANs

5 Predictions

Energy-Energy Correlators

(e^+e^- collisions)



$$\text{EEC}(\chi) = \frac{d\sigma}{d\chi} = \sum_{i,j} \int d\sigma_{e^+e^- \rightarrow ij+\chi} \frac{E_i E_j}{Q^2} \delta(\cos \theta_{ij} - \cos \chi)$$

$$\chi \longleftrightarrow z = \frac{1 - \cos(\chi)}{2}$$

Back-to-back limit

$z \rightarrow 1$ ($\chi \rightarrow 180^\circ$)

- Fixed order perturbative result: $EEC \sim \log(1-z)$ LARGE LOGARITHMS !!
- All orders resummation:

$$\underbrace{\lim_{z \rightarrow 1} \frac{d\sigma}{dz} \leftrightarrow \lim_{q_\perp \rightarrow 0} \frac{d\sigma}{dx_i dx_j d^2 \vec{q}_\perp}} \quad \& \quad \text{TMD factorization}$$

$\Downarrow$

Factorization theorem

$$\frac{d\sigma}{dz} = \frac{\hat{\sigma}_0}{8} \sum_{f, \bar{f}} H_{f\bar{f}}(Q, \mu) \int_0^\infty d(b_T Q)^2 J_0(b_T Q \sqrt{1-z}) J_f(b_T, \mu, \zeta) J_{\bar{f}}(b_T, \mu, \bar{\zeta})$$

- * $\hat{\sigma}_0$ Born cross section
- * $H_{f\bar{f}}(Q, \mu)$ hard function $\leftrightarrow \mathcal{O}(\alpha_s^3)$ [4-loops] $\overbrace{\text{TMDFF}}$
- * Jet function $J_f(b_T, \mu, \zeta) \equiv \sum_h \int_0^1 dz z^3 \overbrace{D_{f \rightarrow h}(z, b_T, \mu, \zeta)} \leftrightarrow \text{N}^4\text{LL} + \mathcal{O}(\alpha_s^3)$

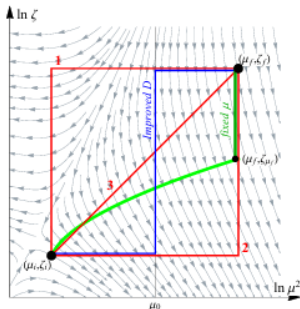
$$d\sigma \sim \int \text{Bessel}_J \times \text{TMD} \times \text{TMD} \implies \boxed{\text{ARTEMIDE}} \quad [\text{JHEP05(2024)036}] \quad [\text{arxiv, 2503.11201}]$$

Outline

- 1 Introduction
- 2 ARTEMIDE implementation
- 3 Data
- 4 Results
 - Fits
 - SCANs
- 5 Predictions

ζ -prescription

[I.Scimemi, A. Vladimirov]



$$\mu^2 \frac{d}{d\mu^2} D_{f \rightarrow h}(z, b_T, \mu, \zeta) = -\frac{\gamma_D^f(\mu, \zeta)}{2} D_{f \rightarrow h}(z, b_T, \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} D_{f \rightarrow h}(z, b_T, \mu, \zeta) = -\mathcal{D}^f(\mu, b_T) D_{f \rightarrow h}(z, b_T, \mu, \zeta)$$

* γ_D^f , $\mathcal{D}^f \leftrightarrow$ TMDFF, rapidity anomalous dimensions

Null-evolution lines: $\zeta_\mu(b_T)$

$$\Rightarrow D_{f \rightarrow h}(z, b_T, \mu, \zeta) = \left(\frac{\zeta}{\zeta_\mu(b_T)} \right)^{-\mathcal{D}^f(\mu, b_T)} \underbrace{D_{f \rightarrow h}(z, b_T, \mu, \zeta_\mu(b_T))}_{D_{f \rightarrow h}(z, b_T) \text{ OPTIMAL TMDFF}}$$

Non-Perturbative models

Collins-Soper kernel

$$\mathcal{D}(\mu, b_T) = \mathcal{D}_{\text{pert}}(b^*, \mu^*) + \int_{\mu^*}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\text{cusp}}(\mu') + \mathcal{D}_{\text{NP}}(b_T)$$

- b^* -prescription:

$$b^*(b_T) = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{B_{\text{NP}}^2}}}, \quad \mu^*(b_T) = \frac{2e^{-\gamma_E}}{b^*(b_T)}$$

* $B_{\text{NP}} \equiv 1.5\text{GeV}^{-1}$

- Non-perturbative correction:

$$\mathcal{D}_{\text{NP}}(b_T) = b_T b^* \left[c_0 + c_1 \ln \left(\frac{b^*}{B_{\text{NP}}} \right) \right]$$

* c_0 and c_1 free parameters

Non-Perturbative models

(Optimal) Jet function

→ Optimal Jet function:

$$J_f(b_T) \equiv \overbrace{\sum_h}^{\text{sum over all hadrons}} \int_0^1 dz z^3 \overbrace{D_{f \rightarrow h}(z, b_T)}^{\text{Optimal TMDFF}}$$

→ Optimal TMD fragmentation function: LO OPE $\otimes$ model

$$D_{f \rightarrow h}(z, b_T) \equiv \frac{1}{z^2} \int_z^1 \frac{dy}{y} \sum_{f'} y^2 \overbrace{\mathbb{C}_{f \rightarrow f'}(y, b, \mu_{\text{OPE}})}^{\text{perturbative coeff.}} \overbrace{d_{1, f' \rightarrow h}\left(\frac{z}{y}, \mu_{\text{OPE}}\right)}^{\text{Collinear FF (DATA)}} \overbrace{D_{\text{NP}}^h(z, b)}^{\text{NP model}}$$

✗ $J = \sum_h \int \text{LO OPE} \otimes \text{model}$

Collinear FF data sets do not satisfy the sum rule:

$$\sum_{h \in \text{Data}} \int_0^1 dz z d_{f \rightarrow h}(z, \mu) \sim 0.5 \neq 1$$

Non-Perturbative models

(Optimal) Jet function

$$I) \quad J^{\text{pert.}}(b_T) \equiv \lim_{b \rightarrow 0} J_f(b_T) |_{f=q, \bar{q}}$$

$$\left. \lim_{b \rightarrow 0} D_{\text{NP}}^h(z, b) = 1 \right\}_{\sum_h \int_0^1 dz \, z \, d_{f \rightarrow h}(z, \mu) = 1} \Rightarrow J^{\text{pert.}}(b_T) = \sum_{f'} \int_0^1 dy \, y^3 \mathbb{C}_{f \rightarrow f'}(y, b, \mu_{\text{OPE}}) |_{f=q, \bar{q}}$$

$$\begin{aligned} J^{\text{pert.}}(b_T) = & 1 + \left(\frac{16}{3} - 2\pi^2 \right) \alpha_s + \frac{\alpha_s^2}{486} \left[2n_f (936\zeta_3 + 594\pi^2 - 989) + 3 (2736\zeta_3 + 516\pi^4 - 9150\pi^2 + 10777) \right] \\ & + \frac{\alpha_s^3}{1377810} \left\{ -28n_f^2 (1332720\zeta_3 + 3618\pi^4 + 157140\pi^2 - 114055) \right. \\ & - 420n_f (-1846062\zeta_3 + 30\pi^2(2313\zeta_3 - 23912) - 372600\zeta_5 + 24222\pi^4 + 730687) \\ & - 15 [126\pi^2(1664467 - 383310\zeta_3) - 21(216\zeta_3(1577\zeta_3 - 20116) - 20741616\zeta_5 + 5925635) \\ & \left. + 438352\pi^6 - 15926337\pi^4] \right\} + \mathcal{O}(\alpha_s^4) = J^{\text{pert.}}(b_T, \mu, \zeta_\mu) \quad ([M.A.Ebert, B.Mistlberger, G.Vita]) \end{aligned}$$

$$* \quad \text{with } \alpha_s \equiv \alpha_s \left(\frac{2e^{-\gamma_E}}{b_T} \right)$$

Non-Perturbative models

(Optimal) Jet function

$$\text{II) } J_f(b_T) \equiv J^{\text{pert.}}_f(b_J^*) J_f^{\text{NP}}(b_T)$$

- b_J^* -prescription:

$$b_J^*(b_T) = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{B_J^{*2}}}}$$

- * $B_J^* \equiv 0.22 \text{ GeV}^{-1} \Rightarrow \frac{2e^{-\gamma_E}}{b_J^*} > 5 \text{ GeV}$ (Avoid numerical fluctuations below bottom threshold m_b)

- Non-perturbative correction:

$$\rightarrow \lim_{b_T \rightarrow 0} J_f^{\text{NP}}(b_T) = 1$$

- Numerical justification: [H.T. Li, Y. Makris, I. Vitev]

$$\left. \begin{aligned} J_f^{\text{NP}}(b) &\sim \sum_h \int_0^1 dz \, z \, d_{f \rightarrow h}(z, \mu) D_{\text{NP}}^h(z, b)|_{\mu=\mu_0} \\ D_{\text{NP}}^h(z, b) &\sim \text{Gaussian} \end{aligned} \right\} \Rightarrow J_f^{\text{NP}}(b_T) \sim e^{-a b_T} \quad (\sim f\text{-independent})$$

- Similar model used in [Z.-B. Kang, J. Penttala and C. Zhang]

Non-Perturbative models

(Optimal) Jet function

MODEL 1: Exponential model:

$$J^{\text{NP}}(b_T) = e^{-a_1 b_T}$$

- * One free parameter a_1
- * Enough to describe shape of the data

MODEL 2:

$$J^{\text{NP}}(b_T) = e^{-a_1 b_T} \frac{1 + a_2 b_T^2}{1 + a_3 b_T^2}$$

- * Quadratic corrections a_2, a_3
- * $a_3 > 0$

Outline

1 Introduction

2 ARTEMIDE implementation

3 Data

4 Results

- Fits
- SCANs

5 Predictions

Data

Summary

Experiment	Q (GeV)	References
OPAL	91.0, 91.2, 90.7-91.7	PLB [1990 , 1992], [ZPC 1993]
SLD	91.2	[PRD 1995]
TOPAZ	53.3, 59.5	[PLB 1989]
MARKII	29	[PRD 1988]
TASSO	14, 22, 34.8, 43.5	[ZPC 1987]
MAC	29	[PRD 1985]
PLUTO	7.7, 9.4, 12, 13, 17, 22, 27.6, 27.6-31.6	[PLB 1981]
CELLO	22, 34	[ZPC 1982]
JADE	14, 22, 34	[ZPC 1984]
DELPHI	91.2	ZPC [1992 , 1993]

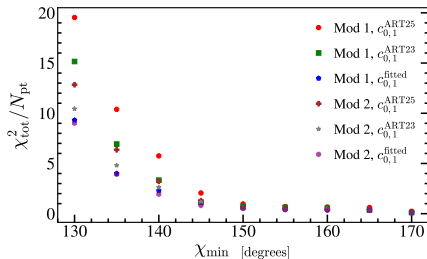
Data

Comments

- Publication dates: 1981-1995 $\Rightarrow$ Data from >30 years ago
- Intended for α_s extractions from central region (EEC used in conjunction with results for event shapes)
- Few details on error estimation
- Focus on shape $\Rightarrow$ include set-by-set normalization (best χ^2)
- Fit range: (validity of back-to-back factorization theorem)

$$\chi_{\min} \equiv 150^\circ$$

$$\left(\sqrt{1-z} \sim \frac{q\tau}{Q} \sim 0.25 \Rightarrow \chi \sim 150^\circ \right)$$



- Total number of points 208

- 3 different fits for each J^{NP} :
 - I CS kernel fixed to ART25 values (obtained from DY+SIDIS data)
 - II CS kernel fixed to ART23 values (obtained from DY data)
 - III CS kernel fitted
- 500 replicas of data for parameters error. For error propagation in ART kernels, variation:
 - EEC data replica
 - c_0 , c_1 results from ART replicas

Outline

- 1 Introduction
- 2 ARTEMIDE implementation
- 3 Data
- 4 Results
 - Fits
 - SCANs
- 5 Predictions

Results

Parameters

Model 1

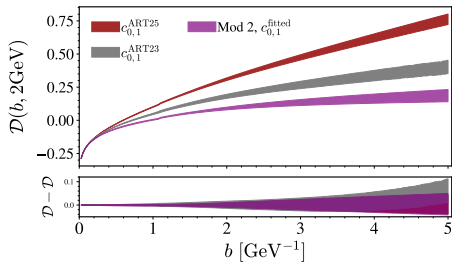
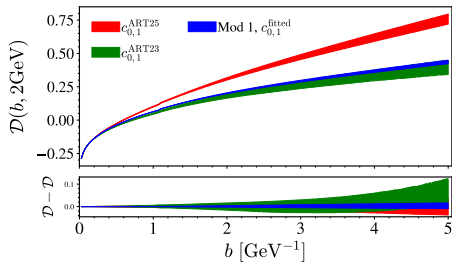
$c_{0,1}^{\text{ART25}}$	$c_{0,1}^{\text{ART23}}$	$c_{0,1}^{\text{fitted}}$
$c_0^{\text{ART25}} = 0.0859^{+0.002}_{-0.002} \text{ GeV}^{-2}$ $c_1^{\text{ART25}} = 0.0303^{+0.004}_{-0.004} \text{ GeV}^{-2}$ $a_1 = 0.903^{+0.014}_{-0.014} \text{ GeV}$	$c_0^{\text{ART23}} = 0.0369^{+0.007}_{-0.006} \text{ GeV}^{-2}$ $c_1^{\text{ART23}} = 0.0582^{+0.006}_{-0.009} \text{ GeV}^{-2}$ $a_1 = 1.004^{+0.006}_{-0.015} \text{ GeV}$	$c_0 = 0.040^{+0.002}_{-0.001} \text{ GeV}^{-2}$ $c_1 = 0.030^{+0.003}_{-0.002} \text{ GeV}^{-2}$ $a_1 = 0.949^{+0.004}_{-0.002} \text{ GeV}$
$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.969$	$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.685$	$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.590$

Model 2

$c_{0,1}^{\text{ART25}}$	$c_{0,1}^{\text{ART23}}$	$c_{0,1}^{\text{fitted}}$
$c_0^{\text{ART25}} = 0.0859^{+0.002}_{-0.002} \text{ GeV}^{-2}$ $c_1^{\text{ART25}} = 0.0303^{+0.004}_{-0.004} \text{ GeV}^{-2}$ $a_1 = 0.983^{+0.012}_{-0.012} \text{ GeV}$ $a_2 = 0.219^{+0.017}_{-0.016} \text{ GeV}^2$ $a_3 = (3.55^{+11.75}_{-3.08}) \times 10^{-7} \text{ GeV}^2$	$c_0^{\text{ART23}} = 0.0369^{+0.007}_{-0.006} \text{ GeV}^{-2}$ $c_1^{\text{ART23}} = 0.0582^{+0.006}_{-0.009} \text{ GeV}^{-2}$ $a_1 = 0.940^{+0.031}_{-0.018} \text{ GeV}$ $a_2 = 1.044^{+0.633}_{-1.052} \text{ GeV}^2$ $a_3 = 1.255^{+0.723}_{-1.204} \text{ GeV}^2$	$c_0 = 0.0080^{+0.0075}_{-0.0064} \text{ GeV}^{-2}$ $c_1 = 0.094^{+0.018}_{-0.013} \text{ GeV}^{-2}$ $a_1 = 0.910^{+0.002}_{-0.003} \text{ GeV}$ $a_2 = 0.945^{+0.233}_{-0.182} \text{ GeV}^2$ $a_3 = 1.531^{+0.335}_{-0.250} \text{ GeV}^2$
$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.608$	$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.548$	$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.511$

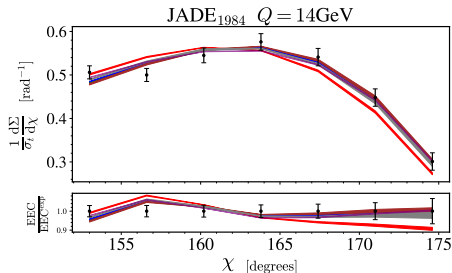
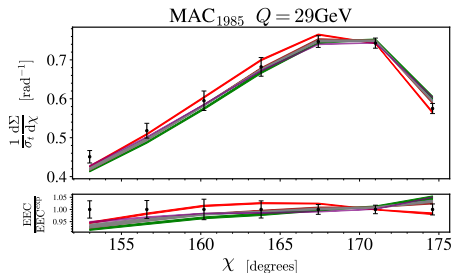
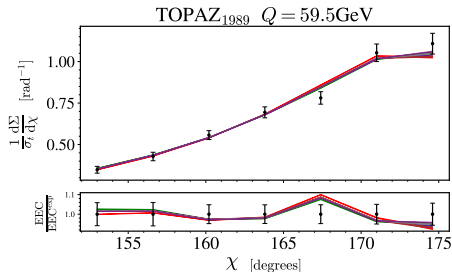
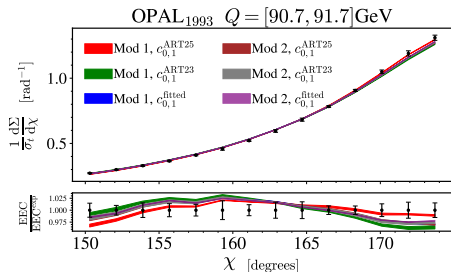
Results

Collins-Soper kernel



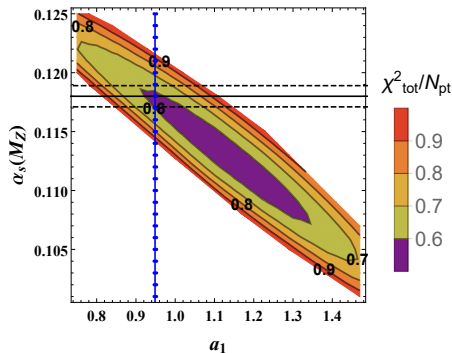
Results

Cross Section

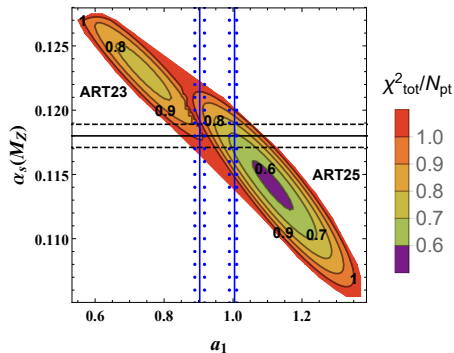


Results

SCANs (Model 1)



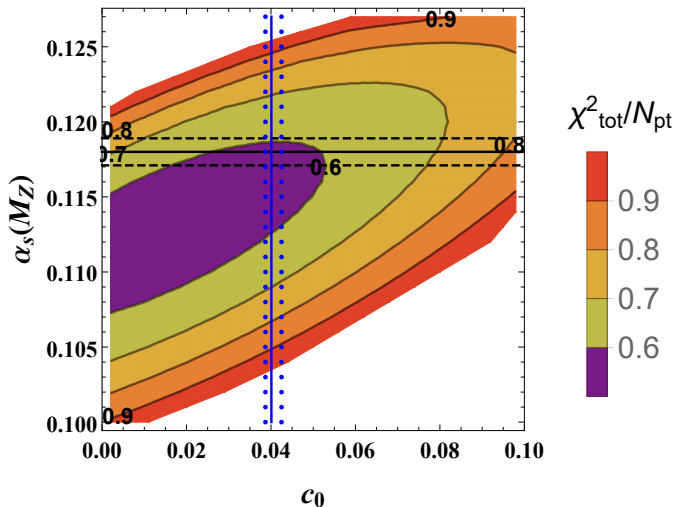
(Collis-Soper kernel fitted)



(Collis-Soper kernel fixed to ART.)

Results

SCANs (Model 1)



(a_1 & c_1 fitted)

Outline

1 Introduction

2 ARTEMIDE implementation

3 Data

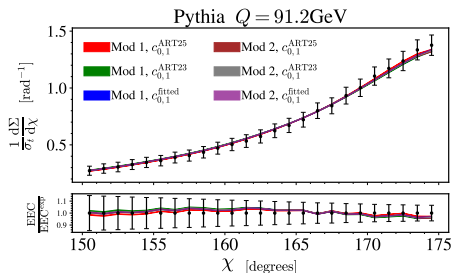
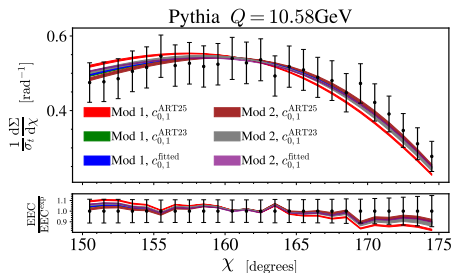
4 Results

- Fits
- SCANs

5 Predictions

Predictions

Comparison with [Pythia 8.3 (ffbar2gmZ)]

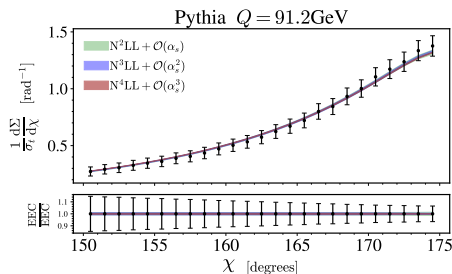
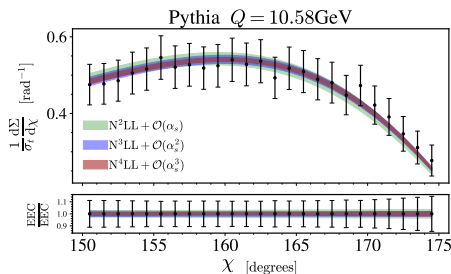


Predictions

Perturbative convergence

$$\mu^* \in \{1, 2\} \times \frac{2e^{-\gamma_E}}{b_J^*} ;$$

$$\mu \in \{0.5, 1, 2\} \times Q$$



Pert. error Mod 1 $c_{0,1}^{\text{fitted}}$

Conclusions

- We implemented the back-to-back EEC factorization theorem at $N^4\text{LL} + \mathcal{O}(\alpha_s^3)$ order within **ARTEMIDE**'s framework
- Two different models for the non-perturbative part of the jet functions were proposed: one-parameter exponential and including corrections to this function.
- The free parameters were fitted using, to the best of our knowledge, all data available (being the most recent set with more than 30 years).
- A preliminary study was made through different scans to determine the viability of an α_s extraction within this setup, turning out to be a very inaccurate procedure.
- The replica method seems to underestimate errors when having very low χ^2 fit.
- Predictions were given for *Belle* (at $Q = 10.58\text{GeV}$) and for experiments at $Q = M_Z$, accounting for the errors coming from our extraction of the non-perturbative parameters and also showing some of the uncertainties due to the truncation of the perturbative series, where we compared different orders.

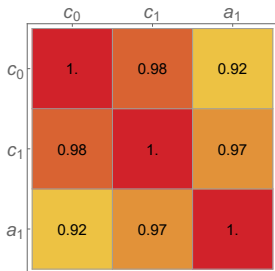
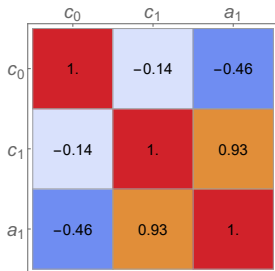
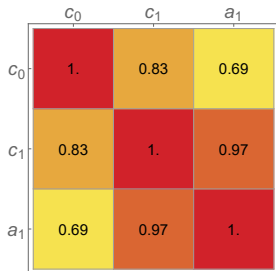
Thank you

Backup slides

Results

Fits. Model 1: $J^{\text{NP}}(b_T) = e^{-a_1 b_T}$

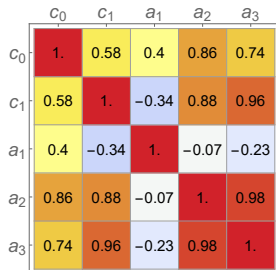
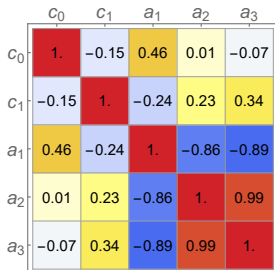
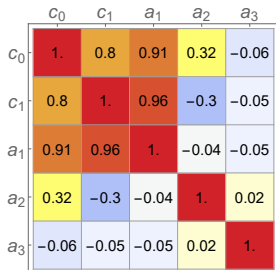
$c_{0,1}^{\text{ART25}}$	$c_{0,1}^{\text{ART23}}$	$c_{0,1}^{\text{fitted}}$
$c_0^{\text{ART25}} = 0.0859^{+0.002}_{-0.002} \text{ GeV}^{-2}$ $c_1^{\text{ART25}} = 0.0303^{+0.004}_{-0.004} \text{ GeV}^{-2}$ $a_1 = 0.903^{+0.014}_{-0.014} \text{ GeV}$	$c_0^{\text{ART23}} = 0.0369^{+0.007}_{-0.006} \text{ GeV}^{-2}$ $c_1^{\text{ART23}} = 0.0582^{+0.006}_{-0.009} \text{ GeV}^{-2}$ $a_1 = 1.004^{+0.006}_{-0.015} \text{ GeV}$	$c_0 = 0.040^{+0.002}_{-0.001} \text{ GeV}^{-2}$ $c_1 = 0.030^{+0.003}_{-0.002} \text{ GeV}^{-2}$ $a_1 = 0.949^{+0.004}_{-0.002} \text{ GeV}$
$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.969$	$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.685$	$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.590$



Results

Fits. Model 2: $J^{\text{NP}}(b_T) = e^{-a_1 b_T} \frac{1+a_2 b_T^2}{1+a_3 b_T^2}$

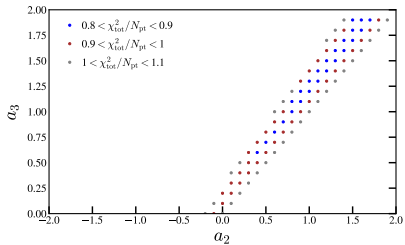
$c_{0,1}^{\text{ART25}}$	$c_{0,1}^{\text{ART23}}$	$c_{0,1}^{\text{fitted}}$
$c_0^{\text{ART25}} = 0.0859^{+0.002}_{-0.002} \text{ GeV}^{-2}$ $c_1^{\text{ART25}} = 0.0303^{+0.004}_{-0.004} \text{ GeV}^{-2}$ $a_1 = 0.983^{+0.012}_{-0.012} \text{ GeV}$ $a_2 = 0.219^{+0.017}_{-0.016} \text{ GeV}^2$ $a_3 = (3.55^{+11.75}_{-3.08}) \times 10^{-7} \text{ GeV}^2$	$c_0^{\text{ART23}} = 0.0369^{+0.007}_{-0.006} \text{ GeV}^{-2}$ $c_1^{\text{ART23}} = 0.0582^{+0.006}_{-0.009} \text{ GeV}^{-2}$ $a_1 = 0.940^{+0.031}_{-0.018} \text{ GeV}$ $a_2 = 1.044^{+0.633}_{-1.052} \text{ GeV}^2$ $a_3 = 1.255^{+0.723}_{-1.204} \text{ GeV}^2$	$c_0 = 0.0080^{+0.0075}_{-0.0064} \text{ GeV}^{-2}$ $c_1 = 0.094^{+0.018}_{-0.013} \text{ GeV}^{-2}$ $a_1 = 0.910^{+0.002}_{-0.003} \text{ GeV}$ $a_2 = 0.945^{+0.233}_{-0.182} \text{ GeV}^2$ $a_3 = 1.531^{+0.335}_{-0.250} \text{ GeV}^2$
$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.608$	$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.548$	$\chi^2_{\text{tot}}/N_{\text{pt}} = 0.511$



⇒ SCAN $a_2 - a_3$:
Integral
normalization

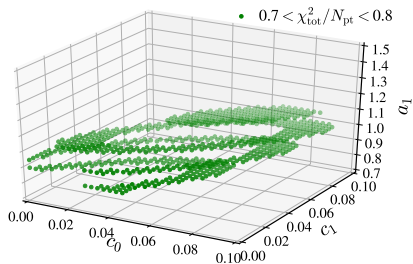
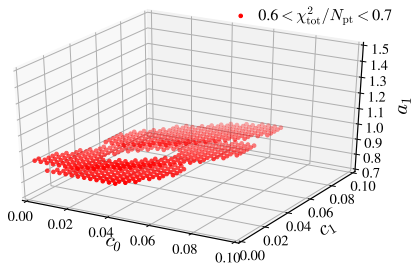
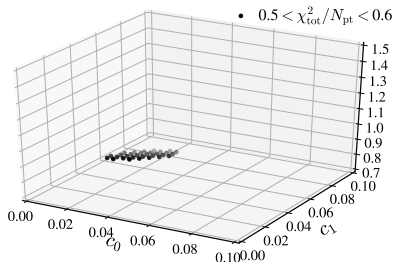
→ $c_{0,1}^{\text{ART23}}$

→ a_1^{fitted} for each
point



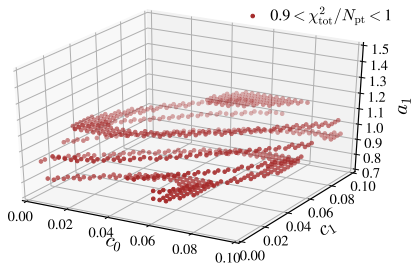
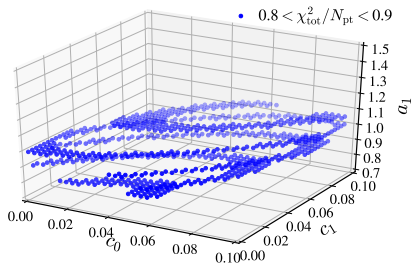
Results

SCANs (Model 1)



Results

SCANs (Model 1)



* $\alpha_s(M_Z) \equiv 0.118$ (fixed)